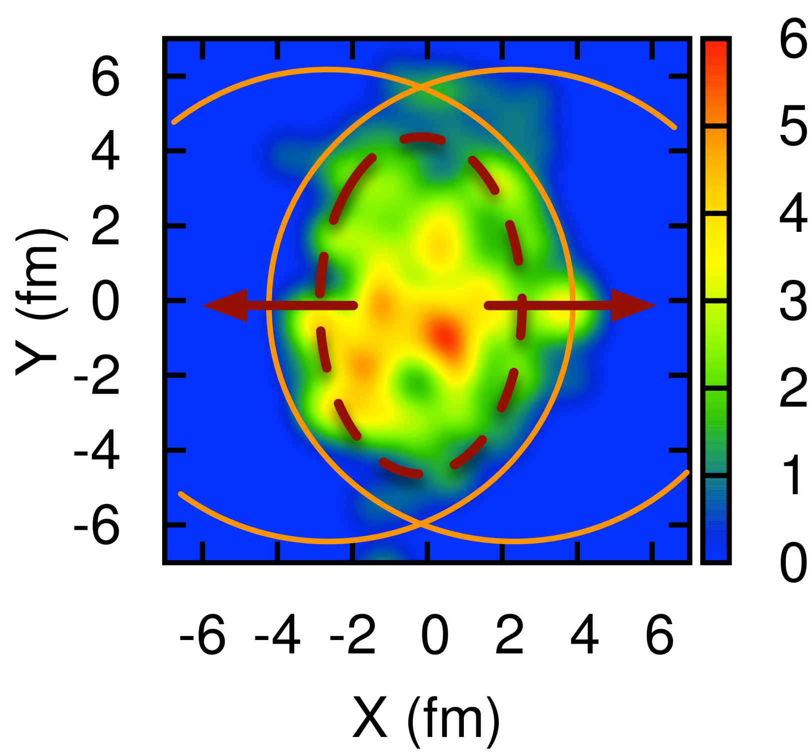
Hydrodynamics: Opening the Box!

Derek Teaney
Stony Brook University



Ideal hydro is the most important:

$$\partial_\mu T^{\mu\nu} = 0$$

What is $T^{\mu\nu}$?

1. There is a rest frame $u^\mu(x) = (\gamma, \gamma\mathbf{v})$ we are in local perfect equilibrium:

$$T^{\mu\nu} = e(x)u^\mu(x)u^\nu(x) + \underbrace{\mathcal{P}(e)}_{\text{equation of state}} (\eta^{\mu\nu} + u^\mu(x)u^\nu(x)) +$$

... Corrections

2. From $u_\nu \partial_\mu T^{\mu\nu} = 0$ and the EOS derive:

$$\underbrace{\partial_\mu (s u^\mu) = 0}_{\text{The final particle yield is determined by the initial entropy}}$$

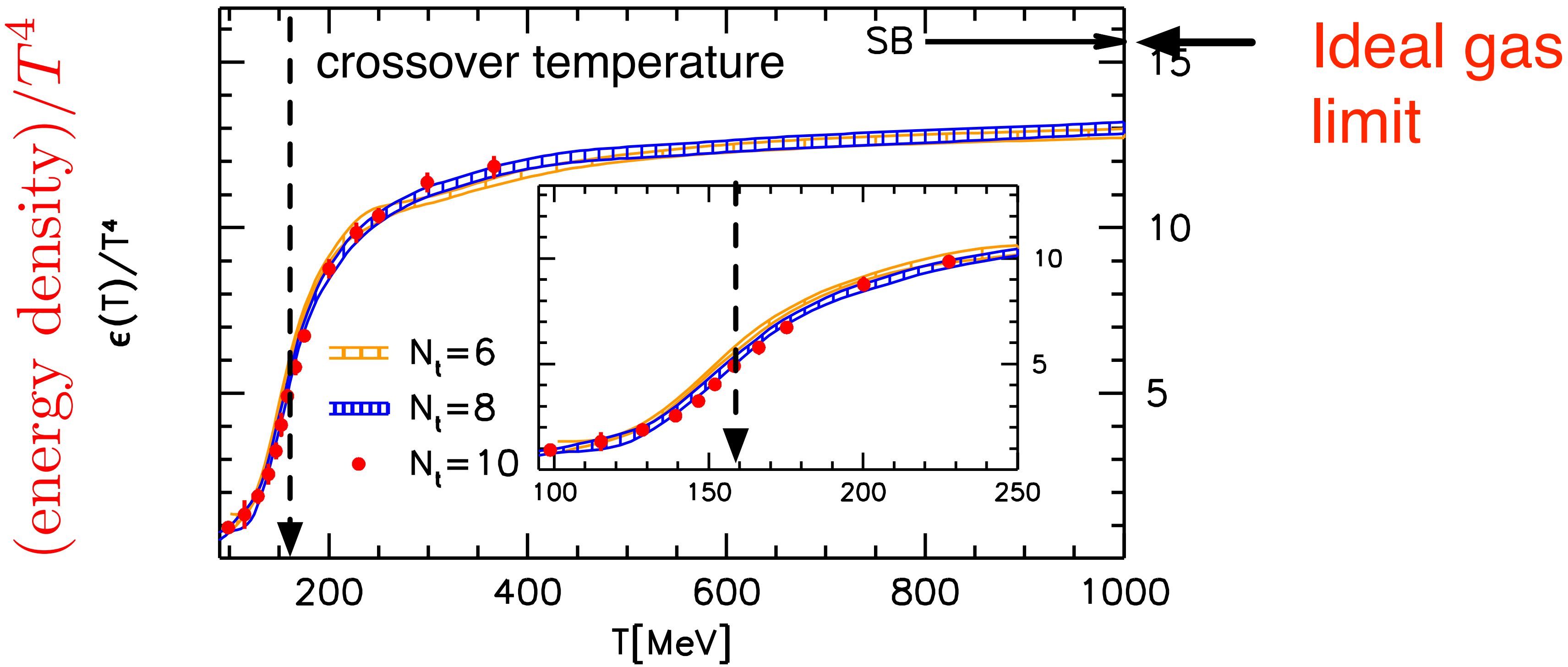
The Equation of State

Lattice QCD and the QCD equation of state:

Compute the equation of state by sampling fields with the statistical weight:

$$Z \sim \int [DA] e^{-S_{QCD}[A]}$$

The QCD Equation of State (Budapest-Wuppertal Collaboration)



1. The "critical" energy density and temperature are

$$e_c \simeq 1\,\text{GeV}/\text{fm}^3 \qquad T_c \simeq 160\,\text{MeV}$$

Need reach an energy density of e_c over a *Large* volume for *Long* enough.

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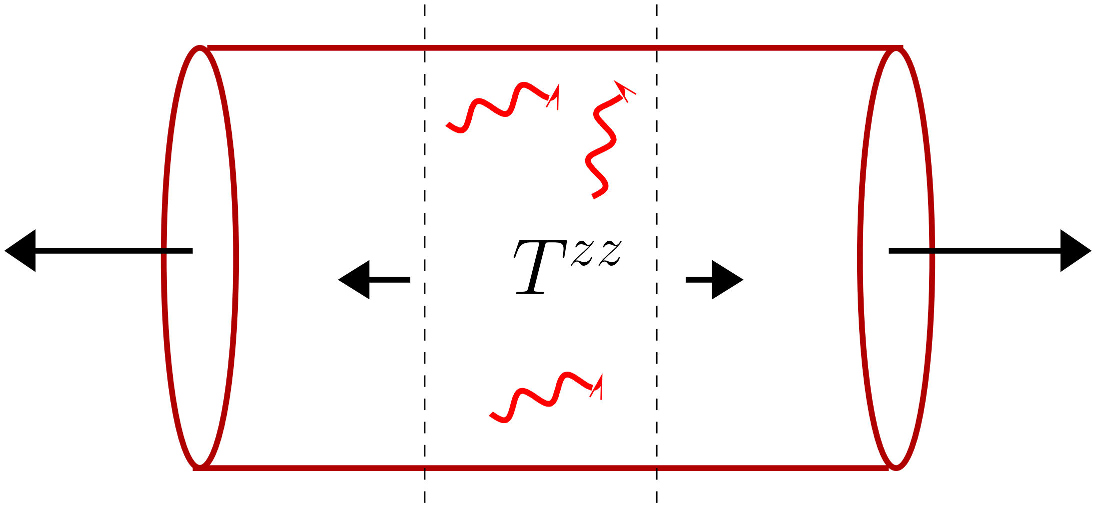
$$\ldots + \text{Corrections}$$

