Higgs production in gluon fusion: recent developments

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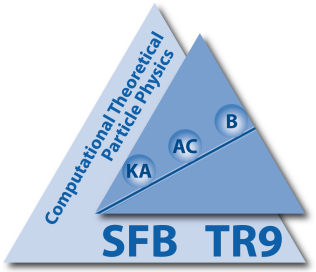


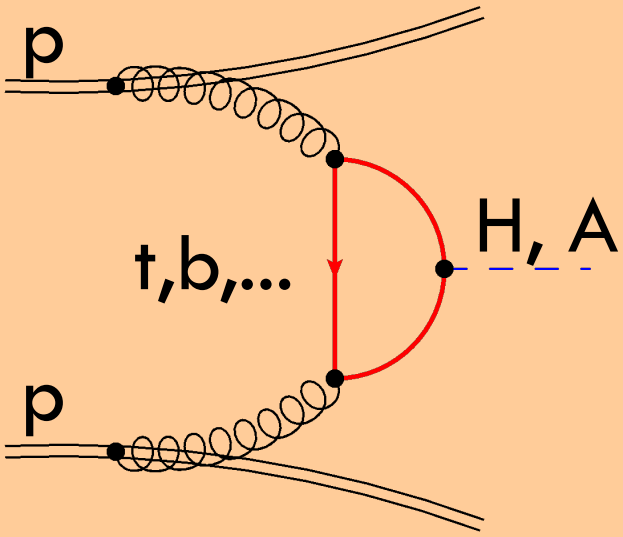
Outline

- Introduction to gluon fusion and top mass effects
- NNLO QCD pseudo-scalar Higgs production:

  Lagrangian, γ^5 definition, calculation, partonic and Hadronic cross-sections
- MSSM effects in NNLO QCD Higgs production:

  Effective theory, DRED, ε-scalar, matching coefficient, scale dependence, hadronic results
- Summary

Gluon fusion



$$\sqrt{s} = 100 - 14000\ GeV$$
$$m_t = 173\ GeV$$
$$m_H = 100 - 300\ GeV$$
$$m_A = 100 - ?$$

residual NNLO scale uncertainty: O(few %)

SM calculations:

- LO: [Geordi et al '78] (exact)
- NLO: [Dawson; Djouadi, Spira, Zerwas '91], [Spira et al '95] (exact) → O(70%)
- NNLO - heavy top limit results: [Harlander, Kilgore '02], [Anastasiou, Melnikov '02], [Ravindran, Smith, van Neerven '03] → O(10%)
- Also available – non-QCD and beyond FOPT: EW, QCD-EW, NNLO+NNLL, N^{3}LO threshold enhanced corrections, π^2-resummation, NNLO differential distributions, ..., due to: Catani, de Florian, Grazzini, Nason; Ahrens, Becher, Neubert, Yang; Actis, Passarino, Sturm, Uccirati; Anastasiou, Boughezal, Petriello; de Florian, Grazzini, ...

NNLO results and mass dependence

S – scalar Higgs, H
P – pseudo-scalar Higgs, A

leading logs
S: [Marzani et al '08; Harlander et al '09]
P: [Caola, Marzani '11]

effective theory calculations, infinitely heavy top quark
S: [Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran et al '03]
P: [Anastasiou, Melnikov '02; Ravindran et al '03]

only NLO results available

$m_t/\sqrt{s}=0$ zero mass limit

$m_t \sim \sqrt{s}$ threshold region

$m_t=\infty$ heavy top limit

expansion in powers of $1/m_t^2$ + interpolation to zero mass limit

S: [Harlander, Ozeren '09; Pak, Rogal, Steinhauser '10]
P: this work: expand to $1/m_t^8$, check effective theory

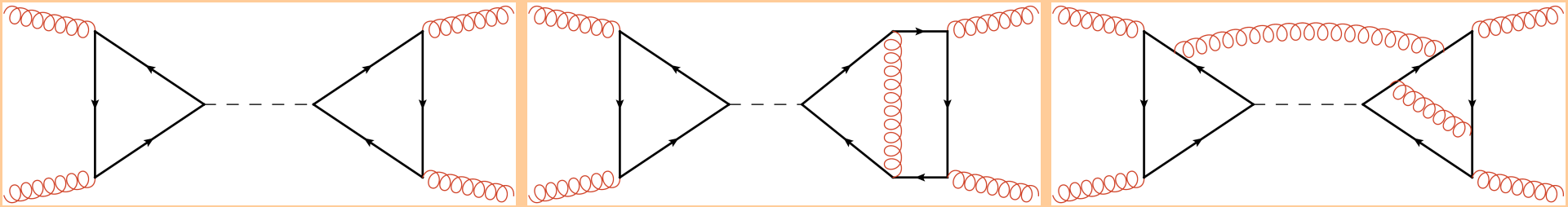
Pseudo-scalar Higgs production

Lagrangian:

$$L_Y = -m_q \frac{H}{v^0} \bar{q}\, q - g^Y m_q \frac{A}{v^0} \bar{q}\, i\gamma^5 q$$

appears in many extensions of SM

Use optical theorem (same diagrams as for scalar Higgs):



+ special treatment of imaginary parts for NLO x NLO virtual diagrams

Larin's prescription:

$$\gamma^5 = \frac{\epsilon^{\mu\nu\rho\sigma}}{24} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma, \qquad \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\delta\kappa} = \begin{vmatrix} g^{\mu\alpha} & g^{\mu\beta} & g^{\mu\delta} & g^{\mu\kappa} \\ g^{\nu\alpha} & g^{\nu\beta} & g^{\nu\delta} & g^{\nu\kappa} \\ g^{\rho\alpha} & g^{\rho\beta} & g^{\rho\delta} & g^{\rho\kappa} \\ g^{\sigma\alpha} & g^{\sigma\beta} & g^{\sigma\delta} & g^{\sigma\kappa} \end{vmatrix}$$

+ finite renormalization of pseudo-scalar current, renormalization of t-quark current

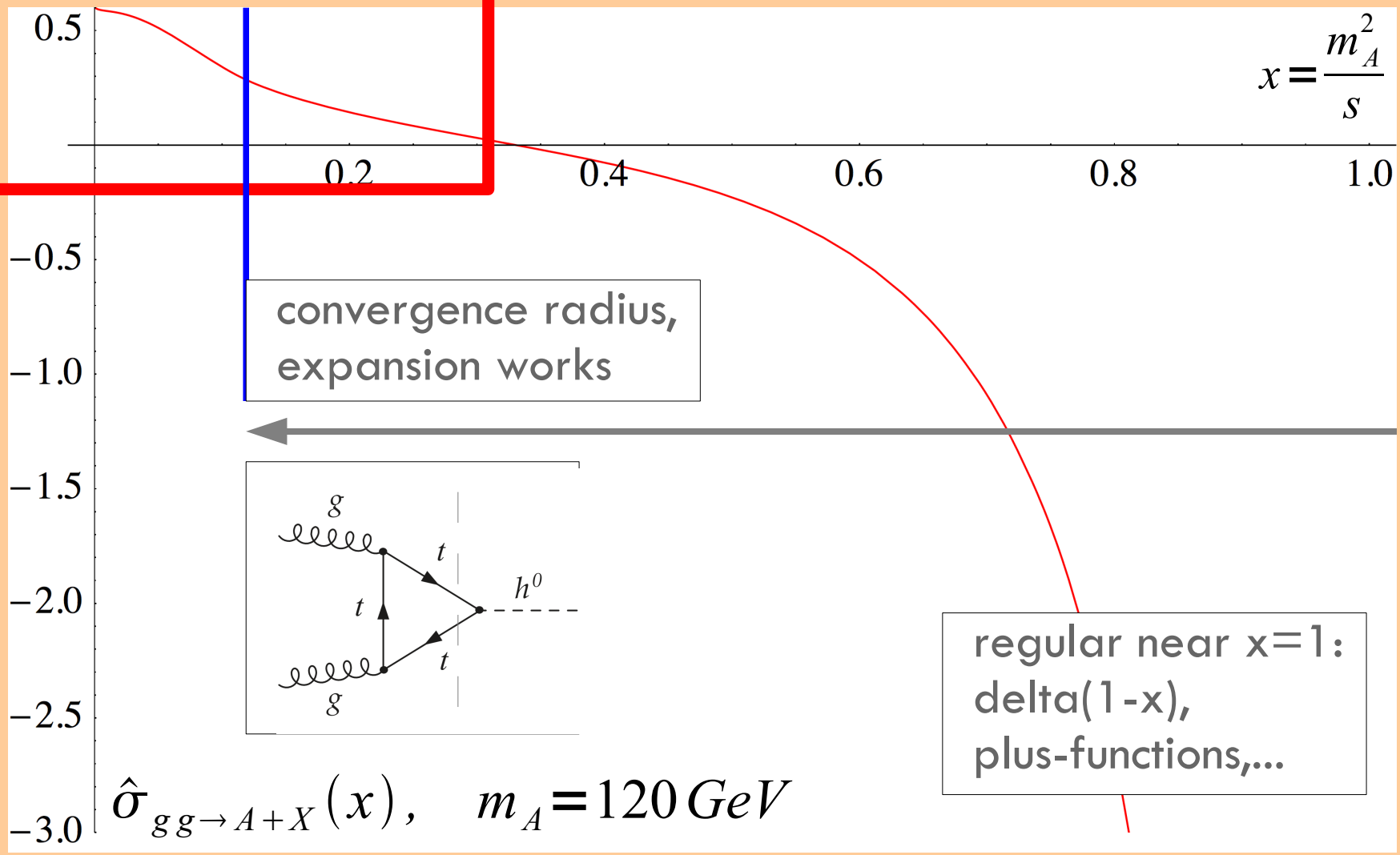
Calculation

- QGRAF, FORM, q2e/exp + Perl program, own Laporta
- ~20000 4-loop diagrams, ~1 month at TTP cluster, cross-checks, all partonic results to $(m_A/m_t)^8$, SU(N)
- Prepare to N^3LO calculation: master integrals extended by 1-2 orders in ε
- Convolution of plus-functions and HPLs done in Mellin space, implemented as Mathematica module
- Prepare to N^3LO calculation: evaluated Mellin images of HPLs to weight 5

