From QCD kinetics to hydrodynamics

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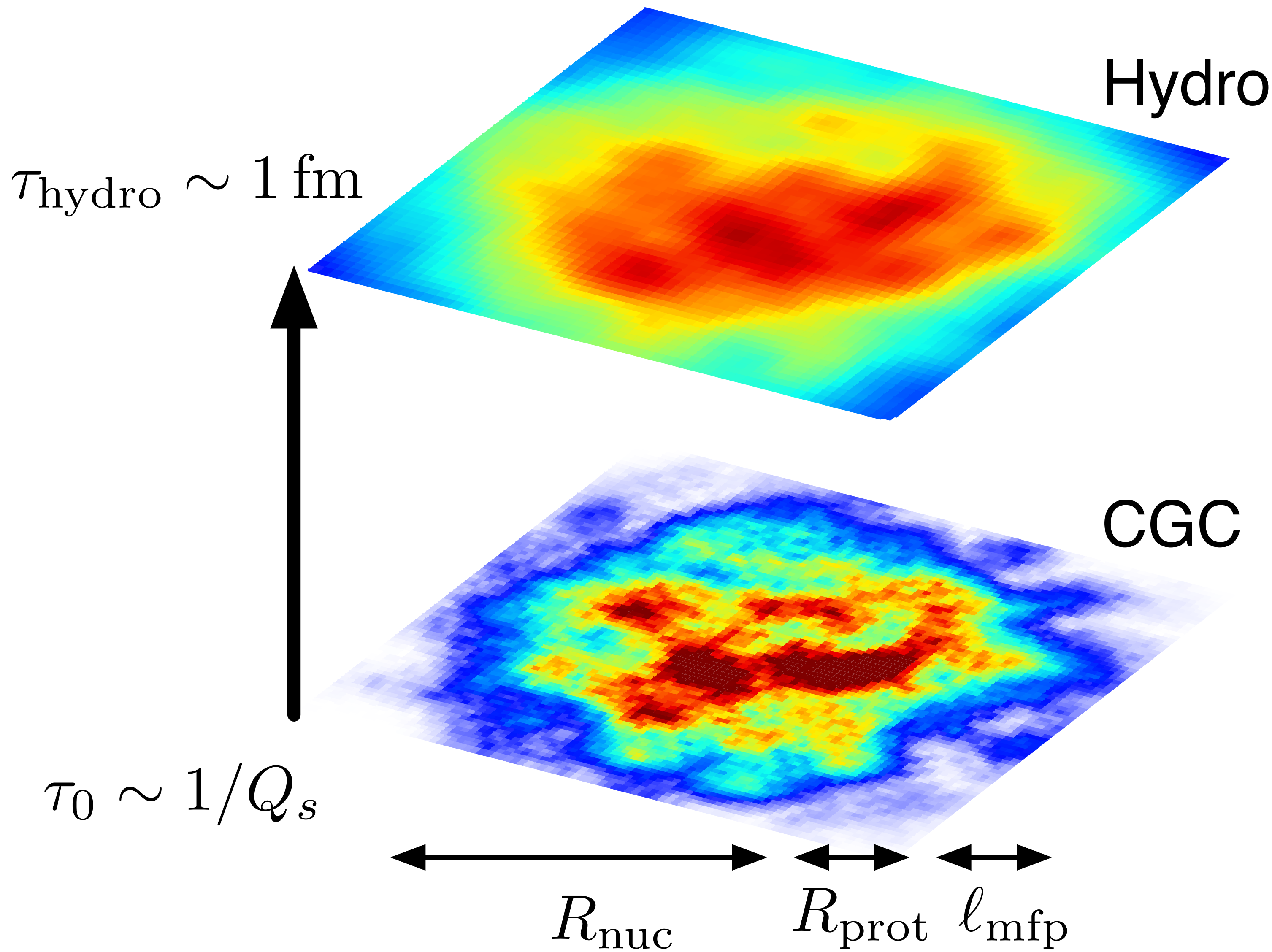
1. L. Keegan, A. Kurkela, A. Mazeliauskas, DT, JHEP (2016)

2. A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, DT, 65 pages, almost done

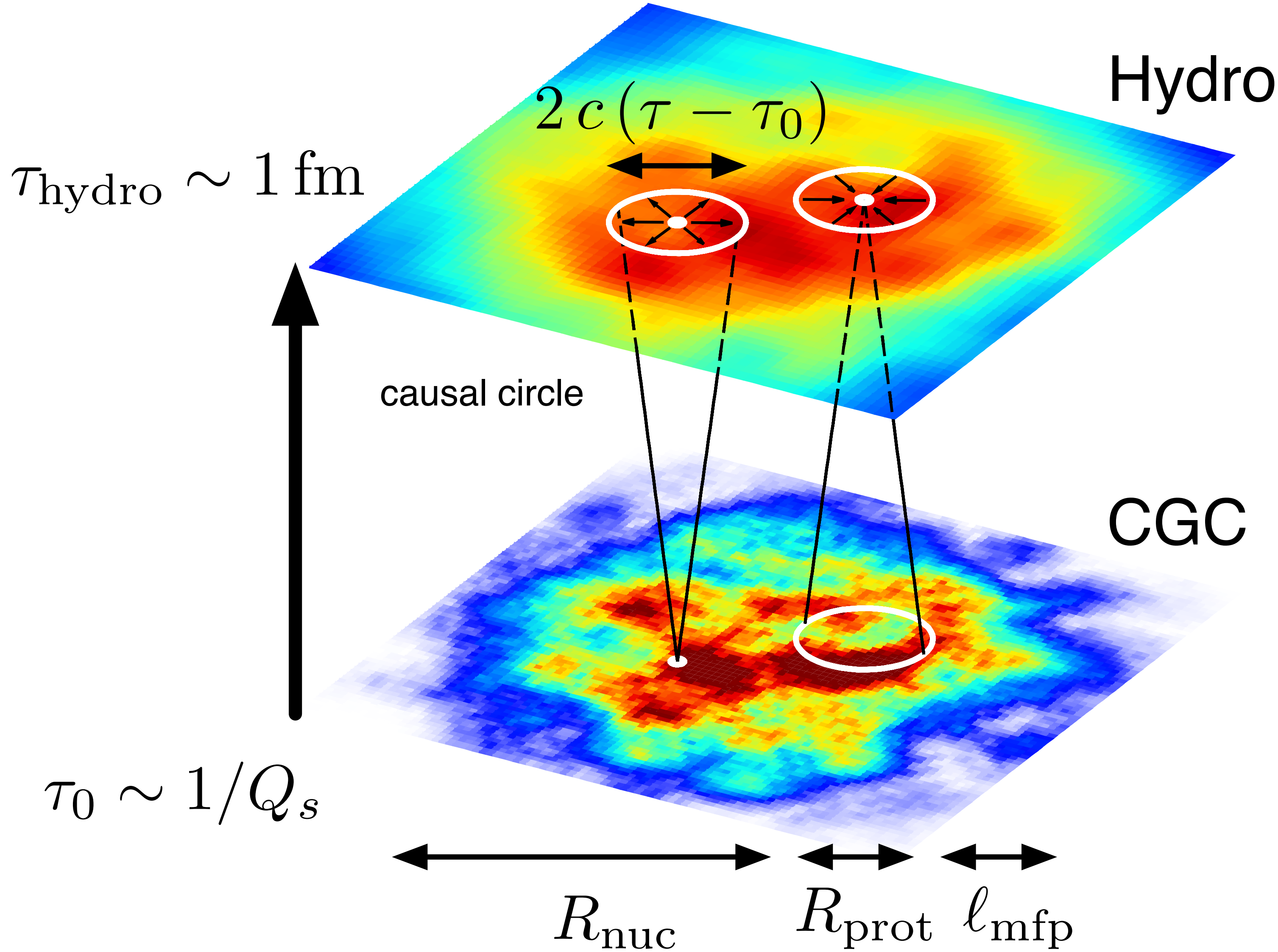
Mapping the CGC fluctuating initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics with approximations:

$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \gg 1/Q_s$$

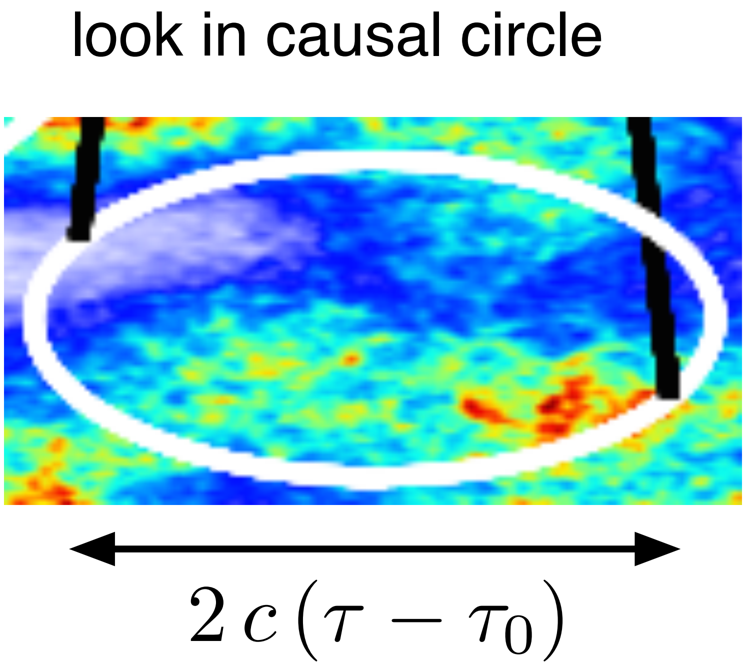
Mapping the CGC fluctuating initial conditions to hydro

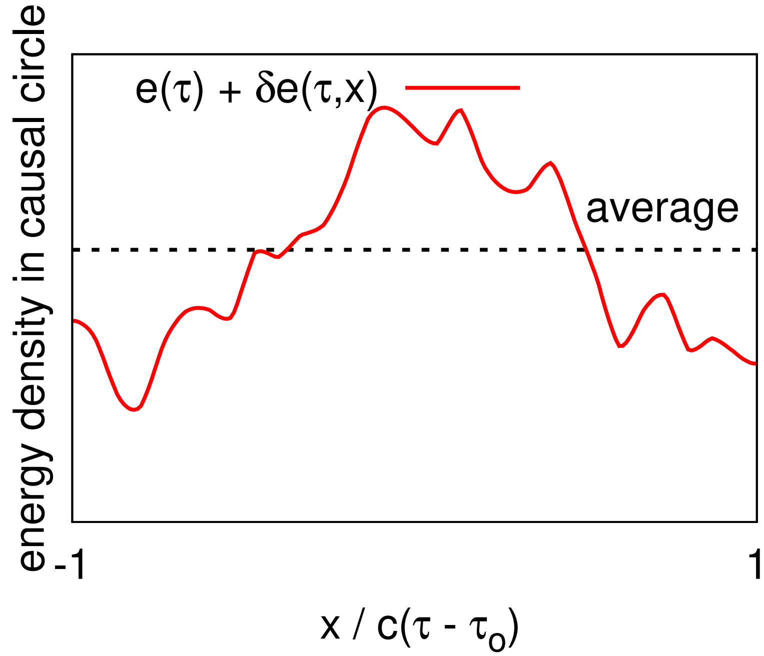


Causality limits the equilibration dynamics within a causal circle

$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \sim c\tau_{\rm hydro} \gg 1/Q_s$$

An approximation scheme for the equilibration dynamics:





1. Determine the evolution of the average (homogeneous) background

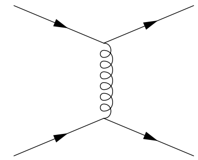
Bottom-Up Thermalization!

