NLO QCD corrections to $\mathrm{W^+W^-b\bar{b}}$ production at hadron colliders

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Outline

1. Introduction
   - Motivation
   - Full Description vs. Narrow-Width Approximation
2. Technical aspects of the calculation
   - NLO QCD calculation with Feynman Diagrams
   - Complex-Mass Scheme for Unstable Particles
3. Numerical Results for the Tevatron and the LHC
   - Integrated WWbb cross sections
   - Differential WWbb distributions
4. Conclusions

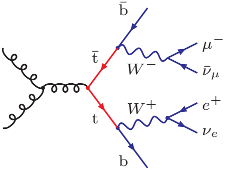
NLO priority list (Les Houches '05): completed 2 → 4 calculations

- **Two calculations for** $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$
  - arXiv:0905.0110 and arXiv:1001.4006 by Bredenstein, Denner, Dittmaier, and Pozzorini
    Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau, and Worek
    OPP reduction and HELAC
- **Two calculations for** $\mathrm{pp} \to Vjjj$
  - arXiv:0906.1445 by Ellis, Melnikov, and Zanderighi
    D-dimensional unitarity (leading colour)
  - arXiv:0907.1984 ($Wjjj$) and arXiv:1004.1659 ($Zjjj$) by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, and Maitre
    generalized unitarity (full colour)
- $q\bar{q}$**-channel contribution to** $\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$
  - arXiv:0910.4379 by Binoth, Greiner, Guffanti, Reuter, Guillet, and Reiter
    Feynman diagrams and tensor integrals (GOLEM)
- **First result for** $\mathrm{pp} \to \mathrm{t\bar{t}}jj$
  - arXiv:1002.4009 by Bevilacqua, Czakon, Papadopoulos, and Worek
    OPP reduction and HELAC

NLO priority list (Les Houches '05): completed $2 \to 4$ calculations

- **One calculation for** $\mathrm{pp} \to W^+W^\pm jj$
  - arXiv:1007.5313 and arXiv:1104.2327 by Melia, Melnikov, Rontsch, and Zanderighi
    D-dimensional unitarity
- **First result for** $\mathrm{pp} \to W + 4j$
  - arXiv:1009.2338 by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, and Maitre
    generalized unitarity (leading colour)
- **Two calculations for** $\mathrm{pp} \to W^+W^-\mathrm{b}\bar{\mathrm{b}}$
  - arXiv:1012.3975 by Denner, Dittmaier, SK, and Pozzorini
    Feynman diagrams and tensor integrals
  - arXiv:1012.4230 by Bevilacqua, Czakon, van Hameren, Papadopoulos, and Worek
    OPP reduction and HELAC

Why $\mathrm{W^+W^-b\bar{b}}$ production at NLO?



Full description of $\mathrm{t\bar{t}}$ prod×decay

- off-shell tops and non-resonant backgr.
- $\mathrm{W} \to l\nu$ decays in spin-correlated NWA

Huge $\mathrm{t\bar{t}}$ samples at hadron colliders

- Tevatron: few 10^4 events $\Rightarrow \frac{\delta\sigma}{\sigma} < 10\%$
- LHC at 7(14) TeV: $1.5(9) \times 10^5$ events per $\mathrm{fb}^{-1} \Rightarrow \frac{\delta\sigma}{\sigma} = \mathrm{few}\,\%$

Crucial measurements and tests

- precise studies of rich variety of (differential) observables
- checks and tuning of many theoretical/experimental tools
- $\delta m_\mathrm{t}^\mathrm{exp} \sim 1\,\mathrm{GeV}$ measurements

Relevance for discoveries

- typical discovery signature: leptons + jets + missing E_T (SUSY, $\mathrm{H} \to \mathrm{W^+W^-}, \ldots$)
- heavy resonances decaying into $\mathrm{t\bar{t}}$ in various BSM scenarios

Precise predictions for hadronic $\mathrm{t\bar{t}}$ production (and decay)

NLO QCD corrections

Beenakker, Dawson, Ellis, Frixione, Kuijf, Meng, Nason, van Neerven, Schuler, Smith

Electroweak NLO corrections

Beenakker, Bernreuther, Denner, Fücker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackeroth, Yuan

From LL to NNLL resummations

Ahrens, Beneke, Berger, Bonciani, Catani, Contopanagos, Czakon, Falgari, Ferroglia, Frixione, Kidonakis, Kiyo, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak, Ridolfi, Schwinn, Sterman, Uwer, Vogt, Yang

Towards full NNLO predictions

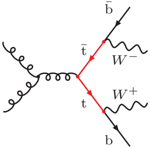
Anastasiou, Aybat, Bonciani, Czakon, Dittmaier, Ferroglia, Gehrmann, Gerhmann–De Ridder, Kniehl, Körner, Langenfeld, Maitre, Merebashvili, Mitov, Moch, Ritzmann, Rogal, Studerus, von Manteuffel, Uwer, Weinzierl

NLO $\mathrm{t\bar{t}}$ production×decay in spin-correlated narrow-width approx.