The Hydrodynamic Effective Theory

Viscous Bjorken expansion at 0th, 1st, 2nd order

(Bjorken, Gyulassy, BRSSS)

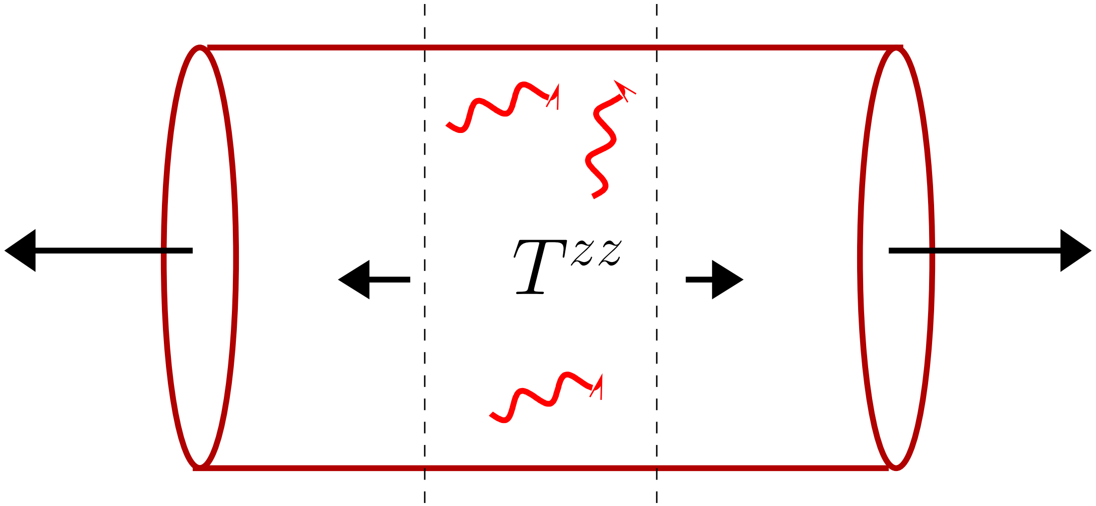


$$\frac{\langle T^{zz}\rangle}{e+p} = \Big[\underbrace{\frac{p}{e+p}}_{\sim 1} - \underbrace{\frac{4}{3}\frac{\gamma_\eta}{\tau}}_{\text{1st order}} + \underbrace{\frac{(\lambda_1 - \eta\tau_\pi)}{e+p}\frac{8}{9\tau^2}}_{\text{2nd order}} + \ldots \Big]$$

$$\gamma_\eta \equiv \eta/(e+p)$$

Viscous Bjorken expansion at 1st and 2nd order + 1 loop!

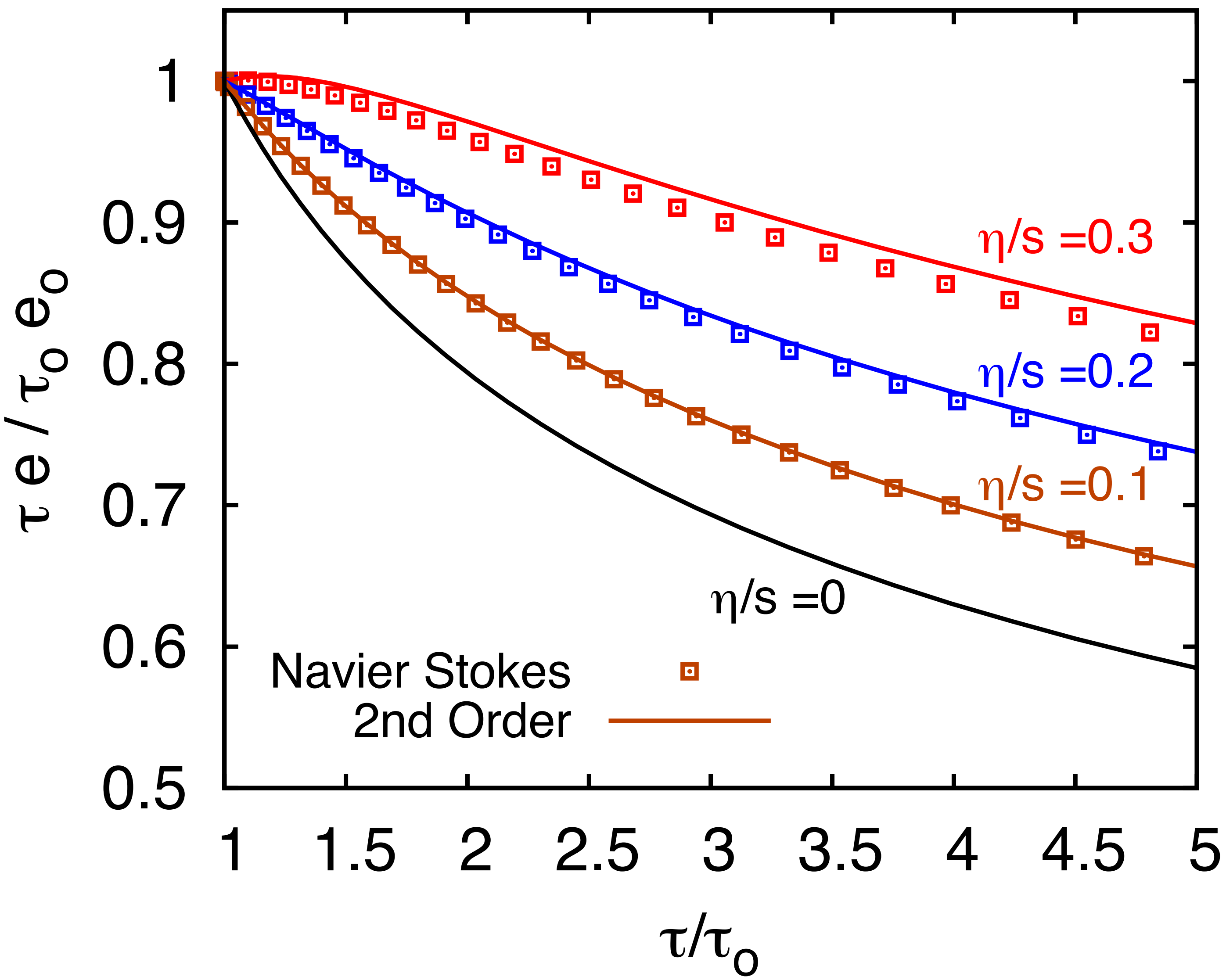
(Bjorken; Gyulassy; BRSSS; Akamatsu,Mazeliauskas,Teaney)



$$\frac{\langle T^{zz}\rangle}{e+p} = \Big[\underbrace{\frac{p}{e+p}}_{\sim 1} - \underbrace{\frac{4}{3}\frac{\gamma_\eta}{\tau}}_{\text{1st order}} + \underbrace{\frac{1.08318}{s\,(4\pi\gamma_\eta\tau)^{3/2}}}_{\text{3/2 order!}} + \underbrace{\frac{(\lambda_1 - \eta\tau_\pi)}{e+p}\frac{8}{9\tau^2}}_{\text{2nd order}} + \ldots \Big]$$

