Lattice Exploration of Strong Dynamics at the TeV Scale

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Brookhaven Forum “A First Glimpse of the Tera Scale”
October 19 - 21, 2011

Outline

Strong Dynamics at the TeV Scale

- Technicolor (TC) theories:
  strongly coupled gauge theories *may* play a role at the electroweak scale: $\Lambda_{TC} \simeq F_{EW} \approx 246$ GeV. Weinberg & Susskind 1979

- Assume gauge interactions $SU(N_{TC})$ at Λ_{TC}.

- Technifermions possess chiral symmetry, transforming under $SU(N_{f,TC})_L \otimes SU(N_{f,TC})_R$.

- Spontaneous chiral symmetry breaking
  $\rightarrow$ massless Technicolor Goldstone bosons
  $\rightarrow$ three of these provide mass for the EW gauge bosons.

- Extended technicolor at Λ_{ETC} to generate quark and lepton masses. Dimopoulos & Susskind 1979, Eichten & Lane 1979

$$m_{q,l} \simeq \frac{\langle \overline{Q}Q \rangle_{ETC}}{\Lambda_{ETC}^2}$$

- $\Lambda_{ETC} \gg \Lambda_{TC}$ to suppress flavor-changing neutral currents

Difficulties with QCD-like Technicolor

If TC theories are QCD-like

- With $\langle\overline{Q}Q\rangle_{TC} \simeq 4\pi F_{EW}^3$, and

$$\langle\overline{Q}Q\rangle_{ETC} = \langle\overline{Q}Q\rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu}\gamma_m(\mu)\right) \rightarrow \langle\overline{Q}Q\rangle_{TC} \ln(\Lambda_{ETC}/\Lambda_{TC})$$

  $\rightarrow \langle\overline{Q}Q\rangle_{ETC}/\Lambda_{ETC}^2 < 0.1$ MeV
  $\rightarrow$ too small to generate SM quark masses

- The electroweak S parameter would be too big to be consistent with the experimental constraint: $S \approx 0$.

One solution: Walking Technicolor.

Walking Technicolor

There exists a region $\Lambda_{IR} < \mu < \Lambda_{UV}$ where the running coupling $\alpha(\mu)$ evolves very slowly.

- Assume $\gamma(\mu) \sim \gamma$, then Techni-quark condensate gets enhanced:

$$\langle \overline{Q}Q \rangle_{ETC} \sim \langle \overline{Q}Q \rangle_{TC} \left(\frac{\Lambda_{UV}}{\Lambda_{IR}} \right)^{\gamma}$$

  Could be large enough to generate quark masses.

- Could result in a small S parameter.

Such “walking” theories may exist with a large number of massless fermions.

The Conformal Window

- SU(N) gauge theories with N_f massless Dirac fermions, conjectured from two-loop β function

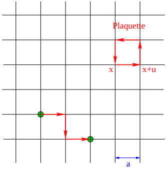
  Banks & Zaks 1981

$N_f < 11N/2$	asymptotically free
$0 \leq N_f < N_f^c$	confinement and SχSB
$N_f^c \leq N_f < 11N/2$	conformal ($\exists$ Infrared fixed point)
$N_f > 11N_c/2$	asymptotic freedom lost

- N_f^c – the conformal window
- "Walking" may appear with $N_f \to N_f^c$.
- The problem: N_f^c not precisely known. Various perturbative calculations and approximations give a wide range of results.
- Lattice calculations are ideal to study such strong interacting theories.

Lattice Formulation

- Discretize field variables onto a space-time grid.



