Quantum Effects in 5D: A 5D NJL Model

Jay Hubisz
working with Don Bunk, Jing Shao, and Philip Tanedo

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4-Fermion Operators

- Augmentation of chirally symmetric fermion theory with irrelevant operator - nontrivial IR behavior
  - NJL - an effective QFT for nucleon masses
  - non-perturbative arguments favor the creation of a gap - broken chiral symmetry

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. Nambu and G. Jona-Lasinio†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.
The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II*

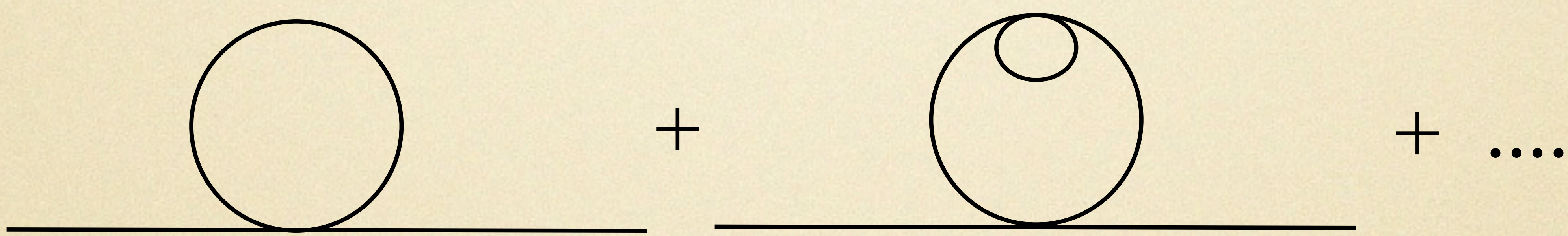
Y. Nambu and G. Jona-Lasinio†
Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois
(Received May 10, 1961)

Continuing the program developed in a previous paper, a "superconductive" solution describing the proton-neutron doublet is obtained from a nonlinear spinor field Lagrangian. We find the pions of finite mass as nucleon-antinucleon bound states by introducing a small bare mass into the Lagrangian which otherwise possesses a certain type of the γ_5 invariance. In addition, heavier mesons and two-nucleon bound states are obtained in the same approximation. On the basis of numerical mass relations, it is suggested that the bare nucleon field is similar to the electron-neutrino field, and further speculations are made concerning the complete description of the baryons and leptons.

The basic NJL Model

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + g_0\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

for positive g_0, have an attractive fermion potential



self consistency relation: $m - m_0 = \Sigma(p, m, g, \Lambda)|_{p\!\!\!/ = m}$

for m_0=0, there exist non-trivial solutions:

$$m = -\frac{g_0 m i}{2\pi^4}\int^{\Lambda}\frac{d^4p}{p^2 - m^2}$$

$m \neq 0$ when $g_0\Lambda^2 > 2\pi^2$

trivial sol'n is unstable

Auxiliary Scalars

we can analyze this theory in a simple way

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\tilde{g}_0^2}{\Lambda^2}\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L$$

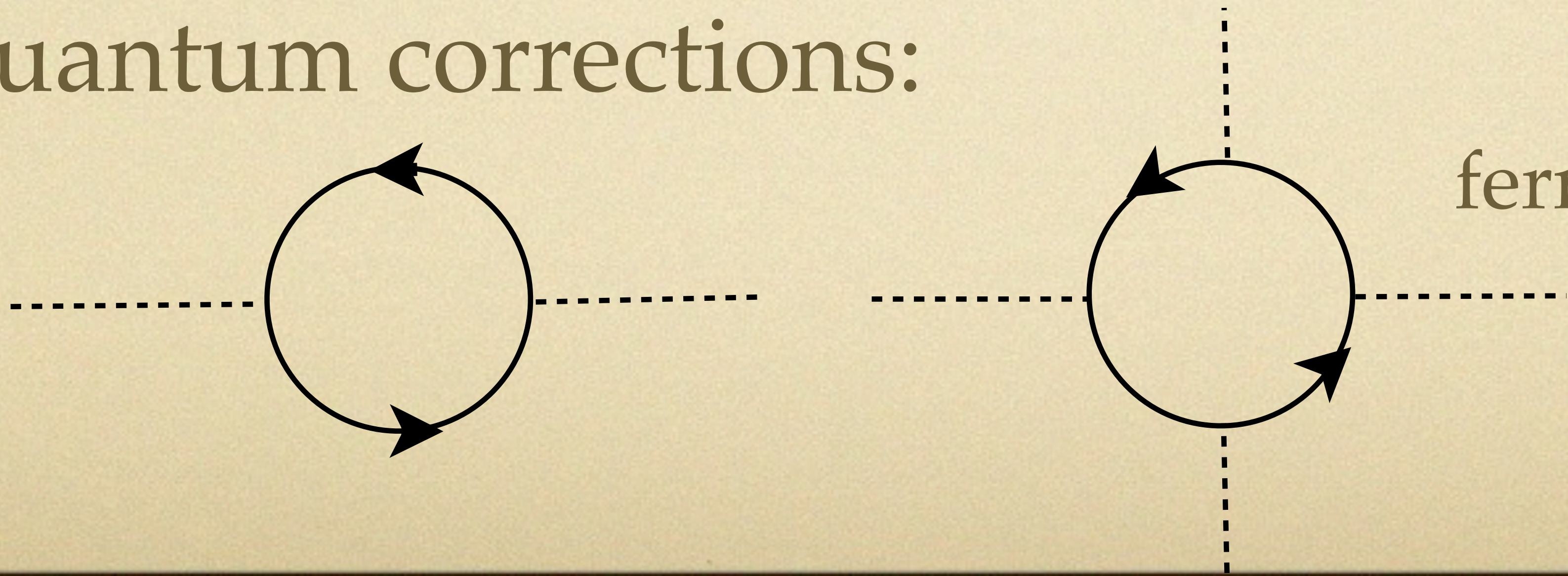
$$\equiv i\bar{\psi}\partial\!\!\!/\psi - \Lambda^2|H|^2 + \tilde{g}_0 H\bar{\psi}_R\psi_L + \text{h.c.}$$

