Applications of the Collinear Anomaly:

q_T Resummation and Jet Broadening

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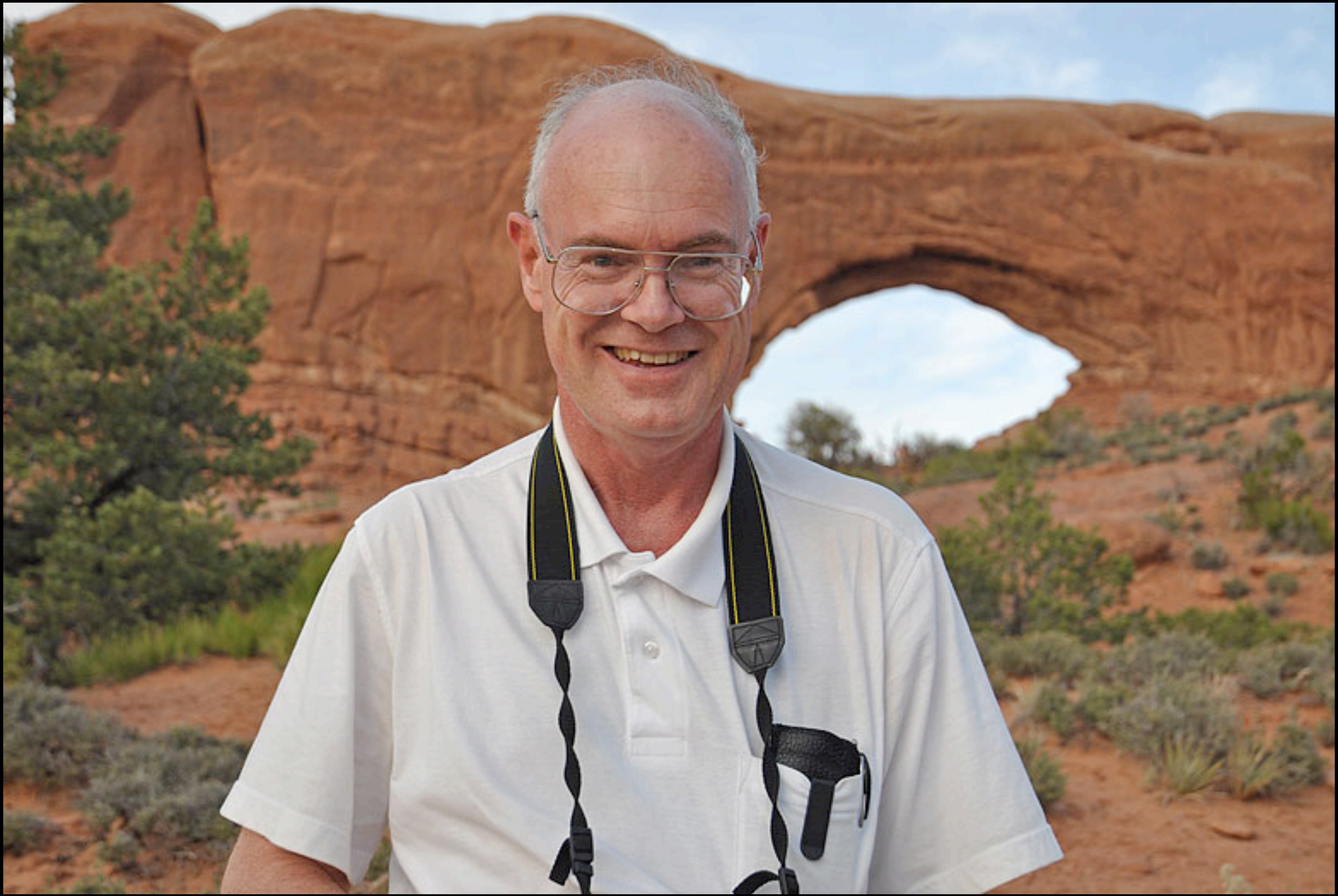


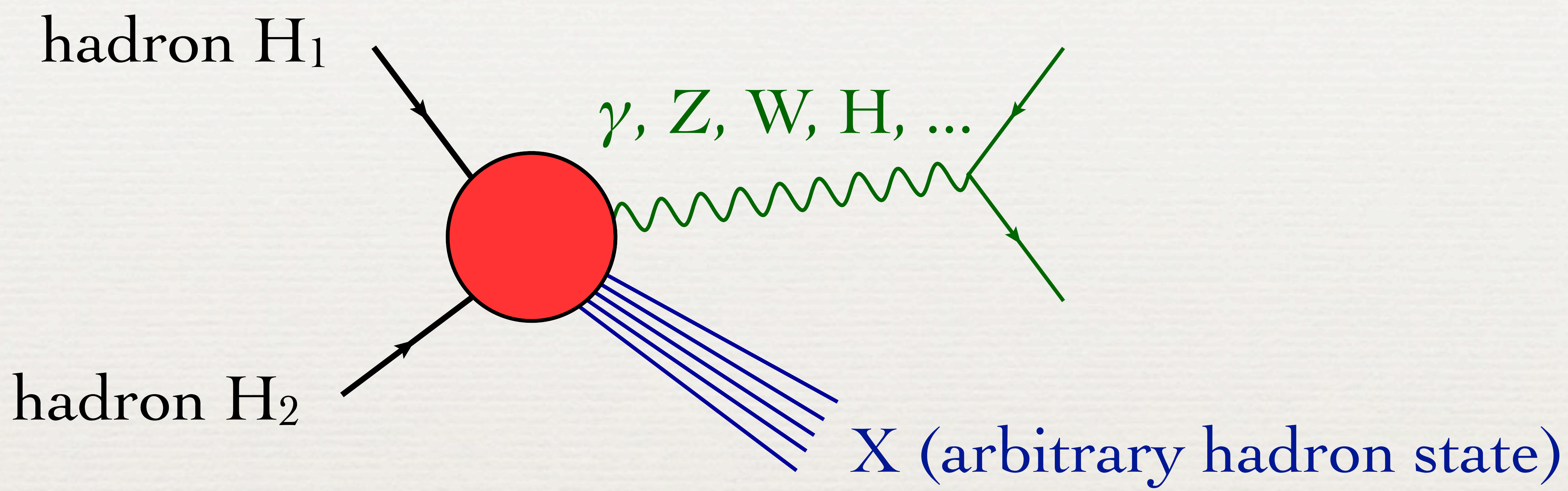
in memory of Uli Bauer

Based on:

- T. Becher and M. Neubert:
  Drell-Yan production at small q_T, transverse parton distributions and the collinear anomaly
  arXiv:1007.4005 (to appear in EPJC)

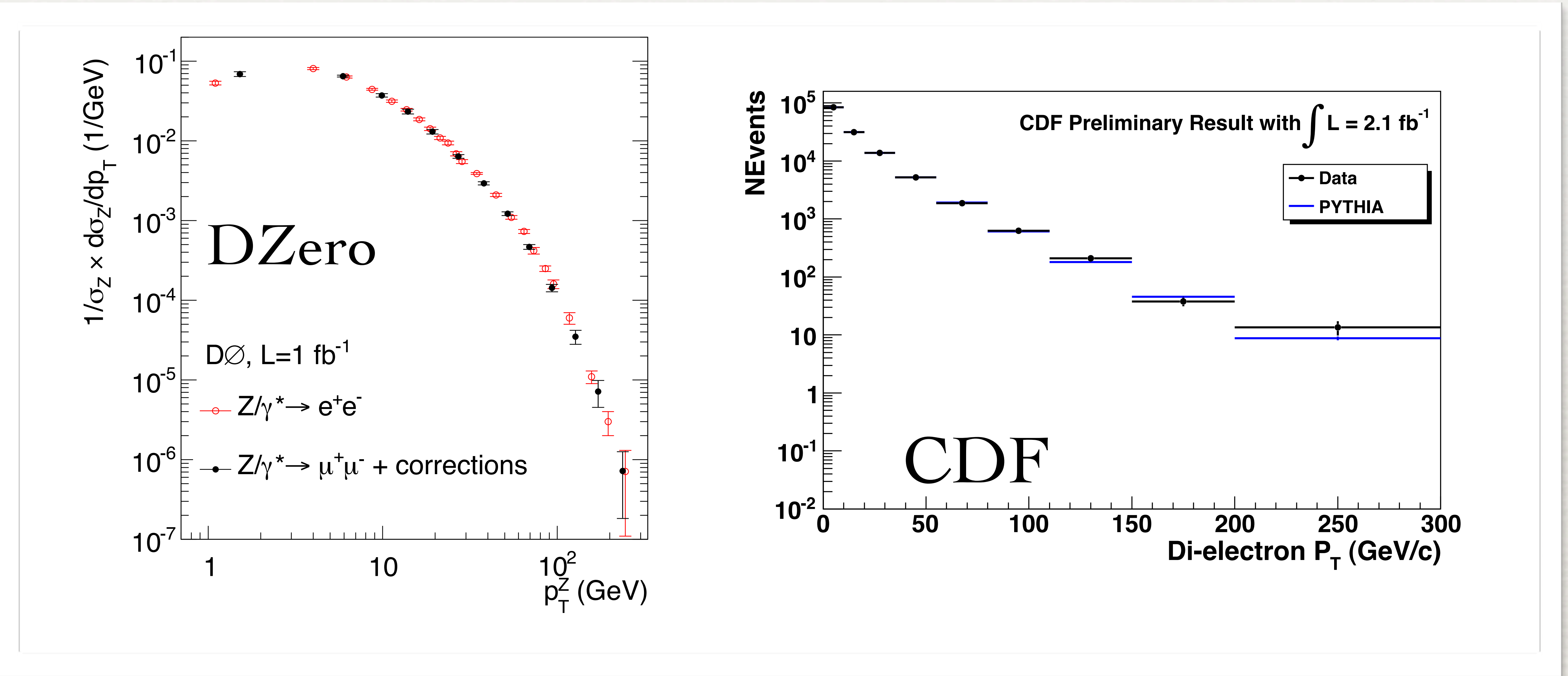
- T. Becher, G. Bell and M. Neubert:
  Factorization and resummation for jet broadening
  arXiv:1104.4108 (submitted to PLB)