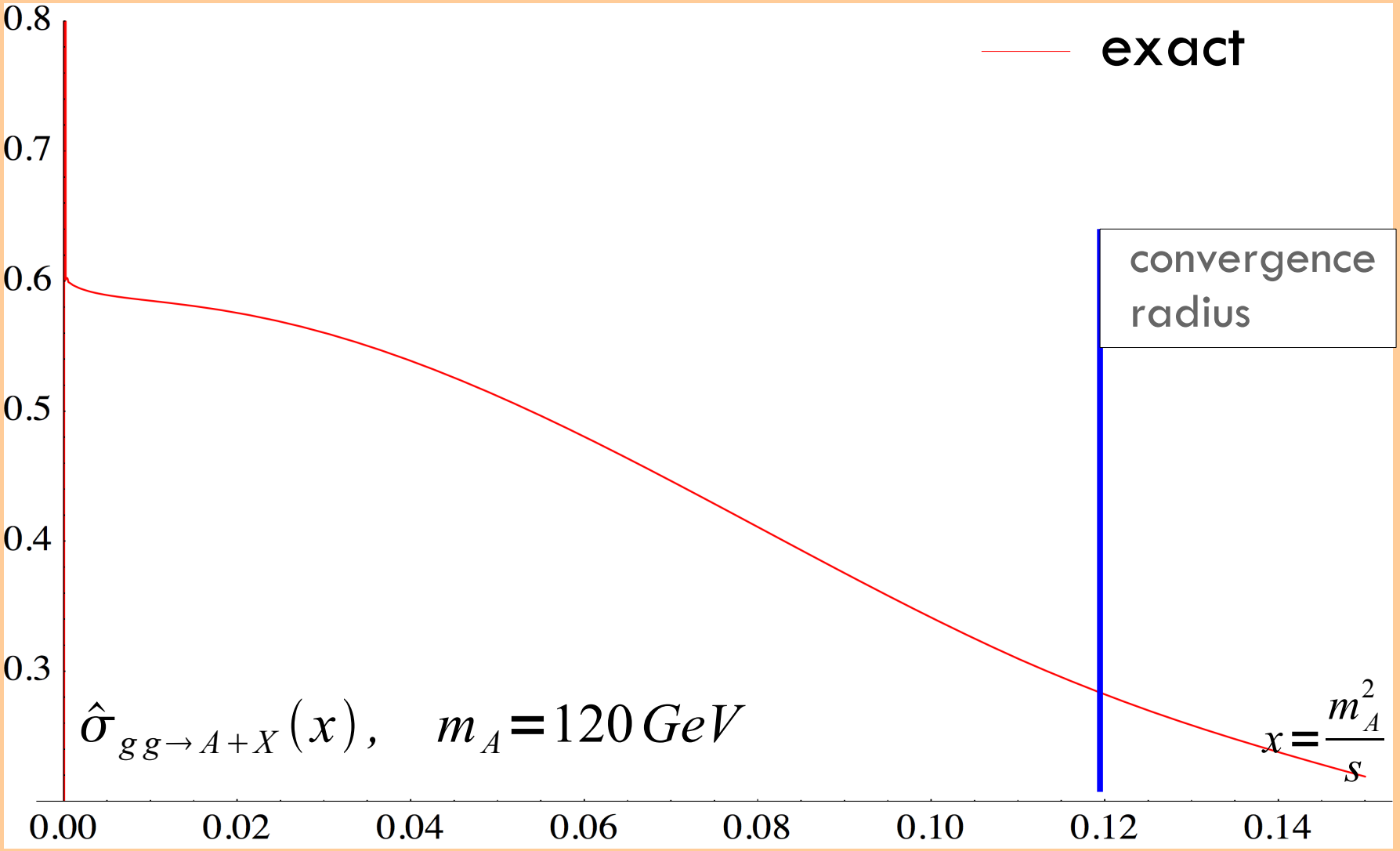
Partonic results, NLO

interesting region, suppressed by luminosity functions

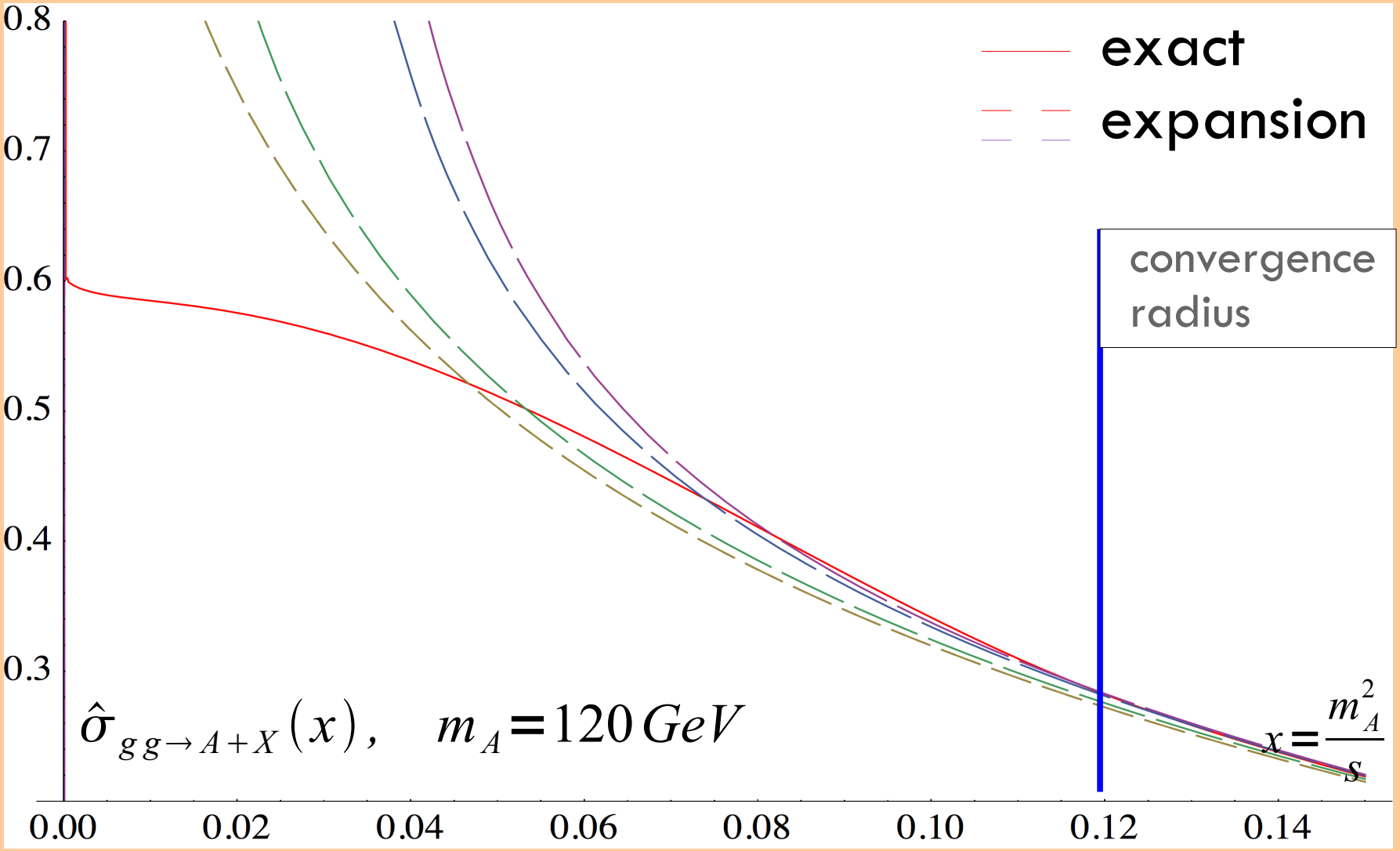
hadronic result: integrate with luminosity functions