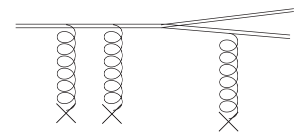
2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}'\, G(\boldsymbol{x}-\boldsymbol{x}';\tau,\tau_o)\; \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

How to compute the background and perturbations:

$$\partial_\tau f + \frac{\mathbf{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]}_{} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}_{} \quad ,$$





Gluon distribution function for background and perturbations

$$f = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp,\mathbf{p}} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}}_{\text{transverse perturbations}} \quad .$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp\cdot\boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\mathbf{p}} = -\delta\mathcal{C}[\bar{f},\delta f] \qquad \text{perturbation}$$

We will discuss the background and perturbations separately

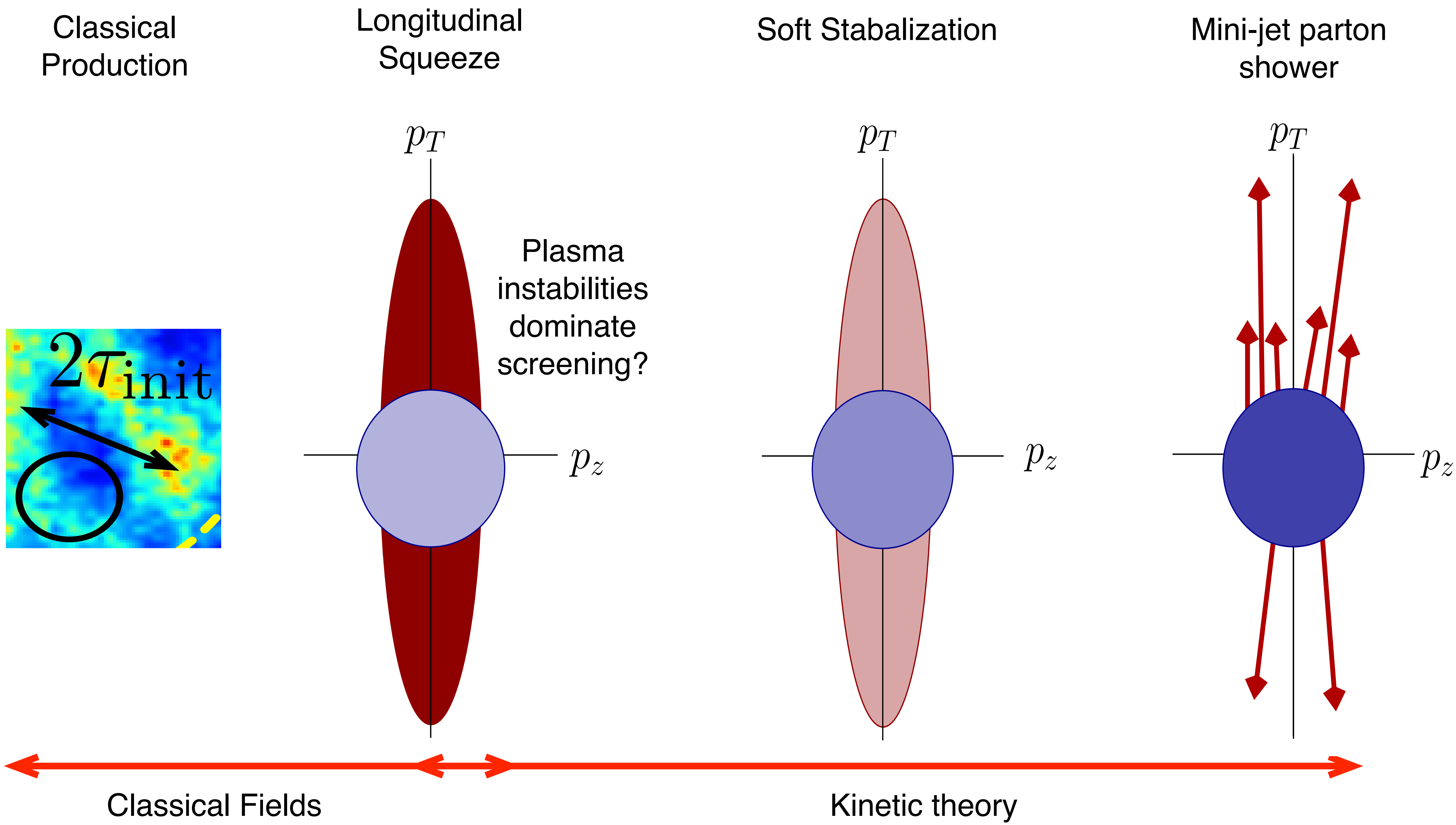
Outline

I. Evolution of the backgound: “bottom-up” thermalization

II. Evolution of the perturbations

The background and "bottom-up" thermalization

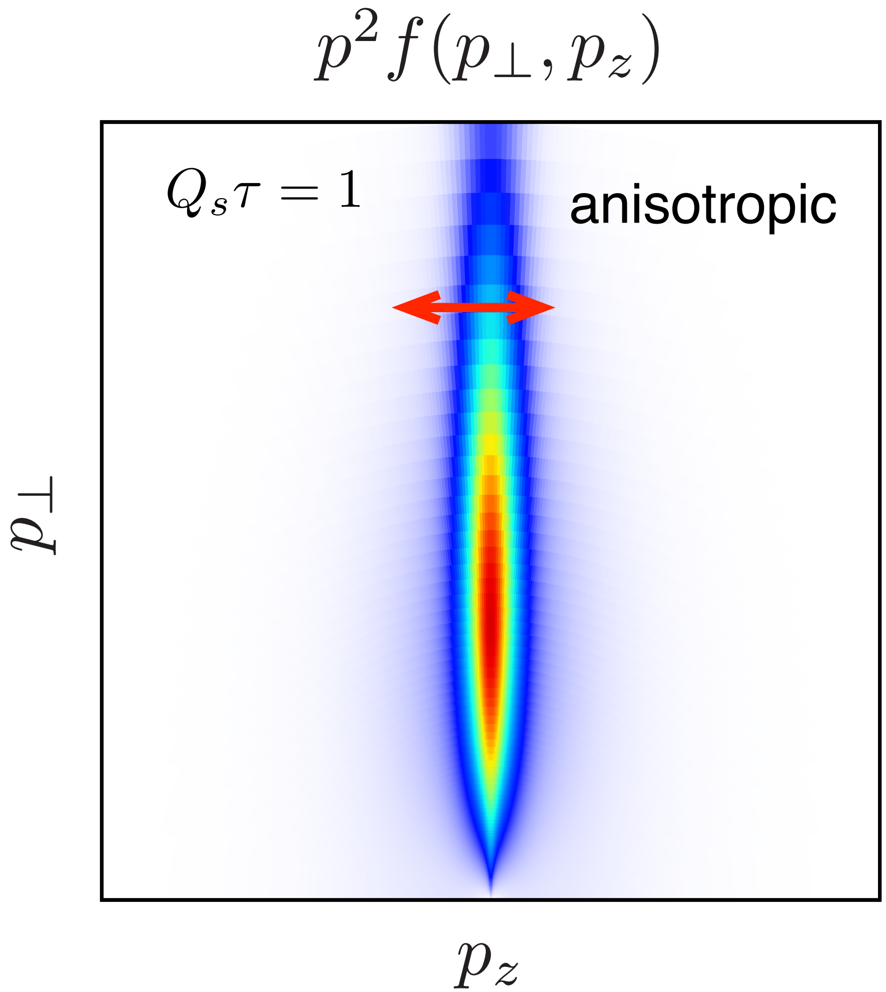
Baier, Mueller, Schiff, Son



Reach a thermal state in $\tau_{\mathrm{hydro}} \sim 1/(\alpha_s^{13/5} Q_s)$

A numerical realization of bottom-up

- Builds upon the first numerical realization — Kurkela, Zhu PRL (2015)



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling constant of $\alpha_s = 0.3$

$$\underbrace{\lambda \equiv 4\pi\alpha_s N_c = 10}_{\text{theorists version of } \alpha_s = 0.3}$$

corresponding to

$$\frac{\eta}{s} = 0.6 = \frac{7.5}{4\pi}$$

We see "Bottom-Up" in the computer code.

