Plasma science - From laboratory-fusion to astrophysical plasmas

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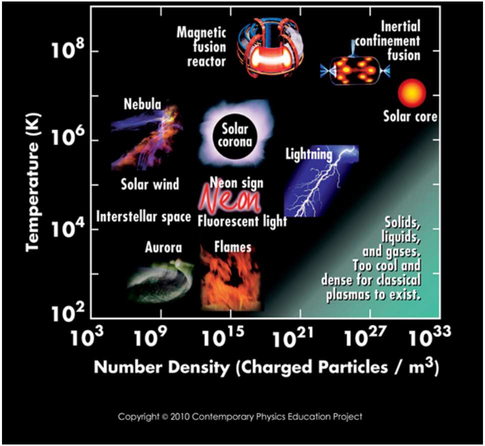
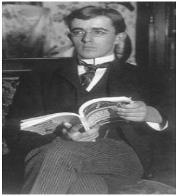
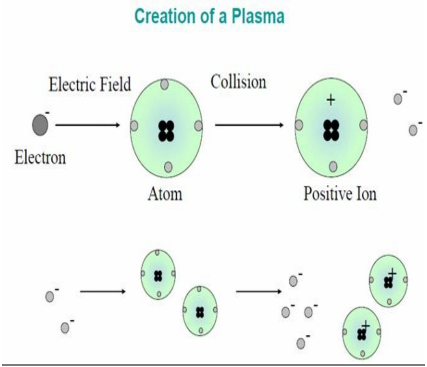




Langmuir first used the term plasma in 1927

About 99% of the matter in the observable universe is in plasma state.



Free cold electrons accelerate by electric field/heating

Plasma: ionized gas

- Collective interactions: particles can interact at long ranges through the electric and magnetic forces
- The electron plasma frequency (Langmuir oscillations) is large compared to the electron-neutral collision frequency
- Could have high conductivity
- Multi-species: electrons, ions, neutrals
- Could have non-Maxwellian velocity distribution

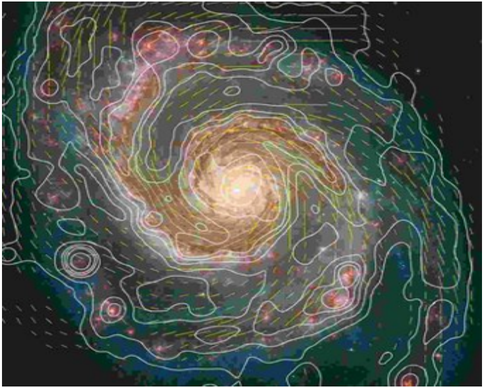
There are outstanding common physics problems in most of these plasmas

- I. Dynamo (generation of magnetic field - energy and flux)
- II. Magnetic reconnection - the rearrangement of magnetic field topology of plasmas. Would magnetic field be annihilated while being generated?
- III. Momentum transport

In all processes turbulence is prevalent

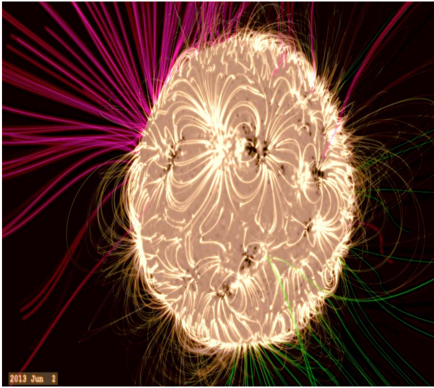
I. Magnetic fields are observed to exist essentially everywhere in the universe - large-scale field generation (dynamo)

Galactic magnetic field



Galactic dynamos must sustain an ordered magnetic field

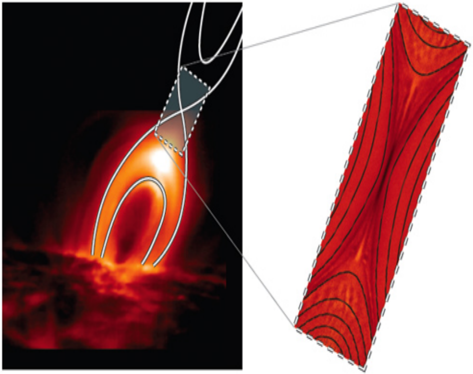
Solar/stellar magnetic field



Sun exhibits dynamo cycles

II. Burst-like phenomena may be due to dissipation of magnetic field (through magnetic reconnection)

Reconnection, the rearrangement of magnetic field topology



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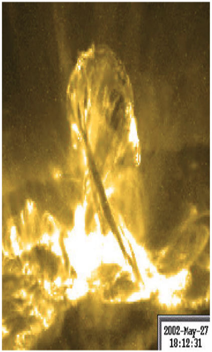
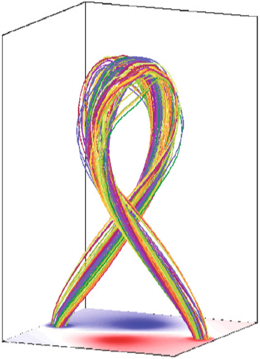
Energizes many processes in nature, solar flares, Earth's magnetosphere, accretion disk flares

Observed twistedness of magnetic field lines in nature can be expressed as global quantity

- What do we know about magnetic fields?
- Energy and flux are the most familiar global quantities of magnetic fields.
- How about the topological property?





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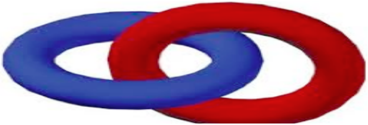
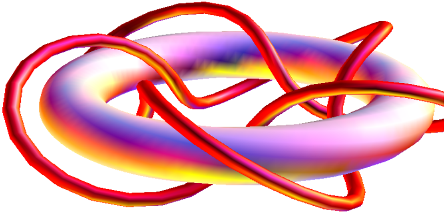
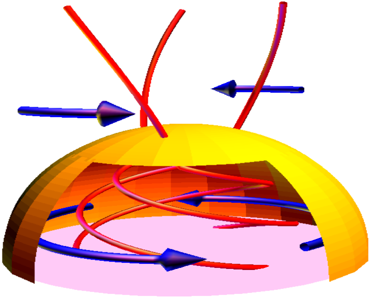
Magnetic helicity measures field linkage in both lab and astrophysical plasmas

Magnetic helicity, a topological property, measures the knottedness and the twistedness of magnetic fields



$$K = \int A \cdot B dV = 2\,\phi\,\phi$$



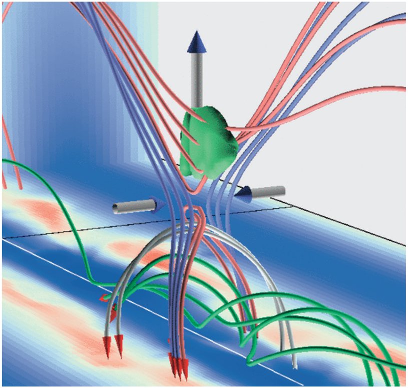


(5,2) field line links the toroidal flux inside.

