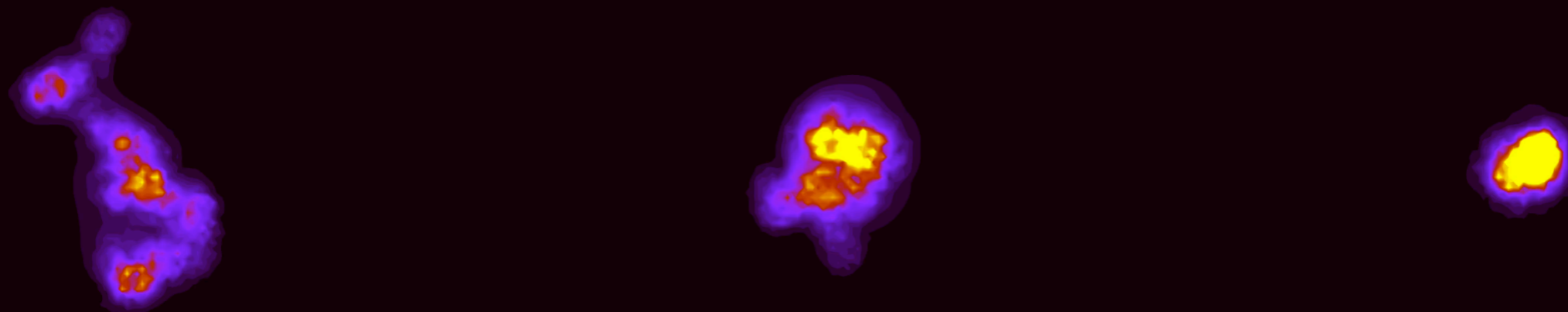


# Hydrodynamics: state of the art

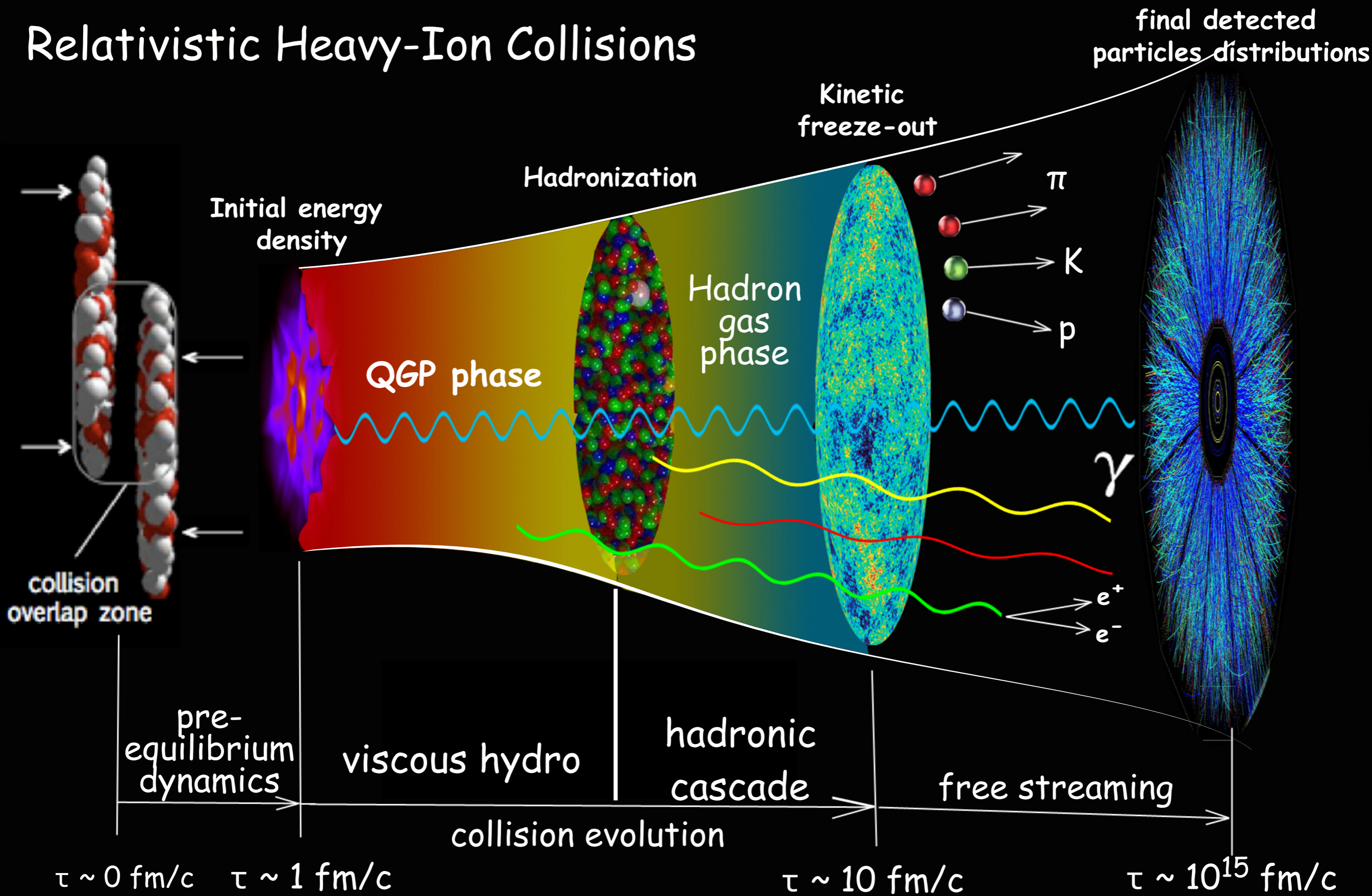
---

Chun Shen

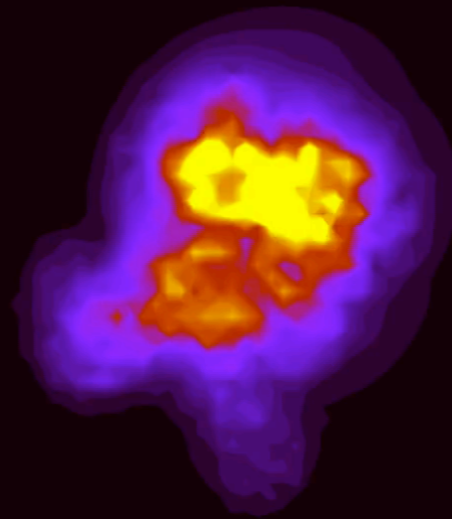
Brookhaven National Lab



# Relativistic Heavy-Ion Collisions

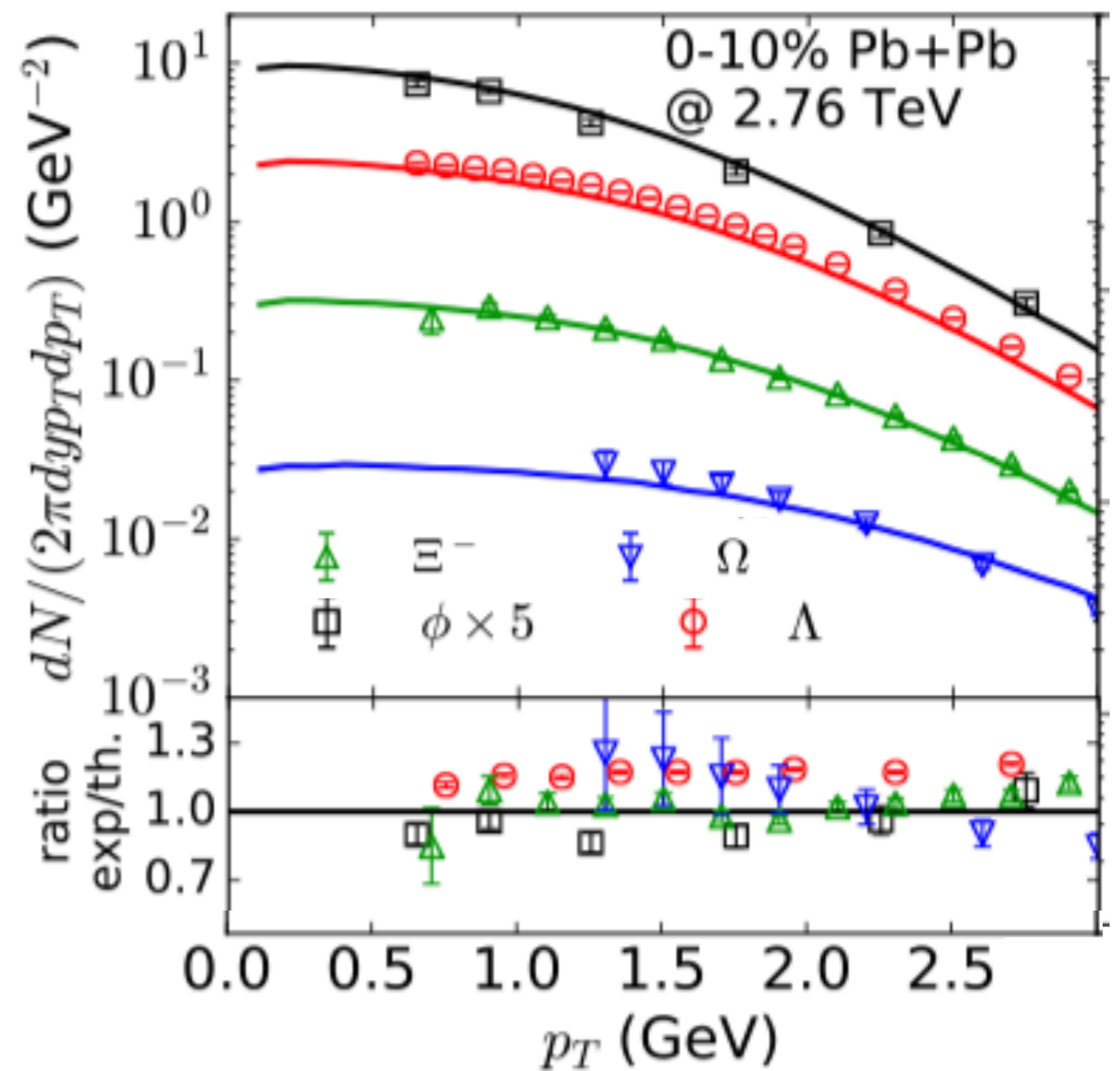
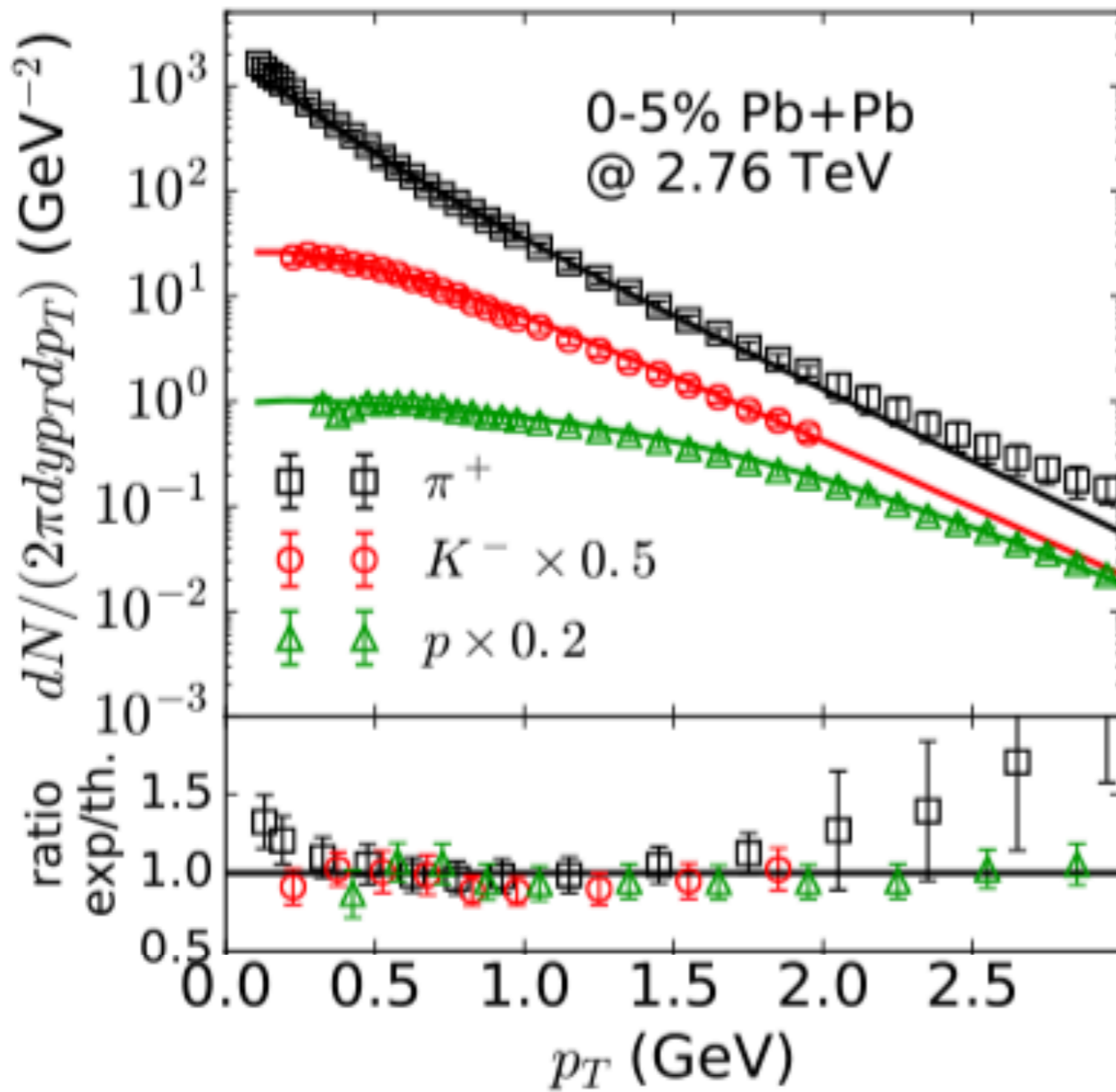


# A blowing wind from hydrodynamics



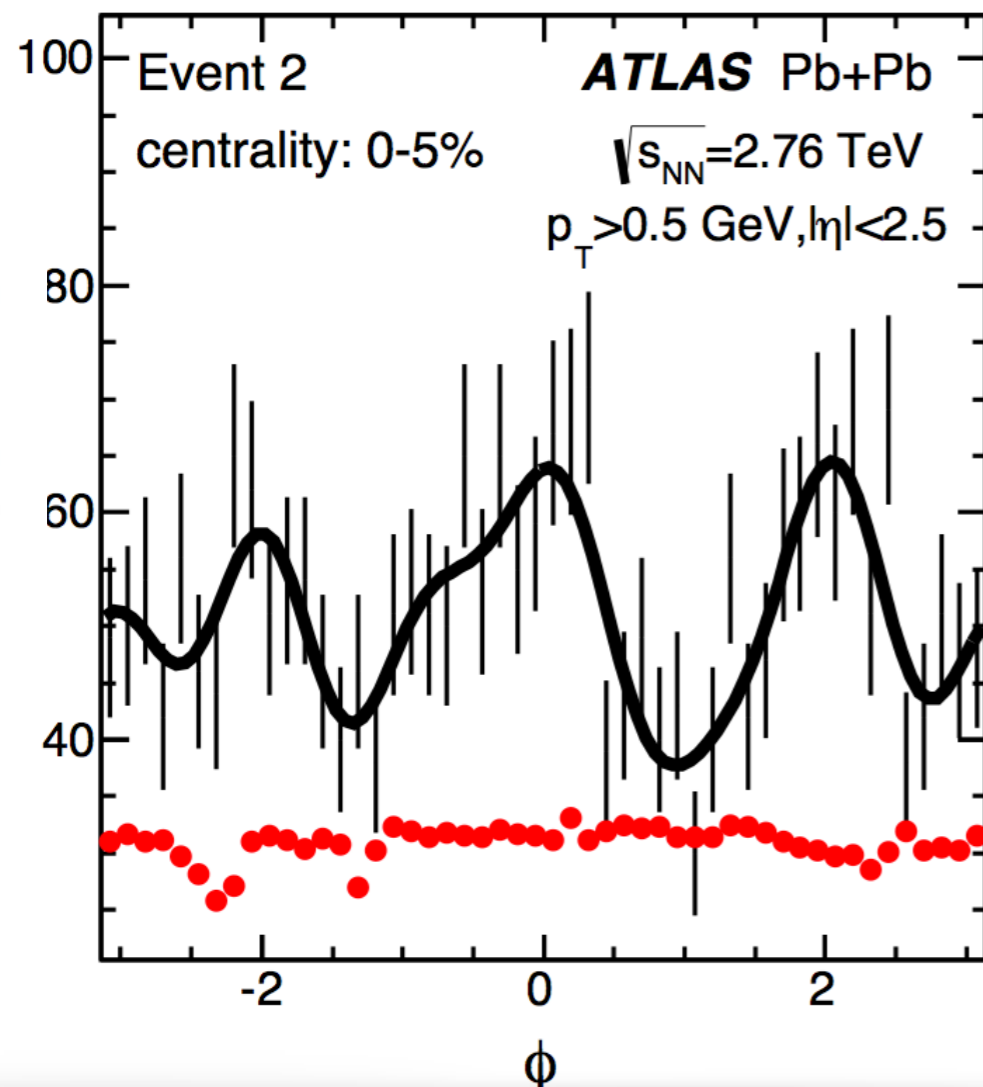
# Identified particle spectra

S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)



- Hydrodynamic simulations can describe a zoo of identified particle spectra within 30% accuracy

# Anisotropic flow



$$v_n(p_T) e^{in\Psi_n} = \frac{\int d\phi_p e^{in\phi_p} dN / (dy p_T dp_T d\phi_p)}{\int d\phi_p dN / (dy p_T dp_T d\phi_p)}$$

**F.T.**  $\rightarrow$

$$N \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi_p - \Psi_n)) \right)$$

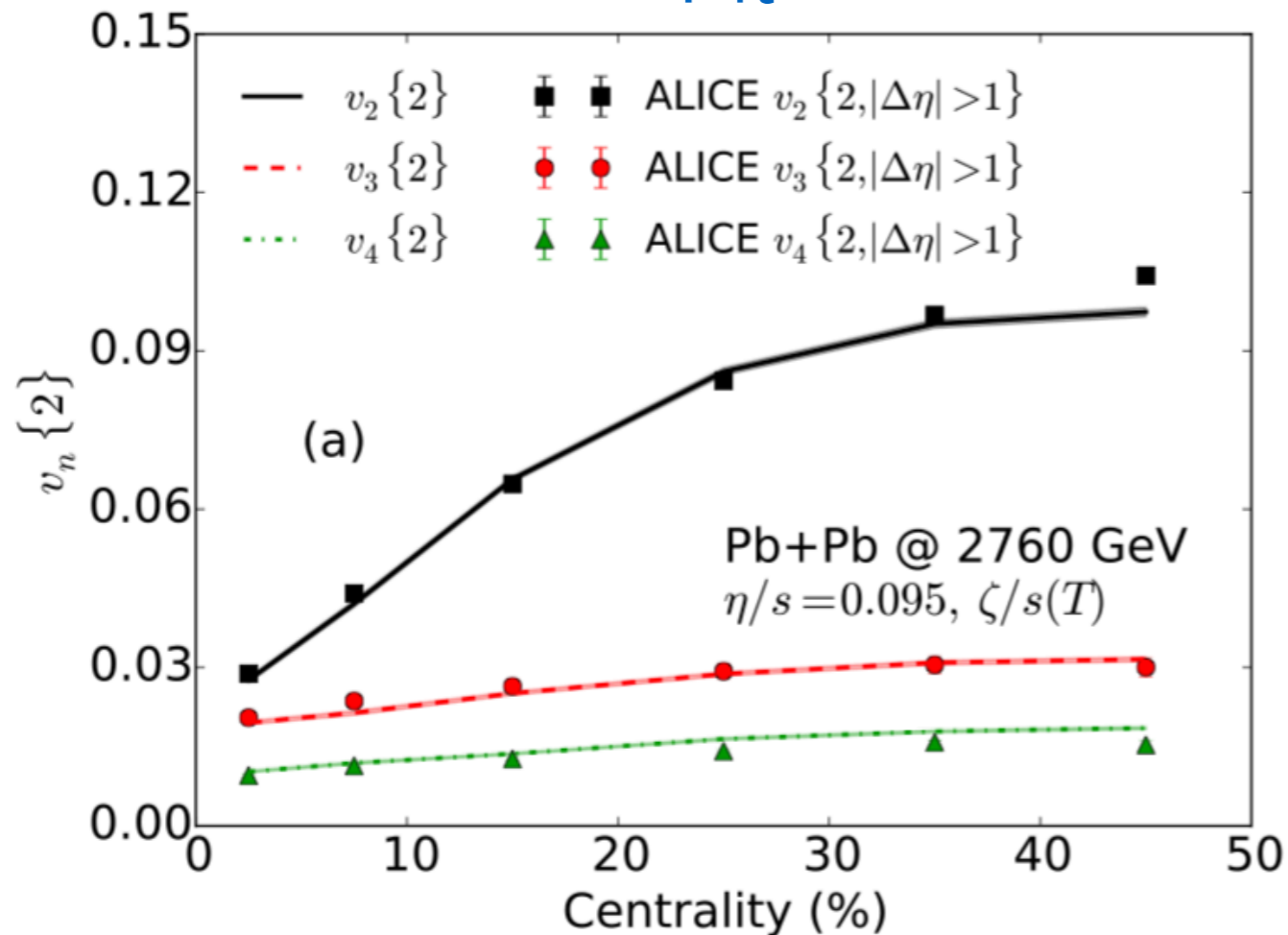
- $\{v_n, \Psi_n\}$  captures the anisotropy in particle azimuthal momentum distribution

G. Aad *et al.* [ATLAS Collaboration], JHEP **1311**, 183 (2013)

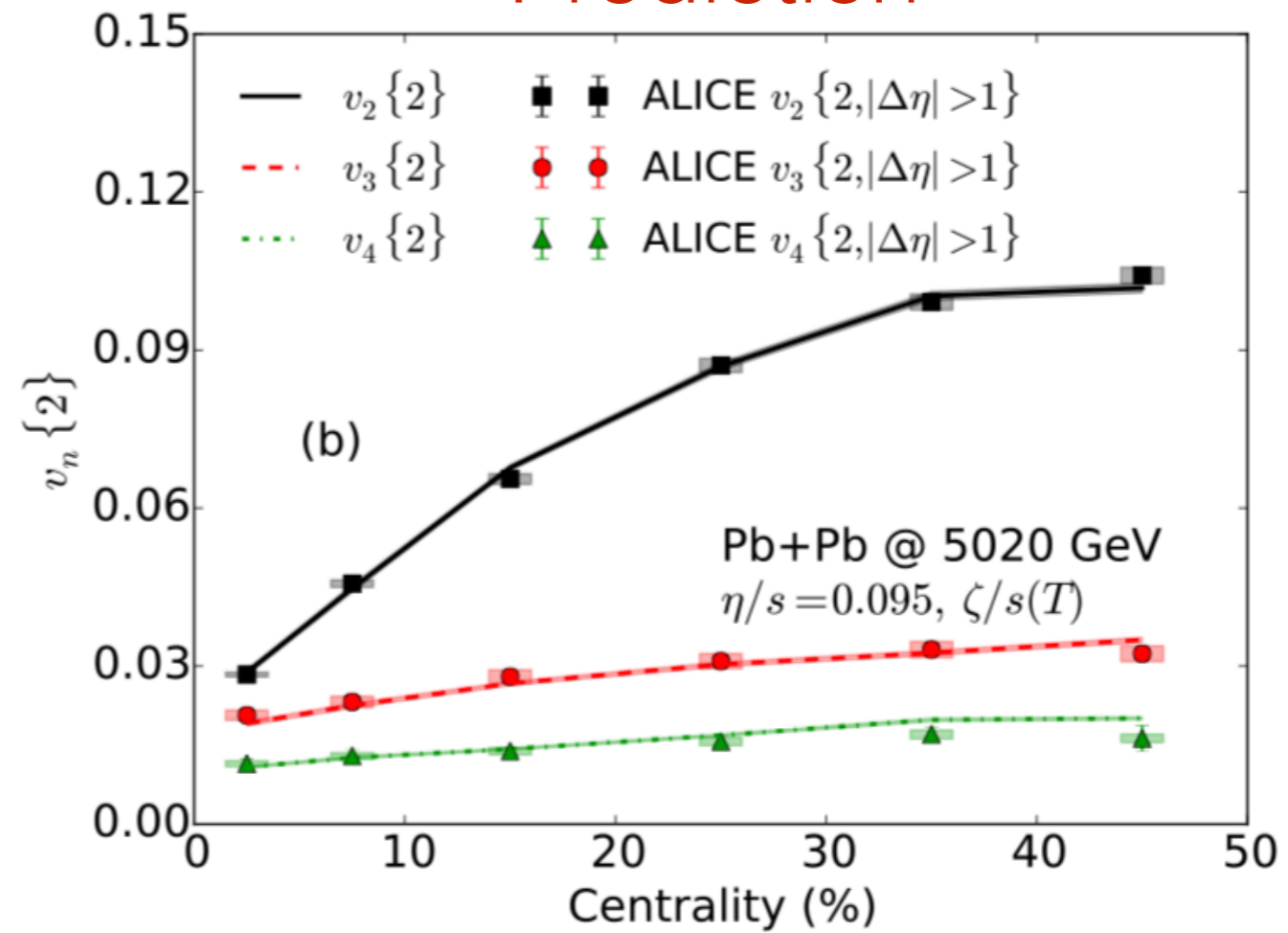
# Charged hadron $v_n$ at the LHC

S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)

Fit



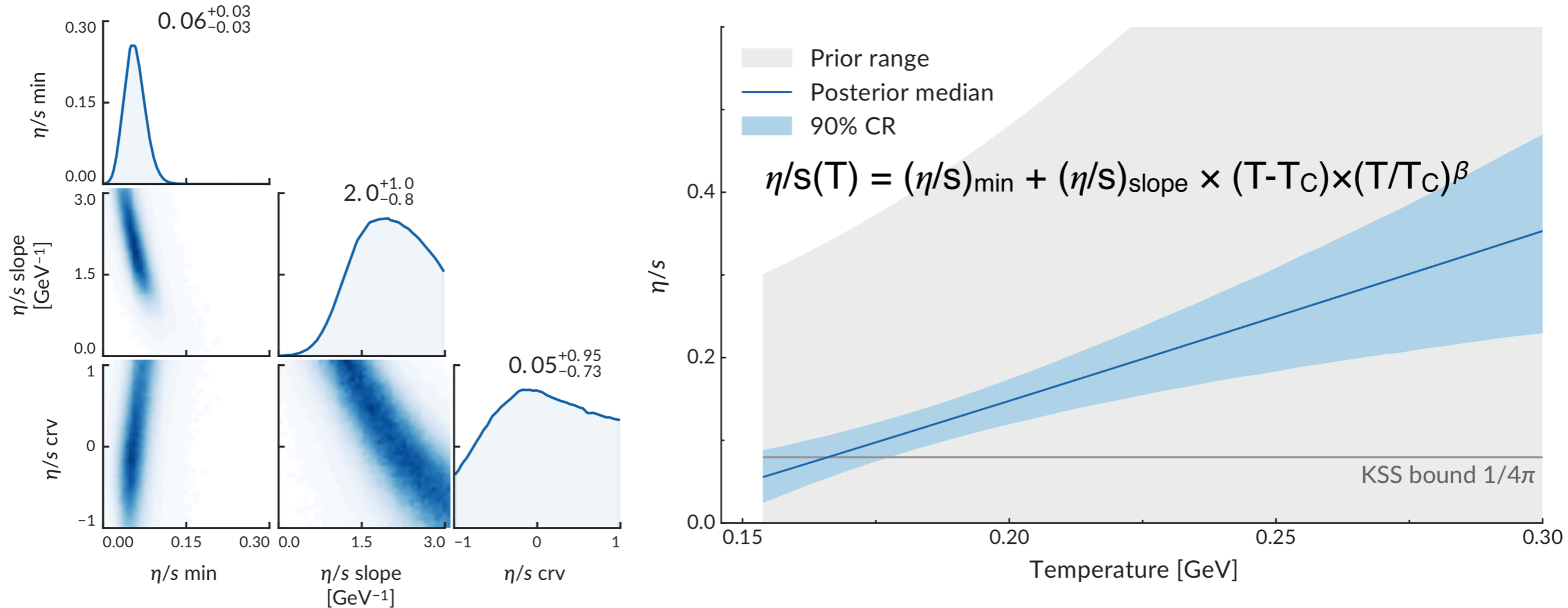
Prediction



- Hydrodynamics can fit and predict anisotropic flow  $v_n$
- The *conversion rate* of initial spatial eccentricity to final momentum anisotropy is controlled by the transport properties of the QGP