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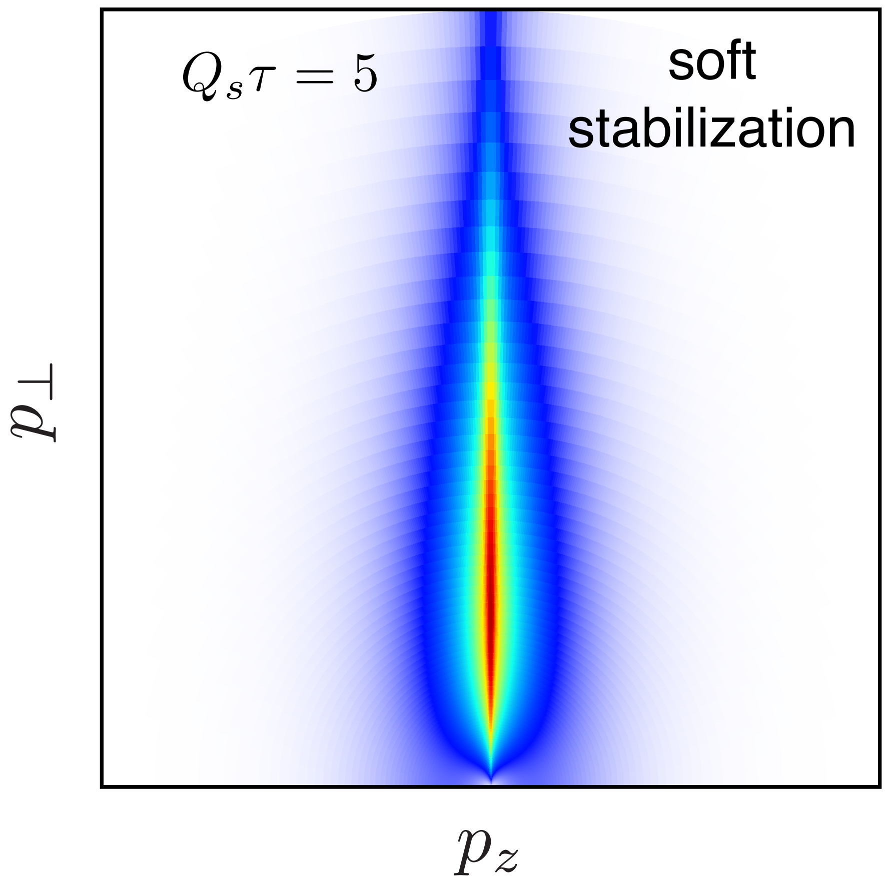
$$p^2 f(p_\perp, p_z)$$



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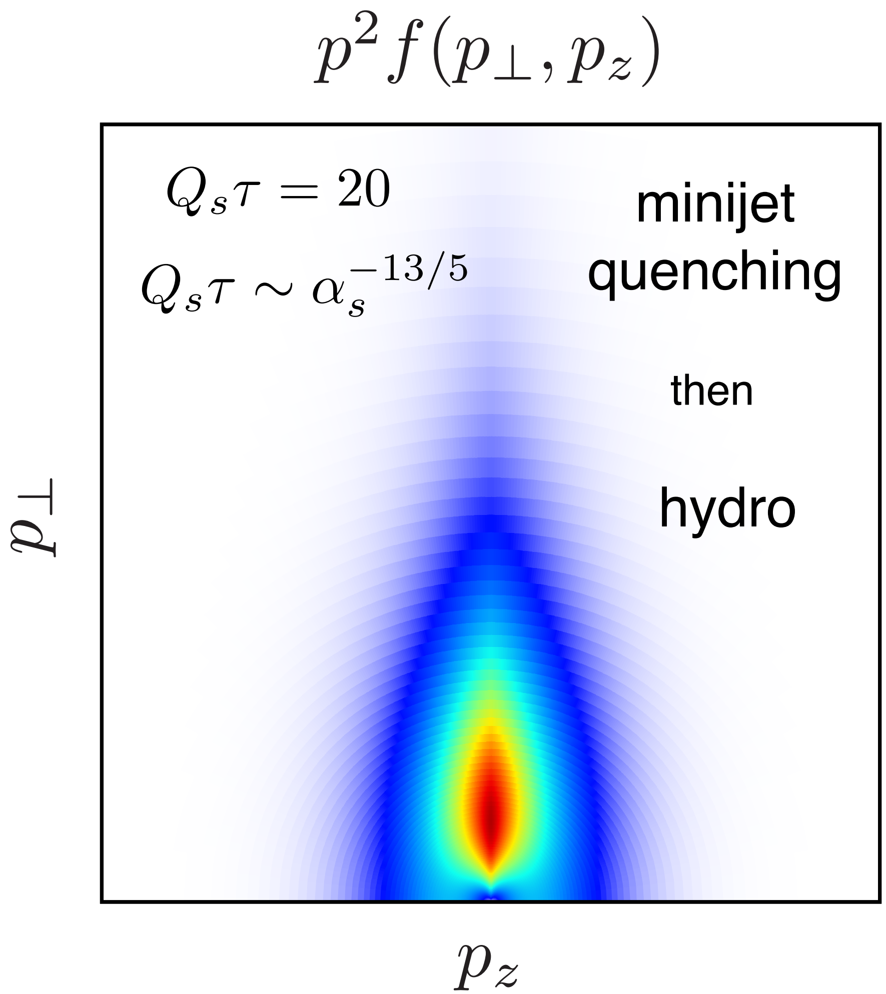

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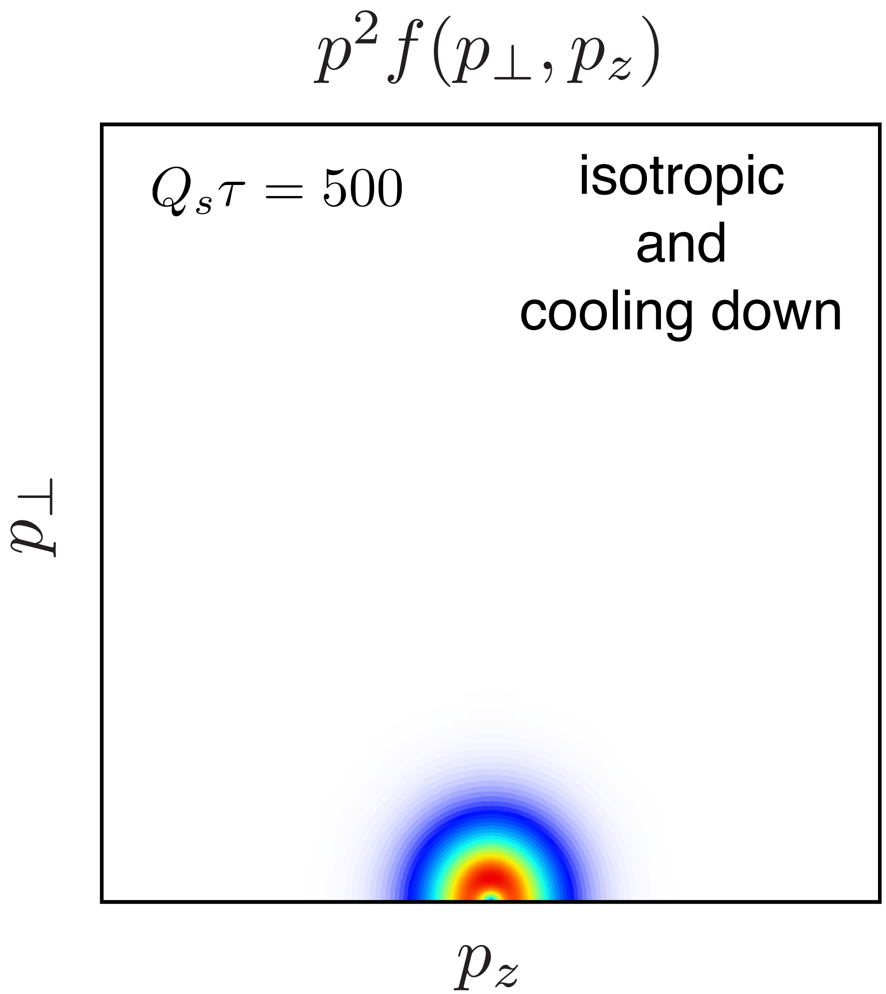

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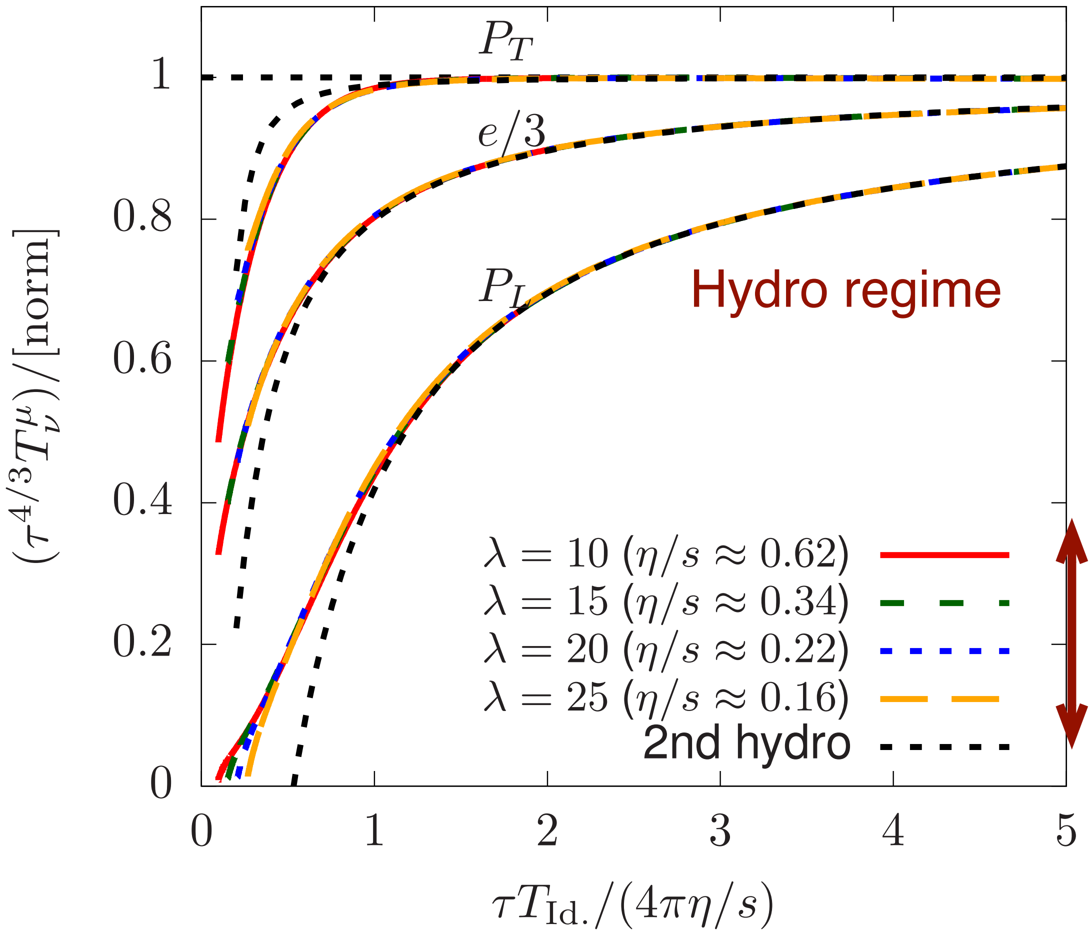
When does the background stress tensor approach second order hydrodynamics?