- Evaluate *Euclidean* Green's functions via Monte Carlo simulations

$$\langle O \rangle = \frac{\int [dA][d\bar{\psi}][d\psi]\, O\, e^{-S^{(E)}(A,\psi,\bar{\psi})}}{Z}$$

$\longrightarrow$ integrating out fermion fields

$$\langle O \rangle = \frac{\int [dU] O[U] \det(\not{D} + m) e^{-S_g[U]}}{Z}$$

U: lattice gauge variable, SU(3) matrices
$\longrightarrow$ Monte Carlo

$$\int [dU] \rightarrow \sum_{\{U\}}$$

Lattice Toolbox

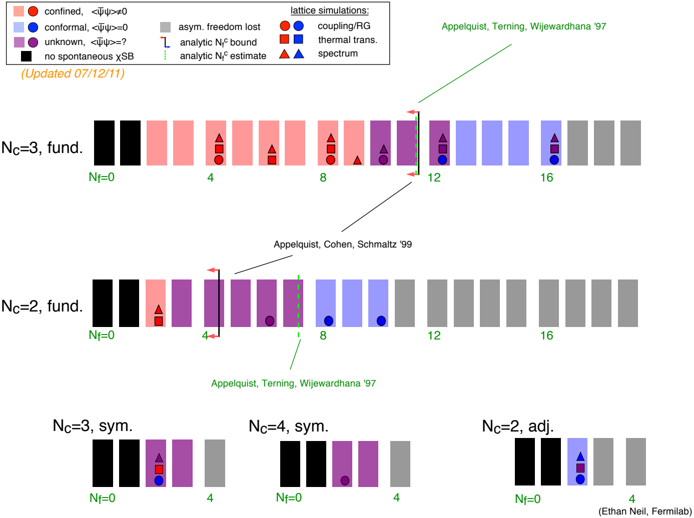
Many existing tools for the QCD studies can be used:

- Non-perturbative running coupling calculations
  How does the running coupling change with the energy scale?
- Hadron and glueball spectrum measurements
  Are the quark mass dependence and state ordering the same as QCD?
- Chiral condensate
  Does the chiral condensate vanish or get enhanced?
- Thermodynamic properties (T_c, EoS, etc.)
  Is there a deconfinement transition? Is EoS different?

A Dozen Lattice BSM Efforts Worldwide

Picture Credit: George Fleming (Origin of Mass 2010)

Current Landscape



Picture Credit: Ethan Neil (Lattice 2011)

The Lattice Strong Dynamics Collaboration

20 people from 9 institutions:

Tom Appelquist	Yale U.
Ron Babich	Boston U.
Rich Brower	Boston U.
Michael Buchoff	LLNL
Michael Cheng	Boston U.
Mike Clark	Harvard U.
Saul Cohen	U. Washington
George Fleming	Yale U.
Joe Kiskis	U. of Carlifornia, Davis
Meifeng Lin	Yale U.
Heechang Na	ANL
Ethan Neil	Fermilab
James Osborn	ANL
Claudio Rebbi	Boston U.
David Schaich	Boston U.
Sergey Syritsyn	LBL
Joe Wasem	LLNL
Oliver Witzel	Boston U.
Gennady Voronov	Yale U.
Pavlos Vranas	LLNL

LSD Program

- Explore the $N_c - N_f$ plane of the SU(N_c) gauge theories non-perturbatively.
- Look for trends with changing N_c, N_f, and perhaps locate N_f^c.
- Chiral symmetry plays an important role.
  - Use domain wall fermions: nearly exact chiral symmetry.
- Start from something familiar on the lattice: SU(3) in the fundamental representation.
  - $N_f = 2$: as a reference point.
  - $N_f = 6$: first focus on QCD-like theories, and move cautiously towards the conformal window.
  - $N_f = 10$: more challenging, can be QCD-like, conformal or walking.
- Want to have sufficient separation between IR and UV scales in a slowly-running theory $\rightarrow$ small lattice spacing:
  - We aim for five times the vector meson mass: $1/a \approx 5M_V$.