Drell-Yan processes

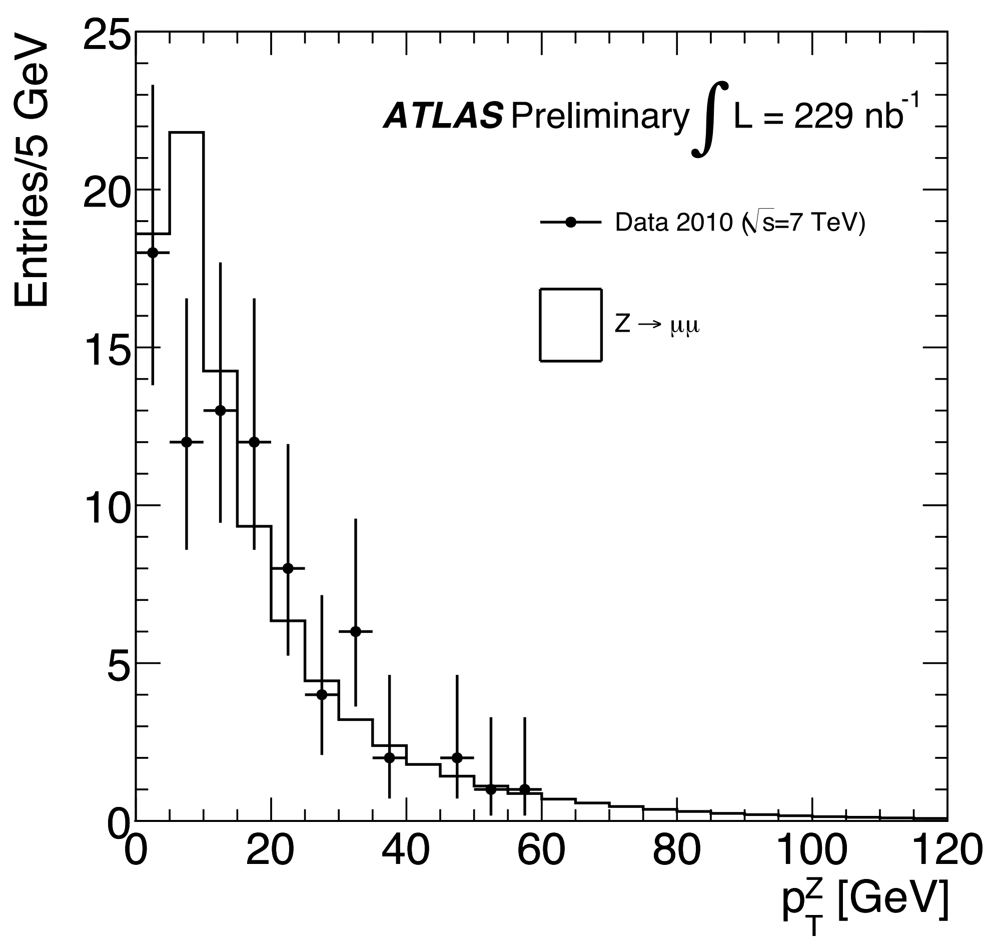


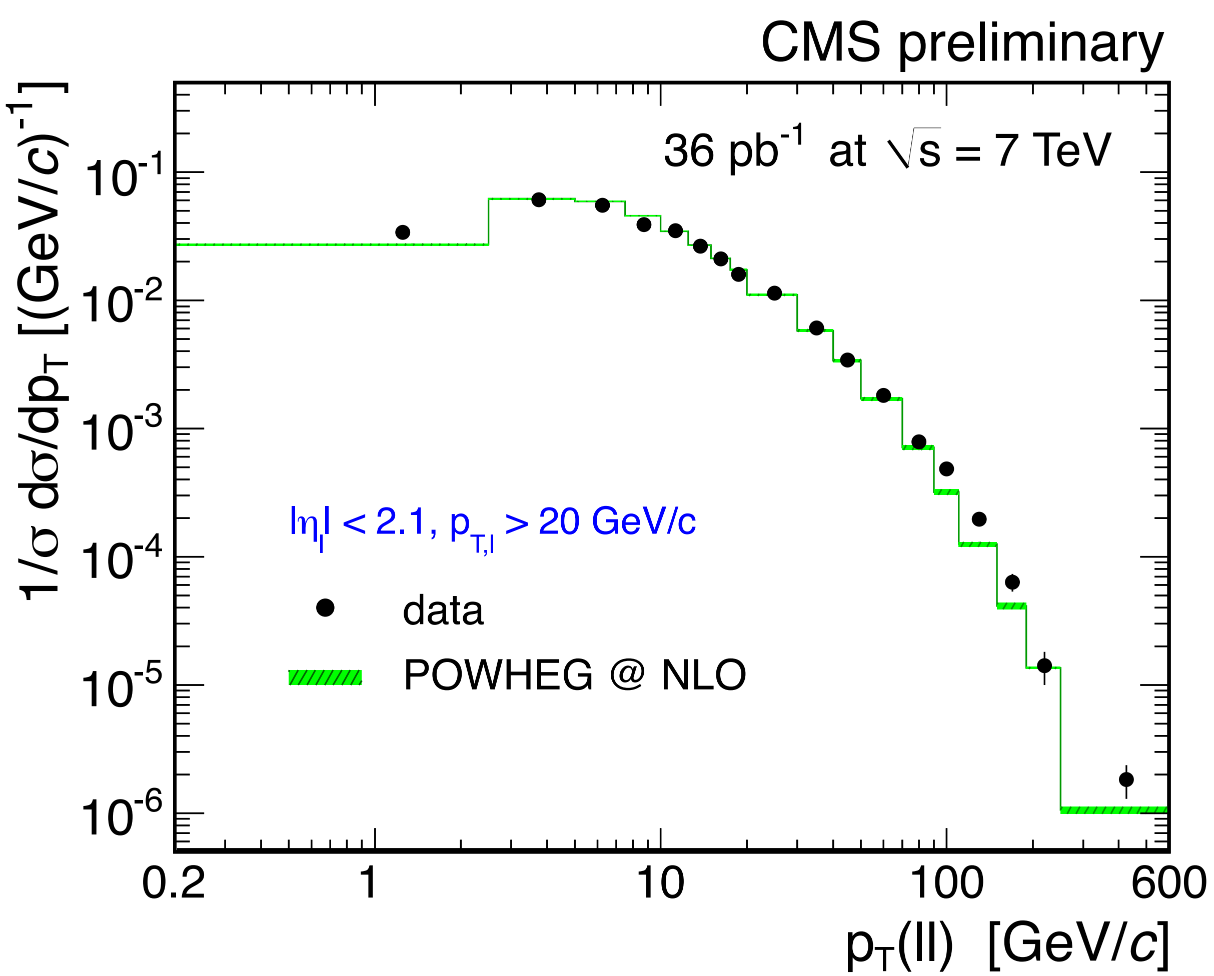
- Used for measurement of W-boson mass and width, PDF determinations, Higgs discovery, background to New Physics searches
- Region of small $q_T \ll M$ particularly relevant to extraction of W mass and reduction of background to Higgs searches

Z-boson production at Tevatron ...