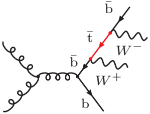
Bernreuther, Brandenburg, Melnikov, Schulze, Si, Uwer

Full $\mathrm{W^+W^-b\bar{b}}$ description vs Narrow-Width Approximation in LO

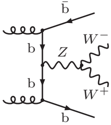
Doubly-resonant (DR)



Singly-resonant (SR)



Non-resonant (NR)



Narrow-Width Approximation

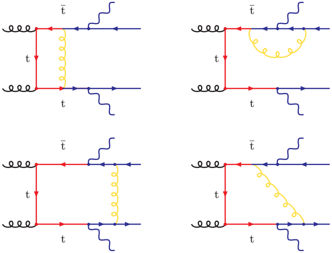
- only DR channels
- narrow-width limit of Breit-Wigner top resonances

$$\lim_{\Gamma_\mathrm{t}\to 0}\left|\frac{1}{p_\mathrm{t}^2 - m_\mathrm{t}^2 + \mathrm{i}\Gamma_\mathrm{t} m_\mathrm{t}}\right|^2 = \frac{\pi}{\Gamma_\mathrm{t} m_\mathrm{t}}\delta(p_\mathrm{t}^2 - m_\mathrm{t}^2)$$

Finite-width contributions to $\mathrm{W^+W^-b\bar{b}}$

- Off-shell corrections to DR channels
- SR+NR channels and interferences
- $\mathcal{O}(\Gamma_\mathrm{t}/m_\mathrm{t})$ corrections to inclusive observables

Full $\mathrm{W^+W^-b\bar{b}}$ description vs Narrow-Width Approximation in NLO



Narrow-Width Approximation

- only factorisable corrections to DR channels
- huge technical simplification

Finite-width contributions to $\mathrm{W^+W^-b\bar{b}}$

- involve pentagons and hexagons
- non-DR and non-factorisable corrections

Soft-gluon enhancements ($\propto \ln(\Gamma_\mathrm{t}/m_\mathrm{t})$) separately contained in virtual and real NF corrections cancel in the sum. [Fadin/Khoze/Martin '94]

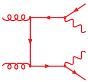
$\hookrightarrow$ Finite-width corrections remain $\mathcal{O}(\Gamma_\mathrm{t}/m_\mathrm{t})$ suppressed for inclusive observables.

Importance of finite-width effects

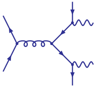
- percent-level precision in σ_incl
- Shape of top resonance and related observables (m_t measurement)
- off-shell regime of $\mathrm{W^+W^-b\bar{b}}$ background

Ingredients of $pp \to W^+W^-b\bar{b}$ at NLO

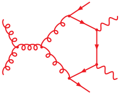
Partonic channels:



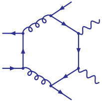
31 trees



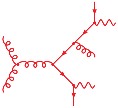
14 trees



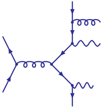
788 loops



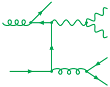
280 loops



222 NLO trees



90 NLO trees



90 NLO trees

Two independent full calculations:

Generation of Feynman diagrams

- `FeynArts 1.0 / 3.2`

Algebraic reduction

- `MATHEMATICA` / `FormCalc` [Hahn]
  + in-house extensions

Tensor integrals & numerics

- `Fortran77` / `C++` executables: ∼ 1 GB

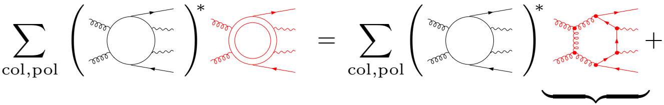
Real emission & IR Subtraction

- `Madgraph` & spinors
- Dipoles [Catani/Seymour '97] &
  `AutoDipole` [Hasegawa/Moch/Uwer '09]

Integration over 11-dim PS

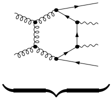
- adaptive multi-channel Monte Carlo with 250–650 mappings per partonic channel

