Transverse Momentum Distributions from Effective Field Theory

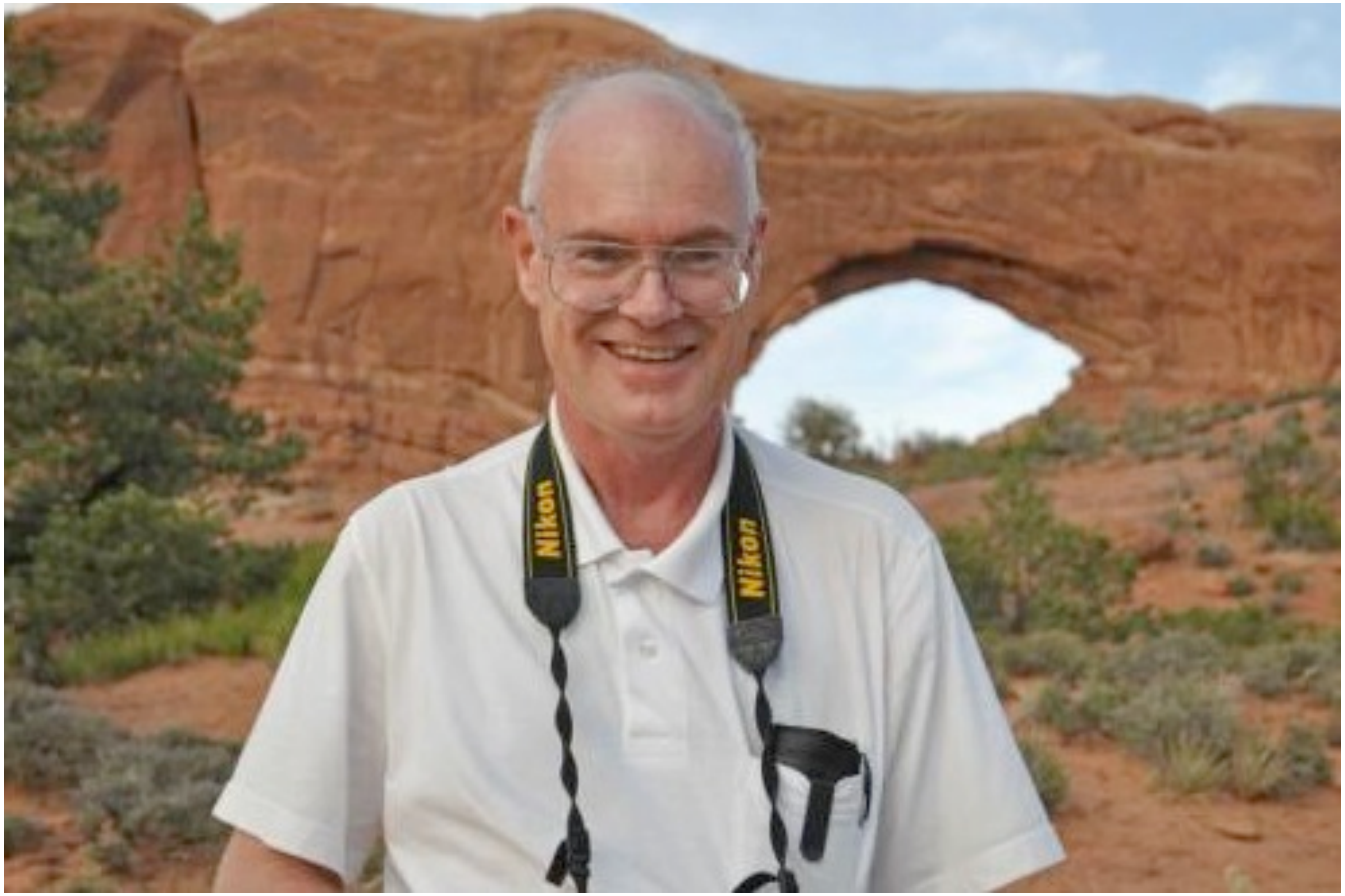
Sonny Mantry
University of Wisconsin at Madison
NPAC Theory Group

In Collaboration with Frank Petriello

arXiv:0911.4135, Phys.Rev.D81:093007, 2010
arXiv:1007.3773, Phys.Rev.D83:053007, 2011
arXiv:1011.0757

(more in progress with Ye Li and Frank Petriello)

LoopFest X: Argonne National Lab and Northwestern University



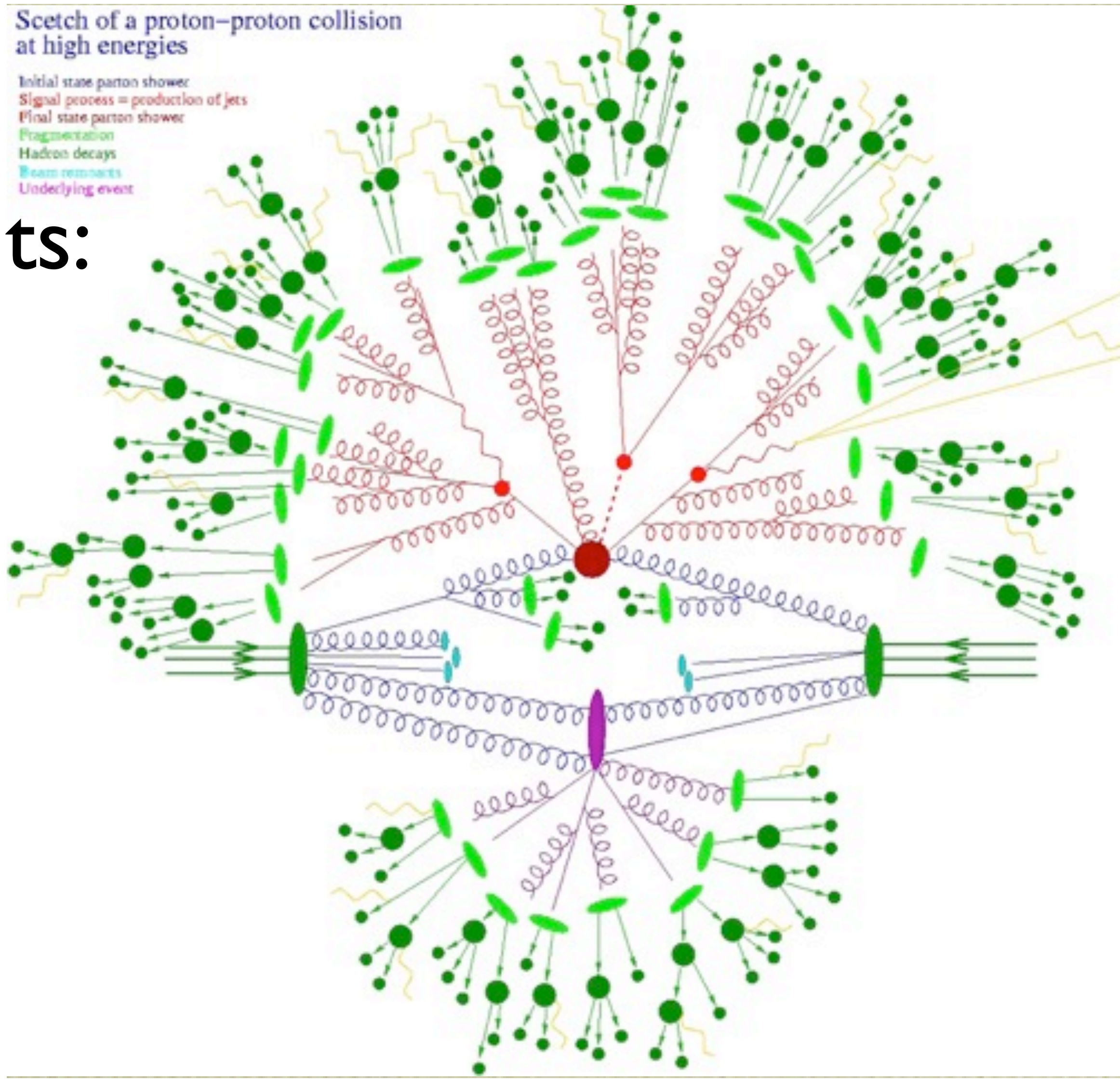
Prof. Uli Baur

(1957-2010)

The Big Picture

- Colliders are complicated environments:

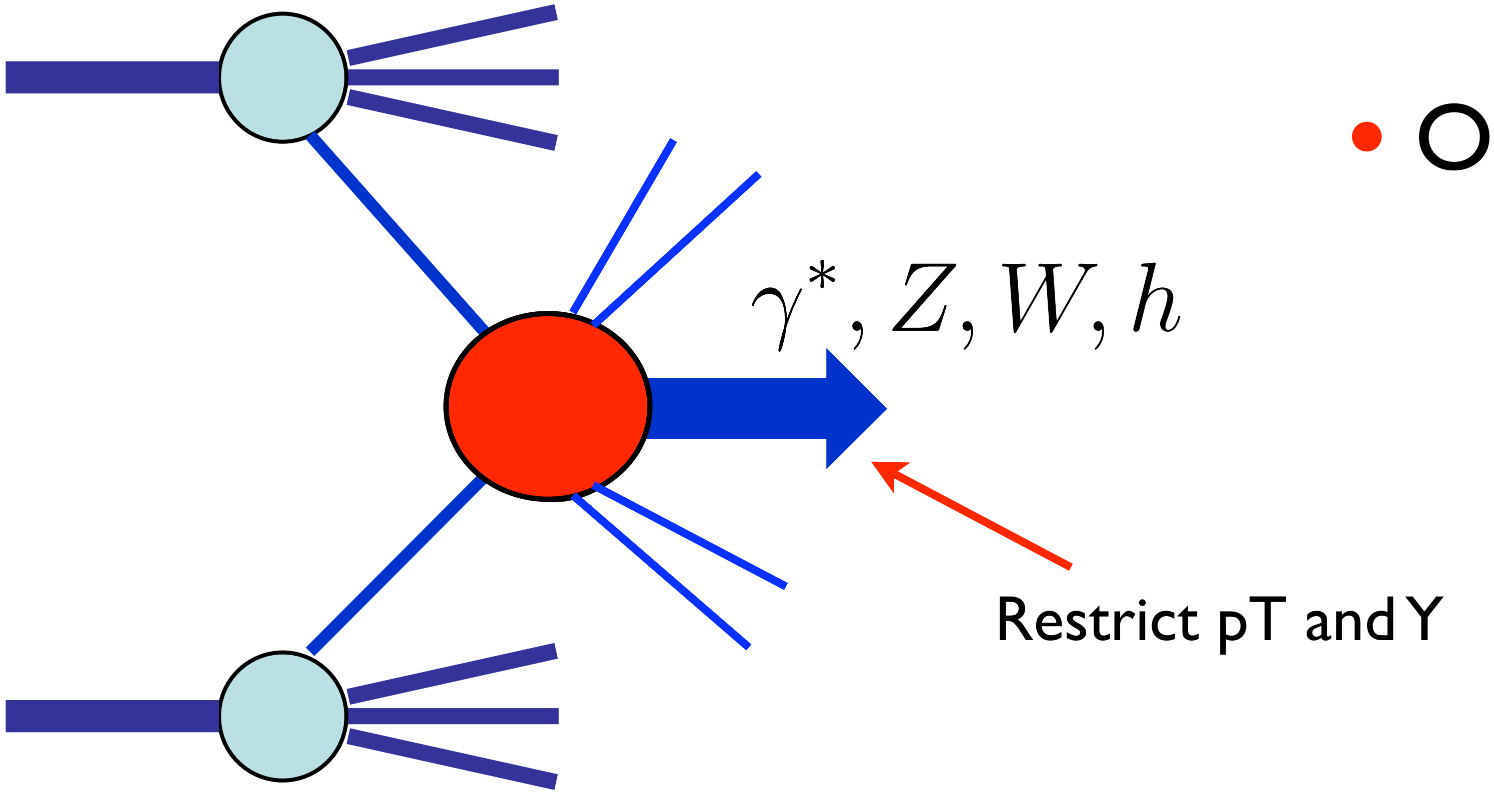
-Initial state parton shower
-Hard interaction of signal process
-Multijet final states
-Underlying events
-Final state parton shower
-Hadronization
-Beam remnants
- ...



- Recent developments in EFT methods:

-Factorization, Resummation, parton showers, jets, ...
- Isolated Drell-Yan, Event shapes, jet vetoes (Stewart, Tackmann, Waalewijn; Kelley, Schwartz,...
- Higgs Production, Threshold resummation (Becher, Neubert, Pecjak,...)
- Fragmentation functions (Procura, Stewart,...)
- Parton Showers (Bauer, Baumgart, Hornig, Schwartz, Stewart, Tackmann, Thaler, ...)
- many others...