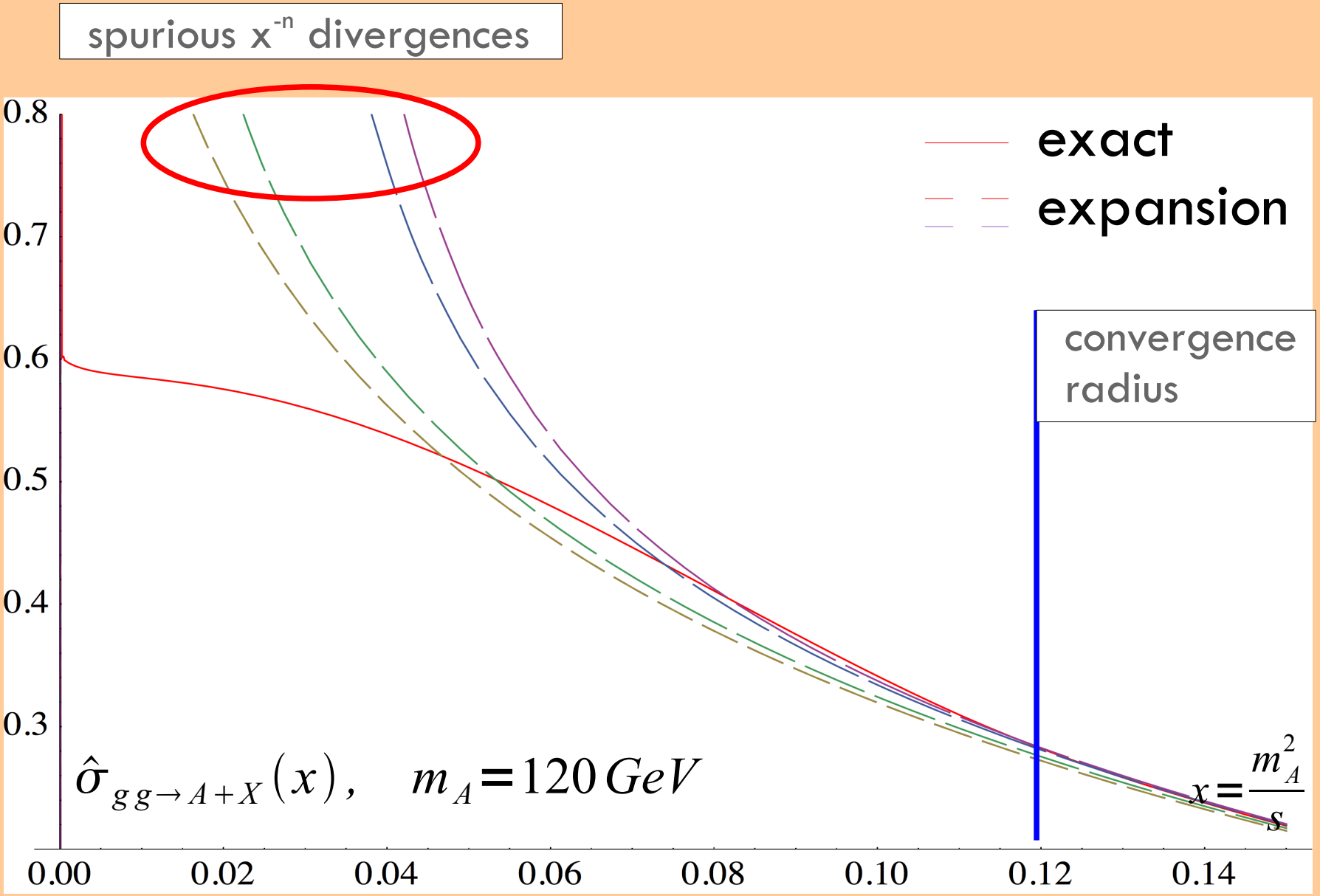
Partonic results, NLO



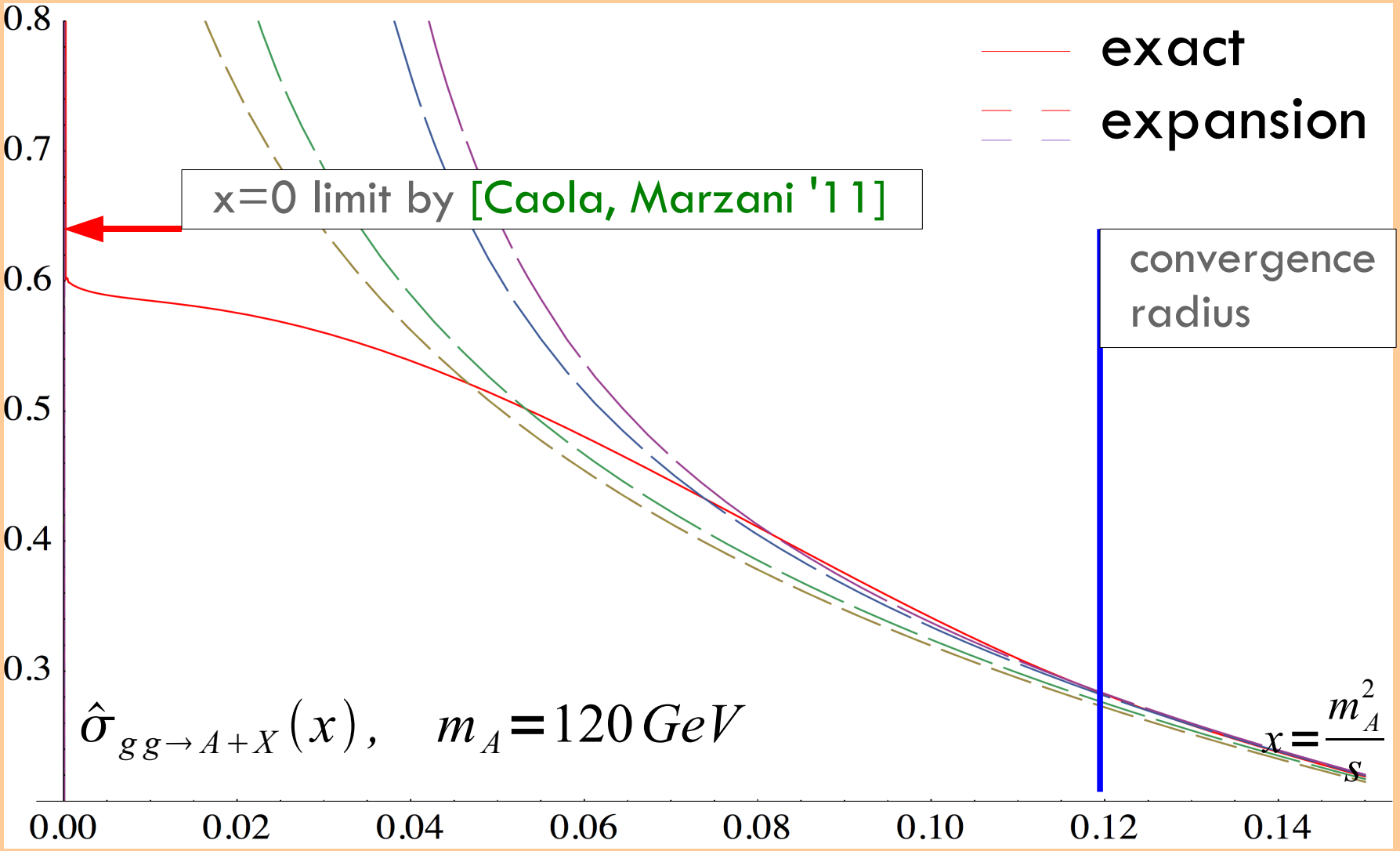
Partonic results, NLO



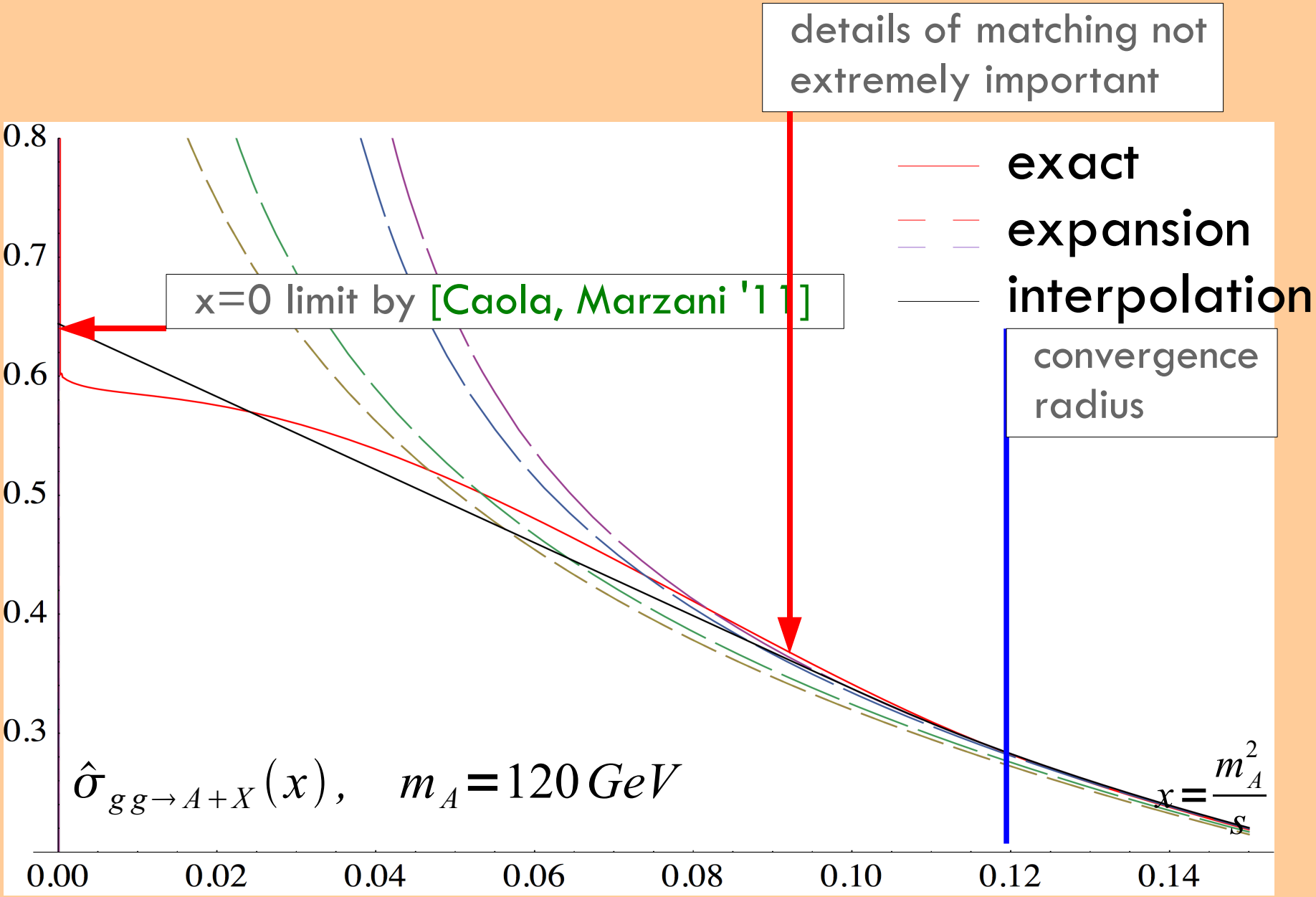
Partonic results, NLO



Partonic results, NLO

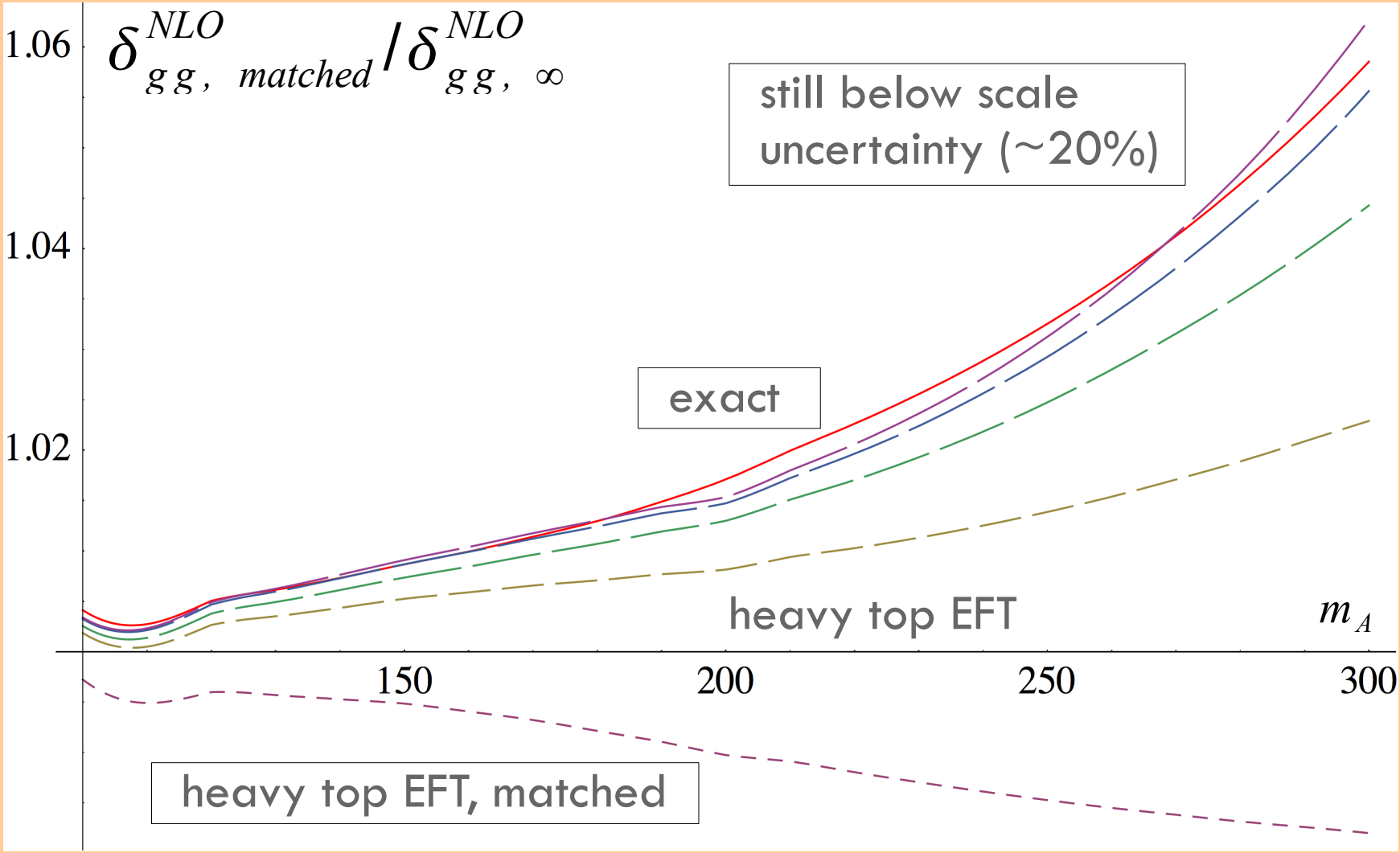