# Extraction the QGP transport property

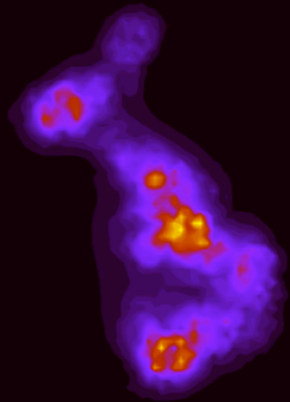
J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu and U. Heinz, Phys. Rev. C 94, 024907 (2016)



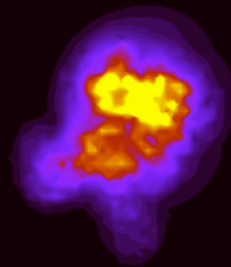
- Hydrodynamic framework is coupled with the Bayesian statistical analysis to provide the state-of-the-art extraction of the QGP shear viscosity

# Universal hydrodynamic response

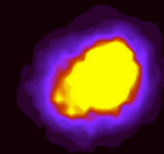
Pb+Pb



Xe+Xe



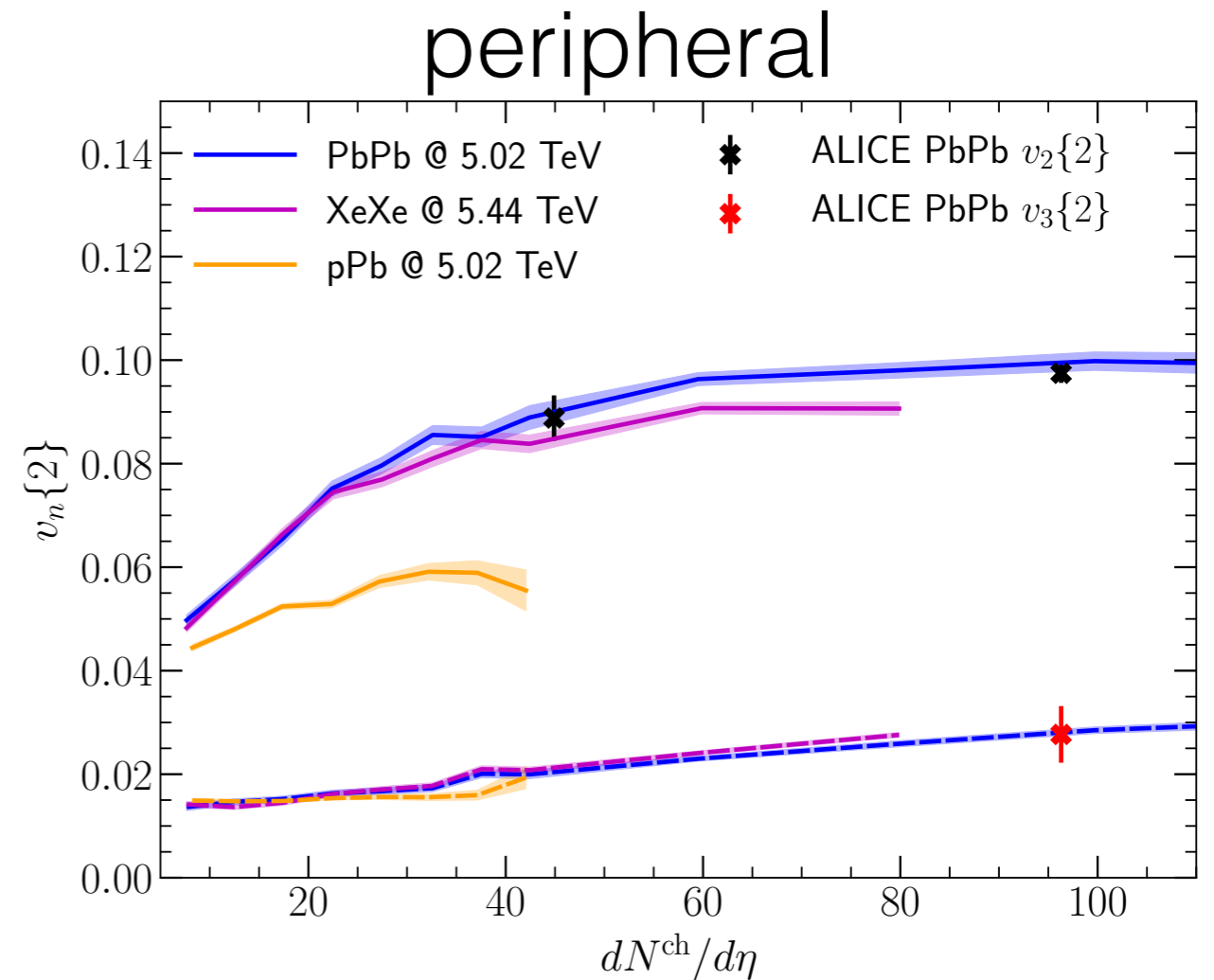
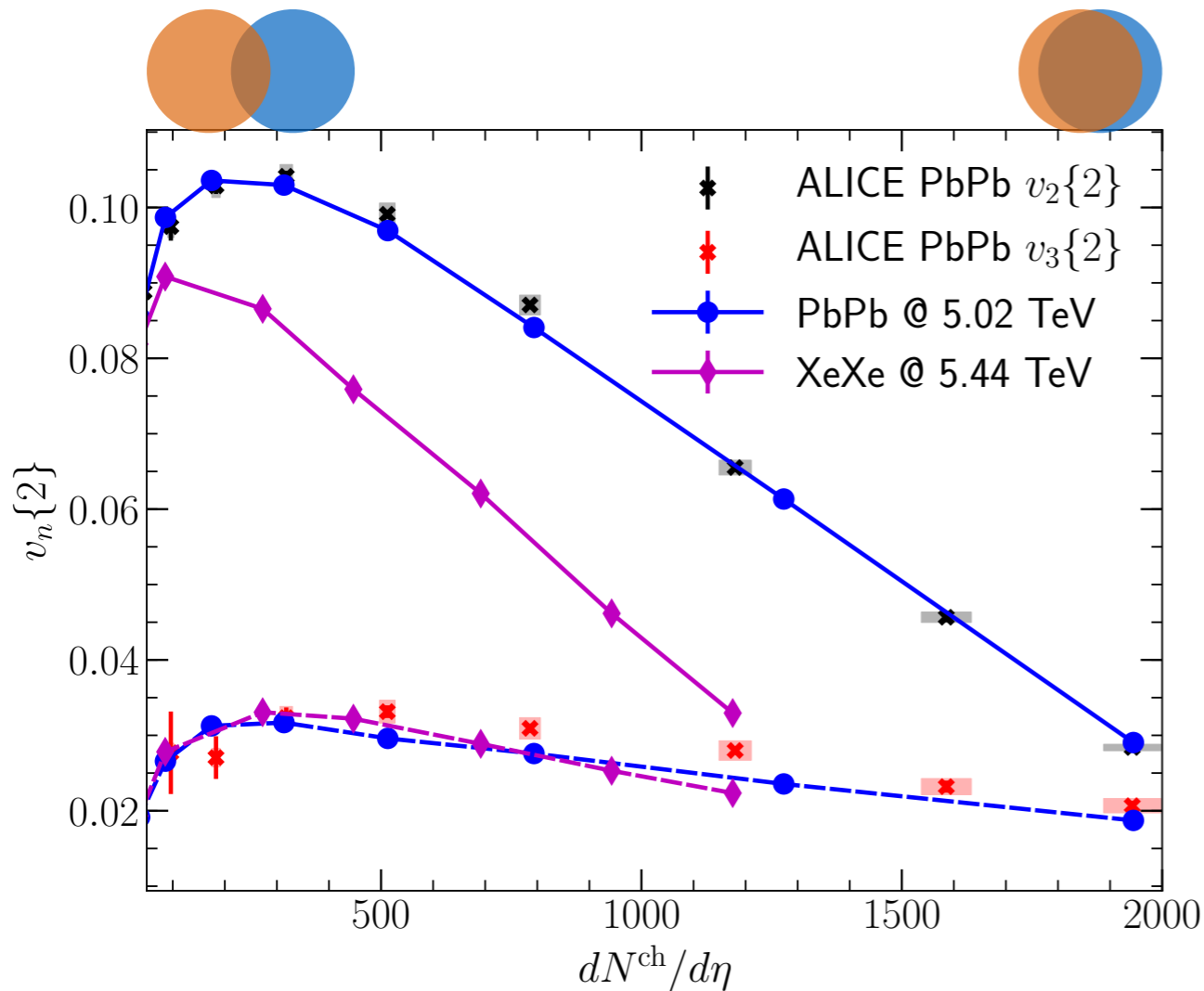
p+Pb





# System size dependence of $v_n$

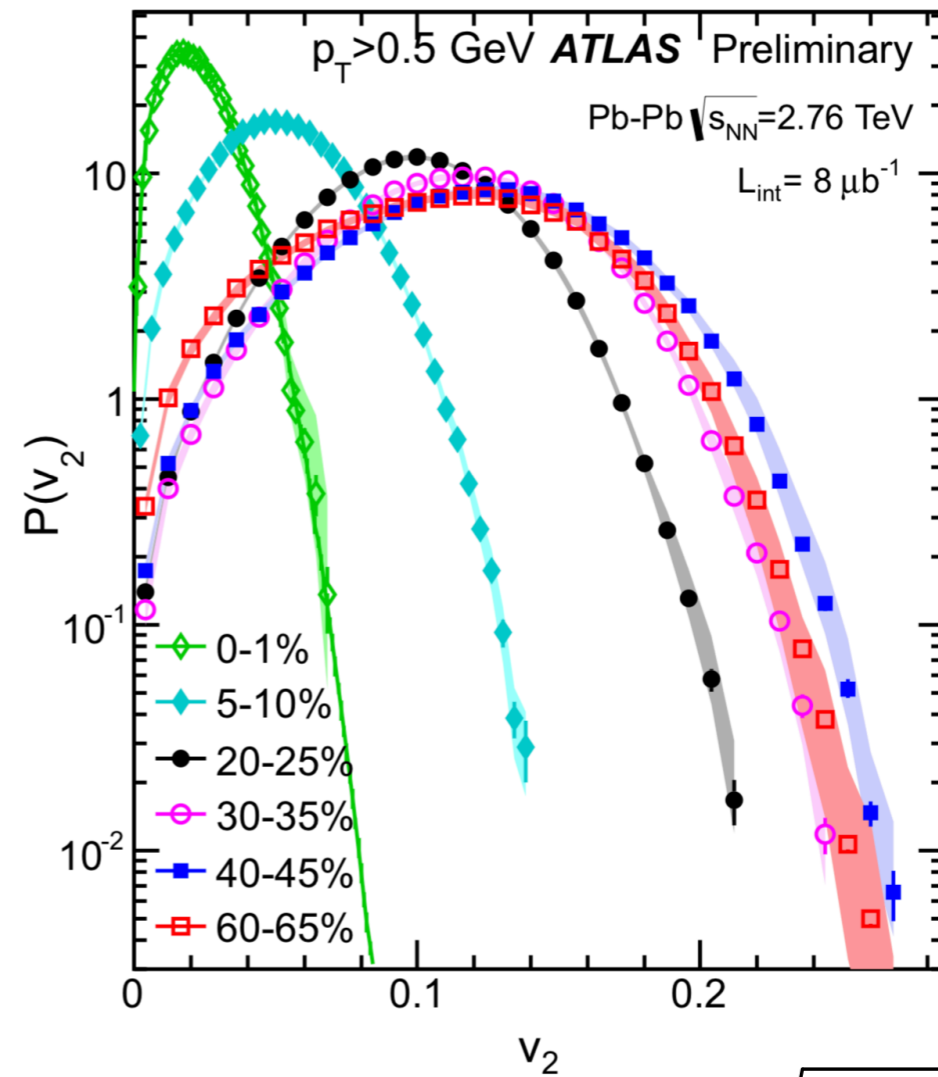
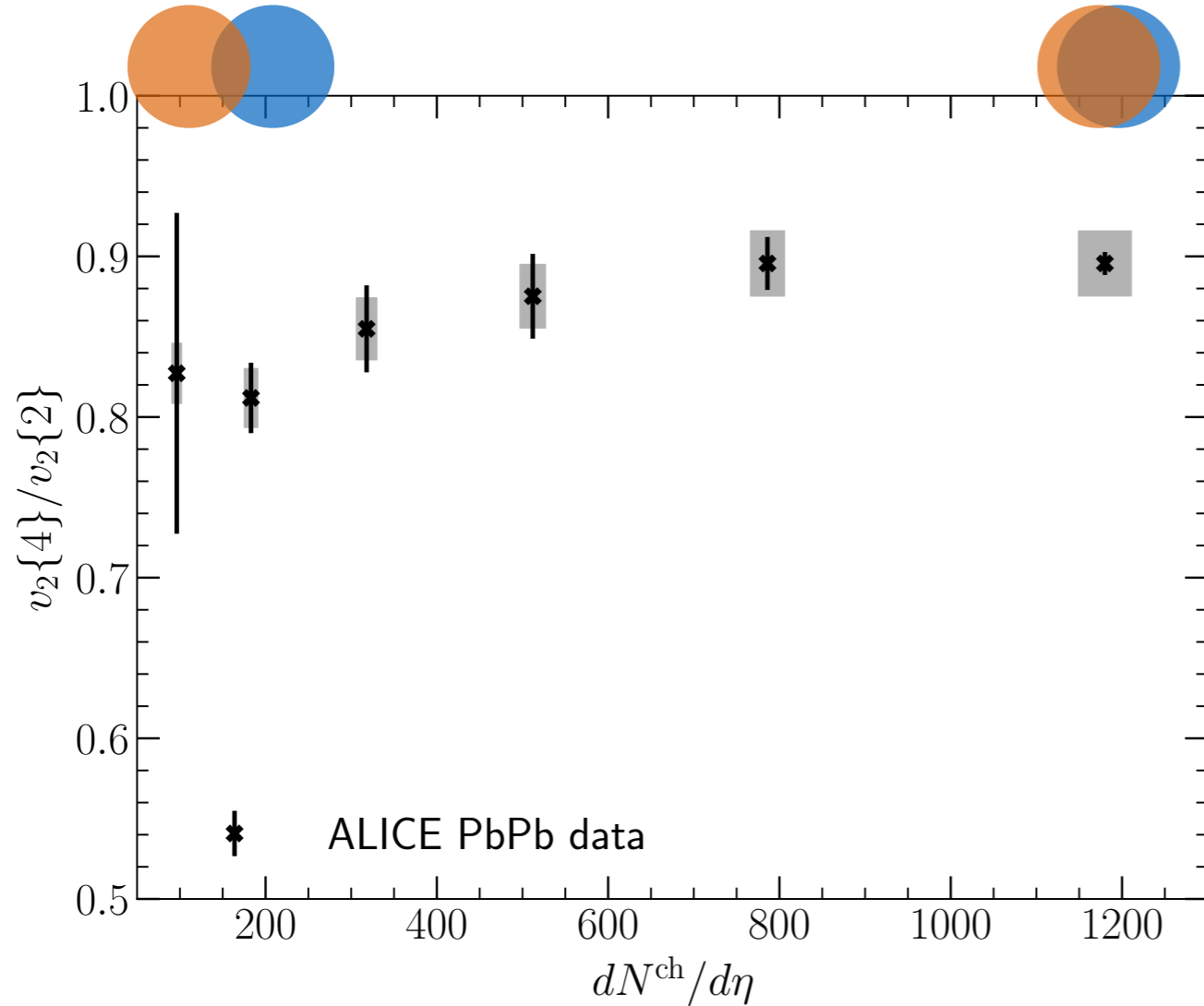
Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



- The IP-Glasma + hydrodynamic framework can reproduce charged hadron  $v_n\{2\}$  from central to peripheral collisions

# Event-by-event fluctuation of $v_n$

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation

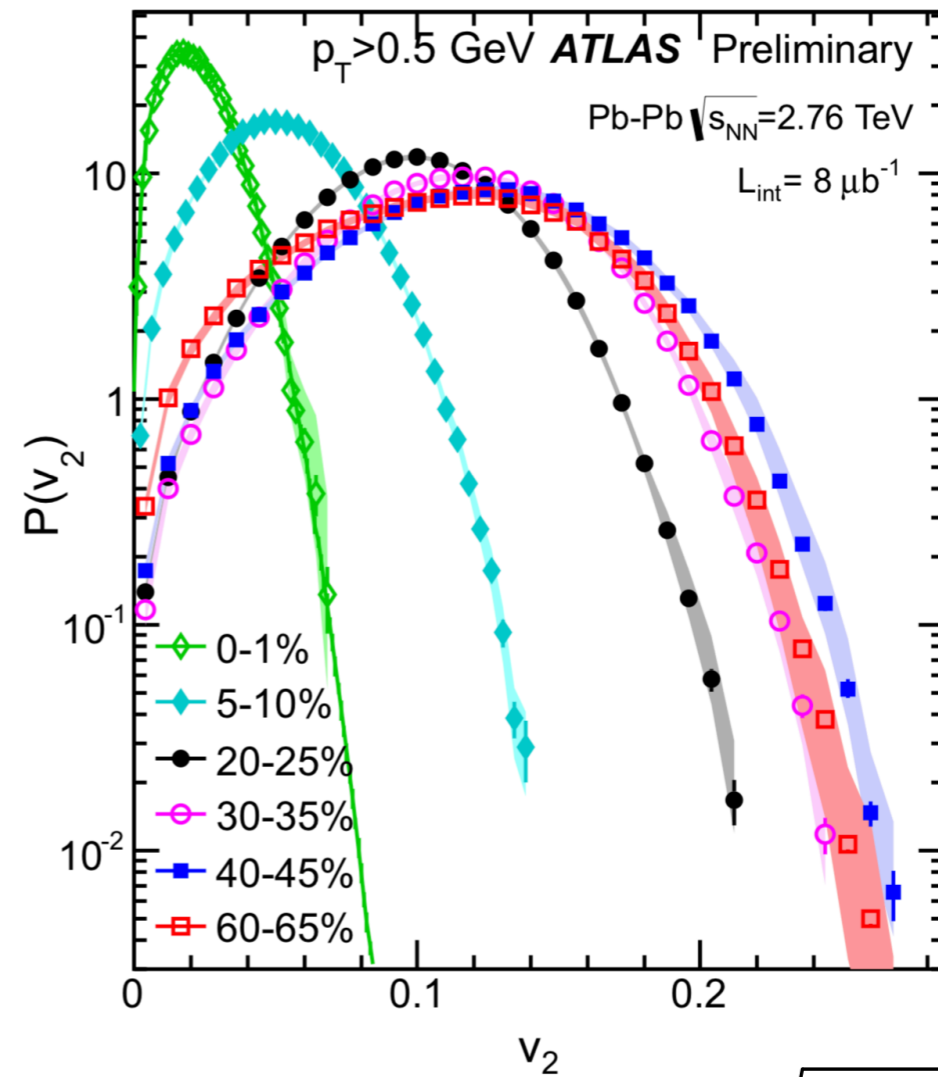
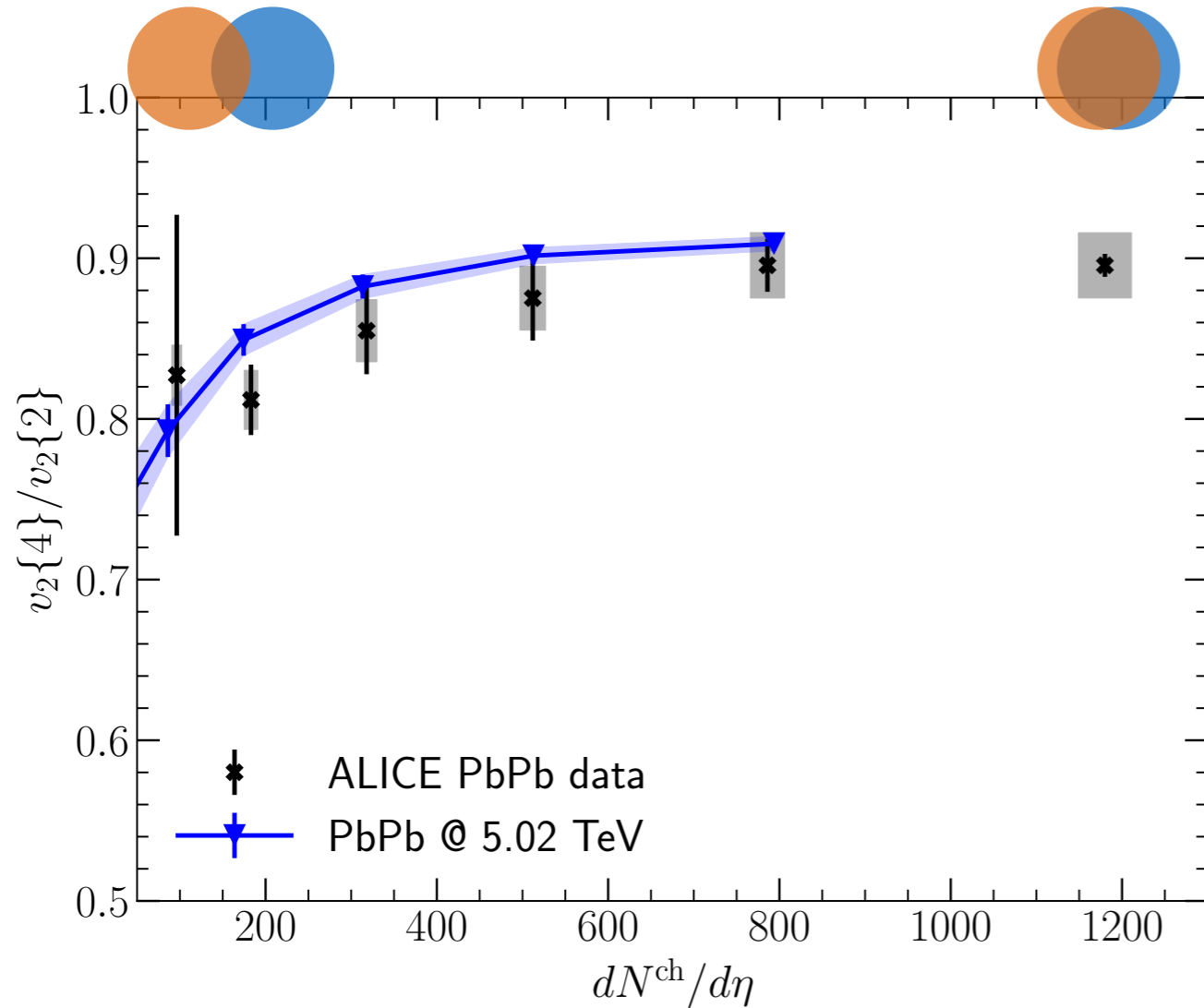


ATLAS-CONF  
-2012-114

- The ratio of  $v_2\{4\}/v_2\{2\}$  measures the variance of the  $v_2$  fluctuations  $\frac{v_2\{4\}}{v_2\{2\}} \rightarrow \sqrt{\frac{1 - (\sigma^2/\bar{v}_2^2)}{1 + (\sigma^2/\bar{v}_2^2)}}$

# Event-by-event fluctuation of $v_n$

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation

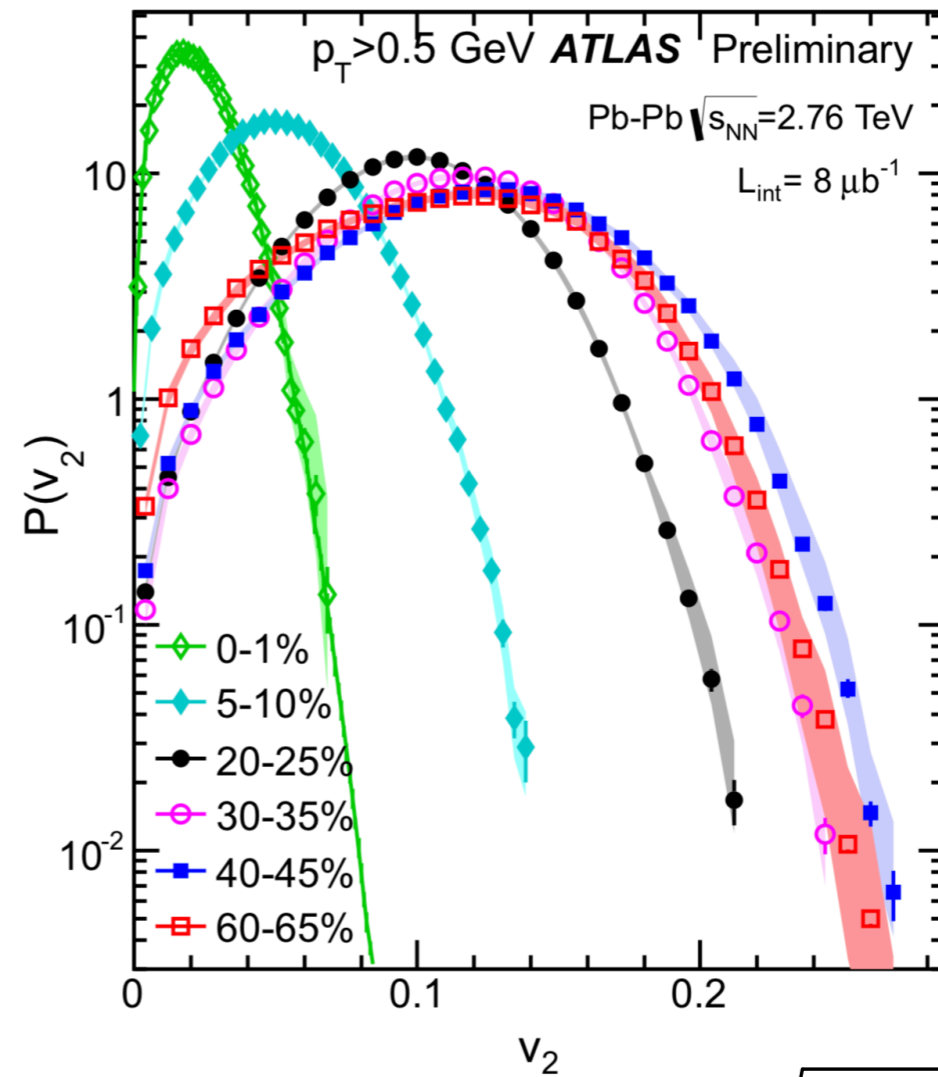
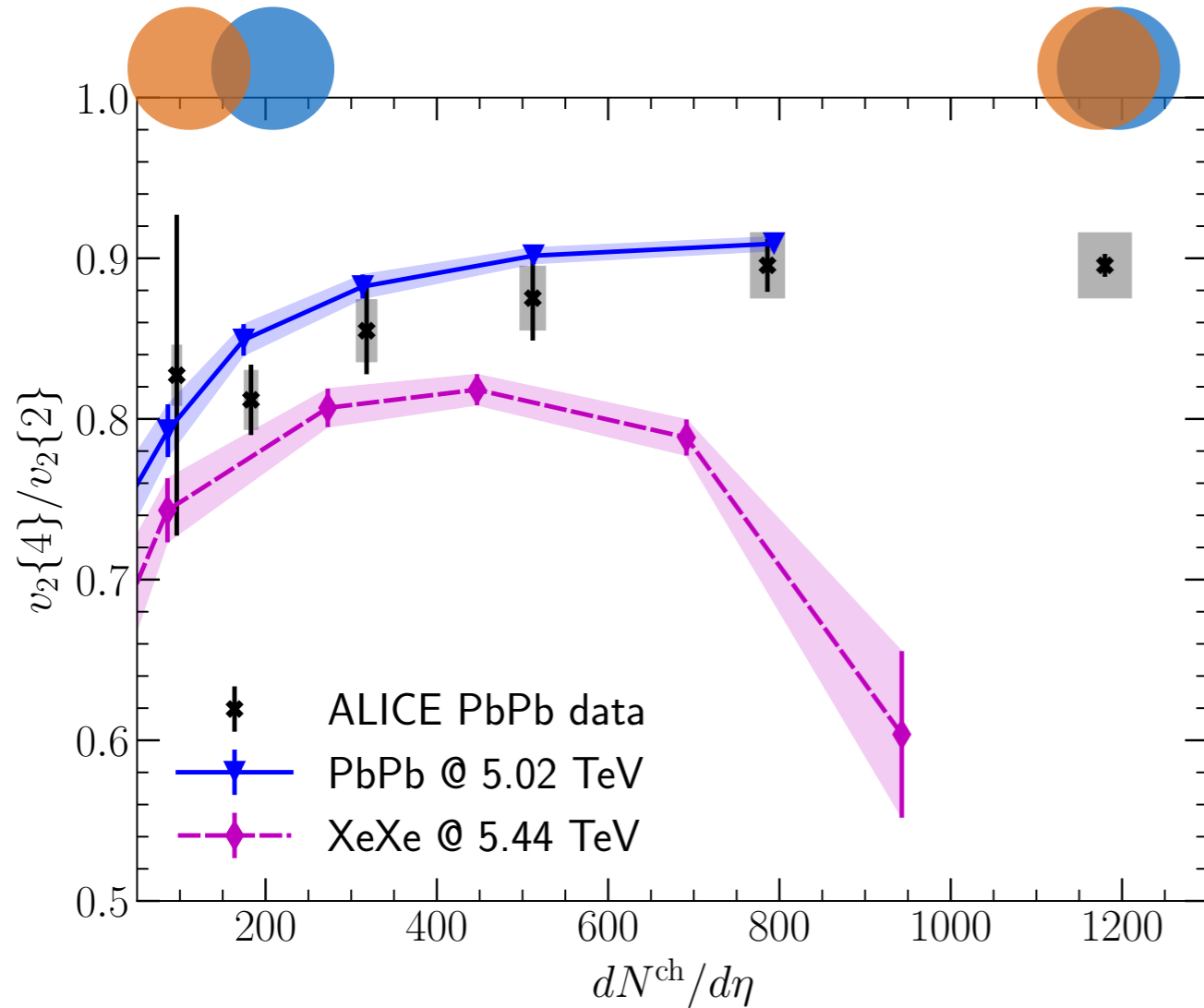