H carries chiral charge

Impose the H eom: $H = \frac{\tilde{g}_0}{\Lambda^2}\bar{\psi}_L\psi_R$

defines theory at scale Λ

Quantum corrections:



fermion bubble approx.

Dynamics in the IR

At an IR scale μ, the theory develops dynamics:

$$\begin{aligned}\mathcal{L}_\mu &= \mathcal{L}_{kinetic} + g\overline{\psi}_L \psi_R H + h.c. \\ &\quad + Z_H |\partial_\nu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2\end{aligned}$$

$$Z_H = \frac{\tilde{g}_0^2}{(4\pi)^2}\log \Lambda^2/\mu^2 \qquad m_H^2 = \Lambda^2 - 2\frac{\tilde{g}_0^2}{(4\pi)^2}\left(\Lambda^2 - \mu^2\right)$$

$$\lambda_0 = \frac{2\tilde{g}_0^4}{(4\pi)^2}\log \Lambda^2/\mu^2$$

mass is driven negative at critical coupling

stabilized by quartic

chiral symmetry spontaneously broken

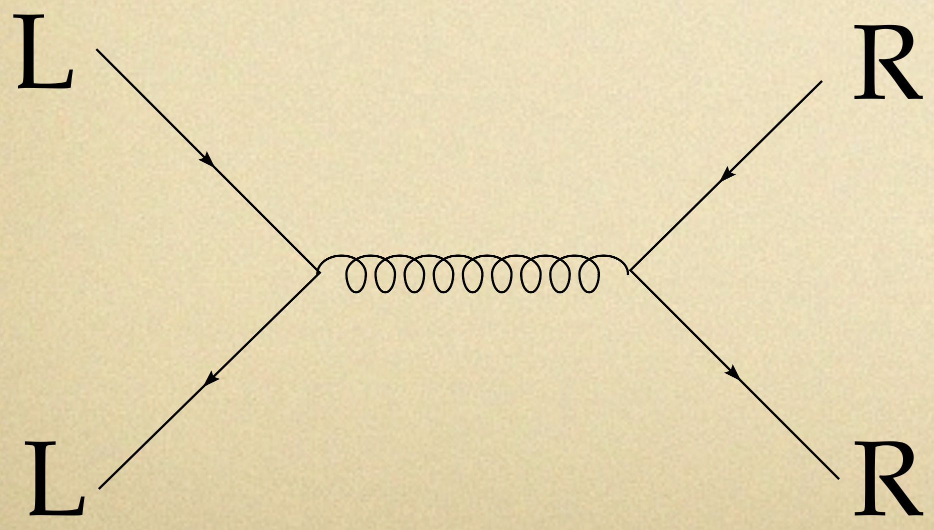
The "minimal" standard model

Bardeen, Hill, Lindner 1990

- Augment a chiral SM with a 4 fermi operator for top quarks

$$\delta\mathcal{L} = \frac{g^2}{\Lambda^2}\bar{t}_L t_R \bar{t}_R t_L$$

could arise from a variety of different UV scenarios
i.e. gauge symmetry with coupling g broken at scale Λ



for super-critical coupling, top quark gains a mass, and weak interactions are spontaneously broken