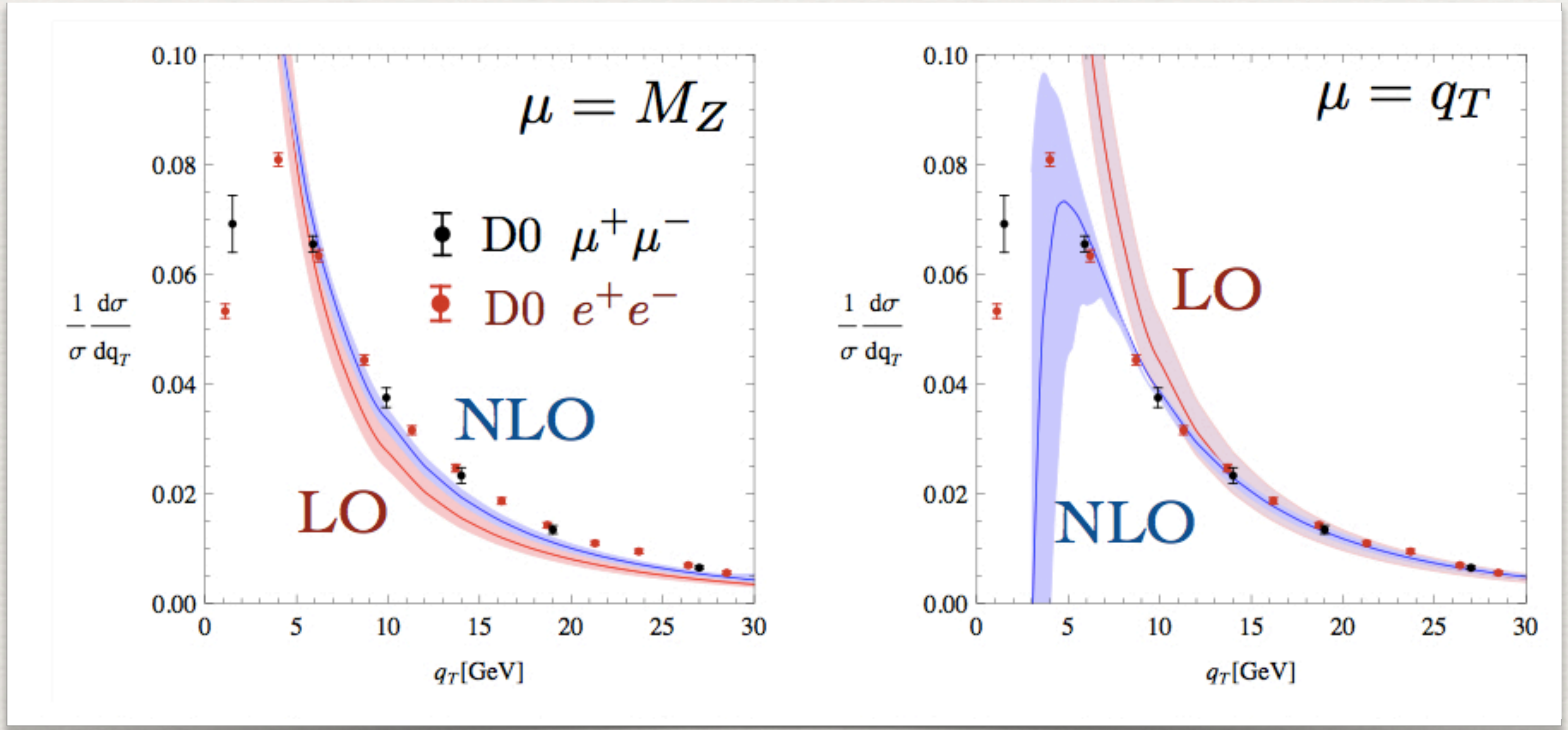
... and at LHC


ATLAS Preliminary $\int$ L = 229 nb^{-1}
Data 2010 ($\sqrt{s}$=7 TeV)
Z → μμ
Entries/5 GeV
p_T^Z [GeV]


CMS preliminary
36 pb^{-1} at $\sqrt{s}$ = 7 TeV
$|\eta_l| < 2.1$, $p_{T,l} > 20$ GeV/c
data
POWHEG @ NLO
$1/\sigma\ d\sigma/dp_T$ [(GeV/c)$^{-1}$]
$p_T(ll)$ [GeV/c]

Drell-Yan processes

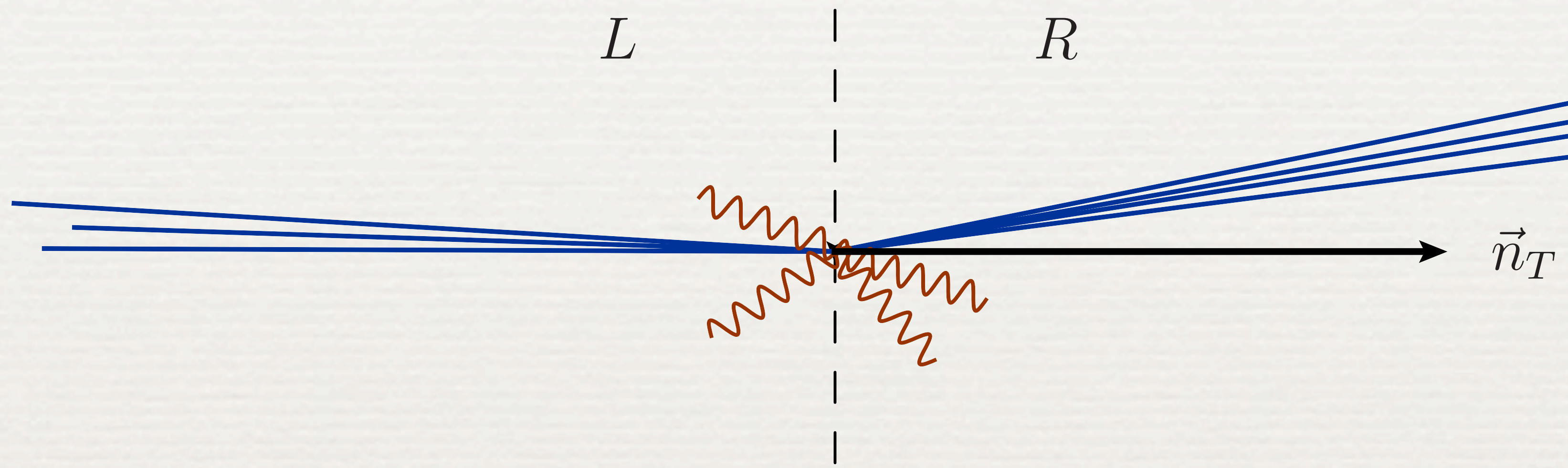
- Classical two-scale problem ($q_T \ll M$), for which large Sudakov logarithms $\sim (\alpha_s \ln^2 M/q_T)^n$ arise that must be resummed



Drell-Yan processes

- Transverse momentum of Drell-Yan object (W, Z, H) due to initial-state radiation (ISR) off collinear partons
- Simple example of beam jets described by beam functions in SCET   Stewart, Tackmann, Waalewijn 2009
- Yet many surprises and subtleties arise, which may be relevant also for other applications of beam functions in jet processes

Jet broadening in e$^+$e$^-$ annihilation



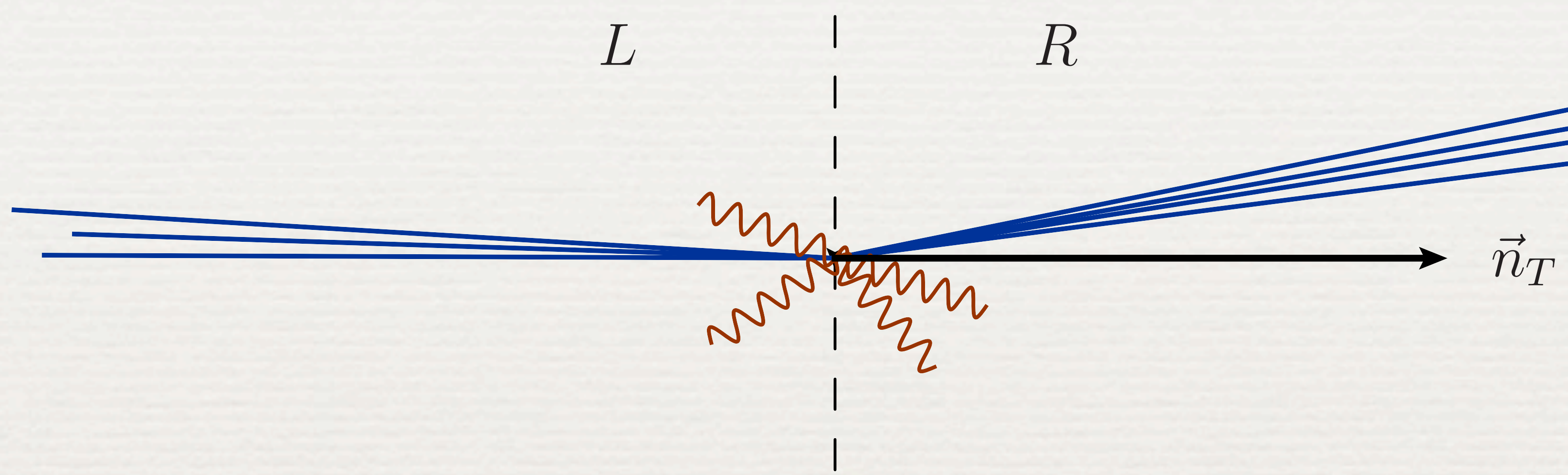
- Broadening measures transverse momenta relative to thrust axis:

$$b_L = \frac{1}{2}\sum_i |\vec{p}_i^{\perp}| = \frac{1}{2}\sum_i |\vec{p}_i \times \vec{n}_T|$$

- Total and wide broadening defined as:

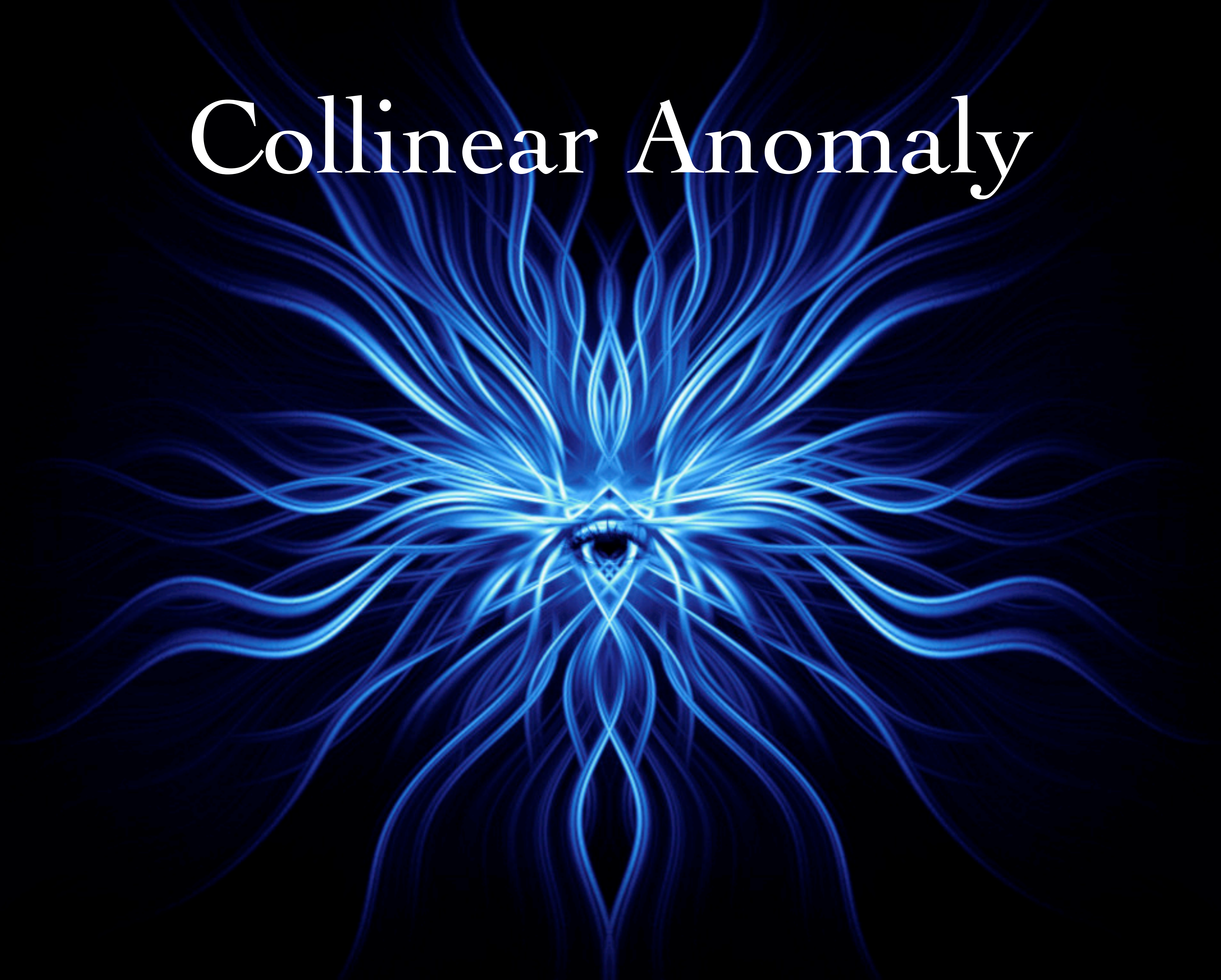
$$b_T = b_L + b_R; \qquad b_W = \max(b_L, b_R)$$

Jet broadening in e^+e^- annihilation



- Important event shape, relevant for precision determination of α_s
- Cross section is largest for $b_{L,R} \ll Q = \sqrt{s}$, where resummation of Sudakov logarithms is required for reliable prediction
- But so far no all-order factorization theorem existed for jet broadening

Collinear Anomaly



Soft-collinear factorization in SCET

- Common to Drell-Yan at small q_T and jet broadening at small $b_{L,R}$ is that observables select final-state partons with small transverse momenta $p_i^\perp = \lambda M\,; \quad \lambda \ll 1$
- Partons can be (anti-)collinear, aligned with initial- or final-state jets, or soft
- Describe these in soft-collinear effective theory (SCET) in terms of (anti-)collinear and soft quark and gluon fields

Soft-collinear factorization in SCET

- Relevant effective theory SCET$_{\text{II}}$ contains collinear, anti-collinear, and soft partons with momenta:

$$p_i^c \sim (\lambda^2, 1, \lambda) M$$
$$p_i^{\bar{c}} \sim (1, \lambda^2, \lambda) M$$
$$p_i^s \sim (\lambda, \lambda, \lambda) M$$

- Classical effective Lagrangian contains no interactions between different modes, implying a complete factorization:

$$\mathcal{L}_{\text{SCET}_{\text{II}}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

Soft-collinear factorization in SCET

- If this was true, then:

Q independent!

$$d\sigma \sim H(Q,\mu)\,\phi_c(q_T,\mu)\,\phi_{\bar c}(q_T,\mu)\,S(q_T,\mu)$$

- But RGE for hard function shows that this cannot be correct:

Sudakov (cusp) logarithm

$$\frac{d}{d\ln\mu}H(Q^2,\mu) = \left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s)\right]H(Q^2,\mu)$$

- RG invariance of cross section implies that soft-collinear part $\phi_c\,\phi_{\bar c}\,S$ must carry some hidden (anomalous) dependence on Q

→ not observed in previous SCET papers on q_T resummation:

Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Soft-collinear factorization in SCET

- At classical level, the SCET$_{\text{II}}$ Lagrangian

$$\mathcal{L}_{\text{SCET}_{\text{II}}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

  exhibits certain symmetries, e.g.:

  - $\mathcal{L}_c$ is invariant under rescalings $\bar{p} \to \bar{\lambda}\bar{p}$ of anti-collinear jet momentum
  - $\mathcal{L}_{\bar{c}}$ is invariant under rescalings $p \to \lambda p$ of collinear jet momentum

- This symmetry is anomalous, not preserved by regularization (broken to subgroup $\lambda\bar{\lambda} = 1$)

"collinear anomaly" Becher, MN 2010

Soft-collinear factorization in SCET

- Not an anomaly of QCD, but of the effective theory relevant to QCD factorization
- In a different context ($B \to \pi$ form factor), Beneke called this the "factorization anomaly"

Dubna lectures 2005

- Fact that additional Q dependence arises from a quantum anomaly gives rise to stringent constraints, which imply that it exponentiates; e.g. for Drell-Yan production at small q_T:

$$(Q^2 x_T^2)^{-F(x_T^2,\mu)} = \exp\left[-\underbrace{F(x_T^2,\mu)}_{\text{calculable if } x_T \ll \Lambda^{-1}} \, \underbrace{\ln\left(Q^2 x_T^2\right)}_{\text{single logarithm}}\right]$$

Soft-collinear factorization in SCET

- There exist many ways to regularize the loop graphs giving rise to the anomaly, but dimensional regularization alone is not sufficient
- Here we use analytic regularization Smirnov 1993
- Other schemes have been proposed, e.g. the "rapidity RG", but their consistency has not yet been demonstrated beyond 1-loop order

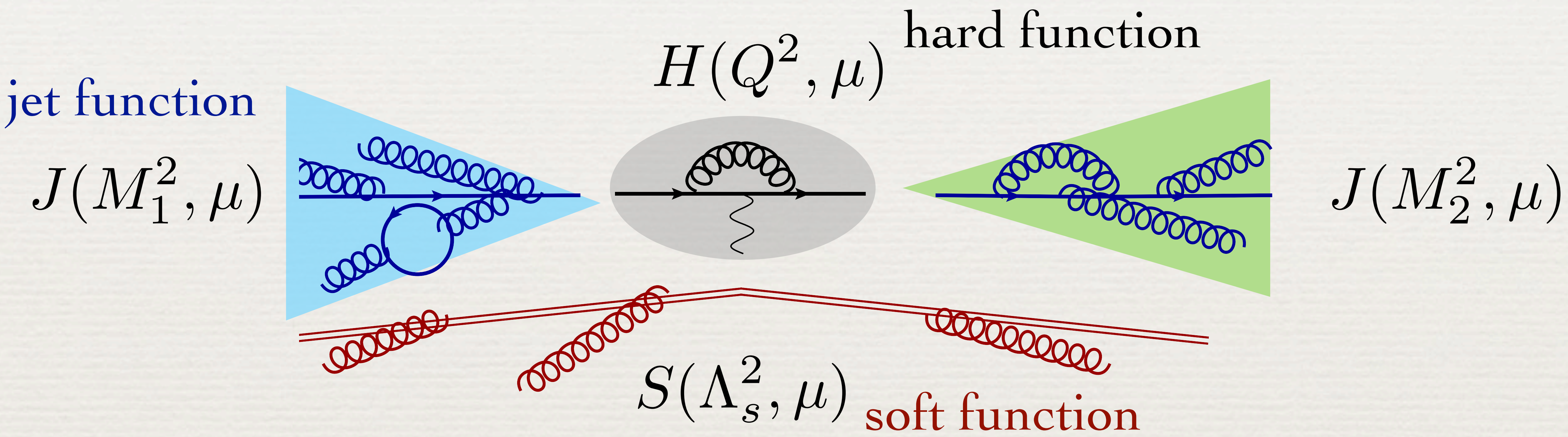
  Chiu, Jain, Neill, Rothstein 2011; see also: Manohar, Stewart 2006
- For any consistent scheme, final results will be independent of the regularization procedure

Factorization and Resummation for the Drell-Yan Cross Section at small q_T

(T. Becher, MN, arXiv:1007.4005)

Drell-Yan cross section in SCET

- Naive soft-collinear factorization:



- In our regularization scheme the soft contribution in this particular case gives rise to scaleless integrals that vanish

Drell-Yan cross section in SCET

Side remark:

- Absence of soft contributions k~(λ,λ,λ) follows after proper multipole expansion using that x~(1,1, λ^{-1}), which implies:

$$(p-k)\cdot x = p\cdot x - k_\perp \cdot x_\perp + \mathcal{O}(\lambda)$$

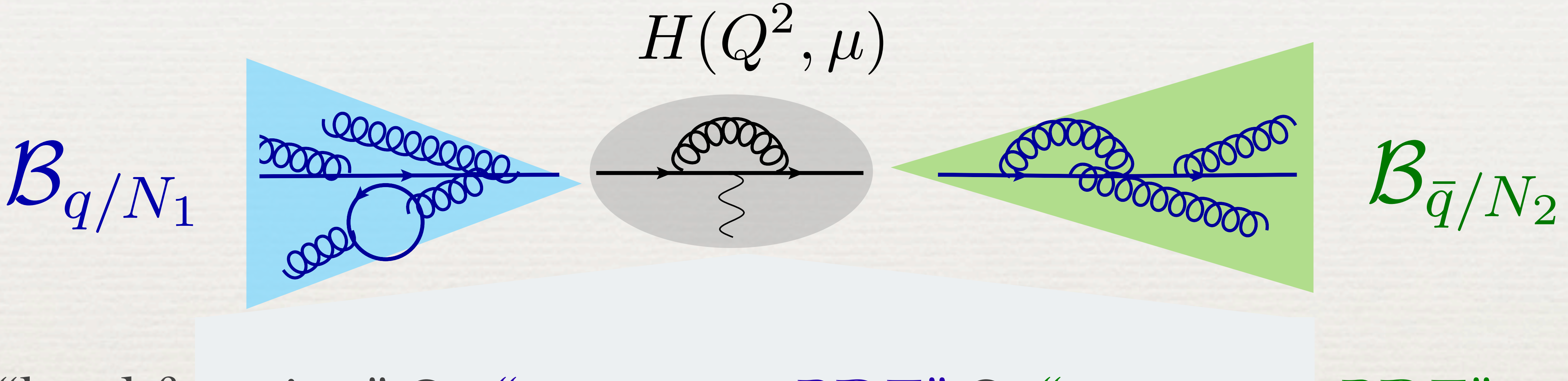
- Relevant loops integrals such as

$$\int d^d k \, \frac{1}{(n\cdot k - i\epsilon)^{1+\alpha}} \, \frac{1}{(\bar{n}\cdot k - i\epsilon)^{1+\beta}} \, \delta(k^2)\,\theta(k^0)\, e^{ip\cdot x - ik_\perp \cdot x_\perp}$$

are scaleless and vanish in dimensional regularization

Drell-Yan cross section in SCET

- Remaining naive factorization formula:



"hard function" ⊗ "transverse PDF" ⊗ "transverse PDF"

- Transverse PDF:

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p) |\, \bar{\chi}(t\bar{n} + x_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi(0)\, | N(p) \rangle$$

This spells trouble: well known that transverse PDF not well defined without additional regulator

Drell-Yan cross section in SCET

- Remaining naive factorization formula:

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\big|H(M^2,\mu)\big|\,\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}$$

$$\times\sum_q e_q^2\left[\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\mathcal{B}_{\bar q/N_2}(\xi_2,x_T^2,\mu)+(q\leftrightarrow\bar q)\right]+\mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

where: $\xi_1=\sqrt{\tau}\,e^{y}\,,\qquad \xi_2=\sqrt{\tau}\,e^{-y}\,,\qquad \text{with}\quad \tau=\frac{m_\perp^2}{s}=\frac{M^2+q_T^2}{s}$

- Resummation would then be accomplished by solving the RGE for the hard function:

$$\frac{d}{d\ln\mu}H(M^2,\mu)=\left[2\Gamma_{\text{cusp}}^F(\alpha_s)\boxed{\ln\frac{M^2}{\mu^2}}+4\gamma^q(\alpha_s)\right]H(M^2,\mu)$$

→ see SCET papers by: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Drell-Yan cross section in SCET

- Remaining naive factorization formula:

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_c M^2 s}\big|H(M^2,\mu)\big|\,\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp} \times \sum_q e_q^2\left[\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(\xi_2,x_T^2,\mu) + (q\leftrightarrow\bar{q})\right] + \mathcal{O}\!\left(\frac{q_T^2}{M^2}\right)$$

where: $\xi_1 = \sqrt{\tau}\,e^{y}\,,\qquad \xi_2 = \sqrt{\tau}\,e^{-y}\,,\qquad$ with $\tau = \dfrac{m_\perp^2}{s} = \dfrac{M^2+q_T^2}{s}$

- Resummation would then be accomplished by solving the RGE for the hard function:

$$\frac{d}{d\ln\mu}\,H(M^2,\mu) = \left[2\Gamma_{\rm cusp}^F(\alpha_s)\,\boxed{\ln\frac{M^2}{\mu^2}} + 4\gamma^q(\alpha_s)\right]H(M^2,\mu)$$

This must be wrong!

→ see SCET papers by: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

Collinear anomaly

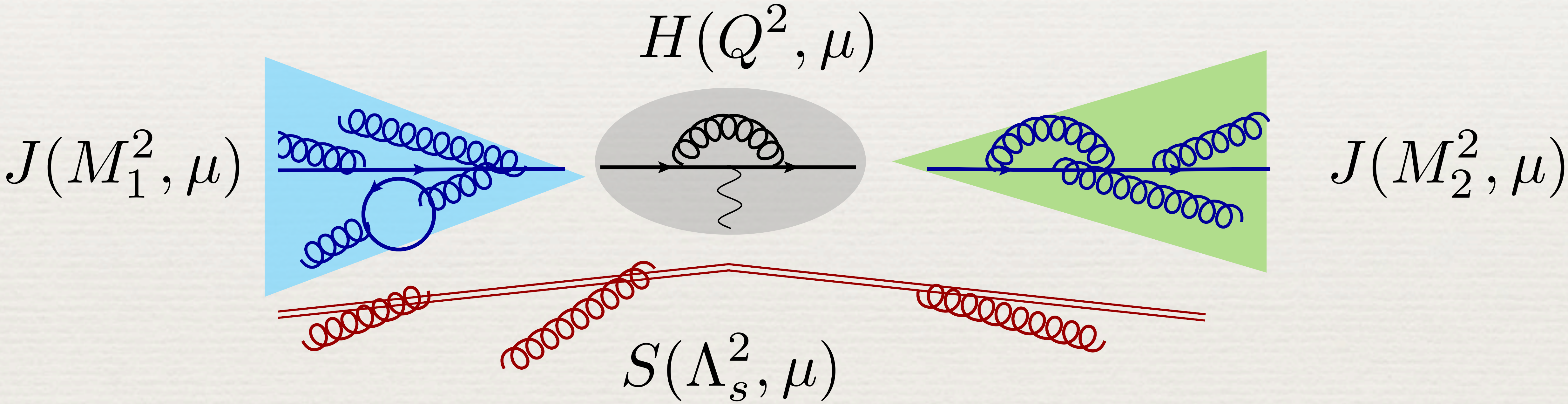
- RG invariance of the cross section requires that the product $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ must contain a hidden M dependence
- Analyzing the relevant diagrams, we find that an additional regulator is needed to make transverse PDFs well defined; in the product of two PDFs this regulator can be removed, but an anomalous M dependence remains:

$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2, \mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu)$$

with: $$\frac{dF_{q\bar{q}}(x_T^2, \mu)}{d \ln \mu} = 2\Gamma_{\text{cusp}}^F(\alpha_s)$$

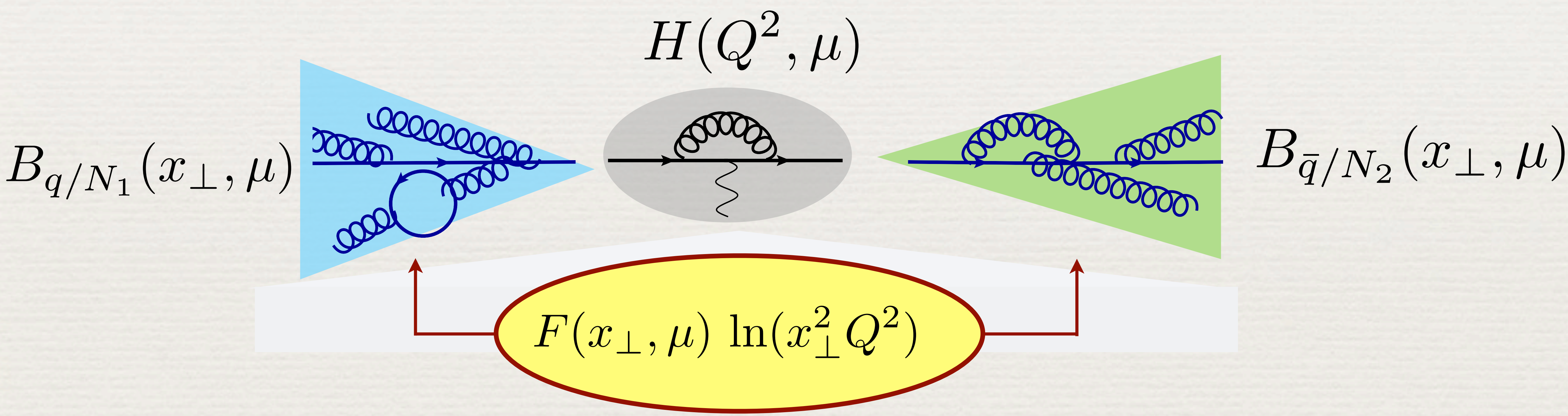
Collinear anomaly

- Regular soft-collinear factorization:



Collinear anomaly

- Anomalous soft-collinear factorization:



Transverse PDFs

"What God has joined together, let no man separate..."