Different values of coupling give different η/s

In terms of eta/s, all couplings thermalize at same scaled time

Keegan, Kurkela, Romatschke,Schee, Zhu

Gives a basis for interpolating from weak coupling results to stronger coupling

Measure time in a physical relaxation time given by $\tau_R \equiv \eta/sT_{\mathrm{eff}}$ instead of α_s:

$$\frac{\tau}{\tau_R} \equiv \frac{\tau T_{\mathrm{eff}}(\tau)}{\eta/s} \qquad \text{with} \qquad \tau_R \equiv \frac{\eta}{sT_{\mathrm{eff}}}$$

Can start hydro when $\tau T_{\mathrm{eff}}/4\pi\eta/s \sim 1$

Translating earliest hydro starting time into physical units:

1. At late times the dynamics is ideal hydro: $T_{\rm eff}(\tau) = \Lambda_T/(\Lambda_T \tau)^{1/3}$

$$\lim_{\tau\to\infty} \tau T^3(\tau) = \Lambda_T^2$$

↕ This integration constant determines dN/dy at the end of hydro

2. Hydro fits to multiplicity give:

$$\left\langle \tau e^{3/4} \right\rangle\Big|_{\tau=1.2\,\rm fm} = \underbrace{1.6\,{\rm GeV}^2 \propto \Lambda_T^2}$$

highly constrained by $\frac{dN}{dy}$!

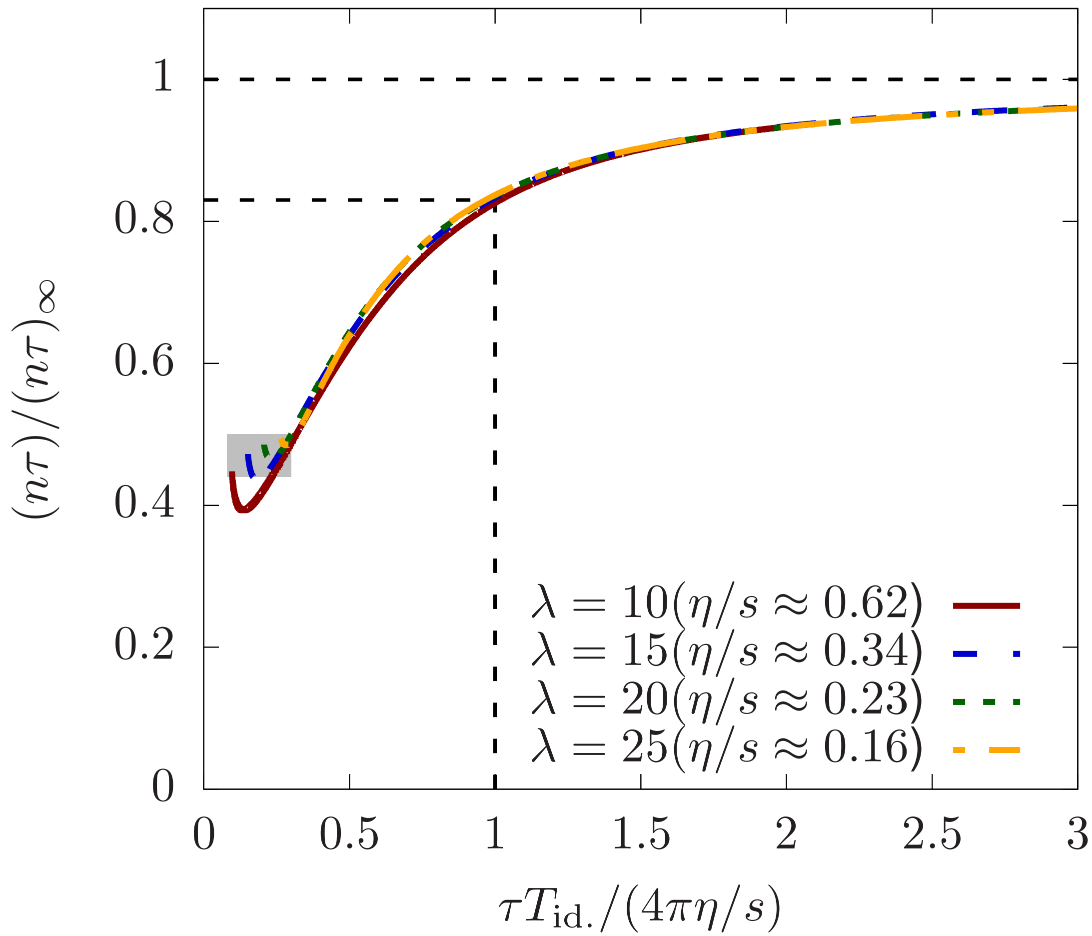
3. The estimate for $\tau_{\rm hydro}$:

$$\frac{\tau_{\rm hydro} T_{\rm eff}(\tau_{\rm hydro})}{4\pi(\eta/s)} = 1$$

Find that hydrodynamics is applicable for times later than:

$$\tau_{\rm hydro} \approx 0.85\,{\rm fm} \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{1.6\,{\rm GeV}}{\langle \tau e^{3/4}\rangle}\right)^{1/2} \left(\frac{\nu_{\rm eff}}{16}\right)^{3/8}$$

How much to gluons multiply during the equilibration process?



$$n\tau = \frac{1}{A}\frac{dN}{dy}$$

The final gluon multiplicity is 2.5 times the initial gluon multiplicity

independent of the coupling or η/s!

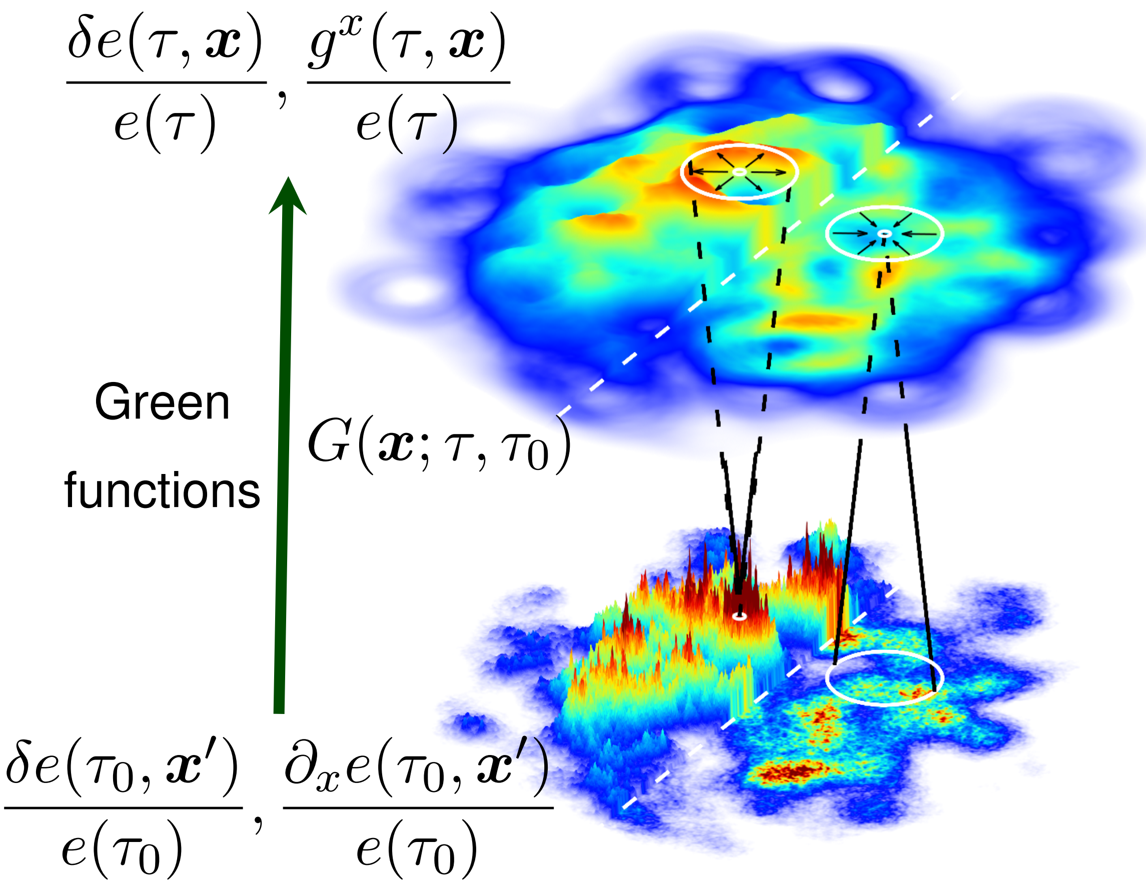
Outline

✓ Evolution of the background: “bottom-up” thermalization

II. Evolution of the perturbations

The Green functions fourier mode by fourier mode:

- Compute the response in "bottom-up" to an initial perturbation, $\delta f_{\boldsymbol{k}} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}$
  - ⋆ Then sum them up



Properties of Green Functions

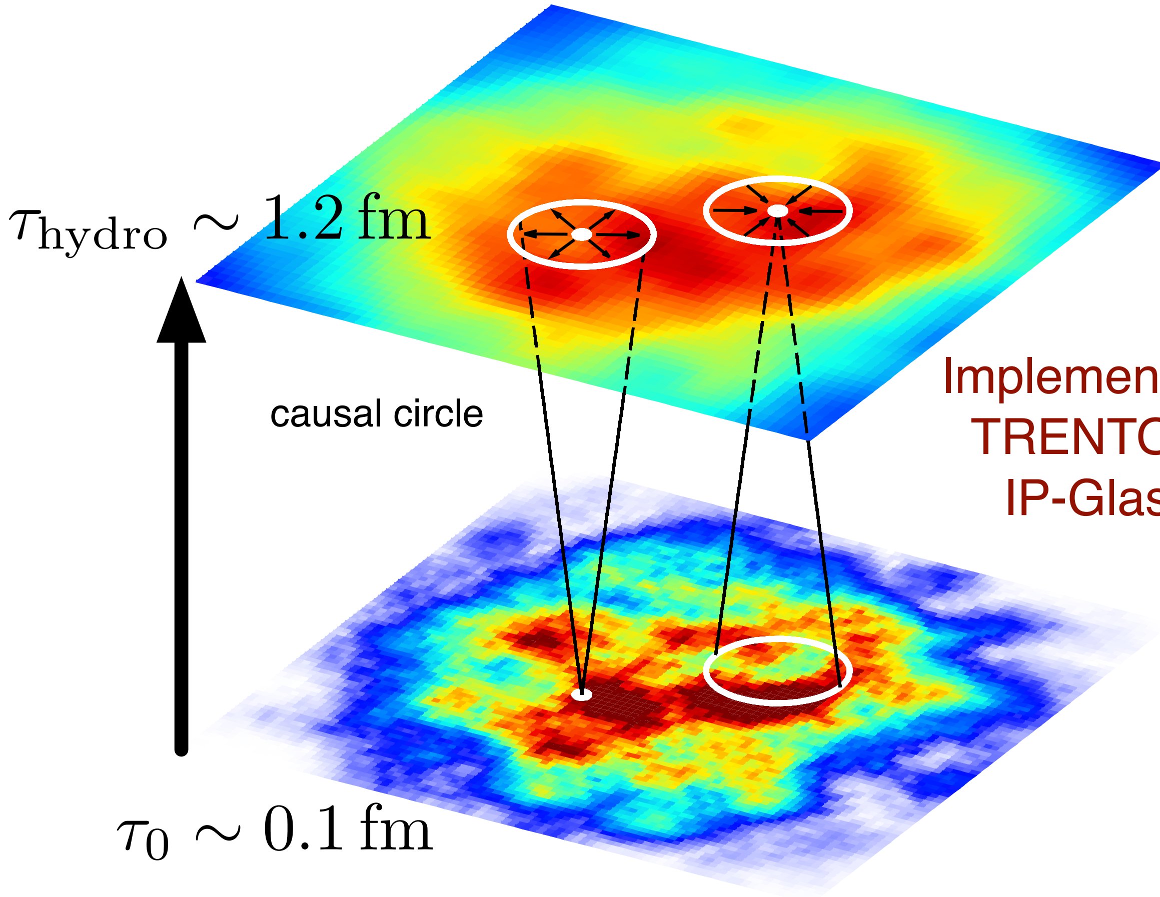
1. Has free streaming for $k \to \infty$
2. Has hydro for $k \to 0$
3. Depends on η/s and time through