Partonic results, NLO

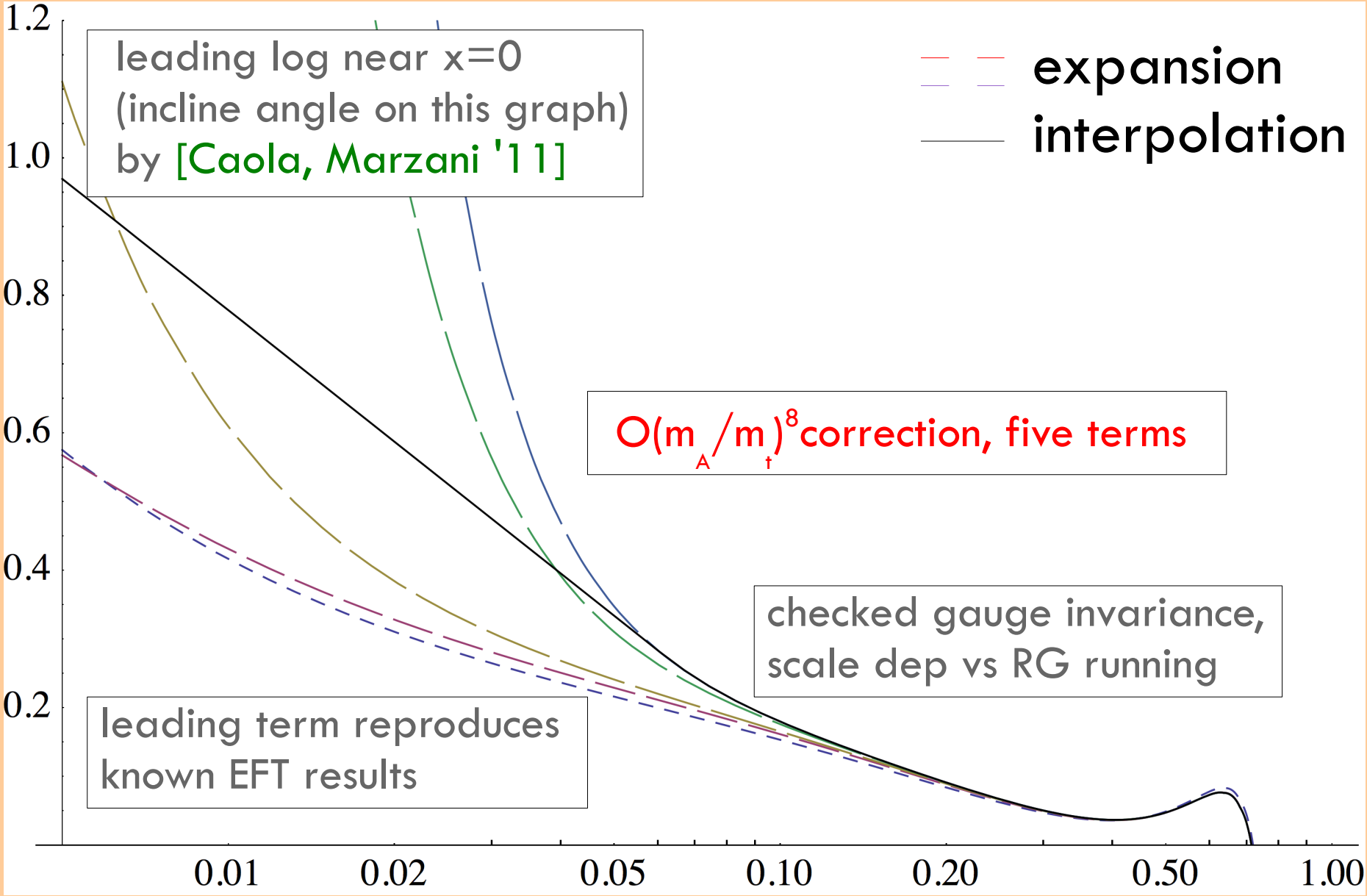


Hadronic results, NLO

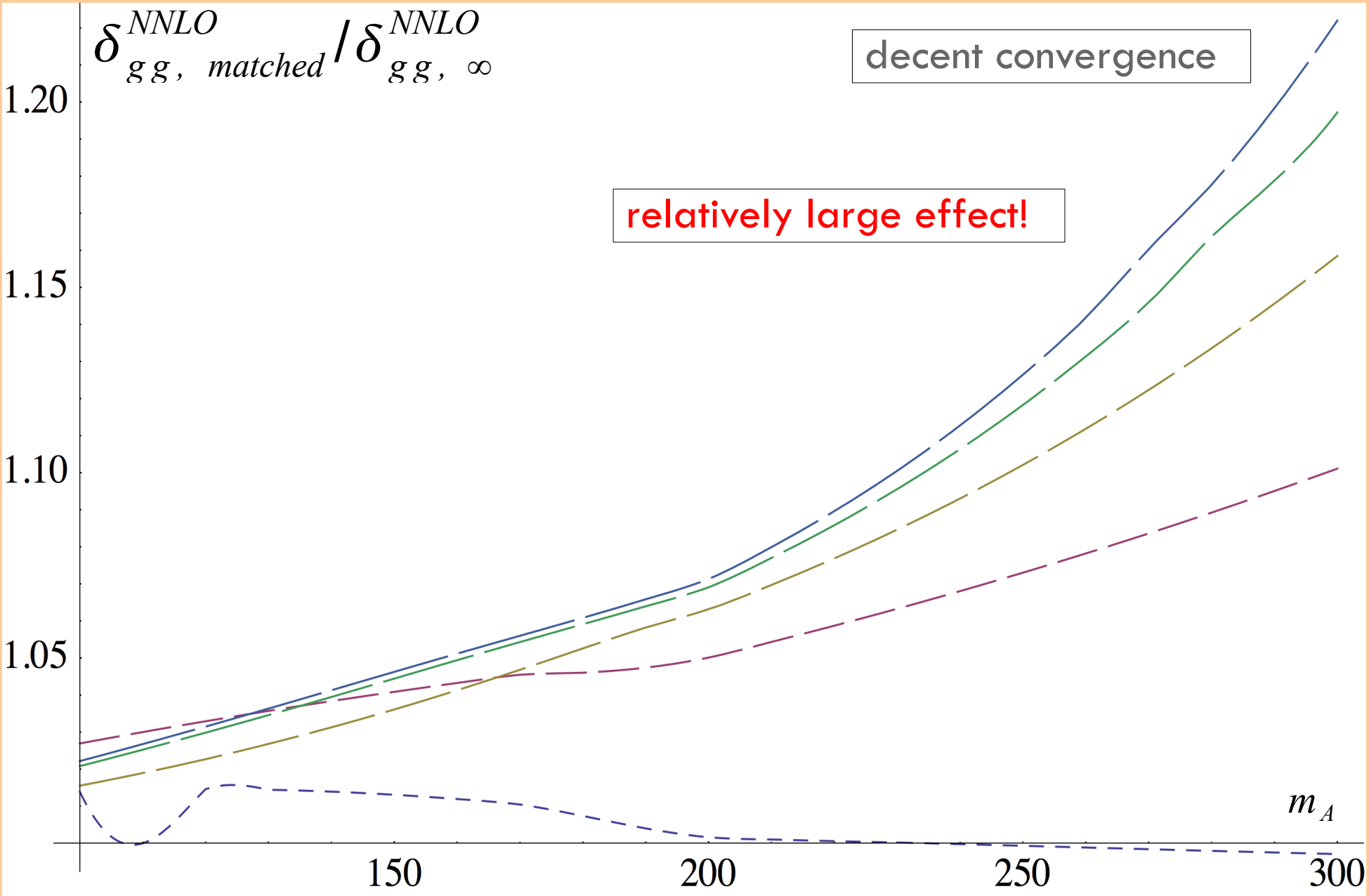
LO factorization: $\sigma_{p\,p\to A+X}(m_A)=\sigma_0(m_A)\left[1+\frac{\alpha_s}{\pi}\delta^{NLO}+\left(\frac{\alpha_s}{\pi}\right)^2\delta^{NNLO}\right]$