Feynman diagrams and tensor integrals



$+\,\mathcal{O}(1000)$ more diagrams

Colour sums at zero cost
thanks to *colour factorisation*



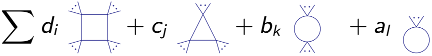
$\times\; f^{a_1 bd} f^{a_2 cd} \left(T^c T^b\right)_{i_5 i_6}$

$$\sum a_{i_1\dots j_P}\;\underbrace{\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}\left[\bar{v}_5\,\gamma_{\mu_5}\dots\gamma_{\mu_k}\,u_6\right]}\;\{g\dots p\}^{\mu_1\dots\nu_P}_{i_1\dots j_P}\;\underbrace{\int \mathrm{d}^D q\,\frac{q_{\nu_1}\dots q_{\nu_P}}{N_0\dots N_{N-1}}}$$

Algebraic reduction of helicity structures
$\mathcal{O}(10^3\text{–}10^4)$ *compact spinor chains* → fast helicity sums

$$\sum\;\underbrace{T^{(N)}_{j_1\dots j_P}}\;\{g\dots p\}_{\substack{j_1\dots j_P,\\ \nu_1\dots\nu_P}}$$

Numerical tensor-integral reduction
avoids *gigantic expressions* and *instabilities*



Reduction of tensor integrals – $e^+e^- \to 4f$ methods [Denner/Dittmaier'05]

(A) **Space-time 4-dim ($N \geq 5$ prop.)** simultaneous prop. & rank reduction
[Melrose '65; Denner/Dittmaier '02&'05; Binoth et. al. '05]

(B) **Lorentz invariance ($N \leq 4$ prop.)** reduction of rank (P)
[Passarino/Veltman '79; Denner '93]

inversion of Gram matrix $Z_{mn} = 2p_m p_n$ unstable when $\det(Z) \to 0$

(C) **General and robust solution of instability problems**
iterative $\det(Z)$-expansion (and various alternative methods)

First physical application up to tensor rank $P = 5$

- **CPU cost of colour/helicity summed $\mathrm{gg} \to \mathrm{W^+W^-b\bar{b}}$ loop amplitudes very low** (450ms per phase-space point) similarly as for $\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$ (180 ms) where $P = 4$.
- σ_{NLO} with **statistical accuracy of $\mathcal{O}(10^{-3})$** requires $\mathcal{O}(10^8)$ events obtained within **5–10 days on single CPU**.
- Total CPU cost at LHC dominated by real and virtual gg-channel corrections.

Treatment of unstable particles

Regularisation of unstable-particle propagators via $\mathrm{Im}[\Sigma(M^2)] = M\Gamma$ resummation

$$\frac{1}{p^2 - M^2 + \mathrm{i}\epsilon} \quad \rightarrow \quad \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

can violate gauge invariance.

$\hookrightarrow$ **Complex mass scheme** (introduced for $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{W}^+\mathrm{W}^- \rightarrow 4f$ [Denner/Dittmaier/Roth/Wieders '05])

- Γ is absorbed into the renormalised pole mass $M^2 \rightarrow \mu^2 = M^2 - \mathrm{i}M\Gamma$ without modifying the bare Lagrangian
- Gauge invariance requires (in general) complex couplings

Technical aspects

- On-shell renormalisation on complex propagator pole: $\hat{\Sigma}(p^2) = 0$ at $p^2 = \mu^2$
- Scalar box integrals with complex masses (subtle analytic continuations!)
  - 't Hooft/Veltman approach: 24 → 108 Li_2 [Nhung/Ninh '09; van Hameren '10]
  - Denner/Nierste/Scharf approach: 16 → 32 Li_2 [Denner/Dittmaier '10]

Setup and input parameters for Tevatron (LHC)

Particle masses and widths ($M_\mathrm{H} = \infty$, $m_\mathrm{b} = 0$)

$$m_\mathrm{t} = 172.0\,\mathrm{GeV} \qquad M_\mathrm{W} = 80.399\,\mathrm{GeV} \qquad M_\mathrm{Z} = 91.1876\,\mathrm{GeV}$$

$$\Gamma_\mathrm{t,LO} = 1.4655\,\mathrm{GeV} \qquad \Gamma_\mathrm{t,NLO} = 1.3376\,\mathrm{GeV} \qquad \Gamma_\mathrm{W,NLO} = 2.0997\,\mathrm{GeV}$$

G_μ-scheme couplings ($G_\mu = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$)

$$\sin^2\theta_\mathrm{w} = 1 - M_\mathrm{W}^2/M_\mathrm{Z}^2, \qquad \alpha = \sqrt{2} G_\mu M_\mathrm{W}^2 \sin^2\theta_\mathrm{w}/\pi$$

PDFs and α_S: MSTW2008NLO(LO) with $1/2 \le \mu_\mathrm{R,F}/m_\mathrm{t} \le 2$ variations