ATLAS-CONF  
-2012-114

- The ratio of  $v_2\{4\}/v_2\{2\}$  measures the variance of the  $v_2$  fluctuations  $\frac{v_2\{4\}}{v_2\{2\}} \rightarrow \sqrt{\frac{1 - (\sigma^2/\bar{v}_2^2)}{1 + (\sigma^2/\bar{v}_2^2)}}$
- The IP-Glasma initial condition captures the  $v_2$  fluctuations from central to peripheral centralities

# Event-by-event fluctuation of $v_n$

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



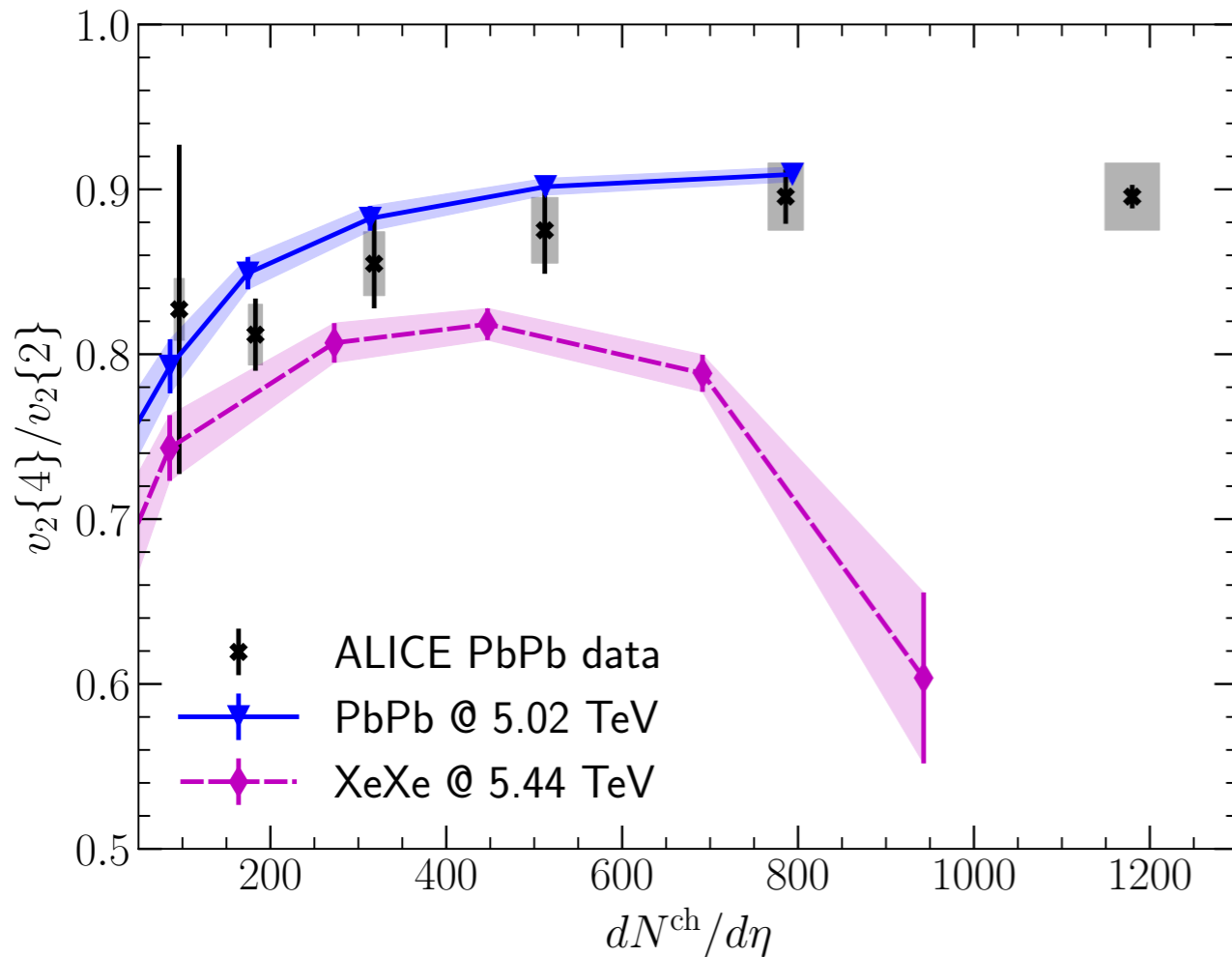
ATLAS-CONF  
-2012-114

- The ratio of  $v_2\{4\}/v_2\{2\}$  measures the variance of the  $v_2$  fluctuations  $\frac{v_2\{4\}}{v_2\{2\}} \rightarrow \sqrt{\frac{1 - (\sigma^2/\bar{v}_2^2)}{1 + (\sigma^2/\bar{v}_2^2)}}$
- The IP-Glasma initial condition captures the  $v_2$  fluctuations from central to peripheral centralities

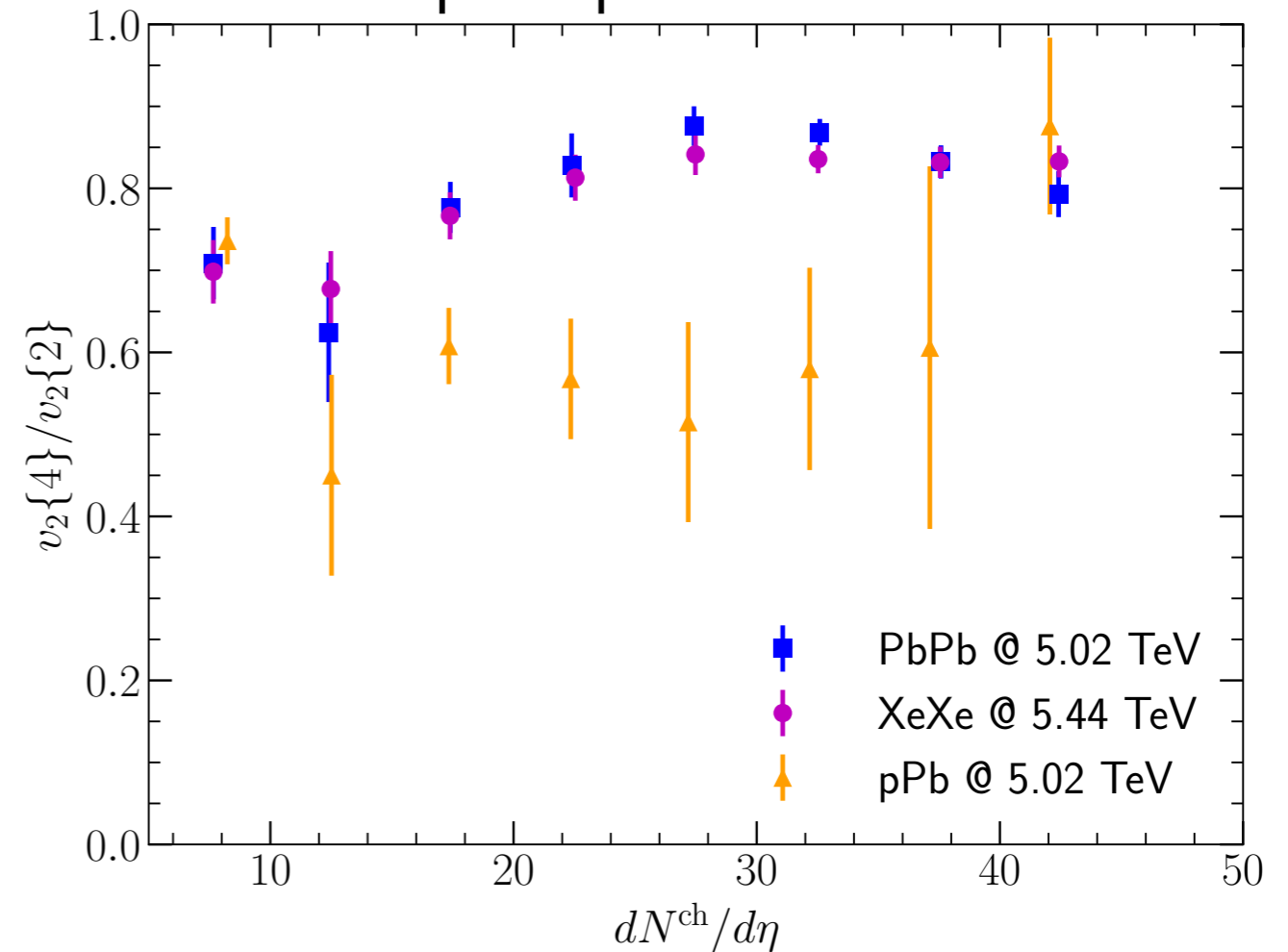
# Event-by-event fluctuation of $v_n$

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation

central



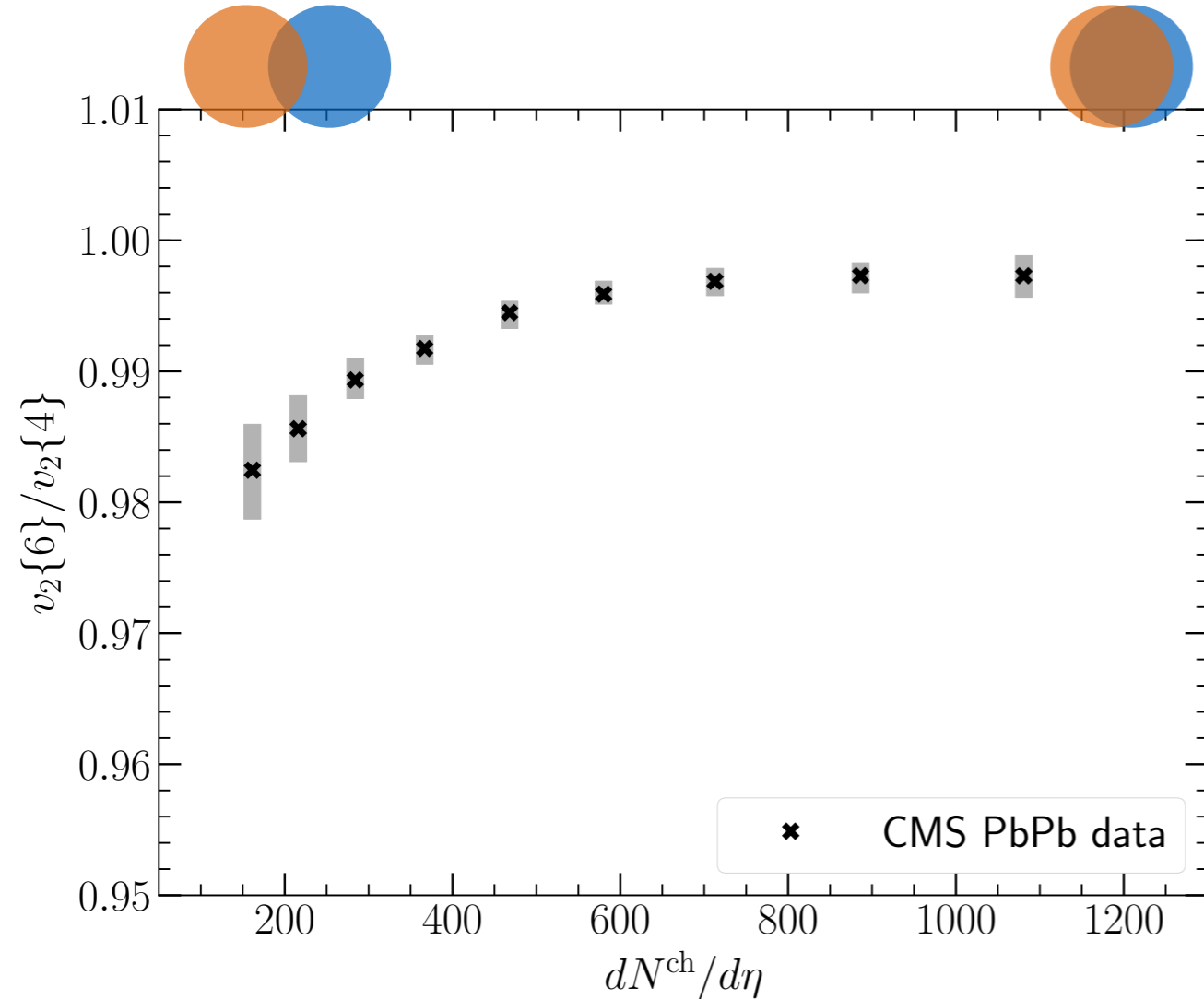
peripheral



- The ratio of  $v_2\{4\}/v_2\{2\}$  measures the variance of the  $v_2$  fluctuations  $\frac{v_2\{4\}}{v_2\{2\}} \rightarrow \sqrt{\frac{1 - (\sigma^2/\bar{v}_2^2)}{1 + (\sigma^2/\bar{v}_2^2)}}$
- A larger  $v_2$  fluctuation is in pPb collisions compared to the larger XeXe and PbPb collisions at a same  $dN^{\text{ch}}/d\eta$

# Non-Gaussianity of $v_n$ fluctuations

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



The Bessel-Gaussian fluctuation gives  $v_2\{4\} = v_2\{6\}$

G. Giacalone, L. Yan, J. Noronha-Hostler and J. Y. Ollitrault, Phys. Rev. C 95, 014913 (2017)

More generally,

$$v_2\{4\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_1 + s_2}{(\bar{v}_2)^2},$$

$$v_2\{6\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_1 + s_2}{(\bar{v}_2)^2},$$

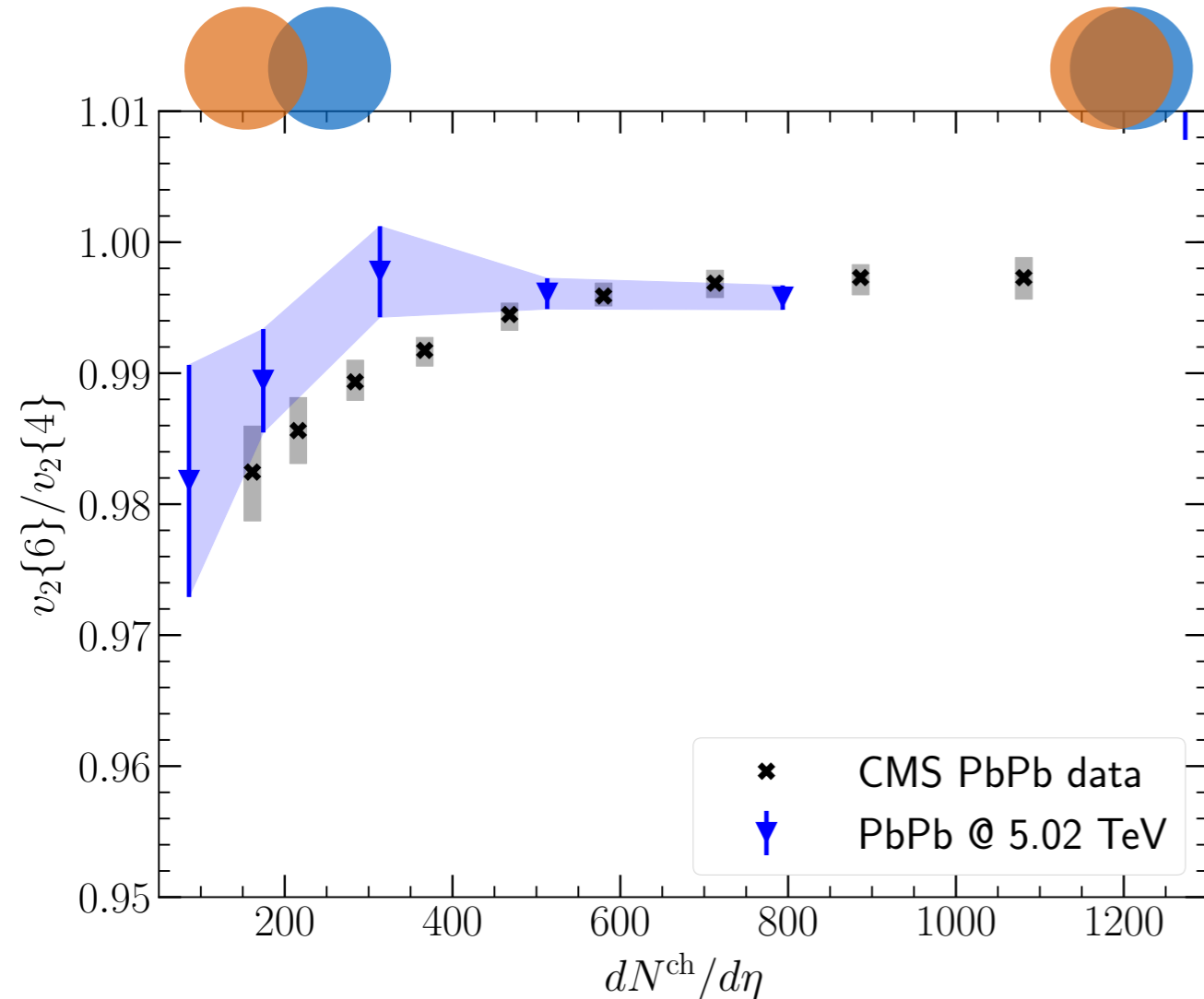
$$s_1 \equiv \langle (v_x - \bar{v}_2)^3 \rangle,$$

$$s_2 \equiv \langle (v_x - \bar{v}_2)v_y^2 \rangle.$$

- The difference between  $v_2\{4\}$  and  $v_2\{6\}$  raises from the skewness of the  $v_2$  distribution

# Non-Gaussianity of $v_n$ fluctuations

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



The Bessel-Gaussian fluctuation gives  $v_2\{4\} = v_2\{6\}$

G. Giacalone, L. Yan, J. Noronha-Hostler and J. Y. Ollitrault, Phys. Rev. C 95, 014913 (2017)

More generally,

$$v_2\{4\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_1 + s_2}{(\bar{v}_2)^2},$$

$$v_2\{6\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_1 + s_2}{(\bar{v}_2)^2},$$

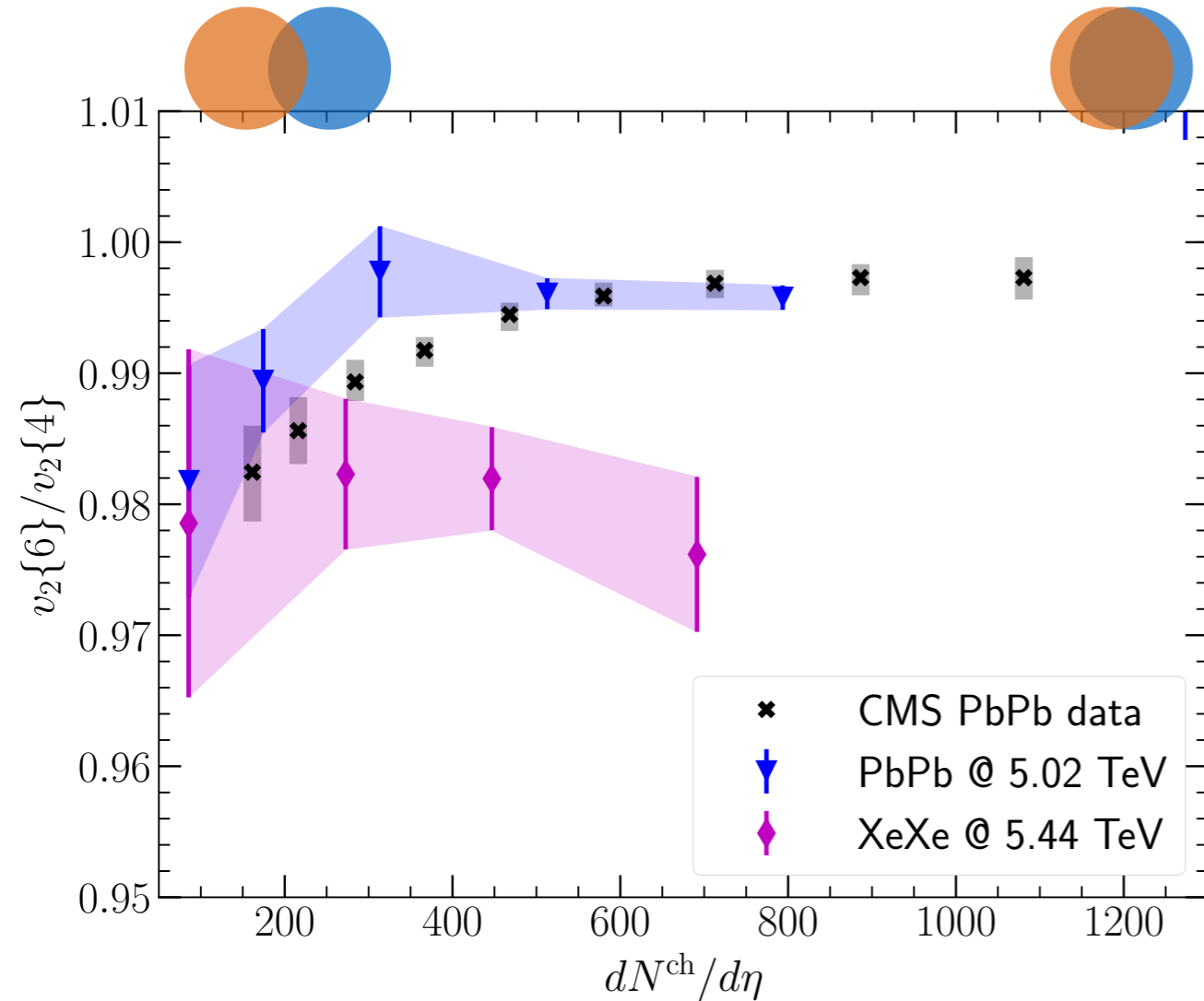
$$s_1 \equiv \langle (v_x - \bar{v}_2)^3 \rangle,$$

$$s_2 \equiv \langle (v_x - \bar{v}_2)v_y^2 \rangle.$$

- The difference between  $v_2\{4\}$  and  $v_2\{6\}$  raises from the skewness of the  $v_2$  distribution
- The IP-Glasma + hydrodynamic framework reproduces the  $v_2\{6\}/v_2\{4\}$  ratio for PbPb collisions

# Non-Gaussianity of $v_n$ fluctuations

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



The Bessel-Gaussian fluctuation gives  $v_2\{4\} = v_2\{6\}$

G. Giacalone, L. Yan, J. Noronha-Hostler and J. Y. Ollitrault, Phys. Rev. C 95, 014913 (2017)

More generally,

$$v_2\{4\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_1 + s_2}{(\bar{v}_2)^2},$$

$$v_2\{6\} \simeq \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_1 + s_2}{(\bar{v}_2)^2},$$

$$s_1 \equiv \langle (v_x - \bar{v}_2)^3 \rangle,$$

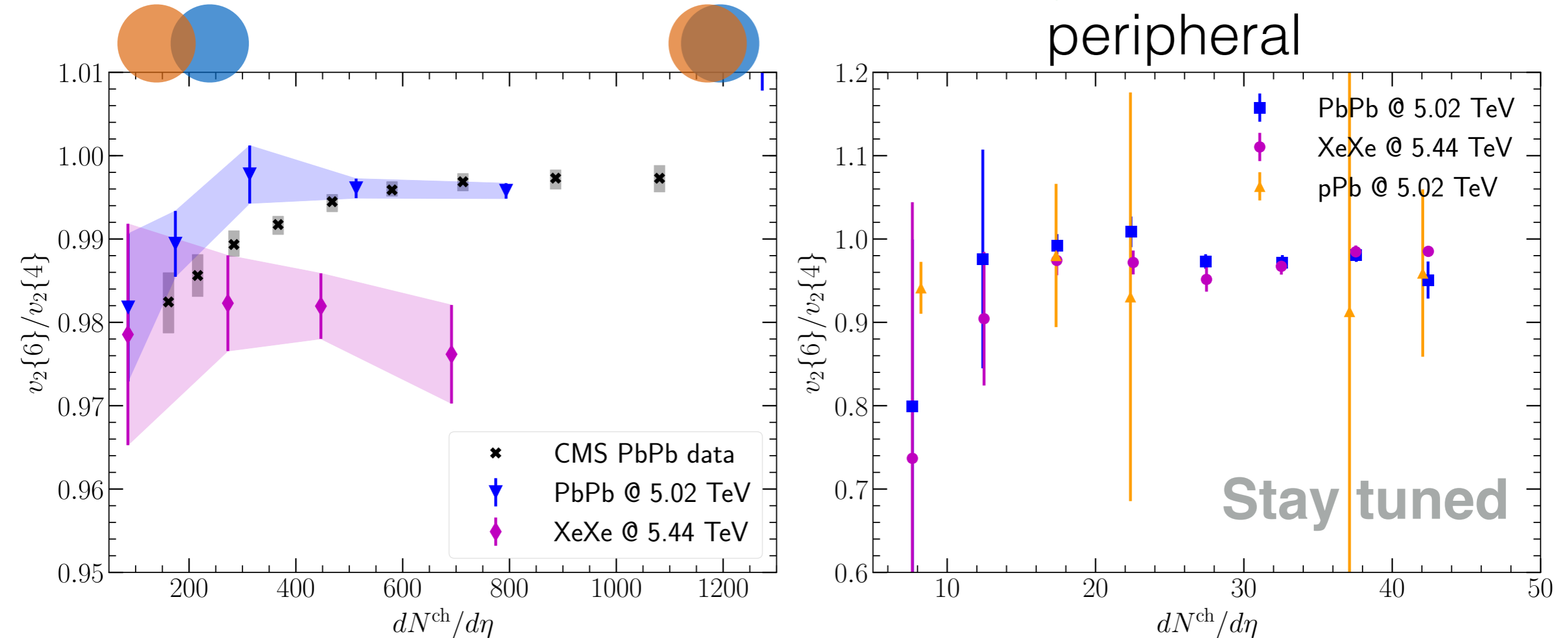
$$s_2 \equiv \langle (v_x - \bar{v}_2)v_y^2 \rangle.$$

- The difference between  $v_2\{4\}$  and  $v_2\{6\}$  raises from the skewness of the  $v_2$  distribution
- The IP-Glasma + hydrodynamic framework reproduces the  $v_2\{6\}/v_2\{4\}$  ratio for PbPb collisions



