# TRANSVERSE MOMENTUM DISTRIBUTIONS FROM EFFECTIVE FIELD THEORY

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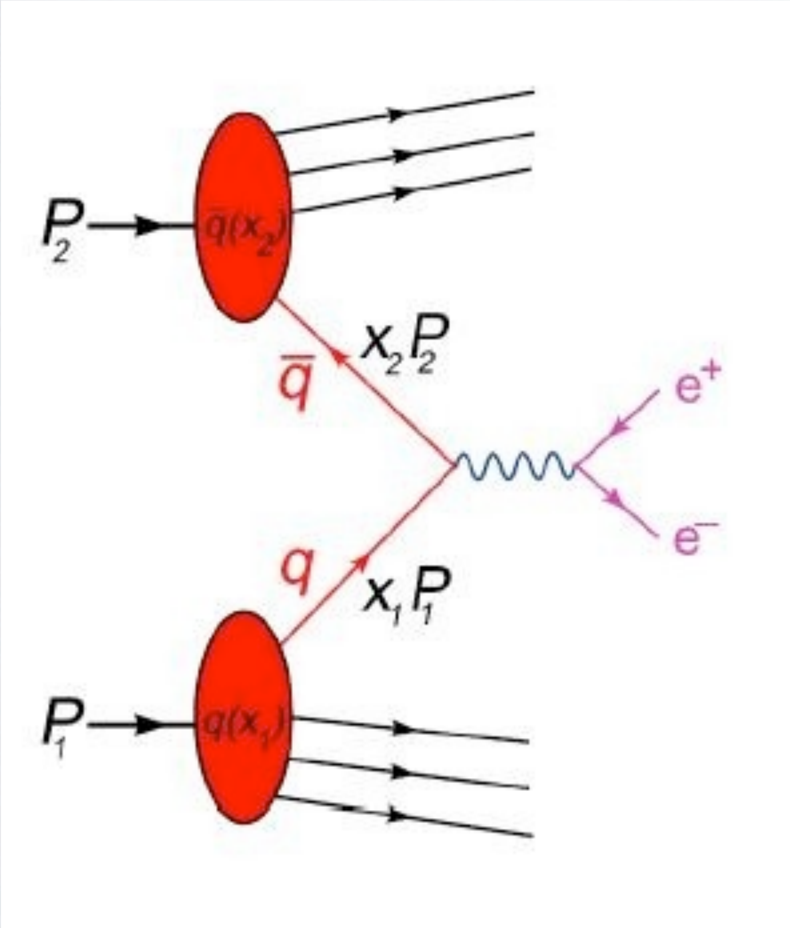
# OUTLINE

- Introduction
- The effective field theory approach
- Numerical results and comparison with data
- Conclusion

# HADRON COLLIDER

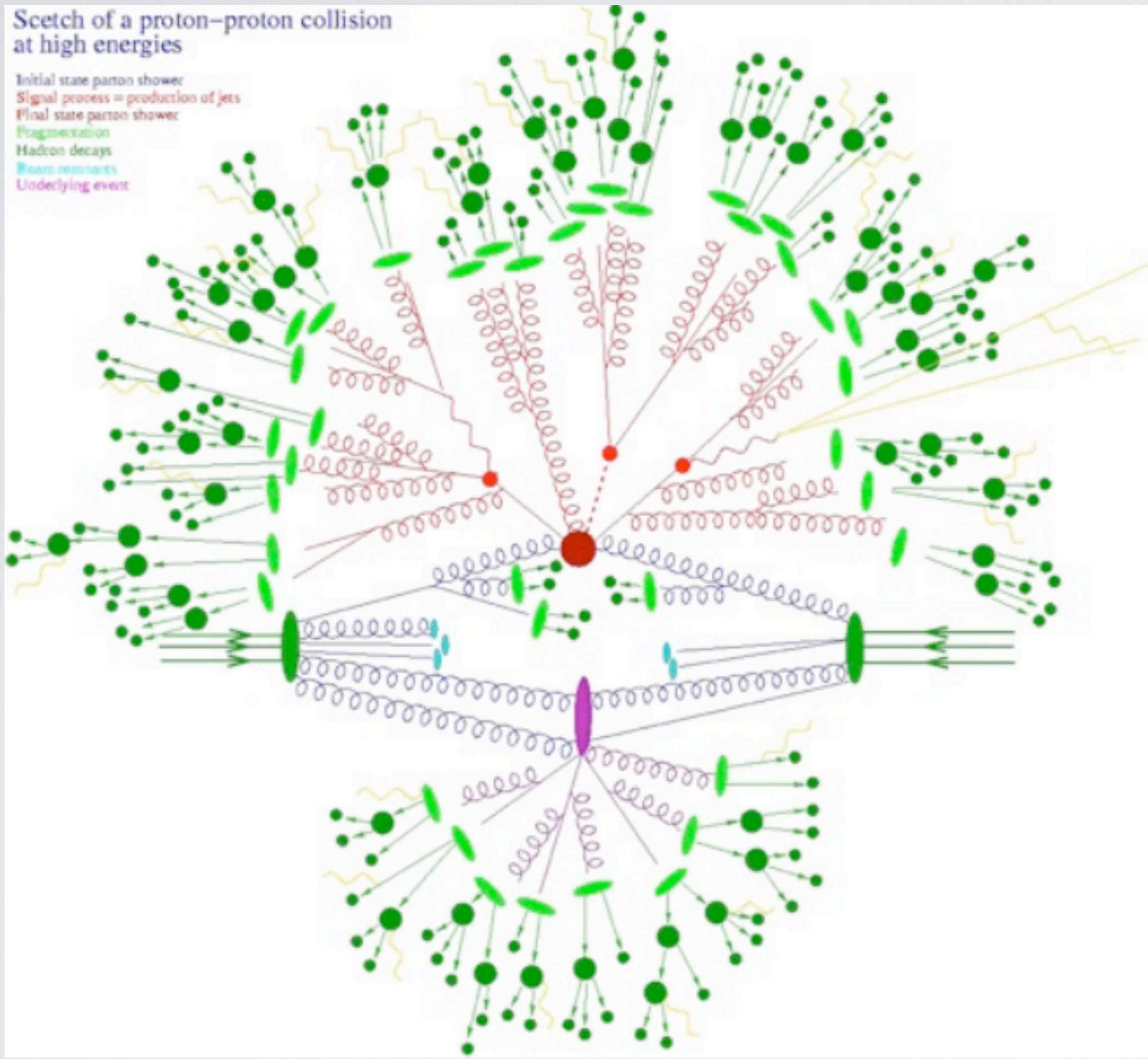
- Useful machine to discover new physics
- Do we really understand what is going on?