The interior of the sun. Differential rotation provides a strong source of helicity injection. (from Berger 99)

Nature tries to get rid of magnetic helicity through magnetic reconnection

Solar flares field lines modeling



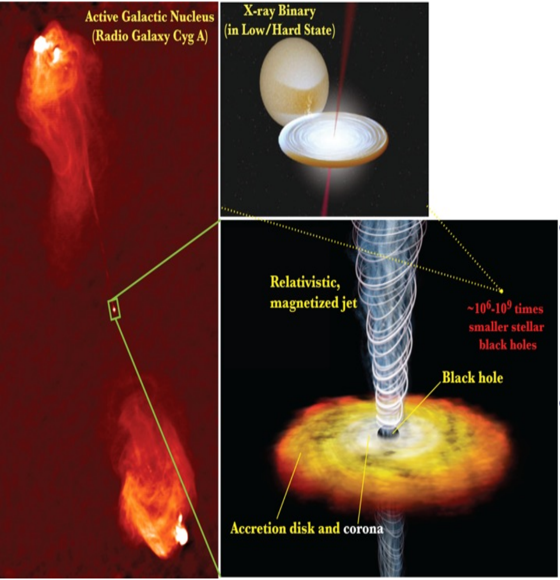
$$\frac{\partial K}{\partial t} = -2\int (\mathbf{A.V})\mathbf{B} \cdot d\mathbf{s}$$

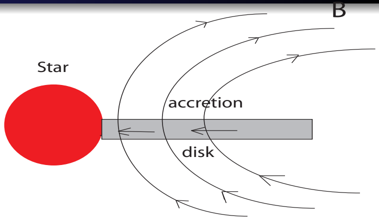
Kusano 2004

Warnecke & Brandenburg 2010

- Magnetic helicity is injected through the twisting of magnetic field lines in the corona region via the relative shear motion of their foot points.
- Field line twisting during helicity injection could bring the oppositely directed field lines together to form current sheets and eventually to cause magnetic reconnection

III. How is momentum transported during accretion process and jet collimation?

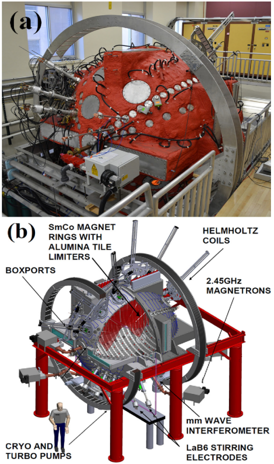




- What causes gas to be drawn in towards black holes, rather than remain in a stable orbit as planets do around the Sun? [S. Balbus]
- Magnetic field is required to cause accretion (the process of angular momentum removal), the collimation of jets and star formation.

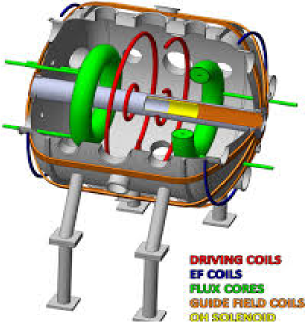
Laboratory experiments are dedicated to explore these fundamental plasma astrophysical problems

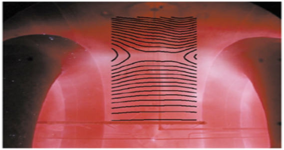
Plasma dynamo



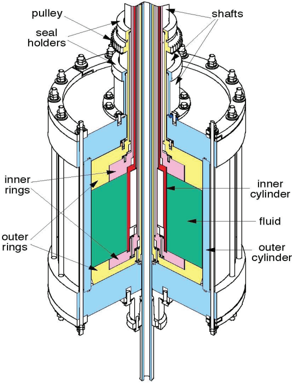
UW-Madison

Flare Experiment Reconnection (PPPL)





Momentum transport

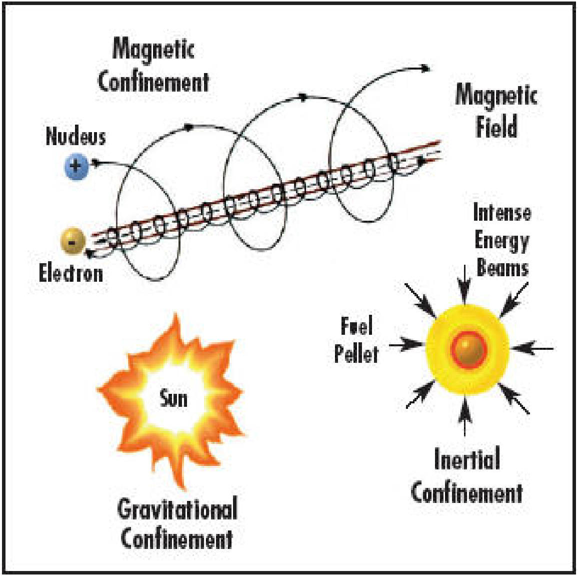


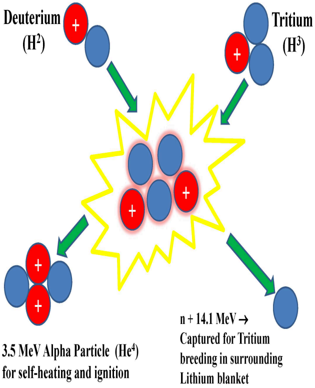
Spinning disks in the lab (PPPL)