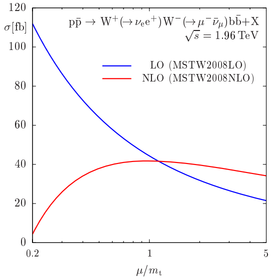
Anti-k_T Jet Algorithm

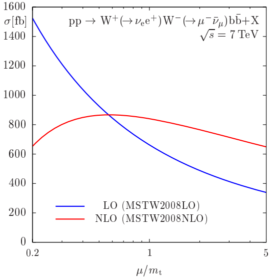
QCD partons with $|\eta| < 5$ ⇒ jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > R = 0.4\,(0.5)$

Typical Tevatron (LHC) cuts

b-jets:	$p_\mathrm{T,b} > 20\,(30)\,\mathrm{GeV}$	$\lvert\eta_\mathrm{b}\rvert \le 2.5$	
leptons:	$p_{\mathrm{T},l} > 20\,\mathrm{GeV}$	$\lvert\eta_l\rvert \le 2.5$	$p_\mathrm{T,miss} > 25\,(20)\,\mathrm{GeV}$

Integrated $\mathrm{W}^+(\to \mathrm{e}^+\nu_\mathrm{e})\mathrm{W}^-(\to \mu^-\bar{\nu}_\mu)\mathrm{b}\bar{\mathrm{b}}$ cross section





Predictions for $\mu_{\mathrm{R,F}} = m_\mathrm{t}$ and $m_\mathrm{t}/2 \leq \mu_{\mathrm{R,F}} \leq 2m_\mathrm{t}$

σ	LO	NLO	NLO/LO
Tevatron	$44.31^{+19.68}_{-12.49}$ fb	$41.75^{+0.00}_{-3.79}$ fb	$0.942^{+0.000}_{-0.085}$
LHC	$662.4^{+263.4}_{-174.1}$ fb	840^{+27}_{-75} fb	$1.27^{+0.04}_{-0.11}$

Scale uncertainty at the Tevatron (LHC)

- 44% (40%) LO uncertainty is mostly due to $\frac{\Delta\sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}} \simeq \frac{\Delta\alpha_\mathrm{S}^2(\mu)}{\alpha_\mathrm{S}^2(\mu)}$ and reduces to 9%(9%) at NLO

NLO corrections

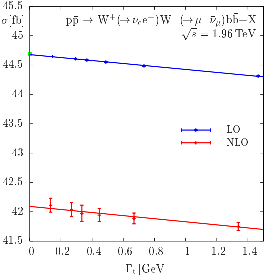
- plots reflect σ_{NLO} stability and moderate corrections
- different sign and size for $q\bar{q}$-dominated $\sigma_{\mathrm{Tevatron}}$ ($K \simeq 0.94$) and gg-dominated σ_{LHC} ($K \simeq 1.27$)

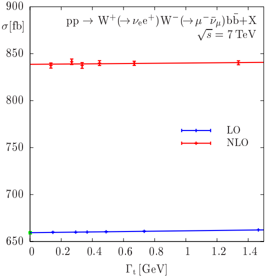
Good agreement with HELAC-OPP calculation [Bevilacqua et al. '10]

Tevatron:

σ	LO	NLO
DDKP	44.310[3] fb	41.75[5] fb
BCHPW	44.32[3] fb	41.86[6] fb

Off-shell and non-resonant contributions to $\sigma_{\rm incl}$





Assessment of finite-width effects $\sigma(\Gamma_t) - \sigma(0)$

- numerical extrapolation to $\Gamma \to 0$ using five rescaled values $\Gamma_t \to \xi\Gamma_t$ with $0.1 \lesssim \xi \leq 1$

Cancellation of soft-gluon $\ln(\Gamma_t/m_t)$ singularities

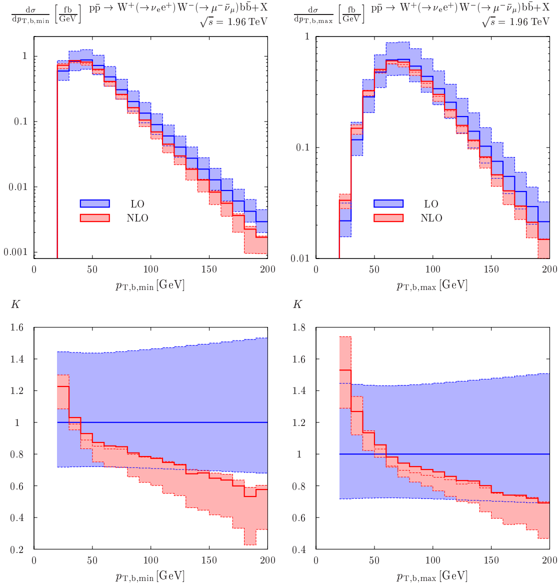
- dipole-subtracted virtual and real parts diverge logarithmically when $\Gamma_t \to 0$
- linear convergence of $\sigma(\Gamma_t) \to \sigma(0)$ provides non-trivial consistency and stability check