$$\frac{\tau T_{\rm eff}(\tau)}{4\pi(\eta/s)}$$

For hydro need:

$$\frac{\tau T_{\rm eff}(\tau)}{4\pi(\eta/s)} > 1$$

A practical algorithm for implementation:



Implemented for TRENTO and IP-Glasma

(i) For each point, average the energy in causal circle

- ⋆ Find the scaling time corresponding to τ_0 and $\tau_{\rm hydro}$ for given η/s and energy

(ii) Propagate background and perturbations in scaled time

- ⋆ Sometimes need to regulate the response

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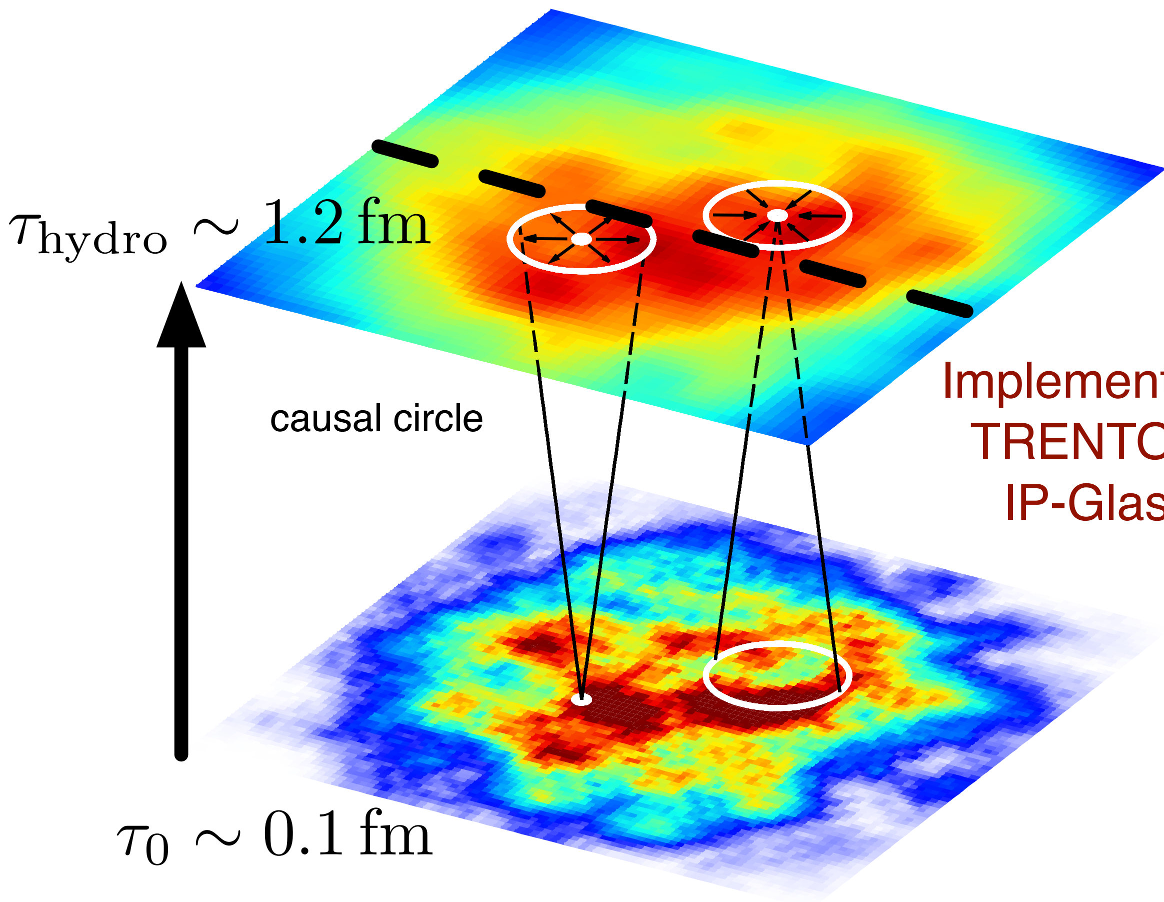

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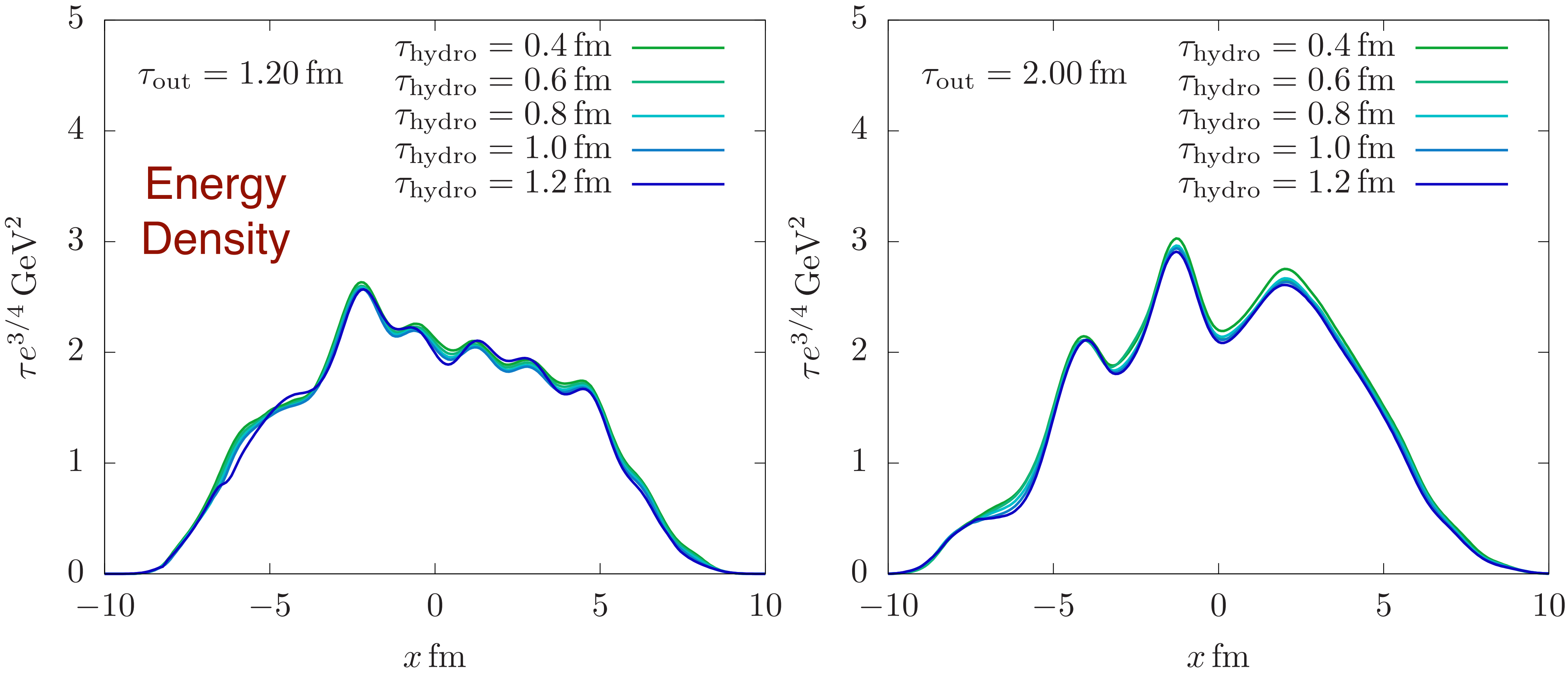
Do hydro results depend on $\tau_{\rm hydro}$?

1. Implementation in TRENTO. $\eta/s = 2/4\pi$. Central LHC.

2. Kinetics runs from $\tau_0 = 0.1$ up to $\tau_{\rm hydro}$, then hydro up to τ_{rmout}.



Remarkably insensitive to $\tau_{\rm hydro}$ as we want !

Do hydro-results depend on τ_{hydro} ?

