Multi-Jet Predictions for the LHC

- A new phase space generator for NLO
  - Why and how
  - Validation
  - K-factors for up to 15 jet events
  - Conclusions and outlook

“A Forward Branching Phase-Space Generator”,
W.T. Giele, G.C. Stavenga and J. Winter,
Fermilab-pub-11-200-T, Cern-ph-th/2011-109
arXiv:1105.xxxx

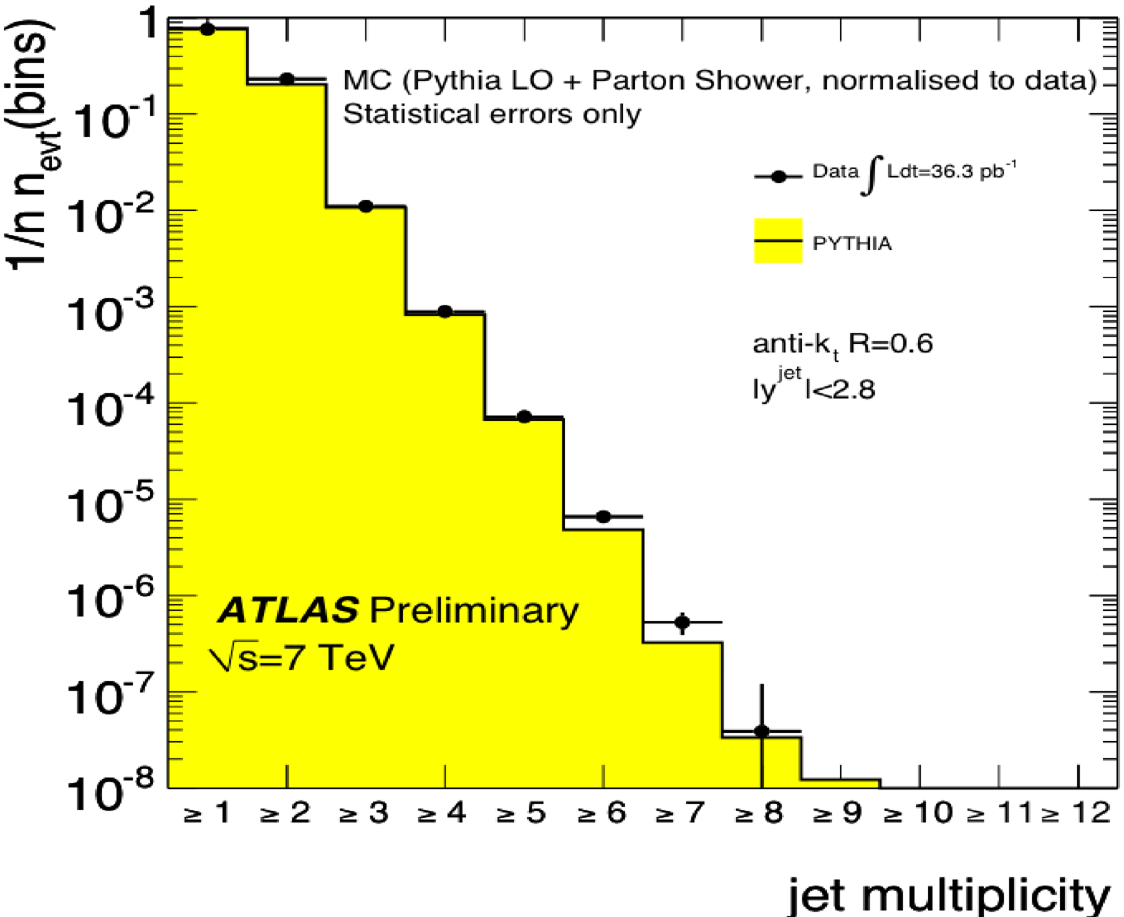
Standard NLO calculation

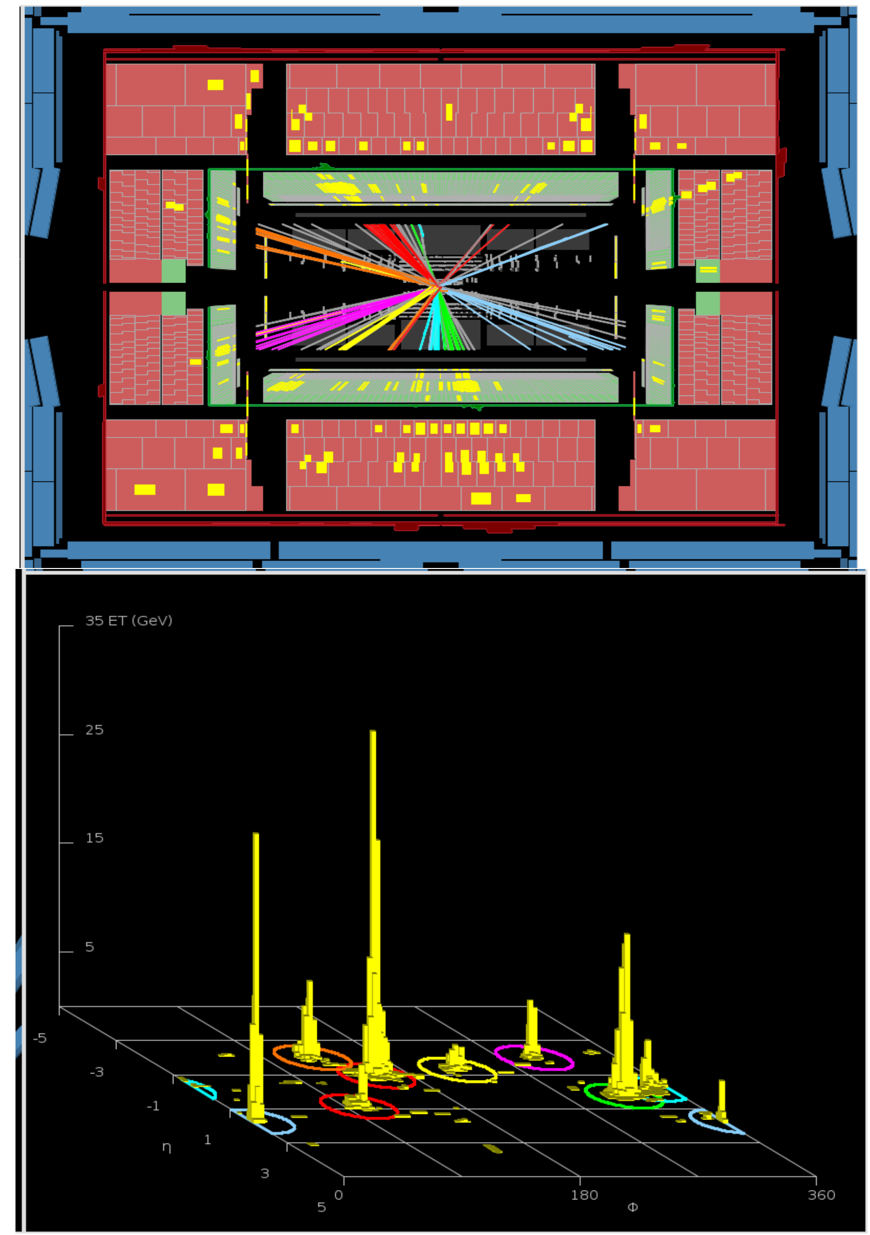
A current standard NLO *n*-jet calculation goes as follows

1. Find a large farm of CPU's
2. Do a MC integration over *n* parton phase space and calculate born+virtual
3. Do a MC integration over *(n+1)* parton phase space using a variant of Catani-Seymour subtraction.
4. Apply a jet algorithm and bin for the observable under study.

You might be able to do *PP-> 4 jets*

How many jets do we need?





- Events at LHC are jetty
- W, Z, Higgs, SUSY,... events will come with lots of bremsstrahlung jets.
- To go from a phenomenological description to a prediction, NLO is needed for events with more than a meager 4 jets

How to go beyond 4 jets

- The problem is the bremsstrahlung phase space MC integration using a subtraction method.

- Algorithmically complicated
- Computer intensive
- Bremsstrahlung generation and virtual generation independent