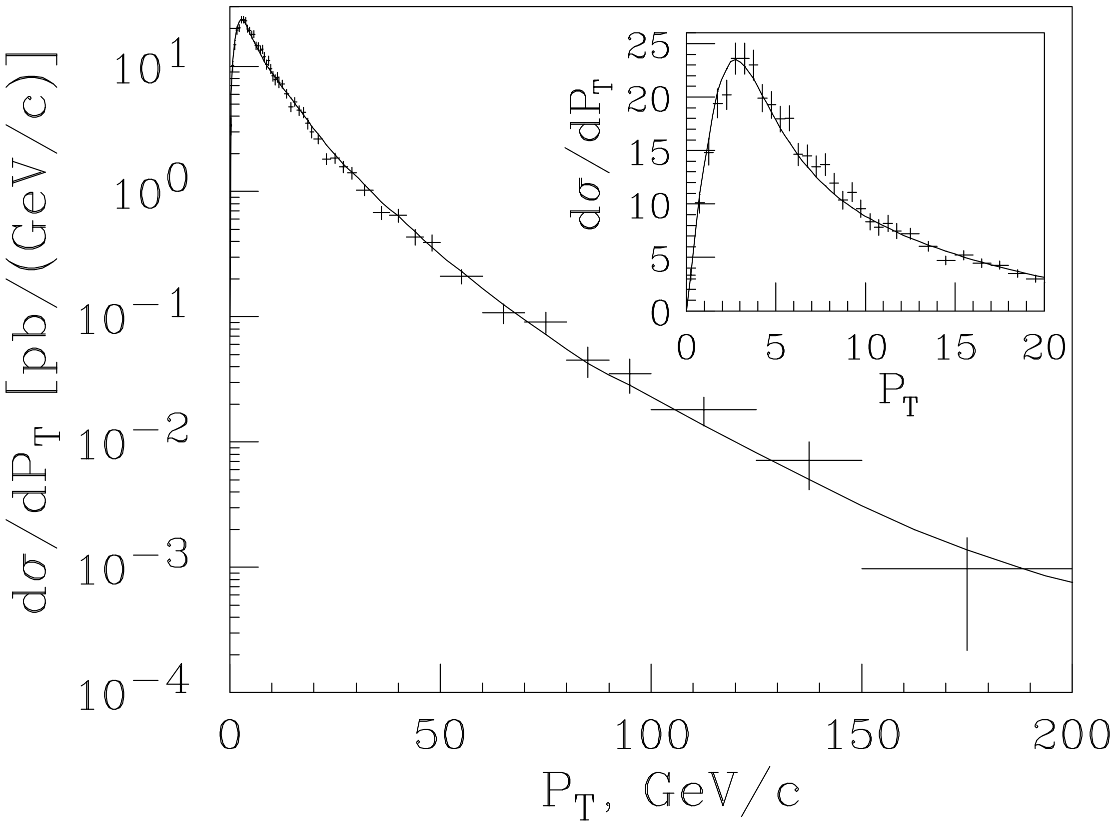
Transverse Momentum Spectrum



Restrict pT and Y

- Observable of interest

$$\frac{d^2\sigma}{dp_T^2 dY}$$



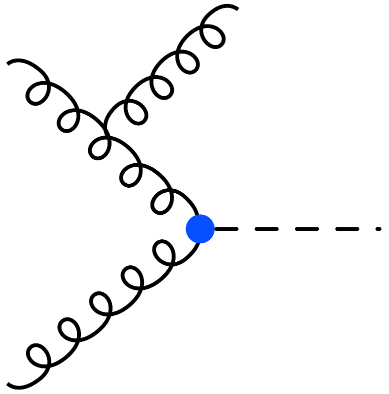
CDF Data
for Z-production

Motivations

- Higgs Boson searches
- W-mass measurement
- Tests of pQCD
- Transverse nucleon structure

Low pT Region

- The schematic perturbative series for the pT distribution for $pp \longrightarrow h + X$



$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_2\alpha_S^2 \ln^3\frac{M^2}{p_T^2} + ... + A_n\alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

Large Logarithms spoil perturbative convergence

- Resummation has been studied in great detail in the Collins-Soper-Sterman formalism.

(Davies, Stirling; Arnold, Kauffman; Berger, Qiu; Ellis, Veseli, Ross, Webber; Brock, Ladinsky Landry, Nadolsky; Yuan; Fai, Zhang; Catani, Emilio, Trentadue; Hinchliffe, Novae; Florian, Grazzini, Cherdnikov, Stefanis; Belitsky, Ji,....)

- Resummation has also been studied recently using the EFT approach.

(Idilbi, Ji, Yuan; Gao, Li, Liu; SM, Petriello; Becher, Neubert)

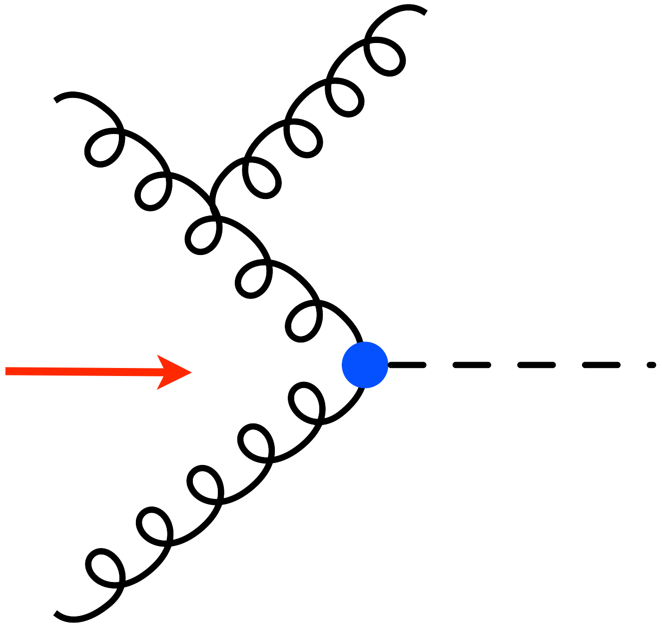
Low pT Region

$$A(P_A) + B(P_B) \to C(Q) + X, \qquad C = \gamma^*, W^\pm, Z, h$$

- The transverse momentum distribution in the CSS formalism is schematically given by:

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

Most singular contribution



Soft or collinear pT emission

Low pT Region

Focus of this talk

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

- Singular as at least Q_T^{-2} as $Q_T \to 0$
- Important in region of small Q_T.
- Treated with resummation.

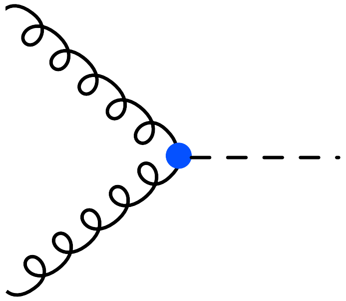
$$\frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{pert})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} - \frac{d\sigma^{(\mathrm{asym})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

- Obtained from fixed order calculation.
- Less Singular terms.
- Important in region of large Q_T.

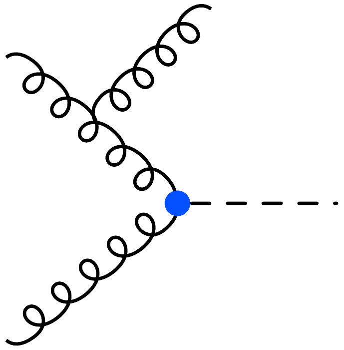
EFT Framework

EFT framework

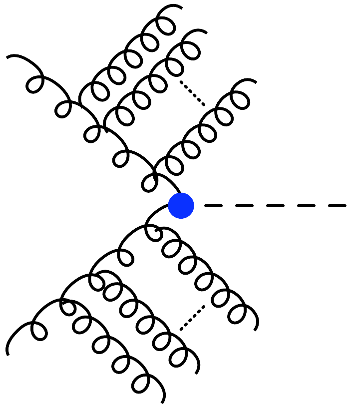
- Low pT region dominated by soft and collinear emissions from initial state:



pT=0



finite pT



finite pT

- Soft and Collinear emissions dominate the low pT distribution:

$$p_n \sim m_h(\eta^2, 1, \eta), \quad p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \quad p_s \sim m_h(\eta, \eta, \eta),$$

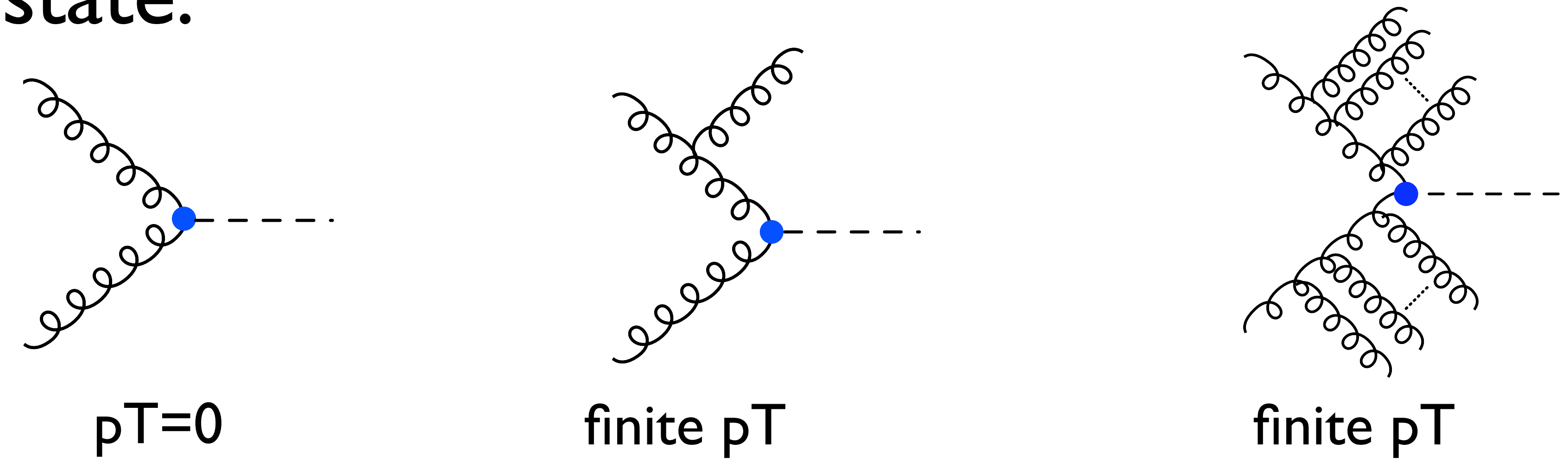
$$\eta \sim \frac{p_T}{m_h}$$

- Hierarchy of scales suggests EFT approach with well defined power counting.

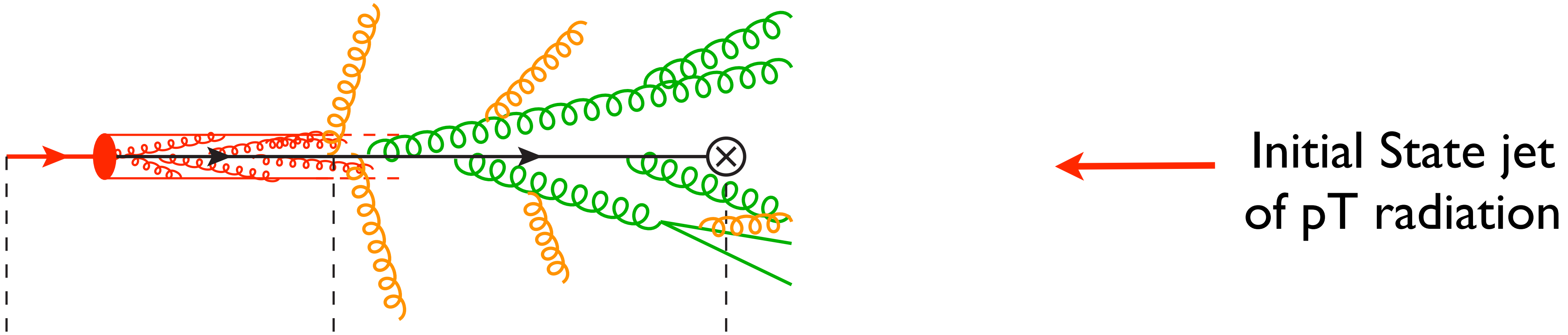
$$m_h \gg p_T \gg \Lambda_{QCD}\,, \qquad p_T \sim \Lambda_{QCD}$$

EFT framework

- Low pT region dominated by soft and collinear emissions from initial state:



- Colliding parton is part of initial state pT radiation beam jet:

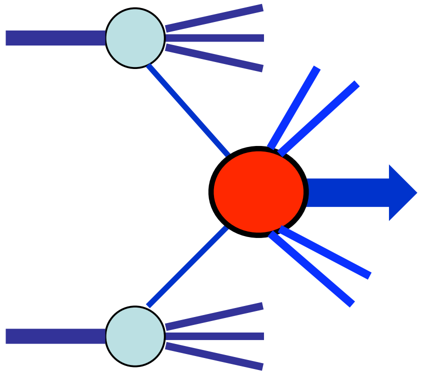


- Gives rise to impact-parameter Beam Functions (iBFs). (SM,Petriello) Analogous beam functions arise in other processes:

(Stewart, Tackmann, Waalewijin; Fleming, Leibovich, Mehen)

- Soft recoil radiation is restricted. Gives rise to a soft function.