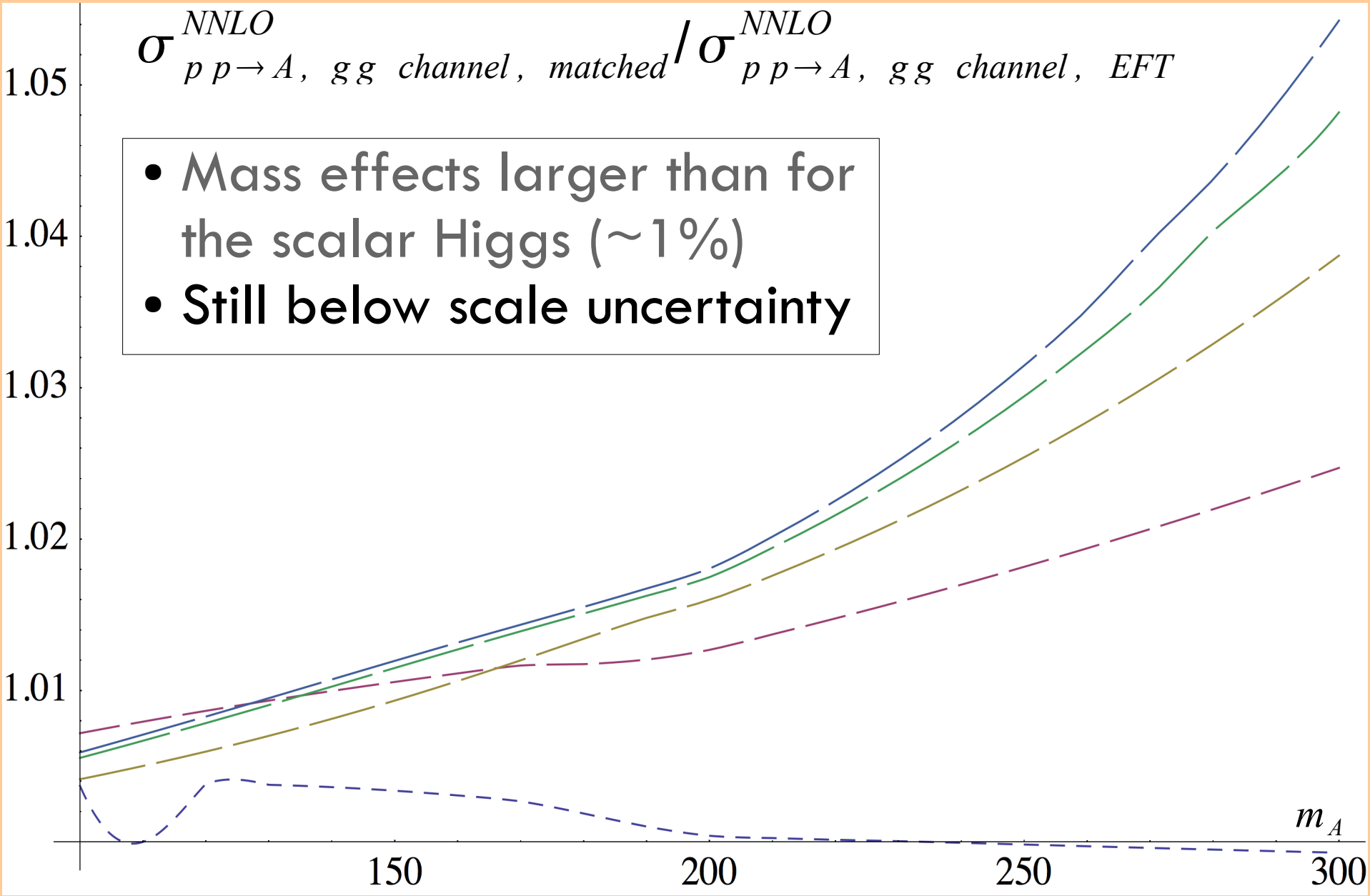
Partonic results, NNLO



Hadronic results, NNLO



Hadronic results, NNLO



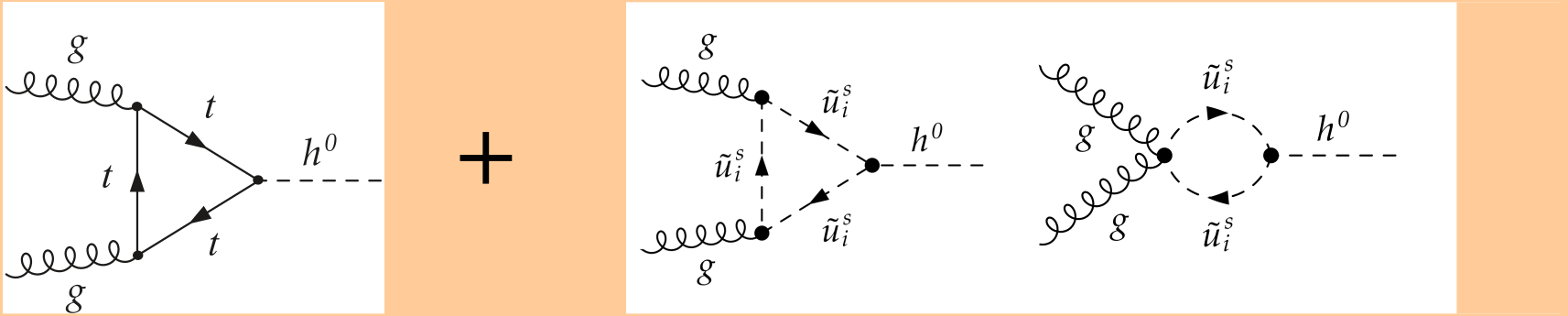
- Mass effects larger than for the scalar Higgs (~1%)
- Still below scale uncertainty