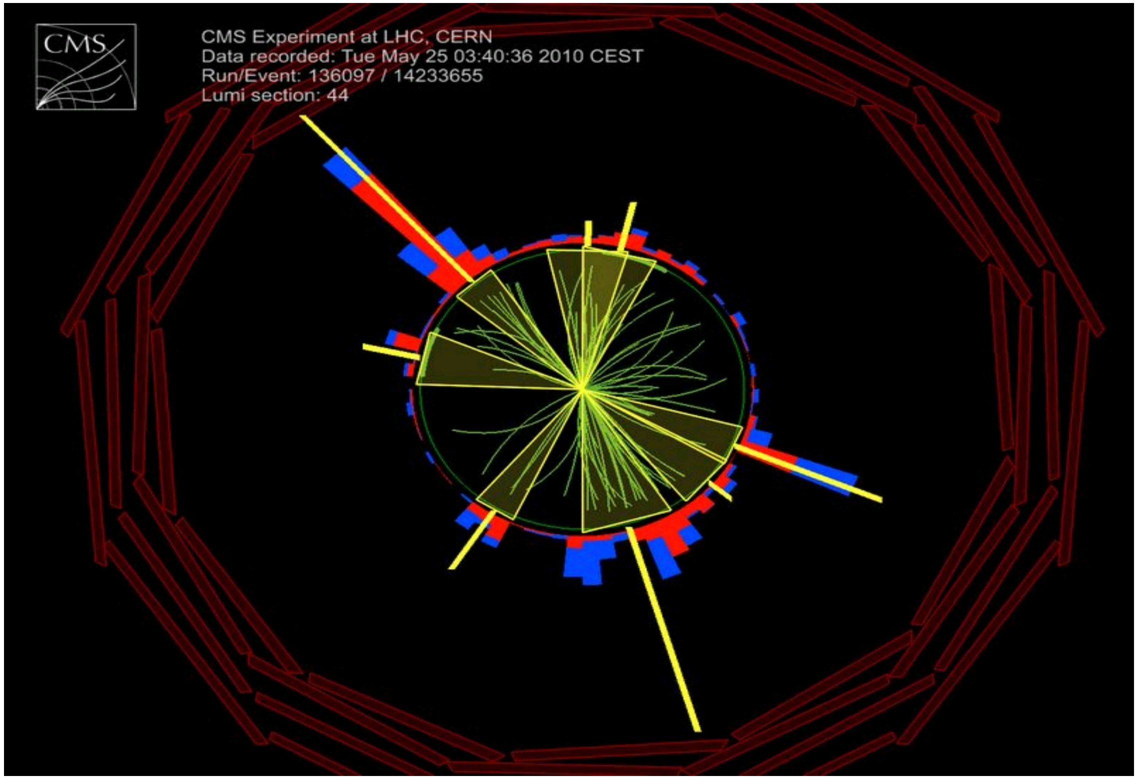
$$d\bar{\sigma}_{i1}^{(n+1)}(r;k_1,k_2) = \frac{\alpha_S}{2\pi}\Bigg\{\overline{P}^{(0)}_{\mathcal{I}_{1\oplus\bar{i}}\mathcal{I}_1}(1-\xi_i)\left[\left(\frac{1}{\xi_i}\right)_c \log\frac{s\delta_I}{2\mu^2} + 2\left(\frac{\log\xi_i}{\xi_i}\right)_c\right.$$
$$- \overline{P}^{(1)}_{\mathcal{I}_{1\oplus\bar{i}}\mathcal{I}_1}(1-\xi_i)\left(\frac{1}{\xi_i}\right)_c - K_{\mathcal{I}_{1\oplus\bar{i}}\mathcal{I}_1}(1-\xi_i)\Bigg\}$$
$$\times\,\mathcal{M}^{(n,0)}\left(r^{1\oplus\bar{i},\not{i}};(1-\xi_i)k_1,k_2\right)\frac{J^{(B)}_{n_L}}{\mathcal{N}(r)}d\phi_n\Big((1-\xi_i)k_1,k_2\Big)d\xi_i\,,$$

$$d\bar{\sigma}_{i2}^{(n+1)}(r;k_1,k_2) = \frac{\alpha_S}{2\pi}\Bigg\{\overline{P}^{(0)}_{\mathcal{I}_{2\oplus\bar{i}}\mathcal{I}_2}(1-\xi_i)\left[\left(\frac{1}{\xi_i}\right)_c \log\frac{s\delta_I}{2\mu^2} + 2\left(\frac{\log\xi_i}{\xi_i}\right)_c\right.$$
$$- \overline{P}^{(1)}_{\mathcal{I}_{2\oplus\bar{i}}\mathcal{I}_2}(1-\xi_i)\left(\frac{1}{\xi_i}\right)_c - K_{\mathcal{I}_{2\oplus\bar{i}}\mathcal{I}_2}(1-\xi_i)\Bigg\}$$
$$\times\,\mathcal{M}^{(n,0)}\left(r^{2\oplus\bar{i},\not{i}};k_1,(1-\xi_i)k_2\right)\frac{J^{(B)}_{n_L}}{\mathcal{N}(r)}d\phi_n\Big(k_1,(1-\xi_i)k_2\Big)d\xi_i\,,$$

where, in analogy with eq. (4.33), we have introduced the distribution

$$\int^{\xi_{\max}} d\xi_i\, f(\xi_i)\left(\frac{\log\xi_i}{\xi_i}\right)_c = \int^{\xi_{\max}} d\xi_i\Big(f(\xi_i) - f(0)\Theta(\xi_{cut}-\xi_i)\Big)\frac{\log\xi_i}{\xi_i}\,.$$

What would be an alternative?