EFT framework



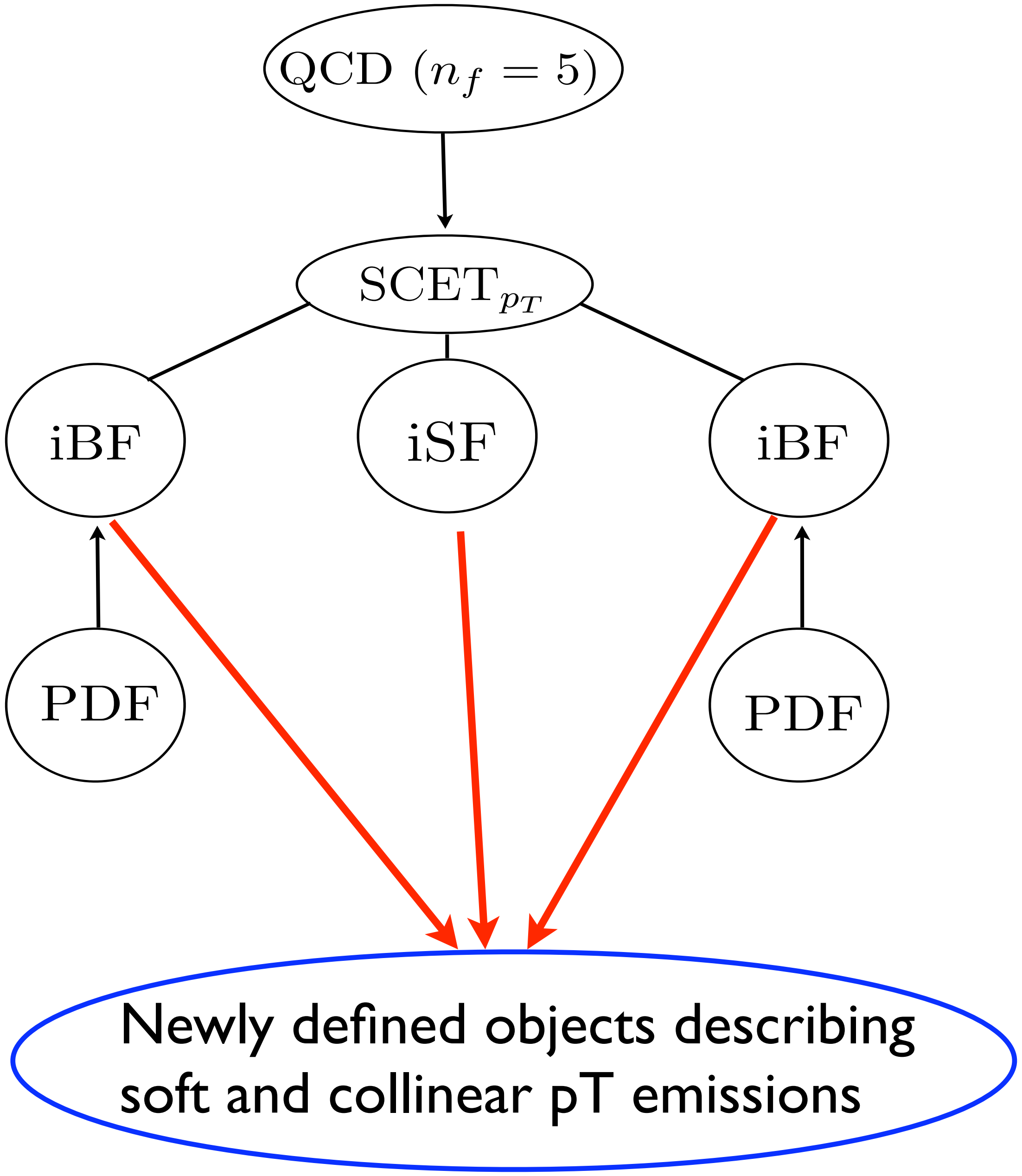
$$\text{QCD}(n_f = 6) \to \text{QCD}(n_f = 5) \to \text{SCET}_{p_T} \to \text{SCET}_{\Lambda_{QCD}}$$

Top quark integrated out.

Matched onto SCET.

Soft-collinear factorization.

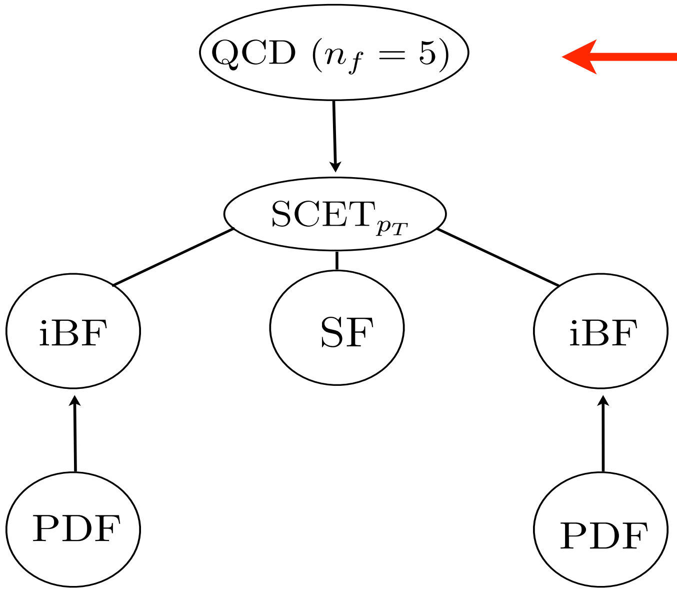
Matching onto PDFs.



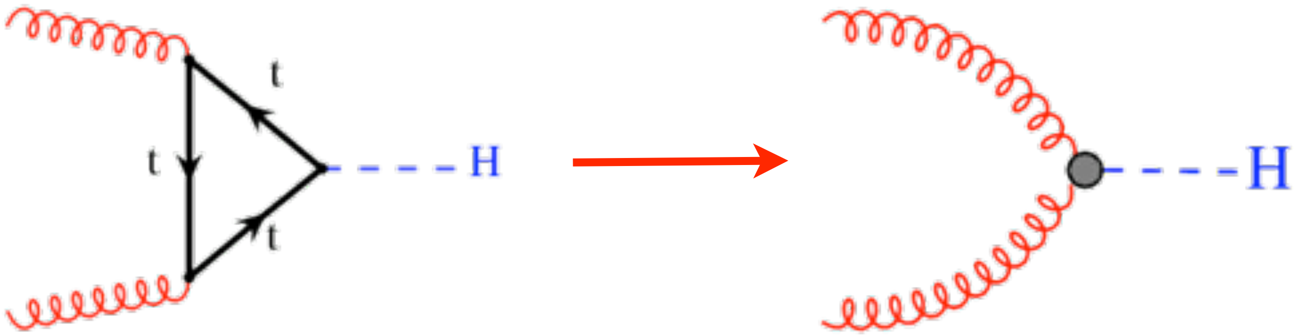
Newly defined objects describing soft and collinear pT emissions

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Integrating out the top



We are here



- Effective Higgs production operator

$$\mathcal{L}_{m_t} = C_{GGh} \frac{h}{v} G^a_{\mu\nu} G^{\mu\nu}_a \quad , \qquad C_{GGh} = \frac{\alpha_s}{12\pi} \left\{ 1 + \frac{11}{4} \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

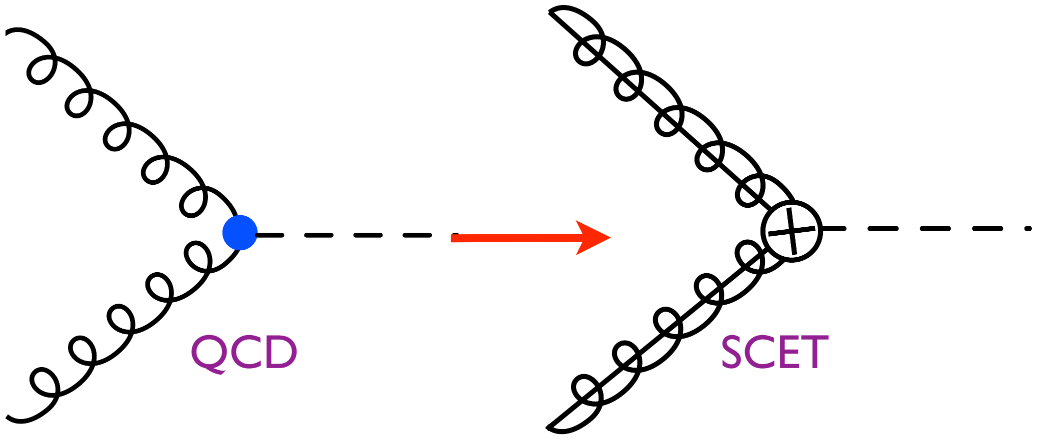
Two loop result for
Wilson coefficient.

(Chetyrkin, Kniehl, Kuhn, Schroder, Steinhauser, Sturm)

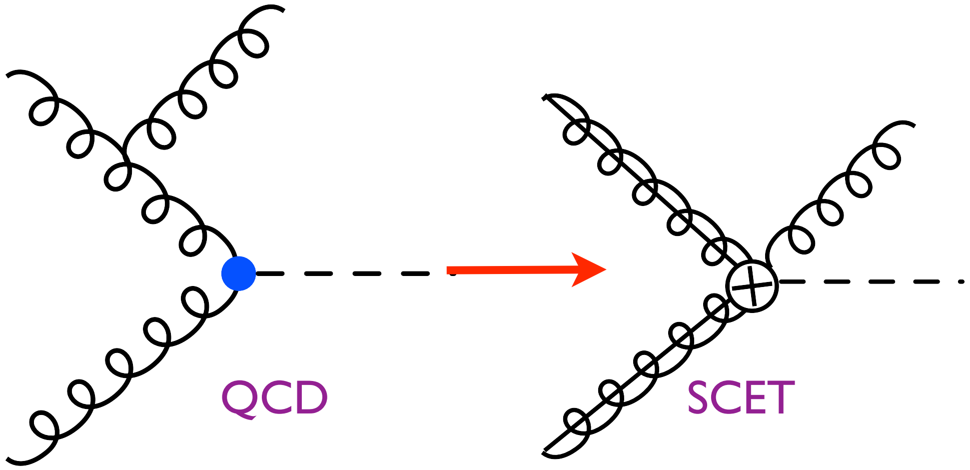
Matching onto SCET

- Matching equation:

$$O_{QCD} = \int d\omega_1 \int d\omega_2 \, C(\omega_1,\omega_2)\, \mathcal{O}(\omega_1,\omega_2)$$

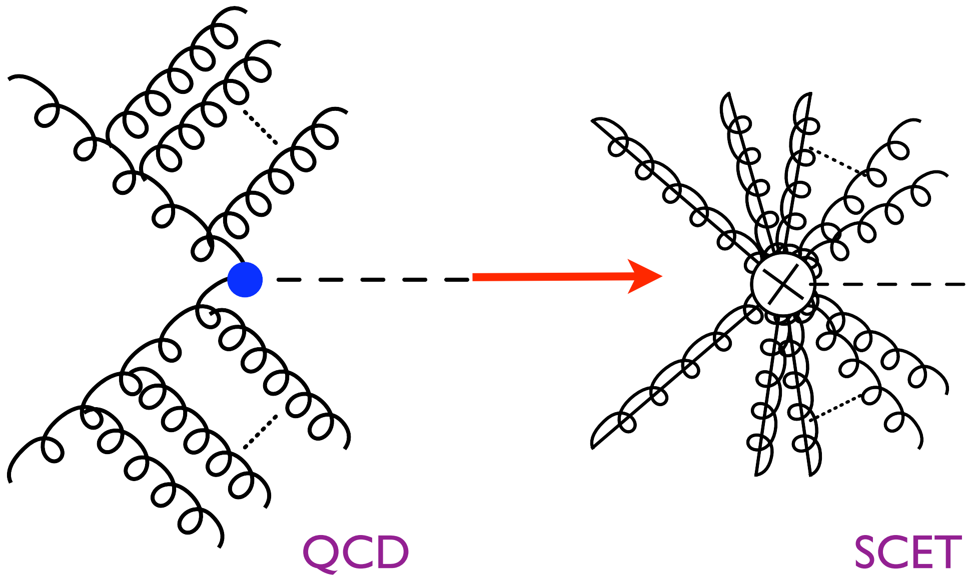


Tree level matching



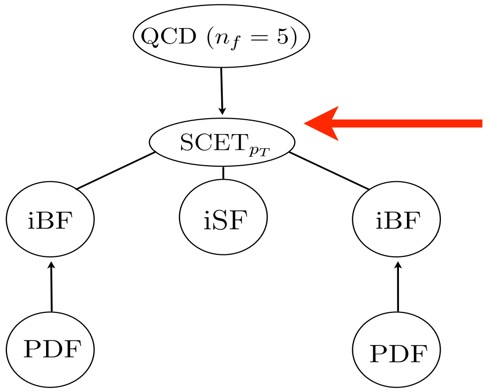
Matching real emission graphs

Soft and Collinear emissions build into Wilson lines determined by soft and collinear gauge invariance of SCET.



- Effective SCET operator:

$$\mathcal{O}(\omega_1,\omega_2) = g_{\mu\nu} h \, T\{\mathrm{Tr}\Big[S_n (gB^{\mu}_{n\perp})_{\omega_1} S_n^\dagger S_{\bar{n}} (gB^{\nu}_{\bar{n}\perp})_{\omega_2} S_{\bar{n}}^\dagger\Big]\}$$



We are here

SCET Cross-Section

- SCET differential cross-section:

$$\frac{d^2\sigma}{du\,dt} = \frac{1}{2Q^2}\left[\frac{1}{4}\right]\int\frac{d^2p_{h_\perp}}{(2\pi)^2}\int\frac{dn\cdot p_h d\bar{n}\cdot p_h}{2(2\pi)^2}(2\pi)\theta(n\cdot p_h+\bar{n}\cdot p_h)\delta(n\cdot p_h\bar{n}\cdot p_h-\vec{p}^{\,2}_{h_\perp}-m_h^2)$$
$$\times\ \delta(u-(p_2-p_h)^2)\delta(t-(p_1-p_h)^2)\sum_{\text{initial pols.}}\sum_X \left|C(\omega_1,\omega_2)\otimes\langle hX_nX_{\bar{n}}X_s|\mathcal{O}(\omega_1,\omega_2)|pp\rangle\right|^2$$
$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-P_{X_n}-P_{X_{\bar{n}}}-P_{X_s}-p_h),$$

- Schematic form of SCET cross-section:

$$\frac{d^2\sigma}{dp_T^2\,dY}\sim\int PS\,|C\otimes\langle\mathcal{O}\rangle|^2$$

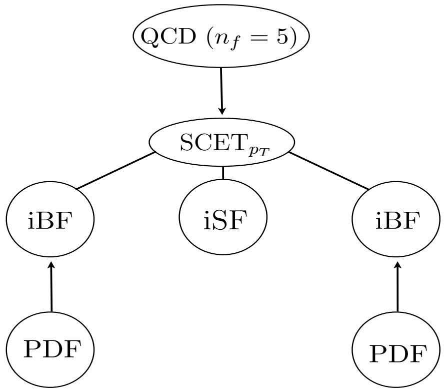
Phase space integrals.