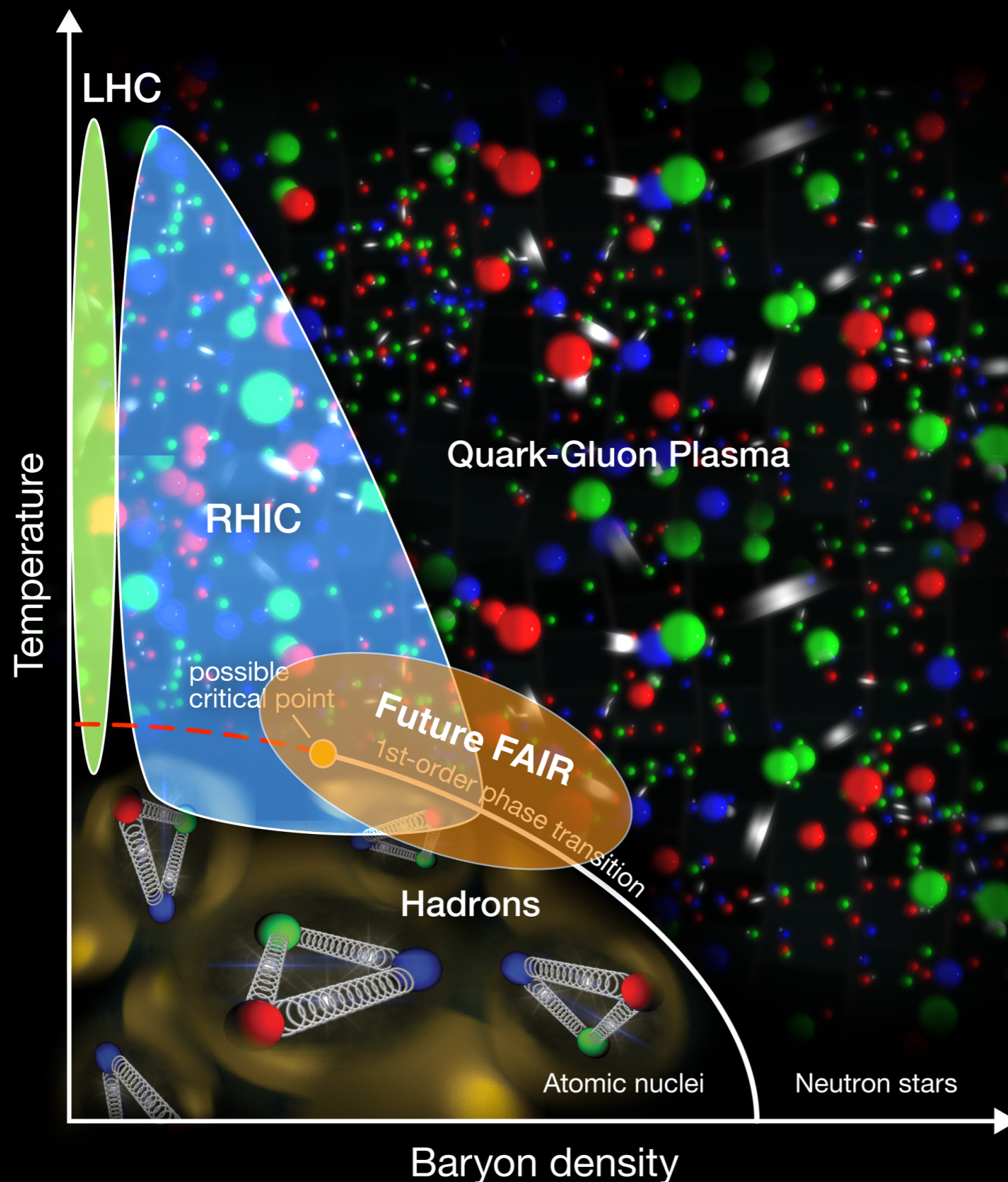
# Non-Gaussianity of $v_n$ fluctuations

Bjoern Schenke, Chun Shen, and Prithwish Tribedy, in preparation



- The difference between  $v_2\{4\}$  and  $v_2\{6\}$  raises from the skewness of the  $v_2$  distribution
- More statistics is needed for comparisons among p+Pb, Xe+Xe, and Pb+Pb collisions at small  $dN^{\text{ch}}/d\eta$

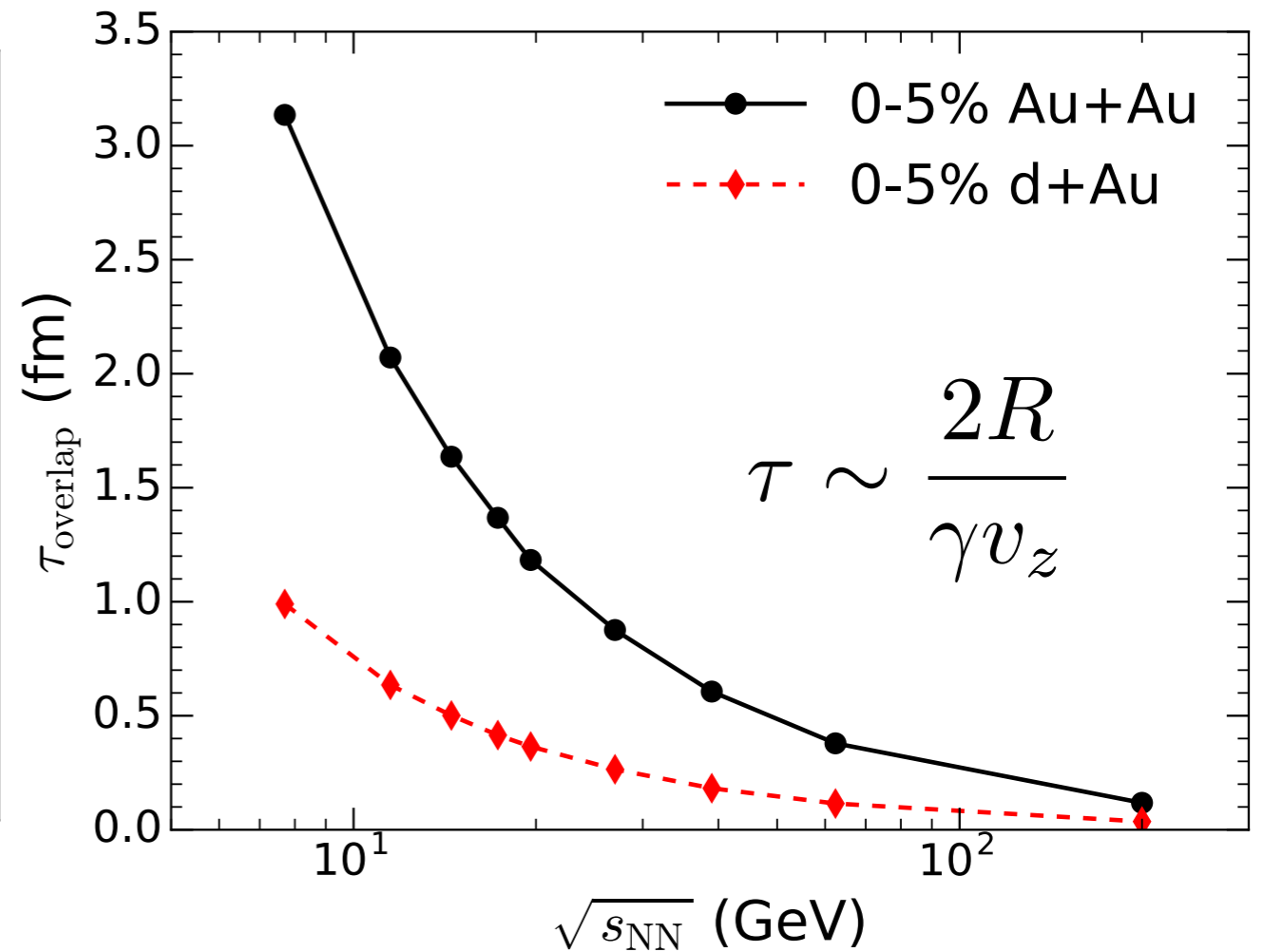
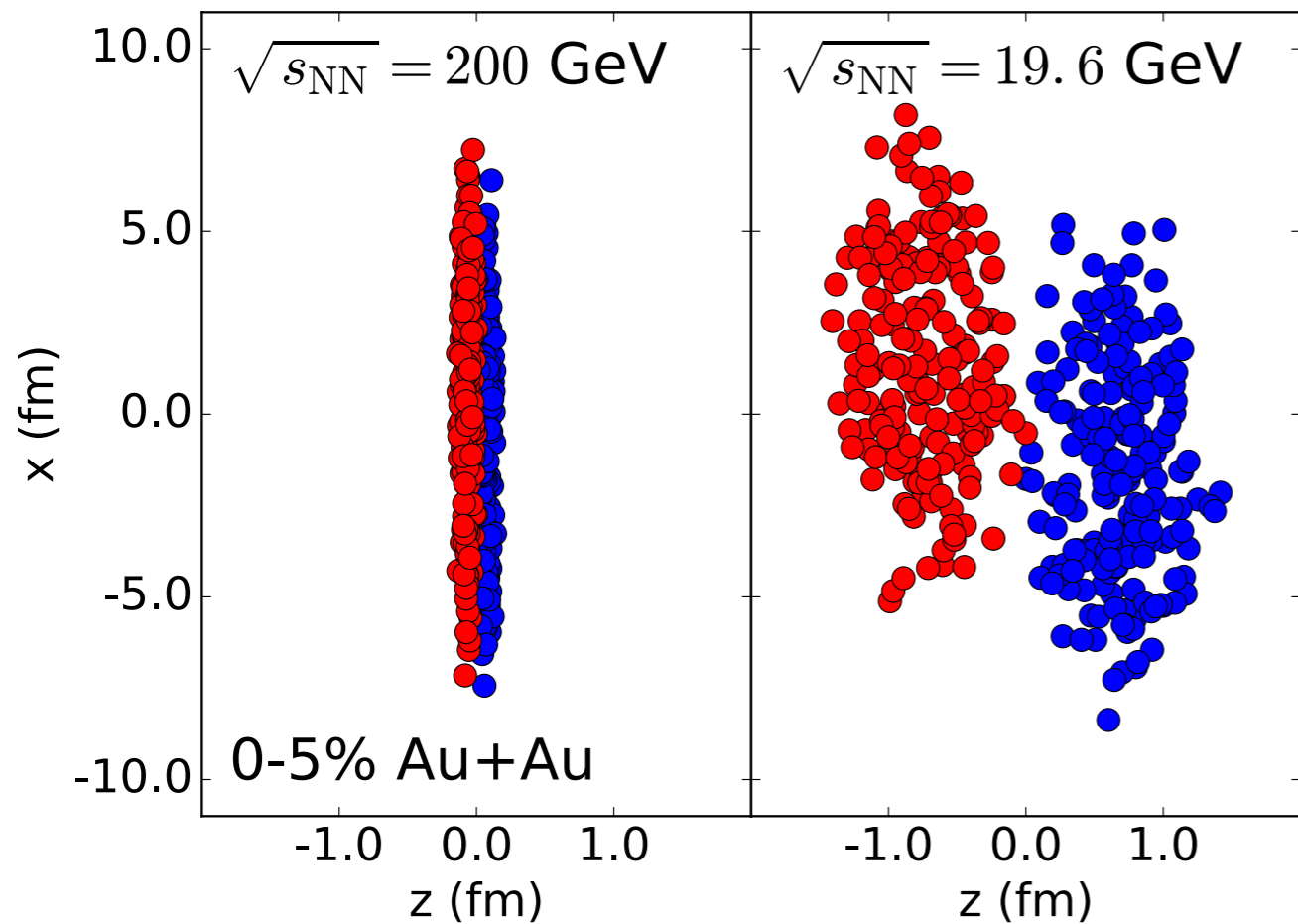
# Exploring the phases of QCD



- Event-by-event fluctuating initial conditions and pre-equilibrium evolution
- (3+1)-d dissipative hydrodynamics with conserved charge currents
- Detailed microscopic description for hadronic phase

# When to start hydrodynamics?

C. Shen and B. Schenke, arXiv:1710.00881 [nucl-th].



- Nuclei overlapping time is **large** at low collision energy
- **Pre-equilibrium dynamics** can play an important role

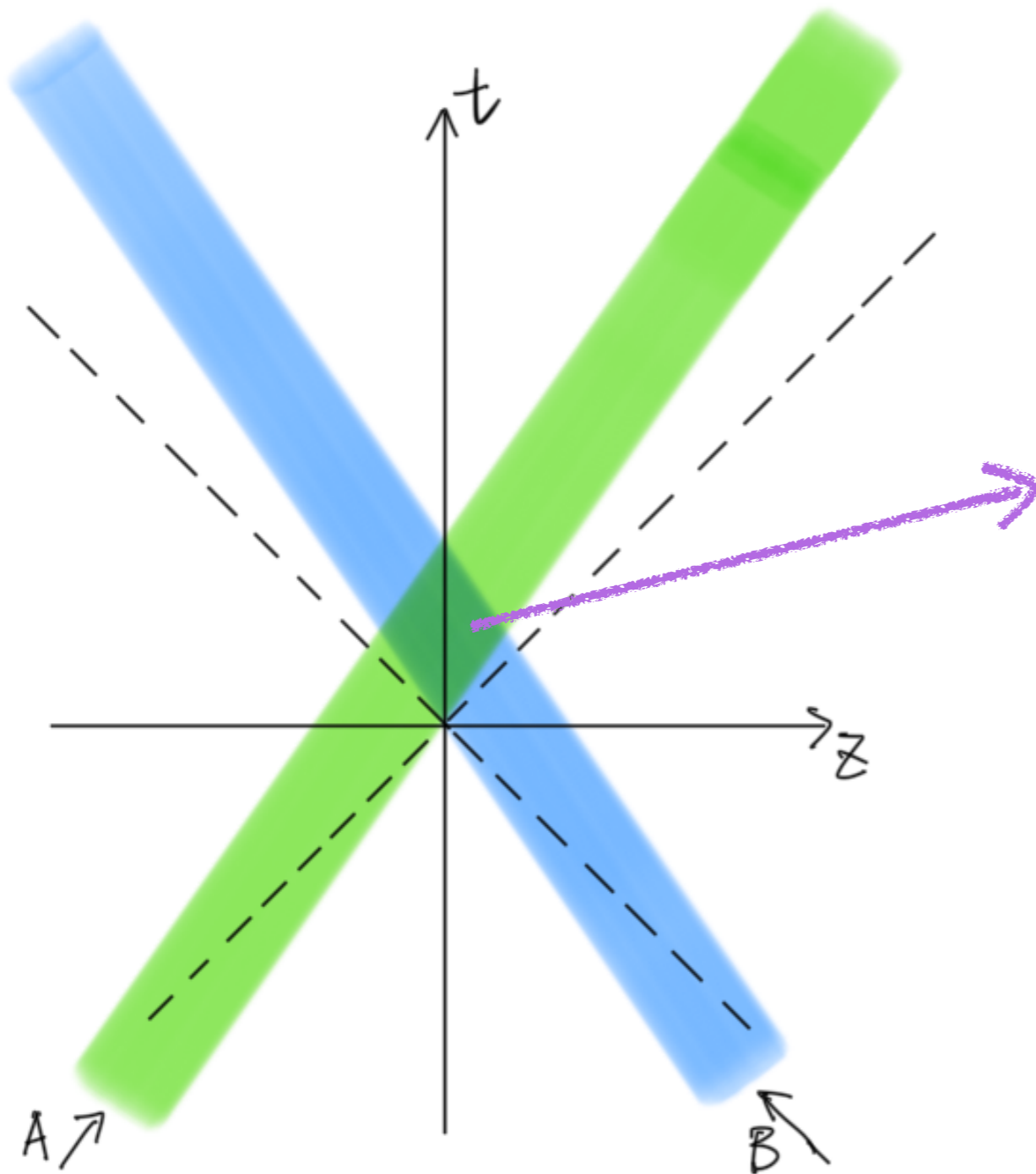
# Go beyond the Bjorken approximation

- The finite widths of the colliding nuclei are taken into account

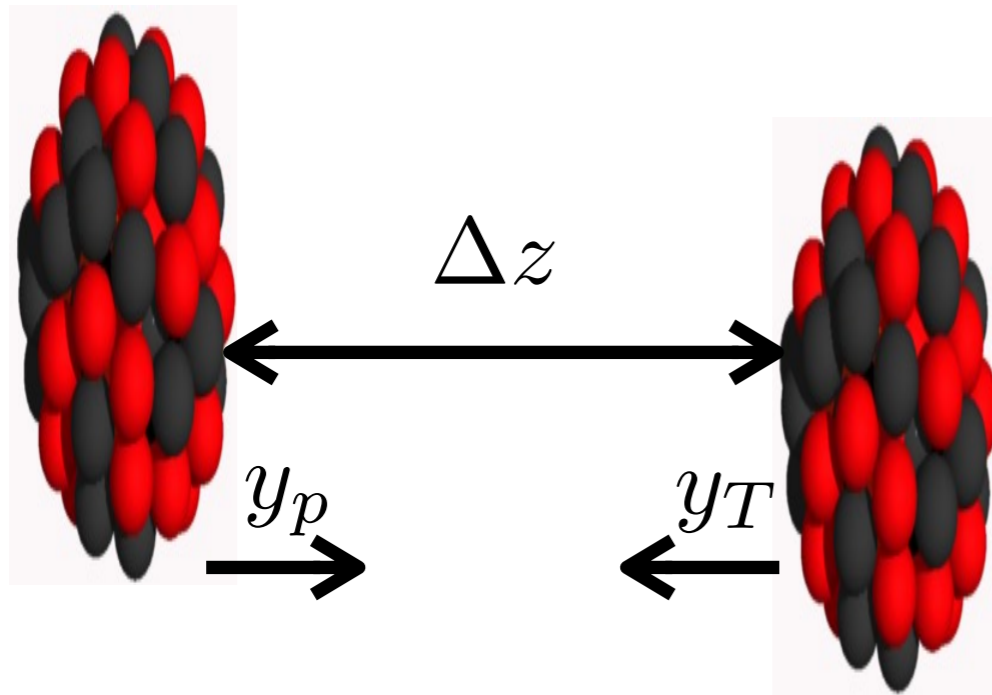
The interaction zone is not point like

$$y \neq \eta_s$$

need full **3D spatial** and **momentum** information

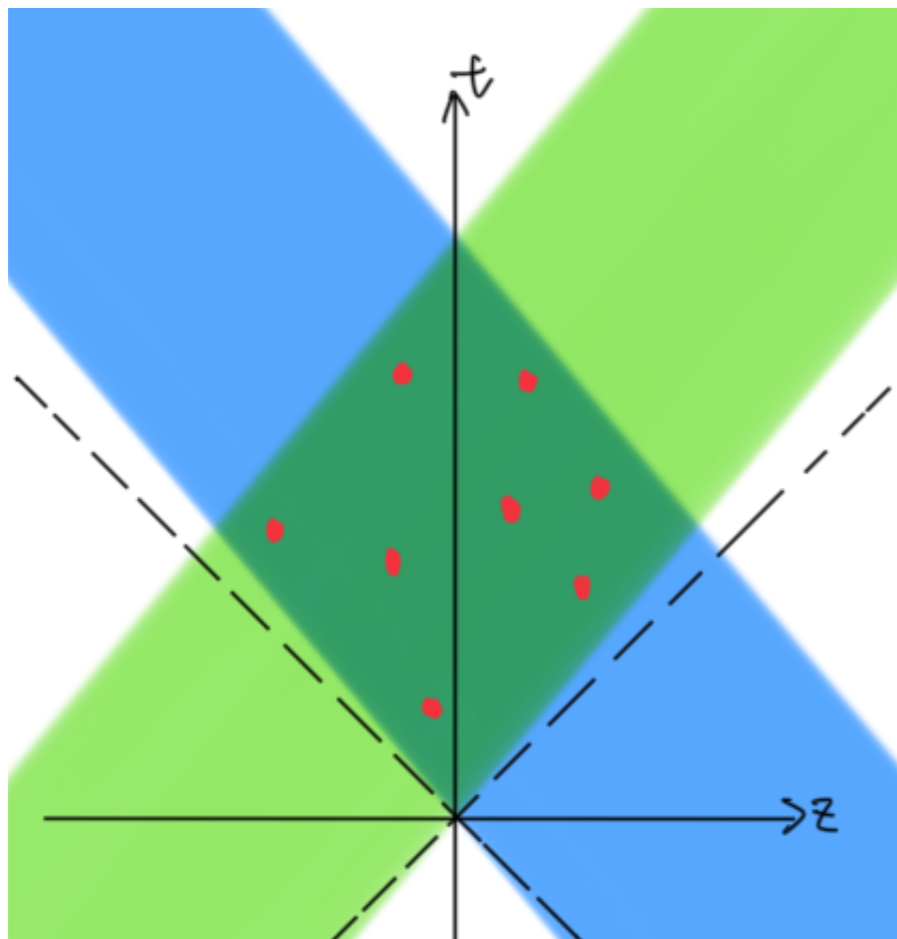
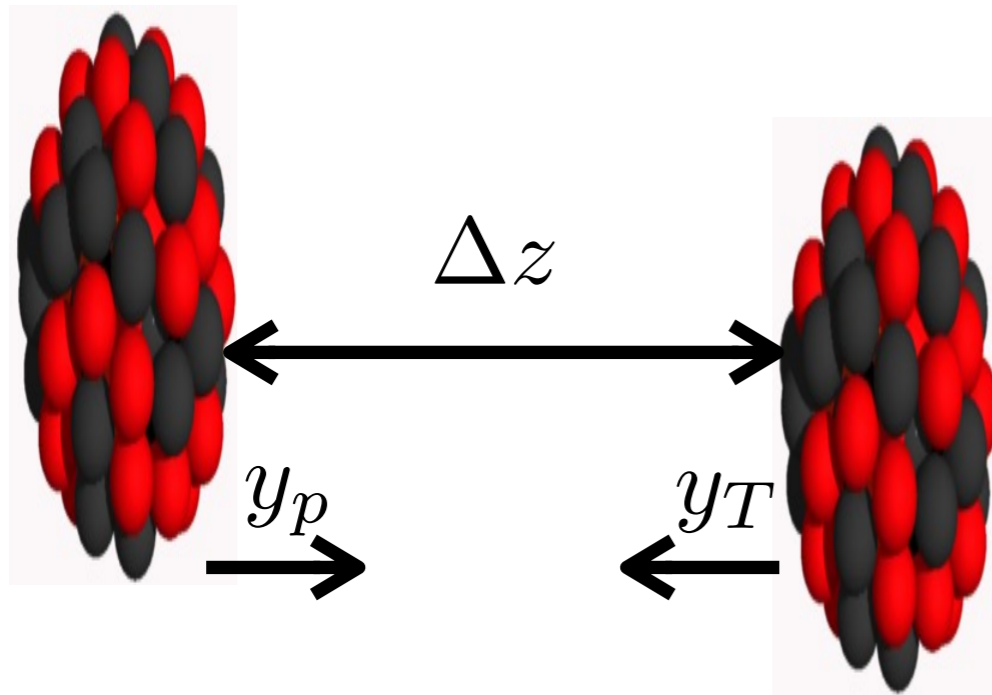


# The 3D MC-Glauber model



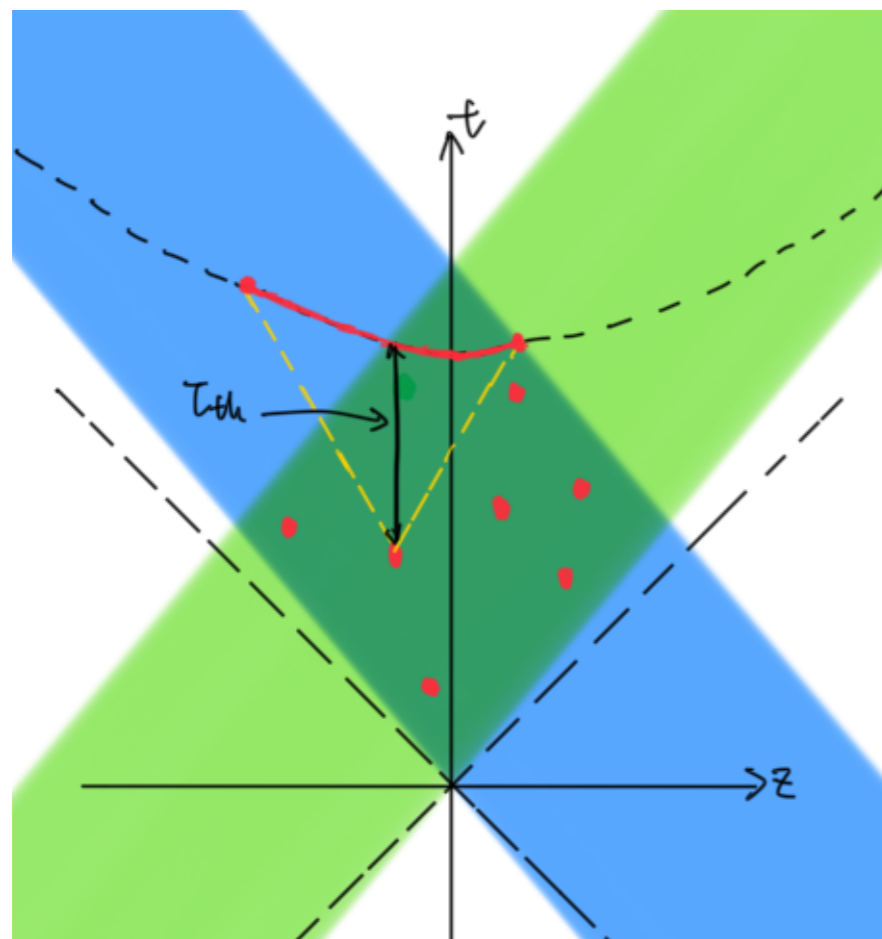
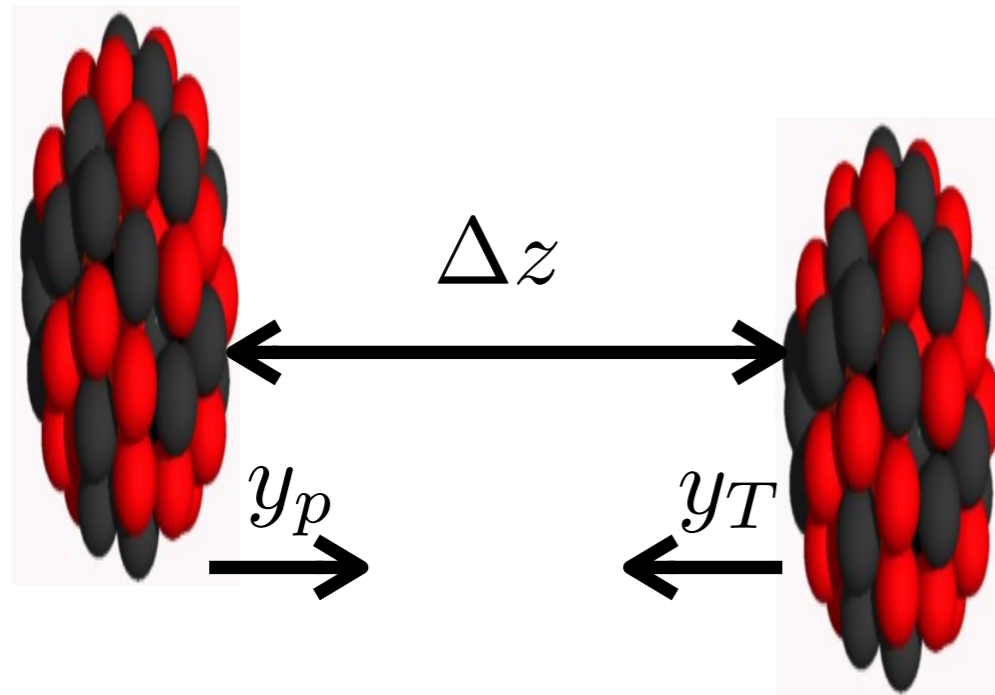
- Collision time and 3D spatial position are determined for every binary collision

# The 3D MC-Glauber model



- Collision time and 3D spatial position are determined for every binary collision
- QCD strings are randomly produced from collision points