Finite-width effects comparable to $\Gamma_t/m_t \simeq 0.8\%$

	$\frac{\sigma_{\rm LO}(\Gamma_t)}{\sigma_{\rm LO}(0)} - 1$	$\frac{\sigma_{\rm NLO}(\Gamma_t)}{\sigma_{\rm NLO}(0)} - 1$
Tevatron	-0.8%	-0.9%
LHC	$+0.4\%$	$+0.2\%$

quantifies precision of NWA for $\sigma_{\rm incl}$

p_{T} distributions of b jets at the Tevatron



Soft b-jet (left)

- saturates cut at 20 GeV
- +20% to −40% corrections

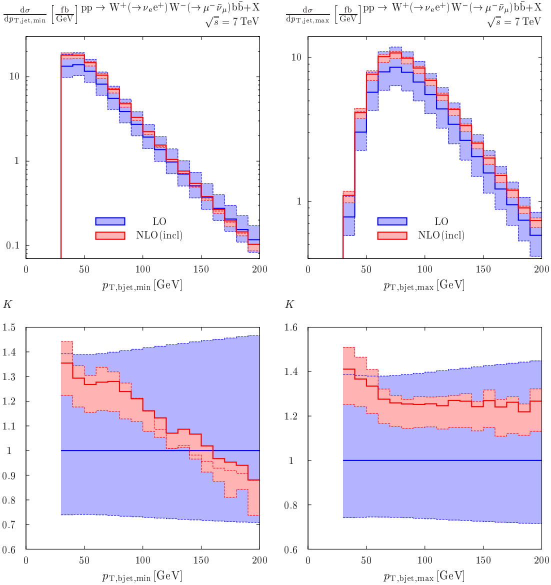
Hard b-jet (right)

- peaked around 80 GeV
- +50% to −30% corrections

Strong shape distortions

- especially at small p_{T} (impact on acceptance!)
- to be compared with parton-shower effects

p_{T} distributions of b jets at the LHC



Soft b-jet (left)

- saturates cut at 30 GeV
- +30% to −10% corrections

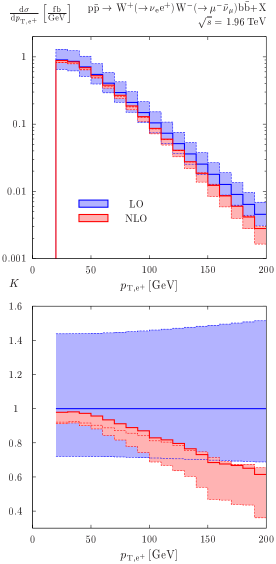
↪ **Strong shape distortions**

Hard b-jet (right)

- peaked around 80 GeV
- +40% to +20% corrections

↪ **Modest shape distortions**

p_{T} distributions of charged leptons at the Tevatron



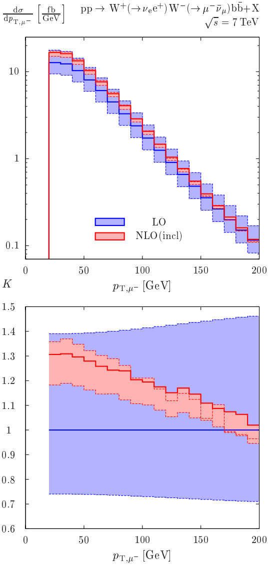
e^+ (μ^-) from W^+ (W^-) decay

- have typically $p_{\mathrm{T}} \lesssim 100\,\mathrm{GeV}$ and tend to saturate the cut at 20 GeV
- corrections range from 0% to -40%

Shape distortion

- mild in the vicinity of the cut, but fairly strong at high p_{T}
- relevant for boosted tops and NP searches
- when $p_{\mathrm{T}} \gtrsim 100\,\mathrm{GeV}$ fixed $\mu = m_{\mathrm{t}}$ should be replaced by dynamical QCD scale

p_{T} distributions of charged leptons at the LHC



μ^- from W^- decay

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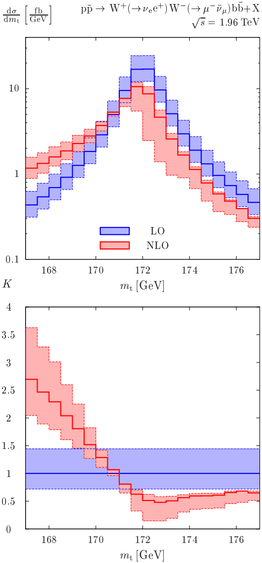
Invariant-mass distribution of the top quark at the Tevatron