?

$$\sigma = \sum_{i,j} \sigma_{i,j}^{part} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

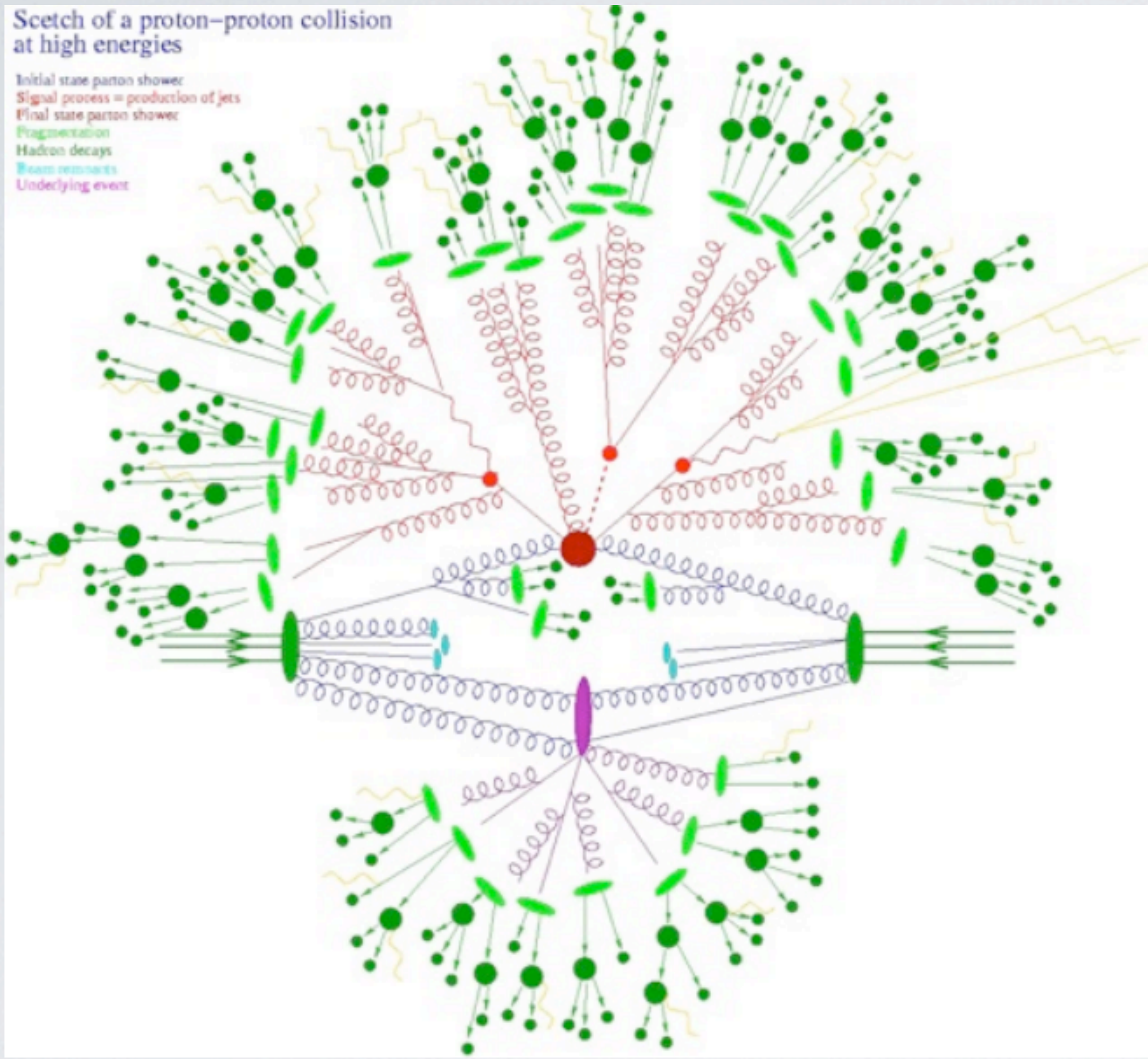
# WHAT IS REALLY GOING ON …




- Initial state parton shower
- Signal process (production of jets)
- Final state parton shower
- Fragmentation (hadronization)
- Hadron decays
- Beam remnants
- Underlying events

A Mess !!! Need Factorization

# FACTORIZATION




- Physics of interest at hard scale $M_H$
- Parton shower evolution from $M_H$ to $\Lambda_{QCD}$
- Final state hadronization at $\Lambda_{QCD}$

**Factorization: separates long distance (low energy) and short distance (high energy) behavior**

# A FAMILIAR EXAMPLE

$$d\sigma = \sum_{i,j} d\sigma_{i,j}^{part} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Live at hard scale

Live at non-perturbative scale

RG evolve to hard scale

- PDFs live at non-perturbative scale and can be measured experimentally
- Partonic cross section can be obtained using perturbative calculation
- Bring two scales together through RG running

# RESUMMATION

$$d\sigma = \sum_{i,j} d\sigma_{i,j}^{part} \otimes f_i(\xi_a) \otimes f_j(\xi_b)$$