Summary, pseudo-scalar Higgs

- NNLO mass effects (for intermediate m_A) computed near heavy top limit to $(m_A/m_t)^8$ for all channels
- Numerically, shift from EFT is ~5%, still within uncertainty
- Effective theory approximation justified!
- Publication is in preparation

MSSM Higgs production: gg → h^0

QCD + SUSY particles, no $h^0 \tilde{b} \tilde{b}$ coupling (small tan β):



NLO SUSY-QCD: up to 3 scales, no exact results

- EFT predictions (light Higgs): [Harlander, Steinhauser '04]
- NLO (s)quark contribution: [Aglietti, Bonciani, Degrassi, Vicini '07; Muelleitner, Spira '08]
- Numerical analysis, full theory: [Anastasiou, Beerli, Daleo '07]
- Complete NLO analysis: [Harlander, Hofmann, Mantler '10]
- Estimate of NNLO correction: [Harlander, Steinhauser 03]
- This work: NNLO EFT calculation, Eur. Phys. J. C71 (2011) 1602

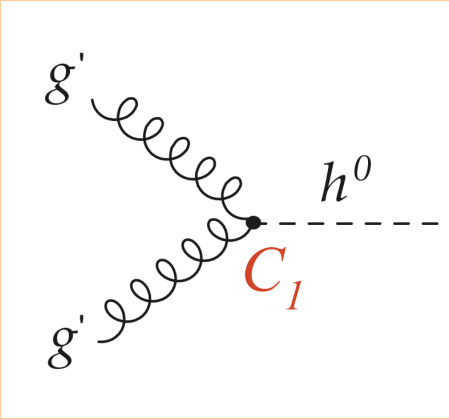
Effective field theory and matching

C_1 hides all heavy masses, is determined by matching

same as QCD with top quark integrated out!

Lagrangian:

$$L_{SUSY-QCD} \to L_{EFT} = \frac{h_0}{v} C_1 O_1$$



EFT	Full MSSM
Dimensional Regularization (DREG)	Dimensional Reduction (DRED)
particles: h_0, g, q, c	$h_0, g, q, c, \tilde{g}, \tilde{q}, t, \tilde{t}, \epsilon$
masses: $m_q = 0, m_{h_0}$	$m_{h_0} \ll m_\epsilon \ll m_t, m_{\tilde{g}}, m_{\tilde{q}}$
coupling: $\alpha_s^{(5),\overline{MS}}$	$\alpha_s^{(6),\overline{DR}}$

ε-scalar

Gluon field decomposed:

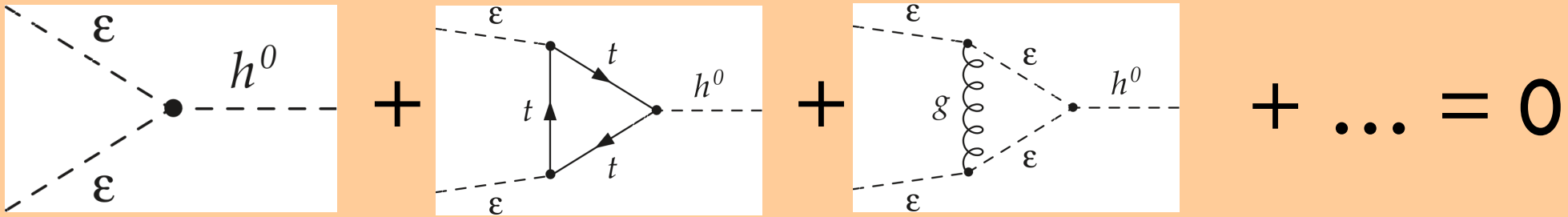
4-D (4-2ε)-D 2ε-D

$$G^\mu = G^\mu + \epsilon^\mu$$

$$L_\epsilon = -\frac{(m_\epsilon^0)^2}{2}\epsilon\epsilon + \frac{h_0}{v}\left(\Lambda_\epsilon^0\right)^2\epsilon\epsilon$$

ε-Higgs coupling, new mass, renormalization...

Require: no new Higgs coupling:



$$\Rightarrow \Lambda_\epsilon^0 = 0 + \frac{\alpha_s}{\pi}\delta\Lambda_\epsilon^1 + \left(\frac{\alpha_s}{\pi}\right)^2 \delta\Lambda_\epsilon^2 + \ldots$$

Require: no ε-scalar in the EFT, no dependence on m_ε:

⇒ Hierarchy: $m_\varepsilon >> m_{h0}$, expand to m_ε^n and check cancellation

Results: matching coefficient

Cross-check: SM QCD

$$C_1^{SM}(DRED, \alpha_s^{(6),\overline{DR}}, \alpha_e) \to C_1^{SM}(DRED, \alpha_s^{(5),\overline{MS}}) = C_1^{SM}(DREG, \alpha_s^{(5),\overline{MS}})$$

MSSM, hierarchy $m_t \ll m_{\hat{g}} = m_{\hat{q}} = m_{\tilde{t}_1} = m_{\tilde{t}_2} \equiv m_{SUSY}$

C_1 to $O(m_t/m_{SUSY})^4$, cross-check: $\lim_{m_{SUSY}\to\infty} C_1^{MSSM} = \frac{\cos\alpha}{\sin\beta} C_1^{SM}$

No stop mixing, fixed scale (mass running is different), limited segment of MSSM parameter space. But: further results include expansion in mass differences, to be published soon.

Cross-section and scale dependence

Full operator: $\frac{d}{d\mu} C_1(\mu) O_1(\mu) = 0$ but $\frac{d}{d\mu} C_1(\mu) \neq 0$

Scale-invariant quantities [Chetyrkin, Kniehl, Steinhauser '97]:

$$O_g = O_1 B^{(5)}, \quad C_g = C_1 / B^{(5)}, \quad B^{(5)} = -\frac{\pi^2 \beta^{(5)}}{\beta_0^{(5)} \alpha_s^{(5)}}$$

Now can separate heavy and soft scales:

$$\sigma_{pp \to h_0 + X}(m_{h_0}, m_t, m_{SUSY}, \mu_s, \mu_h) = \sigma_0 C_g(m_t, m_{SUSY}, \mu_h)^2 \Sigma(m_{h_0}, \mu_s)$$

extracted directly from SM calculation

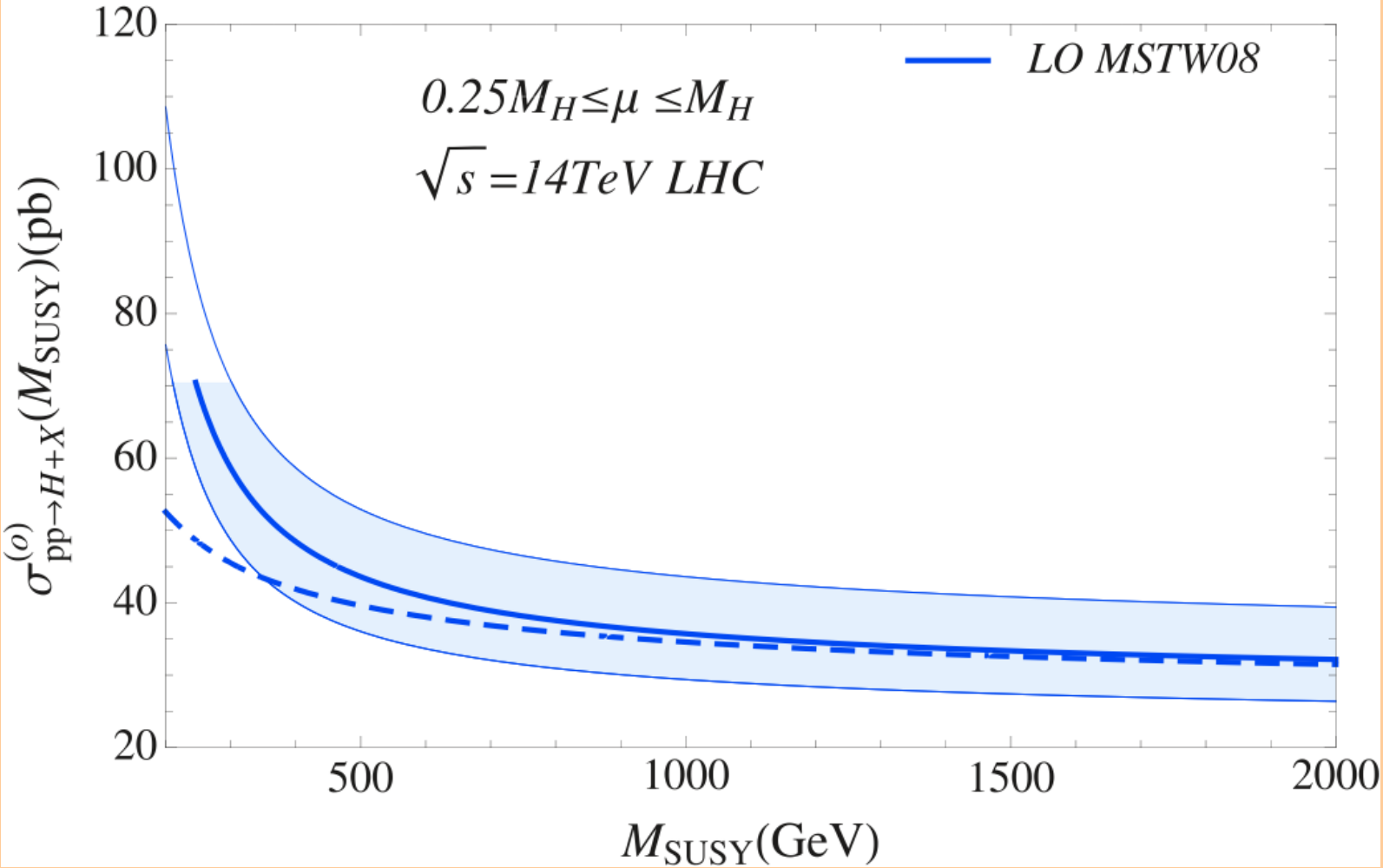
Keep hard scale fixed, $\mu_h \sim m_t$, determine scale dependence by varying $\mu_s \sim m_{h0}/2$