PPPL PRINCETON UNIVERSITY

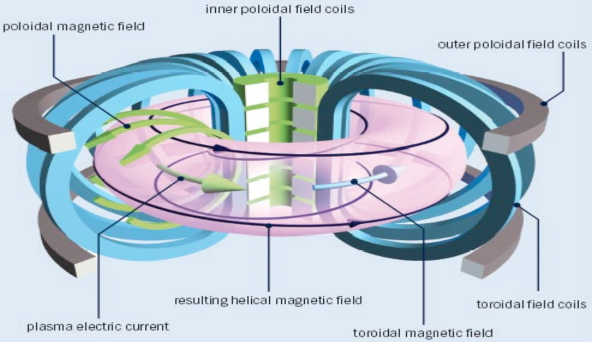
All the physics problems above also exist in fusion plasmas

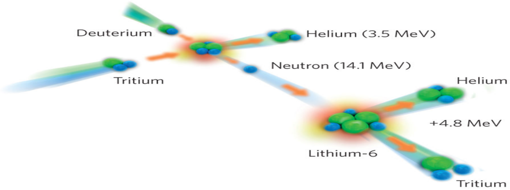
Plasmas need to be confined in laboratory





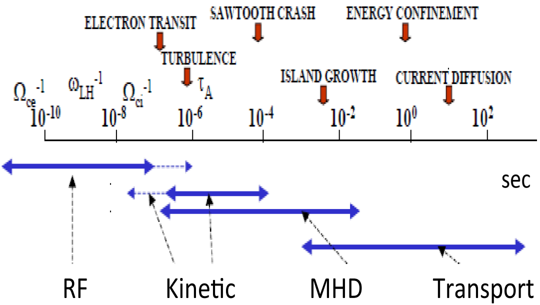
Magnetically confined fusion plasmas

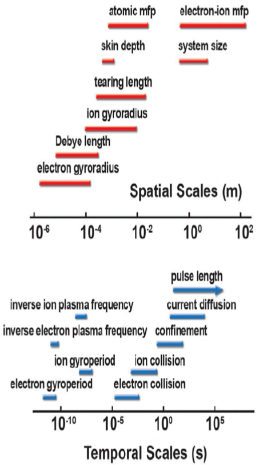




N. Holloway, CCFE.

There are enormous multi-scale challenges for modeling burning tokamak plasma





MHD Magnetohydrodynamics (MHD) is a powerful model for understanding macroscopic plasma dynamics.

Magnetohydrodynamics (MHD) equations describe macroscopic behavior of electrically conducting fluids (including plasmas)

$$\text{Mass} \Rightarrow \frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0$$

$$\text{Momentum} \Rightarrow \rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}.\nabla \mathbf{V}\right) = \underbrace{\mathbf{J} \times \mathbf{B}}_{\text{Lorentz}} - \underbrace{\nabla P}_{\text{pressure gradient}}$$

$$\text{Pressure} \Rightarrow \frac{\partial P}{\partial t} + \mathbf{V}.\nabla P = -\gamma P \nabla.\mathbf{V}$$

$$\text{Faraday's} \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\text{Ohm's} \Rightarrow \mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}$$

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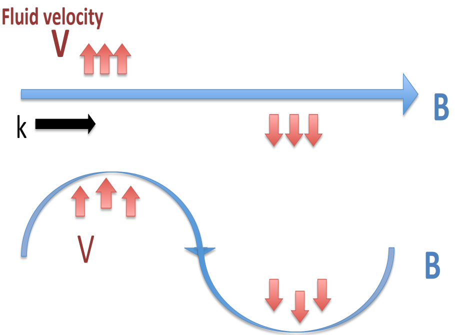
Fluid elements move with the magnetic field.



$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B}) = -(\mathbf{V}.\nabla)\mathbf{B} + \underbrace{(\mathbf{B}.\nabla)\mathbf{V}}$$

bending of lines

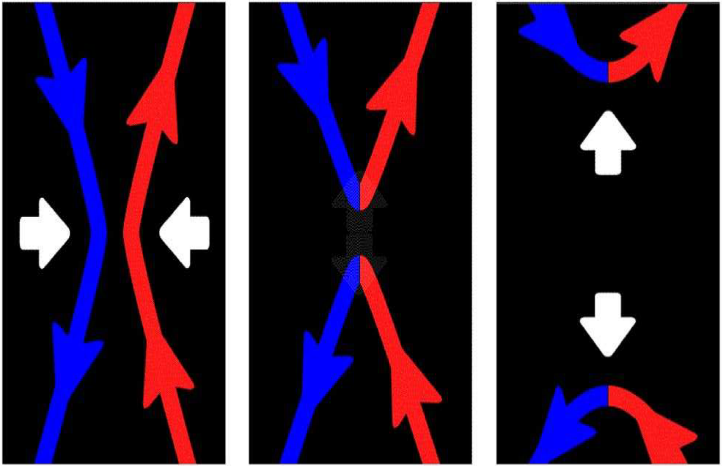
Shear
Alfven waves: $\omega = \mathbf{k} \cdot \mathbf{V}_A$; $\mathbf{V}_A = \frac{\mathbf{B}}{\sqrt{\mu_0 \rho}}$

Hannes Alfven: 1970 Nobel Prize in physics for his pioneering work on MHD

How about when magnetic field is not ideally coupled to the fluid?

Could magnetic field lines break?

Magnetic reconnection is the rearrangement of magnetic field topology of plasmas.



Credits: Center for Visual computing, Univ. of California Riverside

Current-driven magnetic reconnection - Tearing instability
Spontaneous reconnection

THE
PHYSICS OF FLUIDS

VOLUME 6, NUMBER 4 APRIL 1963

Finite-Resistivity Instabilities of a Sheet Pinch

HAROLD P. FURTH AND JOHN KILLEEN
Lawrence Radiation Laboratory, Livermore, California
AND
MARSHALL N. ROSENBLUTH
University of California, San Diego, La Jolla, California, and John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California
(Received 17 September 1962)