# The 3D MC-Glauber model

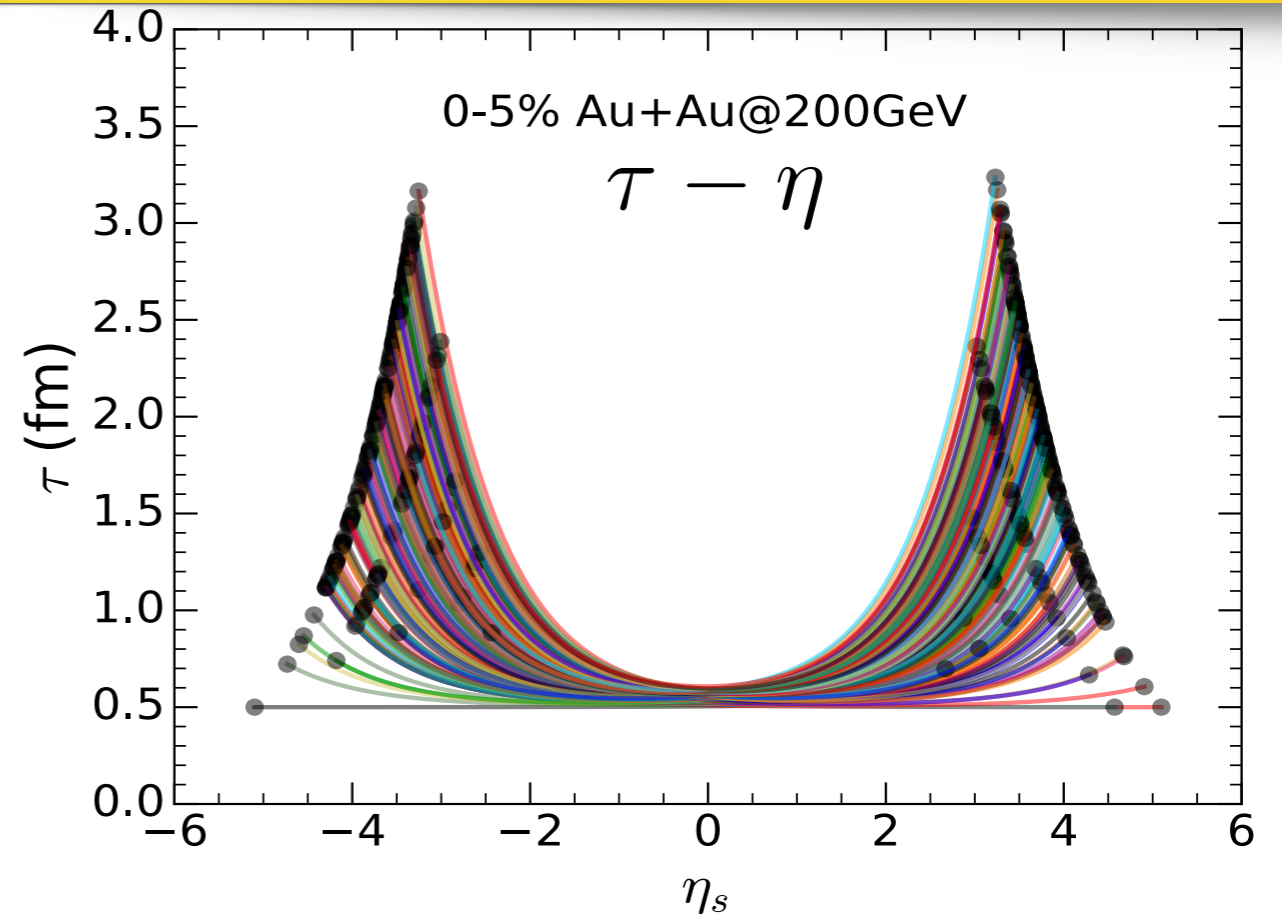
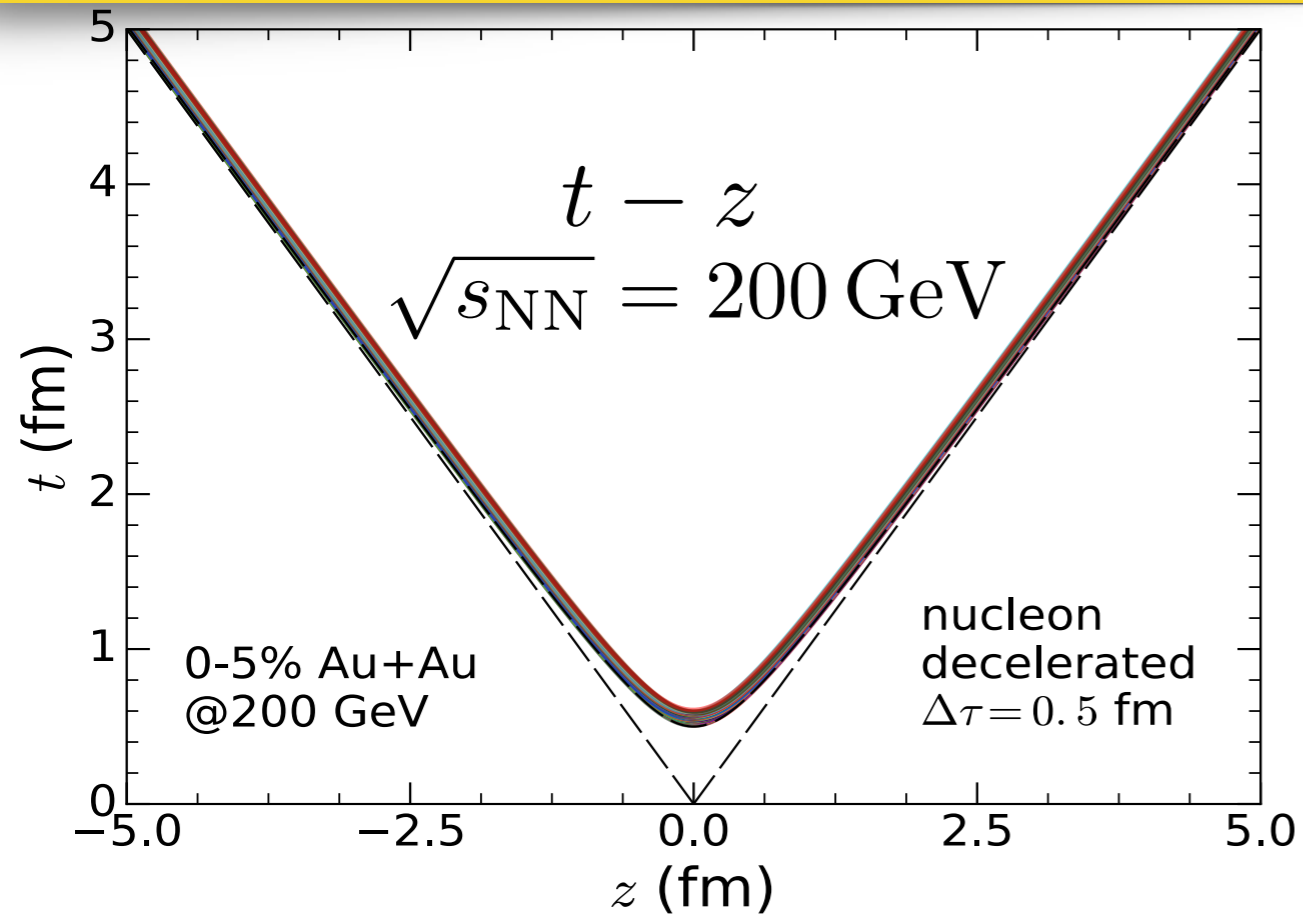


- Collision time and 3D spatial position are determined for every binary collision
- QCD strings are randomly produced from collision points

A. Bialas, A. Bzdak and V. Koch,  
arXiv:1608.07041 [hep-ph]

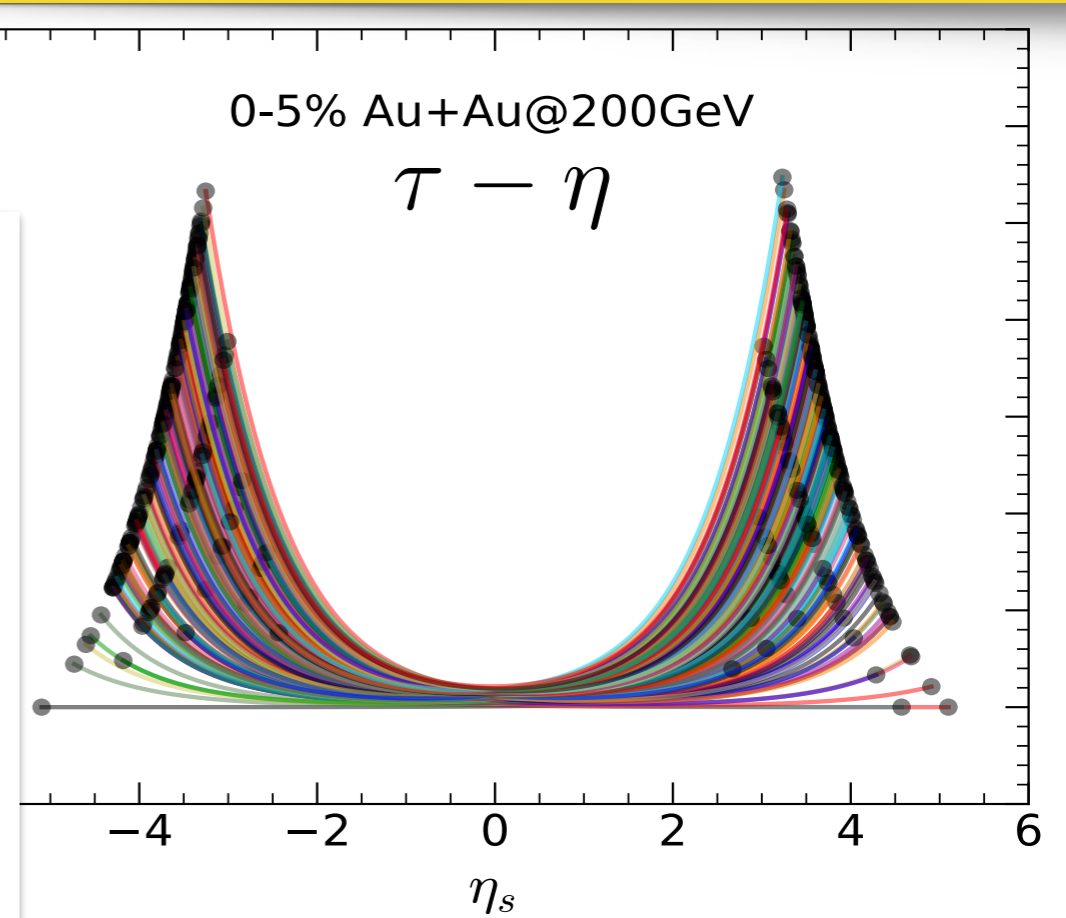
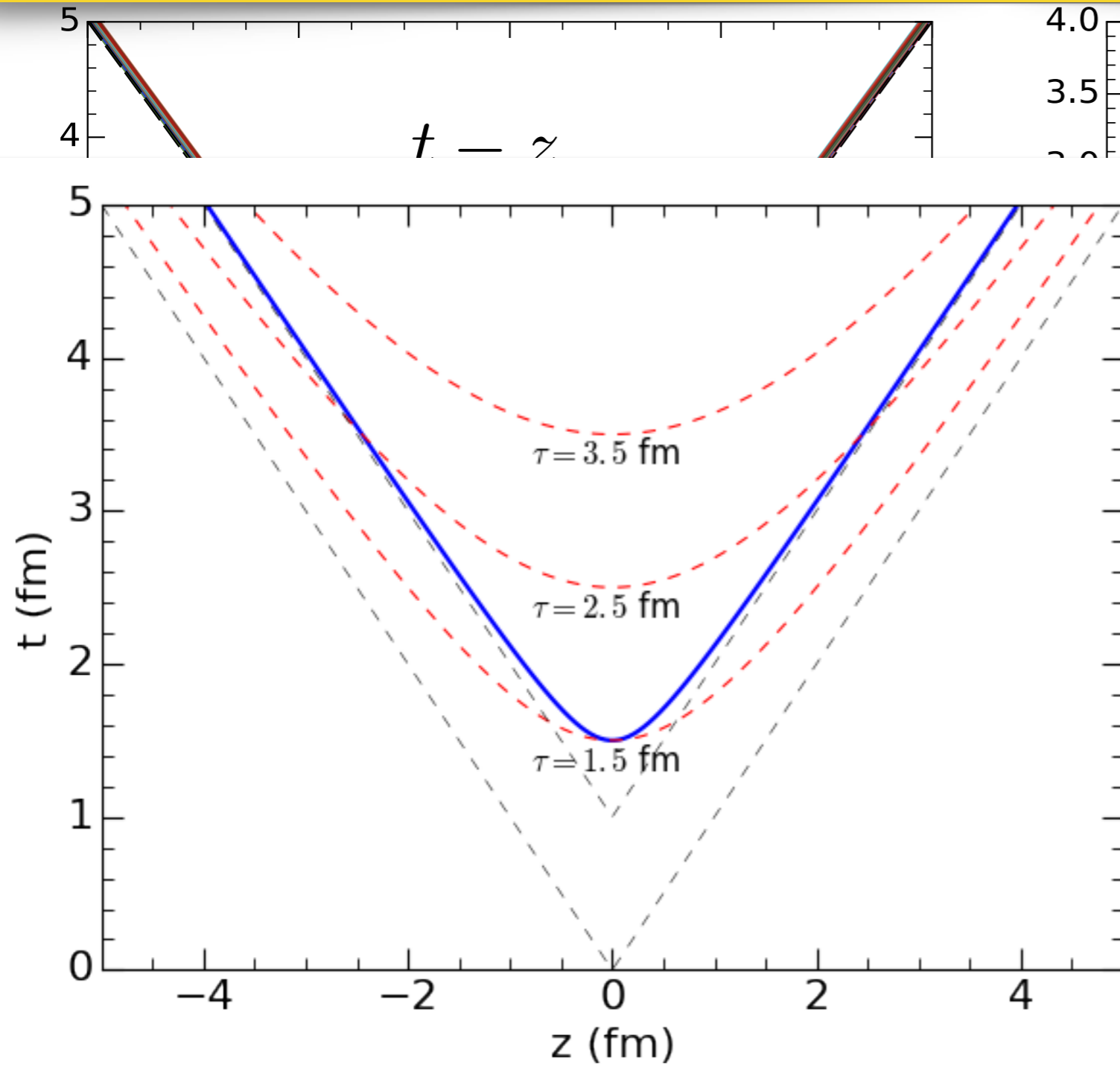
- These strings are decelerated with a constant string tension  $\sigma = 1 \text{ GeV}/\text{fm}$  before thermalized to medium

# String space-time distribution

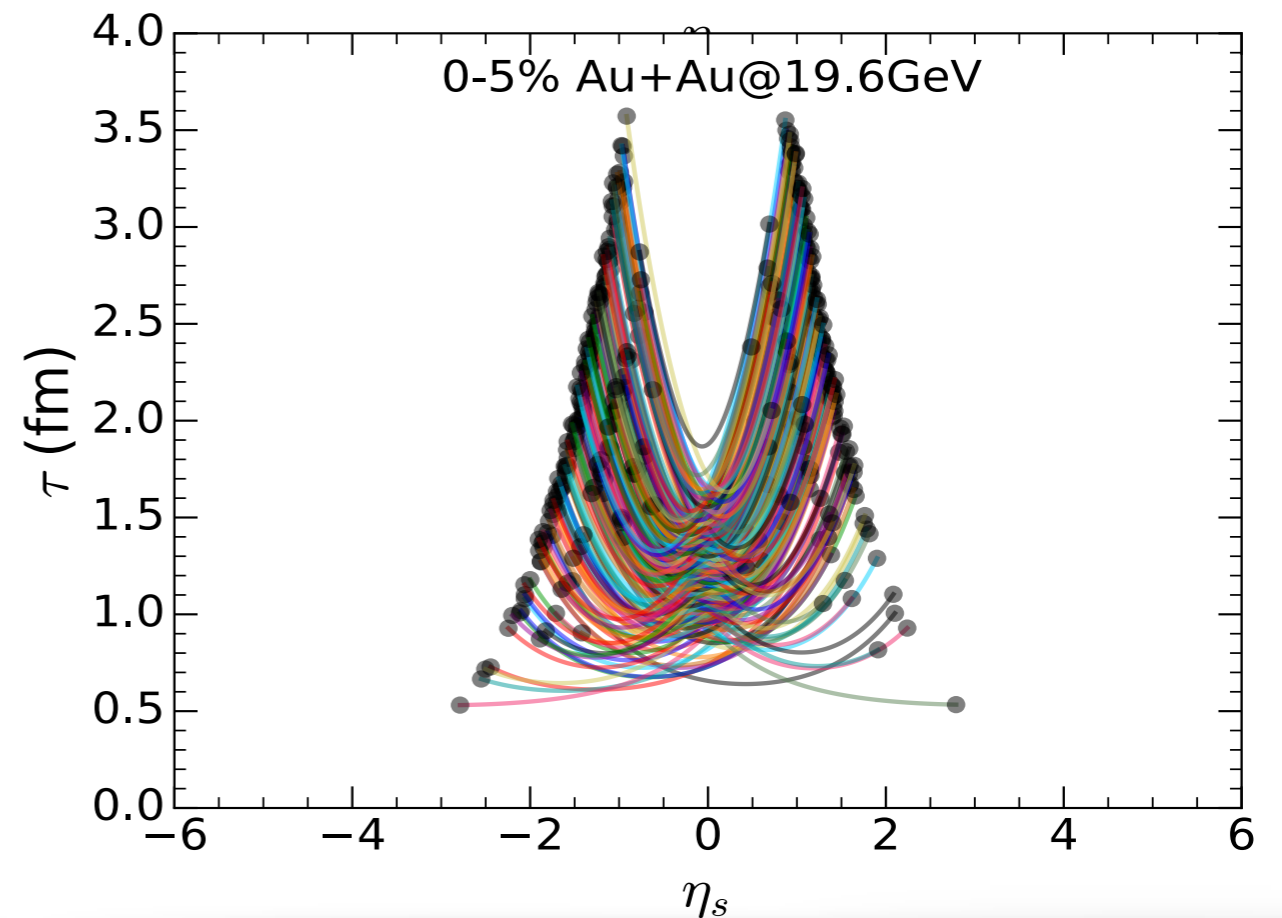
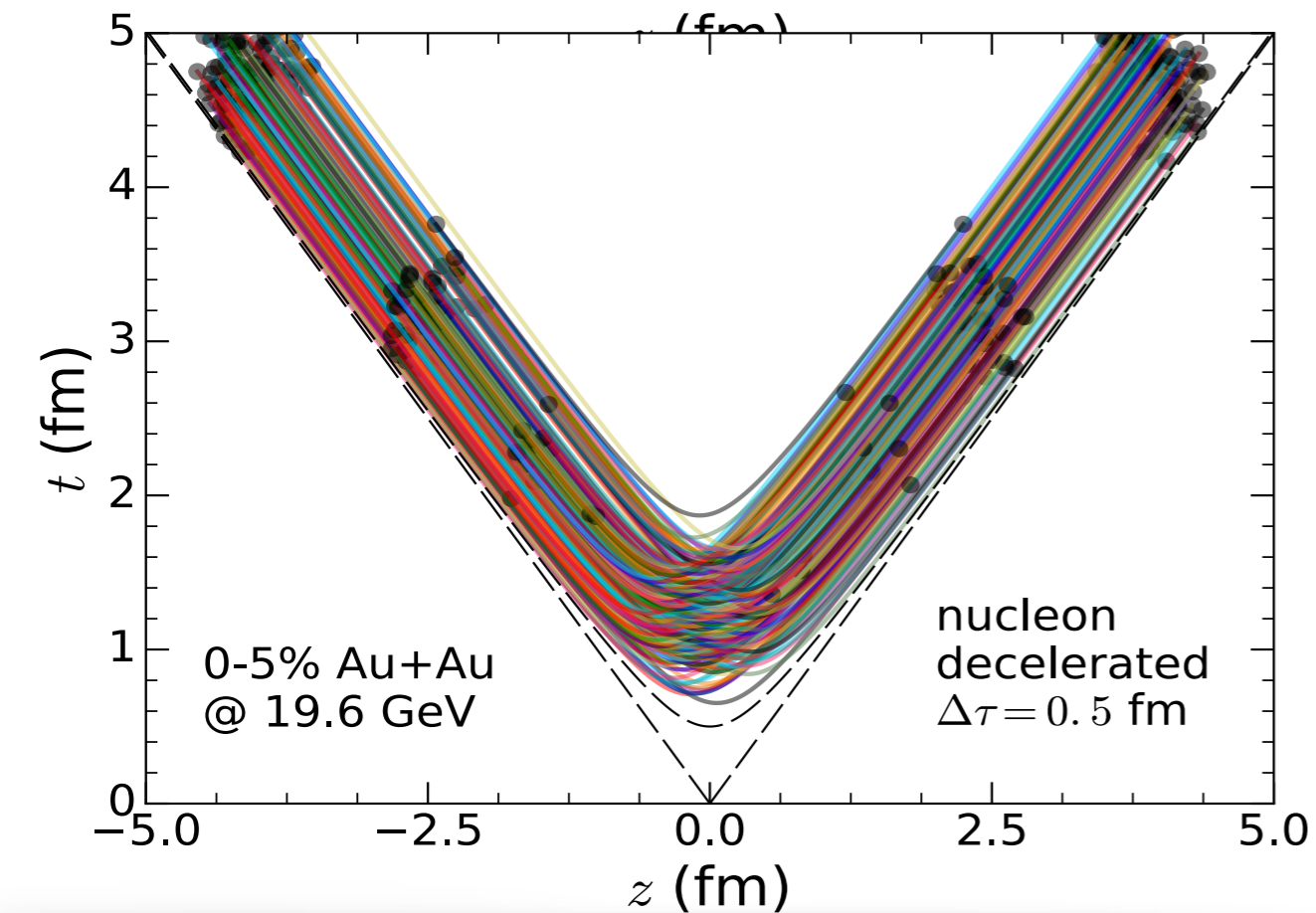
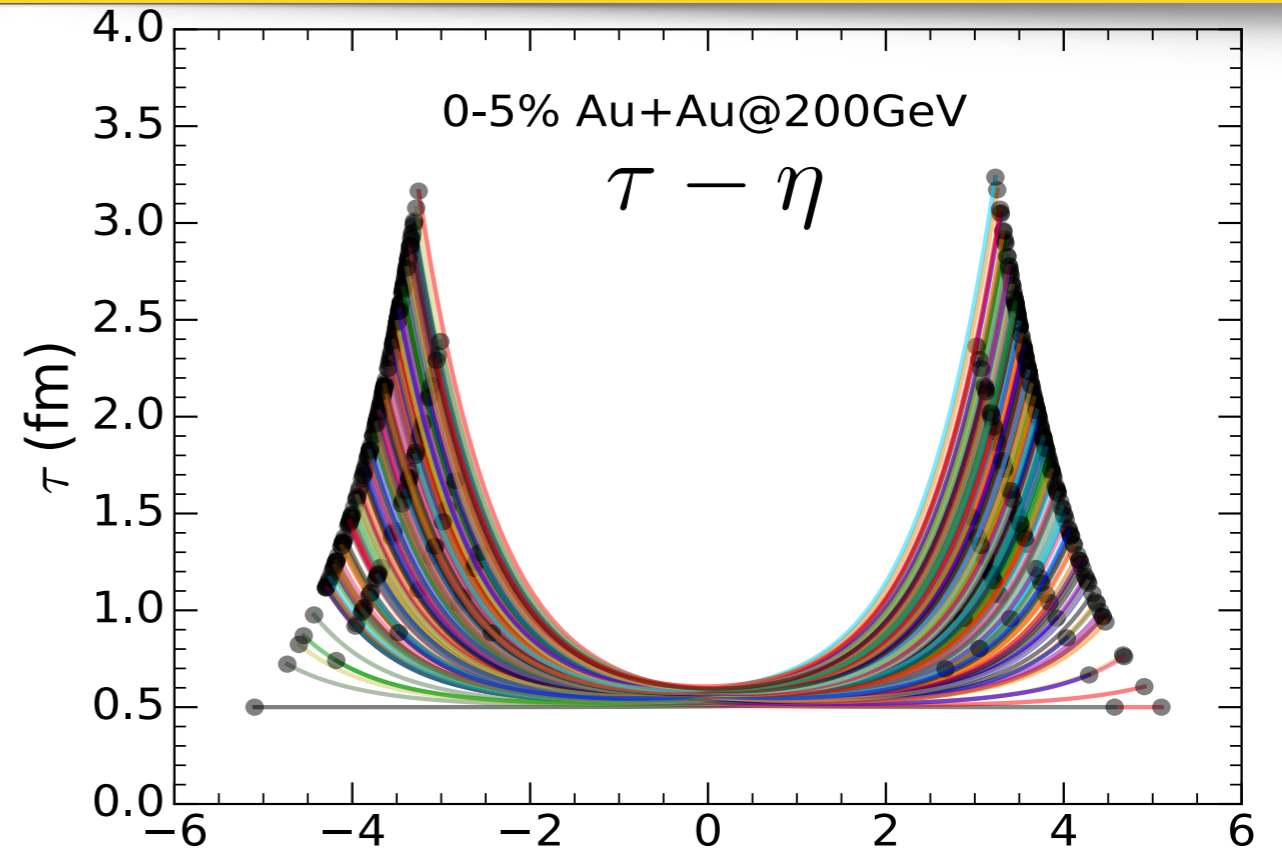
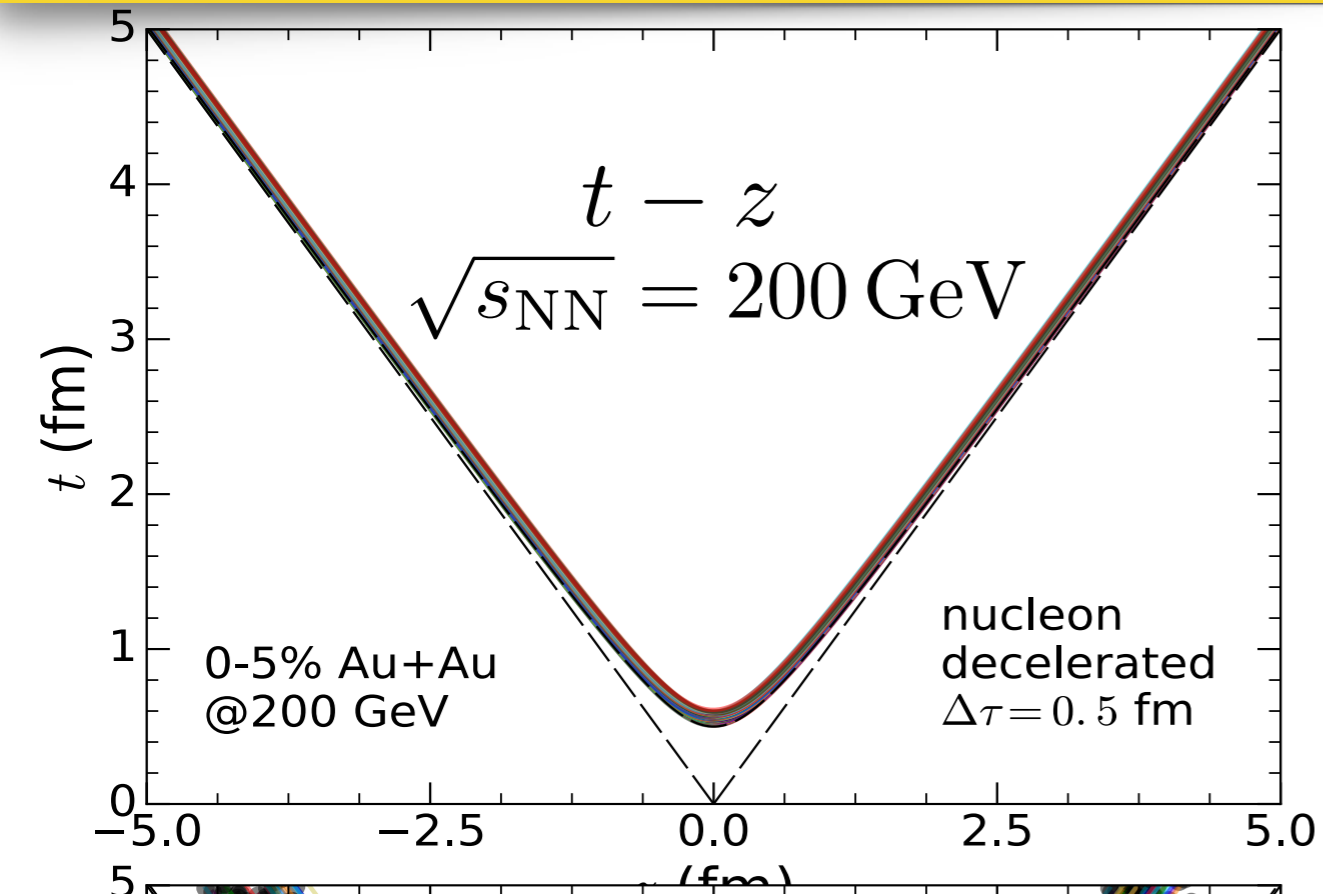