$$\gamma_\eta \equiv \eta/(e+p)$$

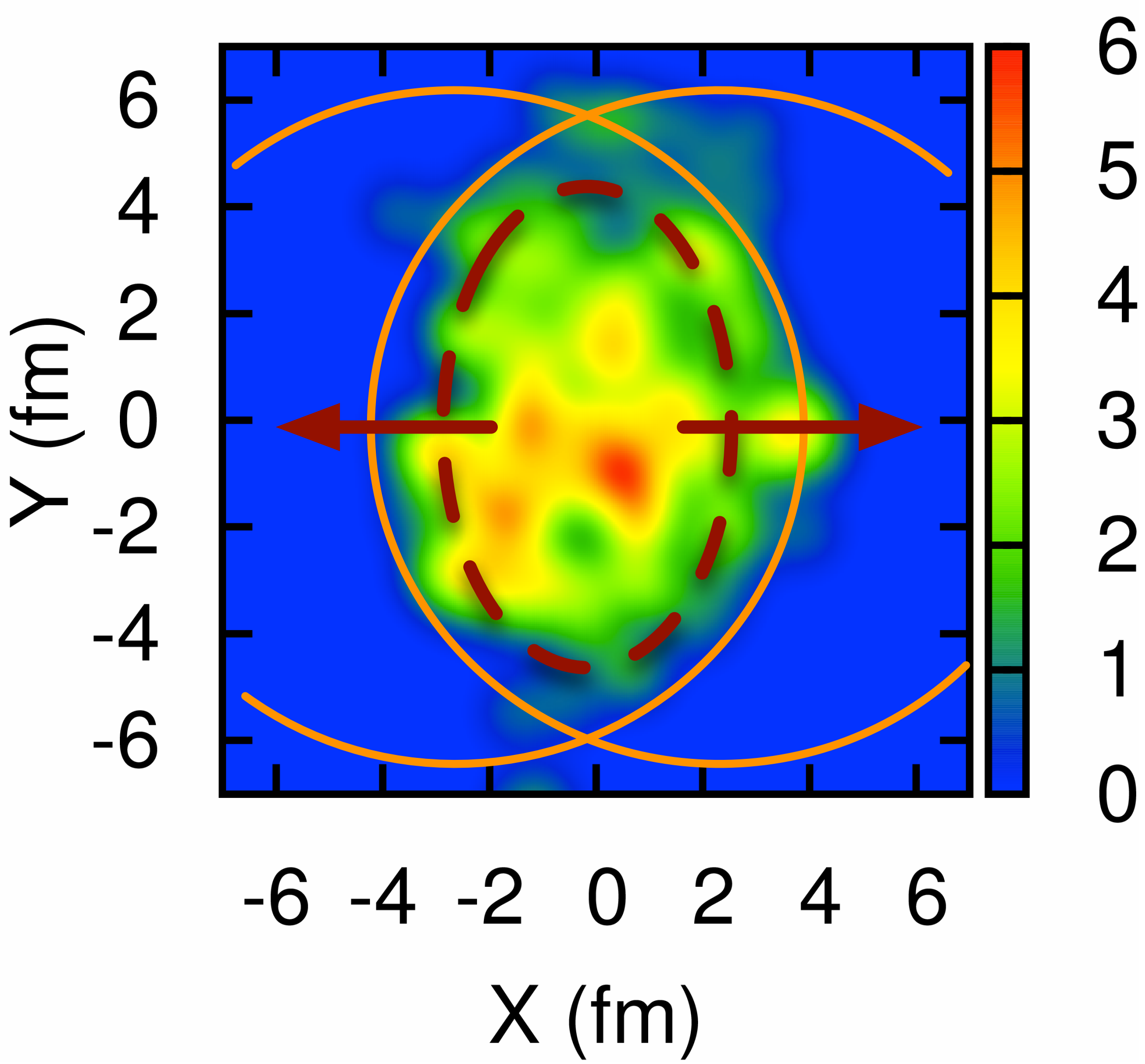
Viscous Bjorken expansion at 1st and 2nd order



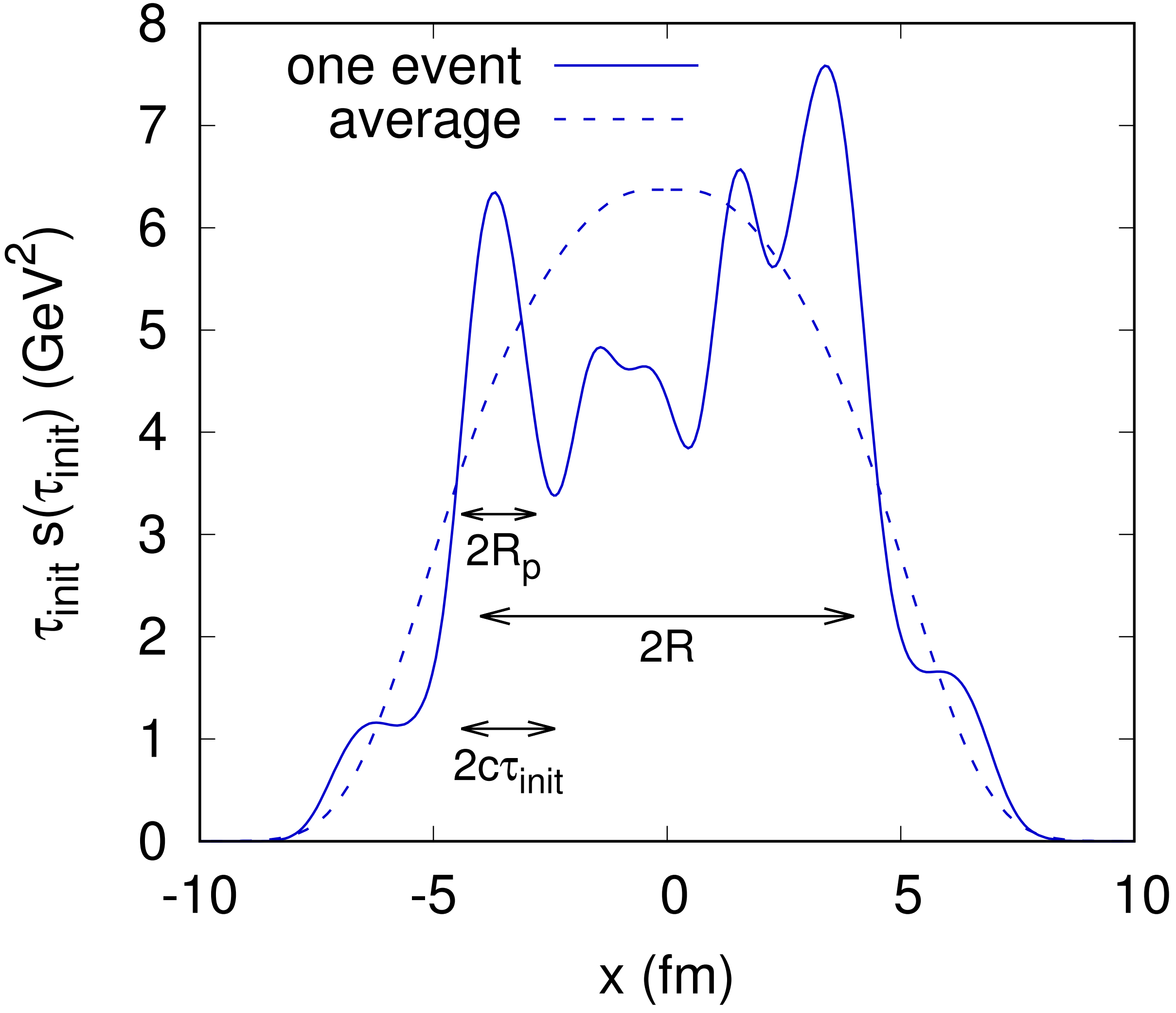
For typical parameters, the hydro effective theory is effective!