5D Orbifolds

e.g. Hall, Nomura 2001
Kawamura 2001

z is extra dimensional coordinate

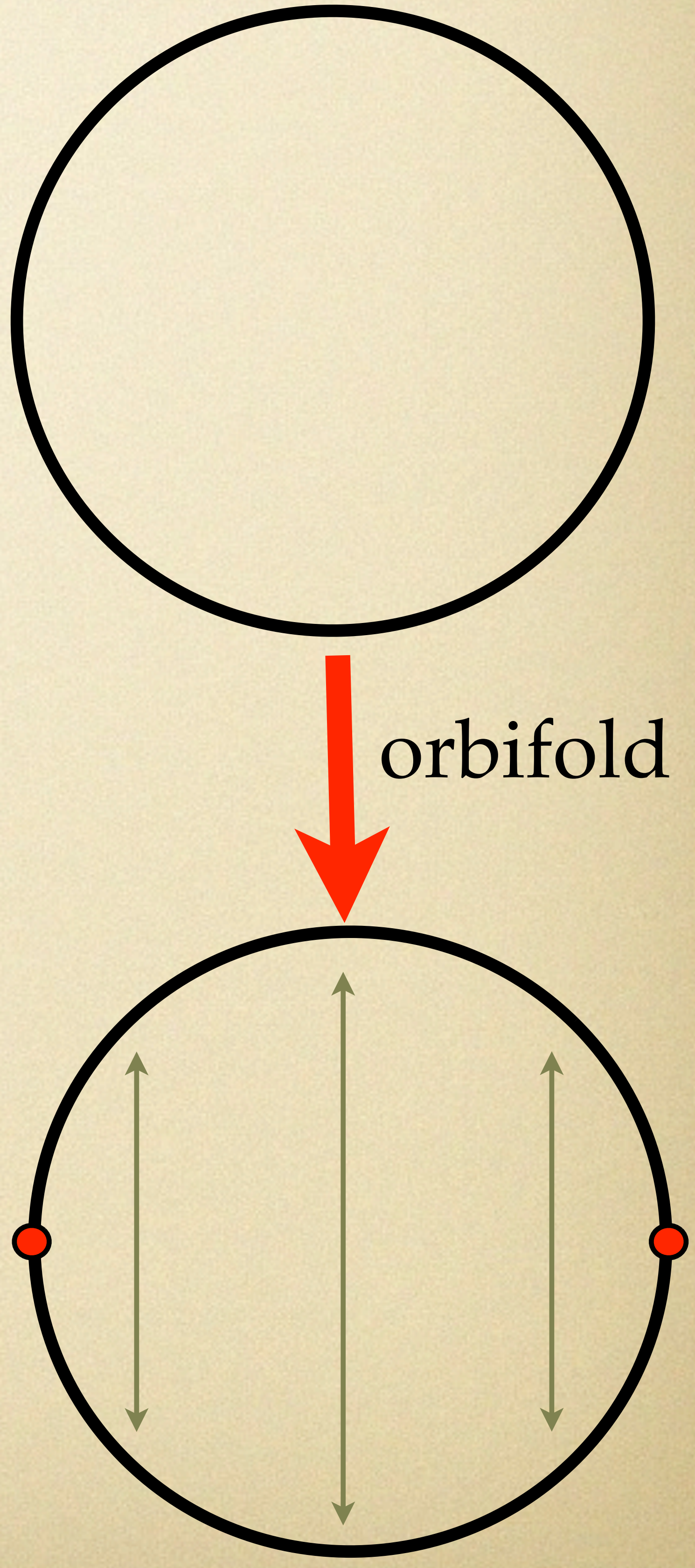
$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dz^2$$

compactified on an S_1/Z_2 orbifold

$$\left(z = \frac{L}{\pi}\theta \right)$$

identify z with -z

fixed points at $z = 0, L$



5D Fermions

Fields may be even or odd under projection, but surviving operators are all even

$$\mathcal{L}_{\text{Dirac}} = \bar{\Psi}_L(x,z)\,(i\partial\!\!\!/ - \gamma_5\partial_z)\,\Psi_L + \bar{\Psi}_R(x,z)\,(i\partial\!\!\!/ - \gamma_5\partial_z)\,\Psi_R$$

Consistent fixed point boundary conditions:

$$\Psi_{L,R}(x,z) = \pm\gamma_5\Psi_{L,R}(x,-z)$$

$\Psi_{L,R}$ contain LH/RH zero mode in KK spectrum

mass terms forbidden: $\bar{\Psi}_L\Psi_L(z) = -\bar{\Psi}_L\Psi_L(-z)$

A Flat 5D NJL Model

$$\mathcal{L} = \mathcal{L}_{\rm Dirac} + \frac{g^2}{\Lambda_0^3}\bar{\Psi}_L\Psi_R\bar{\Psi}_R\Psi_L$$

$$= \mathcal{L}_{\rm Dirac} - \Lambda_0^2|H|^2 + \frac{g}{\sqrt{\Lambda_0}}H\bar{\Psi}_L\Psi_R + \text{h.c.}$$

In low energy 4D effective theory, operator is chirally symmetric NJL

$$H = \frac{g}{\Lambda^{5/2}}\bar{\Psi}_R\Psi_L$$

under orbifold identification: $\bar{\Psi}_L\Psi_R(z) = +\bar{\Psi}_L\Psi_R(-z)$

scalar H is even under orbifold parity

task is to compute quantum corrections in this 5D Yukawa theory and identify the ground state