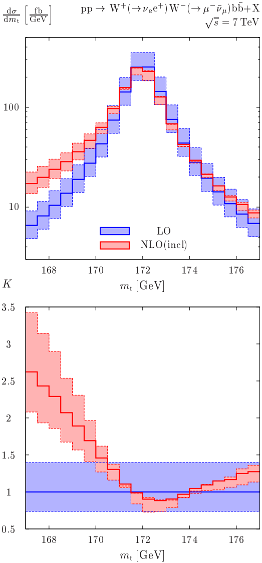


Although not observable $M_t = M_{be^+\nu_e}$ reflects off-shell nature of $2 \to 4$ calculation

- Breit–Wigner shape in the resonance region
- $\delta\Gamma_{NLO}/\Gamma_{LO} \simeq -9\%$ crucial for consistent normalisation of $\sigma_{incl.} \sim 1/\Gamma_t^2$
- Pole of top-quark progagator not shifted in on-shell scheme, but QCD radiation leads to $\mathcal{O}(1\,\mathrm{GeV})$ invariant-mass losses
- m_t-shift depends on jet algorithm

NLO and Γ_t effects will improve description of observables used for m_t determination

Invariant-mass distribution of the top quark at the LHC



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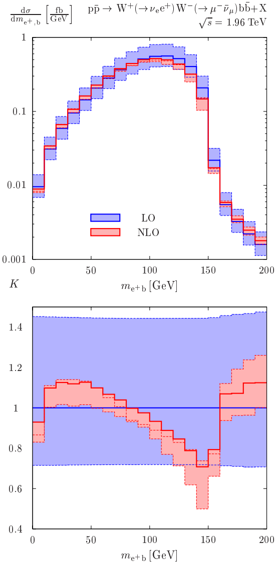
Invariant mass of the b-jet–e^+ pair at the Tevatron



Observable related to m_t measurement

- visible decay products in $t \to bW^+ \to be^+\nu_e$ retain significant fraction of m_t
- good sensitivity to m_t via kinematic bound

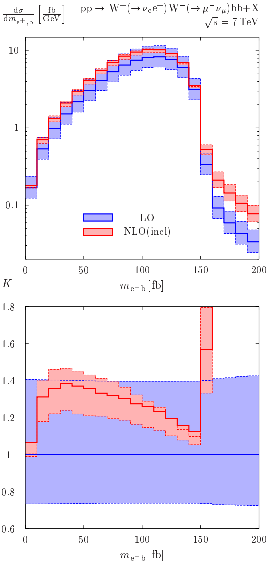
$$M^2_{e^+b} \leq m_t^2 - M_W^2 \simeq (152\,\mathrm{GeV})^2$$

in LO and narrow-width approximation

Off-shell and NLO corrections

- M_{e^+b} bound violated by LO off-shell effects
- additional violation from NLO radiation
- strong NLO shape distortion below the bound: from $+15\%$ to -30% corrections

Invariant mass of the b-jet–e^+ pair at the LHC



Observable related to m_t measurement

- visible decay products in $t \to bW^+ \to be^+\nu_e$ retain significant fraction of m_t
- good sensitivity to m_t via kinematic bound

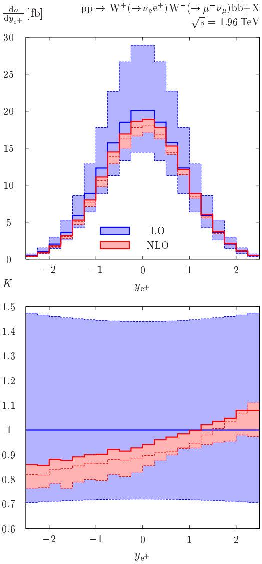
$$M^2_{e^+b} \le m_t^2 - M_W^2 \simeq (152\,\mathrm{GeV})^2$$

in LO and narrow-width approximation

Off-shell and NLO corrections

- M_{e^+b} bound violated by LO off-shell effects
- additional violation from NLO radiation
- strong NLO shape distortion below the bound: from +45% to +5% corrections