- The "operator definition of TMP PDFs is quite problematic [...] and is nowadays under active investigation" Cherednikov, Stefanis 2009

  for a review, see: Collins 2003, 2008
  for an elegant recent definition, see: Collins 2011

- Our result:
  Regularization of individual transverse PDFs is delicate, but the product of two transverse PDFs is well defined and has a specific dependence on hard momentum transfer M^2

Comparison with the CSS formula

- Classic result from Collins-Soper-Sterman: 1985

$$\frac{d^3\sigma}{dM^2\, dq_T^2\, dy} = \frac{4\pi\alpha^2}{3N_c M^2 s}\,\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1 \frac{dz_1}{z_1}\int_{\xi_2}^1 \frac{dz_2}{z_2}$$
$$\times \exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}A\big(\alpha_s(\bar{\mu})\big) + B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$
$$\times \left[\overline{\mathcal{P}}_{q/N_1}(\xi_1, x_T, \mu_b)\,\overline{\mathcal{P}}_{\bar{q}/N_2}(\xi_2, x_T, \mu_b) + (q, i \leftrightarrow \bar{q}, j)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- Disadvantages compared with our approach:
  - $\bar{\mu}$ integral hits the Landau pole of running coupling and requires PDFs at arbitrarily low scales
  - practical calculations employ an x_T-space cutoff, which introduces some ad hoc model dependence

Comparison with the CSS formula

- Classic result from Collins-Soper-Sterman: 1985

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2}$$

$$\times\exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$

$$\times\left[\overline{\mathcal{P}}_{q/N_1}(\xi_1,x_T,\mu_b)\,\overline{\mathcal{P}}_{\bar{q}/N_2}(\xi_2,x_T,\mu_b)+(q,i\leftrightarrow\bar{q},j)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- All-order equivalence to our result, if:

$$A(\alpha_s) = \Gamma_{\text{cusp}}^F(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_1(\alpha_s)}{d\alpha_s}, \qquad g_1(\alpha_s) = F(0,\alpha_s)$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_2(\alpha_s)}{d\alpha_s}, \qquad g_2(\alpha_s) = \ln H(-\mu^2,\mu)$$

$$\overline{\mathcal{P}}_{i/N}(\xi,x_T) = H(-\mu_b^2,\mu_b)\,B_{i/N}(\xi,x_T^2,\mu_b)$$

anomaly contributions

Comparison with the CSS formula

- Only linear dependence on log(Q) in exponent can be made consistent with CSS formula!
- Non-trivial soft function absent in CSS, too!
- Anomaly implies a non-trivial contribution to A, such that $A(\alpha_s) \neq \Gamma^F_{\text{cusp}}(\alpha_s)$ in this case!

→ missed by all previous SCET analyses:
Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

- Can predict unknown 3-loop coefficient of A based on known 2-loop result for B:

$\Gamma^F_2 = 538.2$ while $A^{(3)} = -930.8$ → important effect

Simplification for $x_T \ll \Lambda^{-1}$ (large q_T)

- Can perform operator product expansion:

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

- Only the product of two $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ functions is well defined due to the anomaly:

$$\left[\mathcal{I}_{q\leftarrow i}(z_1, x_T^2, \mu)\, \mathcal{I}_{\bar q\leftarrow j}(z_2, x_T^2, \mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar q}(x_T^2,\mu)} I_{q\leftarrow i}(z_1, x_T^2, \mu)\, I_{\bar q\leftarrow j}(z_2, x_T^2, \mu)$$

anomalous q^2 dependence

- Using analytic regulators in the calculation of these functions is very economical, since it does not introduce any new scales

Simplification for $x_T \ll \Lambda^{-1}$ (large q_T)

- Factorized cross section at small q_T:

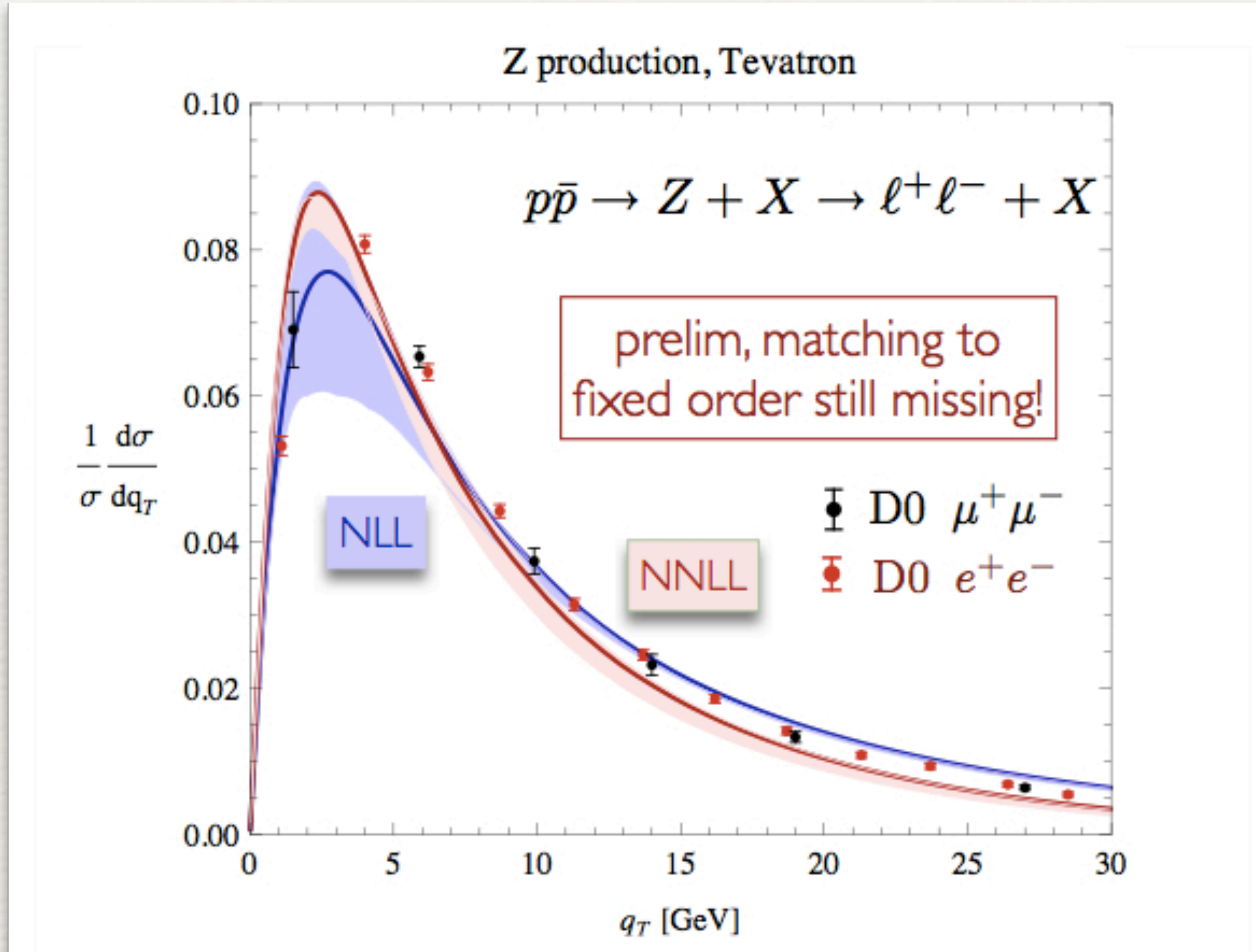
$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1 \frac{dz_1}{z_1}\int_{\xi_2}^1 \frac{dz_2}{z_2}$$
$$\times \left[C_{q\bar{q}\to ij}\left(\frac{\xi_1}{z_1},\frac{\xi_2}{z_2},q_T^2,M^2,\mu\right)\phi_{i/N_1}(z_1,\mu)\,\phi_{j/N_2}(z_2,\mu) + (q,i\leftrightarrow\bar{q},j)\right]$$

- Hard-scattering kernels:

$$C_{q\bar{q}\to ij}(z_1,z_2,q_T^2,M^2,\mu) = H(M^2,\mu)\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$
$$\times\, I_{q\leftarrow i}(z_1,x_T^2,\mu)\,I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)$$

- Two sources of M dependence: hard function and collinear anomaly

Numerical results (preliminary)



Factorization and Resummation for Jet Broadening in e^+e^- Annihilation

(T. Becher, G. Bell, MN, arXiv:1104.4108)

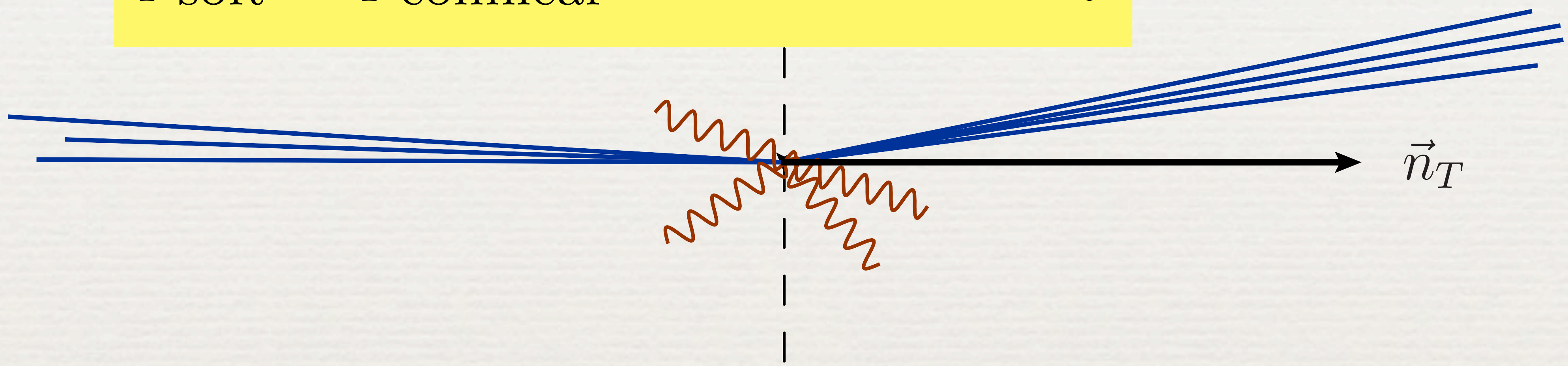
Factorization for jet broadening

Problem that individual jet and soft functions are not well defined without additional regularization also arises in other factorization theorems