Current Status

- $N_f = 2$ and $N_f = 6$, more than three years of running. Currently increasing statistics on existing ensembles.
- Observables under study:
  - hadron spectrum and chiral condensate
  - S parameter
  - $\pi - \pi$ scattering
- Publications for $N_f = 2$ and $N_f = 6$:
  - Condensate enhancement, arXiv:0910.2224 [hep-ph].
  - S parameter and parity doubling, arXiv:1009.5967 [hep-ph]
- $N_f = 10$ in progress.
- SU(2) simulations in the pipeline.

Chiral Condensate Enhancement

- As we increase N_f but below N_f^c, if the theory becomes slowly running, then the chiral condensate should get enhanced.
- On the lattice we can use three ways to measure chiral condensate (from three GMOR ratios)

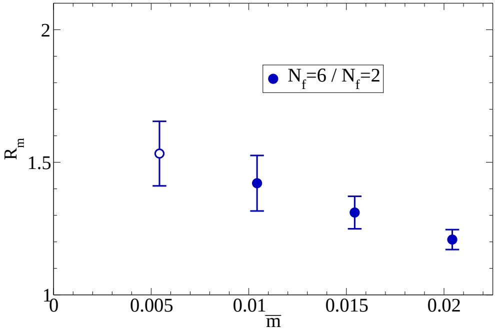
$$R = \frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3} = \frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}} = \frac{M_\pi^2}{2mF_\pi}$$

- Define enhancement ratios w.r.t the SM with $N_f = 2$ at roughly the same lattice cutoff.

$$\mathcal{R}_{XY,\widetilde{m}} \equiv \frac{R^{(N_f)}}{R^{(N_f=2)}}$$

Condensate Enhancement for $N_f = 6$

$$R_m \equiv \frac{M_m^2}{2mF_\pi}_{(N_f=6)} \Big/ \frac{M_m^2}{2mF_\pi}_{(N_f=2)}$$



T. Appelquist *et al.* (LSD Collaboration), PRL104:071601,2010

The S Parameter

- Parametrizes vacuum polarization (oblique) corrections. Peskin and Takeuchi 1992
- Used to contrain new physics beyond the Standard Model.
- Definition

$$S = -4\pi \left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right] - \Delta S_{SM},\ \Pi'(0) = \frac{d\Pi(q^2)}{dq^2}|_{q^2 \to 0}$$

where

$$\Pi^{\mu\nu}_{VV}(q) = \sum_x e^{iq\cdot x} \langle V^\mu(x) V^\nu(0)\rangle,\ \Pi^{\mu\nu}_{AA}(q) = \sum_x e^{iq\cdot x} \langle A^\mu(x) A^\nu(0)\rangle$$

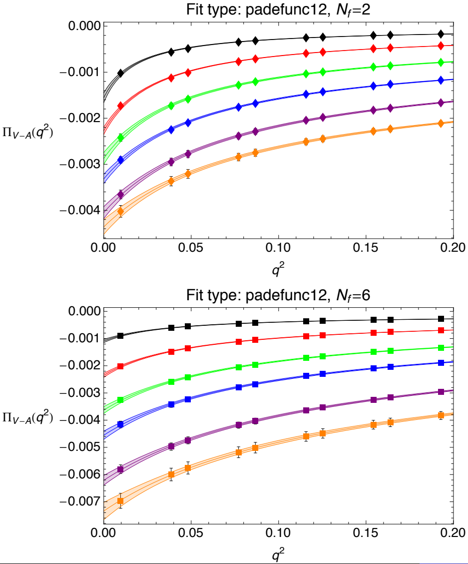
- ΔS_{SM} – Standard Model Higgs contributions

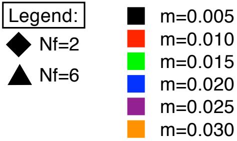
$$\Delta S_{SM} = \frac{1}{4}\int_0^\infty \frac{ds}{s}\left[1 - \left(1 - \frac{m_H^2}{s}\right)^3 \theta(s - m_H^2)\right]$$

m_H – reference Higgs mass.

- Electroweak precision experiments find $S \approx 0$.
Scaled-up QCD with $N_f = 2$ gives $S \approx 0.3$.