Live at hard scale

Live at non-perturbative scale

Kinematic constraints

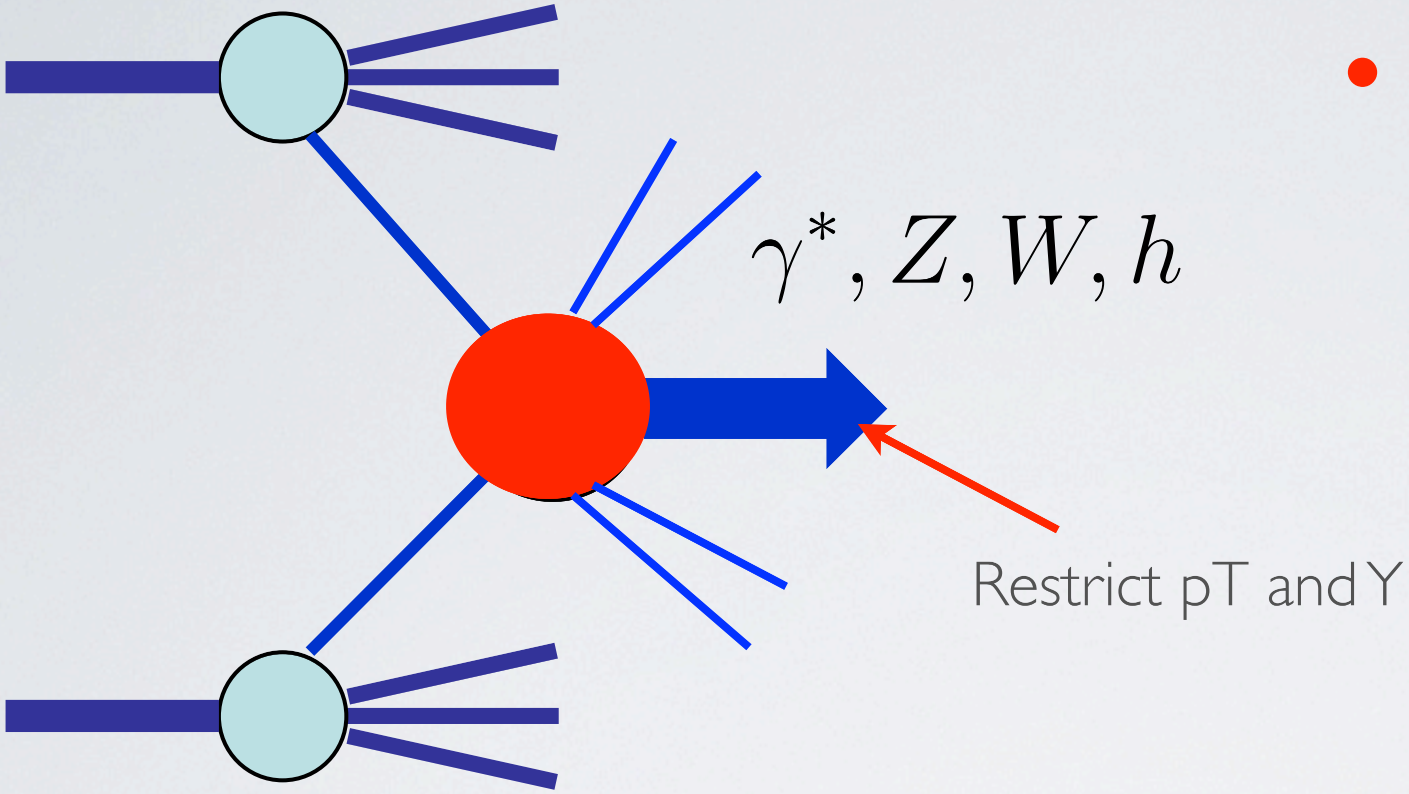
Multiple Disparate scales

RG evolve to hard scale

Additional factorization and resummation required

- Evolution of PDF resums the large logs of hard and non-perturbative scales
- Final state restriction introduces new scales
- Example: low transverse momentum distribution in Drell-Yan process / Higgs production

# TRANSVERSE MOMENTUM



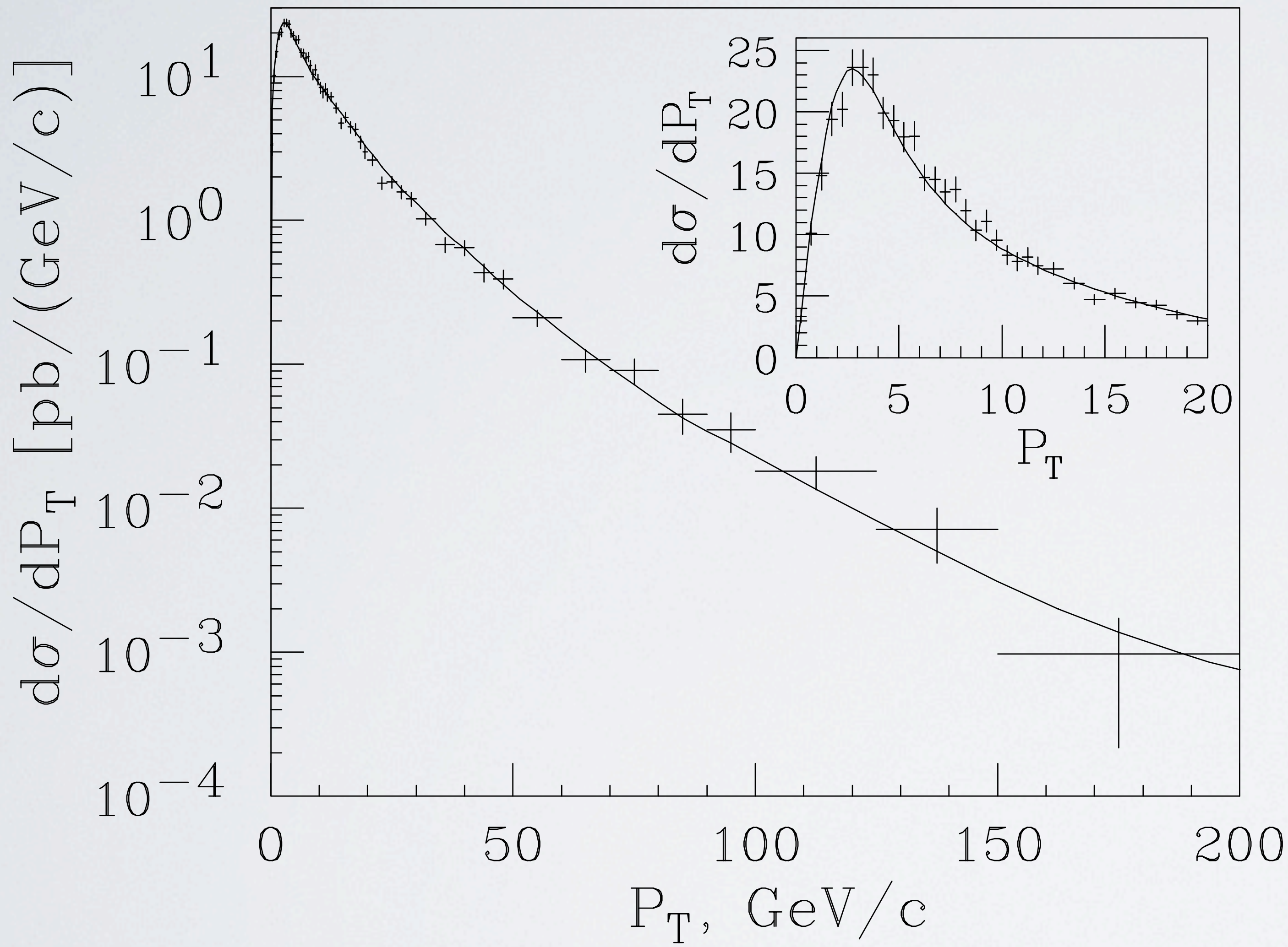

- Observable of interest

$$\frac{d^2\sigma}{dp_T^2 dY}$$

Motivations

- Higgs Boson searches ➔ pT cut introduced by jet veto
- W-mass measurement ➔ transverse mass endpoint smeared by small W pT due to ISR
- Tests of pQCD
- Probe of transverse nucleon structure