- electroweak Sudakov resummation (and any other process at high Q^2 with small but nonzero masses) Chiu, Golf, Kelley, Manohar 2007
- other observables sensitive to transverse momenta, such as jet broadening Becher, Bell, MN 2011

Factorization for jet broadening

$$p_{\text{soft}}^{\perp} \sim p_{\text{collinear}}^{\perp} \sim b_L \sim b_R \ll Q$$



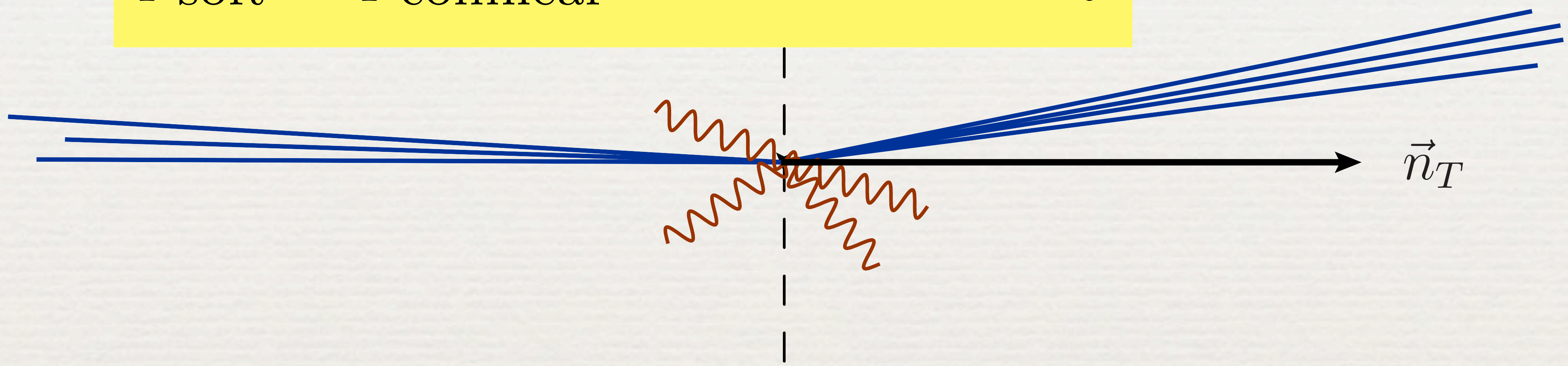
- Naive factorization theorem for broadening, (jets recoil against soft radiation):

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L\,db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^{\perp}\int d^{d-2}p_R^{\perp}$$
$$\times \mathcal{J}_L(b_L - b_L^s, p_L^{\perp}, \mu)\,\mathcal{J}_R(b_R - b_R^s, p_R^{\perp}, \mu)\,\mathcal{S}(b_L^s, b_R^s, -p_L^{\perp}, -p_R^{\perp}, \mu)$$

- Non-trivial soft function arises in this case, since radiation is restricted to hemispheres

Factorization for jet broadening

$$p^{\perp}_{\text{soft}} \sim p^{\perp}_{\text{collinear}} \sim b_L \sim b_R \ll Q$$



- Laplace ($b_{L,R} \to \tau_{L,R}$) and Fourier tranforms ($p^{\perp}_{L,R} \to z_{L,R} = 2|x^{\perp}_{L,R}|/\tau_{L,R}$):

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L\, d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L \int_0^\infty dz_R \boxed{\overline{\mathcal{J}}_L(\tau_L, z_L, \mu)}\,\boxed{\overline{\mathcal{J}}_R(\tau_R, z_R, \mu)}\,\boxed{\overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)}$$

- Jet and soft functions must contain a hidden (anomalous) Q dependence

Anomalous factorization

- Have derived the Q dependence of product

$$P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) = \boxed{\overline{\mathcal{J}}_L(\tau_L, z_L, \mu)}\,\boxed{\overline{\mathcal{J}}_R(\tau_R, z_R, \mu)}\,\boxed{\overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)}$$

using invariance under analytic regularization

- General result:

double logarithm! single logarithms

$$\ln P = \frac{k_2(\mu)}{4} \ln^2\left(Q^2\, \bar{\tau}_L \bar{\tau}_R\right) - F_B(\tau_L, z_L, \mu)\, \ln\left(Q^2 \bar{\tau}_L^2\right) - F_B(\tau_R, z_R, \mu)\, \ln\left(Q^2 \bar{\tau}_R^2\right)$$
$$+ \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

with:

$$\frac{d}{d\ln\mu} k_2(\mu) = 0\,, \qquad \frac{d}{d\ln\mu} F_B(\tau, z, \mu) = \Gamma_{\text{cusp}}(\alpha_s)$$

Anomalous factorization

- General result:

double logarithm! single logarithms

$$\ln P = \frac{k_2(\mu)}{4}\ln^2\left(Q^2\,\bar{\tau}_L\bar{\tau}_R\right) - F_B(\tau_L, z_L, \mu)\,\ln\left(Q^2\bar{\tau}_L^2\right) - F_B(\tau_R, z_R, \mu)\,\ln\left(Q^2\bar{\tau}_R^2\right) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

with:

$$\frac{d}{d\ln\mu}\,k_2(\mu) = 0\,, \qquad \frac{d}{d\ln\mu}\,F_B(\tau, z, \mu) = \Gamma_{\text{cusp}}(\alpha_s)$$

- Perturbative analysis reveals that $k_2 = 0$ (to all orders), and:

$$F_B(\tau, z, \mu) = \frac{C_F\alpha_s}{\pi}\left[\ln(\mu\bar{\tau}) + \ln\frac{\sqrt{1+z^2}+1}{4}\right] + \mathcal{O}(\alpha_s^2)$$

Anomalous factorization

- First all-order factorization formula:

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L \int_0^\infty dz_R\, \left(Q^2\bar{\tau}_L^2\right)^{-F_B(\tau_L,z_L,\mu)}\left(Q^2\bar{\tau}_R^2\right)^{-F_B(\tau_R,z_R,\mu)}$$
$$\times\, W(\tau_L,\tau_R,z_L,z_R,\mu)$$

anomalous Q dependence

- At NLL order, Mellin inversion can be done analytically:

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{db_T} = H(Q^2,\mu)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\frac{1}{b_T}\left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)}$$

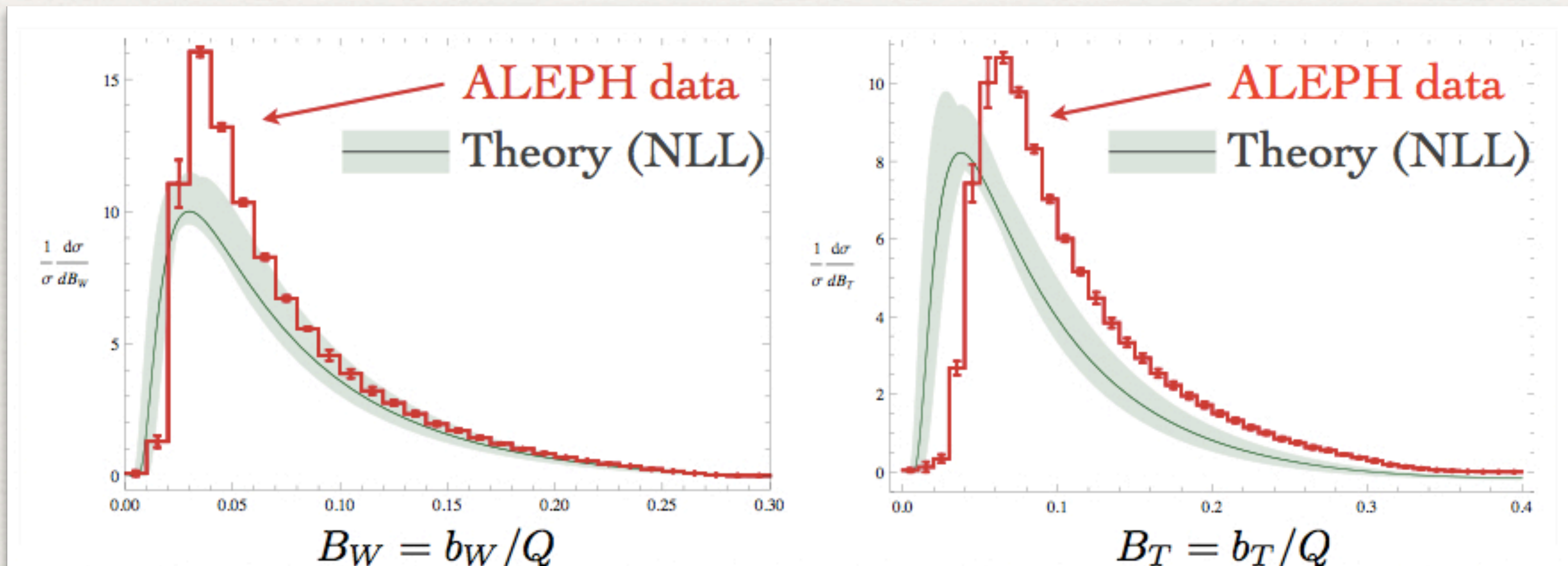
with:

$$I(\eta) = \int_0^\infty dz\,\frac{z}{(1+z^2)^{3/2}}\left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta}, \qquad \eta \equiv \frac{C_F\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2}$$

→ equivalent to: Dokshitzer, Lucenti, Markesini, Salam 1998
[correcting Catani, Turnock, Webber 1992, who missed the $I^2(\eta)$ term]
→ $I^2(\eta)$ term also missed in: Chiu, Jain, Neill, Rothstein 2011

Numerical results (preliminary)

- Comparison with ALEPH data (Q=91.2 GeV)
- Theory predictions at NLL order, still without matching to NLO



- Calculation of NNLL terms desired!

