Quantitative Conclusions from Heavy-Ion Collisions

Scott Pratt, Michigan State University

MADAI Collaboration

Models and Data Analysis Initiative

http://madai.us

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY Duke UNIVERSITY THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL renci

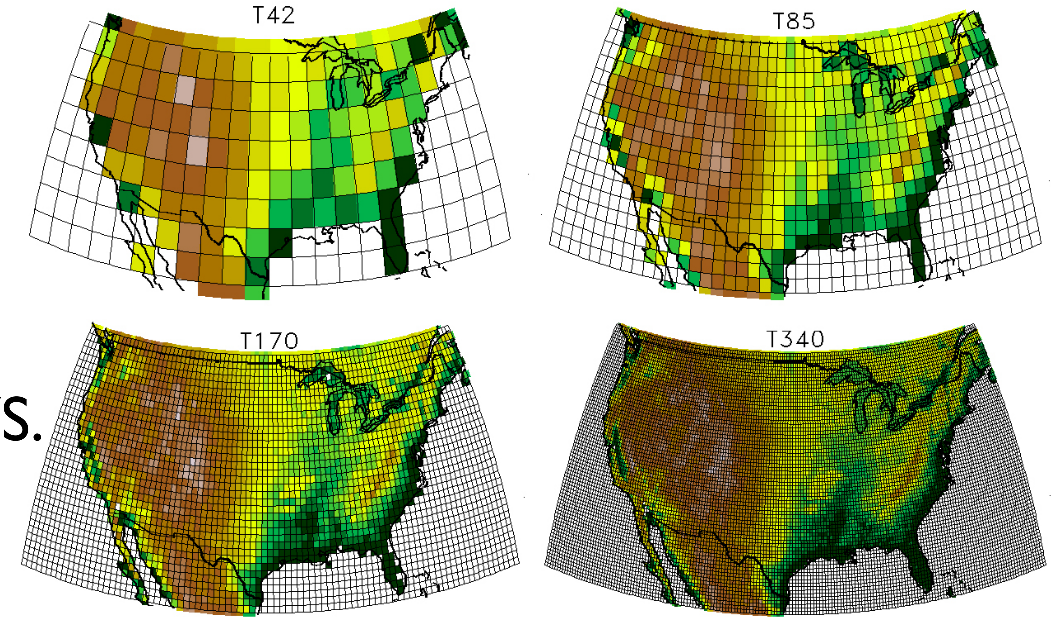
1st MADAI Collaboration Meeting, SANDIA 2010

Common Challenge

BIG Data

Large Heterogenous Data Sets

VS.

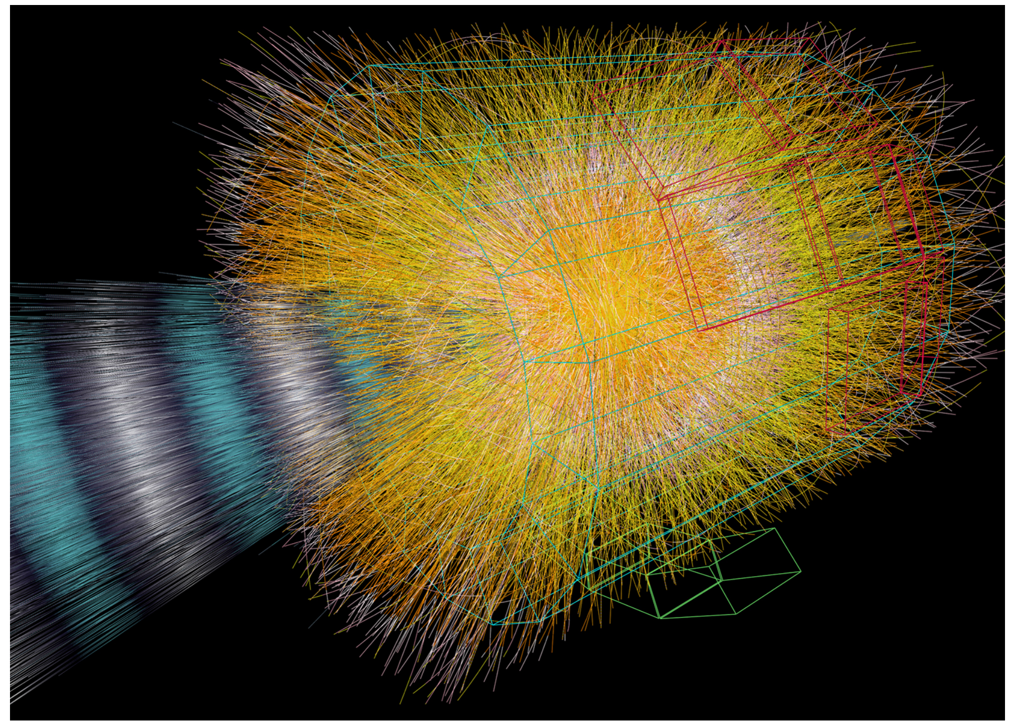


BIG Models
Many parameters
Numerically Intensive

An Example: Relativistic Heavy Ion Physics

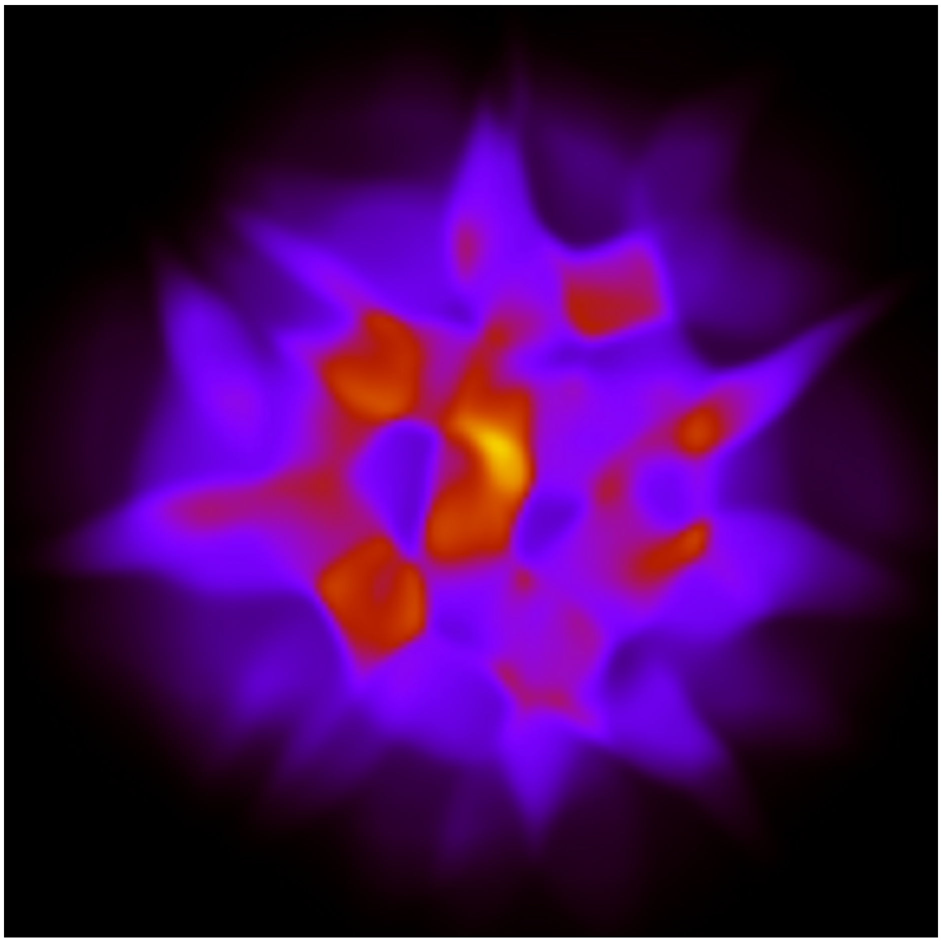
Collisions of Au&Au, Pb+Pb…
at RHIC(BNL) or LHC(CERN)

Numerous Classes of Observables



Goal: Determine properties of super-hadronic matter (**Q**uark-**G**luon **P**lasma)