# String space-time distribution



# String space-time distribution



# Hydrodynamics with sources

Energy-momentum current and net baryon density are feed into hydrodynamic simulation as source terms

$$\partial_\mu T^{\mu\nu} = J_{\text{source}}^\nu$$

$$\partial_\mu J^\mu = \rho_{\text{source}}$$

where

$$J_{\text{source}}^\nu = \delta e u^\nu + (e + P) \delta u^\nu$$

$$\delta u^\nu = \frac{\Delta_{\mu}^{\nu} J_{\text{source}}^{\mu}}{e + P}$$

$\delta e$  heats up the system

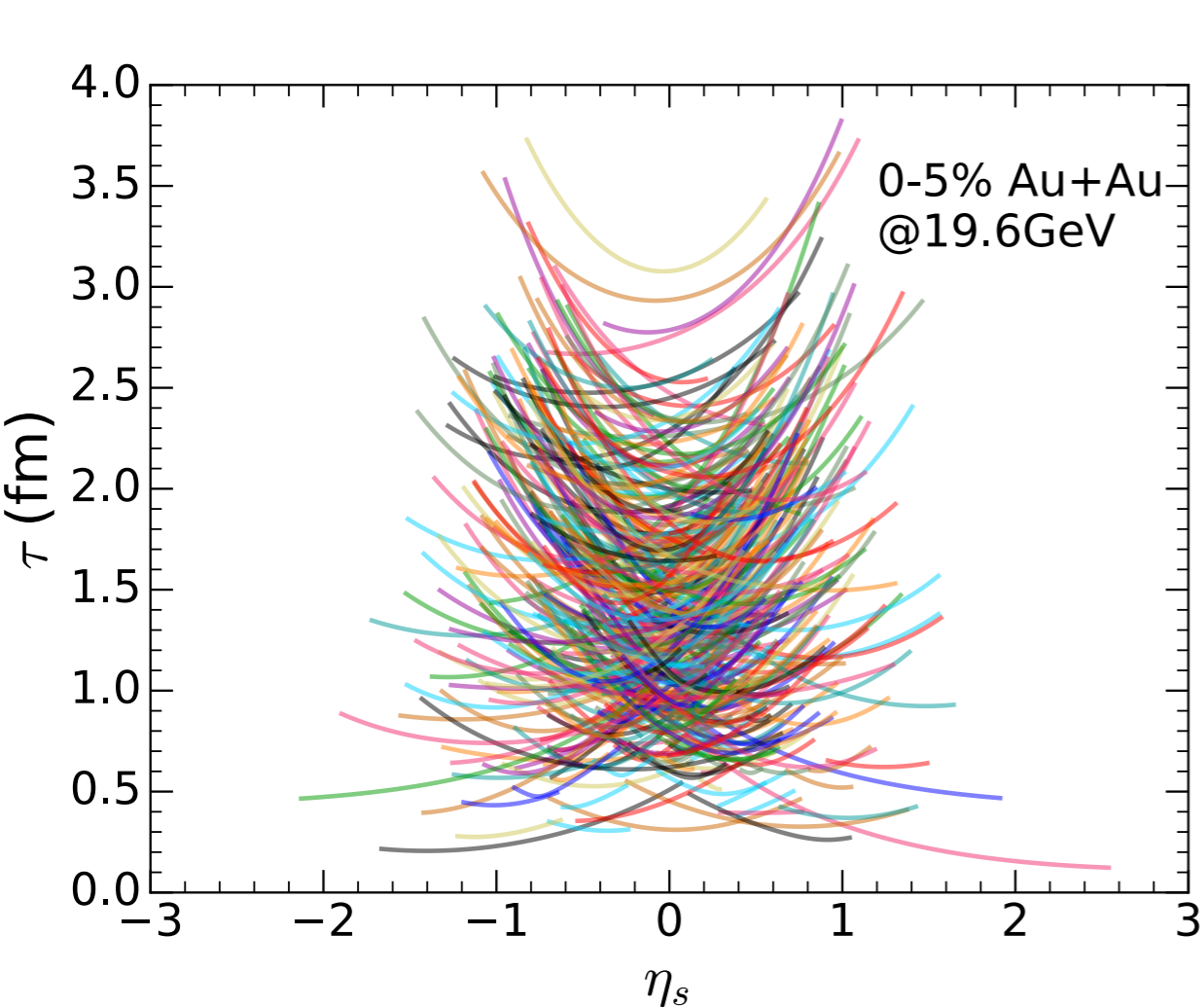
$\delta u^\nu$  accelerates the flow velocity

$\rho_{\text{source}}$  dopes baryon charges into the system

- Source terms are smeared with Gaussians in space and time

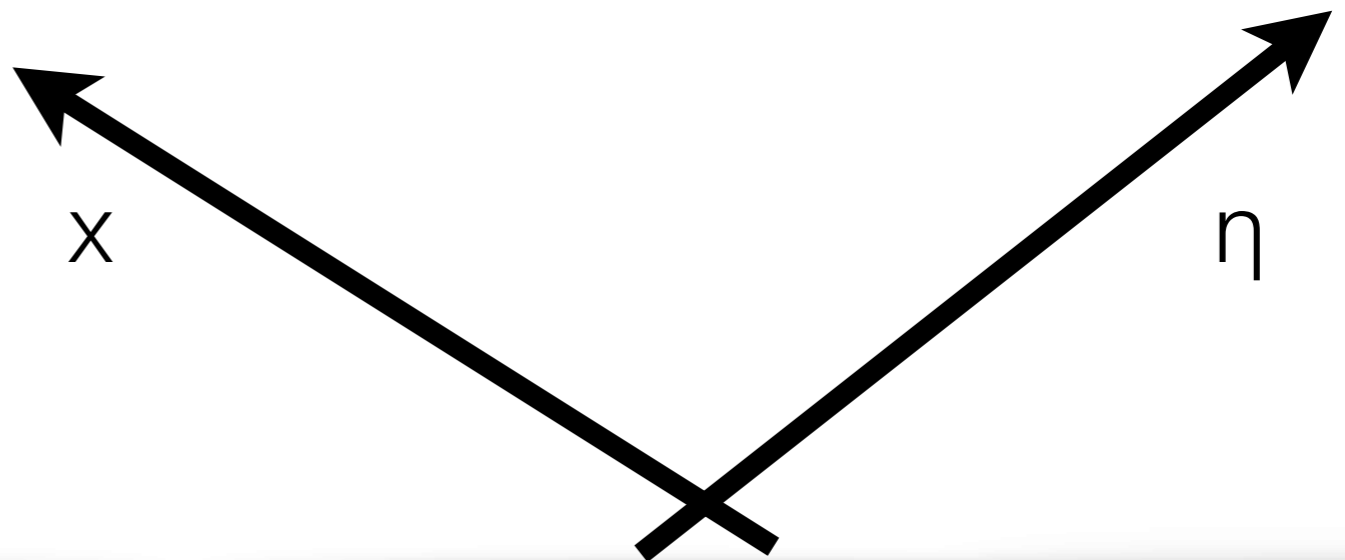
# Hydrodynamical evolution with sources

energy density



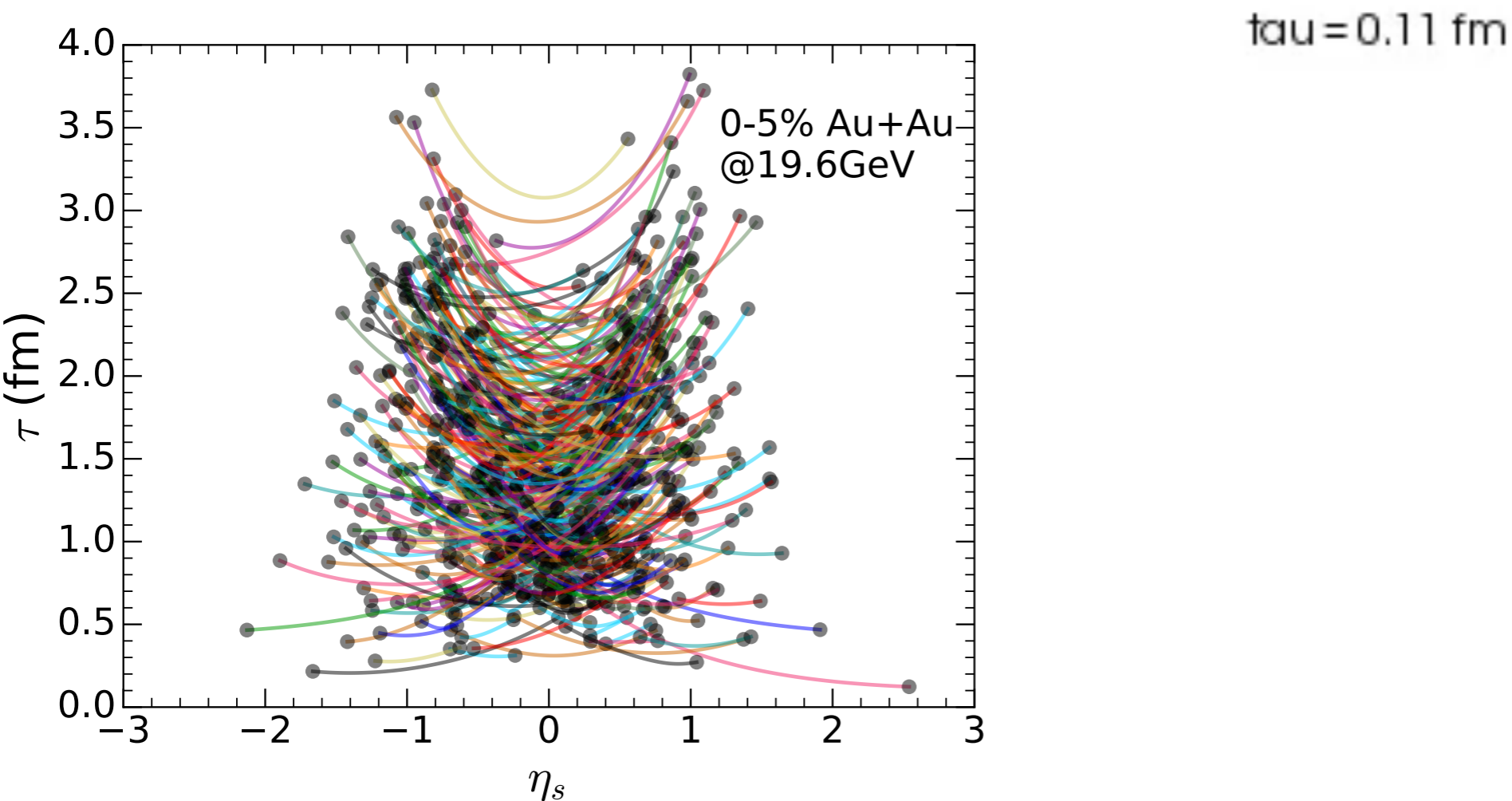
$$\sqrt{s_{\text{NN}}} = 19.6 \text{ GeV}$$

valence quark +  
rapidity loss fluct.



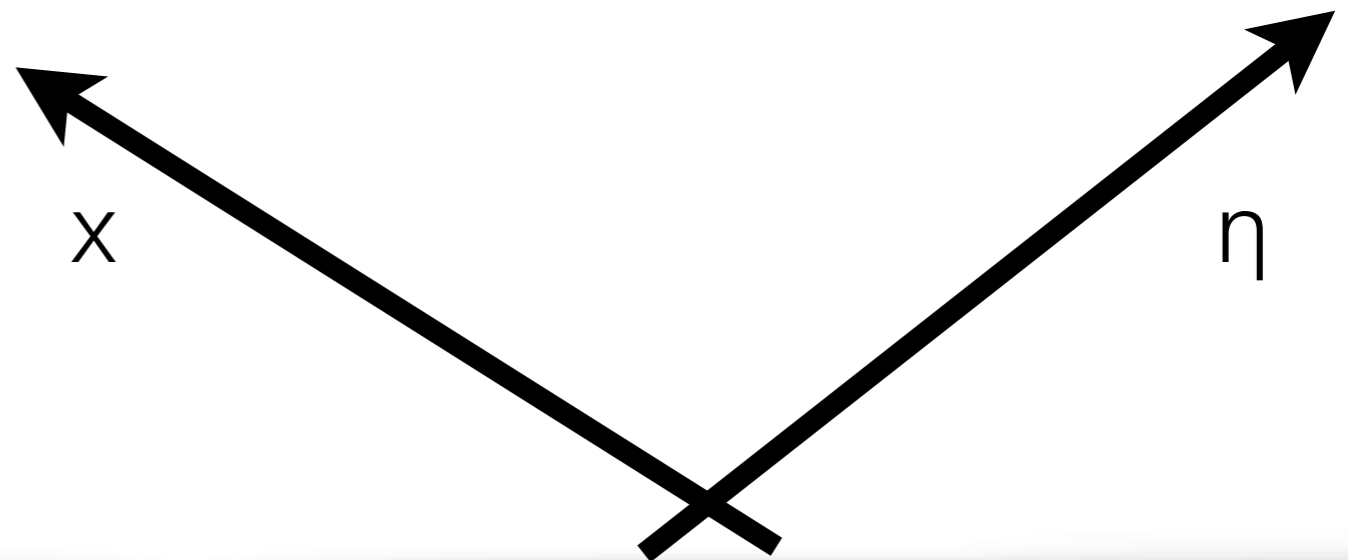
# Hydrodynamical evolution with sources

net baryon density



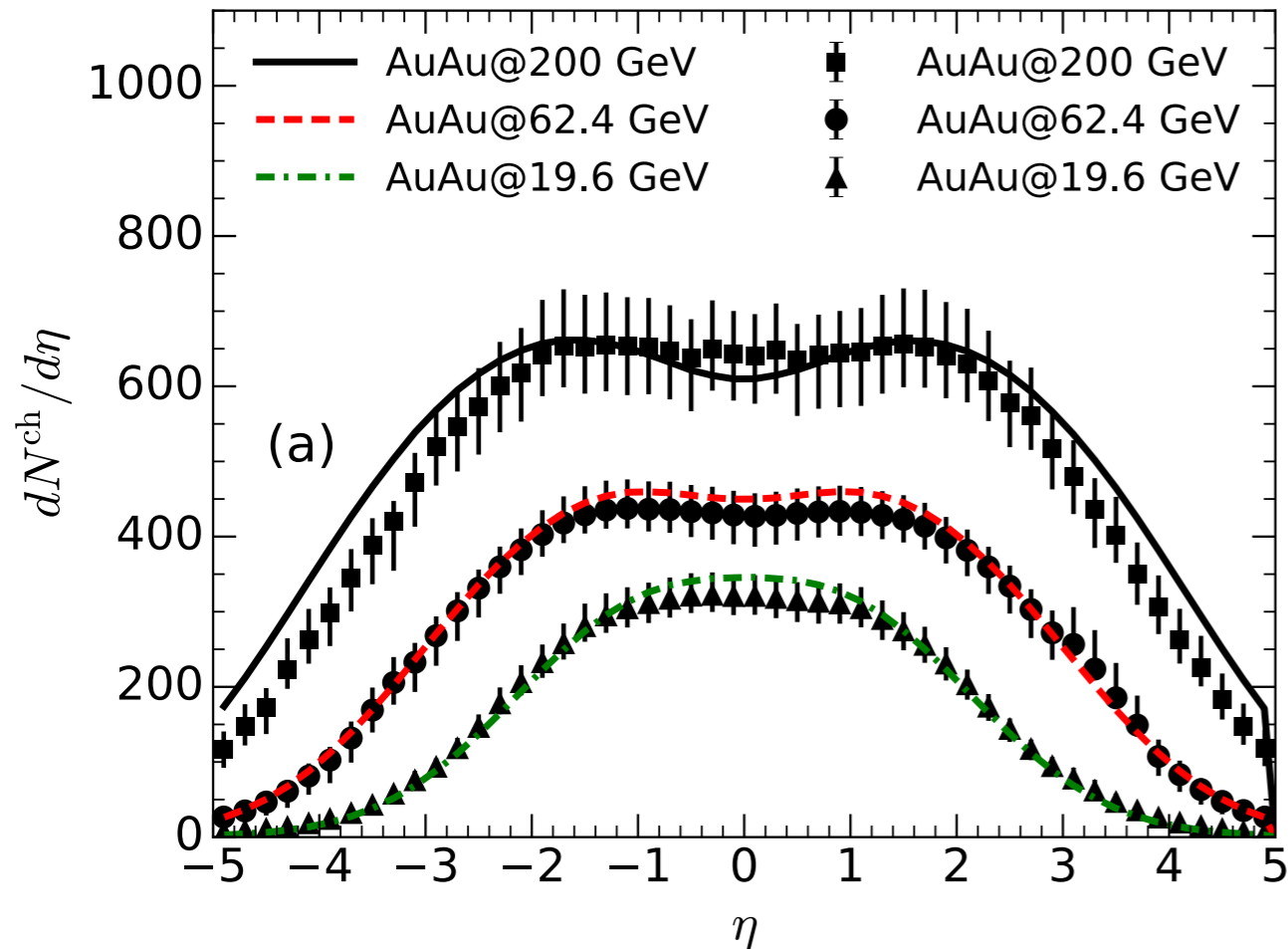
$$\sqrt{s_{\text{NN}}} = 19.6 \text{ GeV}$$

valence quark +  
rapidity loss fluct.



# Particle rapidity distribution

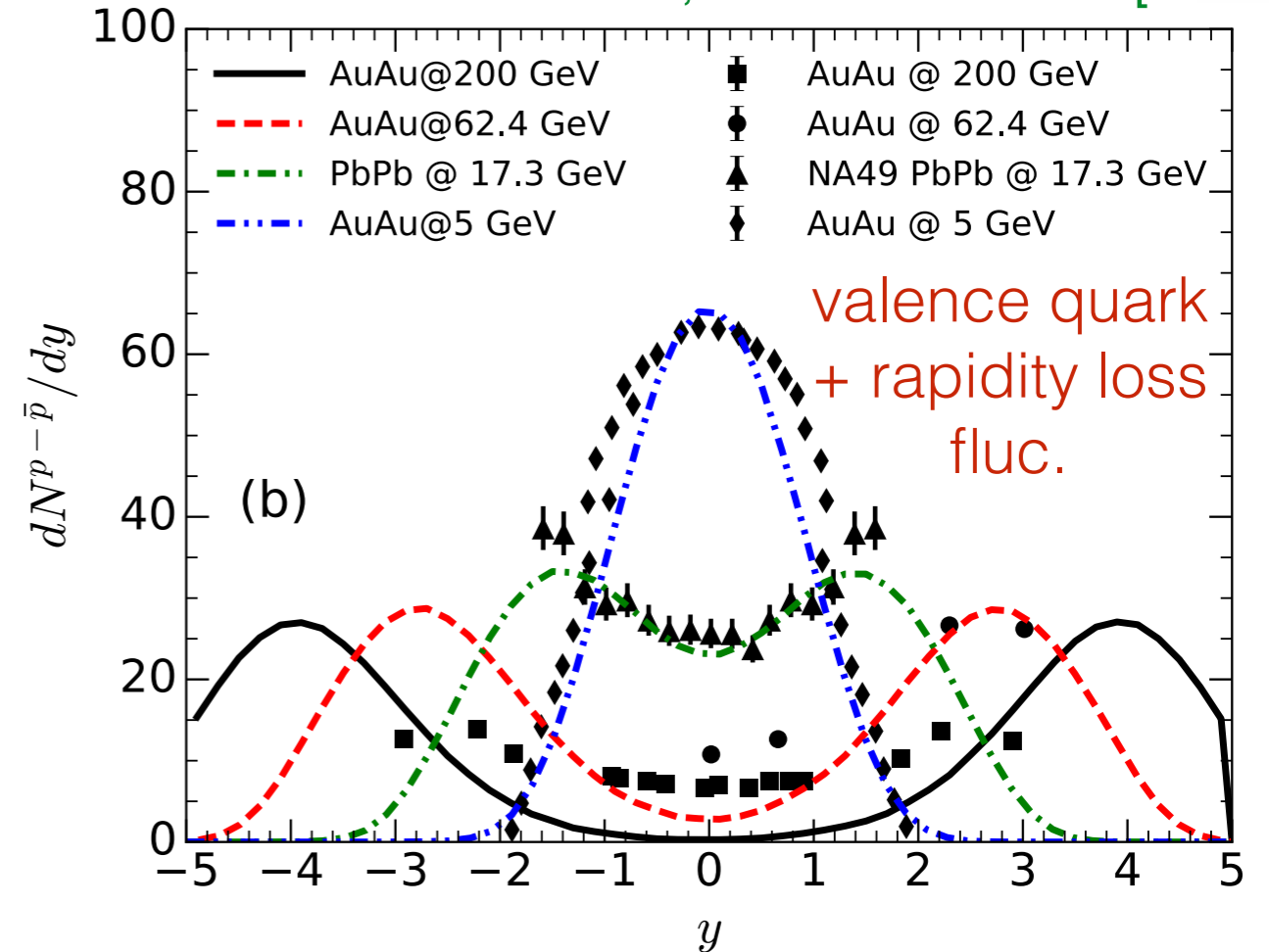
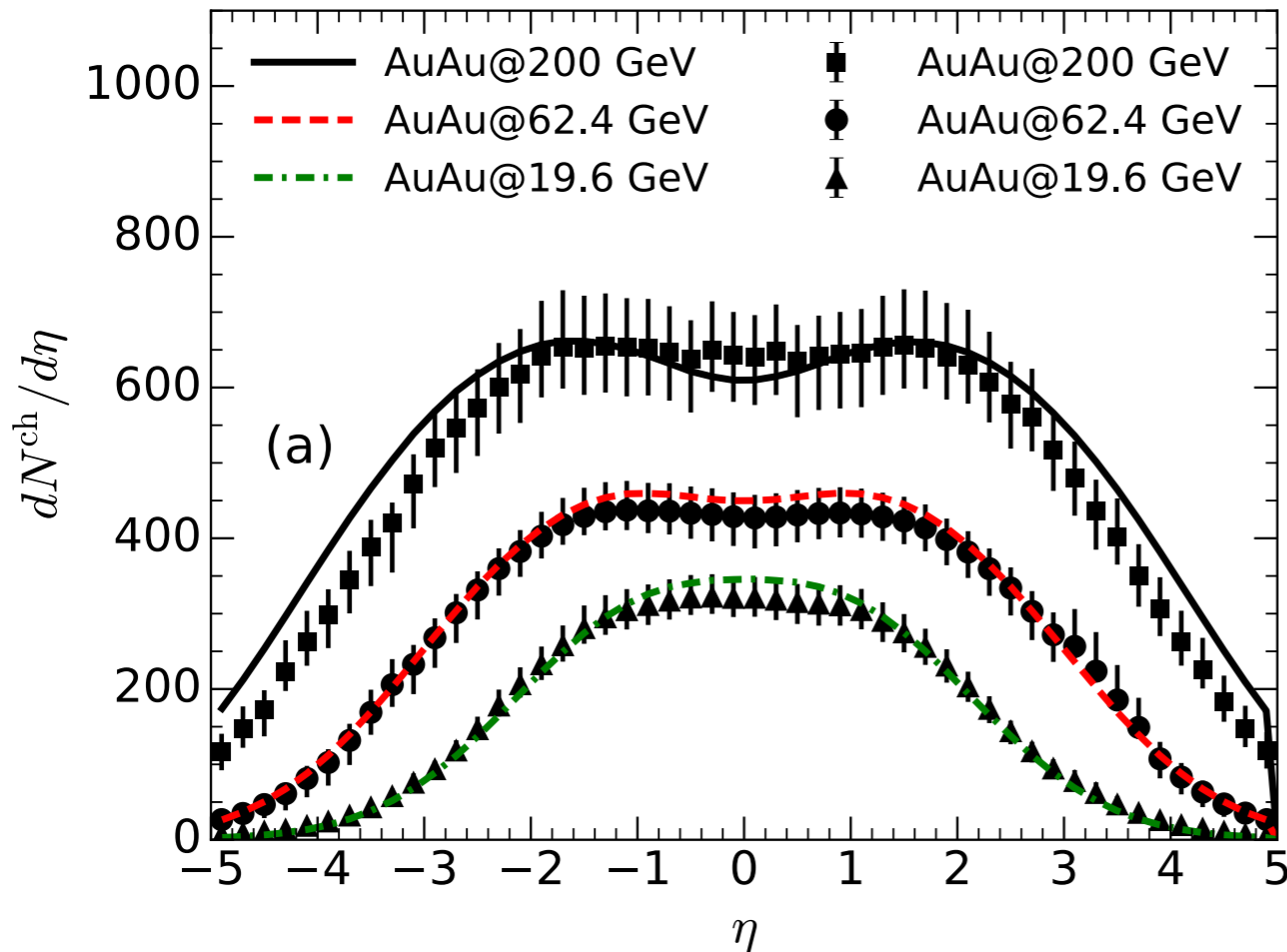
C. Shen and B. Schenke, arXiv:1710.00881 [nucl-th].



- Rapidity distribution of charged hadrons agrees fairly good with the RHIC BES measurements below 62.4 GeV

# Particle rapidity distribution

C. Shen and B. Schenke, arXiv:1710.00881 [nucl-th].



- Rapidity distribution of charged hadrons agrees fairly good with the RHIC BES measurements below 62.4 GeV
- Net proton rapidity distributions are reasonably reproduced at low BES energy; but too low for high energies



Hydrodynamics with baryon diffusion



# Dissipative hydrodynamics

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation

Energy momentum tensor

$$T^{\mu\nu} = e u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Conserved currents

$$J^\mu = n u^\mu + q^\mu$$

**New!**

Equations of motion

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu J^\mu &= 0 \end{aligned} + P(e, n)$$

Dissipative quantities are evolved with 2nd order Israel-Stewart type of equations

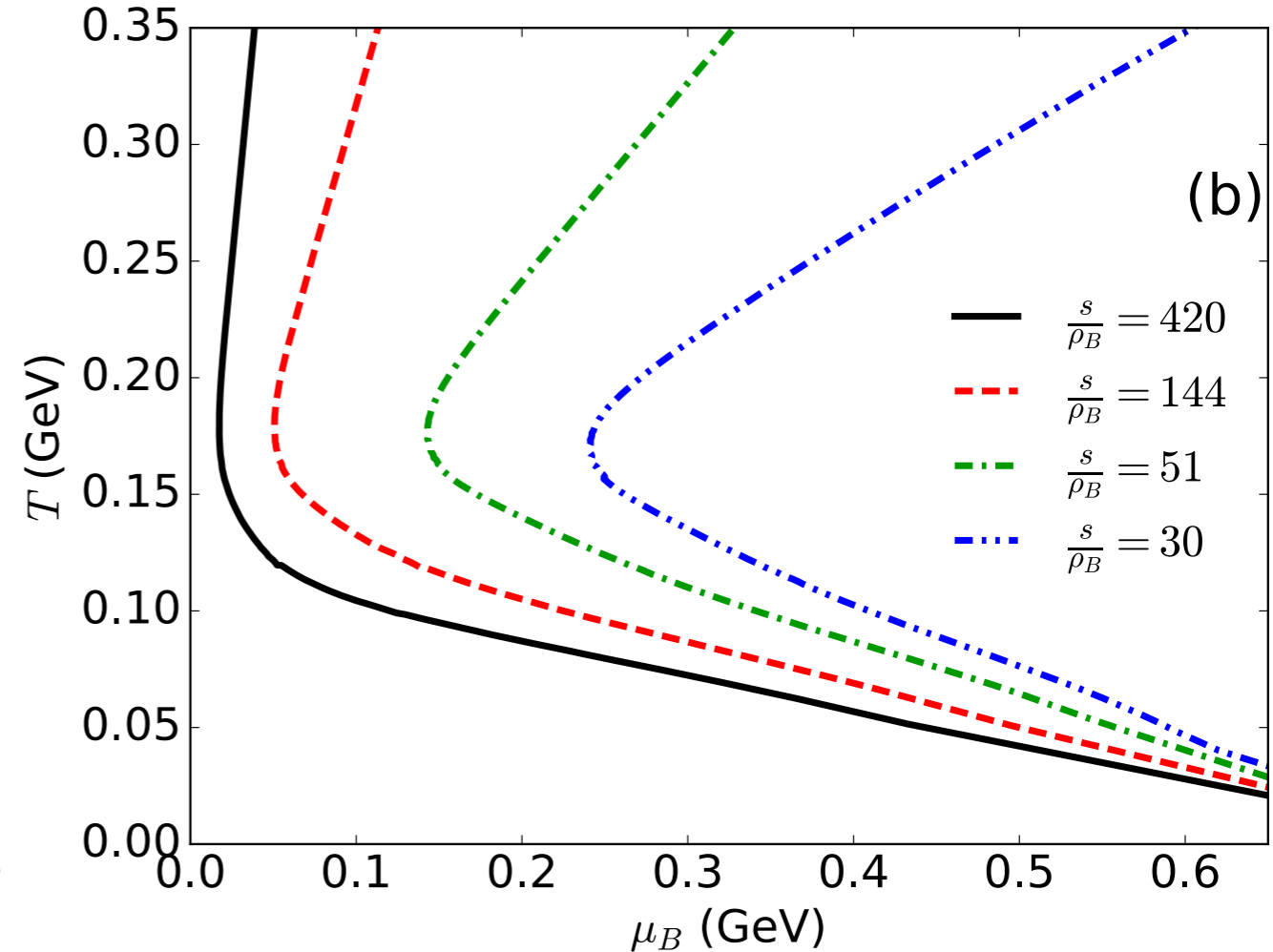
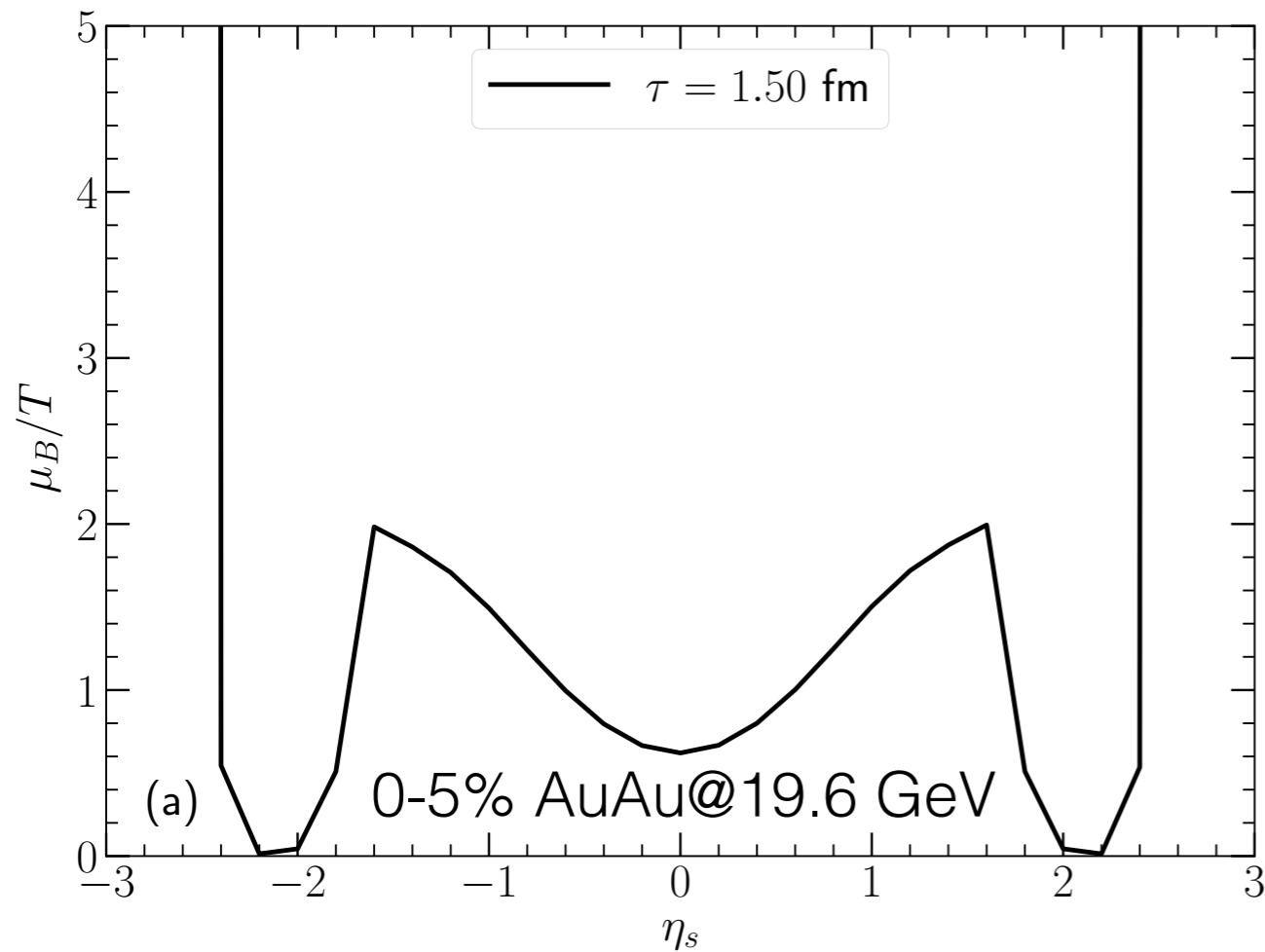
At Navier-Stokes limit,