Choosing a regulator

- Orbifolding breaks 5D Lorentz invariance
  - this is primarily an IR effect
- Want a regulator which respects it in order to trust results
- Dim. reg. + zeta function
  - sum over all KK modes, integrate over all 4D momenta
  - automatically subtracts all power law dependence on cutoff
- 5D implementation of hard cutoff

5D Hard Cutoff

Want to integrate/sum over sphere of Euclidean 5-momentum

$$(k_E^0)^2 + (k_E^1)^2 + (k_E^2)^2 + (k_E^3)^2 + (k_E^5)^2 \leq \Lambda^2$$

Can replace loop sums by integral (Euler-MacLaurin):

$$\sum_{n=a}^{b} f(n) = \int_a^b dn f(n) + \frac{f(a)+f(b)}{2} + \sum_{k=1}^{\infty} \frac{B_{2k}}{2k!} \left(f^{(2k-1)}(b) - f^{(2k-1)}(a) \right)$$

$$\left(\frac{1}{2L}\right) \sum_{k_5} \int_{k_E^2 \leq \Lambda^2 - k_5^2} \frac{d^4 k_E}{(2\pi)^4} I(k_E, k_5) \sim \int_{K_E^2 \leq \Lambda^2} \frac{d^5 K_E}{(2\pi)^5} I(k_E, k_5)$$

Correctly captures divergence structure - matches continuum calculation

Fermion Propagators

In flat space (no warping):

conserves 5D momentum (up to a sign):

$$S_F^{(L,R)}(p;p_5,p_5') = (2L)\frac{i}{2}\left\{\frac{\delta_{p_5,p_5'}}{\not{p}+i\gamma_5 p_5} \pm \frac{\delta_{-p_5,p_5'}}{\not{p}+i\gamma_5 p_5}\gamma_5\right\}$$

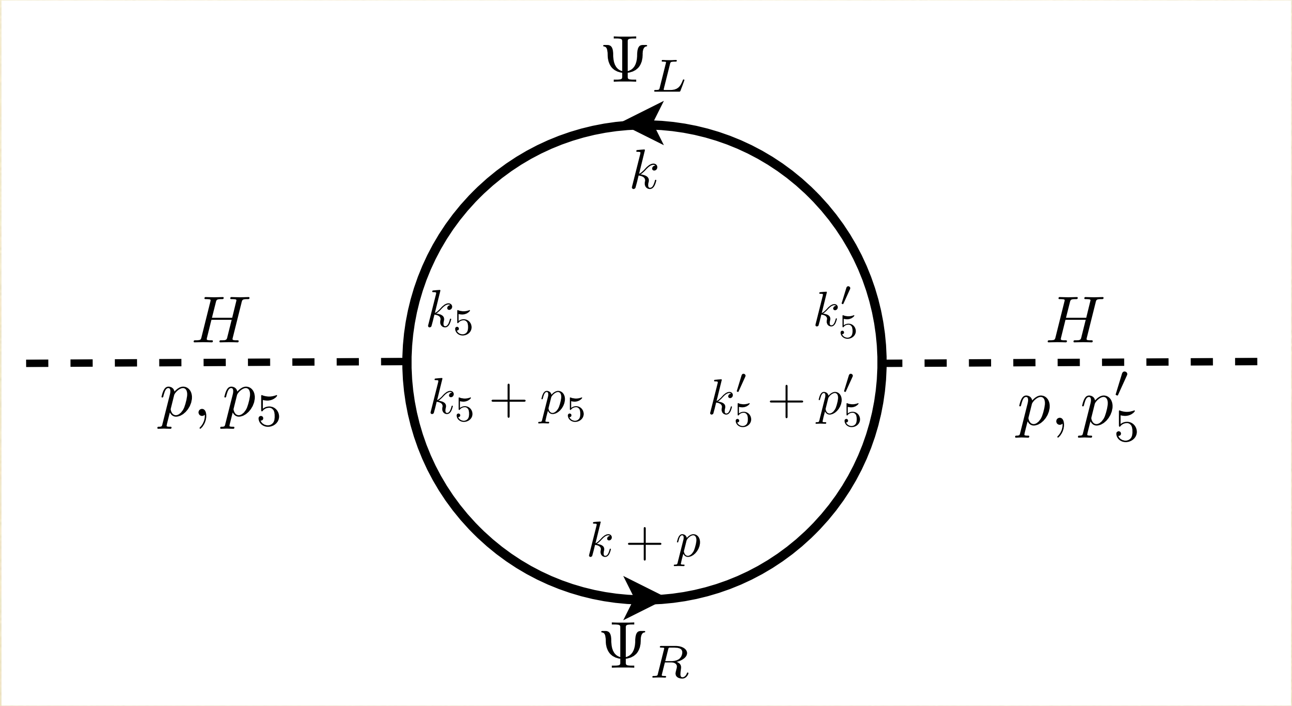
p_5 quantized: $$p_5 = \frac{n\pi}{L}$$

orbifold breaks translation invariance

For a 5D scalar with even/odd orbifold assignments:

$$D(p^2;p_5,p_5') = (2L)\frac{i}{2}\left\{\frac{\delta_{p_5,\,p_5'} \pm \delta_{-p_5,\,p_5'}}{p^2-p_5^2}\right\}$$

