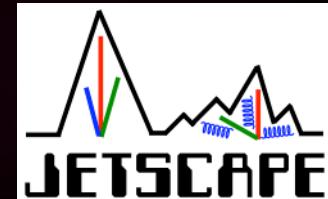


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# 3+1D Free Streaming for Pre-equilibrium Evolution



# Overview

- Initial Conditions for Hydro
- Free streaming approx.
- Formalism
- First results
- Future prospects

# Hydro for Heavy Ion Collisions

- Viscous Hydro evolves conserved currents .
- Applied to heavy ions, yields good agreement; e.g. particle spectra, flow harmonics, ...

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu J_i^\mu = 0$$

# Hydro requires initial conditions

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

- We have initialized hydro ad hoc when using MC Glb, TRENTO,...
- Initial flow velocity set to zero; shear stress, bulk pressure zero or N.S. values

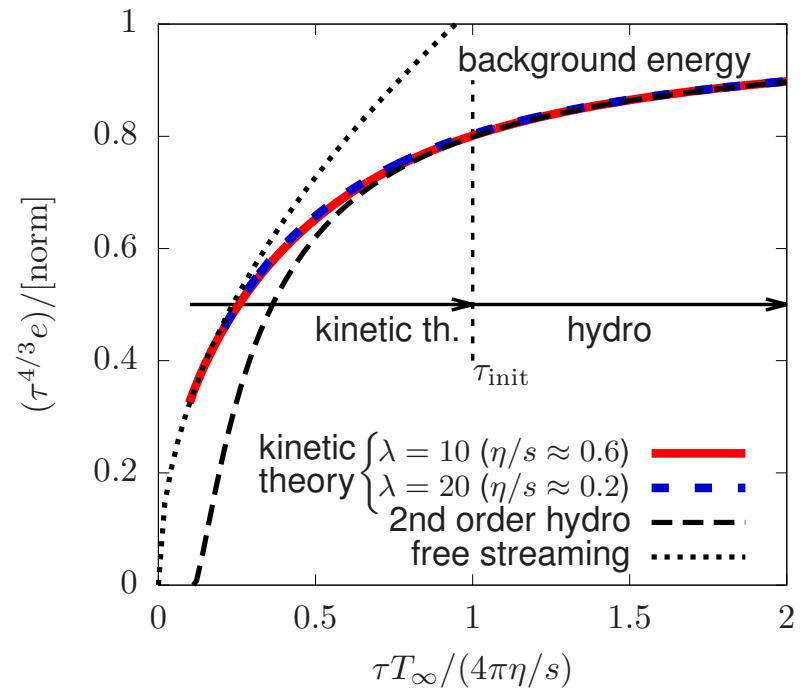
$$u_s^\mu = ?$$

$$\pi_s^{\mu\nu} = ?$$

$$\Pi_s = ?$$

# Pre-equilibrium Dynamics

- Using EKT, Kurkela et al showed early dynamics  $\sim$  free streaming.
- Approximate pre-equilibrium dynamics by free streaming; then Landau match to hydro at  $\tau_s$ .



\*arXiv:1704.05242v1

# Free Streaming Formalism

- Collisionless Boltzmann Eqn. evolves  $f(\tau, \mathbf{x}; p)$ .
- Evolve  $T^{\mu\nu}, J^\mu$  w/o knowing  $f(\tau_0, \mathbf{x}; p)$ , given initial energy, baryon densities from initial state model (MC Glb, TRENTO, ...)
- Possible given assumptions:
  - Massless particles
  - $f(\tau_0, \mathbf{x}; p)$  isotropic in  $\mathbf{p}_T$
  - Assume dependence of  $f(\tau_0, \mathbf{x}; p)$  on rapidity  $\xi$

$$f(\tau_0, x, y, \eta; p_T, \xi) = \exp\left(\frac{(\eta - \xi)^2}{2\sigma^2}\right) \cdot \tilde{f}(\tau_0, x, y, \eta; p_T)$$

# Free Streaming Formalism

$$T_0^{\mu\nu} \rightarrow T_s^{\mu\nu}$$

- Define a momentum dependent function  $G^{\mu\nu}(x; \phi_p, \xi)$ :

$$G^{\mu\nu}(x; \phi_p, \xi) \equiv \int dp_T p_T p^\mu p^\nu f(x; p)$$

$$\implies T^{\mu\nu} = \frac{g}{(2\pi)^3} \int d\xi d\phi_p G^{\mu\nu}(x; \phi_p, \xi)$$