- This 8-jet event has a LO weight.
- It should also have a NLO weight.
- How to calculate this?

$$\frac{d^{(n)}\sigma_{\mathrm{LO}}}{d\,J_1\cdots d\,J_n} = \frac{(2\pi)^{4-3n}}{2x_1x_2S}\frac{1}{\mathcal{S}_n}\sum_{a.b} F_a(x_1)F_b(x_2)\left|\overline{\mathcal{M}}^{(0)}(x_1P_a, x_2P_b, J_1, \ldots, J_n)\right|^2$$

$$\frac{d^{(n)}\sigma_{\mathrm{NLO}}}{d\,J_1\cdots d\,J_n} = K_{\mathrm{NLO}}(J_1, \ldots, J_n) \times \frac{d^{(n)}\sigma_{\mathrm{LO}}}{d\,J_1\cdots d\,J_n}$$

The K-factor approach

$$\frac{d^{(n)}\sigma_{\rm NLO}}{d\,J_1\cdots d\,J_n} = K_{\rm NLO}(J_1,\ldots,J_n)\times\frac{d^{(n)}\sigma_{\rm LO}}{d\,J_1\cdots d\,J_n}$$

Now things are simple:

1. Generate the observable/distribution at LO
2. (Possibly) unweight the events
3. Calculate for each LO event the K-factor, thereby correcting the observable/distribution to NLO

Theoretical advantages

- The partons inside the fixed opaque jets are integrated out.
- The cancellations between virtual and real happen for each jet event.
- The LO phase space is factored out, this means the integration over the bremsstrahlung phase space in the K-factor is 3-dimensional.

However, to work we must have:

NLO jet phase space = LO jet phase space

NLO jets = LO jets

This is not true for any of the current jet algorithms in use. To be true we must have:

- Jets remain massless during clustering.
- There can be no unclustered momenta.

This means

- We need a 3->2 clustering instead of a 2->1 clustering
- If initial state partons are present: need a beam-jet or beam-recoiler.

Phase space partitioning

- We decompose the bremsstrahlung phase space into sectors.
- In each sector a unique triplet of partons has the smallest jet energy resolution and therefore will be clustered to 2 jets.

$$
\begin{aligned}
&\Delta_{\text{JET}}\Big(J_1,\ldots,J_n\Big|p_a,p_b,p_1,\ldots,p_{n+1}\Big) = \qquad (12)\\
&\quad \sum_{irj=\{1,\ldots,n+1\}} \Delta_{\text{JET}}\Big(J_i,J_j\Big|p_i,p_r,p_j\Big)\theta(R_{ir;j}=R_{\text{MIN}})\delta(x_1-\hat{x}_1)\delta(x_2-\hat{x}_2)\prod_{m\neq\{i,j\}}\delta(J_m-\overline{p}_m)\\
&+ \sum_{rj=\{1,\ldots,n+1\}} \Delta_{\text{JET}}\Big(J_j\Big|p_a,p_r,p_j\Big)\theta(R_{ar;j}=R_{\text{MIN}})\delta(x_2-\hat{x}_2)\prod_{m\neq j}\delta(J_m-\overline{p}_m)\\
&+ \sum_{ir=\{1,\ldots,n+1\}} \Delta_{\text{JET}}\Big(J_i\Big|p_i,p_r,p_b\Big)\theta(R_{br;i}=R_{\text{MIN}})\delta(x_1-\hat{x}_1)\prod_{m\neq i}\delta(J_m-\overline{p}_m)
\end{aligned}
$$