$$\pi^{\mu\nu} \sim 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \quad \Pi \sim -\zeta \partial_\mu u^\mu \quad q^\mu \sim \kappa \nabla^\mu \frac{\mu}{T}$$

$$\nabla^\mu = \Delta^{\mu\nu} \partial_\nu$$

# Effects of net baryon diffusion on particle yields

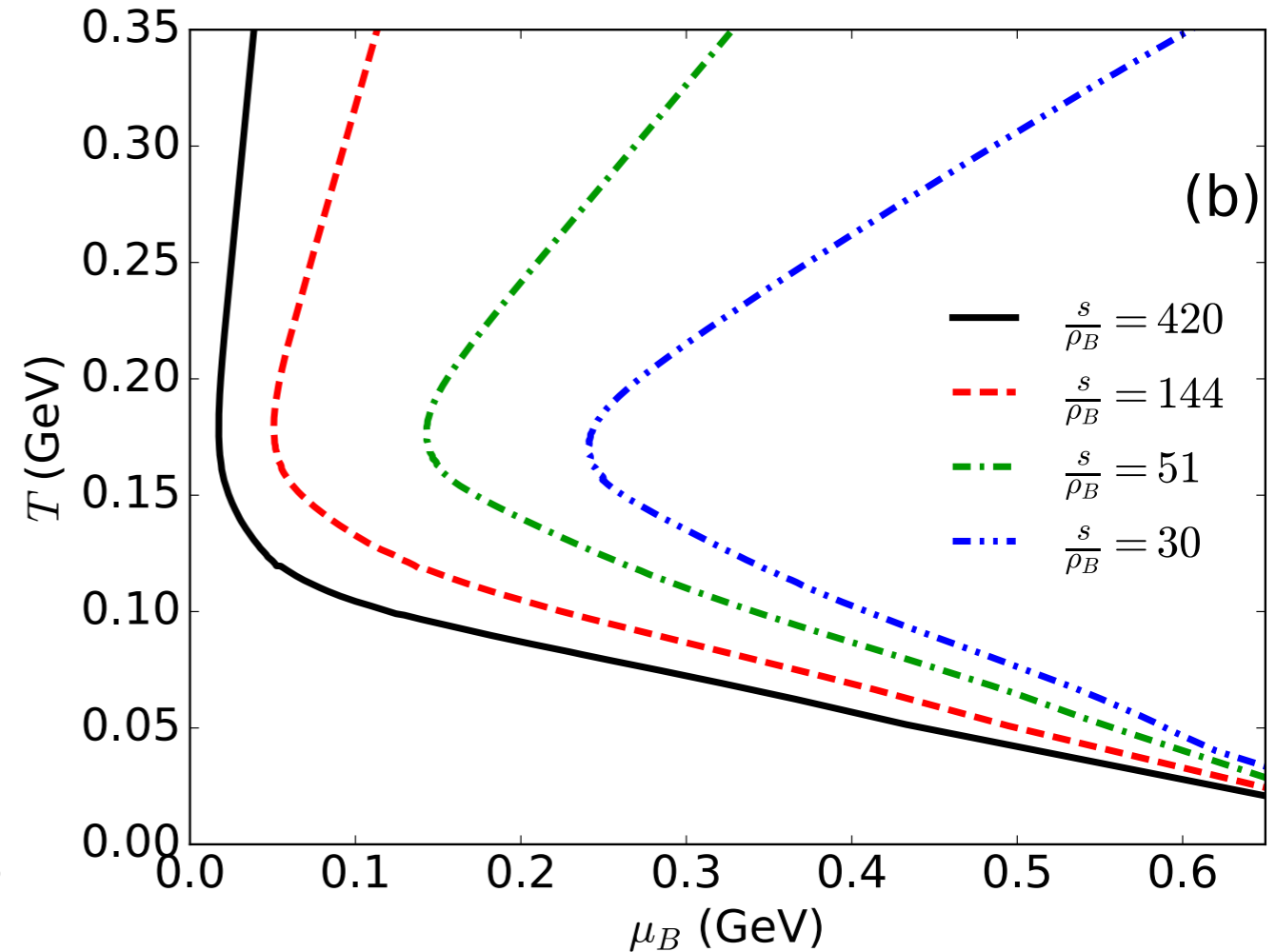
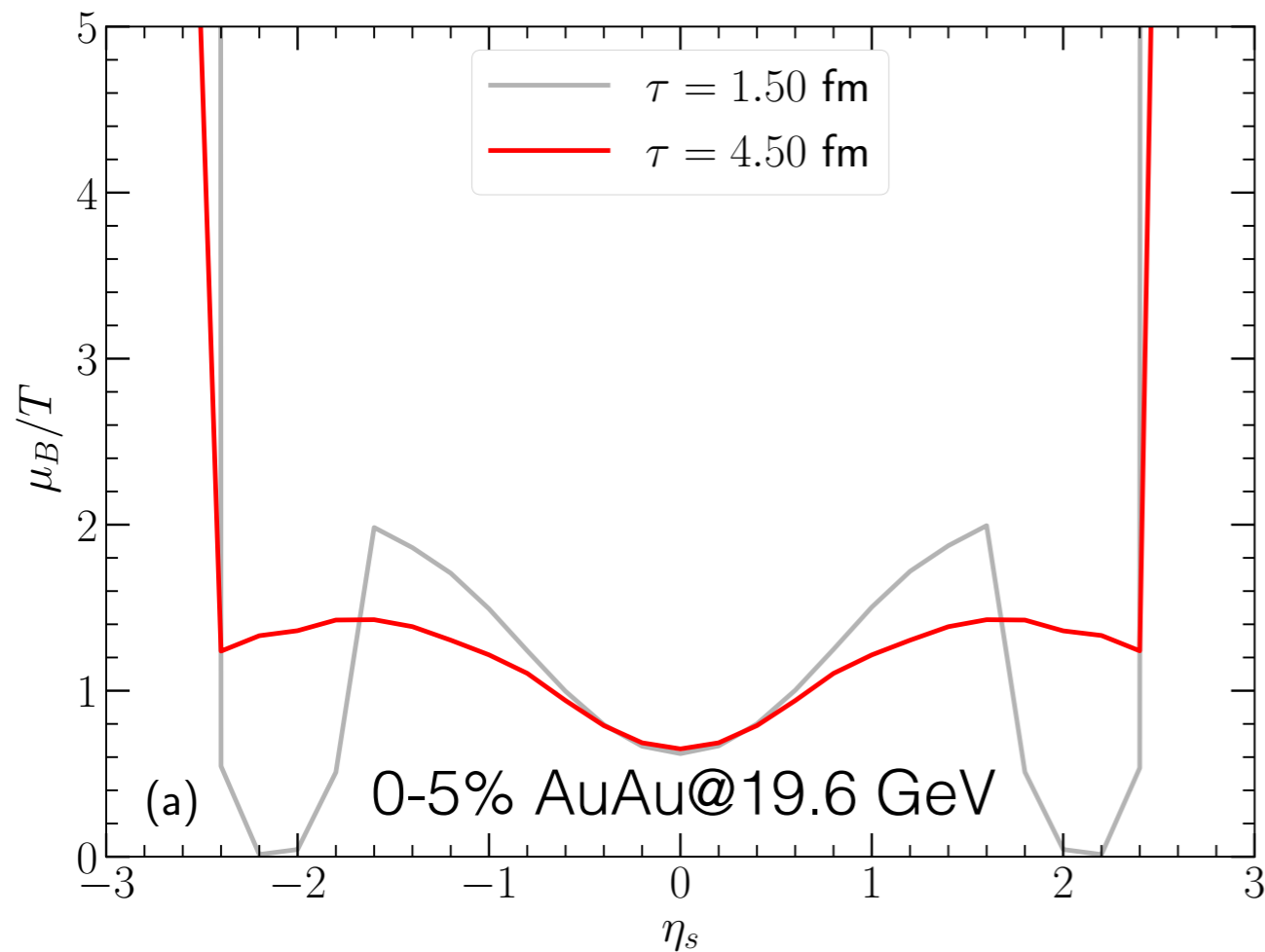
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

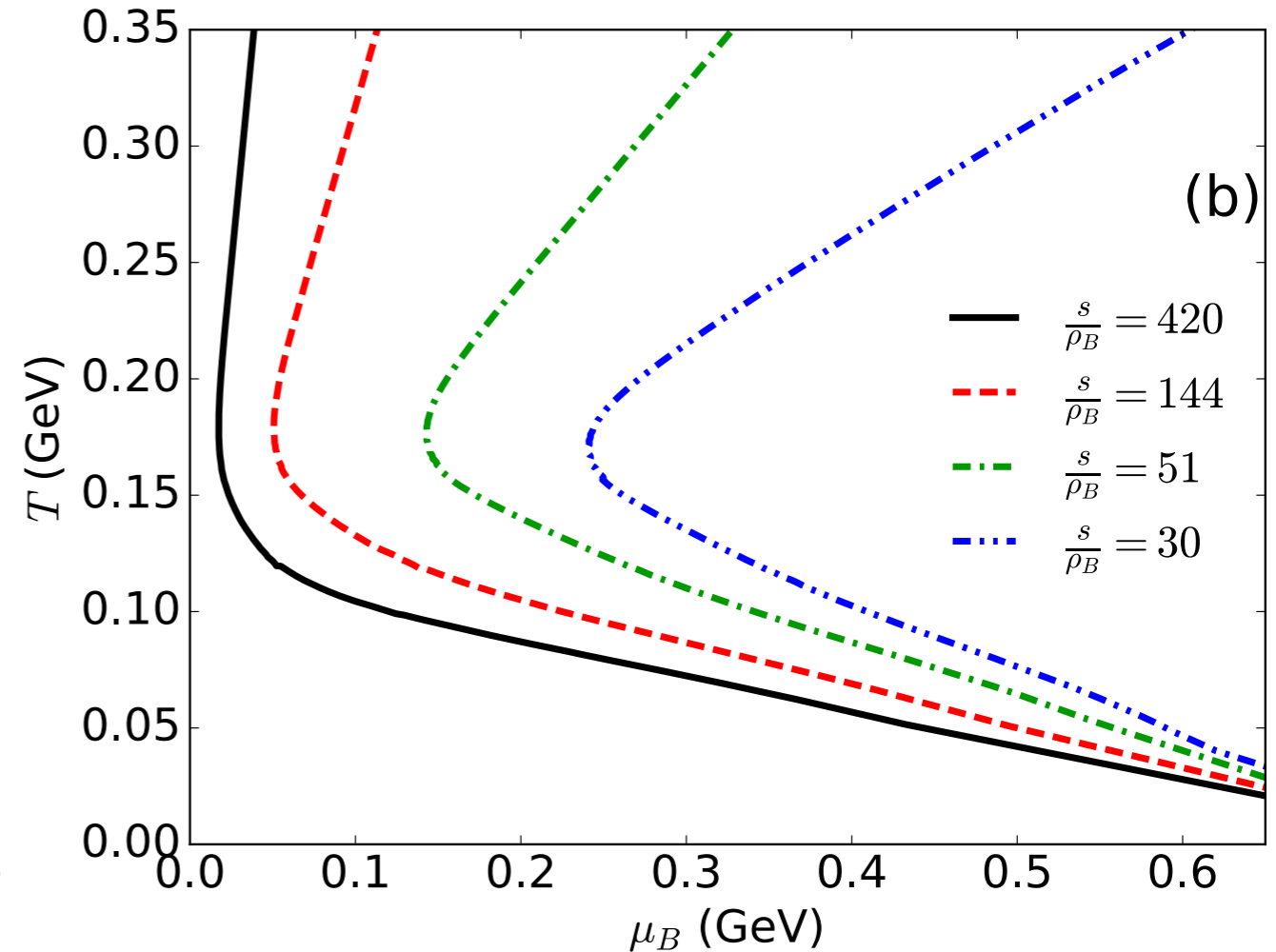
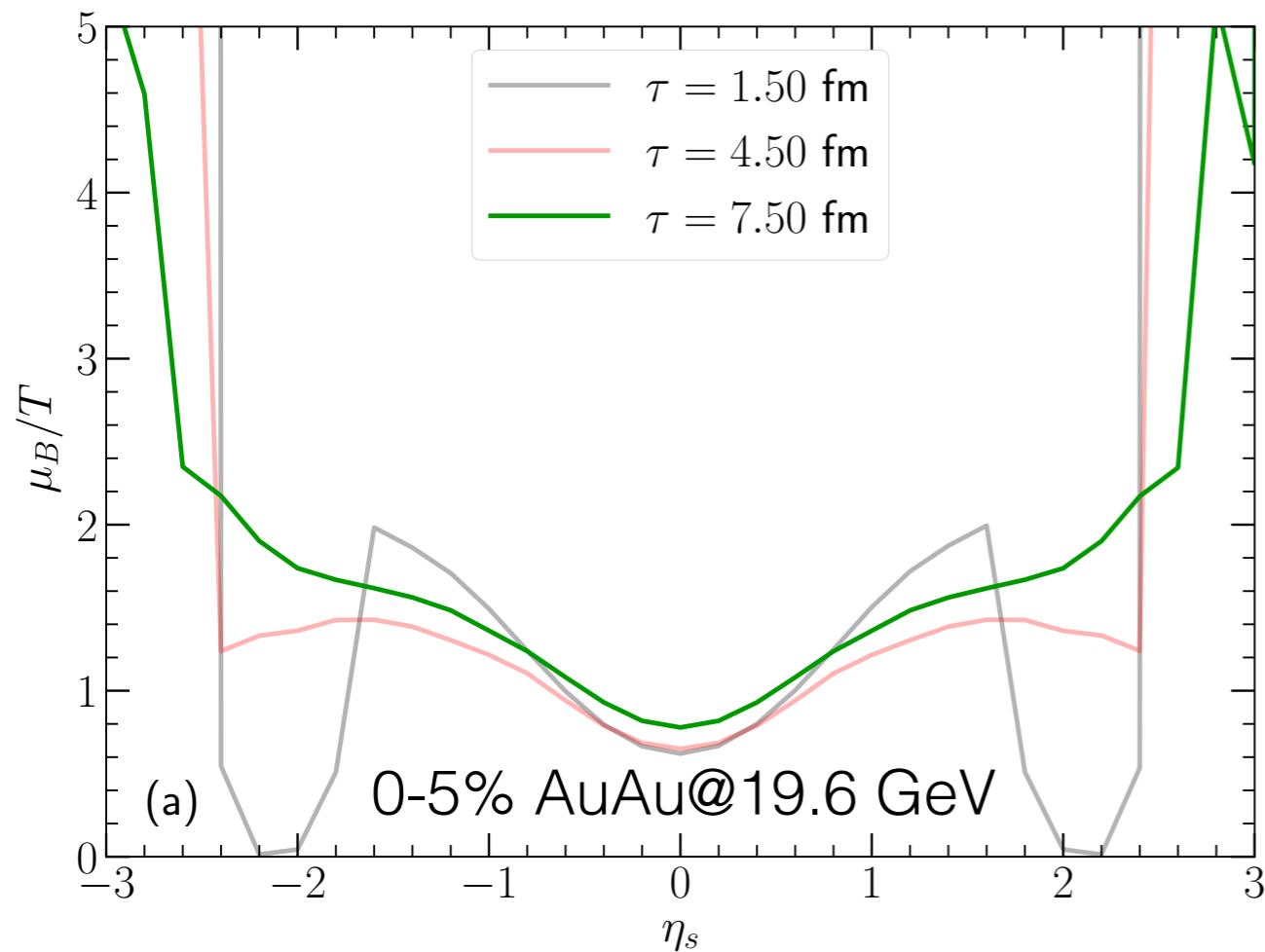
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

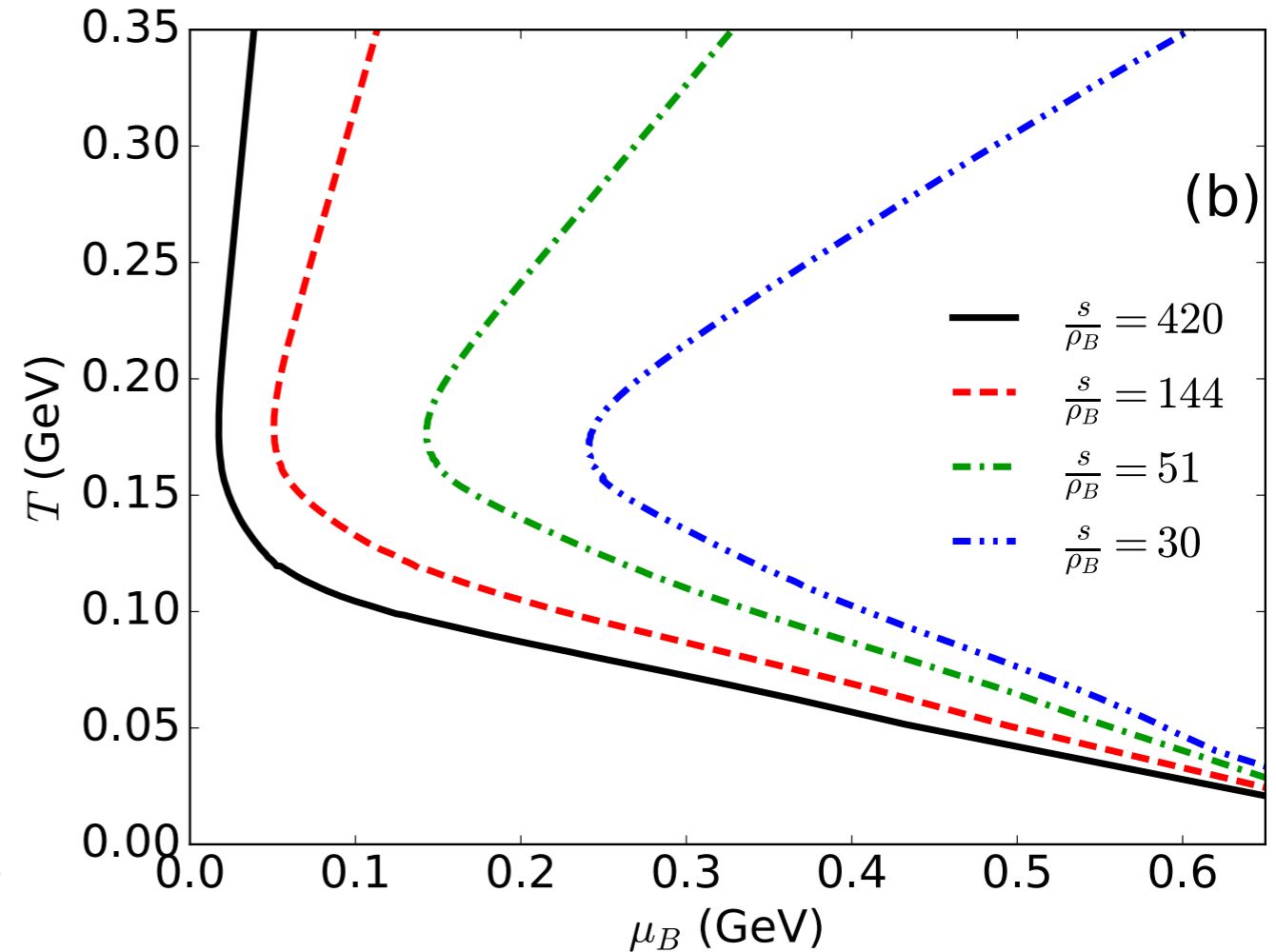
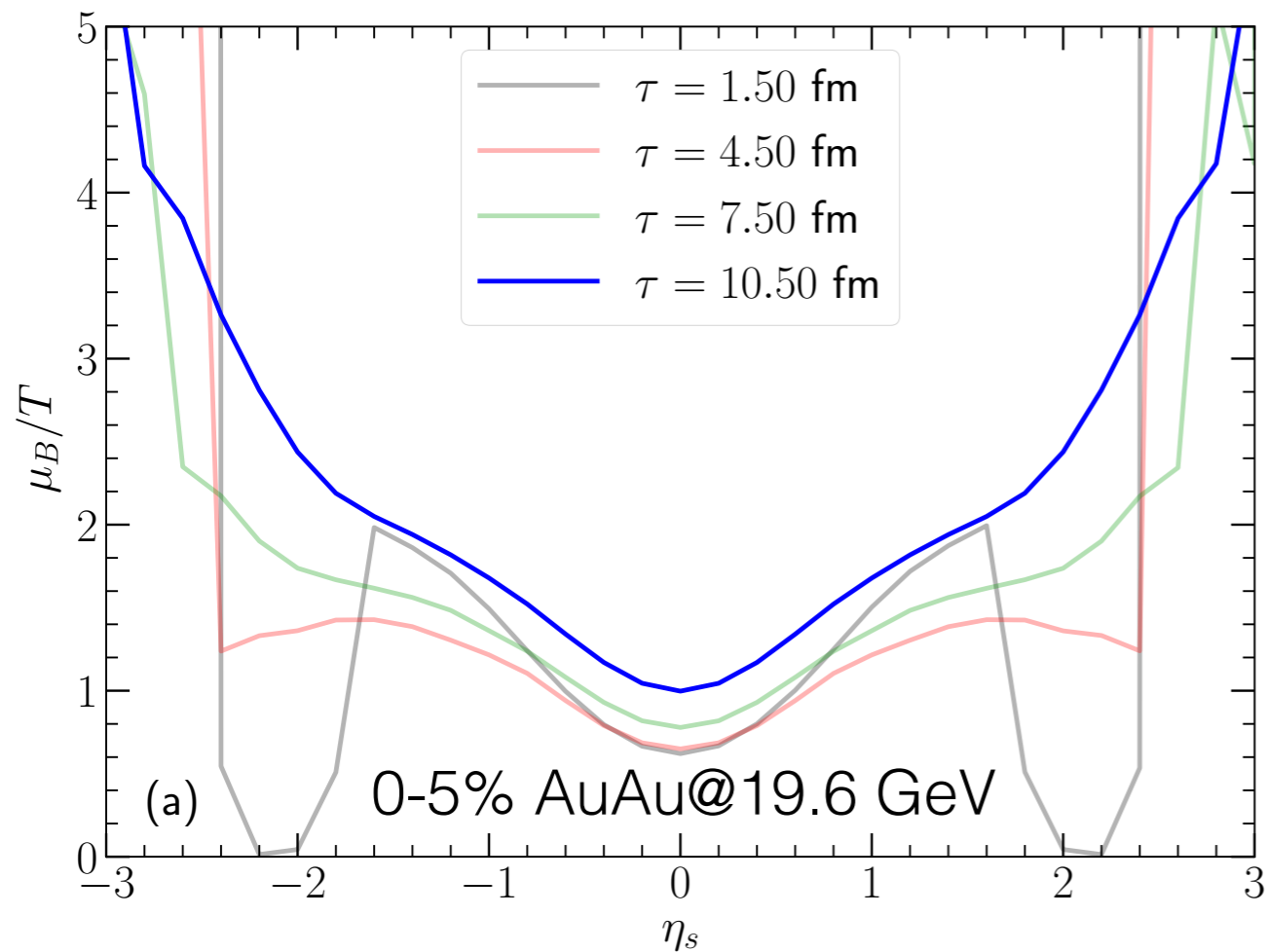
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