(Can define similar function for  $J^\mu$ )

# Free Streaming Formalism

- Solution of collisionless Boltzmann Eqn.

$$f(\tau, \mathbf{x}_T, \eta; p) =$$

$$f(\tau_0, \mathbf{x}_T - \frac{\mathbf{p}_T}{p_T} [\tau \cosh(\xi - \eta) - \tau_0 \cosh(\xi - \eta_0)], \sinh^{-1} [\frac{\tau \sinh(\eta_0 - \xi)}{\tau_0}] + \xi; p)$$

- Free stream function  $G^{\mu\nu}$ , then integrate out momenta to find  $T^{\mu\nu}(\tau, \mathbf{x})$ .

$$G^{\mu\nu}(\tau, \mathbf{x}_T, \eta; p) =$$

$$G^{\mu\nu}(\tau_0, \mathbf{x}_T - \frac{\mathbf{p}_T}{p_T} [\tau \cosh(\xi - \eta) - \tau_0 \cosh(\xi - \eta_0)], \sinh^{-1} [\frac{\tau \sinh(\eta_0 - \xi)}{\tau_0}] + \xi; p)$$

# Landau Matching at $\tau_s$

- Solve eig. problem

$$T^\mu_\nu u^\nu = \epsilon u^\mu$$

- Use E.o.S.