The stability of a plane current layer is analyzed in the hydromagnetic approximation, allowing for finite isotropic resistivity. The effect of a small layer curvature is simulated by a gravitational field. In an incompressible fluid, there can be three basic types of "resistive" instability: a long-wave "tearing" mode, corresponding to breakup of the layer along current-flow lines; a short-wave "rippling" mode, due to the flow of current across the resistivity gradients of the layer; and a low-g gravitational interchange mode that grows in spite of finite magnetic shear. The time scale is set by the resistive diffusion time τ_R and the hydromagnetic transit time τ_H of the layer. For large $S = \tau_R/\tau_H$, the growth rate of the "tearing" and "rippling" modes is of order $\tau_R^{-3/5}\tau_H^{-2/5}$, and that of the gravitational mode is of order $\tau_R^{-1/3}\tau_H^{-2/3}$. As $S \to \infty$, the gravitational effect dominates and may be used to stabilize the two nongravitational modes. If the zero-order configuration is in equilibrium, there are no overstable modes in the incompressible case. Allowance for plasma compressibility somewhat modifies the "rippling" and gravitational modes, and may permit overstable modes to appear. The existence of overstable modes depends also on increasingly large *zero-order* resistivity gradients as $S \to \infty$. The three unstable modes merely require increasingly large gradients of the *first-order* fluid velocity; but even so, the hydromagnetic approximation breaks down as $S \to \infty$. Allowance for isotropic viscosity increases the effective mass density of the fluid, and the growth rates of the "tearing" and "rippling" modes then scale as $\tau_R^{-2/3}\tau_H^{-1/3}$. In plasmas, allowance for thermal conductivity suppresses the "rippling" mode at moderately high values of S. The "tearing" mode can be stabilized by conducting walls. The transition from the low-g "resistive" gravitational mode to the familiar high-g infinite conductivity mode is examined. The extension of the stability analysis to cylindrical geometry is discussed. The relevance of the theory to the results of various plasma experiments is pointed out. A nonhydromagnetic treatment will be needed to achieve rigorous correspondence to the experimental conditions.

I. INTRODUCTION

A PRINCIPAL result of pinch[1,2] and stellarator[3] research has been the observed instability of configurations that the hydromagnetic theory[4,5] would predict to be stable in the limit of high electrical conductivity. In order to establish the cause of this observed instability, the extension of the hydromagnetic analysis to the case of finite conductivity becomes of considerable interest.

A number of particular "resistive" instability modes have been discussed in previous publications. Dungey[6] has shown that, at an x-type neutral point

[1] S. A. Colgate and H. P. Furth, Phys. Fluids 3, 982 (1960).
[2] K. Aitken, R. Bickerton, R. Hardcastle, J. Jukes, P.

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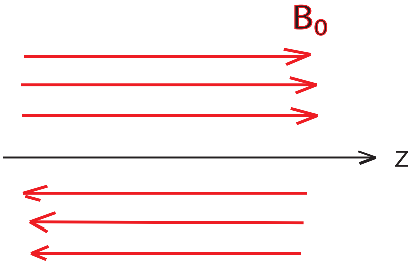
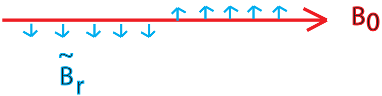
Spontaneous magnetic reconnection as the result of tearing fluctuations

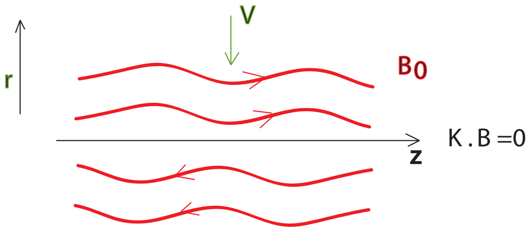


with small perturbation



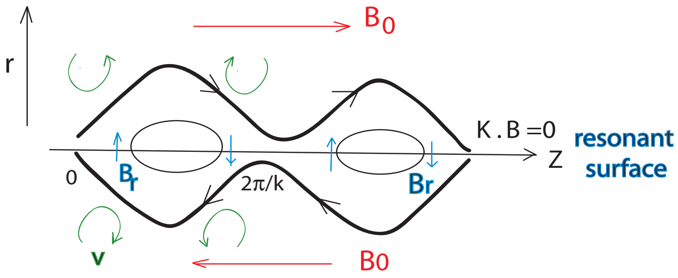
Magnetic reconnection

if $\eta = 0$



$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B})$$

$$\widetilde{V}_r = \gamma \widetilde{B}_r/(k \cdot B) ===> \boxed{\widetilde{B}_r = 0}$$



magnetic diffusion equation:

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{\eta}{\mu_0}\nabla^2\mathbf{B}$$

- $\widetilde{B}_r(\mathbf{r}, t) \sim \widetilde{B}_r(r)\exp(ik_z z - i\omega t)$

Magnetic Reynolds number a measure of how closely the magnetic field is coupled to the fluid

Dimensionless equation

$$\frac{d\mathbf{B}}{dt} = S\nabla \times (\mathbf{V} \times \mathbf{B}) - \eta\nabla^2\mathbf{B}$$

measure of plasma resistivity, Lundquist number $S = \tau_R/\tau_A$

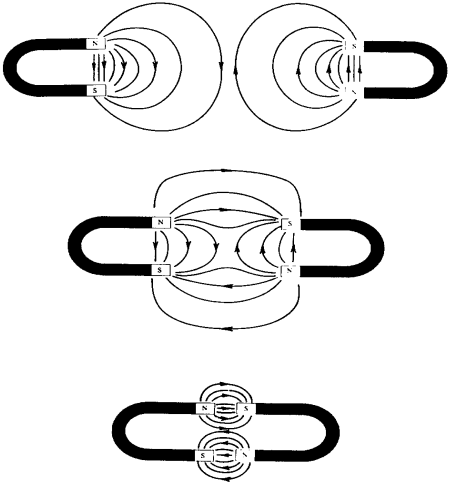
$\tau_R = \mu_0 a^2/\eta \quad \tau_A = a/(B^2/(\mu_0\rho)^{1/2})$

Growth rate is a hybrid between resistive and Alfven times:

$\boxed{\tau_{growth} \sim \tau_R^{-3/5}\tau_A^{-2/5}}$, $\boxed{\gamma\tau_A \propto S^{-3/5}}$

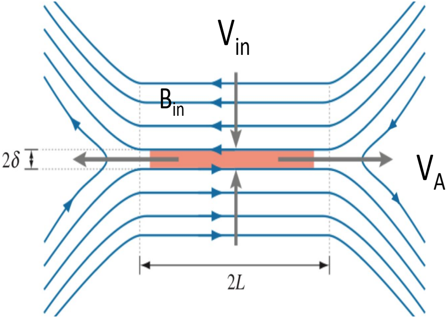
In general magnetic Reynolds number $Rm \equiv LV/\eta$

Magnetic reconnection is the rearrangement of magnetic field topology of plasmas – forced (driven) reconnection



This process occurs in the solar atmosphere, the solar wind, the Earth's magnetosphere, astrophysical disks, turbulence, and laboratory plasmas.