Scalar Self Energy



$$-\frac{g^2}{\Lambda_0}\sum_{k_5,k_5'}\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\left[\frac{(\not{k}+i\gamma_5k_5)(\delta_{k_5,k_5'}-\gamma_5\delta_{k_5,-k_5'})}{k^2-k_5^2}\cdot\frac{(\not{k}+\not{p}+i\gamma_5[k_5'+p_5'])(\delta_{k_5+p_5,k_5'+p_5'}+\gamma_5\delta_{k_5+p_5,-k_5'-p_5'})}{(k+p)^2-(k_5'+p_5')^2}\right]$$

4 structures from kronecker-delta's

$\delta_{p_5,p_5'}$ $\delta_{p_5,-p_5'}$ $\delta_{2k_5+p_5+p_5',0}$ $\delta_{2k_5+p_5-p_5',0}$

Bulk terms Brane terms

Bulk Corrections

In 5D L-R Yukawa theory - scalar is KK even

Coefficients of $\delta_{p_5, p_5'}$ and $\delta_{p_5, -p_5'}$ identical

renormalized propagator retains KK parity even form

$$i\Pi(p^2; p_5, p_5') = iL\left(\delta_{p_5,p_5'} + \delta_{p_5,-p_5'}\right)\left[\frac{g^2\Lambda^3}{18\pi^3\Lambda_0} + \frac{g^2\Lambda}{10\pi^3\Lambda_0}P^2\right]$$

later take $\Lambda = \Lambda_0$

Brane Divergences

- At one loop, the brane action is unchanged
  - remnant of 5D translation invariance protecting it

$$-\frac{g^2}{\Lambda_0}\sum_{k_5,k_5'}\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\left[\frac{(\not k+i\gamma_5k_5)(\delta_{k_5,k_5'}-\gamma_5\delta_{k_5,-k_5'})}{k^2-k_5^2}\cdot\frac{(\not k+\not p+i\gamma_5[k_5'+p_5'])(\delta_{k_5+p_5,k_5'+p_5'}+\gamma_5\delta_{k_5+p_5,-k_5'-p_5'})}{(k+p)^2-(k_5'+p_5')^2}\right]$$

cross terms in kroneckers - odd # of γ's with γ_5

$\delta_{2k_5+p_5+p_5',0}$ $\delta_{2k_5+p_5-p_5',0}$ terms vanish

Remnant of the Circle

Scalar field on circle: $\phi(z,x) = \phi_+(z,x) + \phi_-(z,x)$

In loops, they act together to maintain 5D Lorentz invariance

Orbifold breaks 5D LI by leaving out half this spectrum - brane localized terms generated

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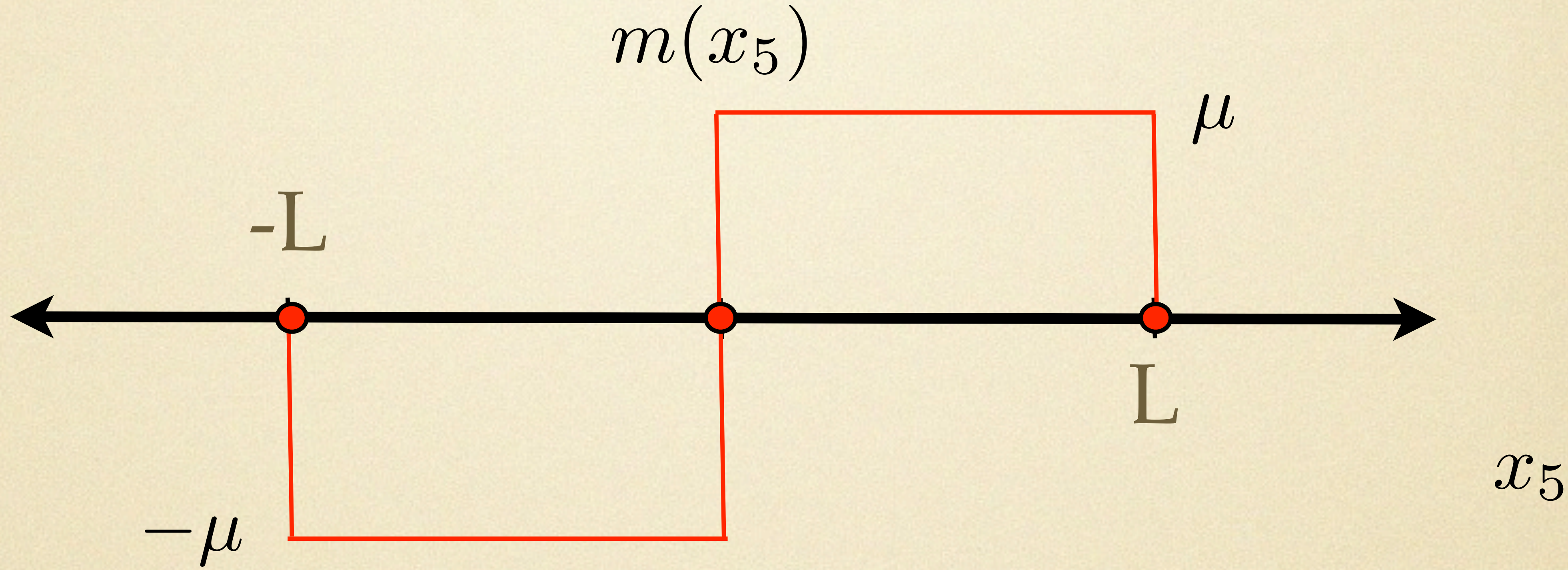
5D fermion: $\Psi(z,x) = \begin{pmatrix} \psi_+ \\ \bar{\chi}_- \end{pmatrix}$