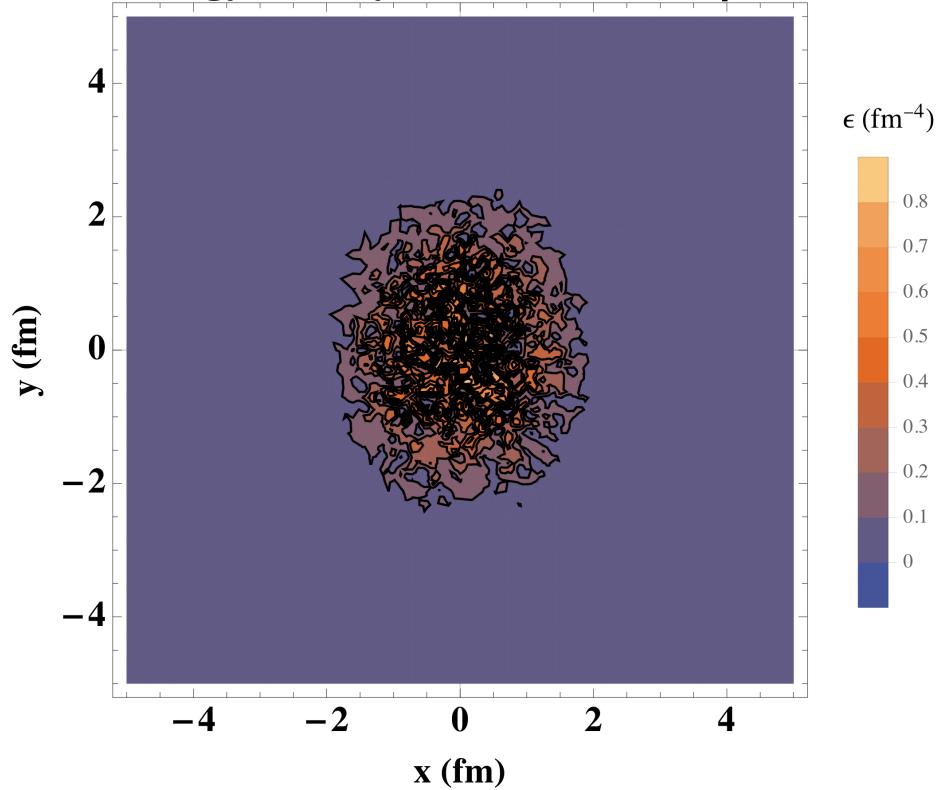
$$p = p(\epsilon)$$

- Find bulk, shear stress

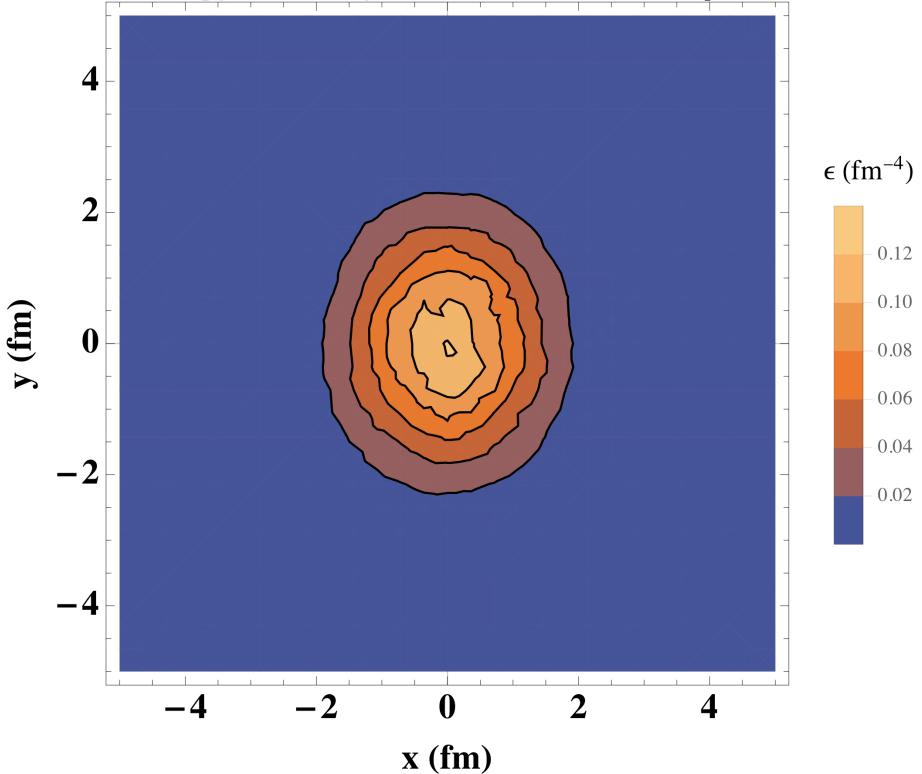
$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

# Results of 3+1D free streaming

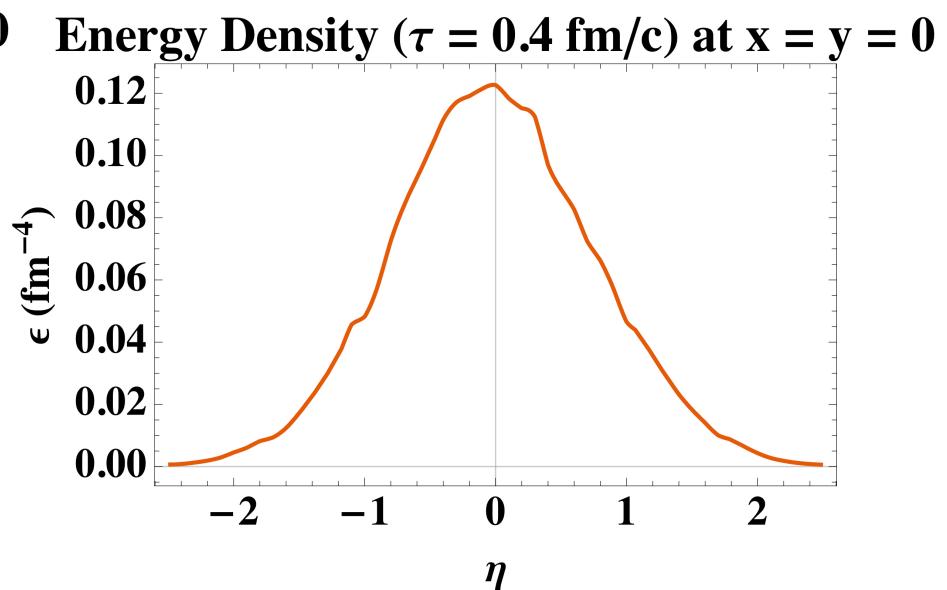
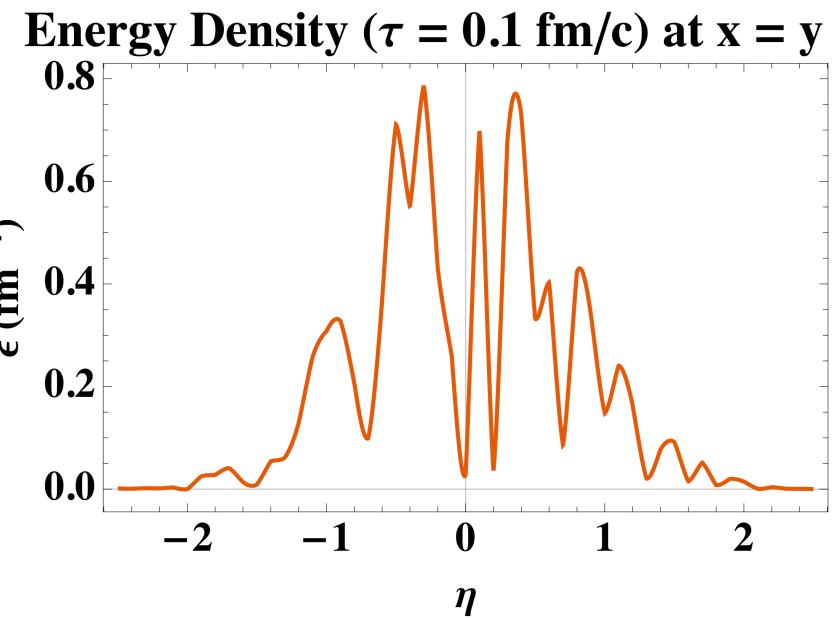
Energy Density ( $\tau = 0.1 \text{ fm}/c$ ) at  $\eta = 0$