Sweet-Parker model (1958) is the simplest description of magnetic reconnection



Recon. Rate $\sim$ flux * $V_{in}/\delta \sim B_{in}\delta V_A S^{-1/2}$
For solar flares, $S \sim 10^{14}$ theory predicts
==> $\tau_{S-P} \sim 2$ months
Flares only last min to an hour
==> Rate is too slow

- Mass conservation
  $V_{in}L \approx V_{out}\delta$
- Energy conservation (acceleration along sheet)
  $V_{out} \approx V_A = B_{in}/\sqrt{(}\mu_0\rho)$
- matching ideal E outside the layer with the resistive E in the layer
  $V_{in}B_{in} \approx \eta J \rightarrow V_{in} \approx \eta/\mu_0\ell$
  (using $J \approx B_{in}/\mu_0\delta$)
- $\boxed{V_{in}/V_{out} = S^{-1/2} = \delta/L}$
  $(S = \mu_0 L V_A/\eta)$

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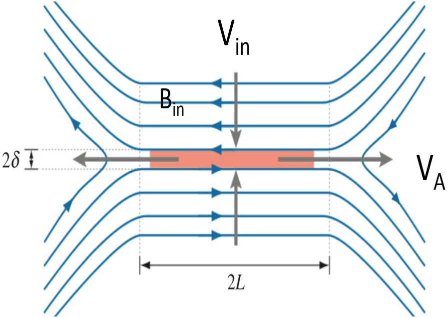

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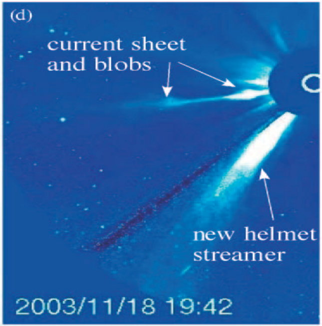
How could one explain fast dynamical reconnecting processes?

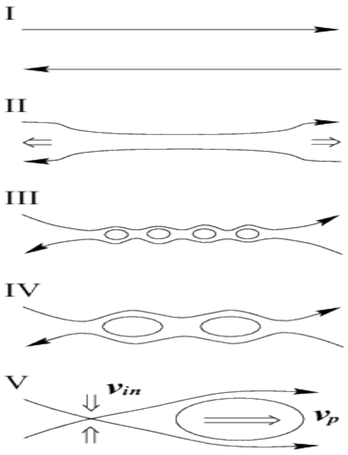
Spontaneous plasmoid instability: tearing instability in a current sheet?



In many fast MHD dynamical processes, plasmoids are essential features

- Elongated current sheet can become tearing unstable at high S. [Biskamp 1986, Shibata & Tanuma 2001]

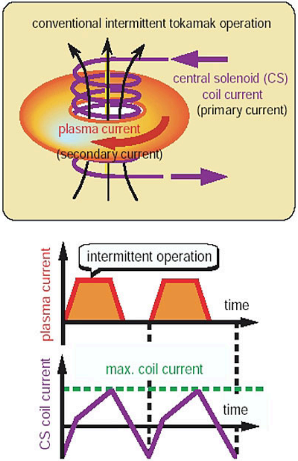
- The scaling properties of a classical linear tearing changes, $(\gamma \sim S^{1/4})$.
  Numerical development: [Tajima & Shibata 1997,Loureiro et al. 2007; Lapenta 2008; Daughton et al. 2009,; Bhattacharjee et al. 2009] **show fast reconnection.** Static linear theory doesn't apply [L. Comisso et al. PoP 2016]



Shibata & Tanuma 2001

Magnetic reconnection in laboratory for current-drive?

In conventional tokamak plasma current is generated inductively.

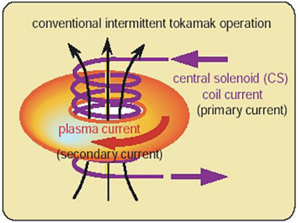


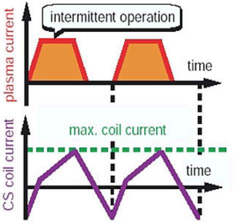
Utilize reconnection for non-inductive current-drive?

- Could plasma current be produced by the process of magnetic reconnetion through detachment of magnetic loops?

System-size plasmoid formation to produce large plasma current in a toroidal fusion device. [Ebrahimi & Raman Nuclear Fusion 2016]

In conventional tokamak plasma current is generated inductively.





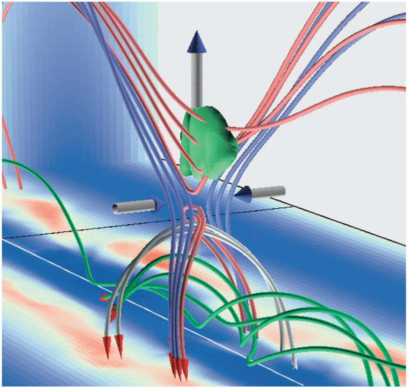
Utilize reconnection for non-inductive current-drive?

- Could plasma current be produced by the process of magnetic reconnetion through detachment of magnetic loops?