# TRANSVERSE MOMENTUM SPECTRUM



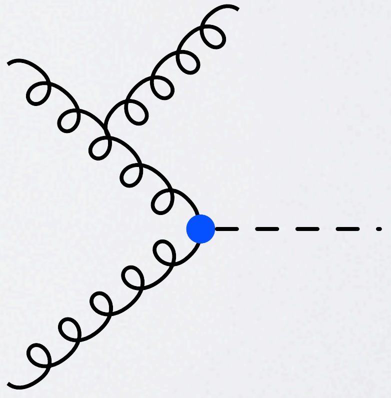

CDF data for Z production:
hep-ex/0001021

Large Logarithms spoil perturbative convergence

- Observable of interest

$$\frac{d\sigma}{dp_T^2} : \frac{1}{p_T^2}\alpha_S^n \ln^k \frac{M_h^2}{p_T^2}$$
$$+ \text{(non-singular)}$$

- High pT region: non-singular term dominates
- Low pT region: perturbation series diverges



# RESUMMATION OF TRANSVERSE MOMENTUM

- Resummation has been studied in great detail in the Collins-Soper-Sterman formalism.

(Davies, Stirling; Arnold, Kauffman; Berger, Qiu; Ellis, Veseli, Ross, Webber; Brock, Ladinsky Landry, Nadolsky; Yuan; Fai, Zhang; Catani, Emilio, Trentadue; Hinchliffe, Novae; Florian, Grazzini, Cherdnikov, Stefanis; Belitsky, Ji,.... )

- Resummation has also been studied recently using the EFT approach.

(Idilbi, Ji, Juan; Gao, Li, Liu; SM, Petriello; Becher, Neubert)

# COLLINS-SOPER-STERMAN FORMALISM

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

$$\frac{d^2\sigma}{dp_T dY} = \sigma_0 \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-ip_T \cdot b_\perp} \sum_{a,b} [C_a \otimes f_{a/P}](x_A, b_0/b_\perp)[C_a \otimes f_{a/P}](x_B, b_0/b_\perp)$$

$$\times \exp\left\{ \int_{b_0^2/b_\perp^2}^{\hat{Q}^2} \frac{d\mu^2}{\mu^2} \left[ \ln\frac{\hat{Q}^2}{\mu^2} A(\alpha_S(\mu^2)) + B(\alpha_S(\mu^2)) \right] \right\}$$

Sudakov Factor

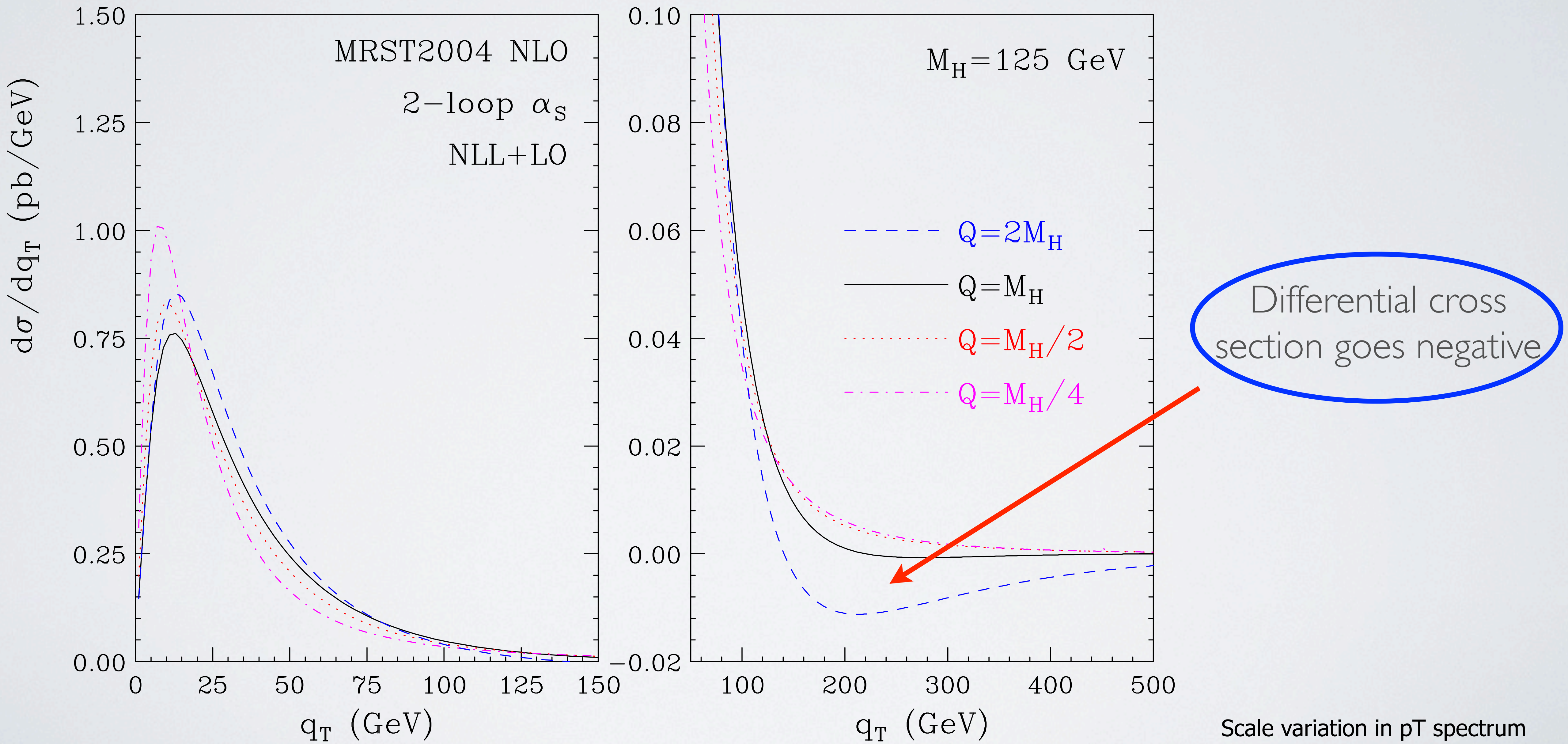
Landau Pole

- Y term neglected for the purpose here
- A,B,C have well-defined perturbative expansions
- Integration of impact parameter b⊥ introduce Landau pole:

a treatment must work for any value of pT

# COLLINS-SOPER-STERMAN FORMALISM

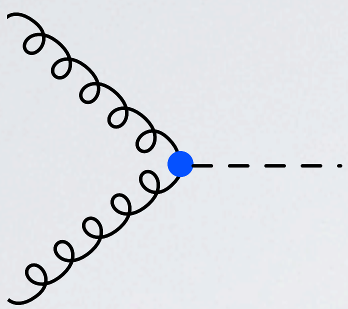
- Resummed exponent in b⊥ space ➔ difficult in matching to fixed order calculation in pT space