Hard matching coefficient.

SCET matrix element.

Apply soft-collinear decoupling

SCET Cross-Section



We are here

$$\frac{d^2\sigma}{dp_T^2\,dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

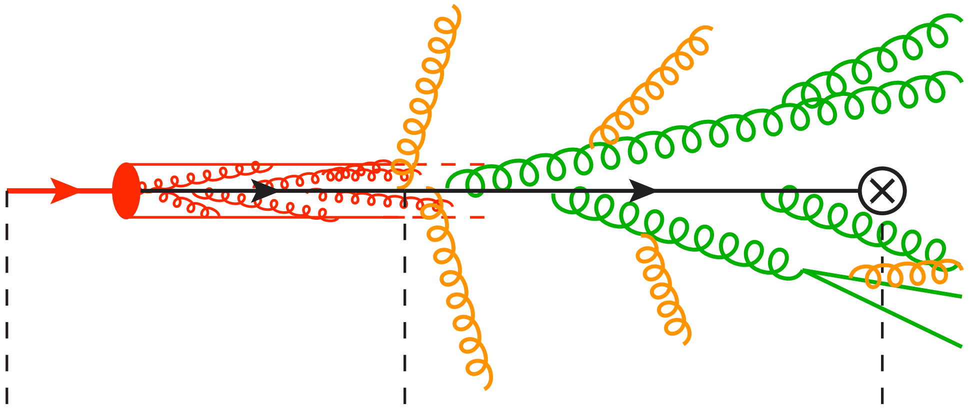
Hard function

Impact-parameter Beam Functions (iBFs)

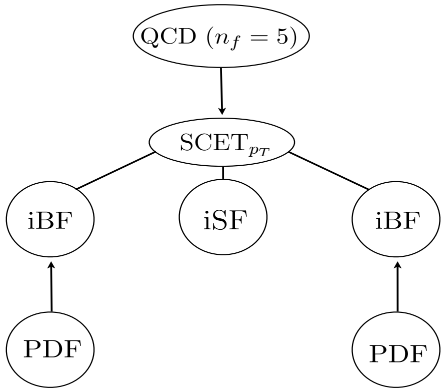
Soft function

Describes collinear pT emissions

Describes soft pT emissions



SCET Cross-Section



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Hard function

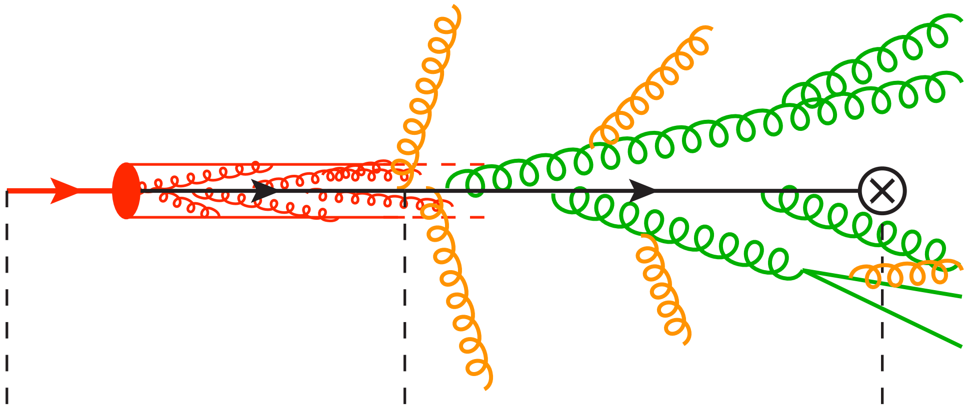
Impact-parameter Beam Functions (iBFs)

Soft function

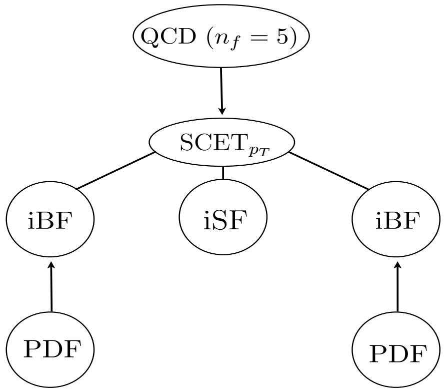
Describes collinear pT emissions

Describes soft pT emissions

Unintegrated nucleon distribution functions



SCET Cross-Section



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Hard function

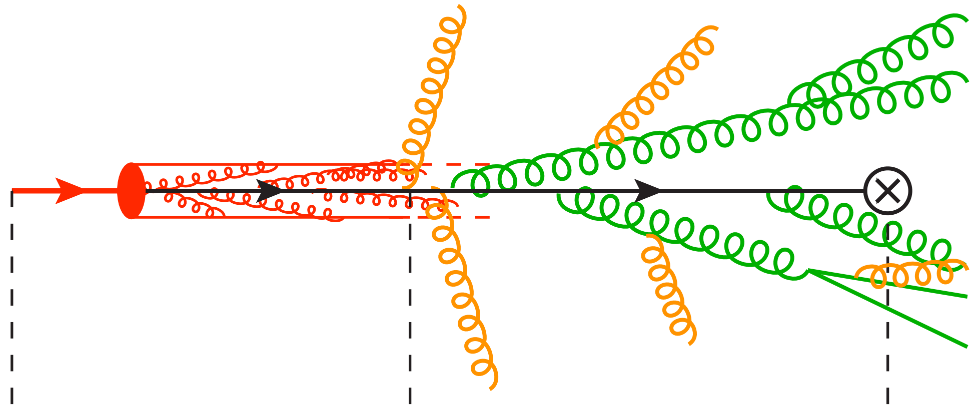
Impact-parameter Beam Functions (iBFs)

Soft function

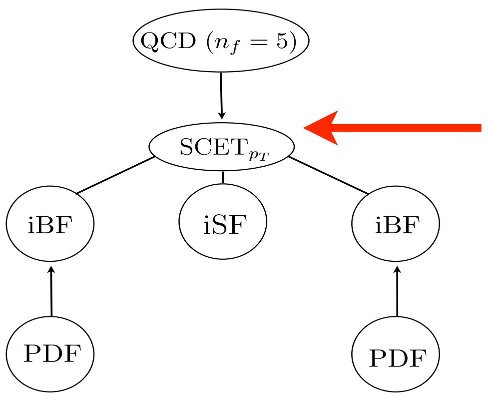
Describes collinear pT emissions

Describes soft pT emissions

Presence of soft function. Plays an important role in the structure of factorization.



SCET Cross-Section



We are here

- Formula in detail:

$$\frac{d^2\sigma}{du\,dt} = \frac{(2\pi)}{(N_c^2-1)^2 8Q^2}\int dp_h^+ dp_h^- \int d^2k_h^\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{k}_h^\perp\cdot\vec{b}_\perp}$$

$$\times\,\delta\left[u-m_h^2+Qp_h^-\right]\delta\left[t-m_h^2+Qp_h^+\right]\delta\left[p_h^+p_h^- - \vec{k}_{h\perp}^2 - m_h^2\right]\int d\omega_1 d\omega_2 \overbrace{|C(\omega_1,\omega_2,\mu)|^2}^{\text{Hard}}$$

$$\times \int dk_n^+ dk_{\bar n}^- \underbrace{B_n^{\alpha\beta}(\omega_1,k_n^+,b_\perp,\mu)}_{\substack{\text{n-collinear}\\ \text{iBF}}} \underbrace{B_{\bar n\alpha\beta}(\omega_2,k_{\bar n}^-,b_\perp,\mu)}_{\substack{\text{bn-collinear}\\ \text{iBF}}} \underbrace{\mathcal{S}(\omega_1-p_h^- -k_{\bar n}^-,\omega_2-p_h^+-k_n^+,b_\perp,\mu)}_{\text{Soft}}$$

- iBFs and soft functions field-theoretically defined as the fourier transform of:

$$J_n^{\alpha\beta}(\omega_1,x^-,x_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_1|\left[gB^A_{1n\perp\beta}(x^-,x_\perp)\delta(\bar{\mathcal{P}}-\omega_1)gB^A_{1n\perp\alpha}(0)\right]|p_1\rangle$$

$$J_{\bar n}^{\alpha\beta}(\omega_1,y^+,y_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_2|\left[gB^A_{1n\perp\beta}(y^+,y_\perp)\delta(\bar{\mathcal{P}}-\omega_2)gB^A_{1n\perp\alpha}(0)\right]|p_2\rangle$$

$$S(z,\mu) = \langle 0|\bar{T}\left[\mathrm{Tr}\left(S_{\bar n}T^D S_{\bar n}^\dagger S_n T^C S_n^\dagger\right)(z)\right] T\left[\mathrm{Tr}\left(S_n T^C S_n^\dagger S_{\bar n}T^D S_{\bar n}^\dagger\right)(0)\right]|0\rangle.$$

Equivalence of Zero-Bin & Soft Subtractions

- Zero-bin iBF reproduces soft graphs. This is the equivalence of zero-bin and soft subtractions in SCET. (Stewart, Hoang; Lee, Sterman; Idilbi, Mehen; Chiu, Fuhrer, Kelly, Hoang, Manohar;...)

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Zero-bin Subtraction in order to avoid double counting the soft region.

(Manohar, Stewart)

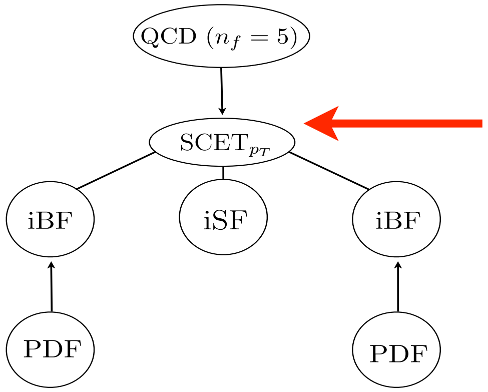
$$B^{\alpha\beta}_{n,\bar{n}}(\omega, k^{\pm}, b_\perp, \mu) = \tilde{B}^{\alpha\beta}_{n,\bar{n}}(\omega, k^{\pm}, b_\perp, \mu) - B^{\alpha\beta}_{\{n0,\bar{n}0\}}(\omega, k^{\pm}, b_\perp, \mu)$$

Purely Collinear iBF

"Naive" iBF

Zero-bin iBF
Equivalent to soft graphs

Factorization in SCET



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Hard function

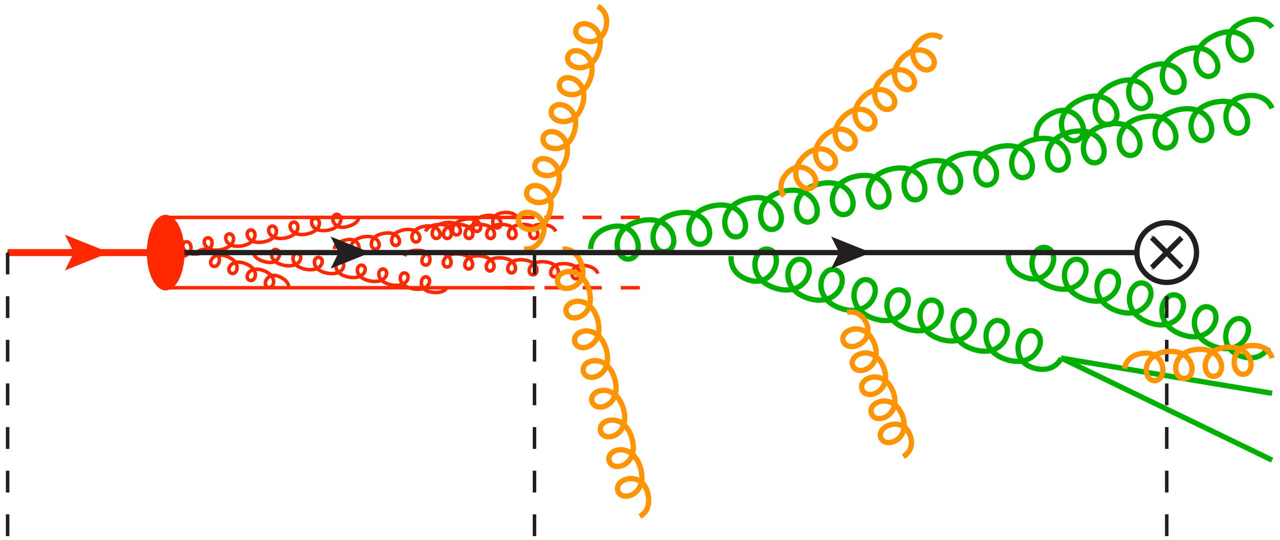
Physics of hard scale.
Sums logs of mh/pT.