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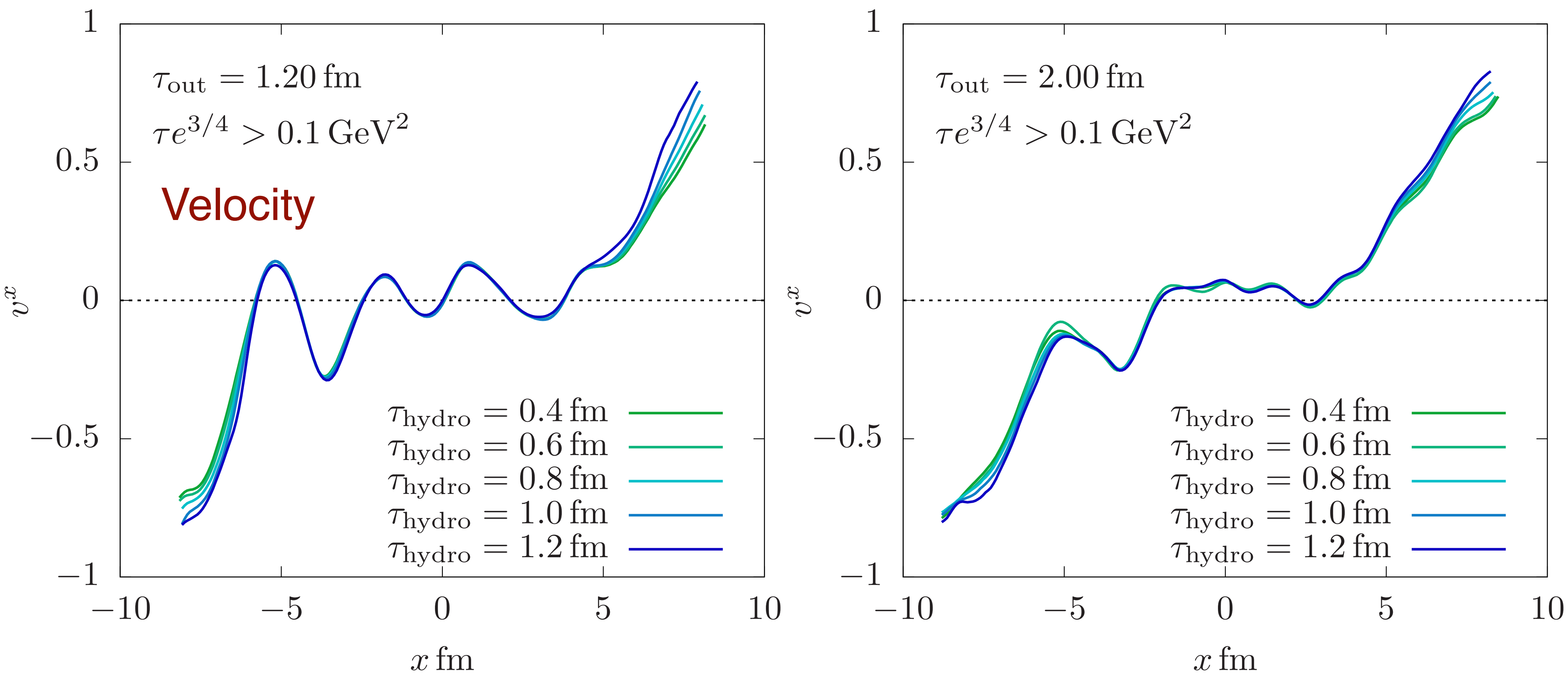
2. Kinetics runs from $\tau_0 = 0.1$ up to τ_{hydro}, then hydro up to τ_{out}

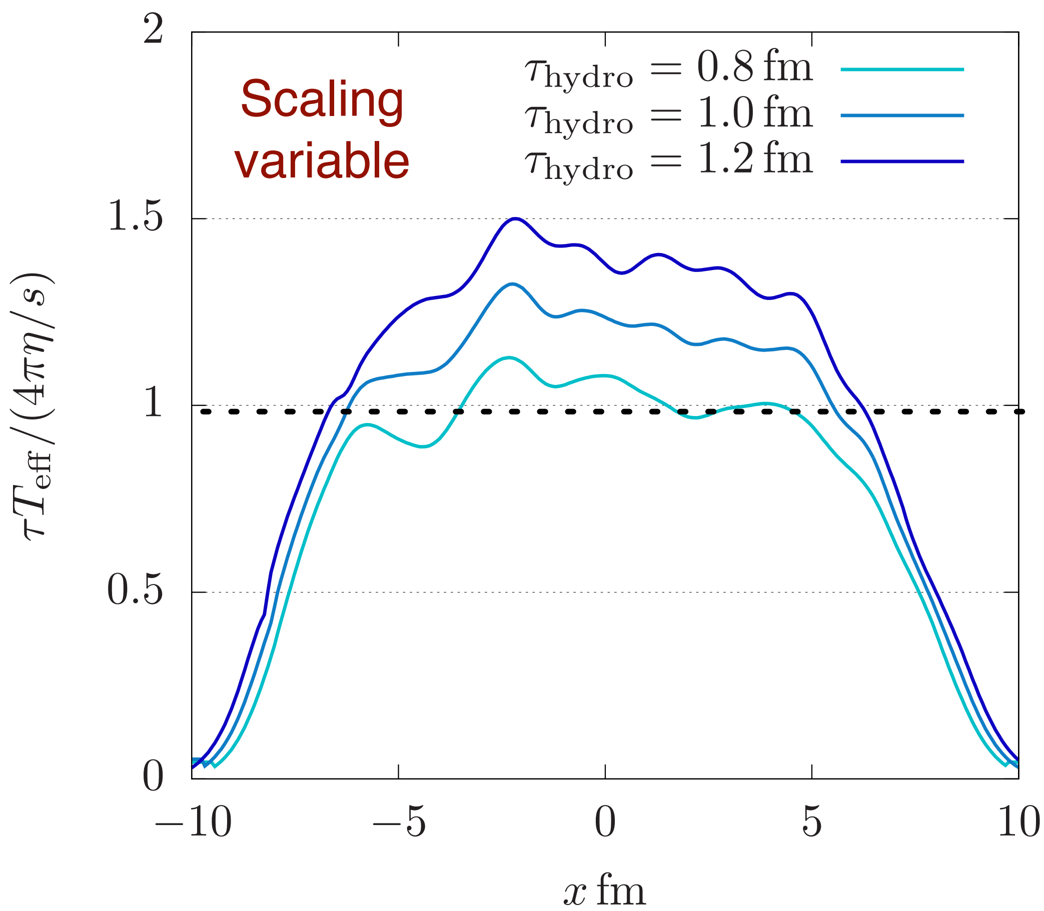


Remarkably insensitive to τ_{hydro} as we want !

Are the constitutive relations are satisfied at late times?

- For times sufficiently late times Navier-Stokes should be valid:

$$\pi^{\mu\nu} = \underbrace{-\eta\sigma^{\mu\nu}}_{\text{navier stokes}} \qquad \text{for} \qquad \frac{\tau T_{\rm eff}}{4\pi\eta/s} > 1$$



Expect the cells near the line to be equilibrated and obey constitutive equations

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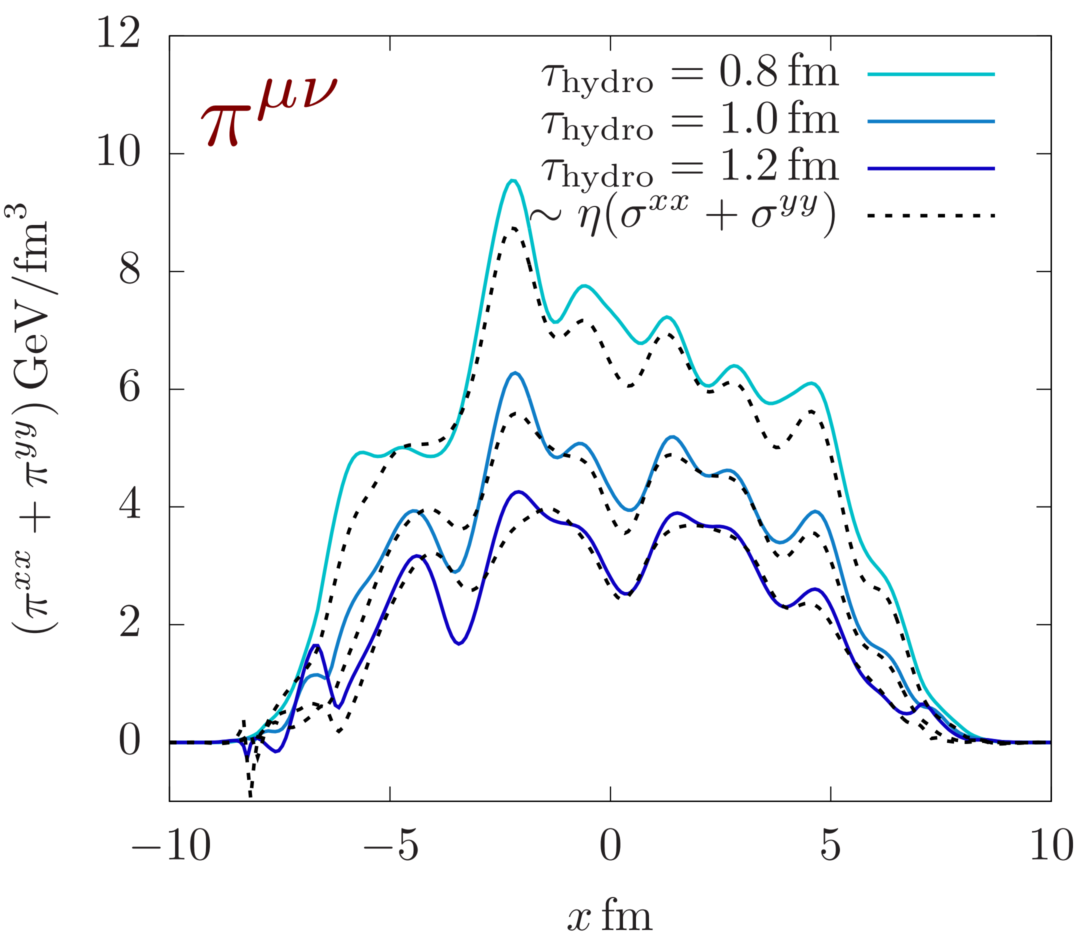
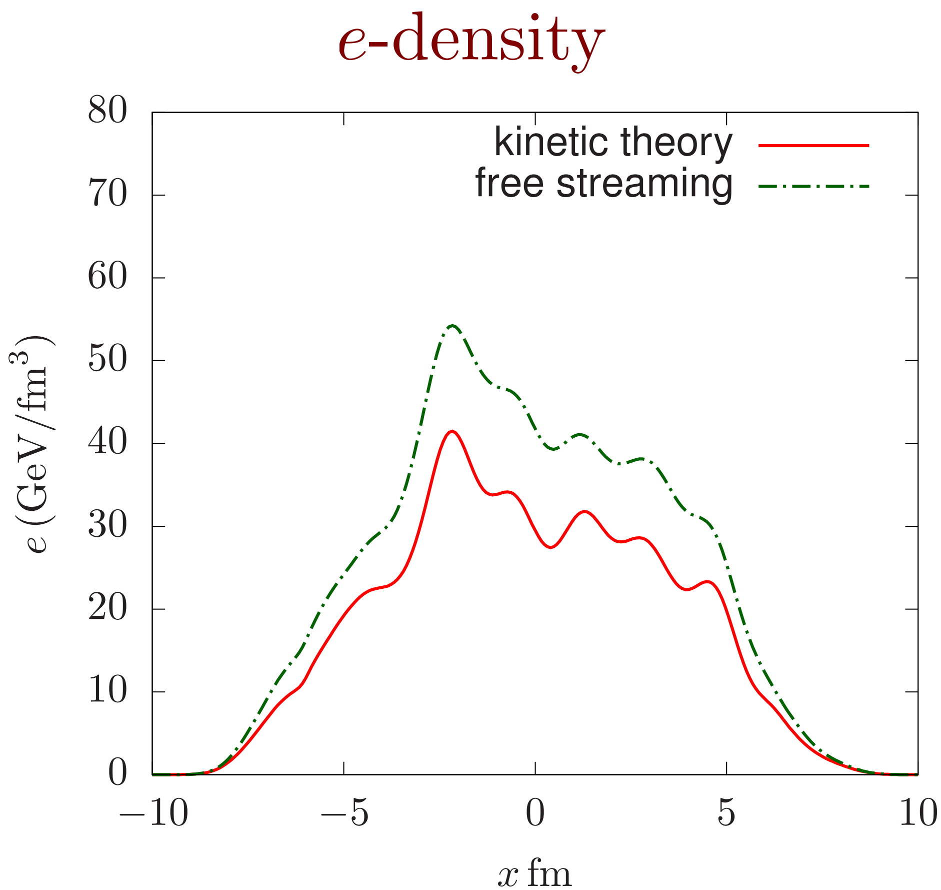