Single 5D fermion on orbifold contains even and odd modes

They *may* conspire together to manifest cancellation

5D fermion masses



tightly kinked scalar domain wall

fermion zero modes survive, but localize exponentially to the domain walls:

$\xi_0(z) \propto e^{\mu z}$ Kaplan, Tait 2001

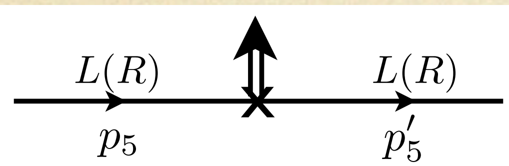
translation invariance on fixed points broken - prop. to μ

Softened divergences

- Brane localized divergences are proportional to this explicit breaking

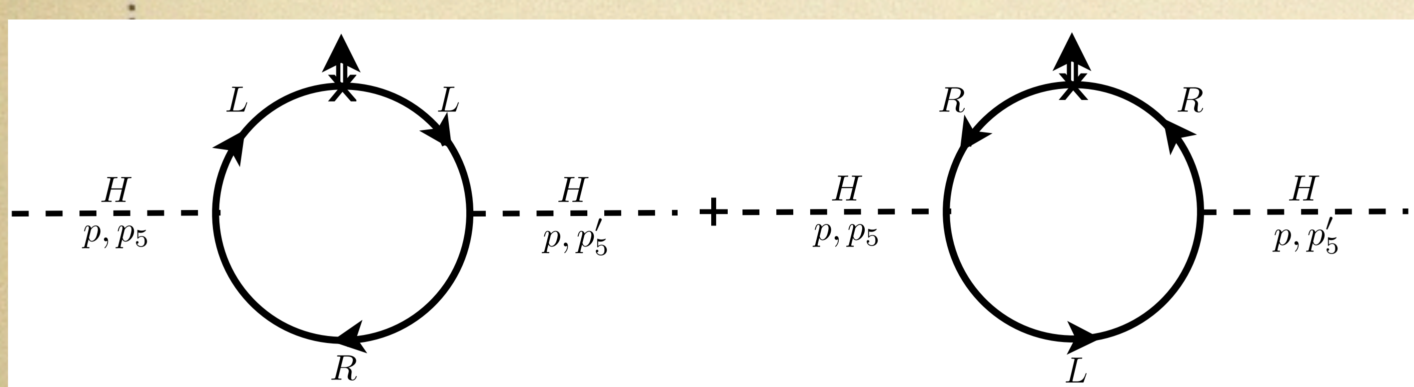
- brane mass correction is linearly divergent

  performed using mass insertion approximation



$$= \frac{4m_{L(R)}}{p'_5 - p_5}\,\delta^{\text{odd}}_{p_5,p'_5}$$

$$\delta^{\text{odd}}_{p_5,p'_5} \equiv \begin{cases} 1 & p_5 + p'_5 \text{ odd multiple of } \pi/L \\ 0 & p_5 + p'_5 \text{ even multiple of } \pi/L \end{cases}$$