Impact-parameter Beam Functions (iBFs)

Describes collinear pT emissions

Inverse soft function (iSF)

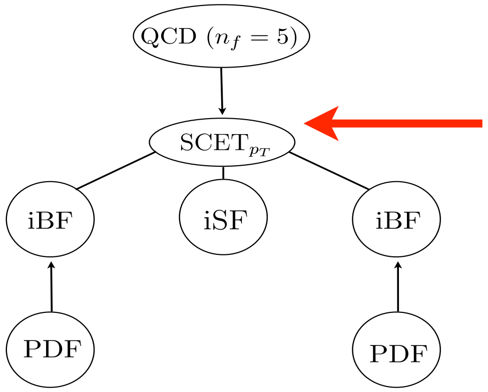
Describes soft pT emissions



Analogous factors and soft-subtractions also appear in TMD-factorization formalism: Drell-Yan, SIDIS

(J.C.Collins, F. Hautmann; X.-d.Ji, J.P.Ma, F.Yuan; Belitsky; Aybat, Rogers,...)

Factorization in SCET



We are here

$$\frac{d^2\sigma}{du\,dt} = \frac{(2\pi)}{(N_c^2-1)^2 8Q^2}\int dn\cdot p_h \int d\bar{n}\cdot p_h \int d^2k_h^\perp \int dk_n^+ d^2k_n^\perp \int dk_{\bar{n}}^- d^2k_{\bar{n}}^\perp \int d^4k_s$$

$$\times \int \frac{dx^- d^2x_\perp}{2(2\pi)^3}\int \frac{dy^- d^2y_\perp}{2(2\pi)^3}\int \frac{d^4z}{(2\pi)^4} e^{\frac{i}{2}k_n^+ x^- - i\vec{k}_n^\perp\cdot x_\perp} e^{\frac{i}{2}k_{\bar{n}}^- y^+ - i\vec{k}_{\bar{n}}^\perp\cdot y_\perp} e^{ik_s\cdot z}$$

$$\times\ \delta\left(u - m_h^2 + Q\bar{n}\cdot p_h\right)\delta\left(t - m_h^2 + Qn\cdot p_h\right)\delta\left(\bar{n}\cdot p_h n\cdot p_h - \vec{k}_{h\perp}^2 - m_h^2\right)$$

$$\times \int d\omega_1 d\omega_2 |C(\omega_1,\omega_2,\mu)|^2 J_n^{\alpha\beta}(\omega_1, x^-, x_\perp, \mu)\, J_{\bar{n}\alpha\beta}(\omega_2, y^+, y_\perp, \mu)\, S(z,\mu)$$

$$\times\ \delta\left(\omega_1 - \bar{n}\cdot p_h - k_{\bar{n}}^- - k_s^-\right)\delta(\omega_2 - p_h^+ - k_n^+ - k_s^+)\delta^{(2)}(k_s^\perp + k_n^\perp + k_{\bar{n}}^\perp + k_h^\perp),$$

Residual light-cone momenta
regulate spurious rapidity
divergences.

- iBFs and iSF are regulated by kinematics of the process and free of rapidity divergences.

- iBFs are fully unintegrated nucleon distributions instead of TMD pdfs.

Comparison with TMD factorization

- SCET formula with Impact Parameter Beam functions:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

- SCET version of formula with TMDPDFs

(Gao, Li, Liu)

$$\frac{d^2\sigma}{du\,dt} = \sum_{qijKL} \frac{\pi F^{KL;q}}{4Q^4 N_c^2} \int d^2k_\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \delta\left[\omega_u \omega_t - \vec{k}_\perp^2 - M_z^2\right] H_Z^{KL;ijq}(\omega_u, \omega_t, \mu_Q; \mu_T)$$
$$\times\ J_n^q(\omega_u, 0, b_\perp, \mu_T) J_{\bar{n}}^{\bar{q}}(\omega_t, 0, b_\perp, \mu_T) S_{qq}(0, 0, b_\perp, \mu_T)$$

← Soft function

- Field-theoretic operator definitions for TMDPDFs and Soft function exist also in the traditional TMD formulation:

(Collins; Aybat, Rogers)

TMDPDF:

$$\tilde{F}_{f/P}^{\mathrm{unsub}}(x, \mathbf{b}_T; \mu; y_P - y_B)$$
$$= \mathrm{Tr}_C \int \frac{dw^-}{2\pi} e^{-ixP^+w^-} \langle P|\bar{\psi}_f(w/2) W(w/2, \infty, n_B)^\dagger \frac{\gamma^+}{2} W(-w/2, \infty, n_B) \psi_f(-w/2)|P\rangle_{c,\text{No S.I.}}.$$

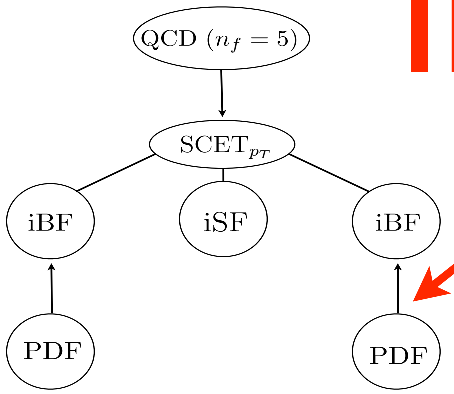
Soft function:

$$\tilde{S}_{(0)}(\mathbf{b}_T; y_A, y_B) = \frac{1}{N_c} \langle 0| W(\mathbf{b_T}/2, \infty; n_B)^\dagger_{ca}\, W(\mathbf{b}_T/2, \infty; n_A)_{ad} W(-\mathbf{b_T}/2, \infty; n_B)_{bc} W(-\mathbf{b_T}/2, \infty; n_A)^\dagger_{db} |0\rangle_{\text{No S.I.}}.$$

(Soft function and operator definitions absent in Becher-Neubert formula)

Perturbative pT

Integrating Out the pT Scale



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

iBFs are proton matrix elements
and sensitive to the
non-perturbative scale

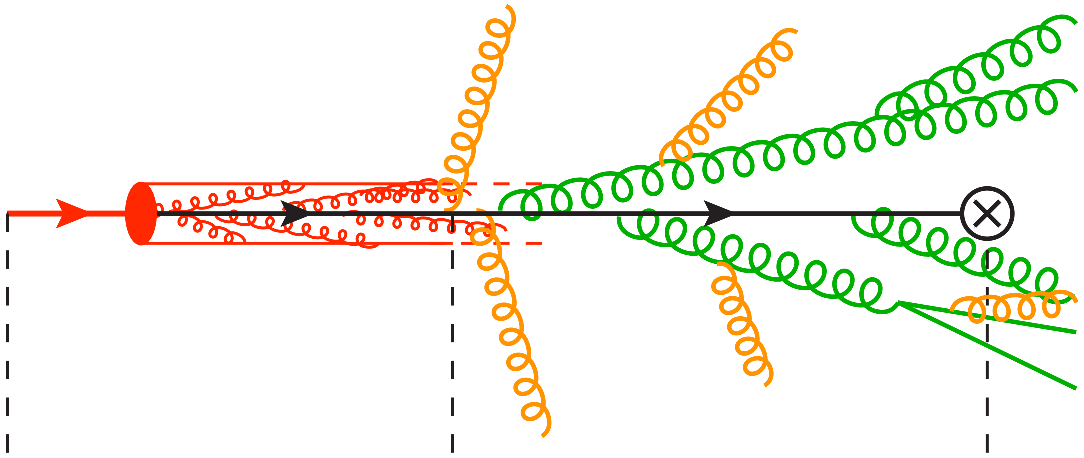
- The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:

$$\tilde{B}_n = \mathcal{I}_{n,i} \otimes f_i ,$$

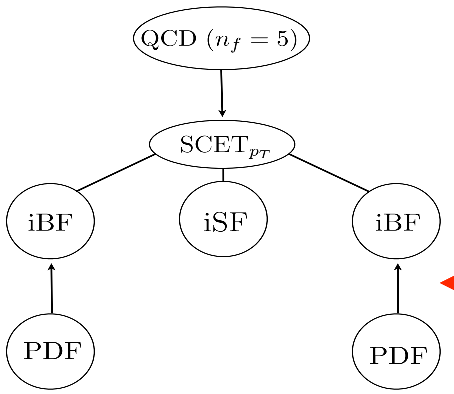
iBF

Matching coefficient

PDF



iBFs to PDFs



We are here

- iBF is matched onto the PDF with matching coefficient defined as:

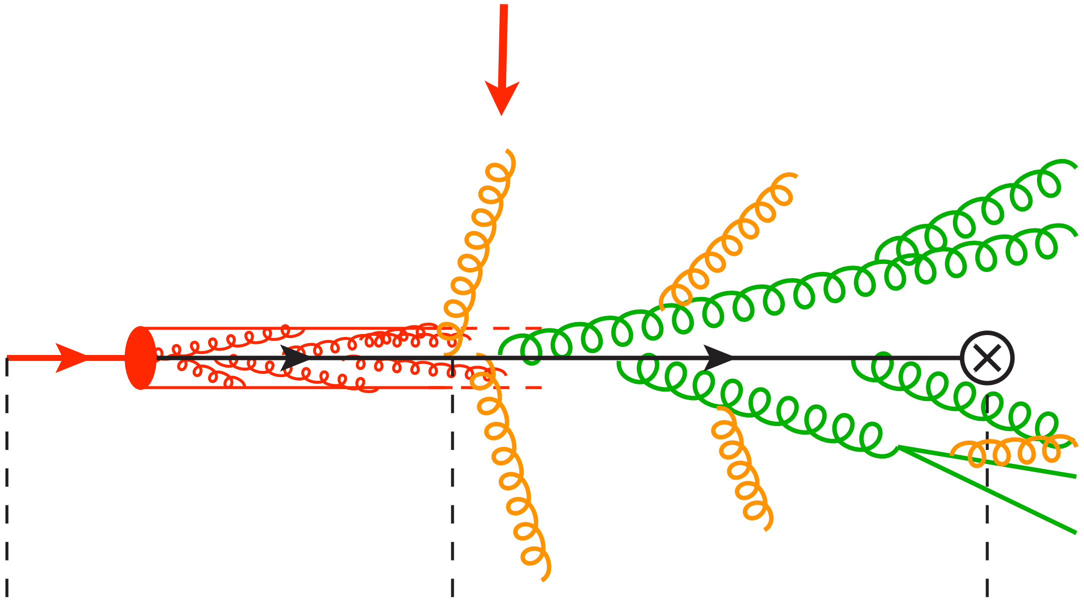
$$\tilde{B}_n^{\alpha\beta}(z, t_n^+, b_\perp, \mu) = -\frac{1}{z}\sum_{i=g,q,\bar{q}}\int_z^1 \frac{dz'}{z'}\mathcal{I}_{n;g,i}^{\alpha\beta}(\frac{z}{z'}, t_n^+, b_\perp, \mu) f_{i/P}(z', \mu)$$

- Tree level matching

$$\mathcal{I}_{n;g,i}^{(0)\beta\alpha}(\frac{z}{z'}, t_n^+, b_\perp, \mu) = g^2 g_\perp^{\alpha\beta}\delta(t_n^+)\delta(1-\frac{z}{z'})$$

- Finite part of iBF in dim-reg gives matching coefficient at higher orders.

Proton fragments into pT radiation beam jet



Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\,dY} = \frac{\pi^2}{4(N_c^2-1)^2Q^2}\int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\int_{x_1}^1\frac{dx_1'}{x_1'}\int_{x_2}^1\frac{dx_2'}{x_2'}$$

$$\times\, H(x_1,x_2,\mu_Q;\mu_T)\,\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T)\,f_{i/P}(x_1',\mu_T)\,f_{j/P}(x_2',\mu_T)$$

Hard function.

Transverse momentum function.

PDFs.