An Example: Relativistic Heavy Ion Physics



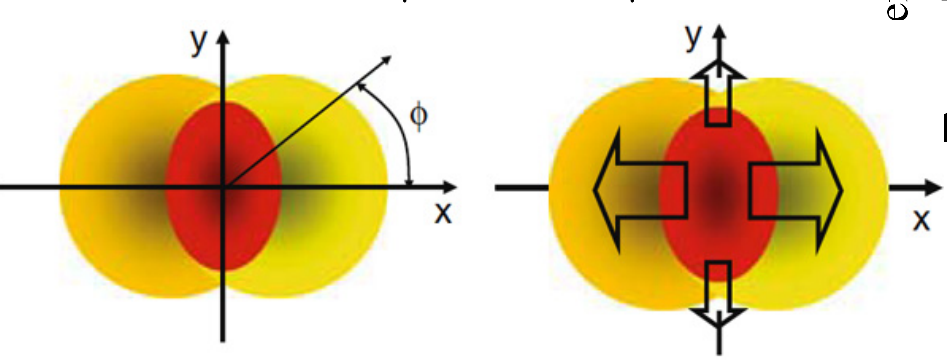
MODEL COMPONENTS

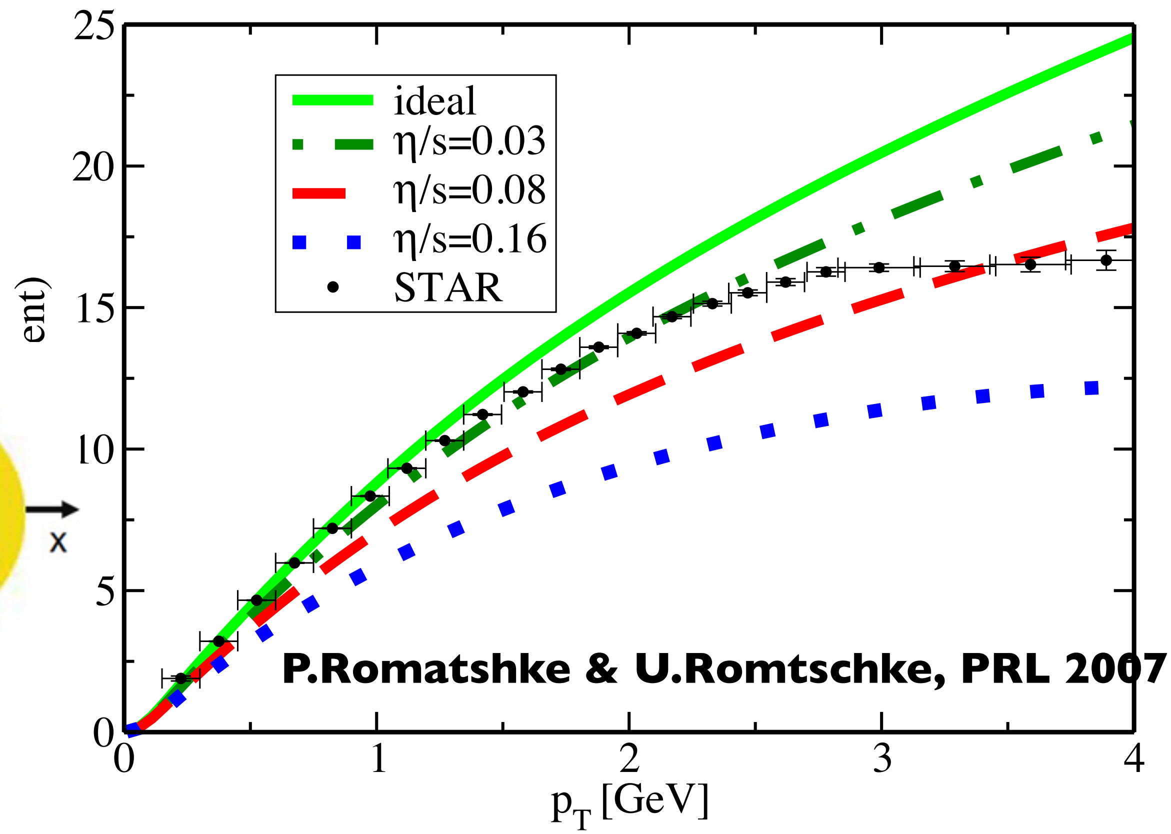
- Thermalization (First fm/c)
- Viscous Hydrodynamics (First ~5-10 fm/c)
- Hadron Simulation (Dissolution & Breakup)
- Numerous parameters (up to few dozen)
- ~Days of CPU to study one point in parameter space

How this was done before (*v*2 and η/s)

Study single parameter vs. single observable

$$v_2 \equiv \langle \cos 2\phi \rangle$$





PROBLEM

v2 depends on ....

- **viscosity**
- **saturation model**
- **pre-thermal flow**
- **Eq. of State**
- **T-dependence of η/s**
- **initial T_{xx}/T_{zz}**
- . . . .

e.g. Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

Correct Way (MCMC)

- Simultaneously vary *N* model parameters $\boldsymbol{x_i}$
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})})^2}{2\sigma_a^2}\right\}$$

- Use all observables $\boldsymbol{y_a}$
- Obtain representative sample of posterior

Difficult Because...

I. Too Many Model Runs
Requires running model ~10^6 times

II. Many Observables
Could be hundreds of plots,
each with dozens of points
Complicated Error Matrices

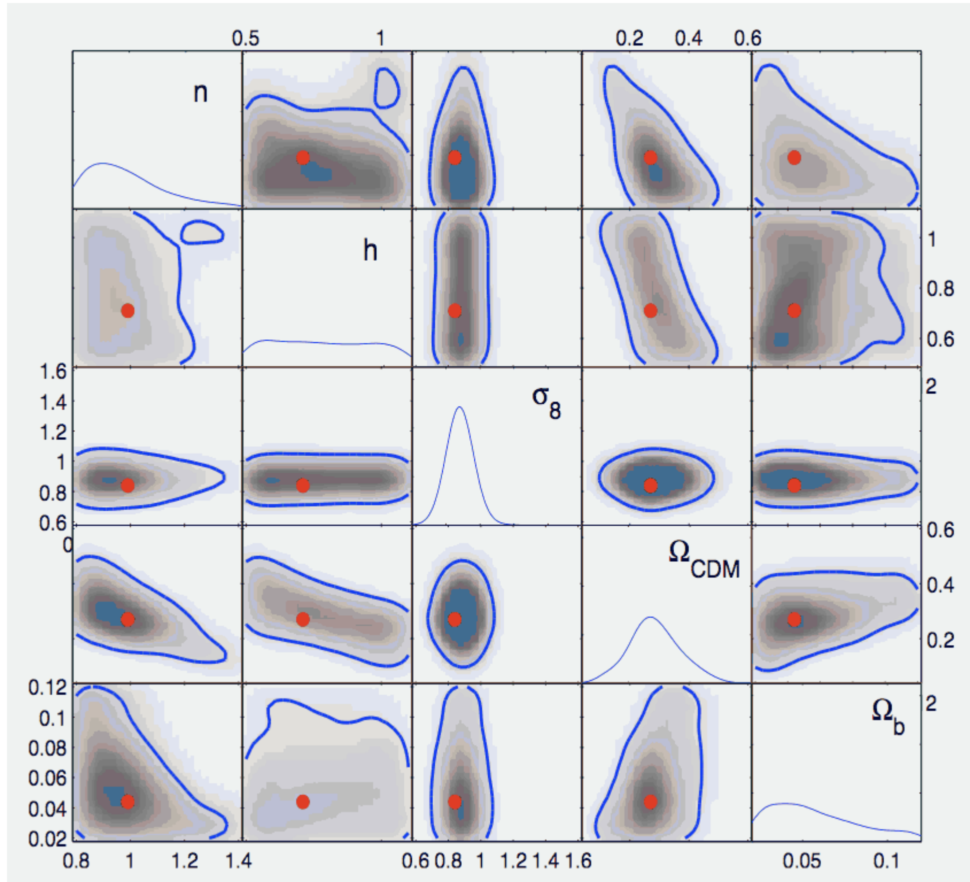
Model Emulators

1. **Run the model ~1000 times**
   Semi-random points (LHS sampling)

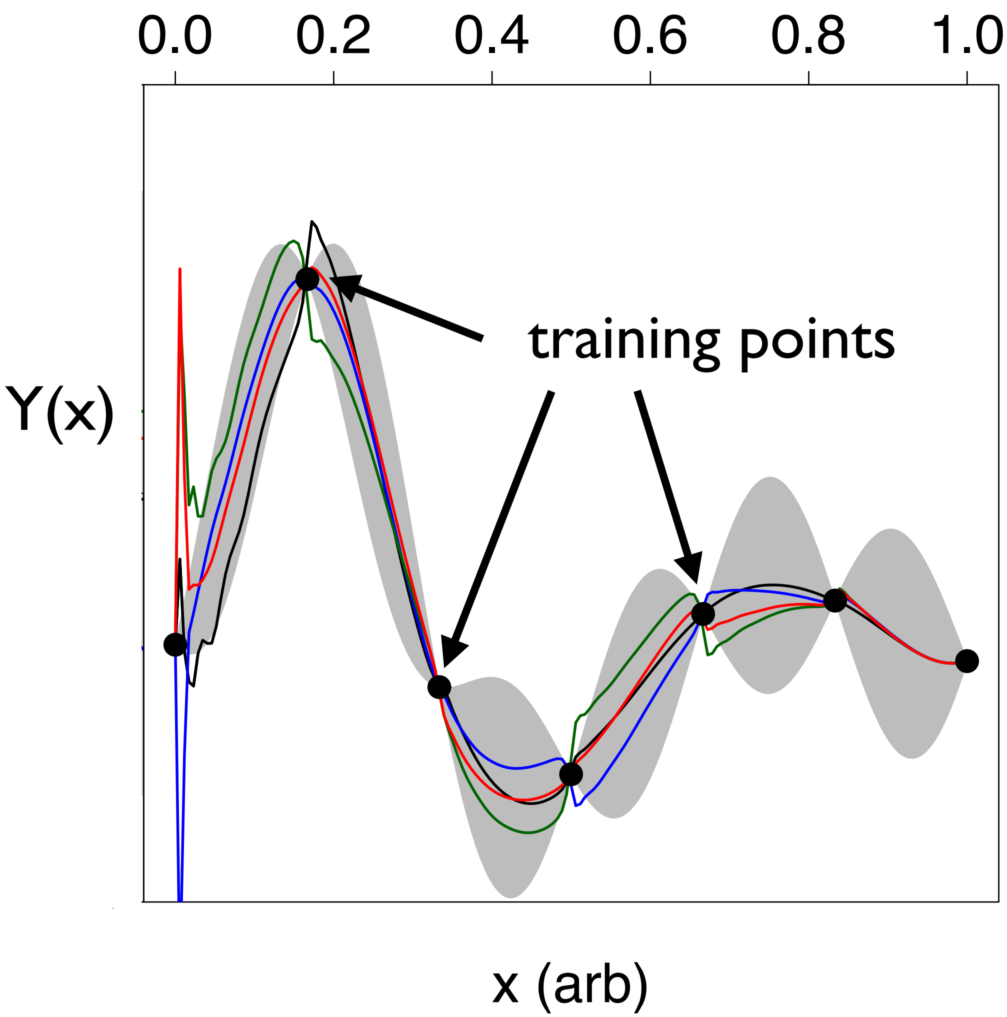
2. **Determine Principal Components**
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

3. **Emulate z_a (Interpolate) for MCMC**
   Gaussian Process…

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\frac{1}{2}\sum_a (z_a^{(\mathrm{emulator})}(\mathbf{x}) - z_a^{(\mathrm{exp})})^2\right\}$$



S. Habib, K. Heitman, D. Higdon, C. Nakhleh & B. Williams, PRD (2007)



Emulator

- **Gaussian Process**
  - **Reproduces training points**
  - **Assumes localized Gaussian covariance**
  - **Must be trained, i.e. find "hyper parameters"**
- **Other methods also work**

14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- π, K, p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Size
- v_2 Weighted by p_t

Initial State Parameters

$$\epsilon(\tau = 0.8\mathrm{fm}/c) = f_{\mathrm{wn}}\epsilon_{\mathrm{wn}} + (1 - f_{\mathrm{wn}})\epsilon_{\mathrm{cgc}},$$

$$\epsilon_{\mathrm{wn}} = \epsilon_0 T_A \frac{\sigma_{\mathrm{nn}}}{2\sigma_{\mathrm{sat}}}\left\{1 - \exp\left(-\sigma_{\mathrm{sat}} T_B\right)\right\} + (A \leftrightarrow B)$$

$$\epsilon_{\mathrm{cgc}} = \epsilon_0 T_{\mathrm{min}} \frac{\sigma_{\mathrm{nn}}}{\sigma_{\mathrm{sat}}}\left\{1 - \exp\left(-\sigma_{\mathrm{sat}} T_{\mathrm{max}}\right)\right\}$$

$$T_{\mathrm{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\mathrm{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

Equation of State and

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

2 parameters for EoS, 2 for η/s

DATA Distillation

1. **Experiments reduce PBs to 100s of plots**

2. **Choose which data to analyze**
   Does physics *factorize*?