Results – hadronic cross-section

$$M_{A_0} = 1000\,GeV\,,\quad \tan\beta = 5,\quad \mu_{SUSY} = 200\,GeV\,,\quad 200 \le m_{SUSY} \le 2000\,GeV$$

Use H3.m [Kant et al '10]:

$$\Rightarrow 89 \le m_{h_0} \le 119\,GeV\,,\quad \frac{\cos\alpha}{\sin\beta} \approx 0.99$$



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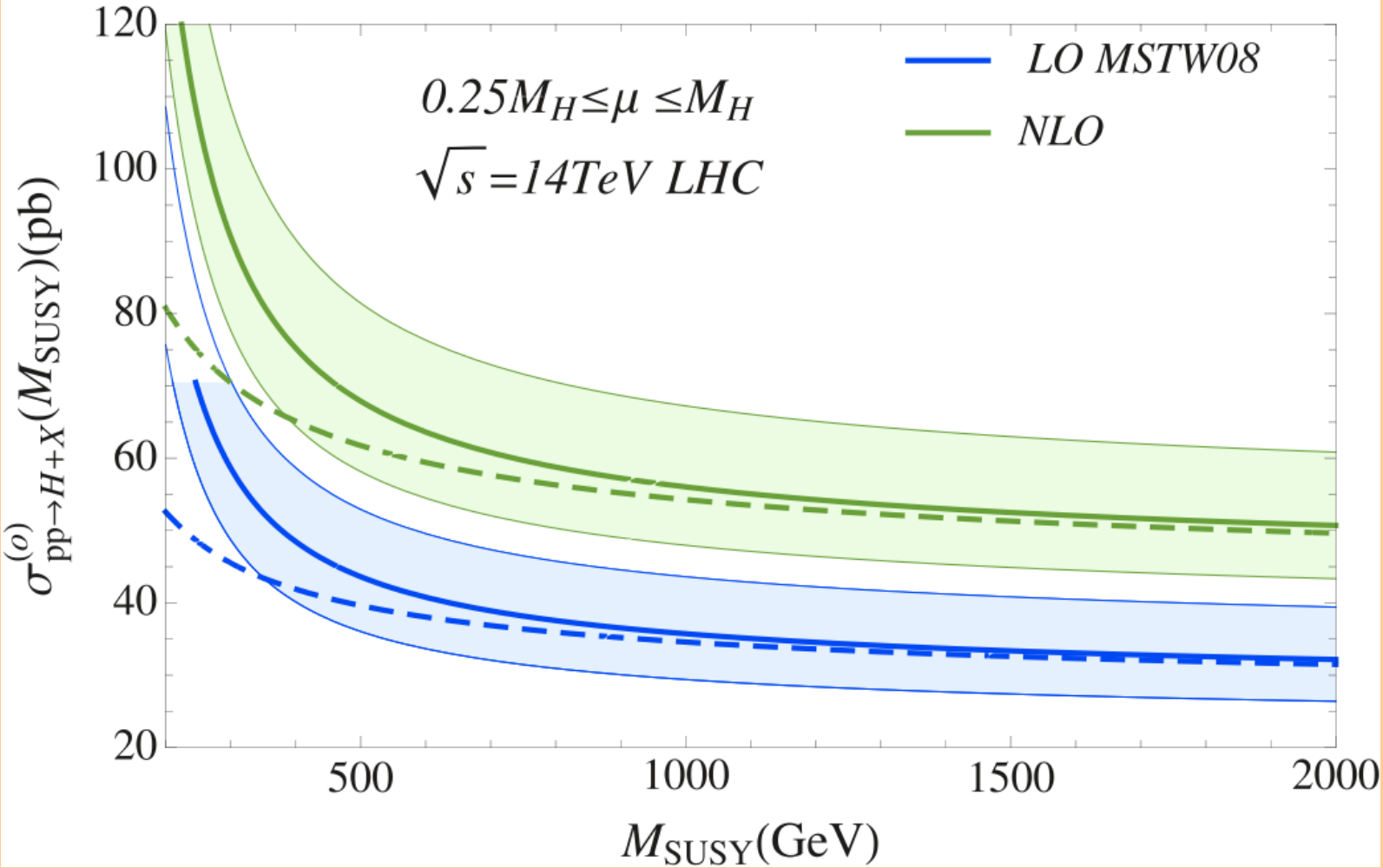

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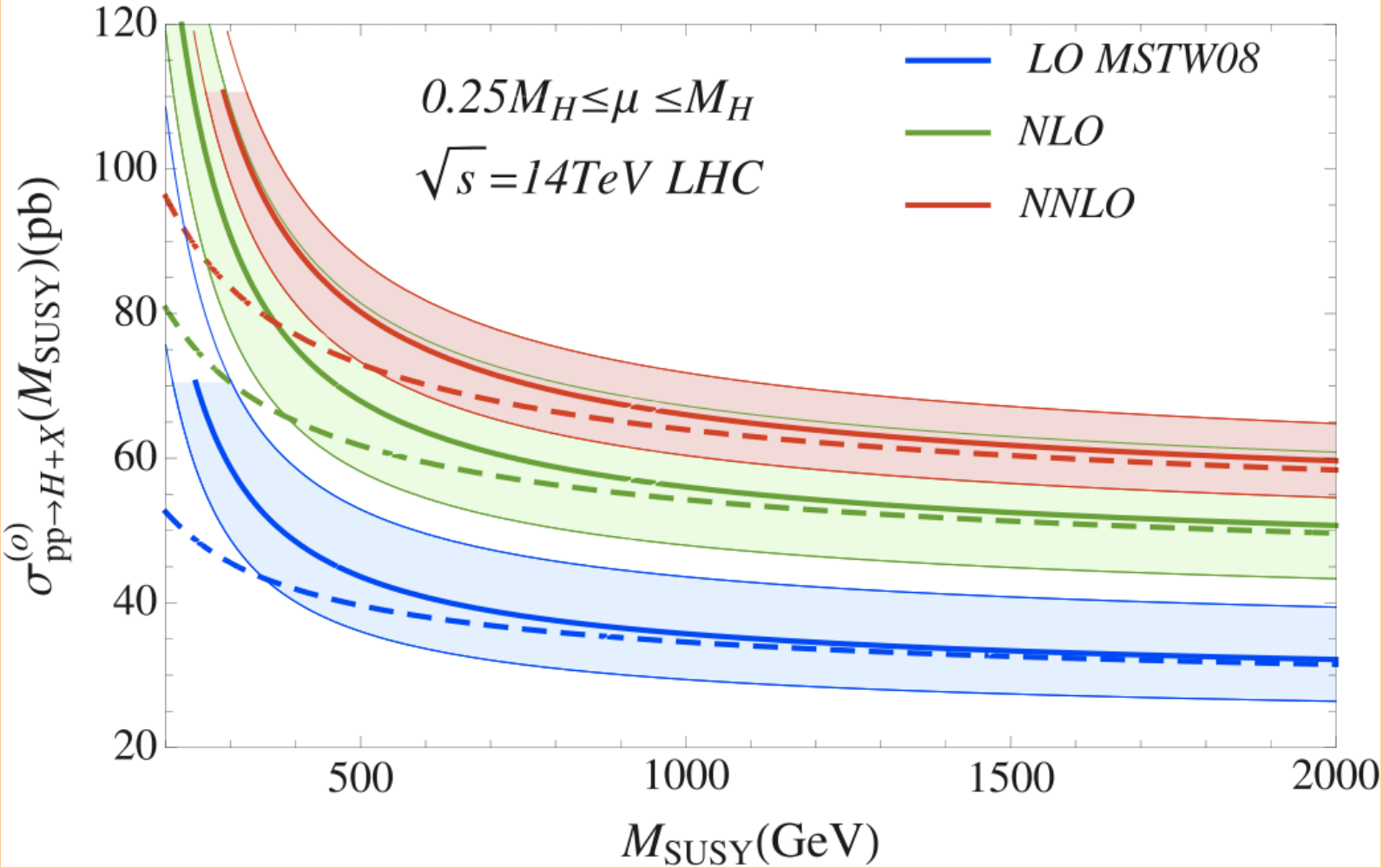


Summary, MSSM Higgs production

- Matching coefficient C_1 computed to 3 loops in DRED, degenerate MSSM hierarchy, to $(m_t/m_{SUSY})^4$
- For $m_{SUSY} > 500$ GeV numerical shift small
- Soon to be published: expansions in mass differences, new hierarchy (two stop masses at different scales), m_Z-suppressed terms from couplings

  $\longrightarrow$ Much larger parameter space coverage!