Initial conditions

Typical (somewhat) misleading color plot

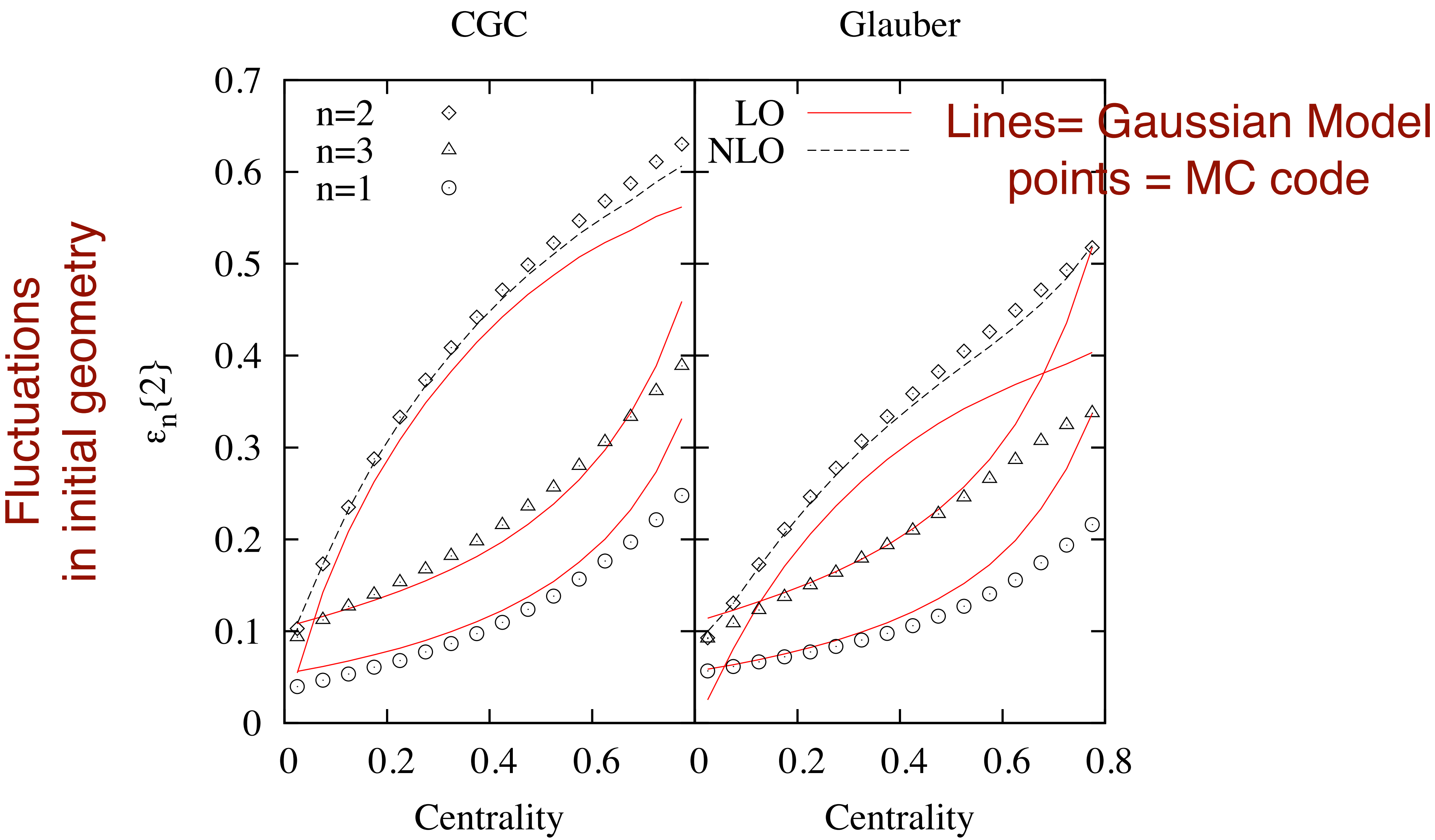


Typical (not-quite-as) misleading plot



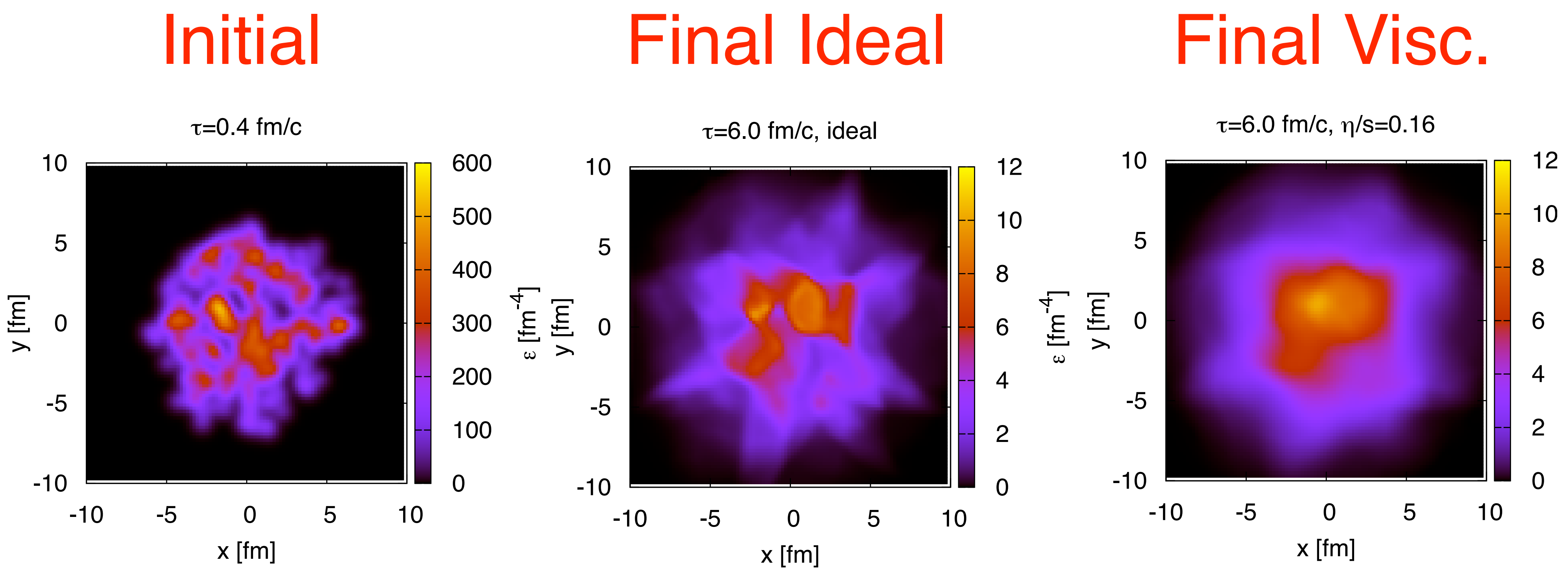
Gaussian model (independent cluster) compared to Glauber codes

Bhalerao, Luzum, Ollitrault

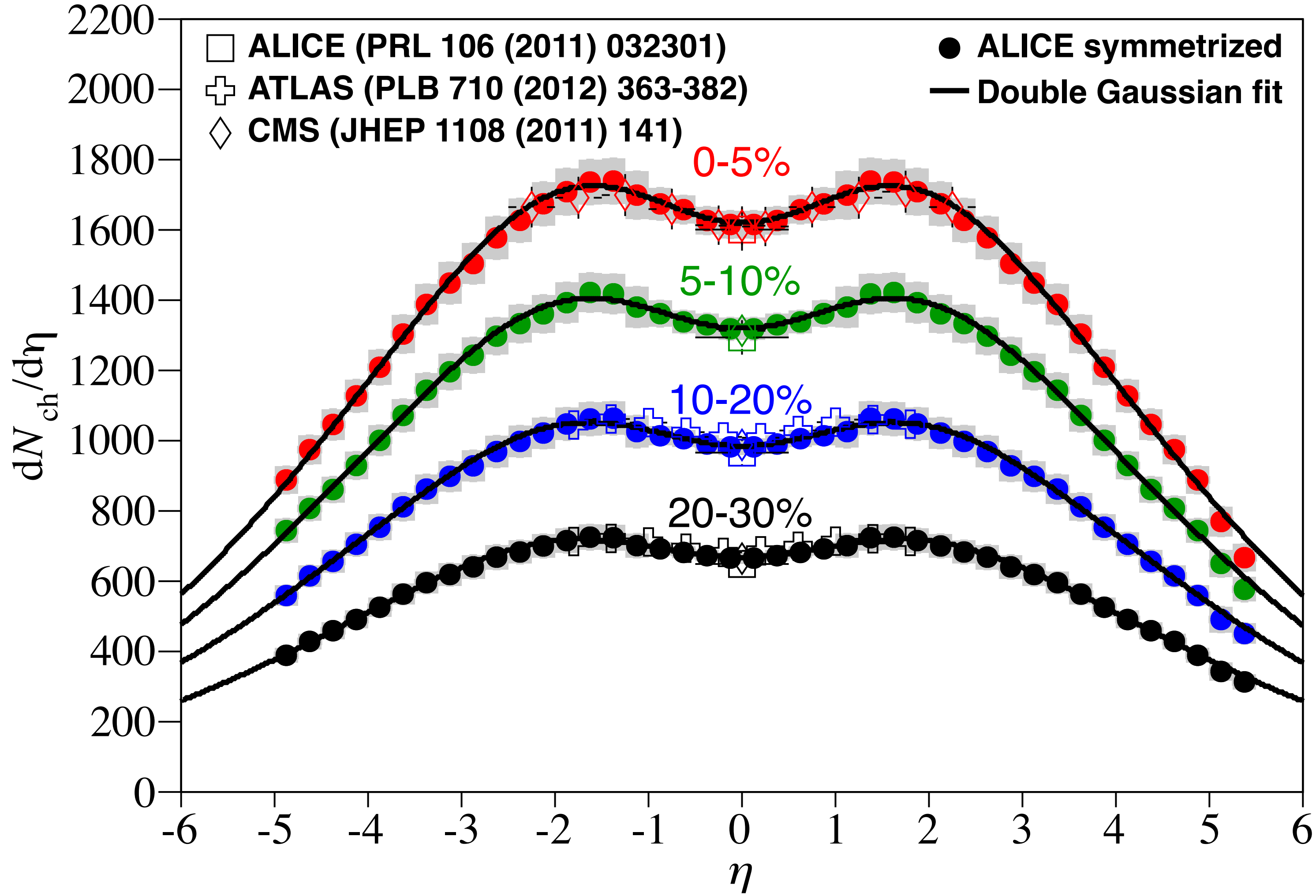


By adjusting the (one) parameter can map any initial state on the Gaussian model!