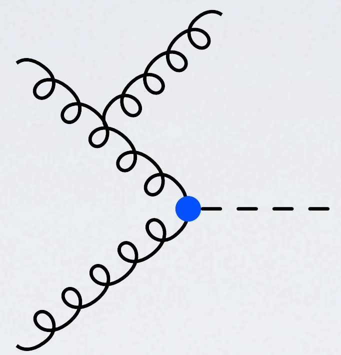

Scale variation in pT spectrum using CSS formalism: hep-ph/0508068

# EFT FRAMEWORK

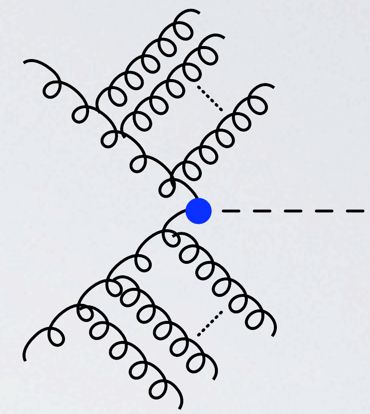
- Low pT region dominated by soft and collinear emissions from initial state:



pT=0



finite pT

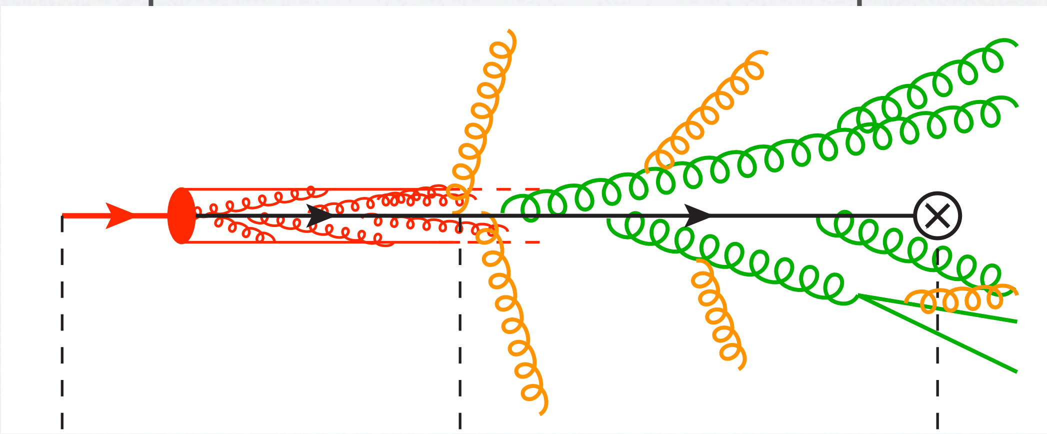


finite pT

- Hierarchy of scales suggests EFT approach with well defined power counting.

$$m_h \gg p_T \gg \Lambda_{QCD}$$

- Colliding parton is part of initial state pT radiation beam jet:



PDF Beam/Soft Function

# SCET CROSS SECTION

- Schematic form of SCET cross-section:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim \int PS\, |C \otimes \langle \mathcal{O} \rangle|^2$$

Wilson coefficient from hard matching

SCET matrix element

- Use soft collinear decoupling to factor out the soft sector

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_n \otimes S \quad \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

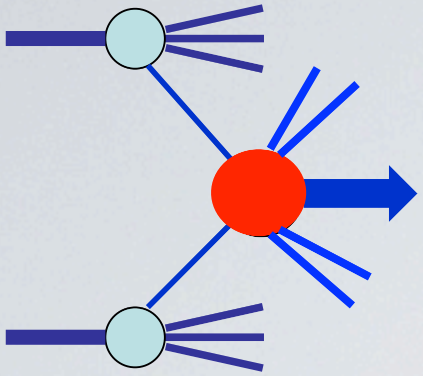
Impact-parameter Beam Functions (iBFs): collinear radiation

Soft function: soft emission

Hard function

Transverse momentum function

- Beam function is essentially unintegrated nucleon distribution function and can be matched onto PDF
- The transverse momentum function is a convolution of the iBF matching coefficient and the soft function