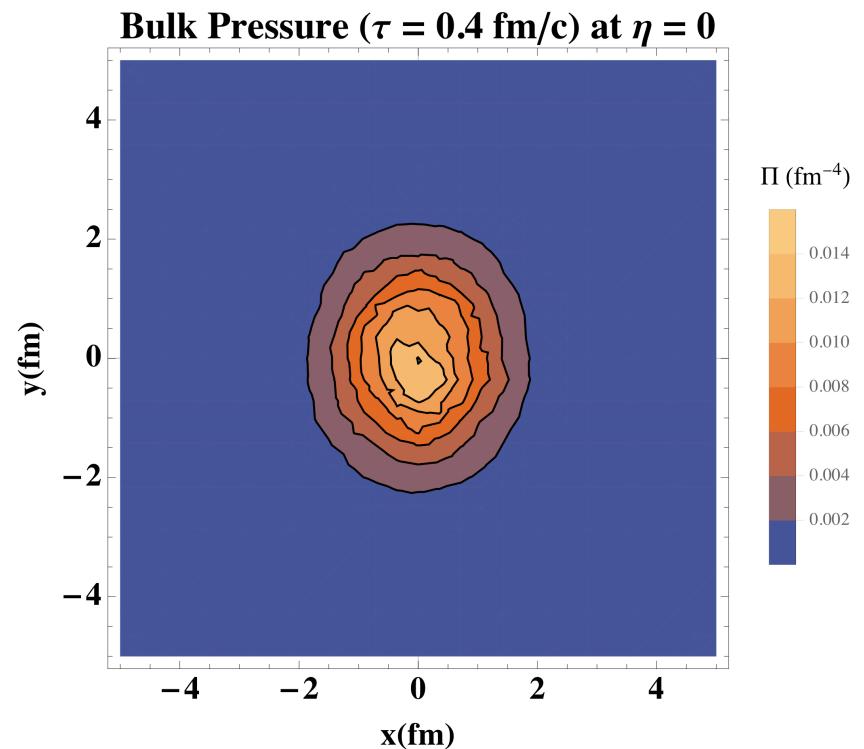
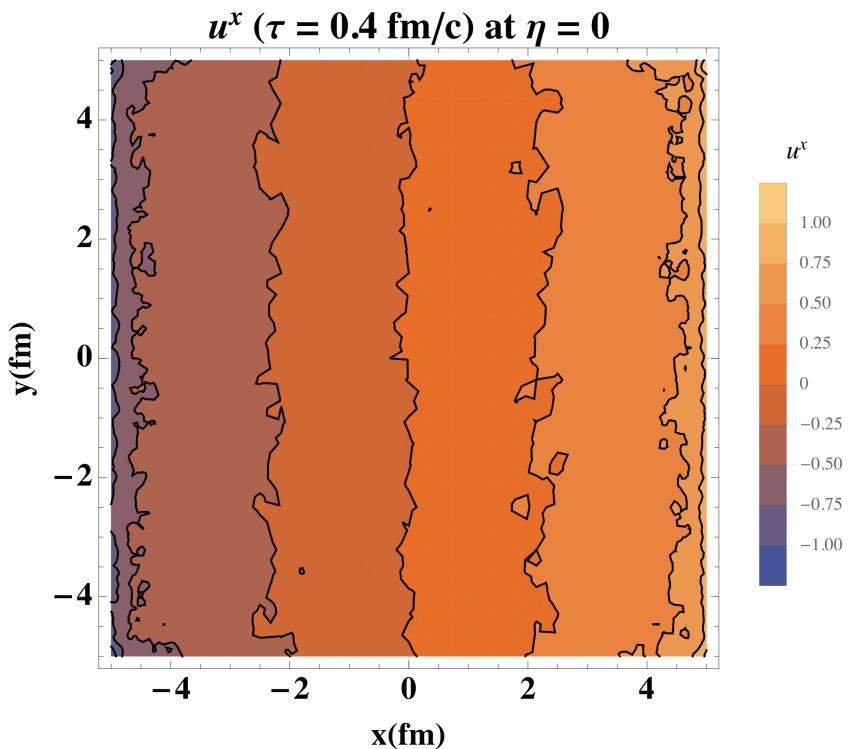
Energy Density ( $\tau = 0.4 \text{ fm}/c$ ) at  $\eta = 0$