Convolution integrals and Mellin transform

$$[f\otimes g](x)=\int_0^1 dx_1\,dx_2\,\delta(x-x_1x_2)\,f(x_1)\,g(x_2)$$

$$M_n[f(x)]=\int_0^1 x^{n-1}f(x)\,dx$$

$$M_n[[f\otimes g](x)]=M_n[f(x)]M_n[g(x)]$$

Plus-functions as derivatives:

$$M_n[\hat{\partial}_x f(x)]=R[f(x)]-(n-1)M_{n-1}[f(x)],$$

$$R[g(x)\ln^a(1-x)+h(x)\ln^b(1-x)+\ldots+r(x)]=r(x)$$

$$\hat{\partial}_x 1=\delta(1-x),$$

$$\hat{\partial}_x H_1(x)=\left[\frac{1}{1-x}\right]_+,$$

$$\hat{\partial}_x H_{11}(x)=-\left[\frac{\ln(1-x)}{1-x}\right]_+,$$

$$\hat{\partial}_x H_{101}(x)=\frac{\pi^2}{6}\left[\frac{1}{1-x}\right]_+ +\frac{H_{01}(x)-\zeta_2}{1-x},$$

Implemented:

- Mellin images of HPLs weight 5
- Algebra of harmonic sums
- "Harmonization" of sums and factors
- Partial fractioning

NNLO result: HPLs weight 4

LO Splitting functions: weight 1

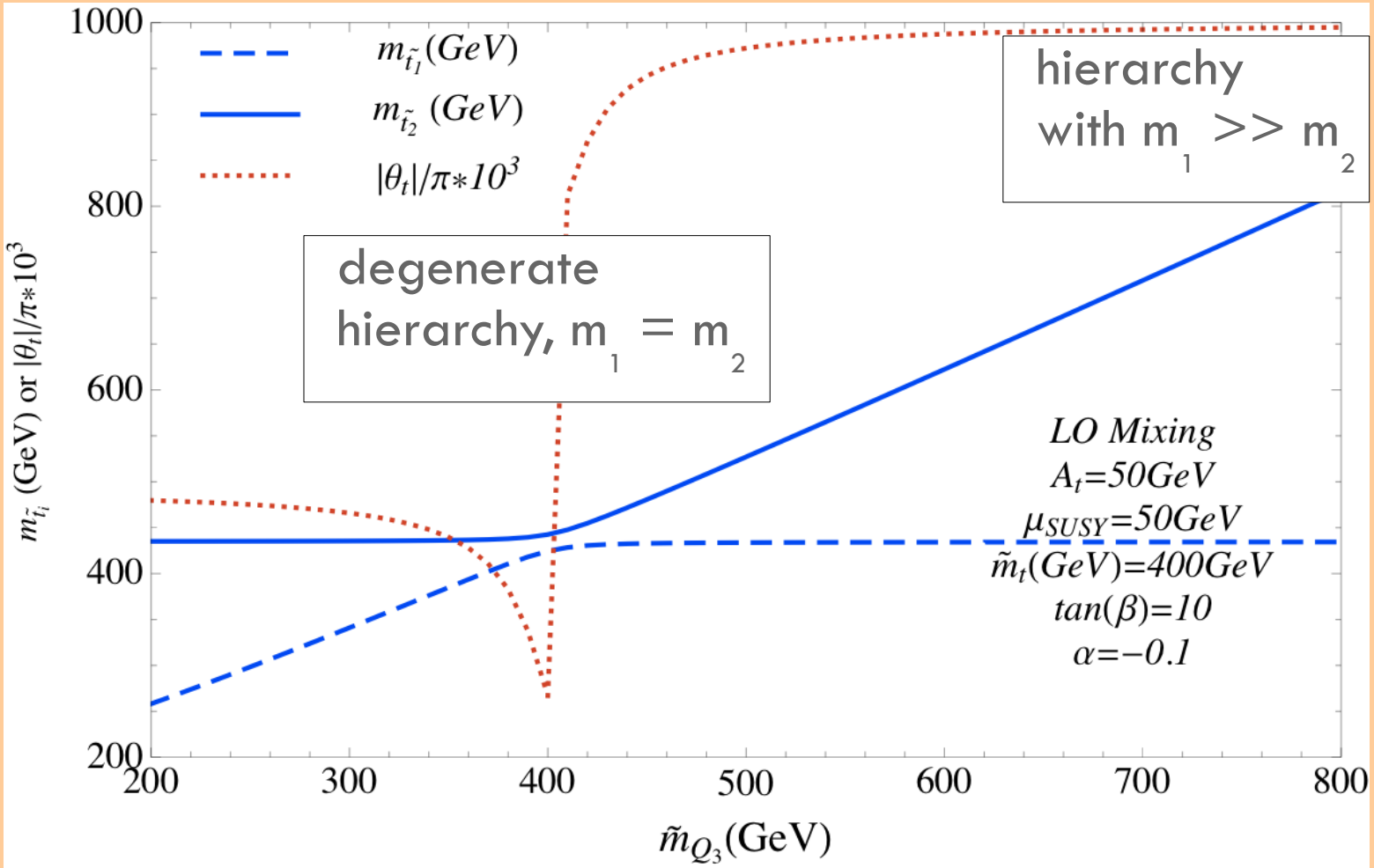
Matching two hierarchies

$$m_t \ll m_{\hat{g}} = m_{\hat{q}} = m_{SUSY}$$

vs

$$m_t \ll m_{\tilde{t}_1} \ll m_{\tilde{g}} = m_{\tilde{q}} = m_{\tilde{t}_2}$$

Evolution of two stop masses:



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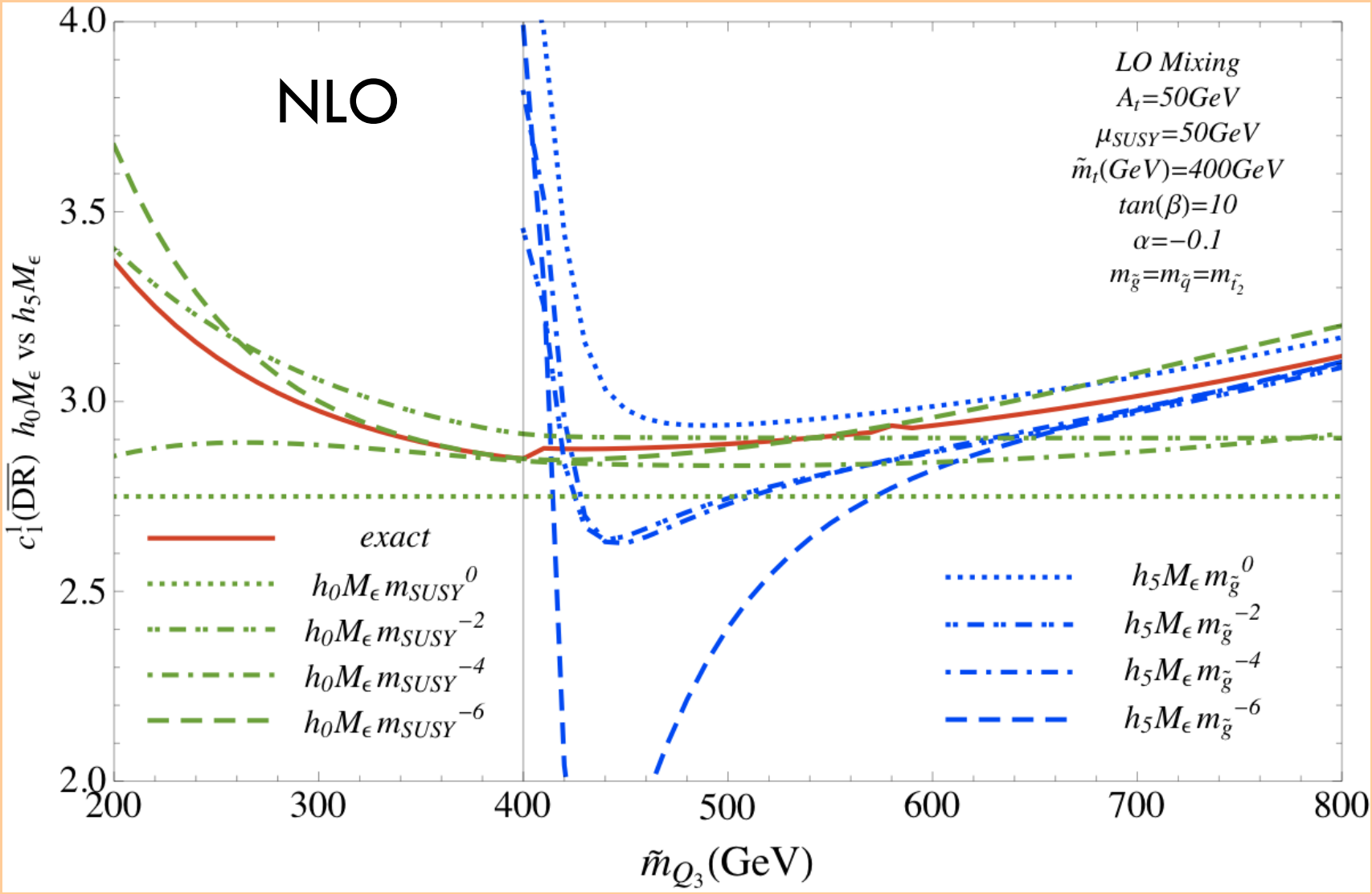

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