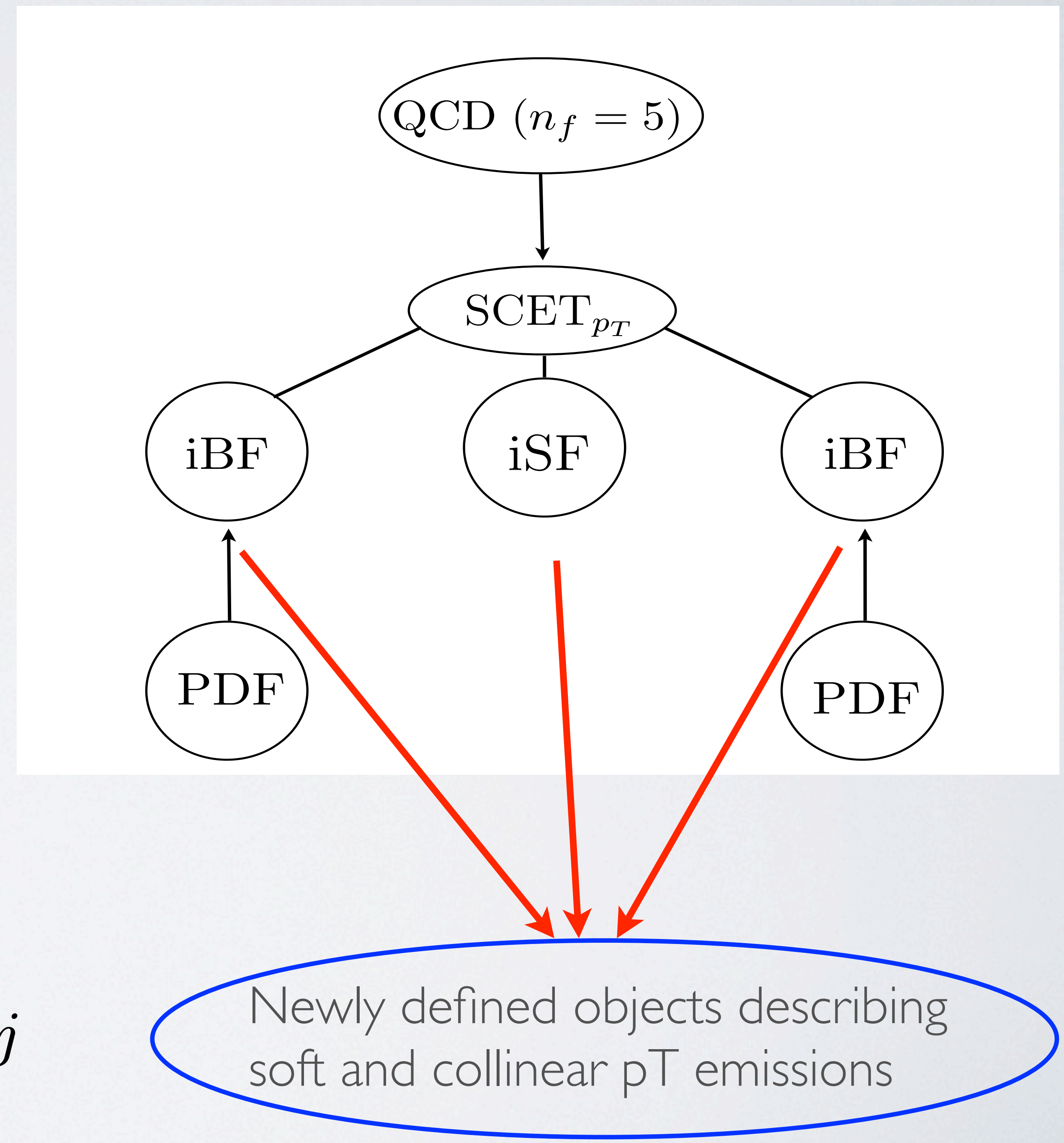
# EFT FRAMEWORK

$$\mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{p_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$$