- The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) = \int dt_n^+\int dt_{\bar{n}}^-\int\frac{d^2b_\perp}{(2\pi)^2}J_0(|\vec{b}_\perp|p_T)$$

Collinear pT emissions →

$$\times\,\mathcal{I}^{\beta\alpha}_{n;g,i}\left(\frac{x_1}{x_1'},t_n^+,b_\perp,\mu_T\right)\mathcal{I}^{\beta\alpha}_{\bar{n};g,j}\left(\frac{x_2}{x_2'},t_{\bar{n}}^-,b_\perp,\mu_T\right)$$

Soft pT emissions →

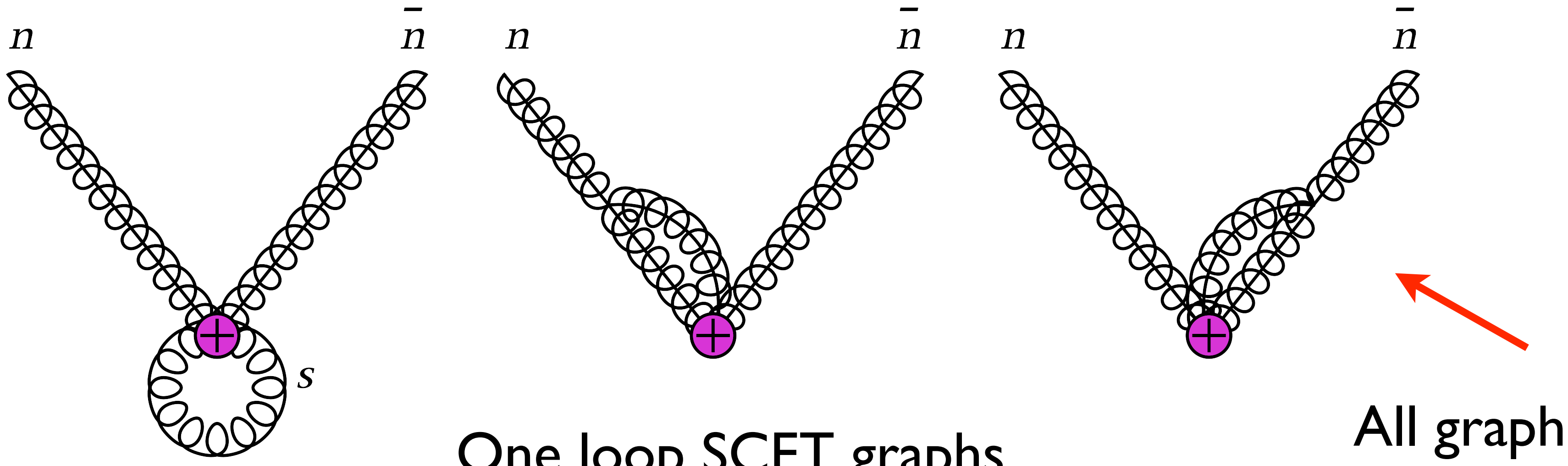
$$\times\,\mathcal{S}^{-1}\left(x_1Q-e^{Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_{\bar{n}}^-}{Q},\,x_2Q-e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_n^+}{Q},\,b_\perp,\mu_T\right)$$

Fixed order and Matching Calculations

Hard Function

$$\frac{d^2\sigma}{dp_T^2 dY} \sim \boxed{H} \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

$$O_{QCD} = \int d\omega_1 \int d\omega_2 \boxed{C(\omega_1,\omega_2)} \mathcal{O}(\omega_1,\omega_2)$$



One loop SCET graphs

All graphs scaless and vanish in dimensional regularization.

- NLO hard Wilson coefficient:

$$C(\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2,\mu) = \frac{c\,\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{v}\left\{1+\frac{\alpha_s}{4\pi}C_A\left[\frac{11}{2}+\frac{\pi^2}{6}-\ln^2\left(-\frac{\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{\mu^2}\right)\right]\right\}$$

- NNLO results known.

(Harlander,Kilgore;Anastasiou,Melnikov;Ravindran,Smith,Van Neerven;Ahrens, Becher, Neubert,Yang;)

Impact Parameter Beam Function (iBF)

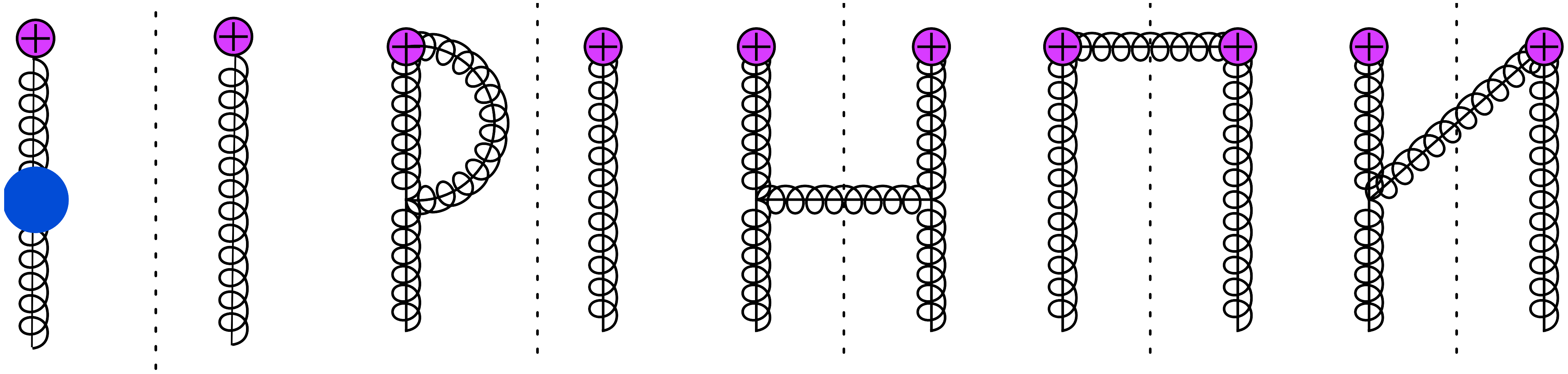
$$\frac{d^2\sigma}{dp_T^2\,dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

"unintegrated Beam Function"
(perturbative region)

"unintegrated PDF"
(non-perturbative region)

- Field-theoretic operator definition:

$$\tilde{B}_n^{\alpha\beta}(x_1, t_n^+, b_\perp, \mu) \;=\; \int \frac{db^-}{4\pi} e^{\frac{i}{2}\frac{t_n^+ b^-}{Q}} \sum_{\text{initial pols.}} \sum_{X_n} \langle p_1 | \big[g B^A_{1n\perp\beta}(b^-, b_\perp) | X_n \rangle \\ \times \; \langle X_n | \delta(\bar{\mathcal{P}} - x_1 \bar{n}\cdot p_1) g B^A_{1n\perp\alpha}(0) \big] | p_1 \rangle,$$



$+ \quad c.c$

- NLO result known. (SM, Petriello)

Inverse Soft function (iSF)

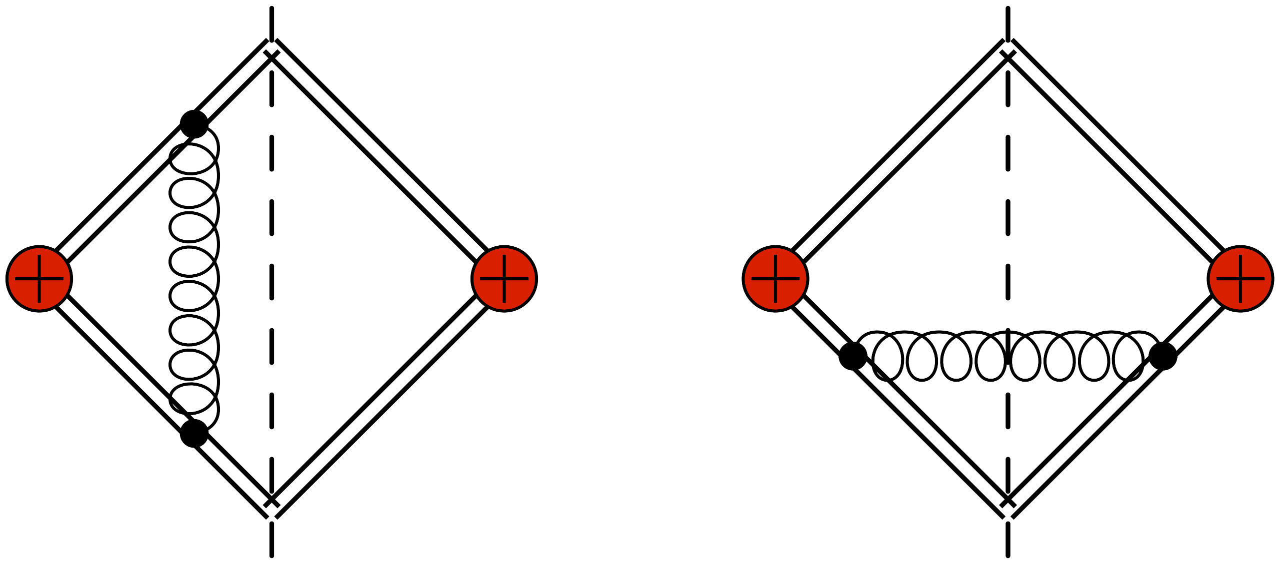
$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes \boxed{S^{-1}}$$

- Field-theoretic operator definition in position space:

$$S(b,\mu) = \sum_{X_s} \langle 0|\bar{T}\left[\mathrm{Tr}\left(S_{\bar{n}} T^D Y_{\bar{n}}^\dagger S_n T^C S_n^\dagger\right)(b)\right]|X_s\rangle\langle X_s|T\left[\mathrm{Tr}\left(S_n T^C S_n^\dagger S_{\bar{n}} T^D S_{\bar{n}}^\dagger\right)(0)\right]|0\rangle$$

- iSF is defined as

$$\mathcal{S}^{-1}(\tilde{\omega}_1, \tilde{\omega}_2, b_\perp, \mu) = \int \frac{db^+ db^-}{16\pi^2} e^{ib^+\tilde{\omega}_1/2} e^{ib^-\tilde{\omega}_2/2} S^{-1}(b^+, b^-, b_\perp)$$



- NLO result known. (SM, Petriello)

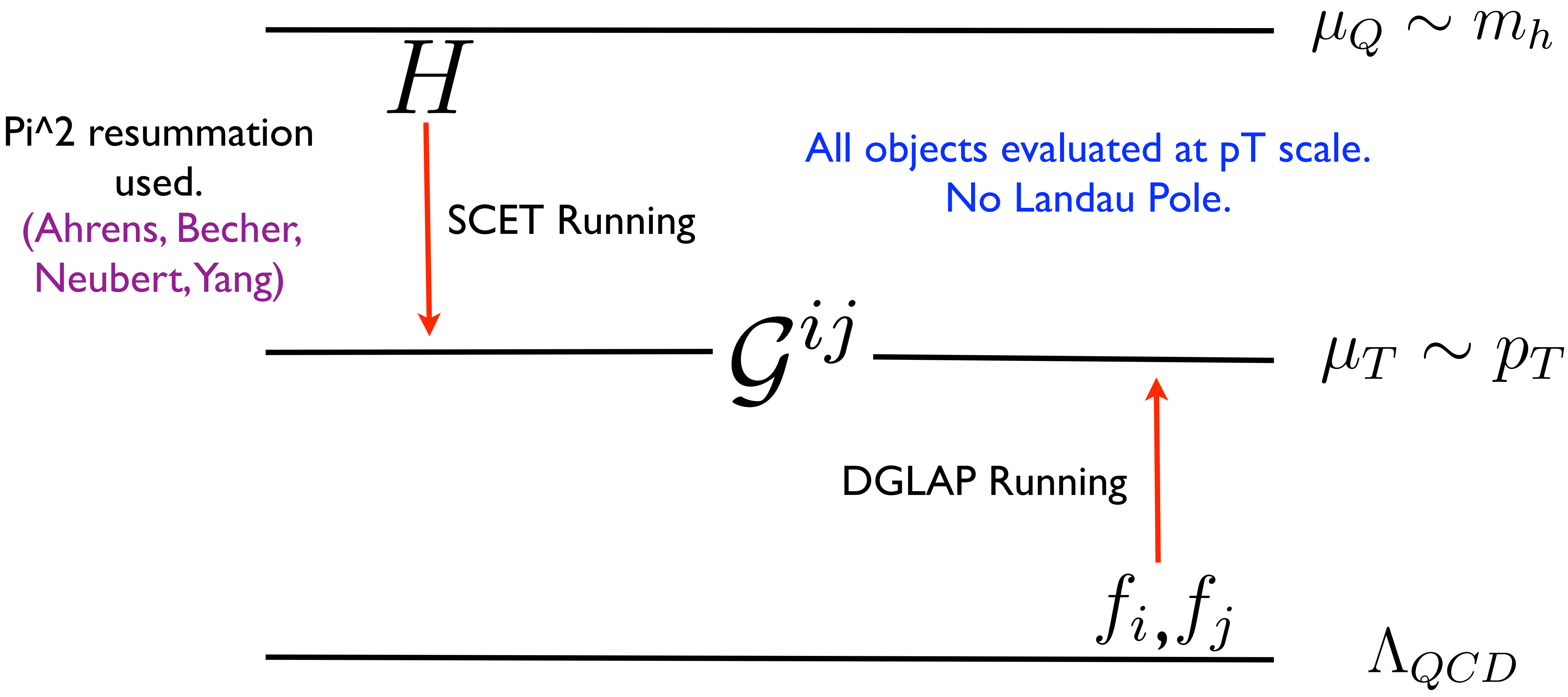
Running

Running

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Schematic picture of running:



$\mu_Q \sim m_h$

H

Pi^2 resummation used.

(Ahrens, Becher, Neubert, Yang)

SCET Running

All objects evaluated at pT scale.
No Landau Pole.