Diffusion, 3+1D Hydro: Schenke, Gale, Jeon



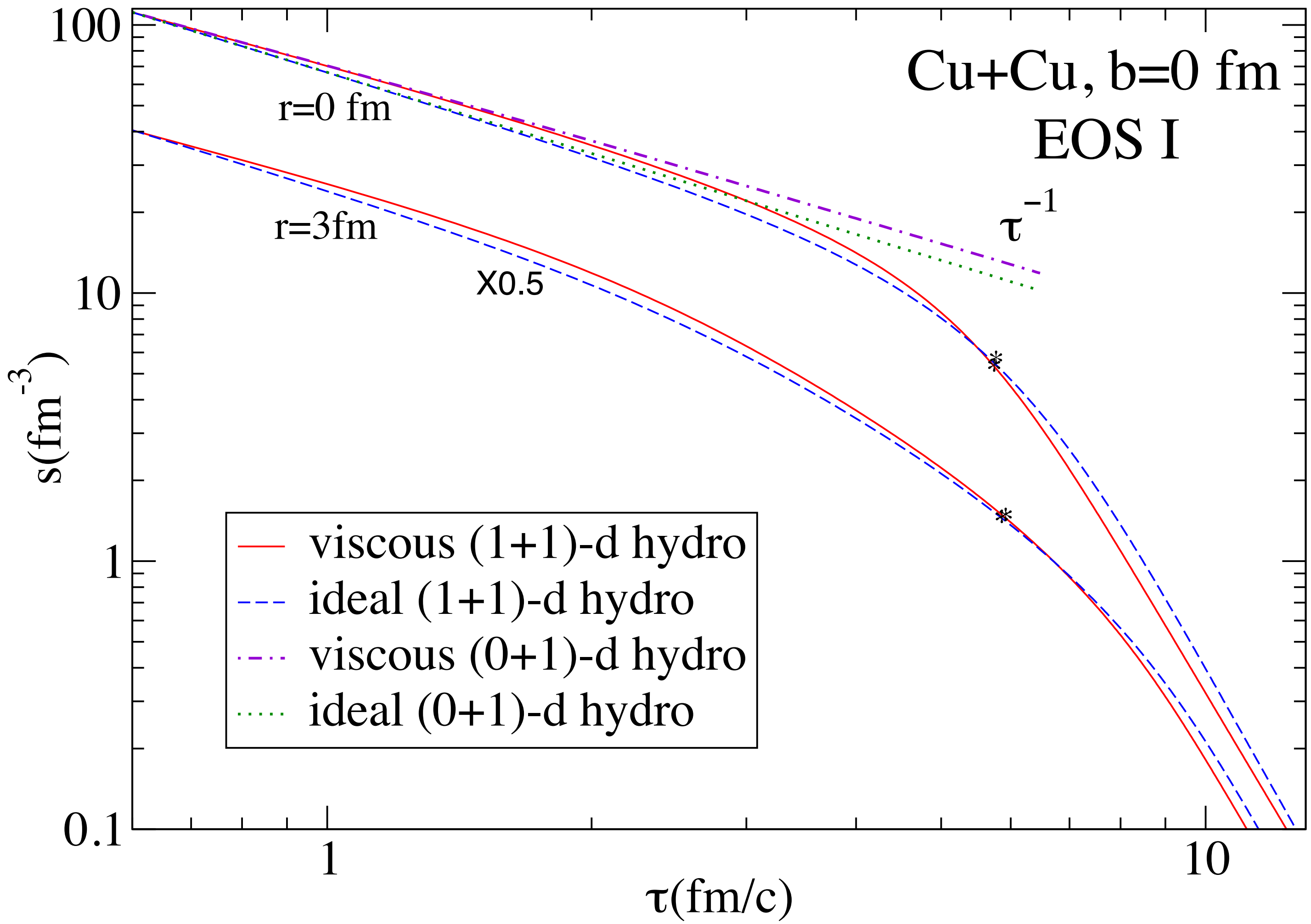
Longitudinal Initial Conditions



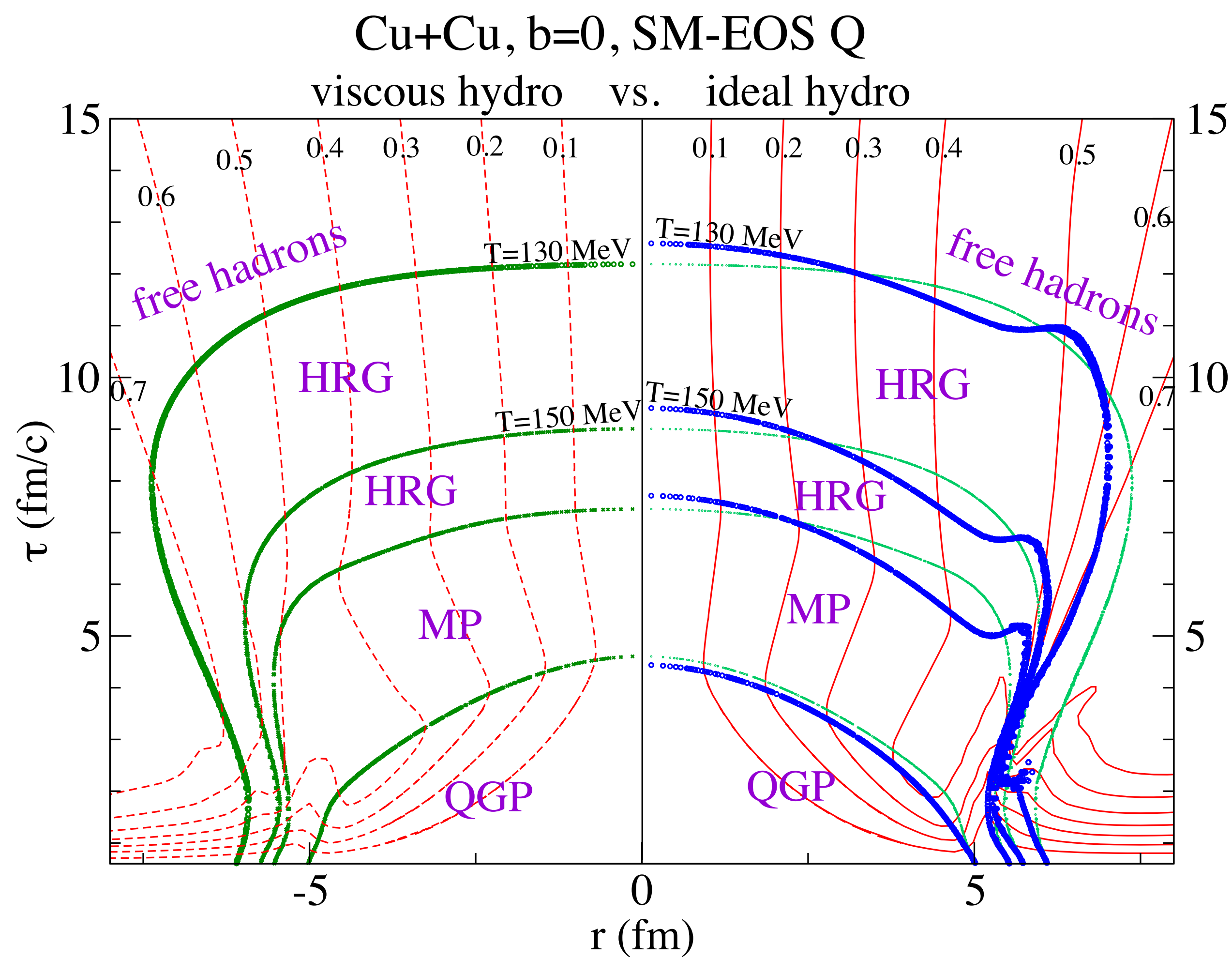
Longitudinal energy density is (almost) frozen during the evolution