Matched onto SCET. →

Soft-collinear factorization. →

Matching onto PDFs. →



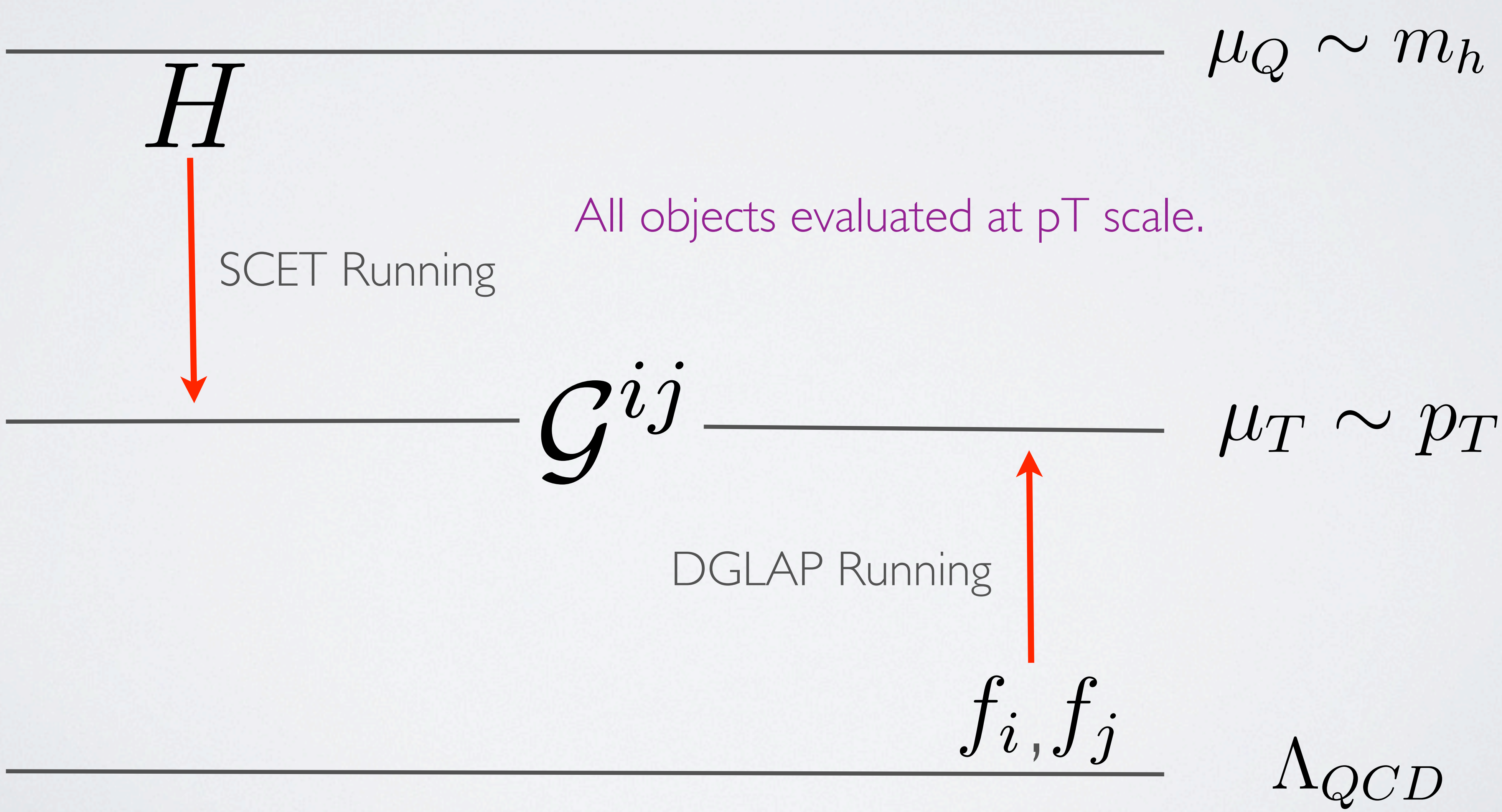

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

# Running

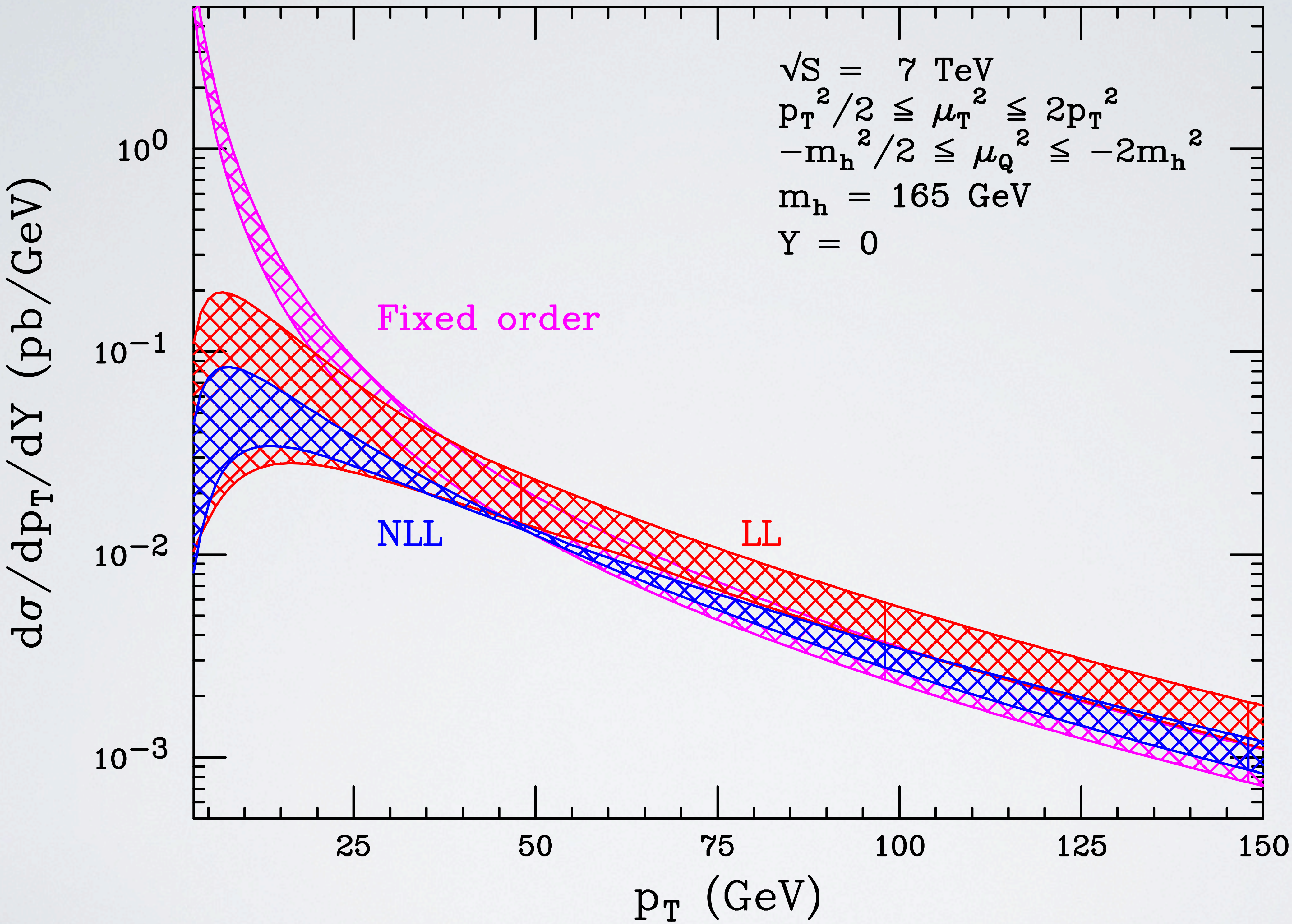
- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Schematic picture of running:

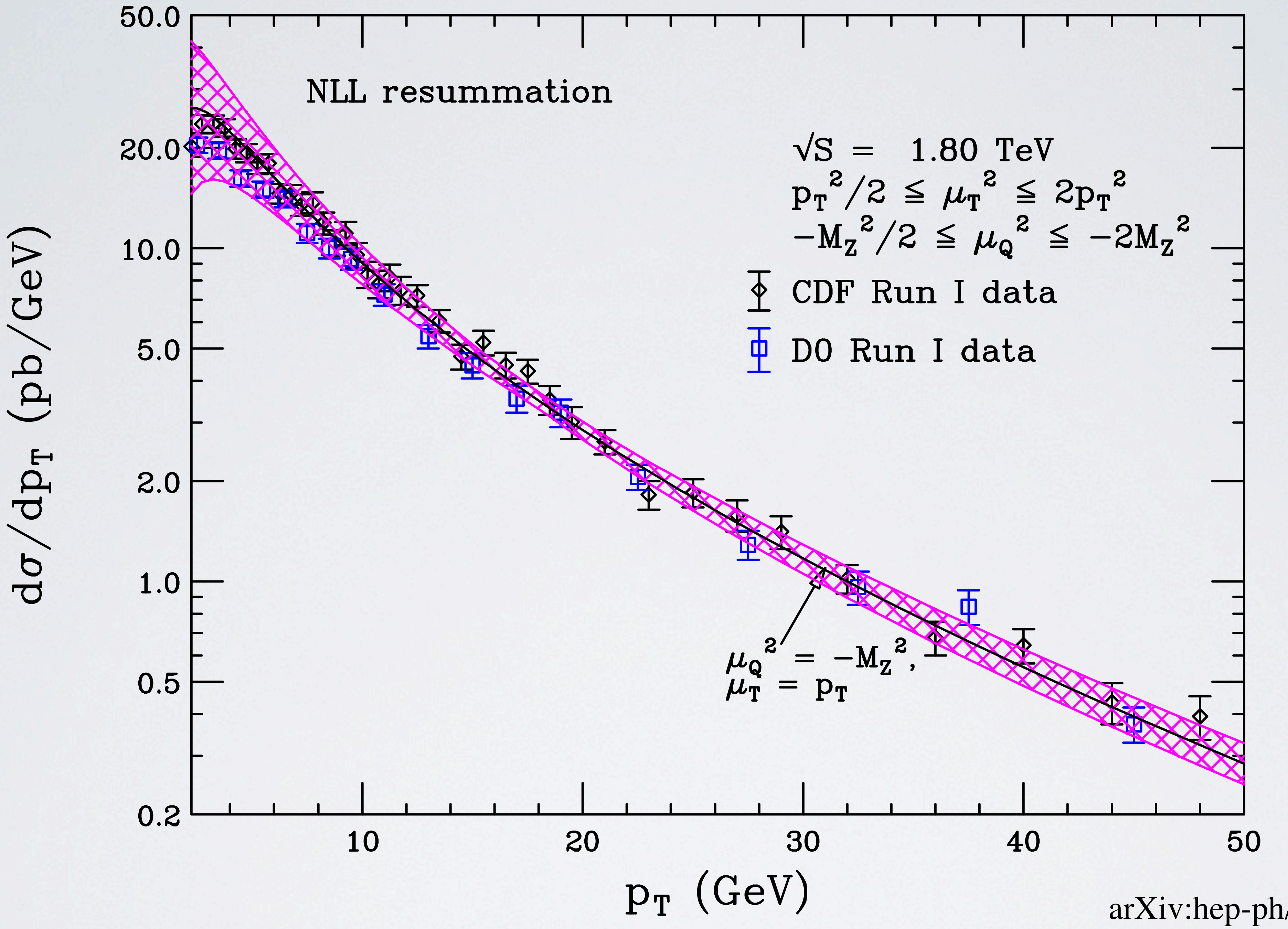


# Higgs pT Distribution




- Prediction for Higgs boson pT distribution.