$\mathcal{G}^{ij}$

$\mu_T \sim p_T$

DGLAP Running

f_i, f_j

Λ_{QCD}

Check of NLL with Fixed Order

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2\,dY} = \frac{4\pi^2}{3}\,\frac{\alpha}{\sin^2\theta_W}\,e^2_{q\bar{q}}\,\frac{1}{s\,p_T^2}\sum_{m,n}\left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m \ln^m\frac{M_Z^2}{p_T^2}$$

$$\text{leading logarithmic} \;:\; \alpha_s^n L^{2n-1},$$

$$\text{next-to-leading logarithmic} \;:\; \alpha_s^n L^{2n-2},$$ (Arnold, Kaufmann; Ellis)

$$\text{next-to-next-to-leading logarithmic} \;:\; \alpha_s^n L^{2n-3}.$$

Checked

LL $\quad {}_1D_1 = A^{(1)} f_A f_B,$

NLL $\quad {}_1D_0 = B^{(1)} f_A f_B + f_B \left(P_{qq}\otimes f\right)_A + f_A \left(P_{qq}\otimes f\right)_B\,,$

LL $\quad {}_2D_3 = -\frac{1}{2}\left[A^{(1)}\right]^2 f_A f_B,$

NLL $\quad {}_2D_2 = -\frac{3}{2}A^{(1)}\left[f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right] f_A f_B,$

Requires two loop iBF and iSF

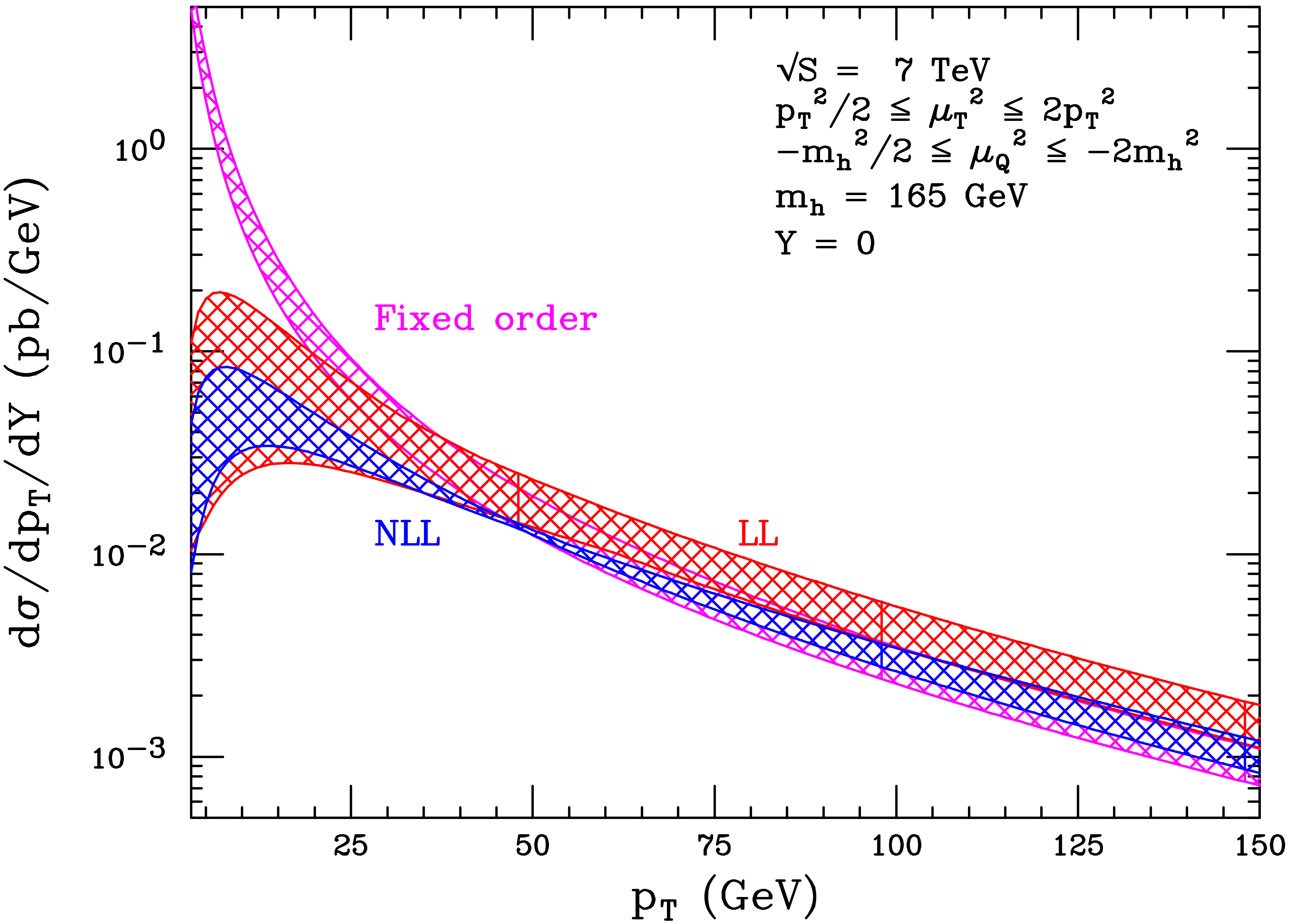
(in progress; Li, SM, Petriello)

NNLL

$$\begin{aligned} {}_2D_1 = &\left\{-A^{(1)} f_B\left(P_{qq}\otimes f\right)_A \ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)} f_B\left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_A f_B\right. \\ &+\frac{\beta_0}{2}A^{(1)} f_A f_B \ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)} f_A f_B - \left(P_{qq}\otimes f\right)_A\left(P_{qq}\otimes f\right)_B \\ &\left. - f_B\left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0\, f_B\left(P_{qq}\otimes f\right)_A\right\} + \left[A\leftrightarrow B\right]. \end{aligned}$$

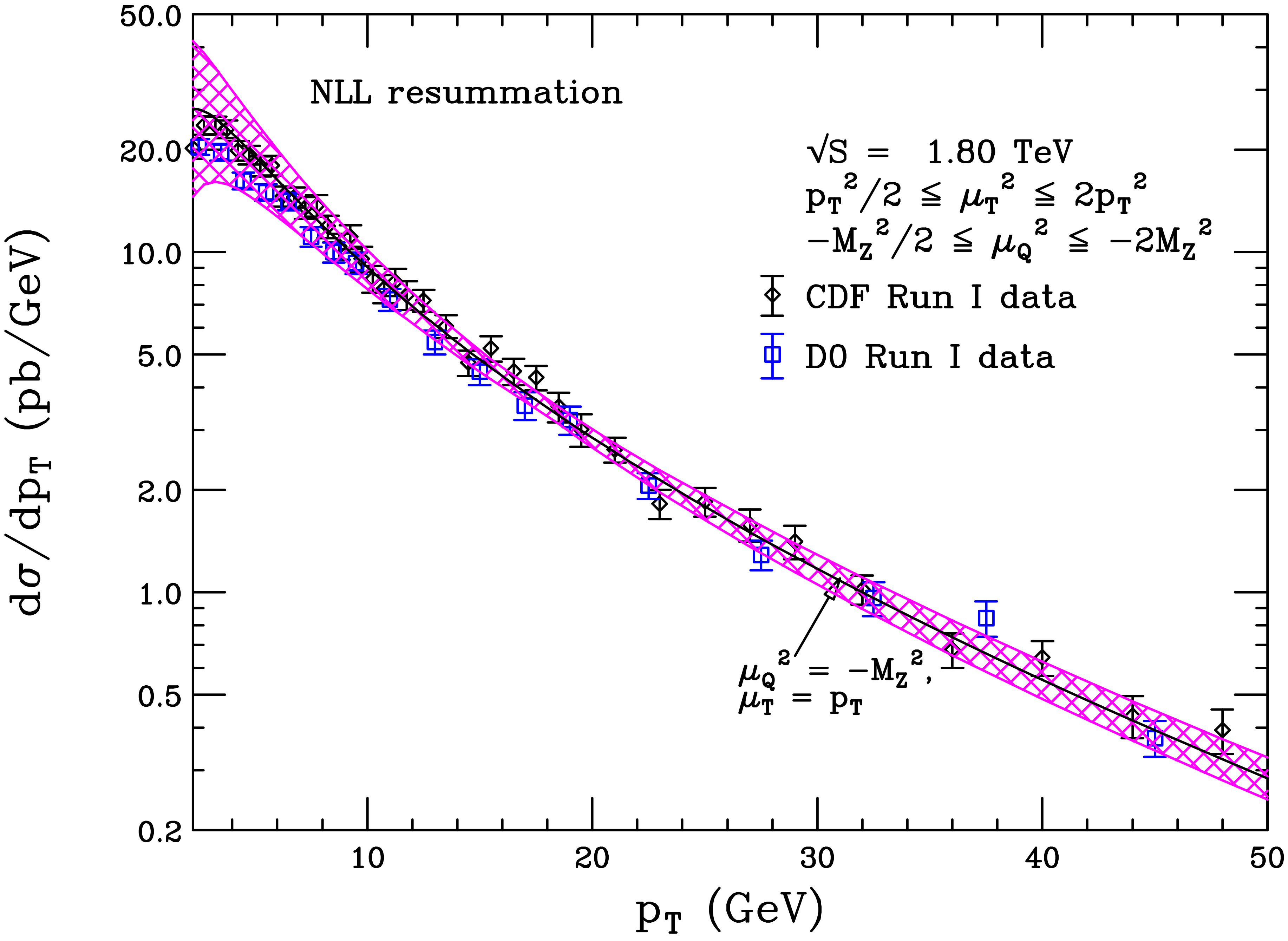
Numerical Results

Higgs pT Distribution



- Prediction for Higgs boson pT distribution.

Z-production: Comparison with Data

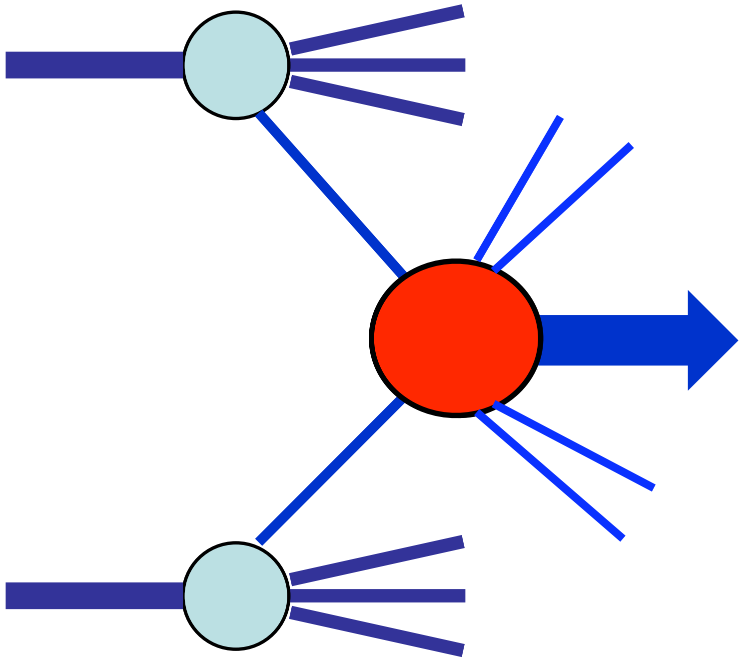


- Good agreement with data.
- Theory curve determined completely by perturbative functions and standard PDFs.

Non-Perturbative pT Region

Non-Perturbative pT Region

- Non-perturbative region of pT:



$$p_T \sim \Lambda_{QCD}$$

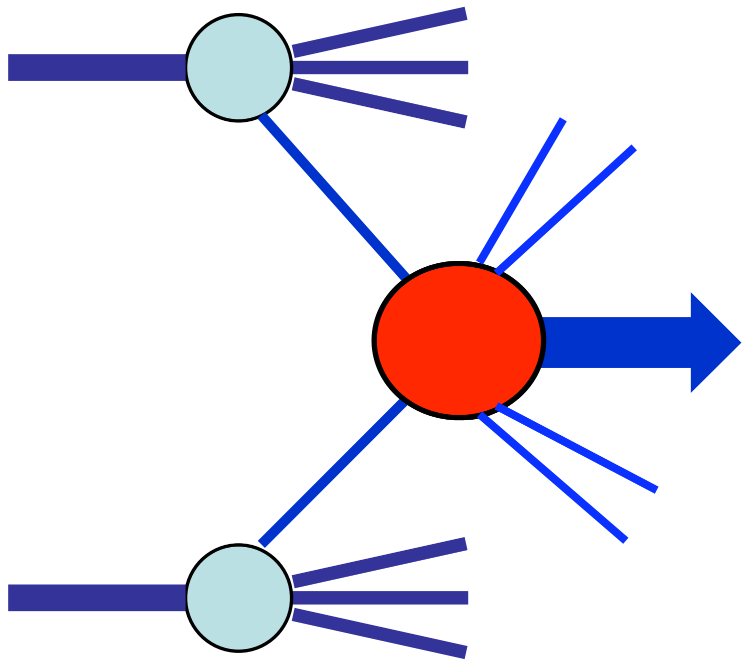
Distribution sensitive to transverse momentum dynamics in nucleon

- iBFs and iSF are non-perturbative:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Non-Perturbative pT Region

- Non-perturbative region of pT:



$$p_T \sim \Lambda_{QCD}$$

Distribution sensitive to transverse momentum dynamics in nucleon

- iBFs and iSF are non-perturbative:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Unintegrated nucleon distribution amplitudes (iBFs)

+

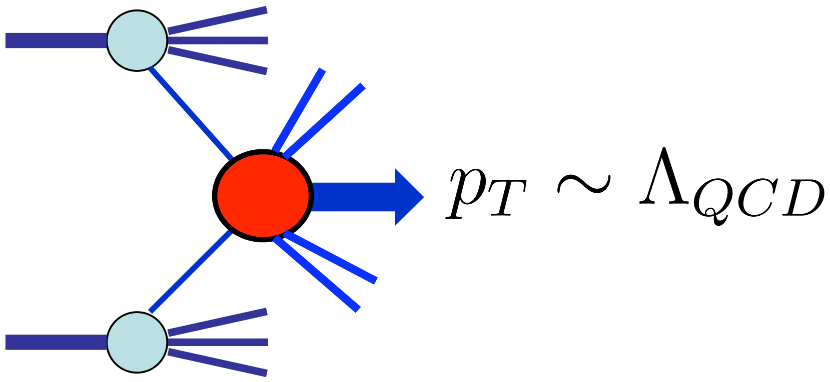
Inverse Soft function (iSF)

- Soft factor can be absorbed into iBFs. This is usually done in the TMDPDF formalism.