Forward Branching Phase Space

The bremsstrahlung contribution to the K-factor is now given in the form of 2->3 branchers (which are the exact inverse of the 3->2 jet clustering):

$$\widetilde{R}(J_1,\ldots,J_n)\times\left|\overline{\mathcal{M}}^{(0)}(x_1P_a,x_2P_b,J_1,\ldots,J_n)\right|^2=\frac{1}{(2\pi)^3}\frac{\mathcal{S}_n}{\mathcal{S}_{n+1}} \tag{13}$$

$$\Bigg(\sum_{irj=\{1,\ldots,n+1\}} d\,\Phi\Big(J_i,J_j\mapsto p_i,p_r,p_j\Big)\left|\overline{\mathcal{M}}^{(0)}\Big(x_1P_a,x_2P_b,p_i,p_r,p_j,\{J_m\}_{m\neq\{i,j\}}\Big)\right|^2$$

$$+\sum_{rj=\{1,\ldots,n+1\}} d\,\Phi\Big(x_1P_a,J_j\mapsto \hat{x}_1P_a,p_r,p_j\Big)\left(\frac{x_1F_a(\hat{x}_1)}{\hat{x}_1F_a(x_1)}\right)\left|\overline{\mathcal{M}}^{(0)}\Big(\hat{x}_1P_a,x_2P_b,p_r,p_j,\{J_m\}_{m\neq j}\Big)\right|^2$$

$$+\sum_{ir=\{1,\ldots,n+1\}} d\,\Phi\Big(x_2P_b,J_i\mapsto \hat{x}_2P_b,p_r,p_i\Big)\left(\frac{x_2F_b(\hat{x}_2)}{\hat{x}_2F_b(x_2)}\right)\left|\overline{\mathcal{M}}^{(0)}\Big(x_1P_a,\hat{x}_2P_b,p_i,p_r,\{J_m\}_{m\neq i}\Big)\right|^2\Bigg)$$

with the final–final antenna phase space given by [46, 52]

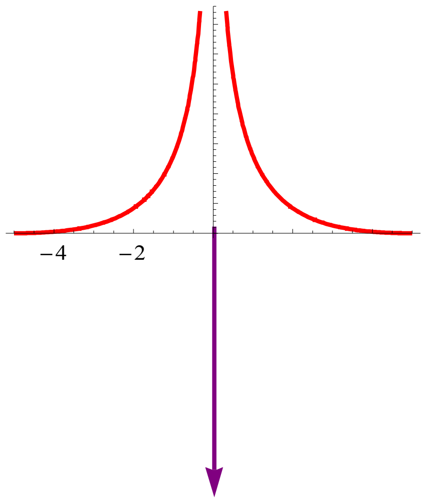
$$d\,\Phi\Big(J_1,J_2\mapsto p_1,p_r,p_2\Big)=\frac{\pi}{2}\frac{1}{s_{1r2}}d\,s_{1r}d\,s_{r2}\frac{d\,\phi}{2\pi}\,\theta(R_{1r;2}=R_{\rm MIN}) \tag{14}$$

and the intial–final state antenna phase space given by [47, 53]

$$d\,\Phi\Big(x_1P_a,J_2\mapsto\hat{x}_1P_a,p_r,p_2\Big)\;=\;\frac{1}{2\pi}\frac{d\,\vec{p}_r}{2E_r}\left(\frac{P_a\cdot J_2}{P_a\cdot J_2-P_a\cdot p_r}\right)\theta(R_{ar;2}=R_{\rm MIN}) \tag{15}$$

Using FBPS with Kt-algorithm

- Given a jet event, the FBPS does not change the jet observable for the 3-->2 jet algorithm (e.g. Ht of the jet event)
- Applying a standard 2-->1 jet algorithm (e.g. anti-kt) produces something like this:



- If this distribution is added in a single bin (i.e. integrated over) it is finite.
- If partly in bin (LO at bin edge) gives fluctuations → a smearing function has to be included.
- However, one can use the FBPS generator in this mode. Still bremsstralung events and virtual are generated fully correlated.
- Sufficient "smearing" has to be added for finite results (binning, resolution,...).
- Cannot be fixed by modifying the FBPS generator.