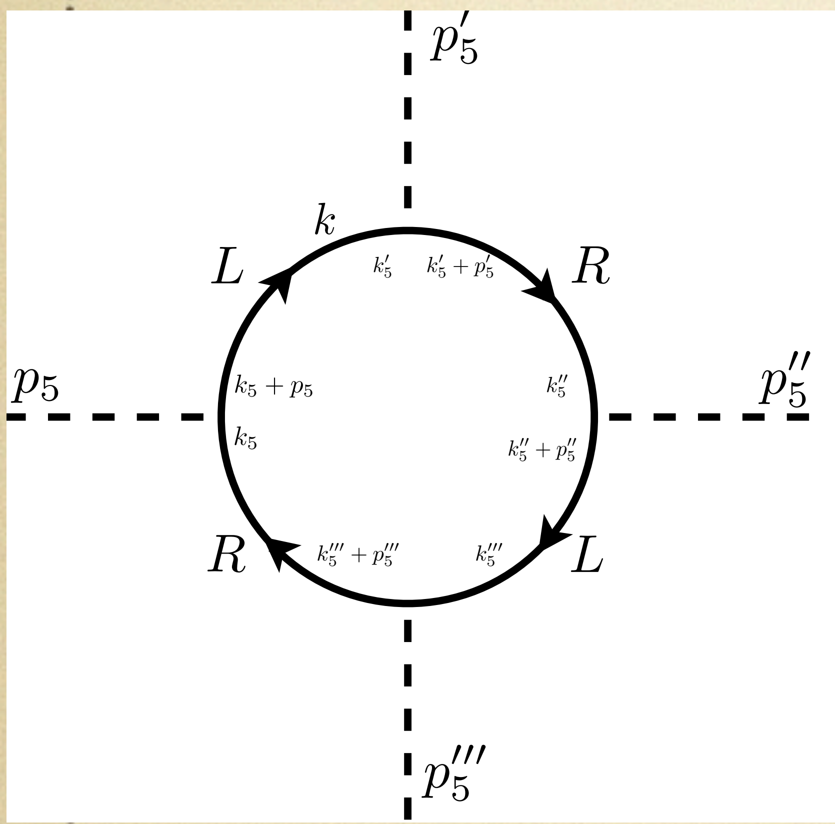


$$i\Pi_M(0; p_5, p'_5) = i\frac{g^2\Lambda}{3\pi^3\Lambda_0}\,(m_L - m_R)\,\delta^{\text{odd}}_{p_5,p'_5}$$

$$\delta^{\text{odd}}_{q_5,q'_5} \to \frac{1}{2}\left[\delta(z) - \delta(z-L)\right]$$

Opposite sign mass on either brane

Quartic Coupling



$$= -\frac{g^4}{\Lambda_0^2} \sum_{\substack{k_5,k_5',\\ k_5'',k_5'''}} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \left[S_F^R(k;k_5,k_5'''+p_5''') S_F^L(k;k_5''',k_5''+p_5'') \right.$$

$$\left. \times\ S_F^R(k;k_5'',k_5'+p_5') S_F^L(k;k_5''',k_5''+p_5'') \right]$$

again, divergent brane localized terms vanish due to trace structure - mass insertions generate only finite corrections

Bulk term is linearly divergent

$$iV_4(0;p_5,p_5',p_5'',p_5''') = \frac{-ig^4\Lambda}{24\pi^3\Lambda_0^2}(2L)\sum_{\pm} \delta_{0,p_5 \pm p_5' \pm p_5'' \pm p_5'''}$$

5D Effective Theory

Canonically normalized 5D fields

$$S = \int d^4x \int_{-L}^{L} dz \left[\bar{\Psi}_L \left(i\partial\!\!\!/ - M_L(z)\right) \Psi_L + \bar{\Psi}_R \left(i\partial\!\!\!/ - M_R(z)\right) t_R + \frac{\widetilde{g}}{\sqrt{\Lambda_0}} H \bar{\Psi}_L \Psi_R + \text{h.c.} \right.$$
$$\left. + \partial_M H \partial^M H^\dagger - \widetilde{m}^2 |H|^2 - \frac{\widetilde{\lambda}}{4\Lambda_0} |H|^4 \right] + \int d^4x \left[\widetilde{m}_0^2 \left. |H|^2 \right|_{z=0} + \widetilde{m}_L^2 \left. |H|^2 \right|_{z=L} \right]$$

$$\widetilde{g}^2 = \frac{10\pi^3}{N_c}$$ ← 5D Yukawa

$$\widetilde{m}^2 = \left(\frac{10\pi^3}{N_c g^2} - \frac{5}{9} \right) \Lambda_0^2$$ ← Bulk mass

$$\widetilde{\lambda} = \frac{100\pi^3}{3N_c}$$ ← Bulk Quartic

$$\widetilde{m}_0^2 = -\widetilde{m}_L^2 = \frac{5}{3}(m_R - m_L).$$ ← Opp. sign brane masses

Chiral Symmetry Breaking

$\langle H(z,x)\rangle \equiv v(z)/(2\sqrt{L})$

$$v''(z) = \widetilde{m}^2 v(z) + \frac{\widetilde{\lambda}}{8\Lambda_0 L} v^3(z)$$

Solutions are Jacobi Elliptic functions

$$v(z) = \sqrt{\frac{8\Lambda_0 L \kappa_-}{\lambda}} \mathrm{sc}\left(|z - z_0| \sqrt{\frac{\kappa_+}{2}} \middle| 1 - \frac{\kappa_-}{\kappa_+} \right)$$

$$\kappa_\pm = \widetilde{m}^2 \pm \sqrt{\widetilde{m}^4 - \frac{\widetilde{\lambda}\widetilde{m}^2 v_0^2}{4\Lambda_0 L}}$$