The hydrodynamic expansion

1D to 3D transition: Huichao Song (circa 2008)



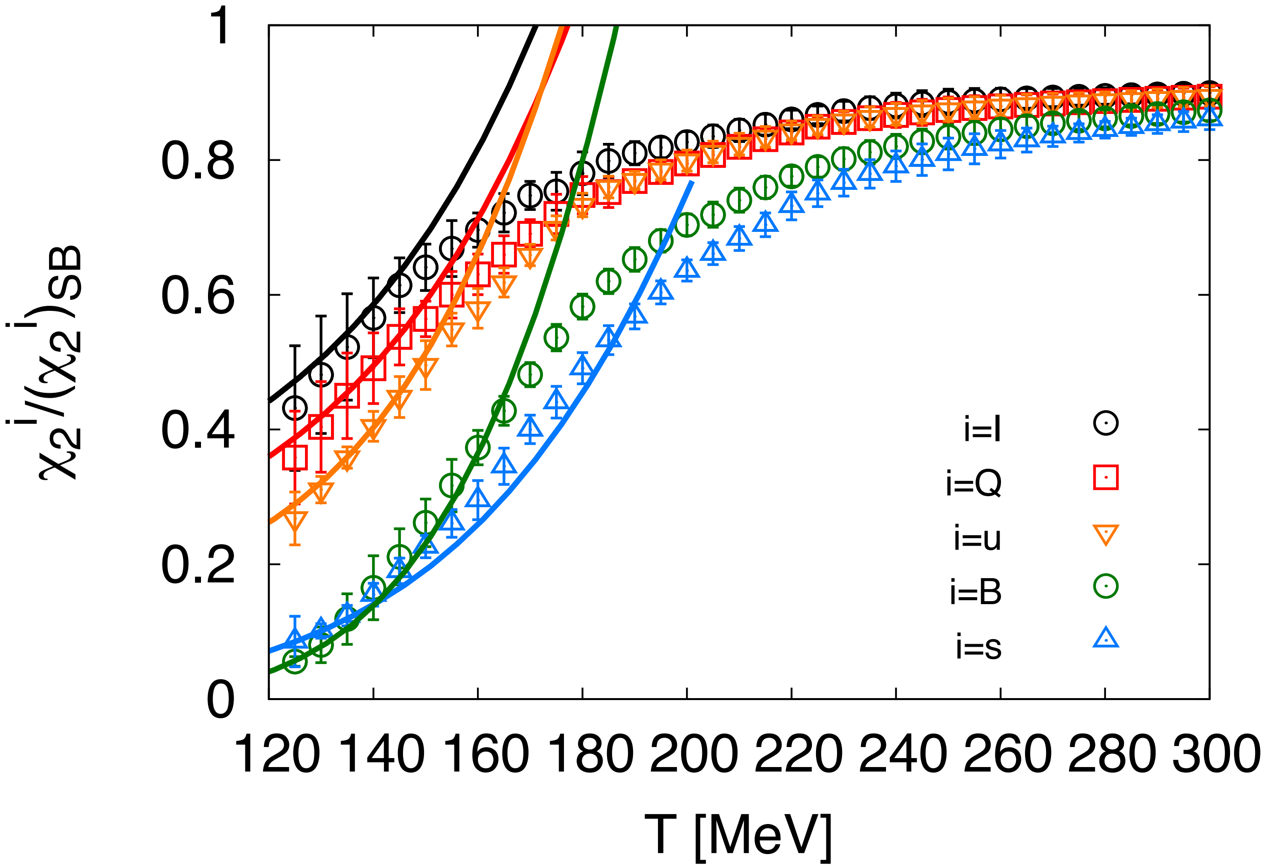
1D to 3D transition – Song and Heinz: 0712.3715



Phase space distributions:

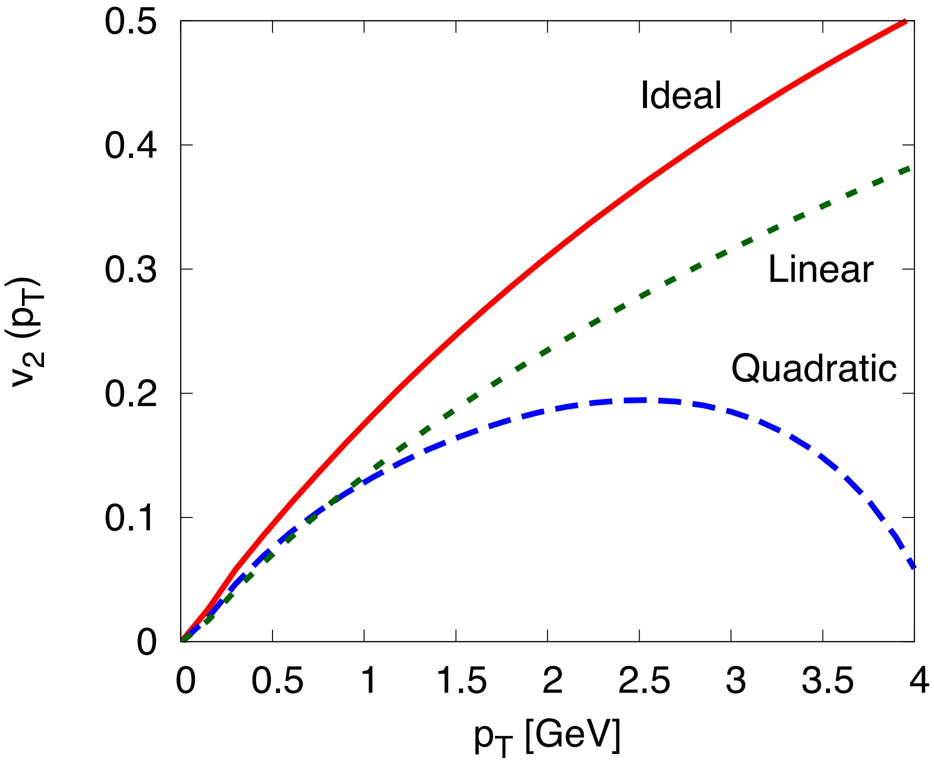
Fluctuations in Lattice QCD: arXiv:1112.4416

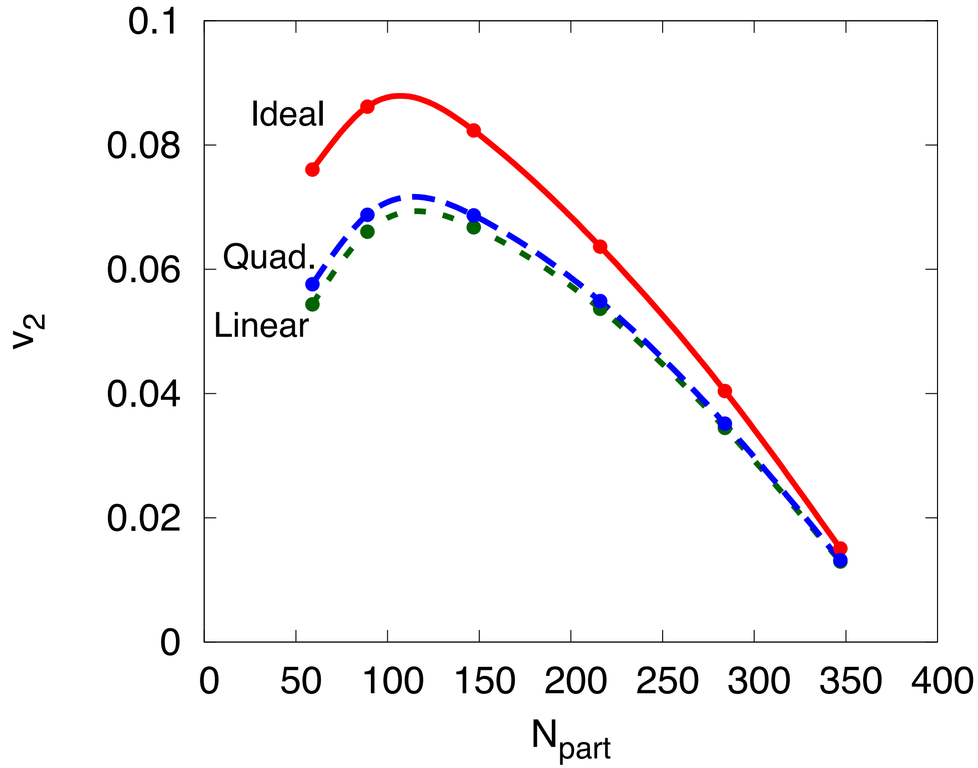
Lattice / Steffan Boltzmann (SB)