3. **Reduce plots to a few representative numbers, y_a**

4. **Transform to principal components, z_a**

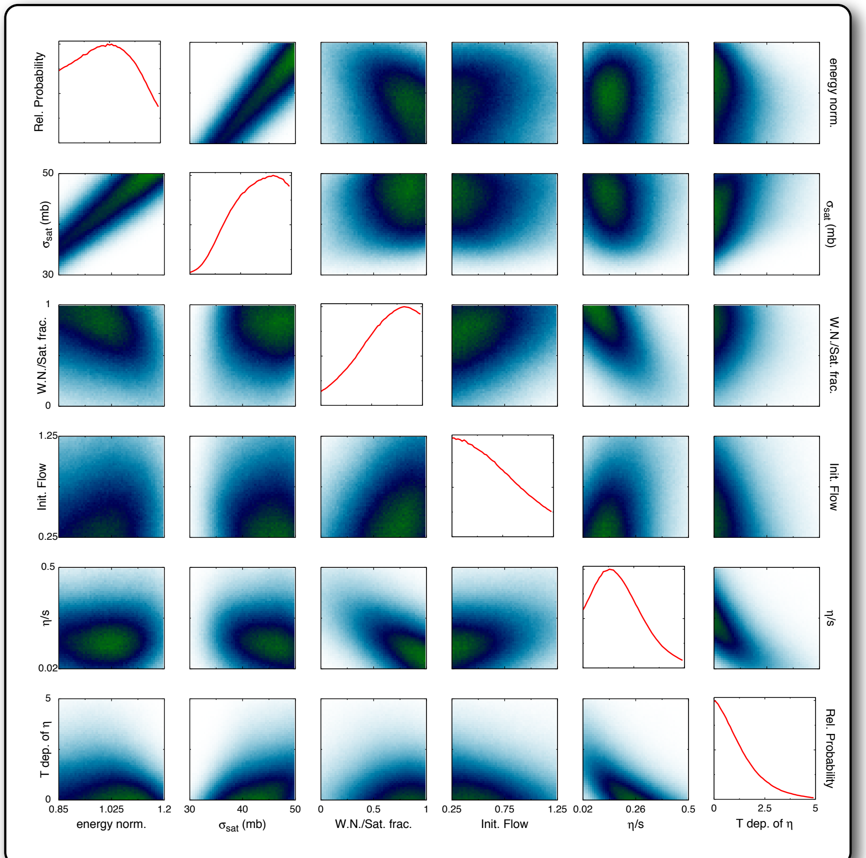
$$\mathcal{L} \sim \exp\left\{ \frac{-1}{2} \sum_a (z_a - z_a^{(\mathrm{exp})})^2 \right\}$$

5. **Resolving power of RHIC/LHC data reduced to $\lesssim$10 numbers!**

Two Calculations

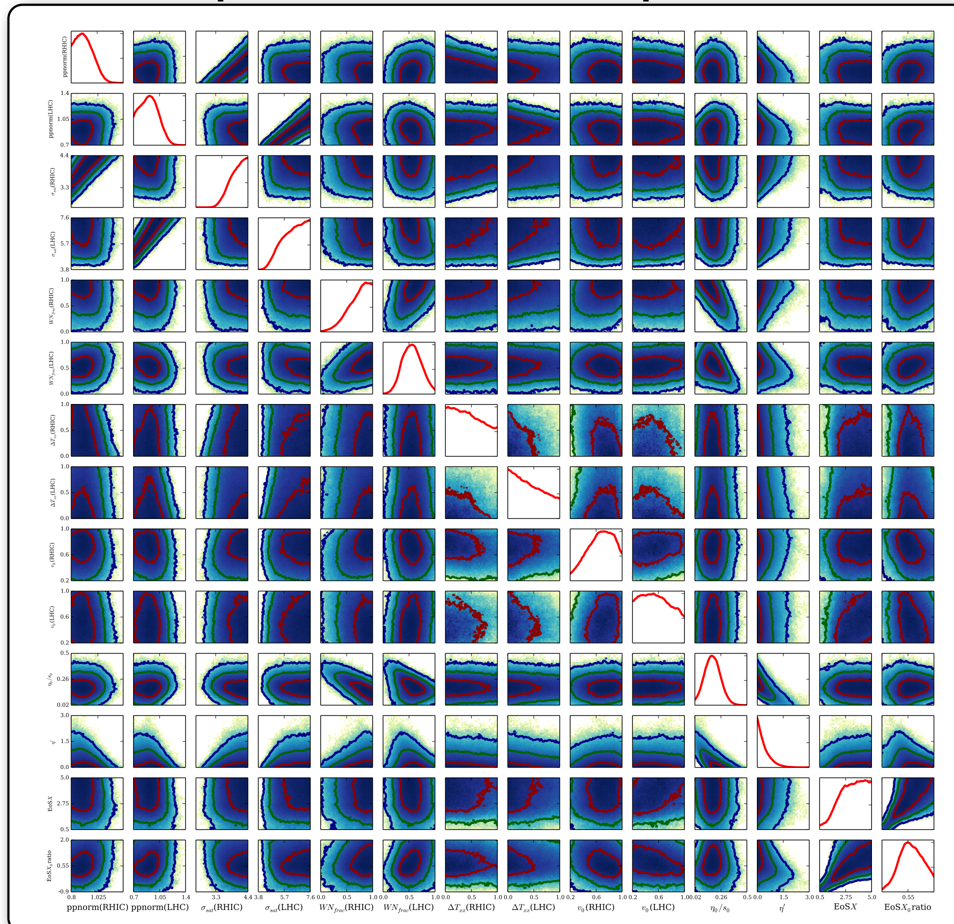
J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, PRC 2014