# Z-production: Comparison with Data




arXiv:hep-ph/1011.0757

- Good agreement with data.
- Theory curve determined completely by perturbative functions and standard PDFs.

# Check to pQCD

- Expand resummed formula to compare to fixed order

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2 dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W} e_{q\bar{q}}^2 \frac{1}{s\,p_T^2}\sum_{m,n}\left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m \ln^m\frac{M_Z^2}{p_T^2}$$

$$\begin{aligned} \text{leading logarithmic} &: \alpha_s^n L^{2n-1}, \\ \text{next-to-leading logarithmic} &: \alpha_s^n L^{2n-2}, \\ \text{next-to-next-to-leading logarithmic} &: \alpha_s^n L^{2n-3}. \end{aligned}$$

Leading Log

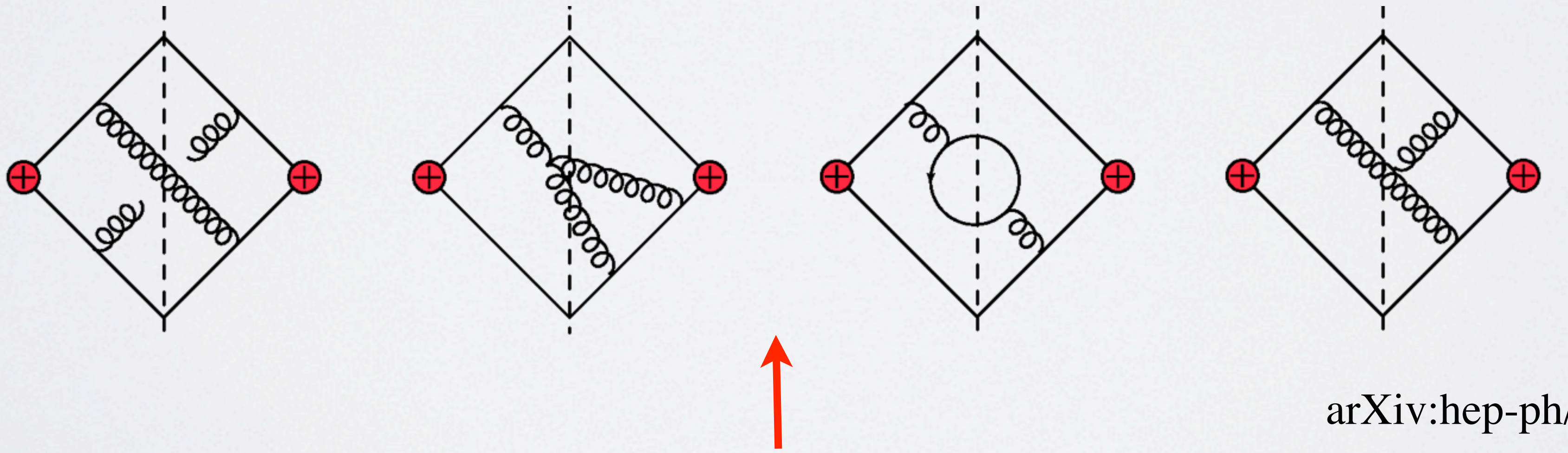
Next-to-Leading Log

$$\begin{aligned} {}_1D_1 &= A^{(1)} f_A f_B, \\ {}_1D_0 &= B^{(1)} f_A f_B + f_B \left(P_{qq}\otimes f\right)_A + f_A \left(P_{qq}\otimes f\right)_B, \\ {}_2D_3 &= -\frac{1}{2}\left[A^{(1)}\right]^2 f_A f_B, \\ {}_2D_2 &= -\frac{3}{2}A^{(1)}\left[f_B\left(P_{qq}\otimes f\right)_A + f_A\left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right] f_A f_B, \\ {}_2D_1 &= \left\{-A^{(1)} f_B \left(P_{qq}\otimes f\right)_A \ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)} f_B\left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_A f_B \right. \\ &\quad + \frac{\beta_0}{2}A^{(1)} f_A f_B \ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)} f_A f_B - \left(P_{qq}\otimes f\right)_A\left(P_{qq}\otimes f\right)_B \\ &\quad \left. - f_B\left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0 f_B\left(P_{qq}\otimes f\right)_A\right\} + \left[A\leftrightarrow B\right]. \end{aligned}$$

Agrees through NLL level
$_2D_1$ requires NNLL

# Next-to-Next-to Leading Logarithm

- NNLO Beam/Soft function required for NNLL resummation
- Soft function worked out as the first step
- NNLO beam function in progress



arXiv:hep-ph/1105.5171

Two loop graphs for soft function

# Soft Function at NNLO

- Anomalous dimensions in position and impact-parameter space
- Old result confirmed: Belitsky (hep-ph/9808389)
- New in impact-parameter space
- New renormalized soft function in full position and impact-parameter space

Define $L = -\frac{b^+ b^- \mu^2 e^{2\gamma_E}}{4}$

$$\gamma_s^{(1)}(b) = 2\frac{\alpha_s}{\pi} C_F \ln(L)$$

$$\gamma_s^{(2)}(b) = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ C_F N_F \left[ -\frac{5}{9}\ln(L) + \frac{\pi^2}{36} - \frac{14}{27} \right] + C_F C_A \left[ \left( -\frac{\pi^2}{6} + \frac{67}{18} \right) \ln(L) - \frac{7}{2}\zeta(3) - \frac{11\pi^2}{72} + \frac{101}{27} \right] \right\}$$

Define $L_{0,0} = \delta(q^-)\delta(q^+)$

and $L_{0,1} = \frac{1}{\mu}\left[\frac{\mu}{q^+}\right]_+ \delta(q^-) + \frac{1}{\mu}\left[\frac{\mu}{q^-}\right]_+ \delta(q^+)$

$$\gamma_s^{(1)}(q^-, q^+) = -2\frac{\alpha_s}{\pi} C_F L_{0,1}$$

$$\gamma_s^{(2)}(q^-, q^+) = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ C_F N_F \left[ \frac{5}{9} L_{0,1} + \left( \frac{\pi^2}{36} - \frac{14}{27} \right) L_{0,0} \right] + C_F C_A \left[ \left( \frac{\pi^2}{6} - \frac{67}{18} \right) L_{0,1} - \left( \frac{7}{2}\zeta(3) + \frac{11\pi^2}{72} - \frac{101}{27} \right) L_{0,0} \right] \right\}$$

# Summary

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Perturbative pT distribution given in terms of perturbatively calculable functions and the standard PDFs.

- Performed NLL resummation and found good agreement with data.

- Next step: NNLL resummation
  - Soft function done
  - Beam function in progress