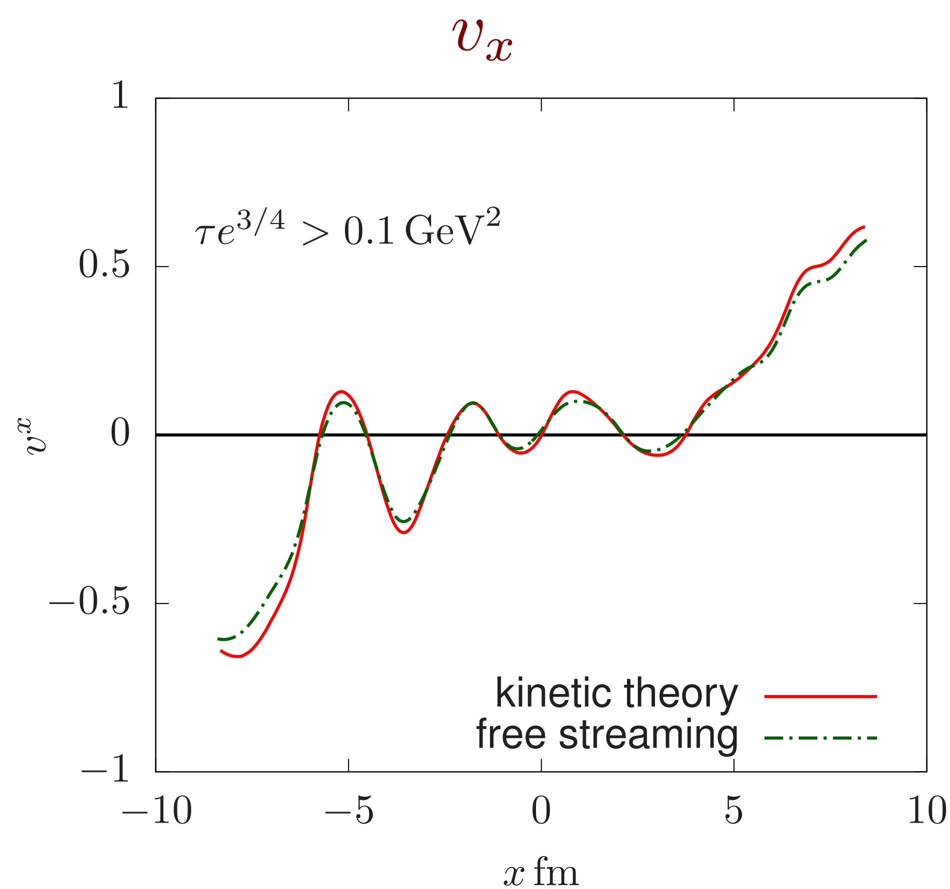
Black lines are navier stokes.

Color lines are kinetics

Constitutive relations are satisfied!

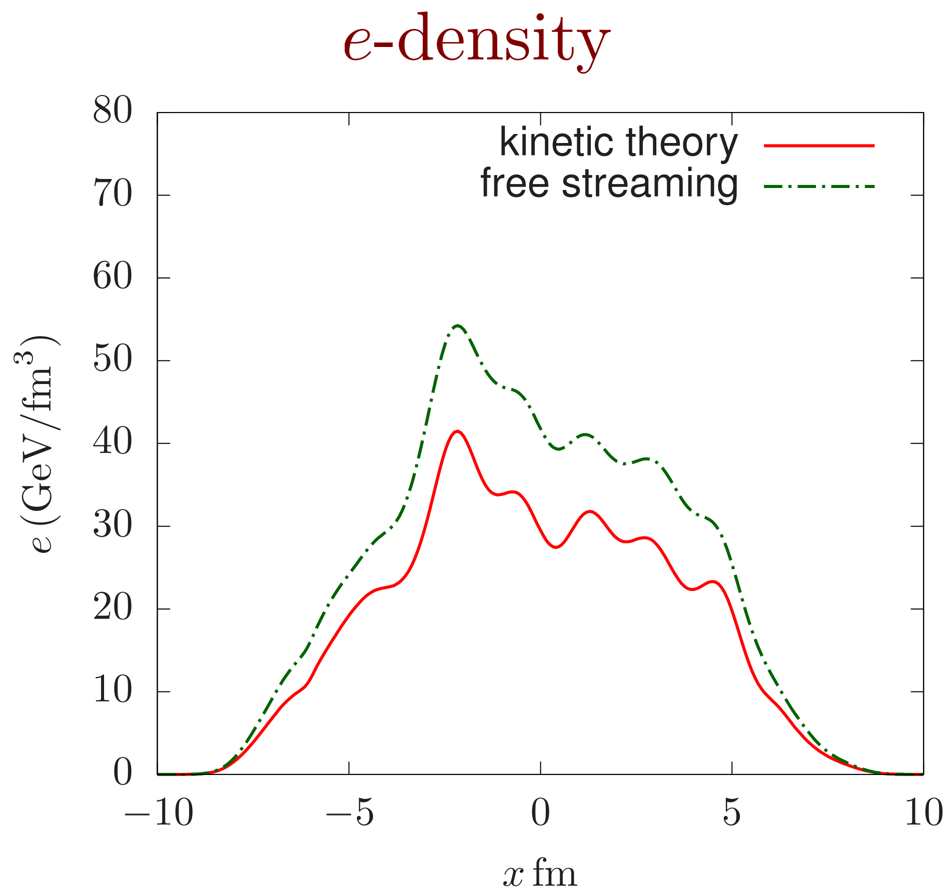
Comparison with free streaming to kinetics at $\tau_{\text{hydro}} = 1.2$:

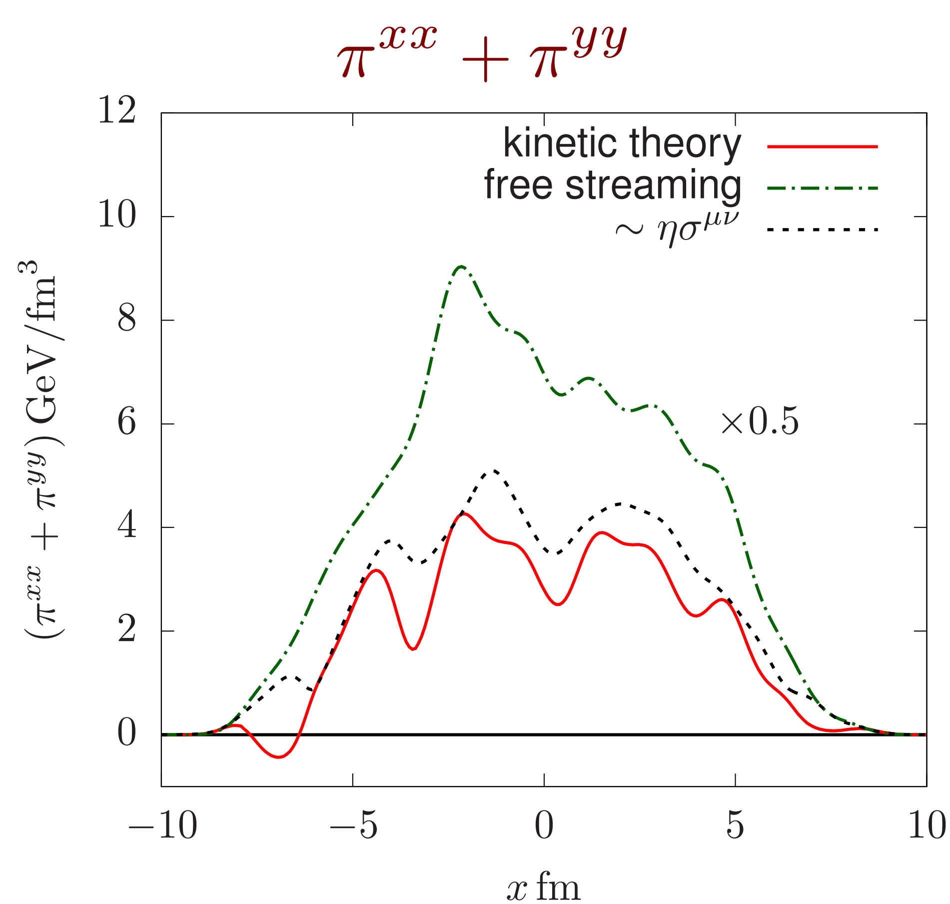




In free streaming + hydro we readjust the initial energy density to reproduce dN/dy, leading to an ambiguity in the early-time energy density