Extension to NNLL?

- Have operator definitions of jet and soft functions, e.g.:

$$\frac{\pi}{2}\,(\not{n})_{\alpha\beta}\,\mathcal{J}_L(b,p^\perp,\mu) = \sum\!\!\!\!\!\!\!\int_X (2\pi)^d\,\delta(\bar{n}\cdot p_X - Q)\,\delta^{d-2}(p_X^\perp - p^\perp)$$
$$\times\,\delta\Big(b - \frac{1}{2}\sum_{i\in X}|p_i^\perp|\Big)\,\langle 0|\chi_\alpha(0)|X\rangle\,\langle X|\bar{\chi}_\beta(0)|0\rangle$$

- For NNLL accuracy we need one-loop jet and soft functions (latter is known) and two-loop anomaly function $F_B(\tau,z,\mu)$
- Appears doable and worthwhile

Conclusions

- Have derived all-order resummed expression for Drell-Yan cross section at small $q_T \ll M$
- Naive factorization broken by collinear anomaly
- Correct SCET analysis reproduces CSS formula with a nontrivial relation between A and $\Gamma_{\rm cusp}$; predicted $A^{(3)}$, last missing ingredient for NNLL
- Transverse PDFs do not exist as individual objects;*) only products of two PDFs are well defined, and carry an anomalous M dependence

*) They are gauge dependent in the standard treatment and affected by (dim. unregularized) "rapidity divergences"

Conclusions

- Extending these methods, we have derived the first all-order resummation formula for jet broadening in e^+e^- annihilations
- Features non-trivial anomalous Q dependence due to anomaly
- NLL results agree with (the correct) known expressions in literature
- Calculations necessary to achieve NNLL resummation appear feasible
- Phenomenology in progress

BACKUP SLIDES:
Analytic regulators at work

Short-distance expansion for $x_T \ll \Lambda_{\rm QCD}^{-1}$

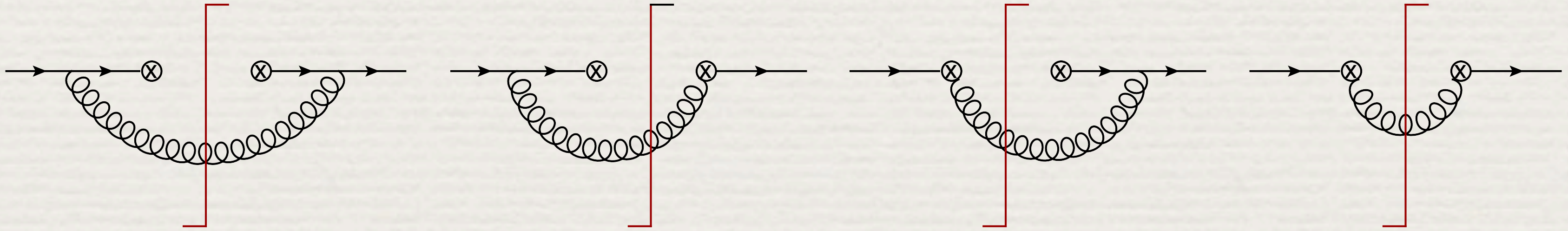
- Generalized PDFs at small transverse separation can be expanded in usual PDFs:

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

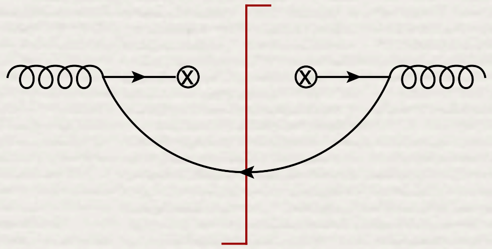
$$B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, I_{i\leftarrow j}(\xi/z, x_T^2, \mu)\, \phi_{j/N}(z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

- Expansion kernels are obtained from matching calculation

$\mathcal{I}_{q\leftarrow q}$:



$\mathcal{I}_{q\leftarrow g}$:

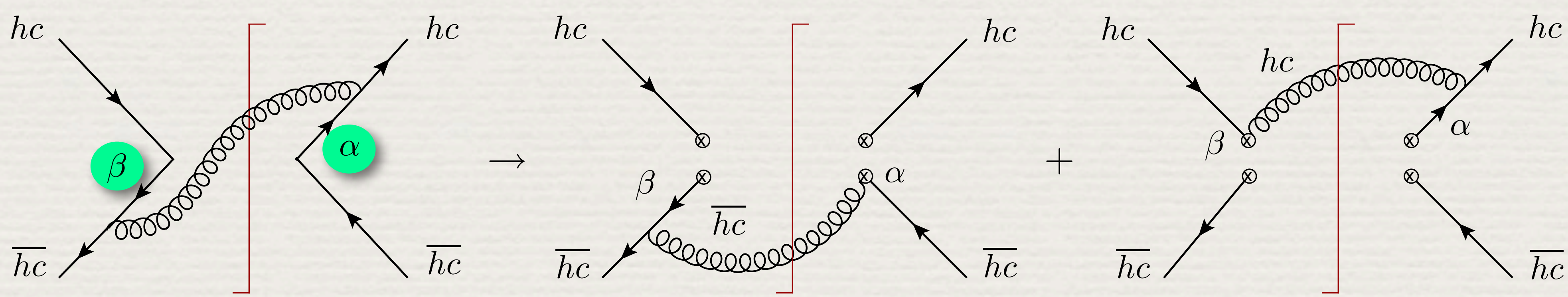


Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- Collinear loops are not defined and require a regulator beyond dimensional regularization
- Most economic possibility is to use analytic regularization scheme: Smirnov 1993

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- Adaption to SCET collinear propagators:



breaks rescaling invariance

$$\frac{n^\mu}{n \cdot k - i\varepsilon} \to \frac{\nu_1^{2\alpha}\, n^\mu\, \bar{n} \cdot p}{(n \cdot k\, \bar{n} \cdot p - i\varepsilon)^{1+\alpha}}$$

regularized Wilson lines

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

regularized propagator

Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- Introducing analogous regulator β in anti-collinear sector, we find:

$$\mathcal{I}_{q\leftarrow q}(z,x_T^2,\mu)\Big|_{\alpha\text{ reg.}} = -\frac{C_F\alpha_s}{2\pi}\left\{\left(\frac{1}{\epsilon}+L_\perp\right)\left[\left(\boxed{\frac{2}{\alpha}-2\ln\frac{\mu^2}{\nu_1^2}}\right)\delta(1-z)+\frac{1+z^2}{(1-z)_+}\right]\right.$$
$$\left.+\,\delta(1-z)\left(-\frac{2}{\epsilon^2}+L_\perp^2+\frac{\pi^2}{6}\right)-(1-z)\right\}.$$

$$L_\perp = \ln\frac{x_T^2\mu^2}{4e^{-2\gamma_E}}$$

$$\mathcal{I}_{q\leftarrow q}(z,x_T^2,\mu)\Big|_{\beta\text{ reg.}} = -\frac{C_F\alpha_s}{2\pi}\left\{\left(\frac{1}{\epsilon}+L_\perp\right)\left[\left(\boxed{-\frac{2}{\beta}+2\ln\frac{q^2}{\nu_2^2}}\right)\delta(1-z)+\frac{1+z^2}{(1-z)_+}\right]-(1-z)\right\}$$

- The product of **two** such functions is regulator independent:

anomalous hard logarithm

$$\left[\mathcal{I}_{q\leftarrow q}(z_1,x_T^2,\mu)\,\mathcal{I}_{\bar q\leftarrow\bar q}(z_2,x_T^2,\mu)\right]_{q^2}$$
$$=\delta(1-z_1)\,\delta(1-z_2)\left[1-\frac{C_F\alpha_s}{2\pi}\left(2L_\perp\boxed{\ln\frac{q^2}{\mu^2}}+L_\perp^2-3L_\perp+\frac{\pi^2}{6}\right)\right]$$
$$-\frac{C_F\alpha_s}{2\pi}\left\{\delta(1-z_1)\left[L_\perp\left(\frac{1+z_2^2}{1-z_2}\right)_+-(1-z_2)\right]+(z_1\leftrightarrow z_2)\right\}+\mathcal{O}(\alpha_s^2)$$

Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- From previous result we read off:

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2)$$

$$I_{q\leftarrow q}(z, L_\perp, \alpha_s) = \delta(1-z)\left[1 + \frac{C_F\alpha_s}{4\pi}\left(L_\perp^2 + 3L_\perp - \frac{\pi^2}{6}\right)\right] - \frac{C_F\alpha_s}{2\pi}\left[L_\perp P_{q\leftarrow q}(z) - (1-z)\right] + \mathcal{O}(\alpha_s^2)$$

$$I_{q\leftarrow g}(z, L_\perp, \alpha_s) = -\frac{T_F\alpha_s}{2\pi}\left[L_\perp P_{q\leftarrow g}(z) - 2z(1-z)\right] + \mathcal{O}(\alpha_s^2)$$

Altarelli-Parisi splitting functions

- Two-loop result for $F_{q\bar{q}}(L_\perp, \alpha_s) = \sum_{n=1}^{\infty} d_n^q(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$:

$$d_2^q(L_\perp) = \frac{\Gamma_0^F \beta_0}{2} L_\perp^2 + \Gamma_1^F L_\perp + d_2^q\,, \quad d_2^q = C_F C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{27} C_F T_F n_f$$