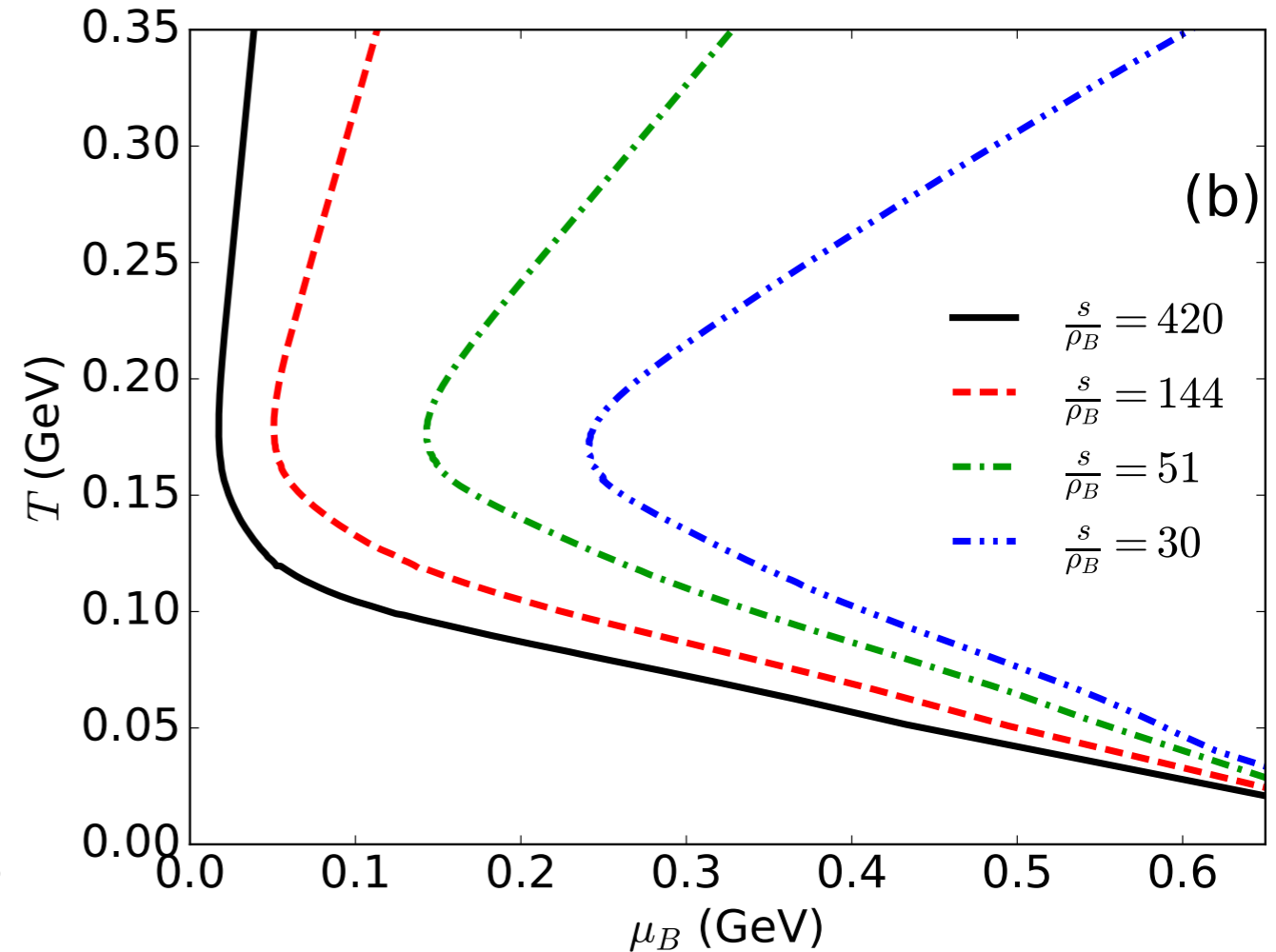
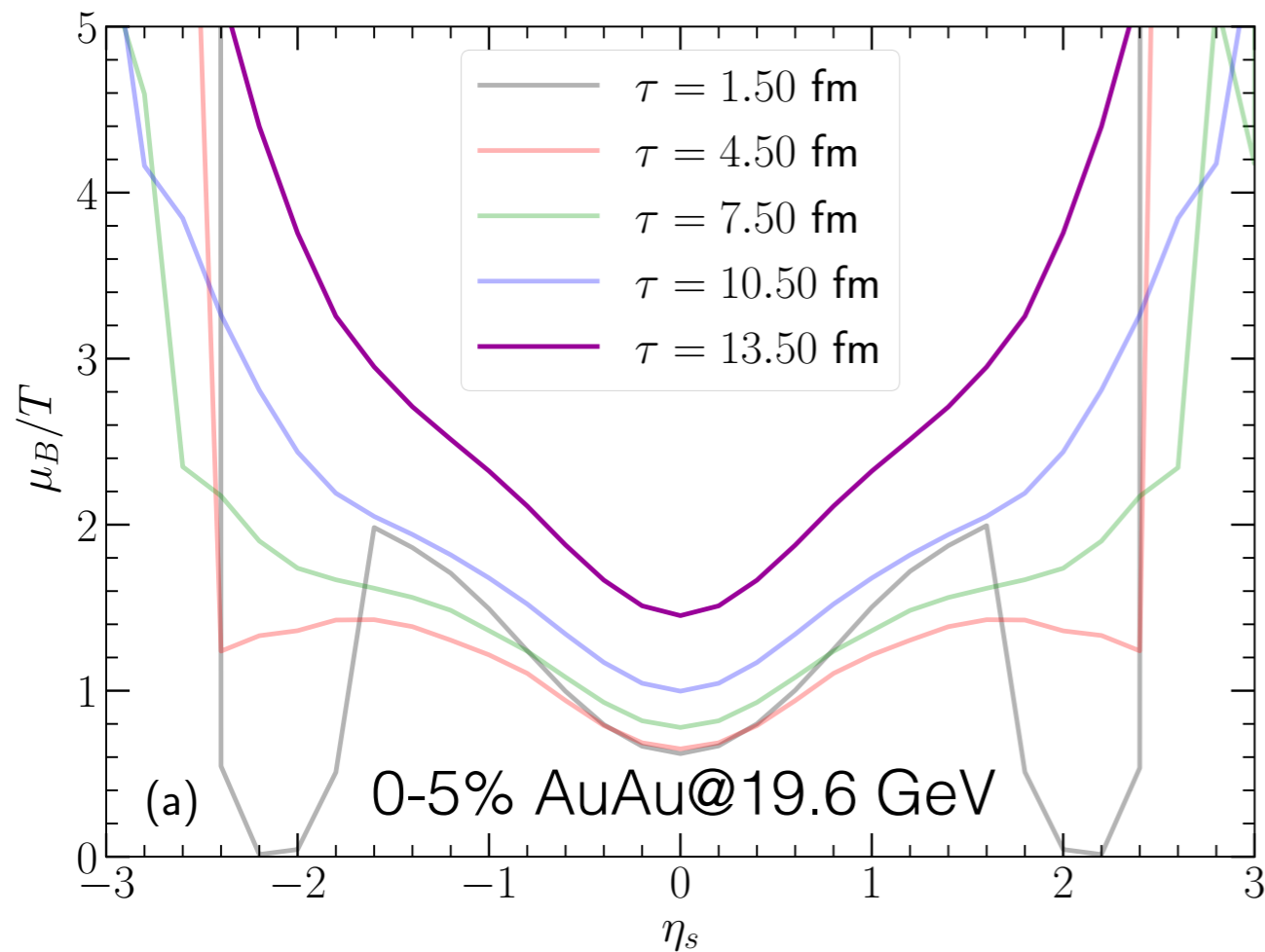
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

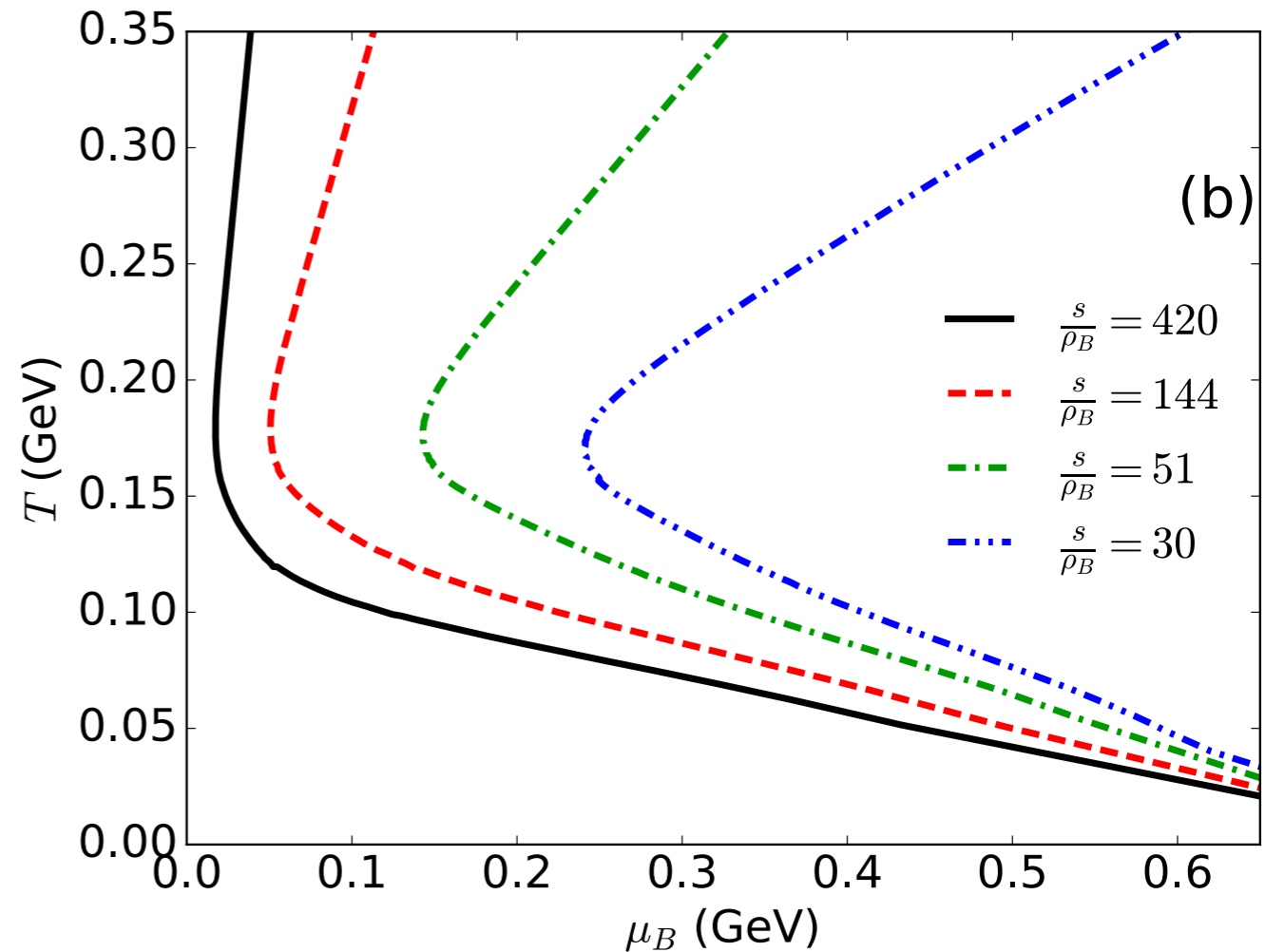
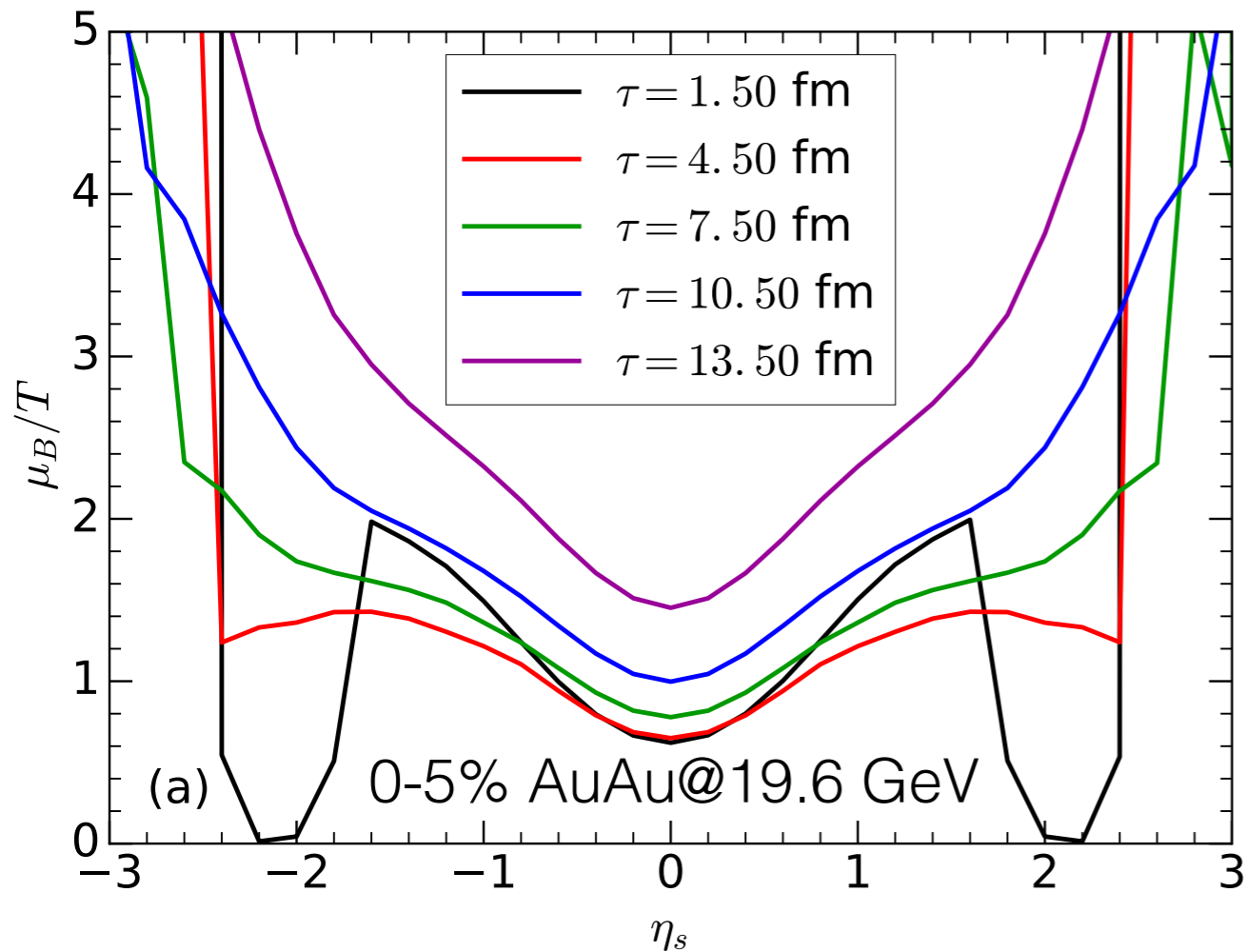
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

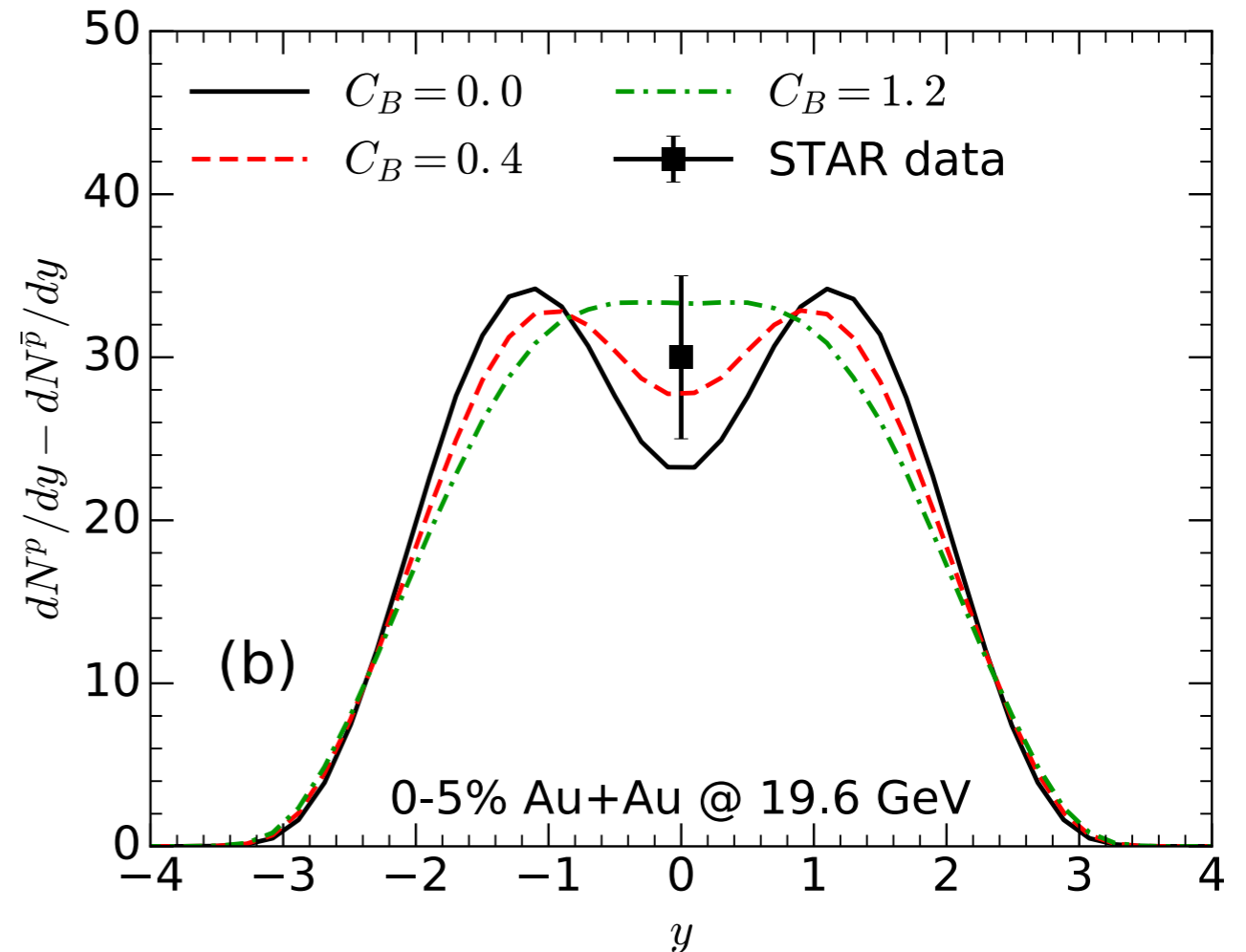
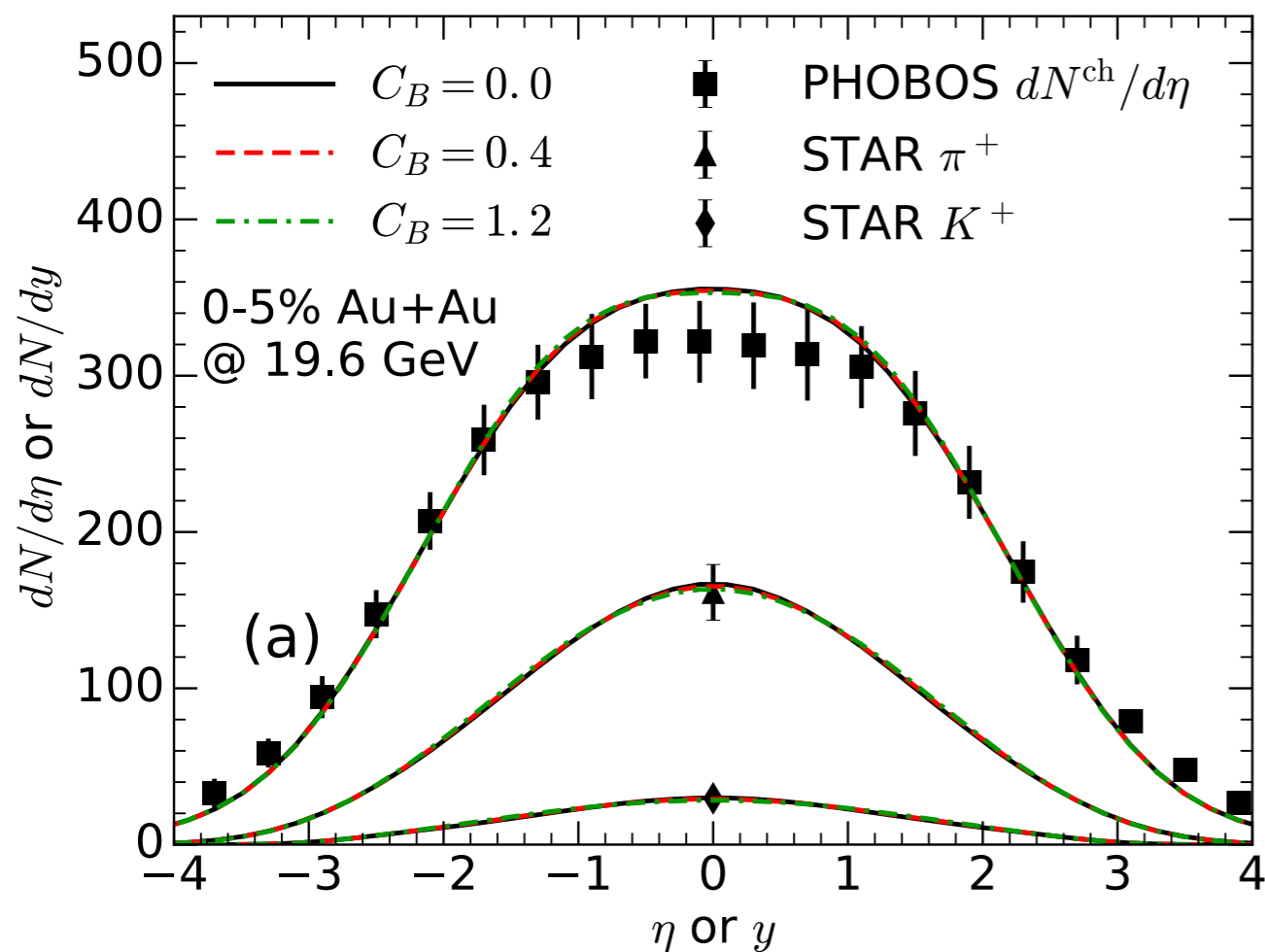
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- The value of  $\mu_B/T$  increases at low density regions
- The spatial gradients of  $\mu_B/T$  drive the net baryon diffusion current to work against the hydrodynamic radial flow

# Effects of net baryon diffusion on particle yields

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



$$\kappa_B = \frac{C_B}{T} \rho_B \left( \frac{1}{3} \coth \left( \frac{\mu_B}{T} \right) - \frac{\rho_B T}{e + P} \right)$$

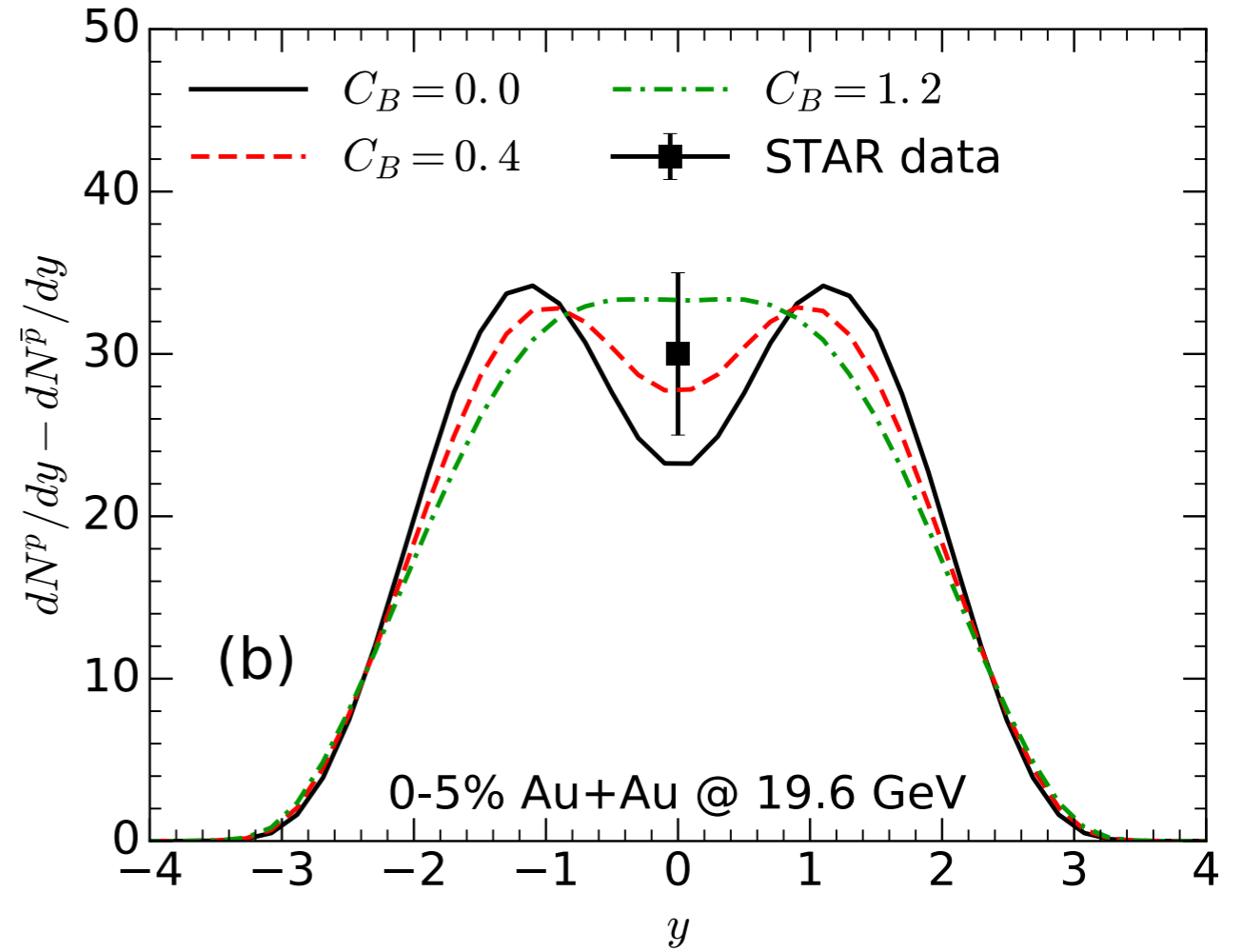
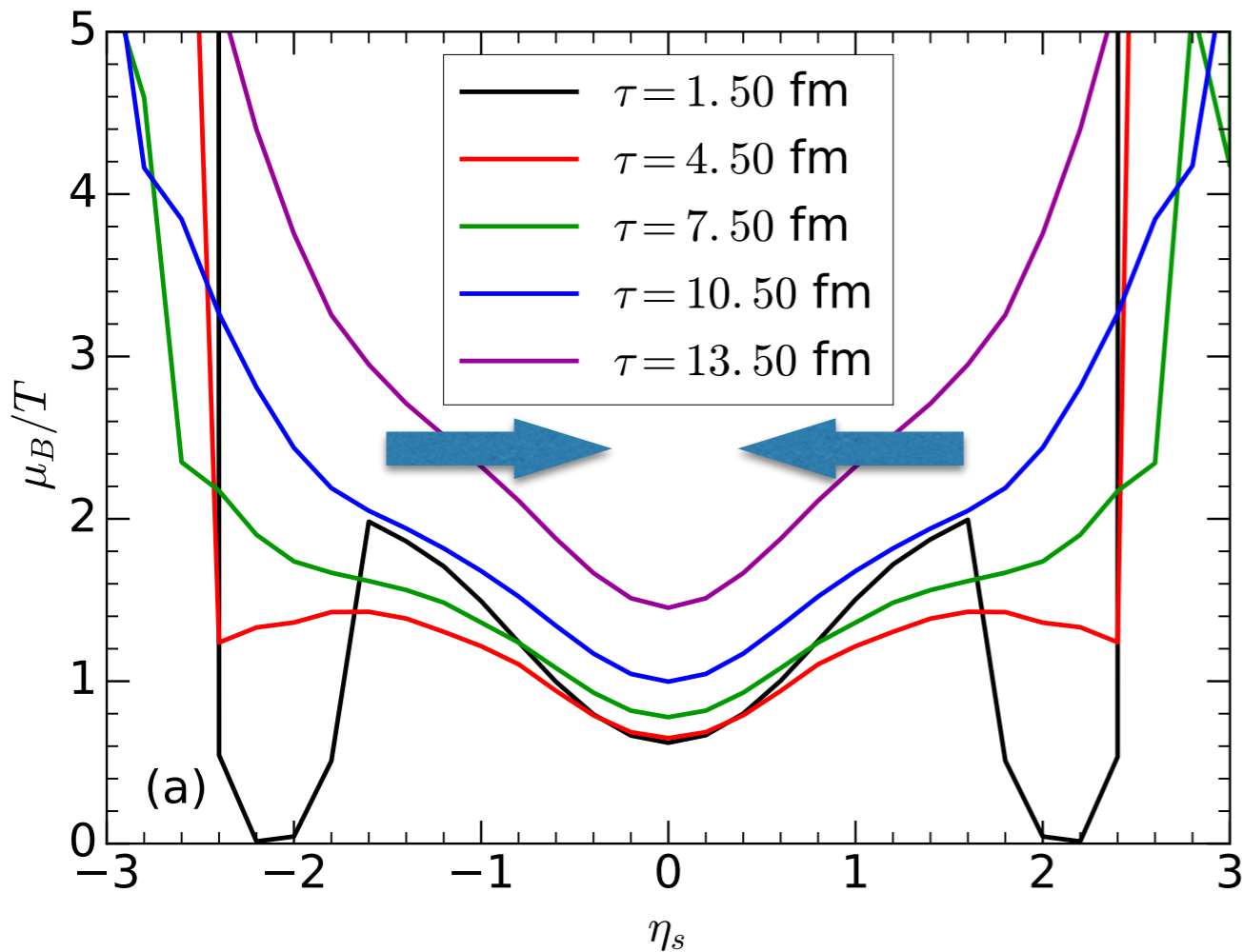
- More net baryon numbers are transported to mid-rapidity with a larger diffusion constant

## Constraints on net baryon diffusion and initial condition



# Effects of net baryon diffusion on particle yields

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



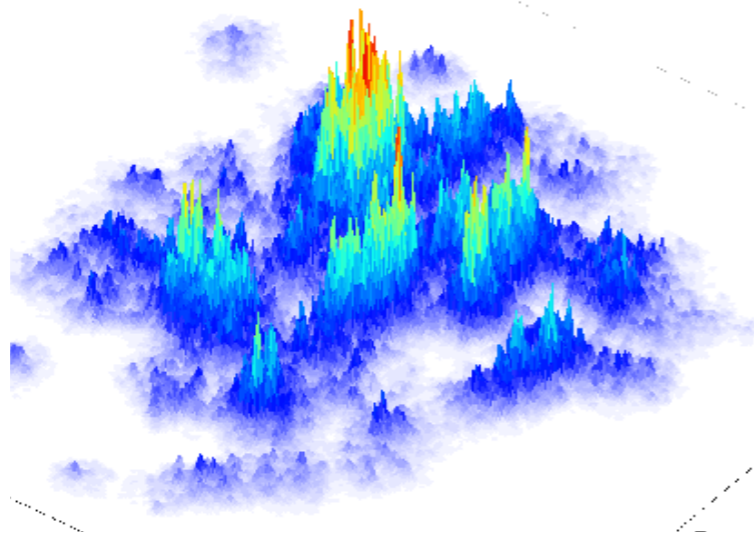
$$\kappa_B = \frac{C_B}{T} \rho_B \left( \frac{1}{3} \coth \left( \frac{\mu_B}{T} \right) - \frac{\rho_B T}{e + P} \right)$$

- More net baryon numbers are transported to mid-rapidity with a larger diffusion constant

## Constraints on net baryon diffusion and initial condition

# Conclusion

- Event-by-event viscous hydrodynamics is an effective macroscopic theory for high energy heavy-ion collisions



$$\frac{\eta}{s}, \frac{\zeta}{s}, \kappa_B, \text{ and more ...}$$

- We develop a **dynamical initialization** framework to study the early time evolution of heavy-ion collisions at the RHIC BES energies

**Baryon stopping**  
**Mapping the QCD phase diagram**