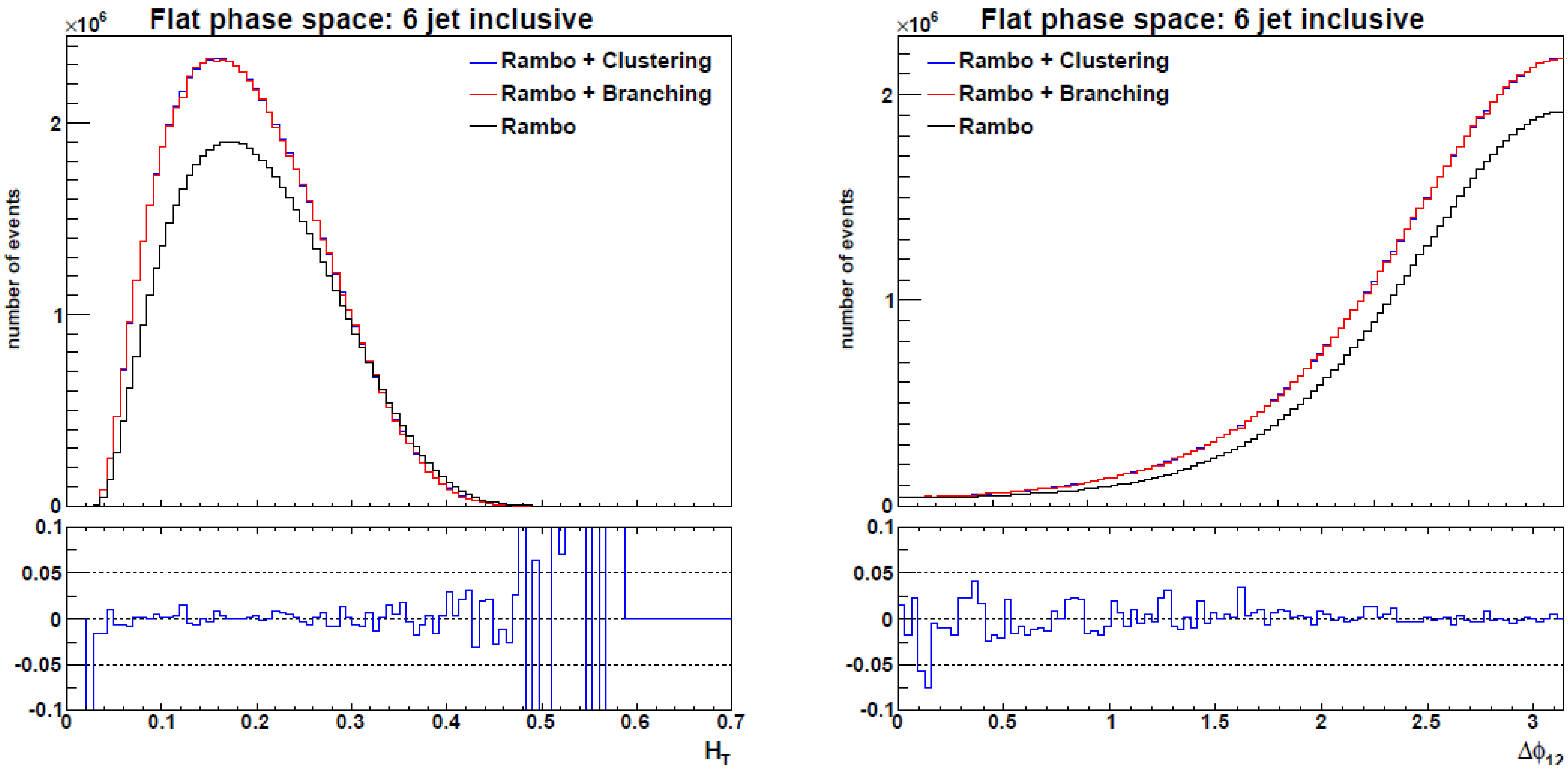
Validation of the FBPS (part 1)

To validate mainly the hard part of phase space we make the following comparison:

- Generate the "LO" *n*-jet phase space using RAMBO and apply the FBPS to calculate the weight to obtain the "NLO" *n*-jet phase space
- Generate the *(n+1)* phase space using RAMBO and apply the jet algorithm to obtain again the "NLO" *n*-jet phase space