RHIC Au+Au Data
6 parameters

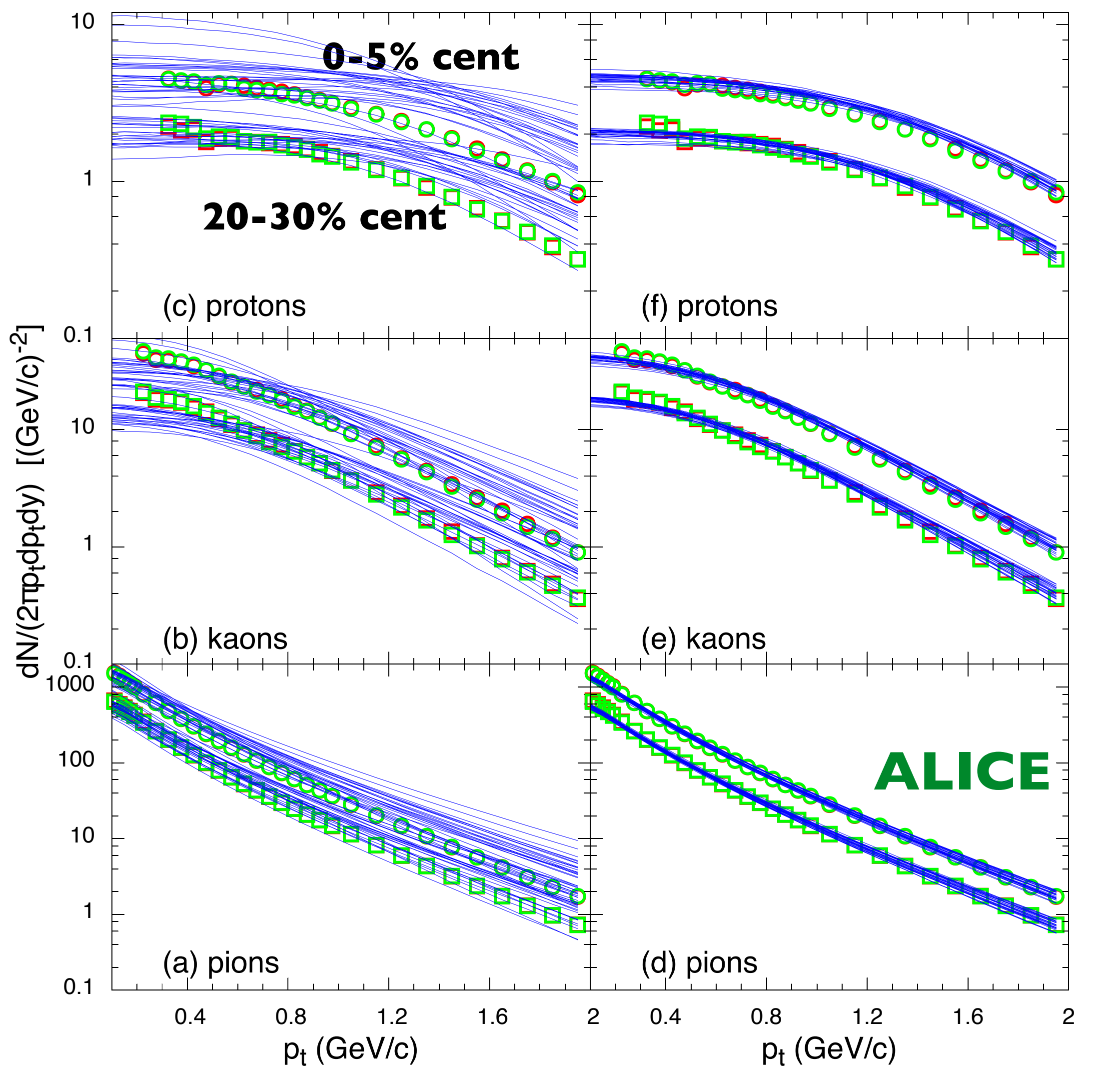


S.P., E.Sangaline, P.Sorensen & H.Wang, PRL 2015

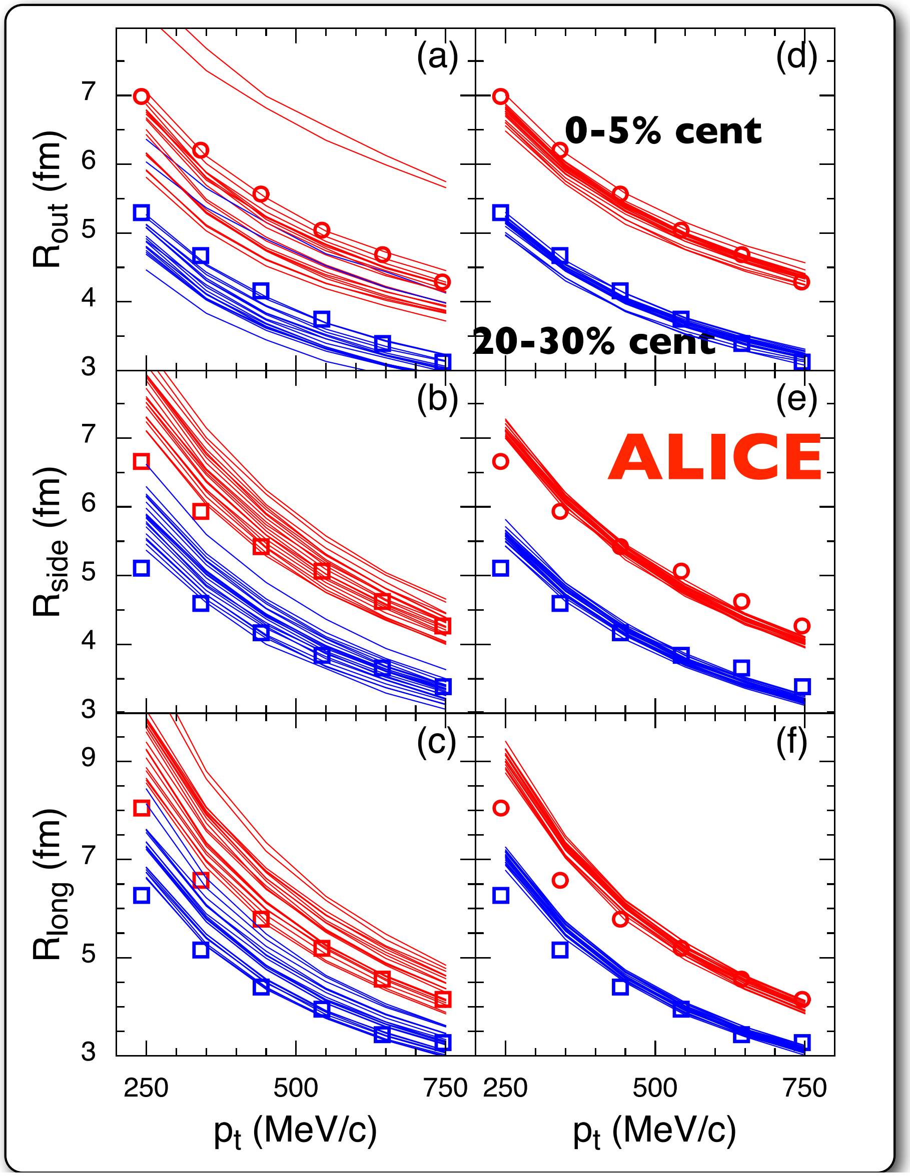
RHIC Au+Au and LHC Pb+Pb Data
14 parameters, include Eq. of State