Agrees with HRG up to $T \simeq 150\,\mathrm{MeV}$

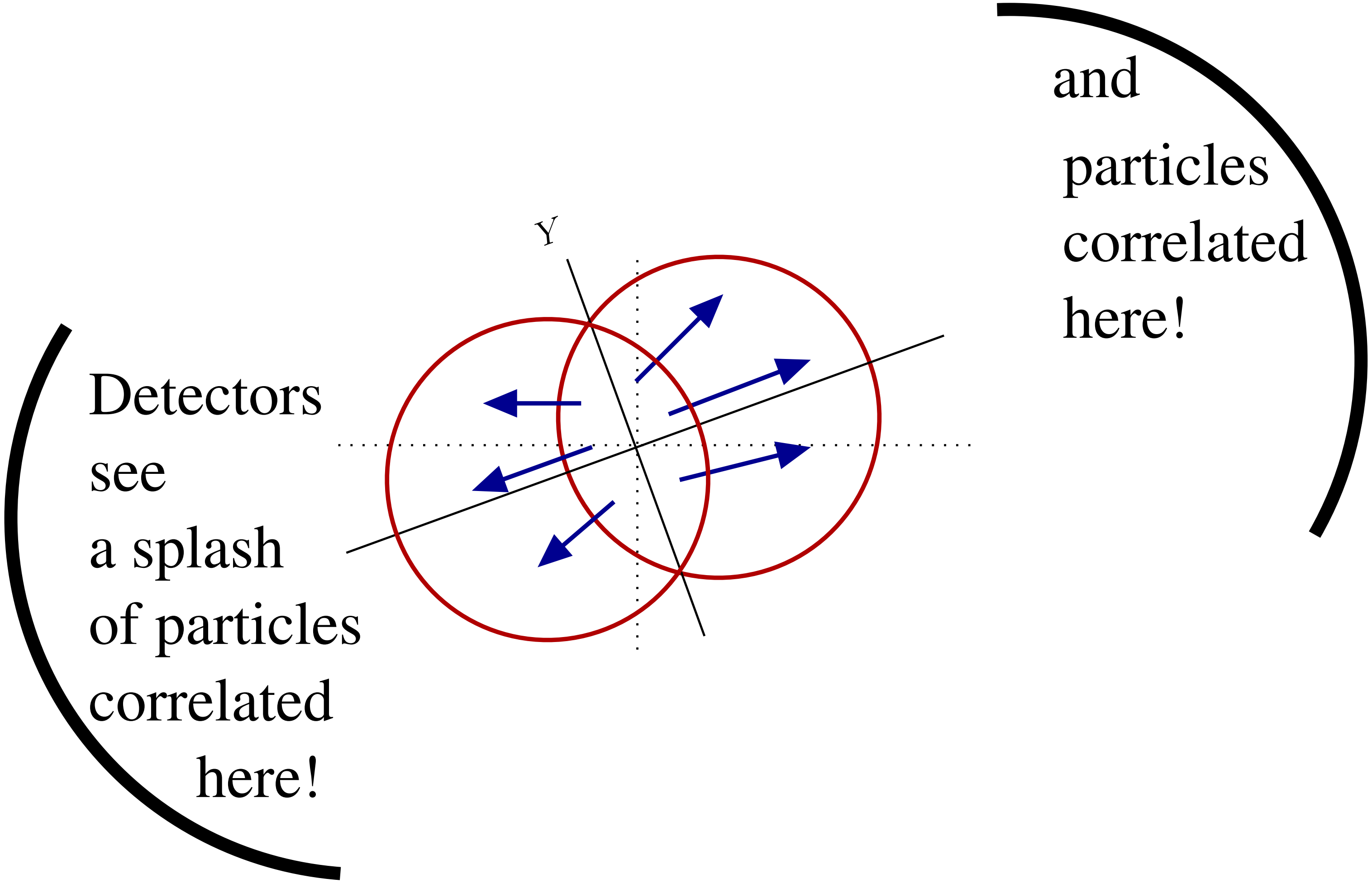
Viscous corrections to v_2 due to δf





Particle correlations

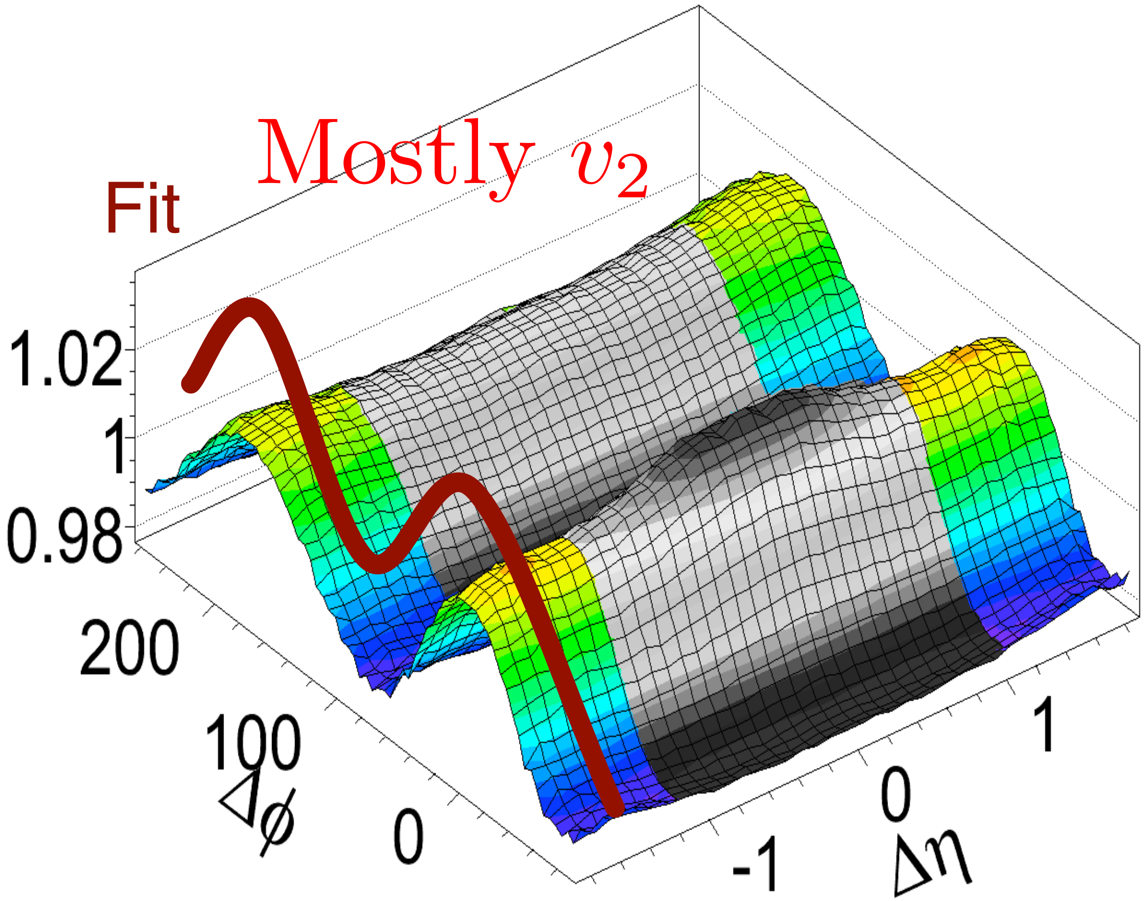
What is seen?