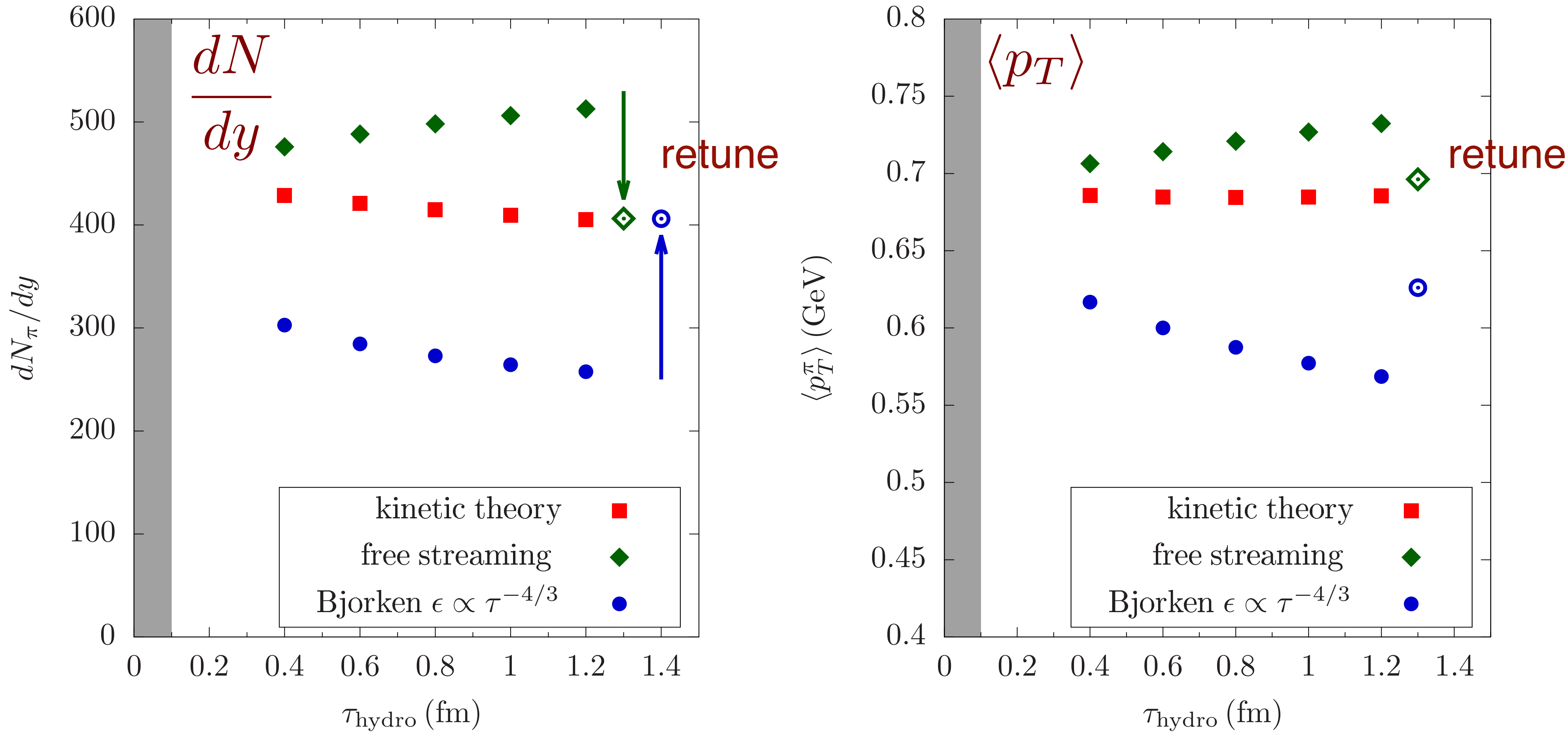
Comparison with free streaming to kinetics at $\tau_{\rm hydro} = 1.2$:





In free streaming + hydro we readjust the initial energy density to reproduce dN/dy, and $\pi^{xx} + \pi^{yy}$ evolves discontinuously.

Hadronic observables are forgiving:



Kinetic theory results are independent of τ_{hydro},

while the free streaming results are (mostly) independent after retune

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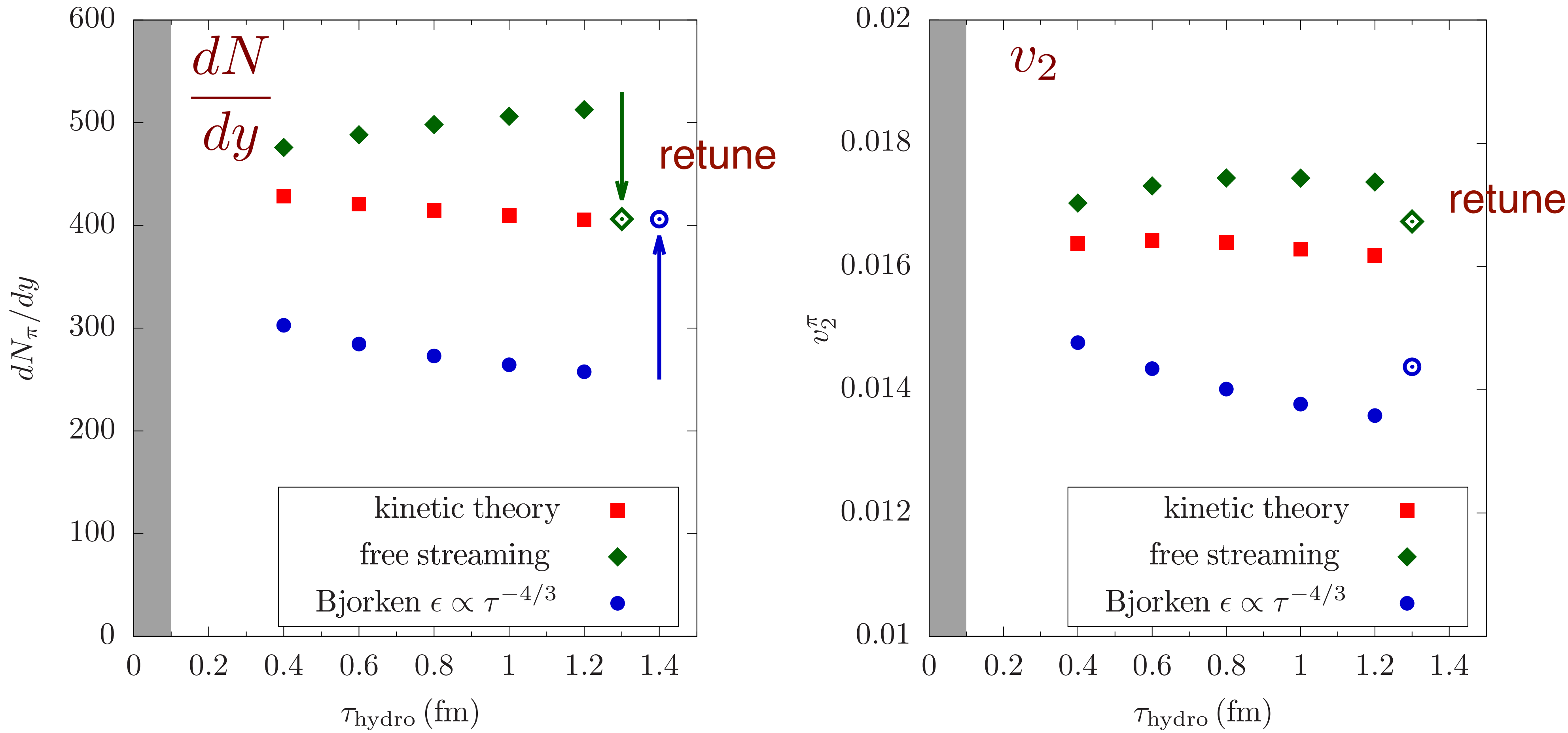

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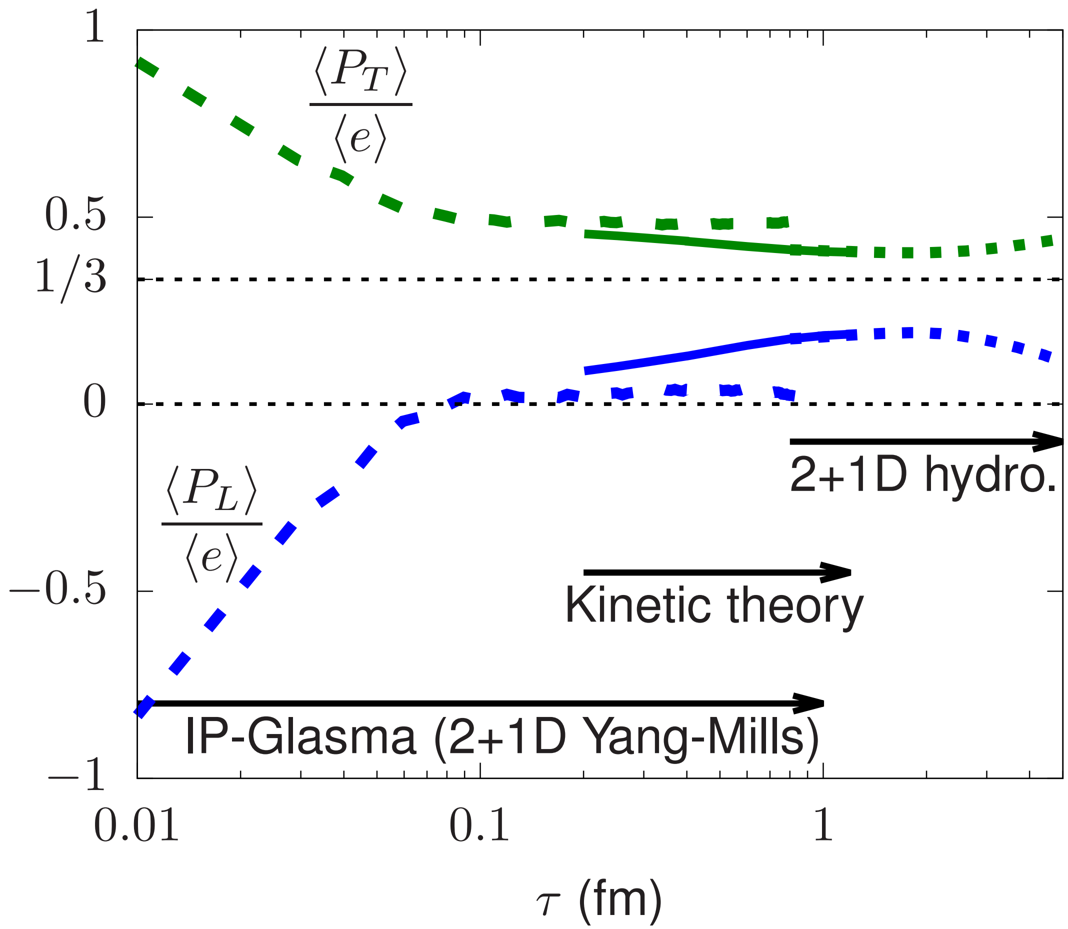
Kinetics give a smooth transition from CGC to Hydro



A leading order smooth matching of effective theories and the whole collision

Summary

1. Still under the spell of "bottom-up" after all these years.

   ⋆ A big next step are non-linear corrections – especially for small systems!

2. The tool is easy to use and fast. Use it!

   ⋆ It gracefully connects any initial state to fully developed hydro

Other items:

1. Hadronic observables – no surprises!

2. Comparison with other approaches:

   ⋆ Free streaming:

   ⋆ The Pratt pre-flow (in super-SONIC) is a low k limit of our results.

Thank You!