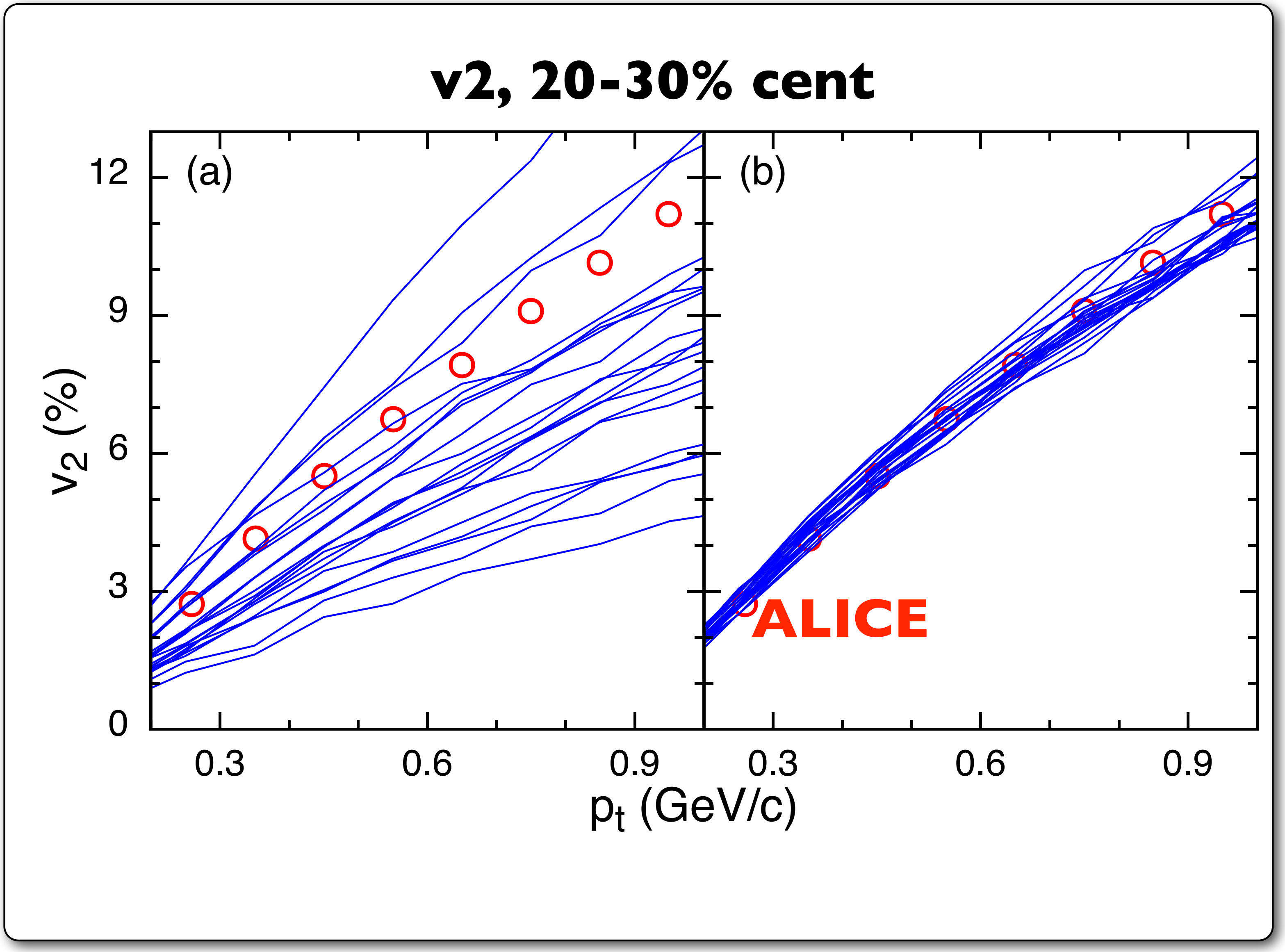
Sample Spectra from Prior and Posterior



Sample HBT from Prior and Posterior



Sample V2 from Prior and Posterior

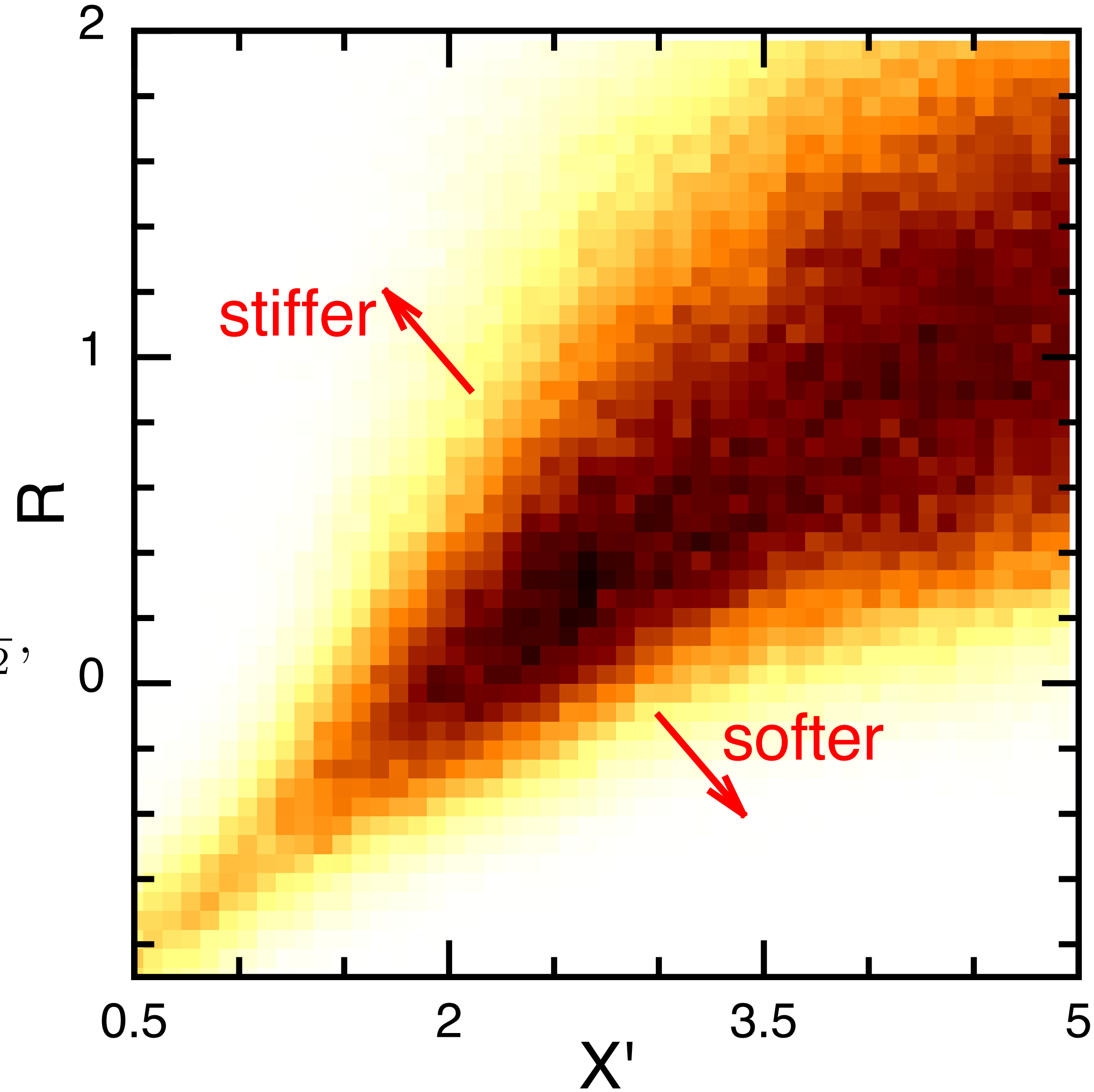


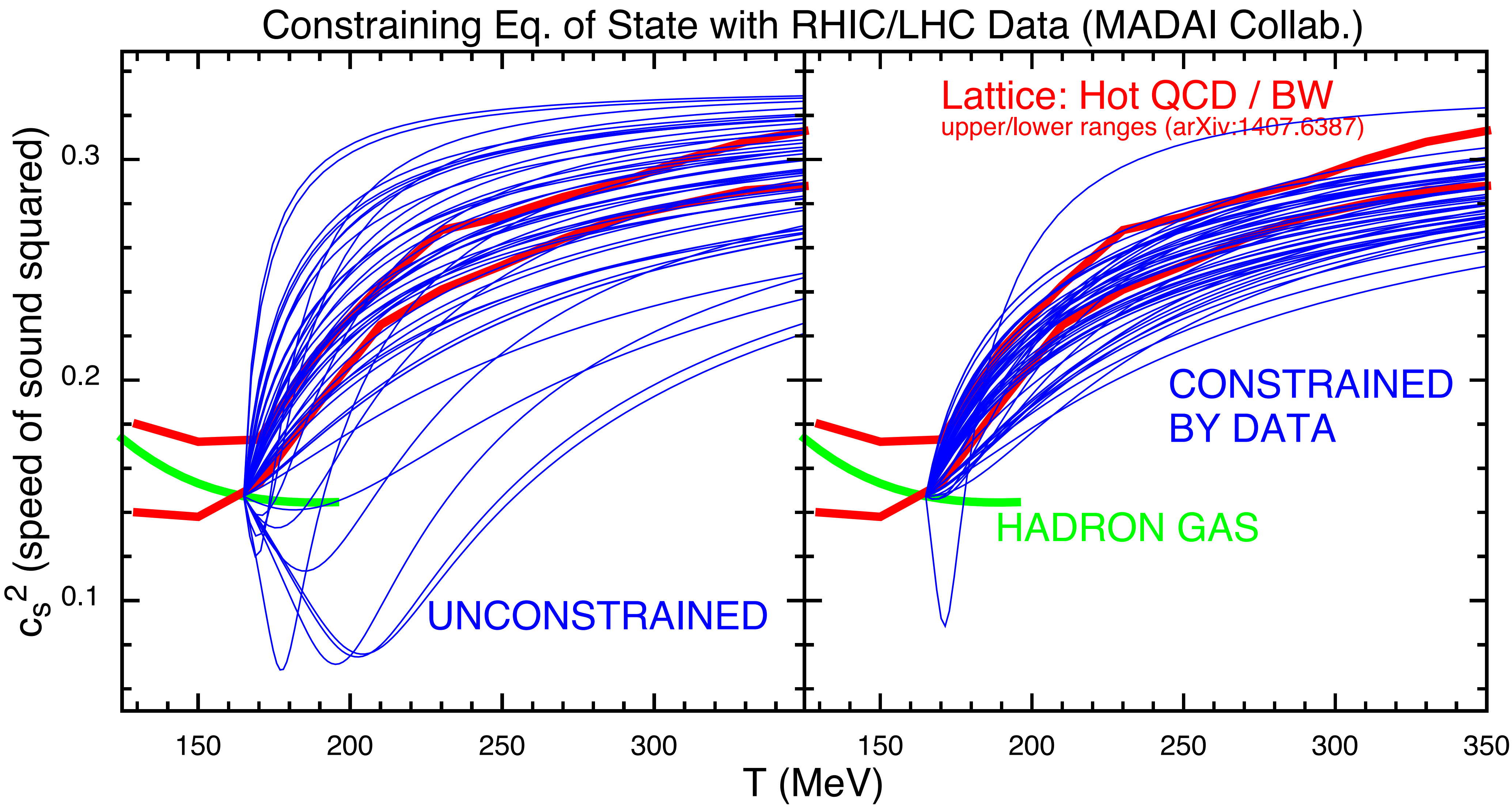
Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon) \sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

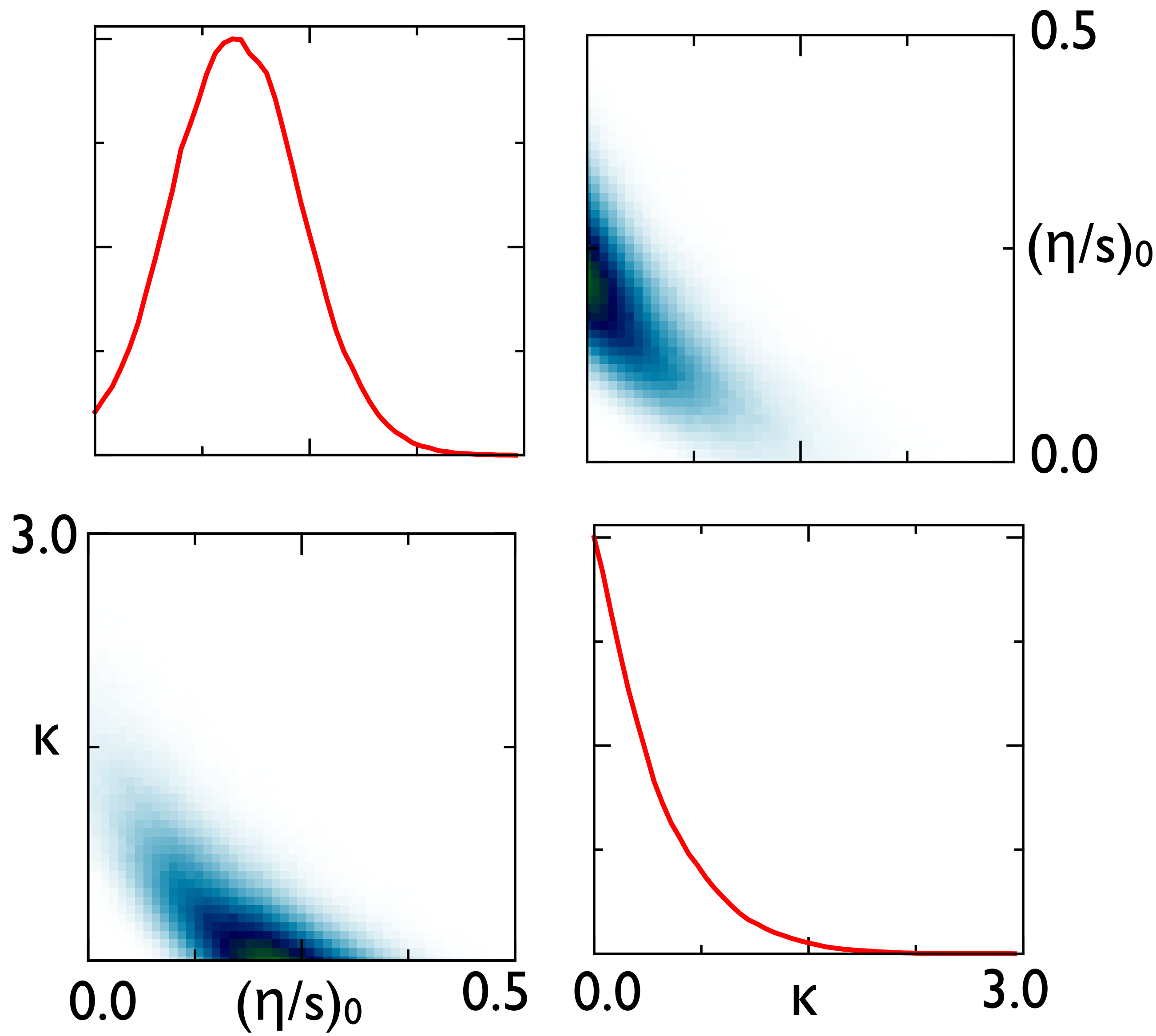




Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
UNCONSTRAINED
CONSTRAINED
BY DATA
HADRON GAS
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

η/s(T)