These two results must agree

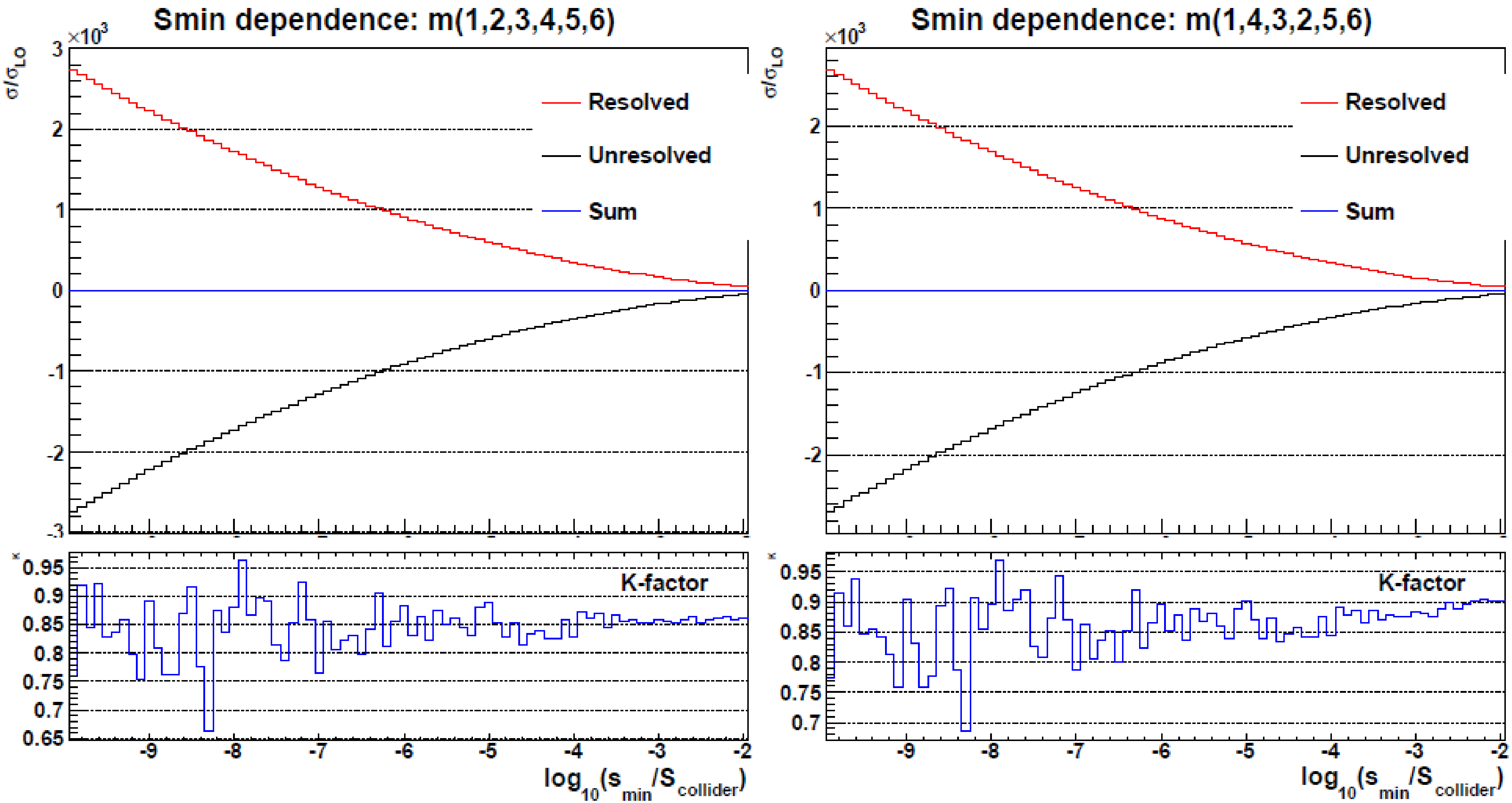
6-"jet" validation



Proof of Principle

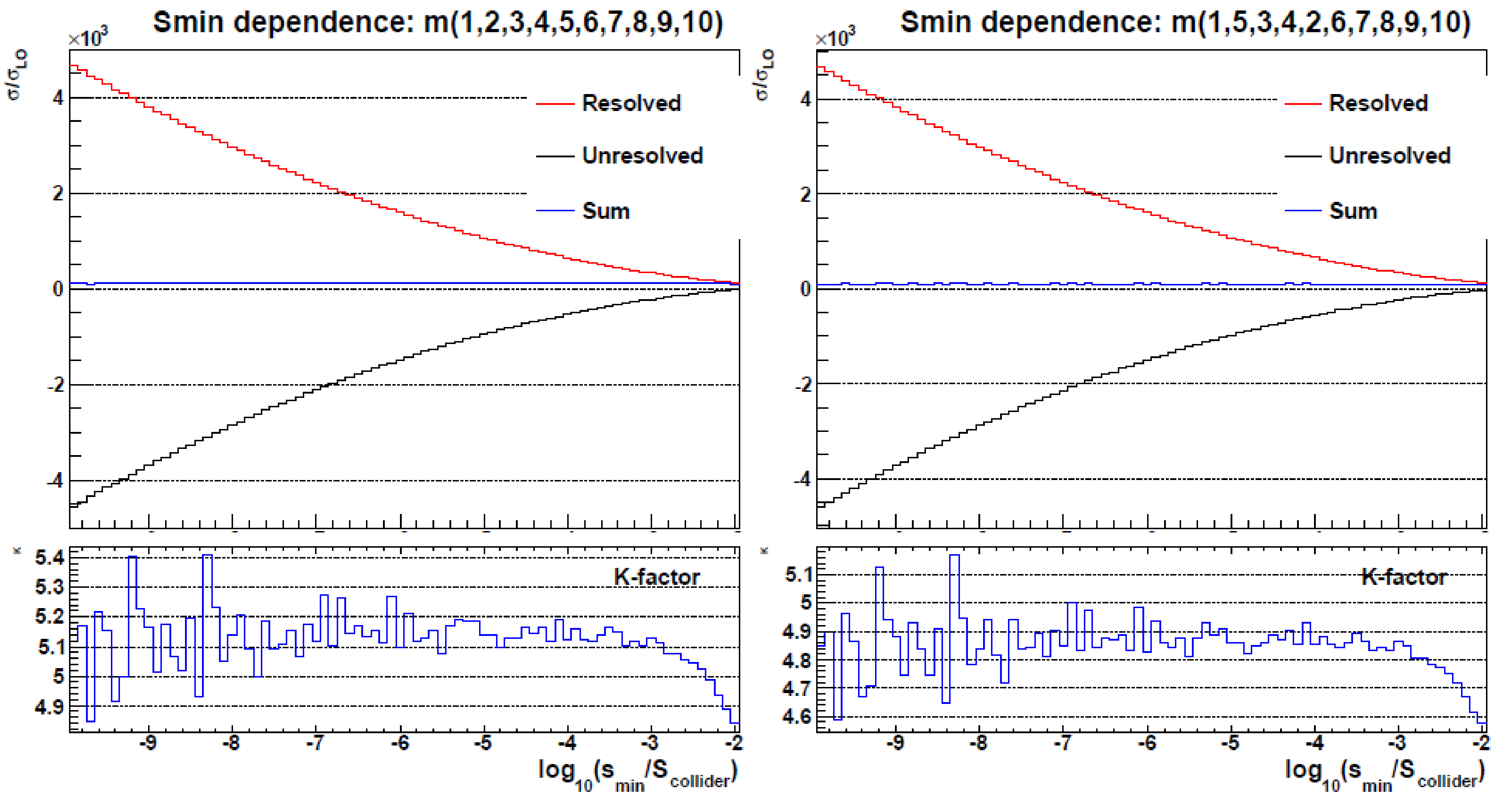
- We use the FBPS to calculate the K-factors for the gluonic contribution to *n*-jet production at LC
- For now we use a simple slicing method to calculate the bremsstrahlung contributions
- This provides a good validation test on the soft/collinear part of the FBPS
- Many virtual packages are available to calculate the one-loop *n*-gluon contribution in generalized unitarity

Validation of the FBPS (part 2)



Note: This is for a single jet event!

Validation for 8 jets



Note: This is for a single jet event!

K-factors for *n* gluonic jets

jets	R-factor	$--++\cdots+$		$---+\cdots++$		$-+-+\cdots-+$	
		$\left\|m^{(0)}\right\|^2$	K-factor	$\left\|m^{(0)}\right\|^2$	K-factor	$\left\|m^{(0)}\right\|^2$	K-factor
2	172± 1	1.72216	1.15± 0.05	1.6×10^{-31}	− − −	0.00552438	1.09± 0.05
3	243± 2	120.638	1.13± 0.08	0.043632	1.18± 0.08	5.98249	1.10± 0.08