System-size plasmoid formation to produce large plasma current in a toroidal fusion device. [Ebrahimi & Raman Nuclear Fusion 2016]

Many fundamentals of reconnection physics can be explored during helicity injection

Solar flares field lines modeling

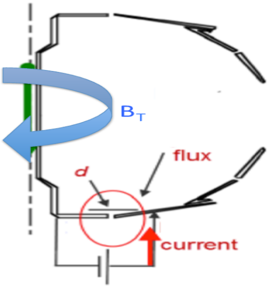


$$\frac{\partial K}{\partial t} = -2 \int (\mathbf{A.V})\mathbf{B} \cdot d\mathbf{s}$$

Kusano 2004

Warnecke & Brandenburg 2010

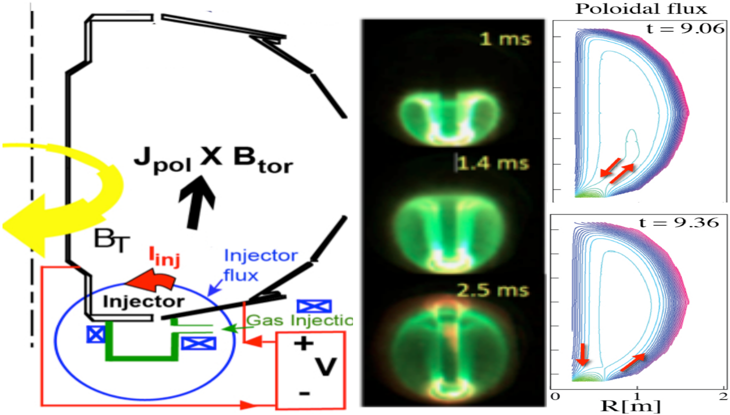
Helicity injection in a lab



$$\frac{\partial K}{\partial t} = -2 \int \Phi \mathbf{B} \cdot d\mathbf{s}$$

Helicity is injected through a surface term.

Transient Coaxial Helicity Injection (CHI), the primary candidate for solenoid- free current start-up in spherical tokamaks



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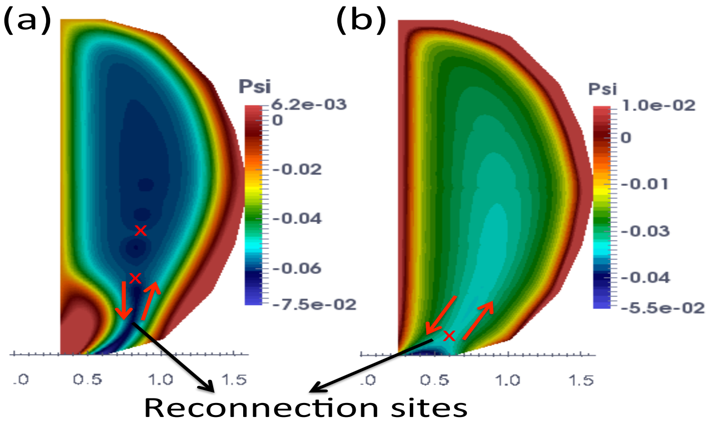
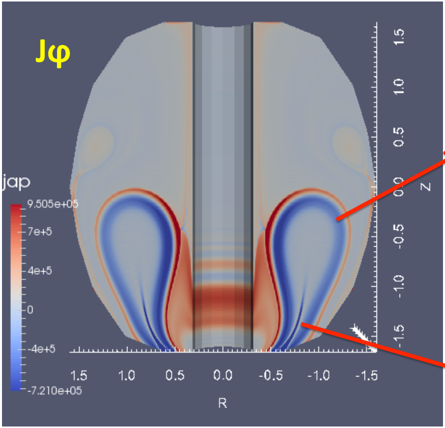
Reconnection could occur during both stages of helicity injection

During injection (V_{inj} on) During decay (V_{inj} =0)



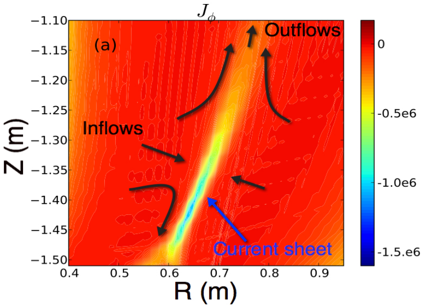
Two types of current sheets are formed during flux expansion/evolution

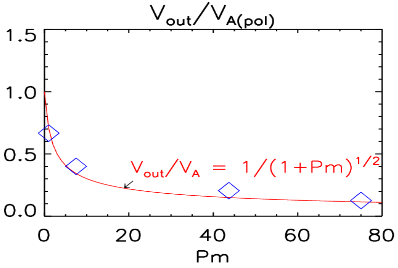


- 1- **Edge current sheet** from the poloidal flux compression near the plasma edge, **leads to 3-D filament structures**
- 2- **Primary reconnecting current sheet** from the oppositely directed field lines in the injector region.

F. Ebrahimi Phys. Plasmas Letters 2016

Forced reconnection in the injection region



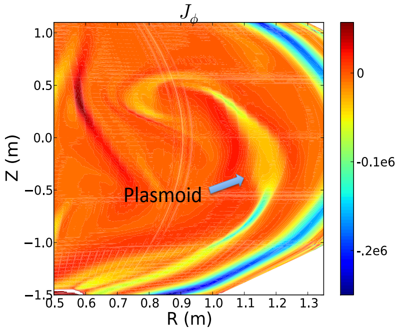


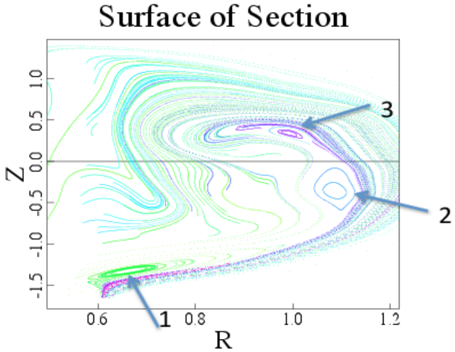
A local 2-D Sweet-Parker type reconnection is triggered in the injection region. Key signatures:

- I - Elongated current sheets, $L > \delta$.
- II - Scaling of the current sheet width $\delta/L \sim (1 + P_m)^{1/4} S^{-1/2} \sim V_{in}/V_{out}$
- III -Pinch inflow and Alfvénic outflow

S = LV_A/η (Alfven velocity based on the reconnecting B, L is the current sheet length) F. Ebrahimi, et al. PoP 2013, 2014

Spontaneous plasmoid reconnection





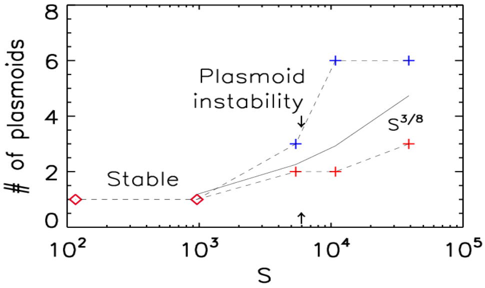
At high S, a transition to a plasmoid instability is demonstrated in the simulations.