Rapidity distributions of charged leptons at the Tevatron



LO y_{e^+} distribution

- e^+ populates central region
- almost exactly symmetric due to $\mathrm{t} \leftrightarrow \bar{\mathrm{t}}$ invariance of $q\bar{q}/\mathrm{gg} \to \mathrm{t}\bar{\mathrm{t}}$

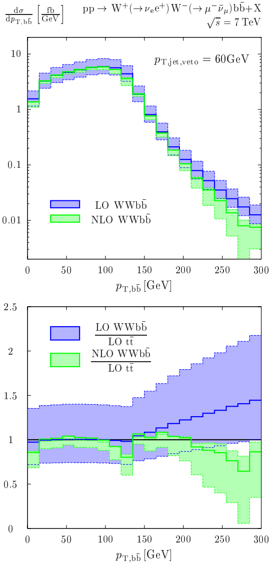
NLO charge and FB asymmetry

- IS–FS gluon exchange induces $\mathrm{t}\bar{\mathrm{t}}$ charge asymmetry
- reflected in y_{e^+} shape distortion (−15% to +10% corrections) and FB asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma(y_{e^+} > 0) - \sigma(y_{e^+} < 0)}{\sigma(y_{e^+} > 0) + \sigma(y_{e^+} < 0)} = 0.035(2)$$

consistent with NWA [Bernreuther/Si '10]

Large off-shell effects: $p_{\mathrm{T,b\bar{b}}}$ distribution at the LHC



$pp\to\mathrm{WH}\to\mathrm{Wb\bar{b}}$ **search** ($M_\mathrm{H}\lesssim 130\,\mathrm{GeV}$)

- huge QCD background suppressed with **boosted Higgs strategy**
- $p_{\mathrm{T,b\bar{b}}}>200\,\mathrm{GeV}$ and $p_{\mathrm{T,jet,veto}}=30\,\mathrm{GeV}$ yield $S/B\approx 1$ and $S/\sqrt{B}\approx 3\sigma$ with $30\,\mathrm{fb}^{-1}$.

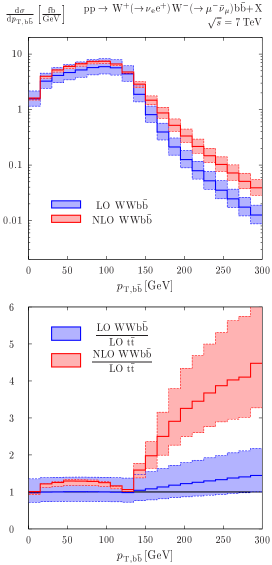
  [Butterworth et al. (2008)]

Suppression of dominant $\mathrm{W^+W^-b\bar{b}}$ background

- 0.4% off-shell effects increase to $\gtrsim$ 30% at LO.
- Strong $\mathrm{W^+W^-b\bar{b}}j$ NLO emission is very sensitive to $p_{\mathrm{T,jet,veto}}$ and barely stable.

Full $2\to 4$ NLO crucial to control $\mathrm{W^+W^-b\bar{b}}$!

Large off-shell effects: $p_{\mathrm{T},\mathrm{b}\bar{\mathrm{b}}}$ distribution at the LHC



$pp \to \mathrm{WH} \to \mathrm{Wb}\bar{\mathrm{b}}$ search ($M_{\mathrm{H}} \lesssim 130\,\mathrm{GeV}$)

- huge QCD background suppressed with **boosted Higgs strategy**
- $p_{\mathrm{T},\mathrm{b}\bar{\mathrm{b}}} > 200\,\mathrm{GeV}$ and $p_{\mathrm{T,jet,veto}} = 30\,\mathrm{GeV}$ yield $S/B \approx 1$ and $S/\sqrt{B} \approx 3\sigma$ with $30\,\mathrm{fm}^{-1}$.

  [Butterworth et al. (2008)]

Suppression of dominant $\mathrm{W}^+\mathrm{W}^-\mathrm{b}\bar{\mathrm{b}}$ background

- 0.4% off-shell effects increase to $\gtrsim$ 30%.
- Strong $\mathrm{W}^+\mathrm{W}^-\mathrm{b}\bar{\mathrm{b}}j$ NLO emission is very sensitive to $p_{\mathrm{T,jet,veto}}$ and barely stable.

Full $2 \to 4$ NLO crucial to control $\mathrm{W}^+\mathrm{W}^-\mathrm{b}\bar{\mathrm{b}}$!