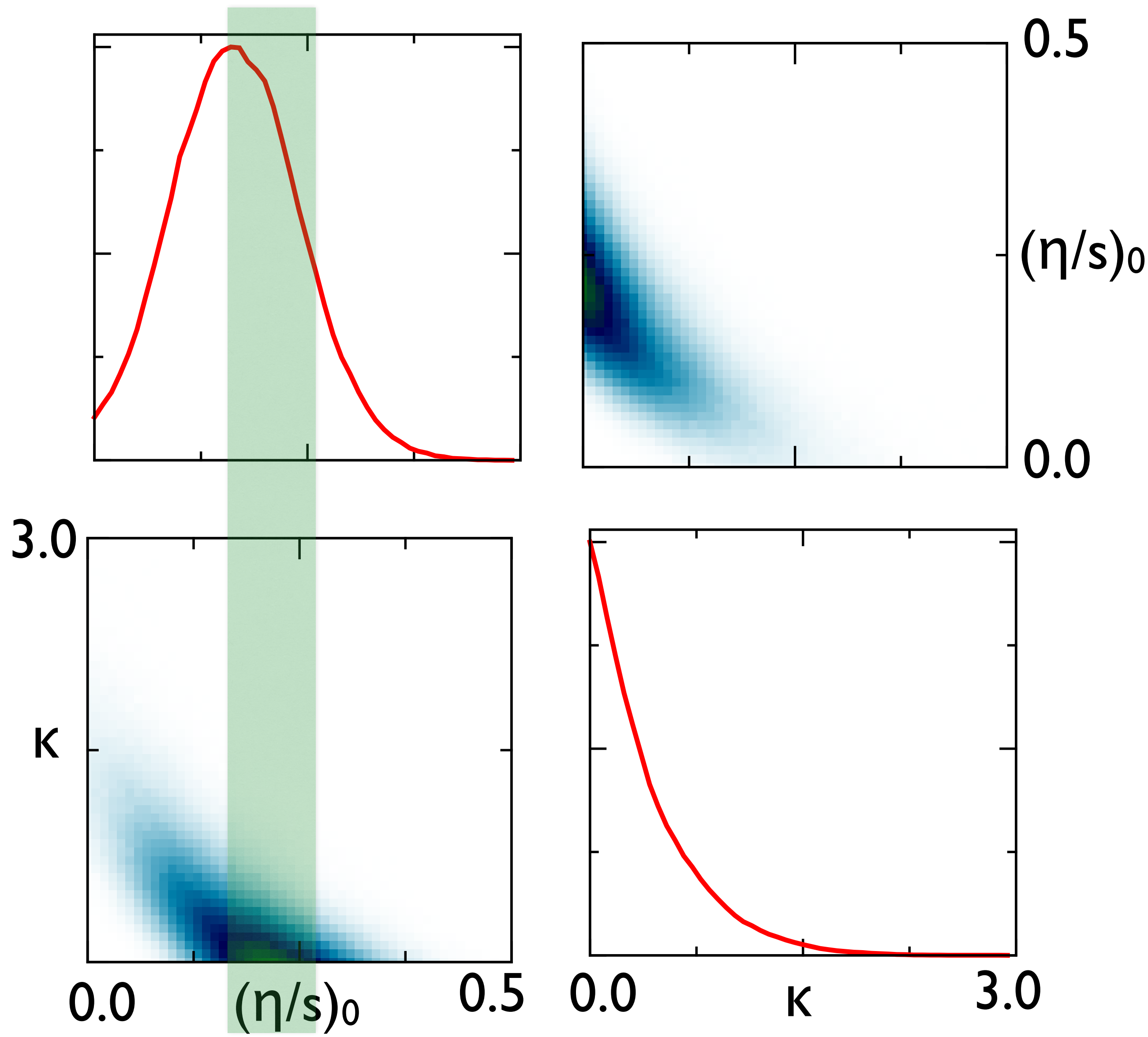
$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$



What should you expect for η/s at T=165 MeV?

- ADS/CFT: 0.08
- Perturbative QCD: > 0.5 ($\sigma \approx$ 3 mb)
- Hadron Gas: $\approx$ 0.2 ($\sigma \approx$ 30 mb)

Extracted η/s at T=165 MeV consistent with expectations for hadron gas!



How does changing $y_{a,\mathrm{exp}}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

We need

$$\frac{\partial}{\partial y_a^{(\mathrm{exp})}} \langle\langle x_i \rangle\rangle$$

NOT

$$\frac{\partial}{\partial x_i} y_a^{(\mathrm{mod})}$$

E.Sangaline and S.P., arXiv 2015

How does changing $y_{a,exp}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

$$\langle\langle x_i \rangle\rangle = \frac{\langle x_i \mathcal{L} \rangle}{\langle \mathcal{L} \rangle}$$

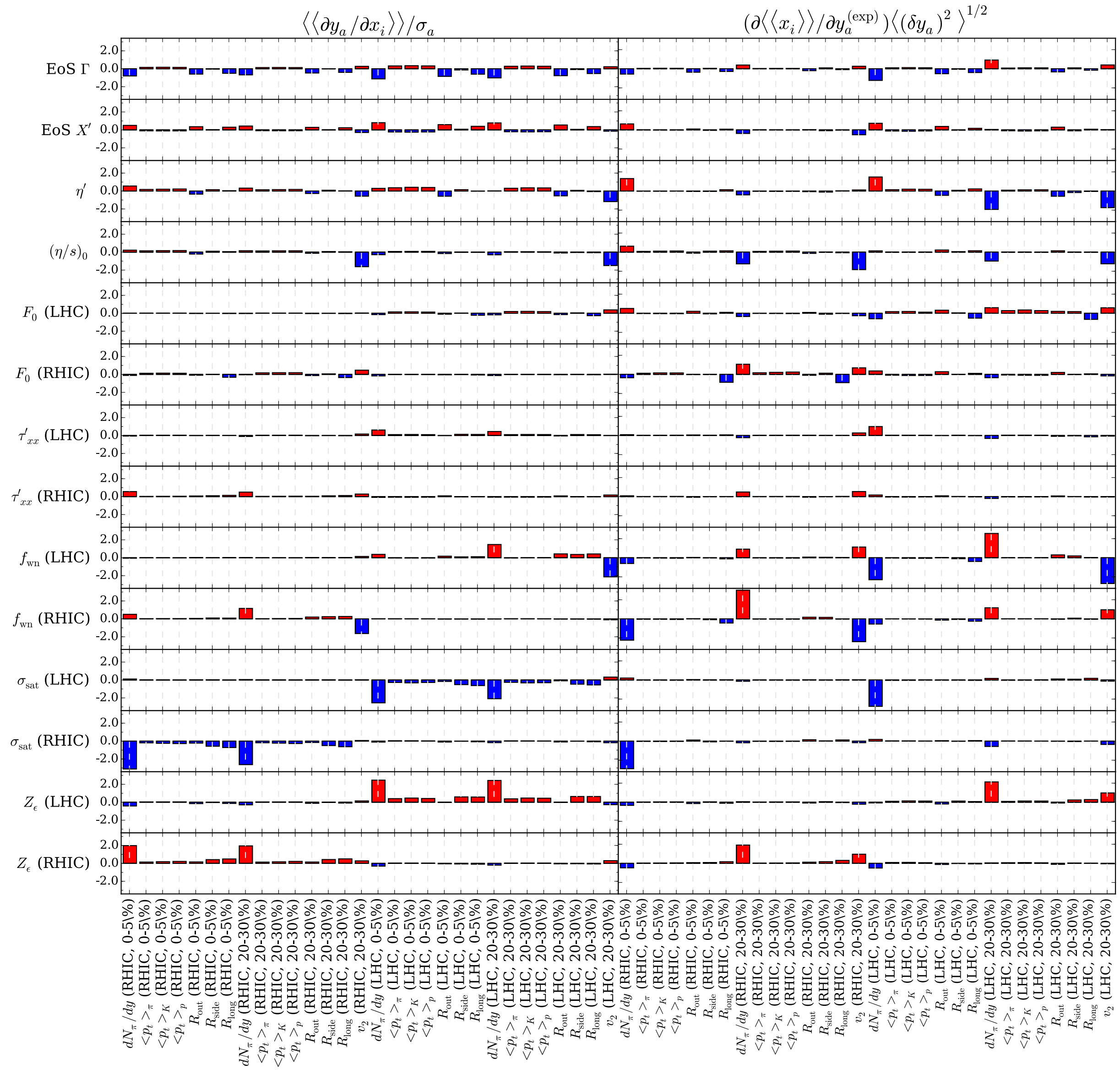
$$\begin{aligned}
\frac{\partial}{\partial y_a^{(\mathrm{exp})}} \langle\langle x_i \rangle\rangle &= \langle\langle x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle - \langle\langle x_i \rangle\rangle \langle\langle (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle \\
&= \langle\langle \delta x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle \\
&= -\Sigma_{ab}^{-1} \langle\langle \delta x_i \delta y_b \rangle\rangle \qquad \text{(for Gaussian)}
\end{aligned}$$

$$\delta x_i = x_i - \langle\langle x_i \rangle\rangle, \quad \delta y_a = y_a - y_a^{(\mathrm{exp})}$$

can find similar relation for $\frac{\partial}{\partial \sigma_a} \langle\langle \delta x_i \delta x_j \rangle\rangle$