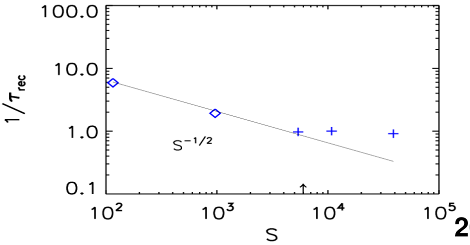
Both small sized transient plasmoids and large system-size plasmoids are formed and co-exist. (S=39000)

Plasmoids merge to form closed flux surfaces. Reconnection rate becomes nearly independent of S.

F. Ebrahimi and R.Raman PRL 2015

At high S, a transition to a plasmoid instability occurs



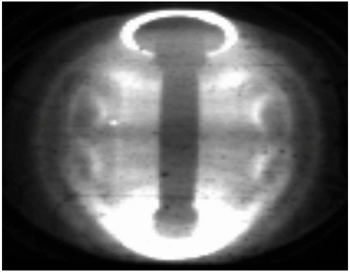


- Number of plasmoids is an increasing function of S. Blue: small sized transient plasmoids Red: large scale and persistent plasmoids
- As the current sheet evolves in time, static linear theory doesn't apply [L. Comisso et al. PoP 2016]
- Reconnection rate becomes nearly independent of S. **[Here with strong guide field]**

F. Ebrahimi & R.Raman PRL 2015

First documentation of plasmoid formation in MHD regime in laboratory.

(Loading poloidalflux.mp4)

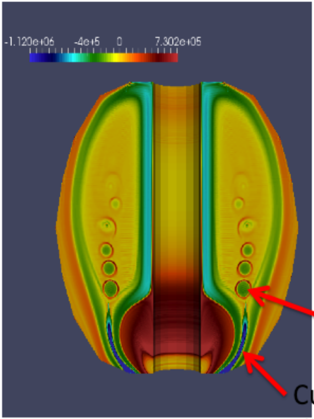


Camera images from NSTX do show the formation of plasmoids that then merge into a larger plasma

[Ebrahimi&Raman PRL 2015]

Plasmoid instability with continued injection of plasmoids is observed during the injection phase (S $\sim$ 29000)

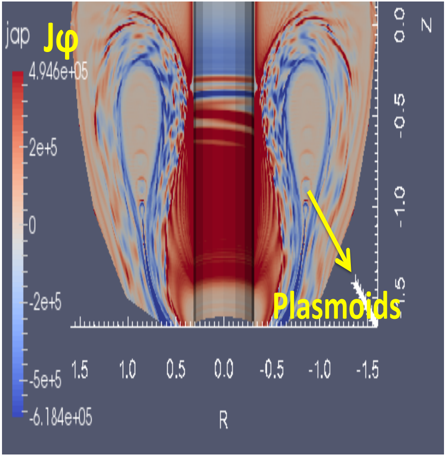
(Loading poloidalflux.mp4)



Dynamo-driven plasmoids formation

- 3D effects
  - Could large-scale dynamo from 3-D fluctuations trigger reconnecting plasmoids?
  - Self-consistent trigger mechanism in 3-D?

Edge current-sheet instabilities are triggered in 3-D, and break the current-sheet



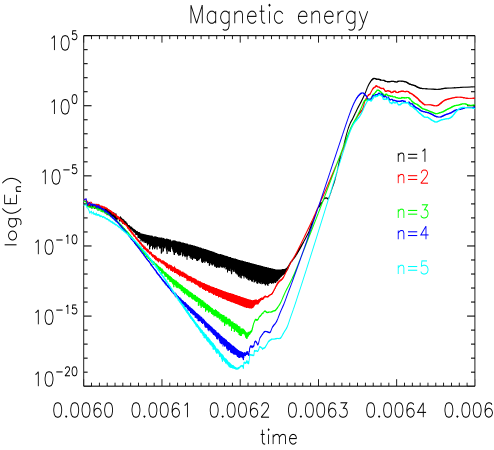
1. I- **Edge-localized modes arising from the asymmetric current-sheet instabilities**
2. II- First observation of nonaxisymmetric edge current
3. III- With 3-D fluctuations, axisymmetric plasmoids are formed, **local S increased to S $\sim$ 15000.** [Ebrahimi PoP Letters 2016]

Edge modes grow on the poloidal Alfven time scales

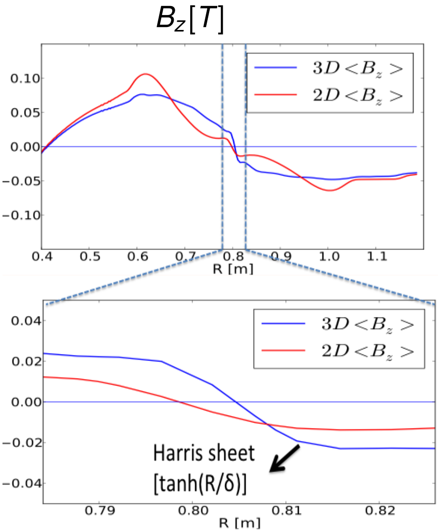
1. These modes grow fast, on the poloidal Alfven time scales, and are **peeling modes with tearing-parity structures.**

2. These modes saturate by modifying and relaxing the edge current sheet

$\gamma\tau_{A(n=1)} = 0.16,$
$\gamma\tau_{A(n=2)} = 0.18,$
$\gamma\tau_{A(n=3)} = 0.2,$
$\gamma\tau_{A(n=4)} = 0.23,$
$\gamma\tau_{A(n=5)} = 0.26.$
$S = 2 \times 10^5$



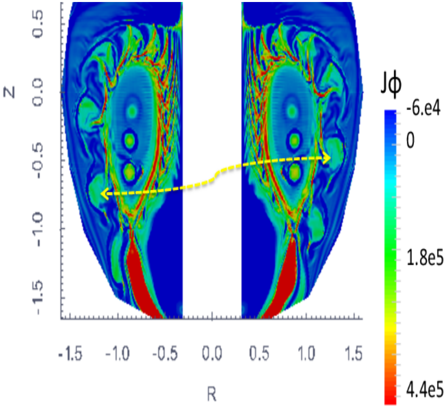
Poloidal flux amplification is observed to trigger axisymmetric reconnecting plasmoids formation

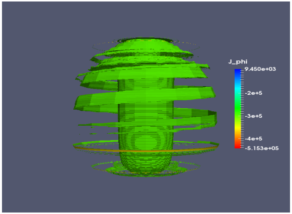


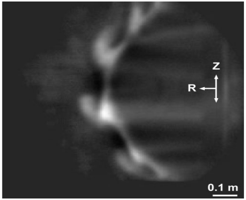
- For the first time a dynamo poloidal flux amplification is observed
- This **fluctuation-induced flux amplification increases the local S** ==> **triggers a plasmoid instability** ==> breaks the primary current sheet.