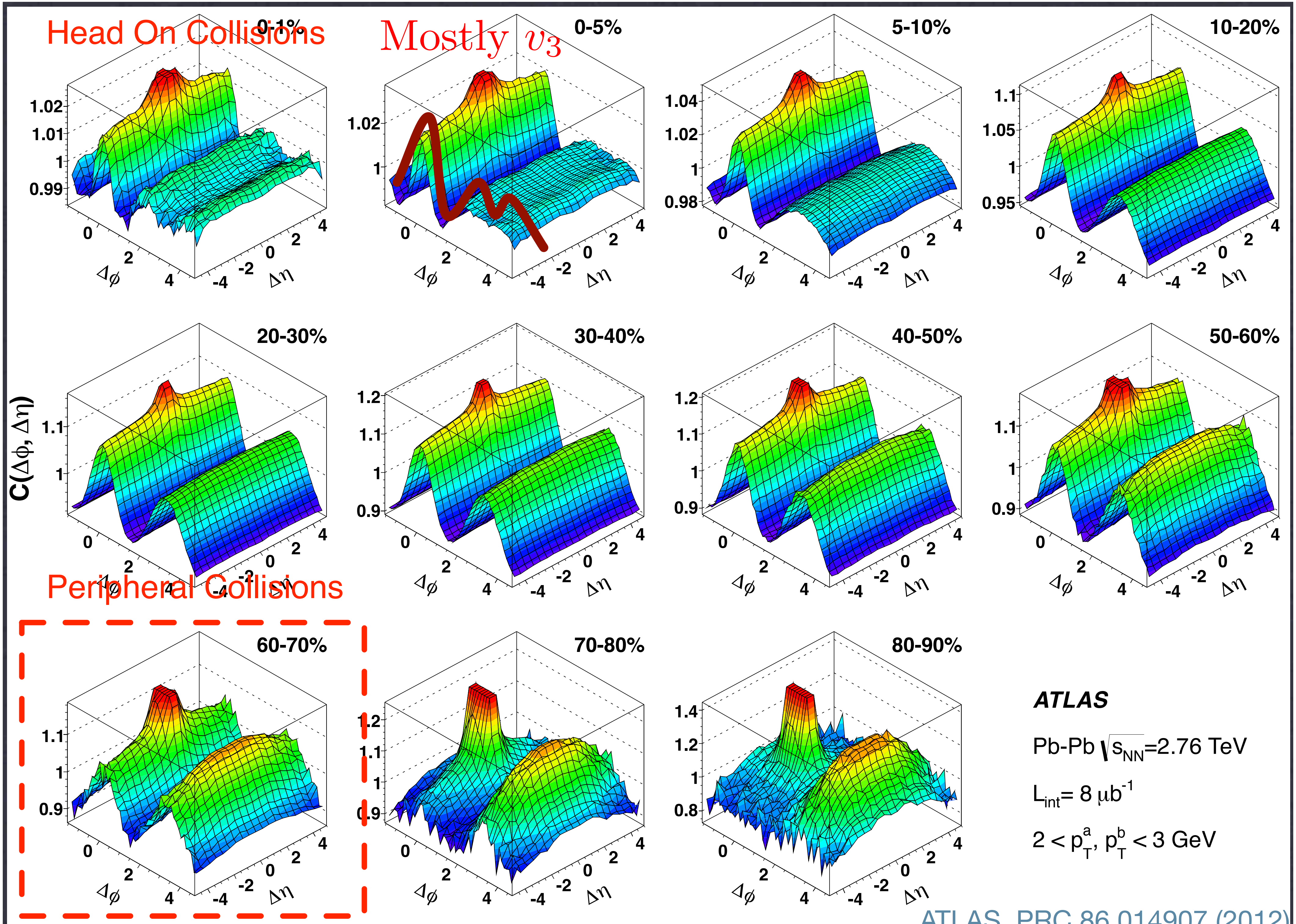
Measure the correlation function and determine the flow!

Inclusive two particle correlation functions – STAR DATA

$$C(\Delta\phi, \Delta\eta) = \frac{1}{\langle N_{\rm pairs}\rangle} \left\langle \frac{dN_{\rm pairs}}{d\Delta\phi d\Delta\eta} \right\rangle \qquad \Delta\phi = \phi_1 - \phi_2, \quad \Delta\eta = \eta_1 - \eta_2.$$

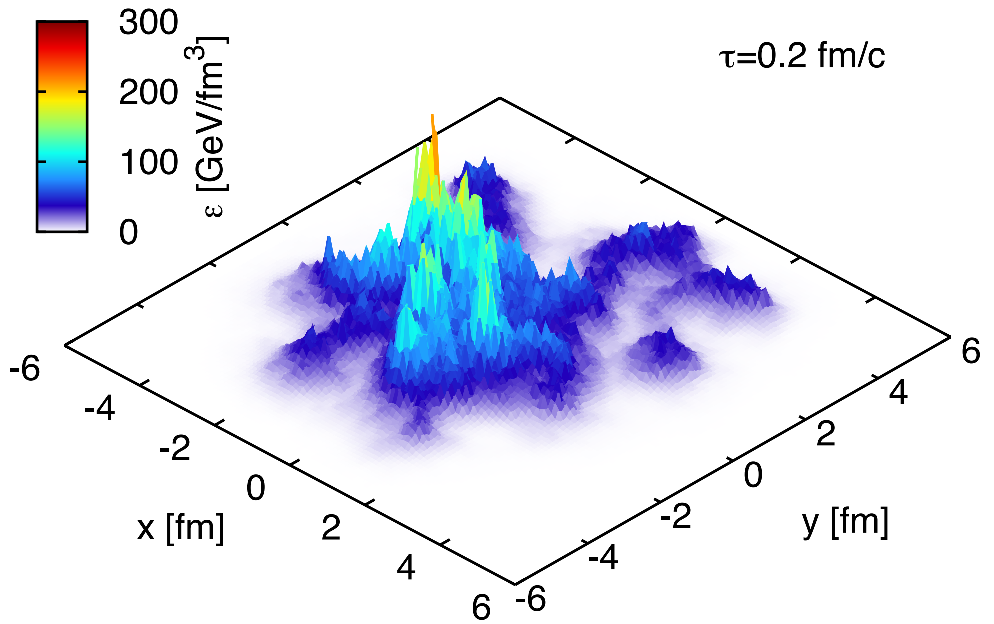


Fit $C(\Delta\phi, \Delta\eta)$ with a fourier series to determine $\langle v_2^2\rangle$ and $\langle v_3^2\rangle$ etc

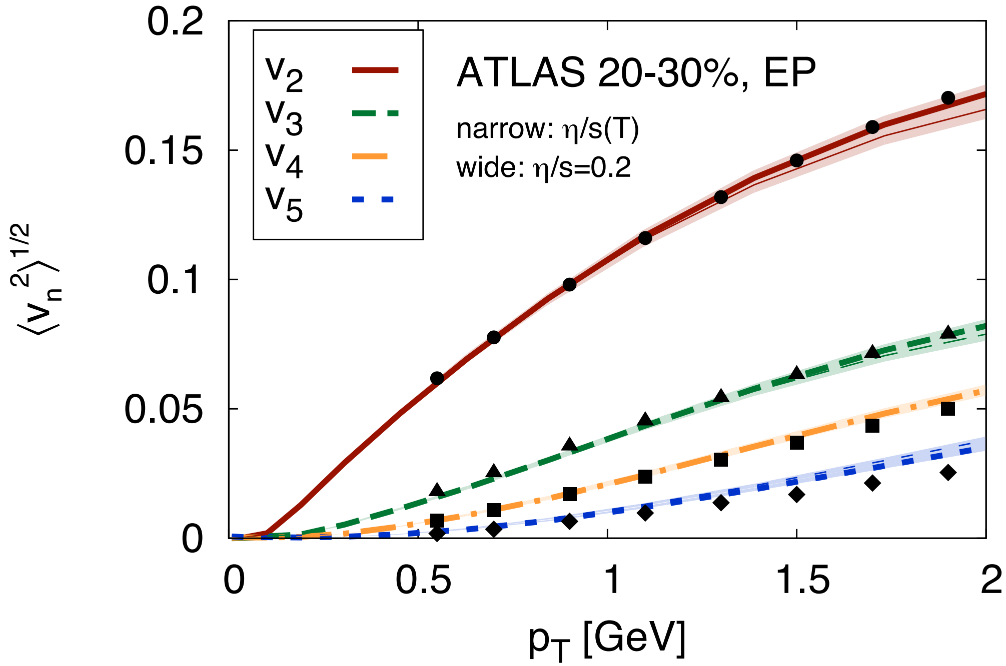

Head On Collisions
Mostly v_3
Peripheral Collisions
0-1%
0-5%
5-10%
10-20%
20-30%
30-40%
40-50%
50-60%
60-70%
70-80%
80-90%
C(Δφ, Δη)
Δφ
Δη
ATLAS
Pb-Pb $\sqrt{s_{NN}}$=2.76 TeV
L_{int}= 8 μb^{-1}
2 < p_T^a, p_T^b < 3 GeV
ATLAS, PRC 86 014907 (2012)

A very complete hydrodynamic simulation 3+1D: Schenke, Jeon, Gale, Venugopalan

Initial Conditions



Result



Hopefully I have opened the box!

Thank You!