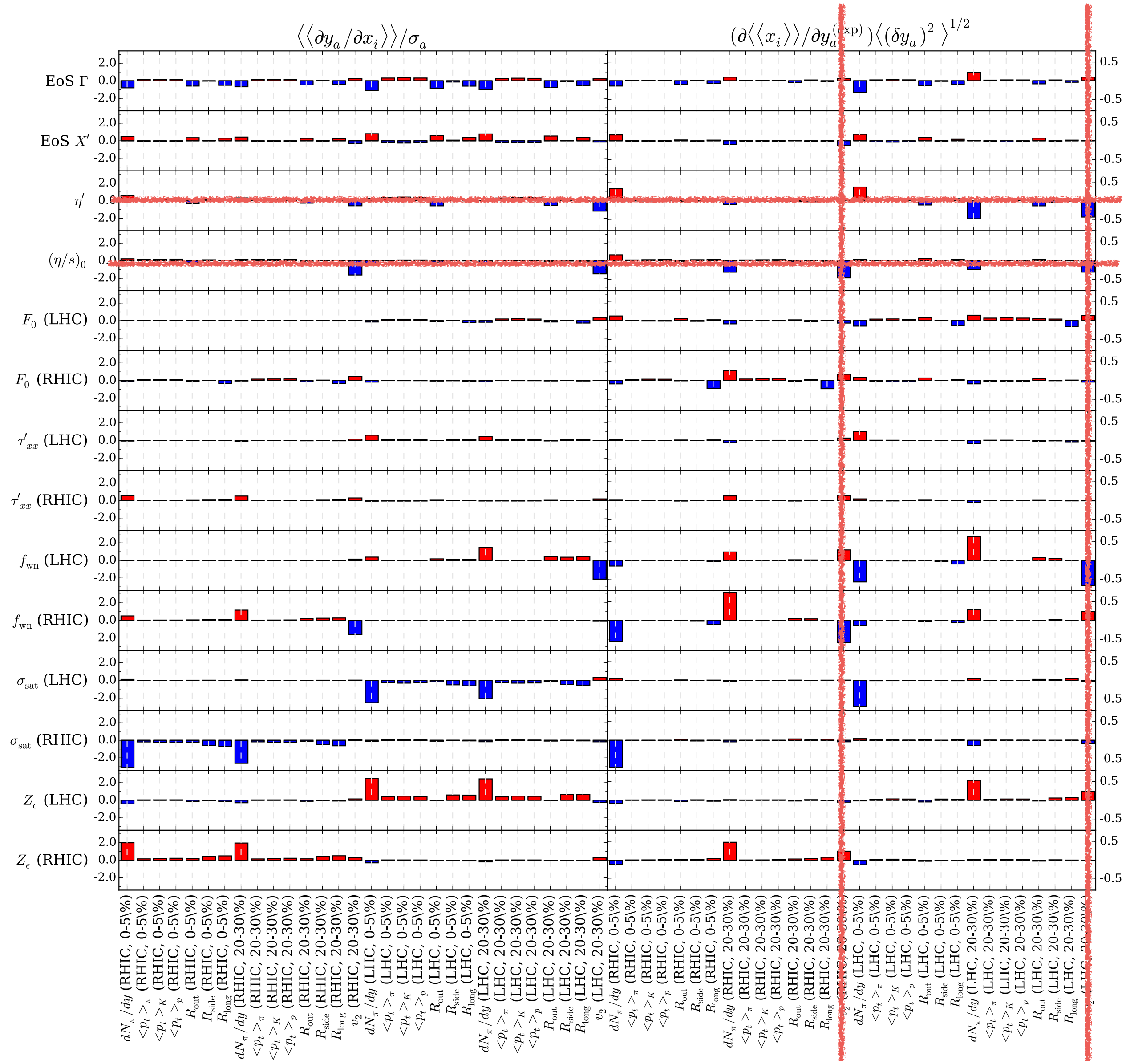
E.Sangaline and S.P., arXiv 2015

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$

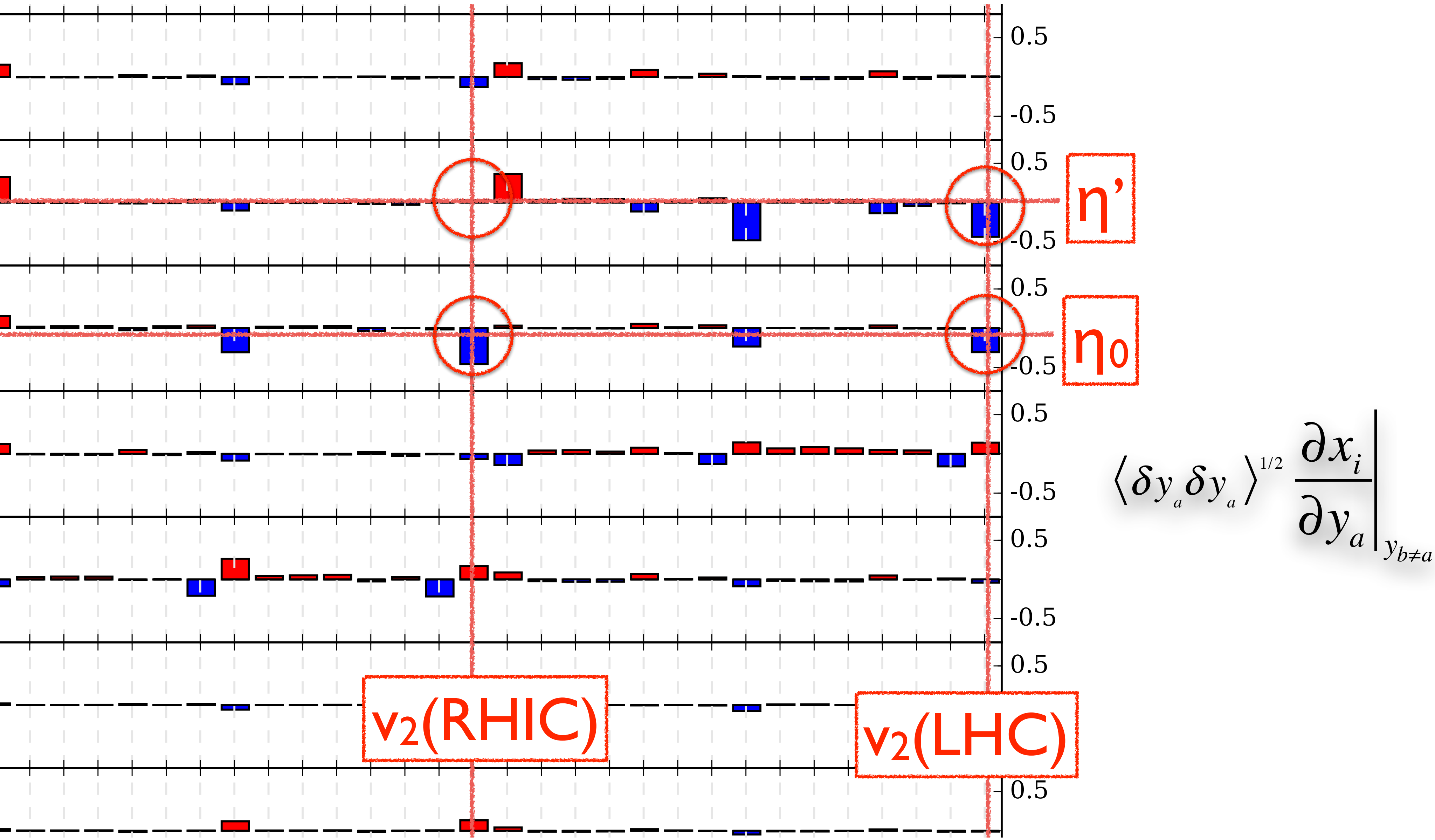


$$\langle \delta y_a \delta y_a \rangle^{1/2}\frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\Big|_{y_{b\neq a}}$$



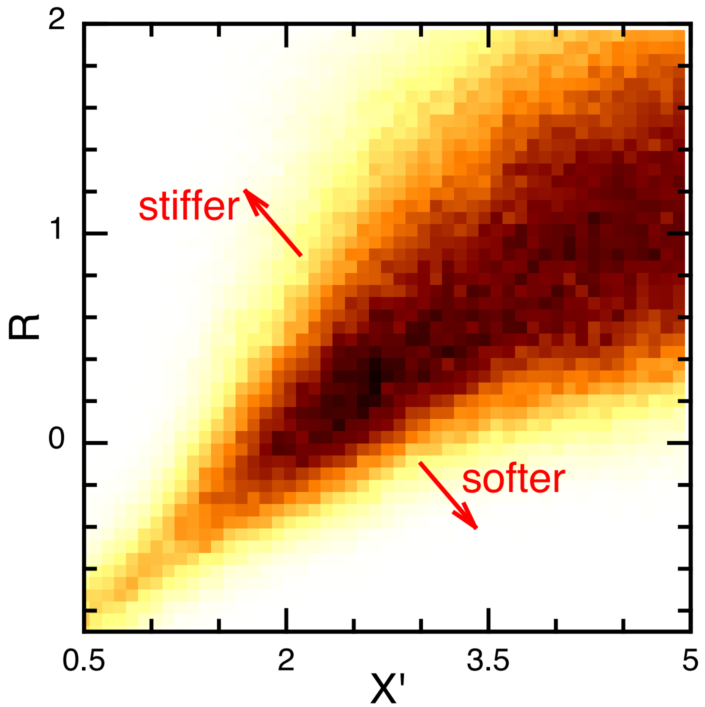
$$\left\langle \delta y_a \delta y_a \right\rangle^{1/2} \frac{\partial x_i}{\partial y_a}\Big|_{y_{b\neq a}}$$