Conclusions

NLO QCD calculation for $\mathrm{W^+W^-b\bar{b}}$ production

- precise description of $\mathrm{t\bar{t}}$ production and decay
- including off-shell effects, non-resonant backgrounds and interferences

Inclusive cross section at the Tevatron (LHC)

- moderate corrections $K{=}0.94$ (1.27) and stable NLO predictions ($\delta\sigma/\sigma \simeq 9\%$)
- quantitative assessment of finite-width effects $\lesssim \Gamma_\mathrm{t}/m_\mathrm{t} = 0.8\%$

NLO corrections to differential distributions at the Tevatron and the LHC

- rich and non-trivial kinematic dependence
- potentially large impact on acceptances and shape-dependent precision measurements (like m_t)

Coming soon: Tuned comparison with NWA, effects of the off-shellness of W's, . . .

Backup

Backup slides

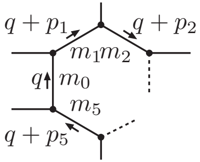
Reduction of tensor integrals – $e^+e^- \to 4f$ methods [Denner/Dittmaier '05]

(A) Space-time 4-dim ($N \geq 5$ prop.)

simultaneous prop. & rank reduction

[Melrose '65; Denner/Dittmaier '02&'05; Binoth et. al. '05]

$$\begin{vmatrix} q^\mu & 2qp_1 & \dots & 2qp_5 \\ p_1^\mu & 2p_1p_1 & \dots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \dots & 2p_4p_5 \\ 0 & f_1 & \dots & f_5 \end{vmatrix} = \mathcal{O}(D-4)$$



(B) Lorentz invariance ($N \leq 4$ prop.)

reduction of rank (P)

[Passarino/Veltman '79; Denner '93]

$$2(D+P-N-1)\ T^{(P)}_{00i_3\dots i_P} = \sum_{k=1}^{N-1} f_k\ T^{(P-1)}_{ki_3\dots i_P} + 2m_0^2\ T^{(P-2)}_{i_3\dots i_P} + \text{lower-point}$$

$$\sum_{n=1}^{N-1} Z_{mn}\ T^{(P)}_{ni_2\dots i_P} = -2\sum_{r=2}^{P} \delta_{mi_r}\ T^{(P)}_{00i_2\dots\hat{i}_r\dots i_P} - f_m\ T^{(P-1)}_{i_2\dots i_P} + \text{lower-point}$$

inversion of Gram matrix $Z_{mn} = 2p_mp_n$ **unstable** when $\det(Z) \to 0$

Reduction of tensor integrals – $e^+e^- \to 4f$ methods [Denner/Dittmaier '05]

(C) **General and robust solution of instability problems**
iterative $\det(Z)$-expansion (and various alternative methods)

$$\begin{aligned}
\tilde{X}_{0j}\, T^{(P)}_{i_1\dots i_P} &= \det(Z)\; T^{(P+1)}_{j i_1\dots i_P} + 2\sum_{n=1}^{N-1} \tilde{Z}_{jn} \sum_{r=1}^{P} \delta_{n i_r}\, T^{(P+1)}_{00 i_1\dots \hat{i}_r\dots i_P} + \text{lower-point} \\
2\tilde{Z}_{kl}\, T^{(P+1)}_{00 i_2\dots i_P} &= \Big\{ -\det(Z)\; T^{(P+1)}_{kl i_2\dots i_P} + 2m_0 \tilde{Z}_{kl}\, T^{(P-1)}_{i_2\dots i_P} + \sum_{n,m=1}^{N-1} \Big[f_n f_m\, T^{(P-1)}_{i_2\dots i_P} + 2\sum_{r=2}^{P} (f_n \delta_{m i_r} + f_m \delta_{n i_r}) \\
&\quad \times T^{(P)}_{00 i_2\dots \hat{i}_r\dots i_P} + 4\sum_{\substack{r,s=2\\ r\neq s}}^{P} \delta_{n i_r}\delta_{m i_s}\, T^{(P+1)}_{0000 i_2\dots \hat{i}_r\dots \hat{i}_s\dots i_P} \Big] \tilde{\tilde{Z}}_{(kn)(lm)} + \text{lower-point} \Big\} \Big(D+1+P-N+\sum_{r=2}^{P} \bar{\delta}_{i_r 0}\Big)^{-1}
\end{aligned}$$

First physical application up to tensor rank $P = 5$

- **CPU cost of colour/helicity summed $gg \to W^+W^-b\bar{b}$ loop amplitudes very low** (450ms) similarly as for $gg \to t\bar{t}b\bar{b}$ (180 ms) where $P = 4$
- σ_{NLO} with **statistical accuracy of $\mathcal{O}(10^{-3})$** requires $\mathcal{O}(10^8)$ events obtained within **5–10 days on single CPU**
- Total CPU cost at LHC dominated by real and virtual gg-channel corrections.