Non-Perturbative pT Region

- Non-perturbative region of pT:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$



- In order to smoothly connect non-perturbative and perturbative regions, we still write

$$\tilde{B}_n = \underset{\substack{\downarrow \\ \text{non-perturbative}}}{\mathcal{I}_{n,i}} \otimes f_i, \qquad \tilde{B}_{\bar{n}} = \underset{\substack{\downarrow \\ \text{non-perturbative}}}{\mathcal{I}_{\bar{n},j}} \otimes f_j$$

Non-Perturbative pT Region

- Transverse momentum function (TMF) is now non-perturbative

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Hard function.

Transverse momentum function (TMF).

PDFs.

Can make non-perturbative model

Field theoretically defined object

Scale dependence and running known

Model for Non-Perturbative TMF

$$\frac{d^2\sigma}{dp_T^2\, dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

$$\begin{aligned}\mathcal{G}^{qrs}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T) \; = \; & \int_0^\infty dp_T'\, \mathcal{G}^{qrs}_{\text{part.}}(x_1, x_2, x_1', x_2', p_T\sqrt{1+(p_T'/p_T)^2}, Y, \mu_T) \\ \times \; & G_{mod}(p_T', a, b, \Lambda),\end{aligned}$$

Model function

Partonic function

(Hoang, Ligeti, Stewart, Tackmann)

- Model function:

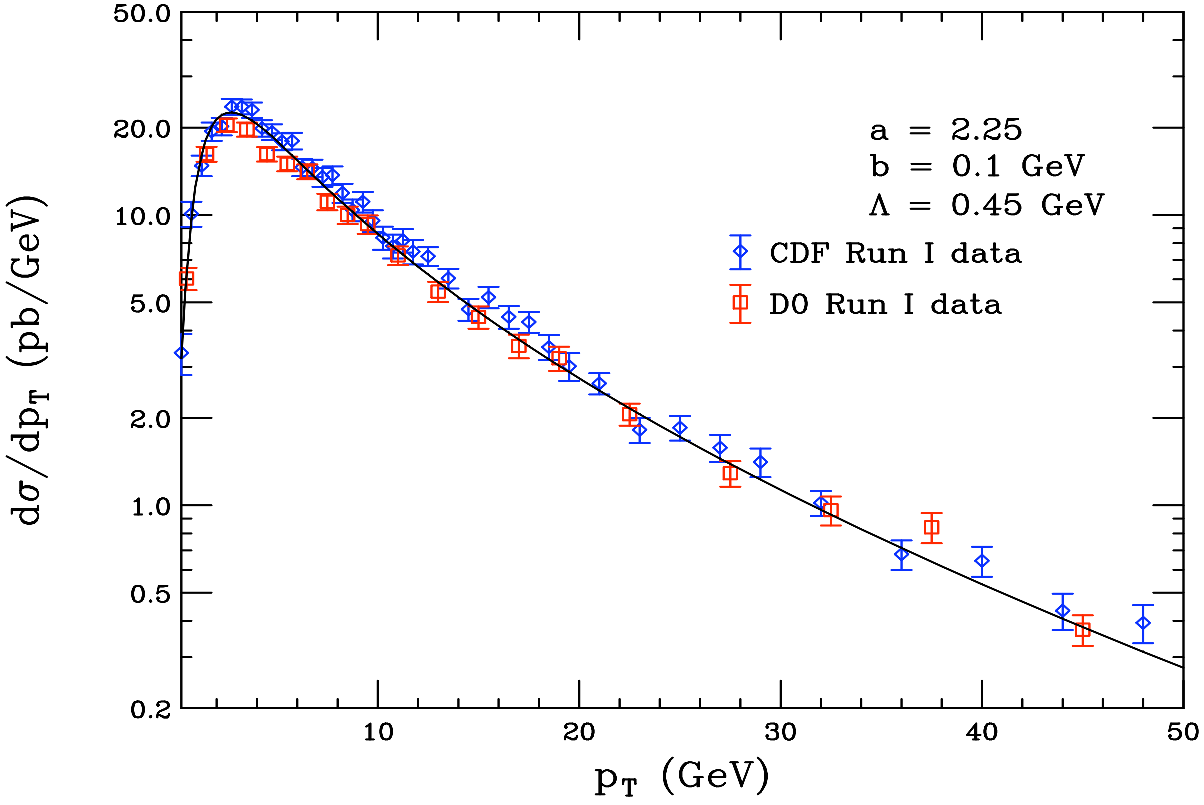
$$G_{mod}(p_T', a, b, \Lambda) \; = \; \frac{N}{\Lambda^2}\left(\frac{p_T'^2}{\Lambda^2}\right)^{a-1} \exp\left[-\frac{(p_T'-b)^2}{2\Lambda^2}\right], \qquad \int_0^\infty dp_T'\, G_{mod}(p_T', a, b, \Lambda) \; = \; 1.$$

- Model reduces to the perturbative result for large pT:

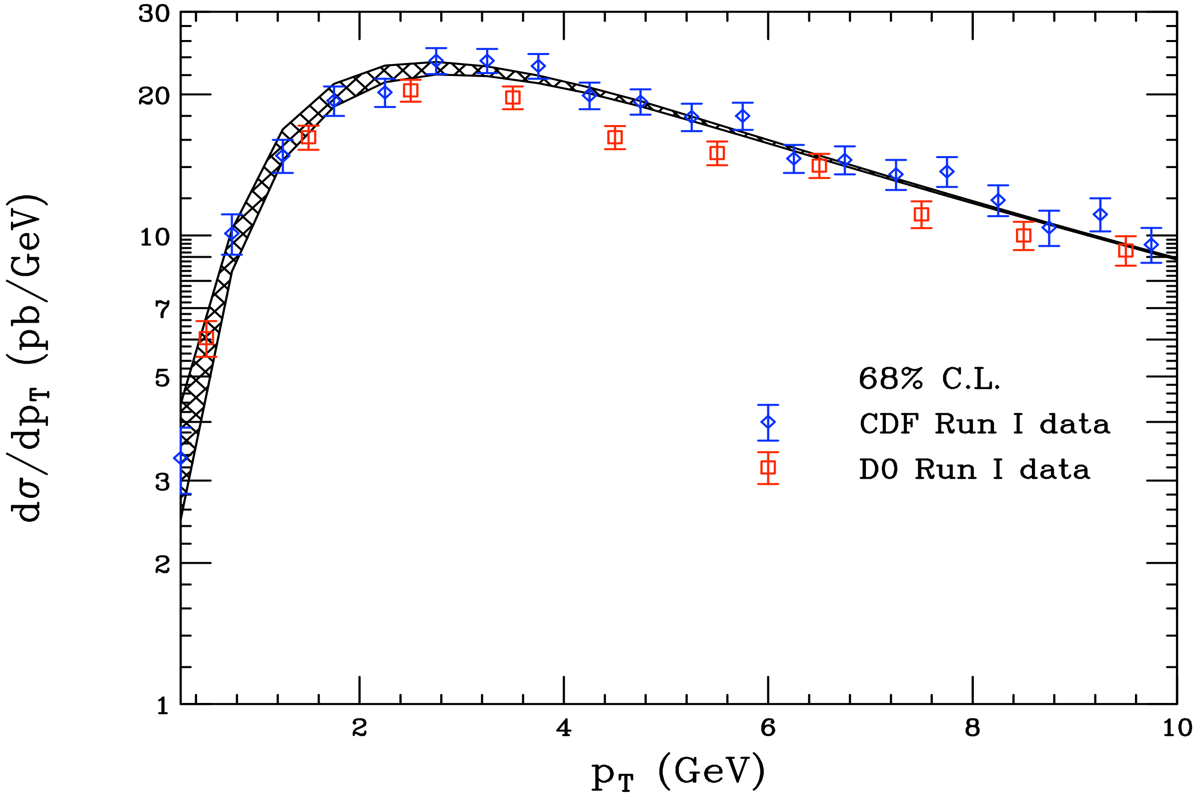
$$\mathcal{G}^{qrs}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T)\Big|_{p_T \gg \Lambda_{QCD}} \; = \; \mathcal{G}^{qrs}_{\text{part.}}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T) + \mathcal{O}(\frac{\Lambda_{QCD}}{p_T}).$$

- Similar to analysis done in CSS with “bmax”.

Including the Non-Perturbative Region



- pT spectrum including the non-perturbative region



- Model dependence restricted only to non-perturbative region as expected.

Summary

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Perturbative pT distribution given in terms of perturbatively calculable functions and the standard PDFs.

- Non-perturbative pT region determined by unintegrated nucleon distributions (iBFs) and inverse soft function (iSF). Interesting objects worth further study; better understand relationship to the TMDPDF formalism

- Smooth transition for spectrum from non-perturbative pT to perturbative pT and large pT.

- Clear and well-defined field theoretic definitions of all objects in the factorization theorem.

Factorization Formula

$$\begin{aligned}\frac{d^2\sigma}{d\mathrm{p}_T^2\,dY} &= \frac{\pi^2}{4(N_c^2-1)^2Q^2}\int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\int_{x_1}^1\frac{dx_1'}{x_1'}\int_{x_2}^1\frac{dx_2'}{x_2'}\\ &\times\ H(x_1,x_2,\mu_Q;\mu_T)\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T)f_{i/P}(x_1',\mu_T)f_{j/P}(x_2',\mu_T)\end{aligned}$$

- One can express the formula entirely in momentum space:

$$\begin{aligned}\mathcal{G}^{ij}(x_1,x_1',x_2,x_2',p_T,Y,\mu_T) &= \frac{1}{2\pi}\int dt_n^+\int dt_{\bar{n}}^-\ \int d^2k_n^\perp\int d^2k_{\bar{n}}^\perp\int d^2k_s^\perp\,\frac{\delta(p_T-|\vec{k}_n^\perp+\vec{k}_{\bar{n}}^\perp+\vec{k}_s^\perp|)}{p_T}\\ &\times\ \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_1}{x_1'},t_n^+,k_n^\perp,\mu_T)\,\mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_2}{x_2'},t_{\bar{n}}^-,k_{\bar{n}}^\perp,\mu_T)\\ &\times\ \mathcal{S}^{-1}(x_1Q-e^Y\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_{\bar{n}}^-}{Q},x_2Q-e^{-Y}\sqrt{\mathrm{p}_T^2+m_h^2}-\frac{t_n^+}{Q},k_s^\perp,\mu_T)\end{aligned}$$

Check of NLL with Fixed Order

$$\begin{aligned}\frac{d^2\sigma_{Z,q\bar q}}{dp_T^2\,dY} &= \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W}e^2_{q\bar q}\frac{\alpha_s(\mu_T)}{2\pi}\frac{1}{s\,p_T^2}\left\{2\,C_F f_{q/P}(x_A,\mu_T)f_{\bar q/P}(x_B,\mu_T)\ln\frac{M_Z^2}{p_T^2}\right.\\ &- 3\,C_F f_{q/P}(x_A,\mu_T)f_{\bar q/P}(x_B,\mu_T) + f_{q/P}(x_A,\mu_T)\left(P_{qq}\otimes f_{\bar q/P}\right)(x_B)\\ &+ \left. f_{\bar q/P}(x_B,\mu_T)\left(P_{qq}\otimes f_{q/P}\right)(x_A)\right\}\left|\exp\left\{\frac{C_F}{4}\frac{\alpha_s}{\pi}\left[-\ln^2\frac{\mu_Q^2}{\mu_T^2}+3\ln\frac{\mu_Q^2}{\mu_T^2}\right]\right\}\right|^2.\end{aligned}$$

$$\frac{d^2\sigma_{Z,q\bar q}}{dp_T^2\,dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W}e^2_{q\bar q}\frac{1}{s\,p_T^2}\sum_{m,n}\left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m\ln^m\frac{M_Z^2}{p_T^2}$$

$$\begin{aligned}\text{leading logarithmic} &: \alpha_s^n L^{2n-1},\\ \text{next-to-leading logarithmic} &: \alpha_s^n L^{2n-2},\\ \text{next-to-next-to-leading logarithmic} &: \alpha_s^n L^{2n-3}.\end{aligned}$$

(Arnold, Kaufmann; Ellis)

Checked

LL $${}_1D_1 = A^{(1)}f_Af_B,$$

NLL $${}_1D_0 = B^{(1)}f_Af_B + f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B,$$

LL $${}_2D_3 = -\frac{1}{2}\left[A^{(1)}\right]^2 f_Af_B,$$

NLL $${}_2D_2 = -\frac{3}{2}A^{(1)}\left[f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right]f_Af_B,$$

Requires
two loop
iBF and iSF

NNLL $$\begin{aligned}{}_2D_1 = &\left\{-A^{(1)}f_B\left(P_{qq}\otimes f\right)_A\ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)}f_B\left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_Af_B\right.\\ &+\frac{\beta_0}{2}A^{(1)}f_Af_B\ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)}f_Af_B - \left(P_{qq}\otimes f\right)_A\left(P_{qq}\otimes f\right)_B\\ &\left.-f_B\left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0\,f_B\left(P_{qq}\otimes f\right)_A\right\} + [A\leftrightarrow B]\,.\end{aligned}$$

(in progress:
Li, SM, Petriello)