4	392± 3	125.234	1.30± 0.13	0.282847	1.17± 0.13	0.0498892	1.18± 0.13
5	366± 4	5941.55	0.94± 0.17	849.054	0.87± 0.17	31.5083	0.80± 0.17
6	529± 5	1202.54	1.15± 0.24	69.0066	1.06± 0.24	0.469815	0.82± 0.24
8	650± 7	26732.0	1.41± 0.34	1364.49	1.32± 0.34	1.41604	1.15± 0.34
10	844± 11	6575.23	1.49± 0.49	579.066	1.26± 0.49	6.09232×10^{-6}	0.97± 0.49
15	1264±20	4690.02	1.39± 0.95	671.554	1.28± 0.95	4.37178×10^{-7}	1.24± 0.95

- These are all for a single event at LC (ordered amplitude) at 7 TeV using CTEQ6M
- The renormalization/factorization scale is half the average di-jet mass. (The di-jet mass is the starting scale for a dipole shower.)

Conclusions/Outlook

- We constructed a new type of NLO phase space generator
- It integrates out all possible partonic configurations in the jet cone, leaving the jet-axis unaltered.
- The generator is constructed for easy future GPU implementation
- We are now positioned to make NLO multi-jet generators for single CPU+GPU systems (no farms) which go up to order 10 jets! (and you can run it again and again with different cuts).