3-D reconnection is being investigated in the edge region of a tokamak

Coherent current-carrying filament formation near the plasma edge

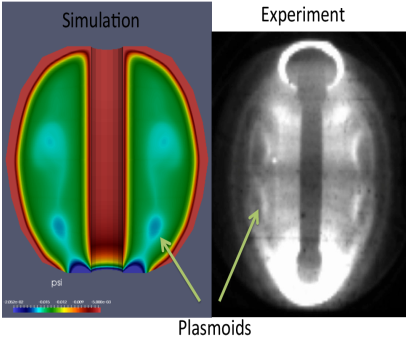




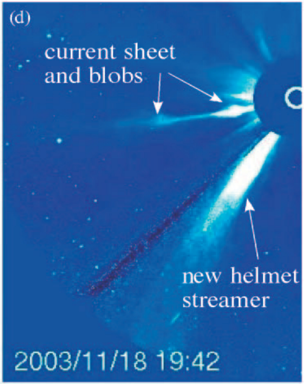


[F. Ebrahimi Phys. Plasmas 2017]

Emerging observational evidence of plasmoid-like structures



Camera images from NSTX do show the formation, and subsequent separation, of smaller plasmoids that then merge into a larger pre-existing plasma.[Ebrahimi&Raman PRL 2015]



Plasmoids (2D) of the reconnected plasma flowing along the current sheet.LASCO/SOHO C3 images [Lin et al. 2005; Reily et al. 2007]

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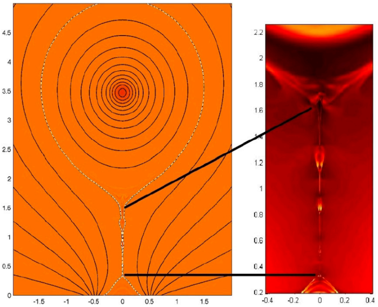
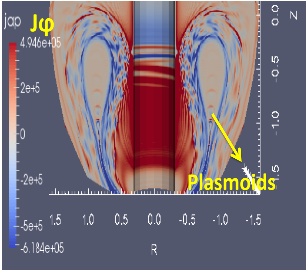
In order to understand the phenomena in a certain plasma region, it is necessary to map not only the magnetic but also the electric field and the electric currents. Space is filled with a network of currents which transfer energy and momentum over large or very large distances. The currents often pinch to filamentary or surface currents. The latter are likely to give space, as also interstellar and intergalactic space, a cellular structure.

Hannes Alfven



Guo et al. APJL (2013)



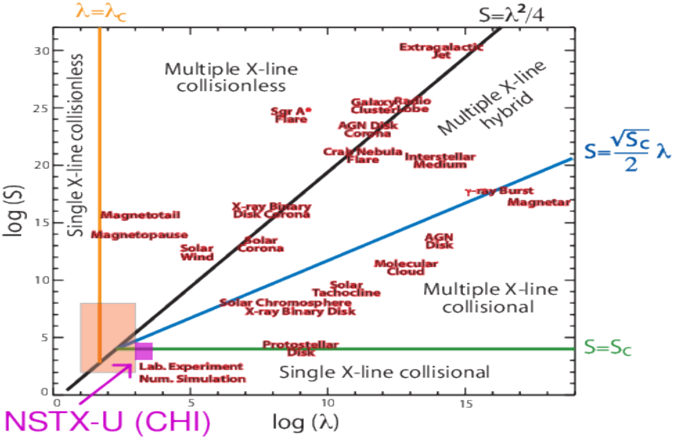
Ebrahimi&Raman PRL 2015, PoP 2016

Helicity injection simulations are performed using the extended-MHD NIMROD code

- Solves the linear and nonlinear MHD equations

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} + \kappa_{divb} \nabla\nabla.\mathbf{B} \\
\mathbf{E} &= -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} \\
\mathbf{J} &= \nabla \times \mathbf{B} \\
\frac{\partial n}{\partial t} + \nabla.(n\mathbf{V}) &= \nabla.D\nabla n \\
\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}.\nabla\mathbf{V}\right) &= \mathbf{J} \times \mathbf{B} - \nabla P - \nabla.\Pi \\
\frac{n}{(\Gamma - 1)}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{V}.\nabla T_\alpha\right) &= -p_\alpha \nabla.\mathbf{V} - \nabla.\mathbf{q}_\alpha + Q
\end{aligned}
$$

- $\mathbf{q} = -n[(\kappa_{||} - \kappa_\perp)\hat{b}\hat{b} + \kappa_\perp \mathbf{I}] \cdot \nabla T$
- Π is the stress tensor (also includes numerical $\rho\nu\nabla V$)
- κ_{divb} and D are magnetic-divergence and density diffusivities for numerical purposes



λ=λc
S=λ²/4
Single X-line collisionless
Multiple X-line collisionless
Multiple X-line hybrid
Multiple X-line collisional
Single X-line collisional
S=√Sc/2 λ
S=Sc
Extragalactic Jet
Galaxy Cluster
Radio Lobe
Sgr A* Flare
AGN Disk Corona
Crab Nebula Flare
Interstellar Medium
γ-ray Burst
Magnetar
Magnetotail
Magnetopause
X-ray Binary Disk Corona
Solar Wind
Solar Corona
AGN Disk
Molecular Cloud
Solar Tachocline
Solar Chromosphere
X-ray Binary Disk
Protostellar Disk
Lab. Experiment
Num. Simulation
NSTX-U (CHI)
log (S)
log (λ)
0
5
10
15
20
25
30
0
5
10
15