# Results of 3+1D free streaming



# Results of 3+1D free streaming



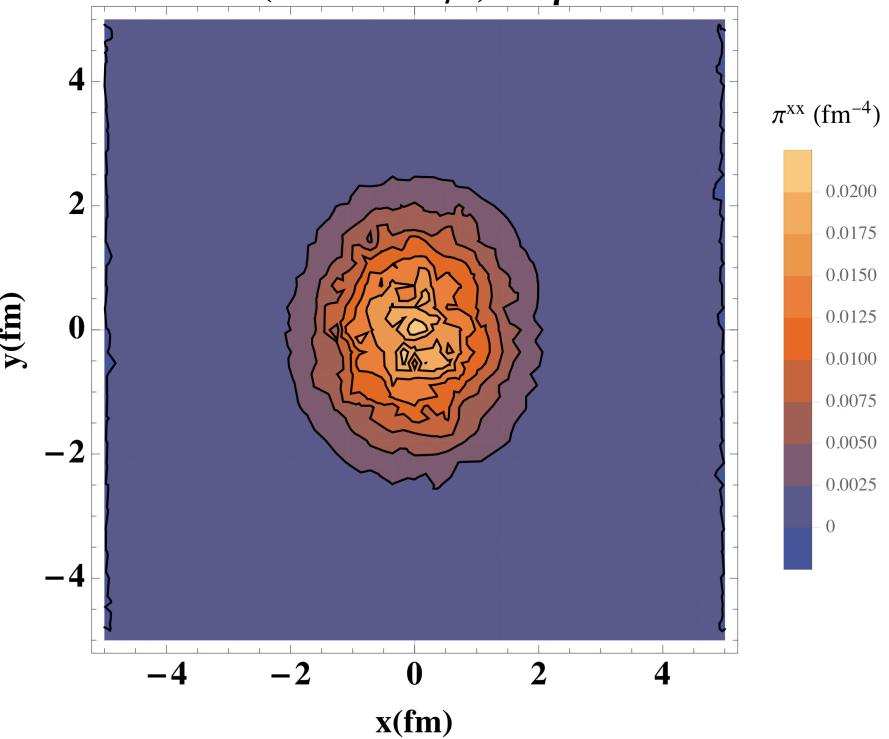
# Coming soon

- 3+1D OSU Hydro (with F.O. and Cooper Frye) on CPU/GPU
- Effects of 3D Free Streaming on Hydro/Spectra

*Thank You*

# More Results

$\pi^{xx}$  ( $\tau = 0.4$  fm/c) at  $\eta = 0$



$\pi^{\eta\eta}$  ( $\tau = 0.4$  fm/c) at  $\eta = 0$