Extracting S from Current Correlators



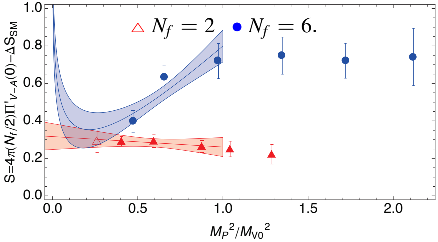


- Need the slopes at $q^2 = 0$.
- Use Padè(1,2) to describe the q^2 dependence.

$$\Pi_{V-A}(q^2) = \frac{a_0 + a_1 q^2}{1 + q^2(b_1 + b_2 q^2)}$$

- Fits describe the data reasonably well in the small q^2 region
- Insensitive to fit ranges: fits are stable.

The S Parameter



T. Appelquist *et al.* (LSD Collaboration), PRL 106 (2011) 231601

- $N_f = 2$ gives $S = 0.32(5)$ with $m_H \sim 1$ TeV, consistent with scaled-up QCD.
- For $N_f = 6$, at small mass, S drops below naive scaled-up QCD value, which would be 3 times the $N_f = 2$ value.
- There can be chiral log contributions, which will eventually make S turn up. (In $N_f = 2$, the chiral logs cancel after the SM subtraction.)
- Other interactions need to be turned on to have a finite S for $N_f = 6$ in the chiral limit.

Is the ten-flavor theory chirally broken, or conformal?

- Hayakawa *et al.* observed evidence for the existence of an infrared fixed point from Schrödinger functional running coupling studies. [Phys. Rev. D 83, 074509 (2011)]
- The hadron mass quark mass dependence is different in these two scenarios.
- In a chirally broken theory, at sufficiently light quark masses, chiral perturbation theory predicts m_f dependence.
  - However, M_P/M_V values in our simulations range from 0.71 to 0.83.
  - Simulated quark masses are likey too heavy to make use of ChPT.
- In a mass-deformed conformal theory, hadron masses are governed by universality,

$$M_X \approx C_X m^{1/(1+\gamma^*)} + \cdots$$

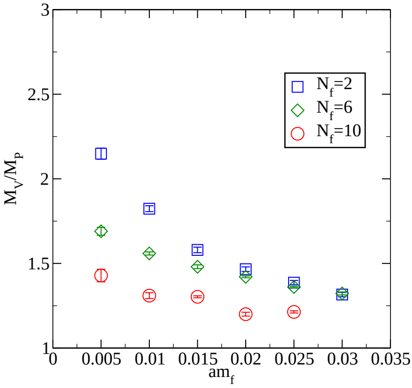
  - Ratios of hadron masses would be roughly independent of quark masses.
  - For DWF, $m \equiv m_f + m_{\text{res}}$.

Can the theory be conformal?

Recall that in a mass-deformed conformal theory

$$M_X = C_X m^{1/(1+\gamma^*)}$$

so the vector-to-pseudoscalar meson mass ratio M_V/M_P should look roughly constant.



- The $N_f = 2$ data are diverging as $m \to 0$, as expected since $M_P \to 0$.
- Similaryly for $N_f = 6$, though not as obvious.
- The $N_f = 10$ data may show an upward trend as $m \to 0$. But since the errorbars are underestimated, it is also possible that the ratios can be roughly constant. More work is in progress.

Summary

- Lattice studies by the LSD collaboration find evidence for condensate enhancement and reduced S parameter with $N_f = 6$ for SU(3) gauge theories.
- $N_f = 10$ studies are in progress. Could be in either the chirally broken or conformal phase.
- Lattice gauge theory has the potential to provide valuable input to theoretical model buidling.

Challenges

- Computionally more demanding than QCD.
- There are still many unresolved issues that need to be understood.
- What is the best way to distinguish conformal from QCD-like?
- How do we distinguish walking from conformal? Larger boxes and smaller lattice spacings?