v_0 and z_0 are integration constants

Determine by imposing boundary conditions

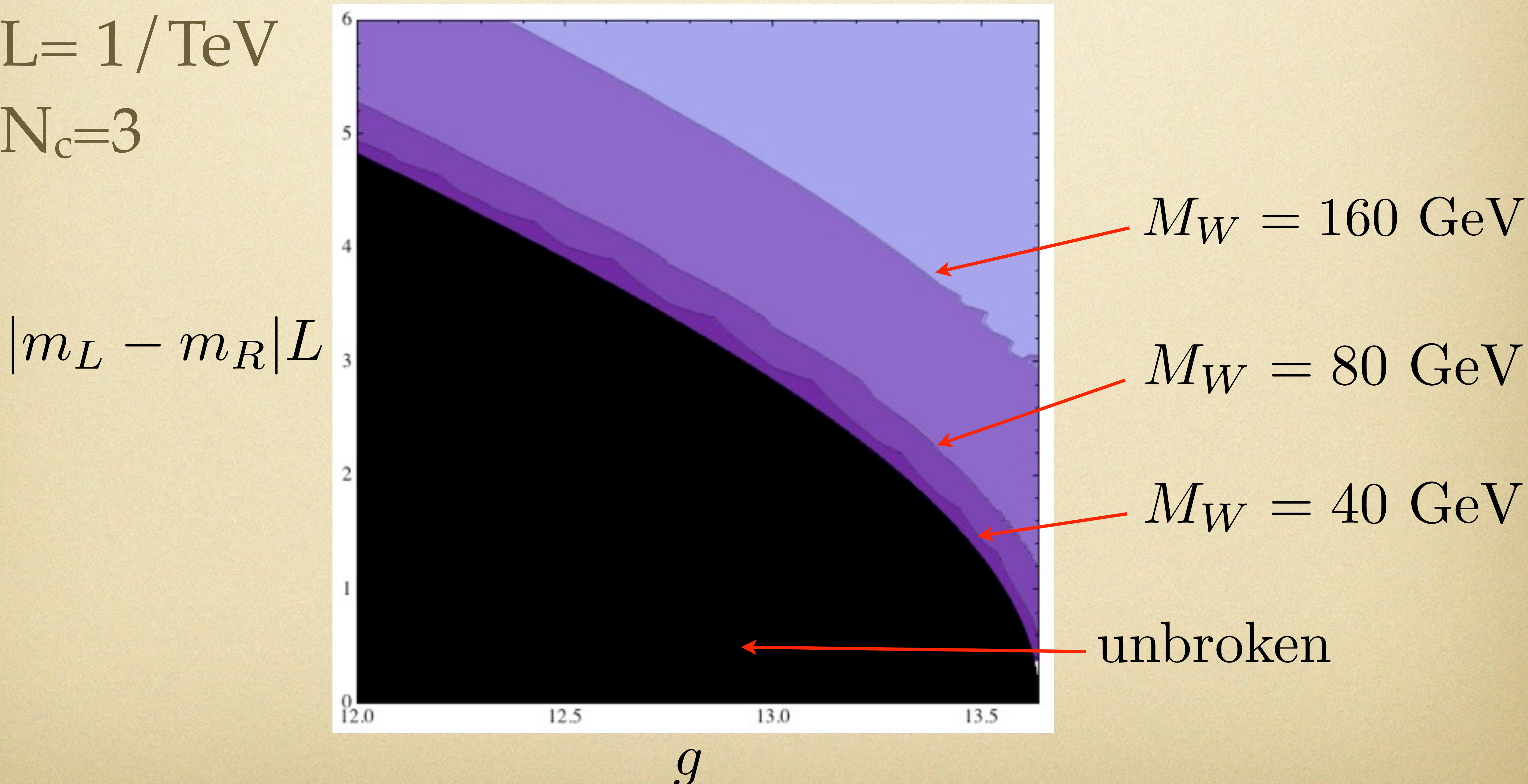
$$\left.\frac{v'(z)}{v(z)}\right|_{z=0} = \frac{1}{2}\widetilde{m}_0^2 \qquad \left.\frac{v'(z)}{v(z)}\right|_{z=L} = -\frac{1}{2}\widetilde{m}_L^2$$

Phase Boundary

derived analytically: $g^2_{\text{critical}} = \frac{18\pi^3}{N_c}\left[1 + \frac{5}{4}\frac{(m_R - m_L)^2}{\Lambda_0^2}\right]^{-1}$

L= 1/TeV

N_c=3



Higgs and top mass

parameters chosen to get correct
W mass and top mass

$(m_R - m_L)$	m_R	m_{Higgs}
1.4	6.9	1.4
3.3	8.5	1.9
4.4	9.5	2.3
1.8	3.6	1.4
3.8	5.6	2.5
4.8	6.6	3

in toy model, Higgs mass too large (perturbative unitarity)

Can increase Λ_0 L, and N_c to decrease m_H

Conclusions

- We are exploring a new method of symmetry breaking in extra dimensional theories
- In flat 5D theories, the NJL prescription carries over straightforwardly, with some interesting results in the UV structure of brane localized terms
  - brane localized running softer than naive estimates - implications for NDA size of brane localized terms in ED theories
- currently carrying over flat space results to warped space - expect better behavior