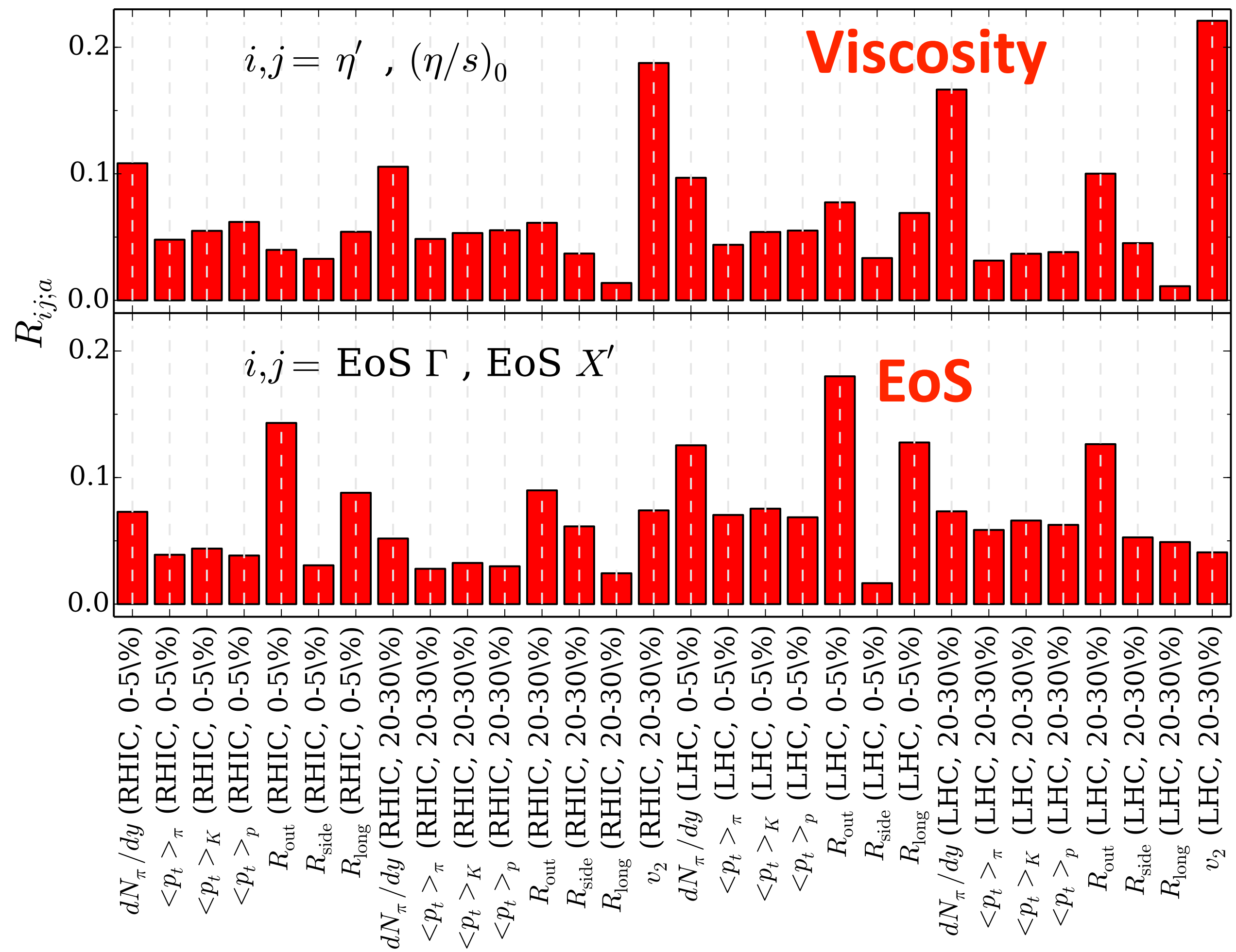


0.5
-0.5
0.5
-0.5
0.5
-0.5
0.5
-0.5
0.5
-0.5
0.5
0.5
η'
η0
$\langle \delta y_a \delta y_a \rangle^{1/2} \left. \frac{\partial x_i}{\partial y_a} \right|_{y_{b \neq a}}$
v2(RHIC)
v2(LHC)

2-Parameter Sensitivity

$$\frac{d}{d\sigma_y}\sqrt{\begin{vmatrix} \langle\langle \delta x_1 \delta x_1 \rangle\rangle & \langle\langle \delta x_1 \delta x_2 \rangle\rangle \\ \langle\langle \delta x_1 \delta x_2 \rangle\rangle & \langle\langle \delta x_2 \delta x_2 \rangle\rangle \end{vmatrix}} \langle \delta y \delta y \rangle^{1/2}$$





What determines viscosity?

- Both v_2 and multiplicities
- T-dependence comes from LHC v_2

What determines EoS?

- Lots of observables
- Femtoscopic radii are important

CONCLUSIONS (not the end of talk)

- **Robust**
- **Emulation works splendidly**
- **Scales well to more parameters & more data**
- **Eq. of State and Viscosity can be extracted from RHIC & LHC data**
- **Other parameters not as well constrained**
- **Some physics still missing (tension with Duke conclusions)**
- **Heavy-Ion Physics can be a Quantitative Science!!!!**

BIGGEST Challenge for JETSCAPE
Choosing "observables" and uncertainties

1. Include "theoretical systematic" error
2. Account for correlated errors

Correlated Errors

$$\mathcal{L} \sim \exp\left\{-\frac{1}{2}(y_a^{(\mathrm{exp})} - y_a^{(\mathrm{emu})})\Sigma_{ab}^{-1}(y_b^{(\mathrm{exp})} - y_b^{(\mathrm{emu})})\right\}$$

Σ_{ab} combines uncertainties:

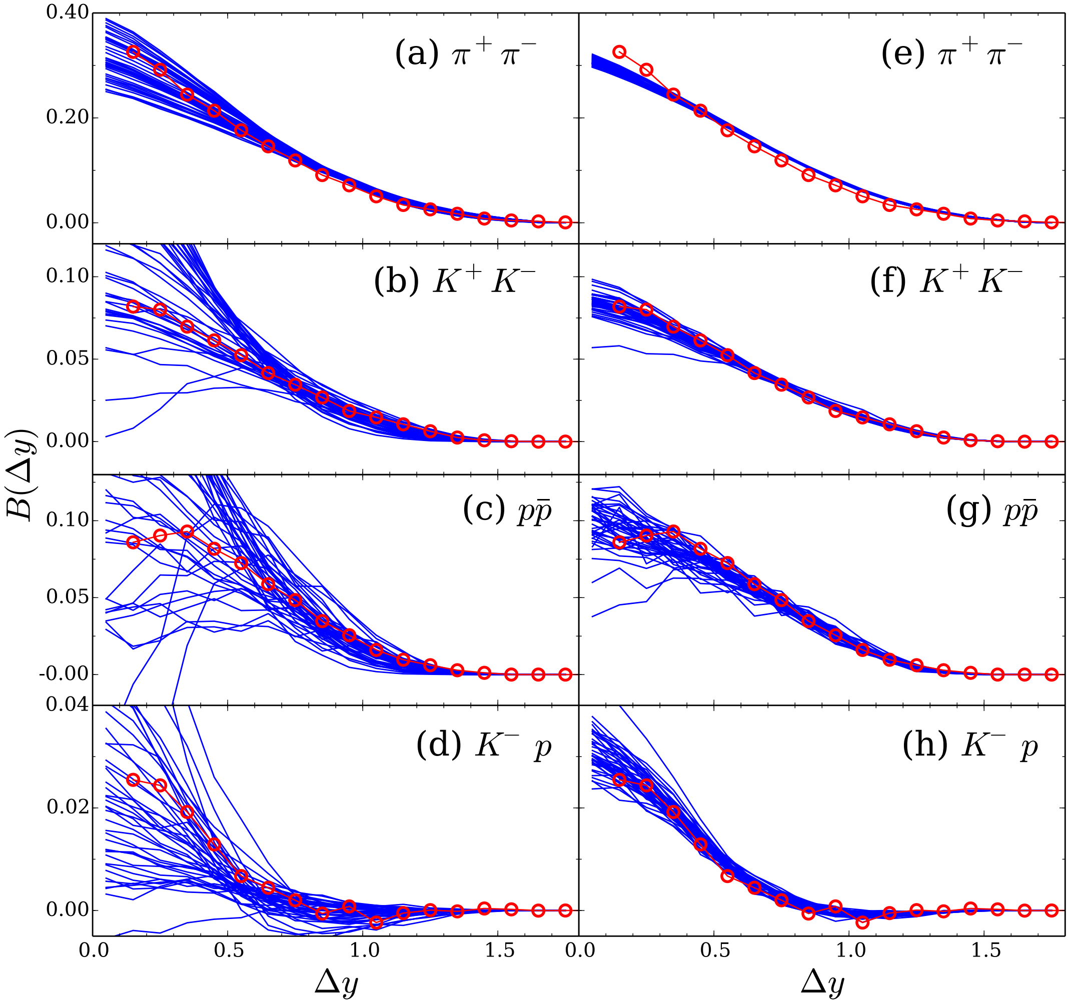
- aleotoric, both theory and experiment
- experimental systematic
- model systematic (missing physics)
- emulator accuracy
- correlated errors (off-diagonal elements)

Bigger uncertainties ➜ less constrained parameter space

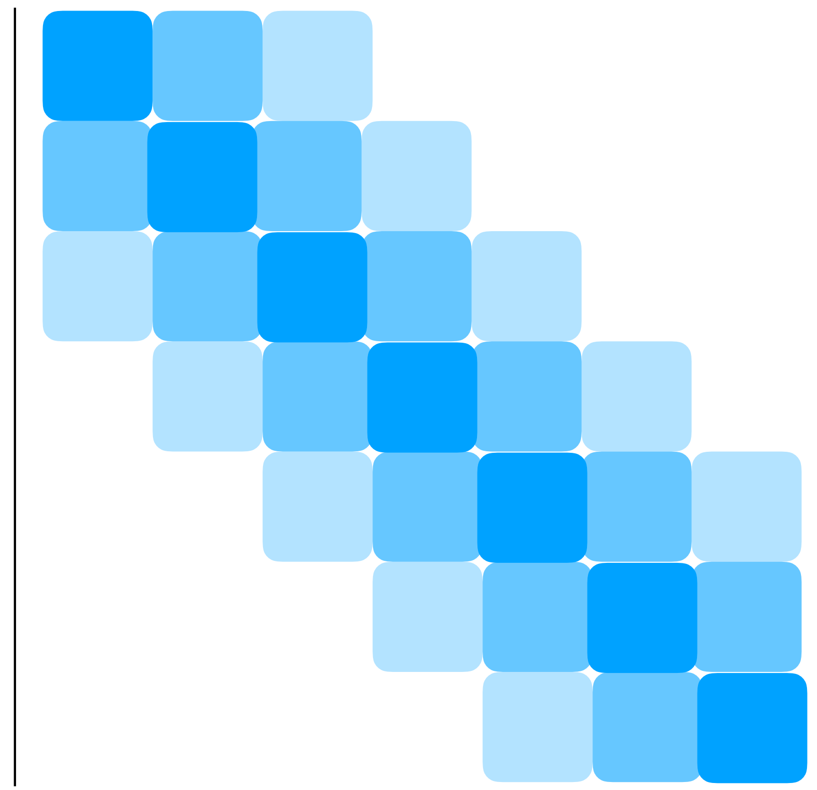
Four Strategies for Correlated Errors

1. Keep full error matrix
2. Exaggerate errors
3. Distillation
4. Nuisance parameters

I. Keep Full Error Matrix



Neighbors correlated



II. Exaggerate Errors

If $y_1, y_2, y_3, \ldots$ are redundant:

$$\Sigma = \Sigma_{aa} \left(\begin{array}{ccc} 1 & \cdots & 1 \\ 1 & \cdots & 1 \\ 1 & \cdots & 1 \end{array} \right)$$

$$\begin{aligned} y &= y_1 = y_2 = y_3 ... = y_n \\ P(y) &\sim \exp\left\{ -y^2/2\sigma^2 - y^2/2\sigma^2 ... - y^2/2\sigma^2 \right\} \\ \langle \delta y^2 \rangle &= \sigma^2/n, \\ \sigma^2 &= n\Sigma_{aa} \end{aligned}$$

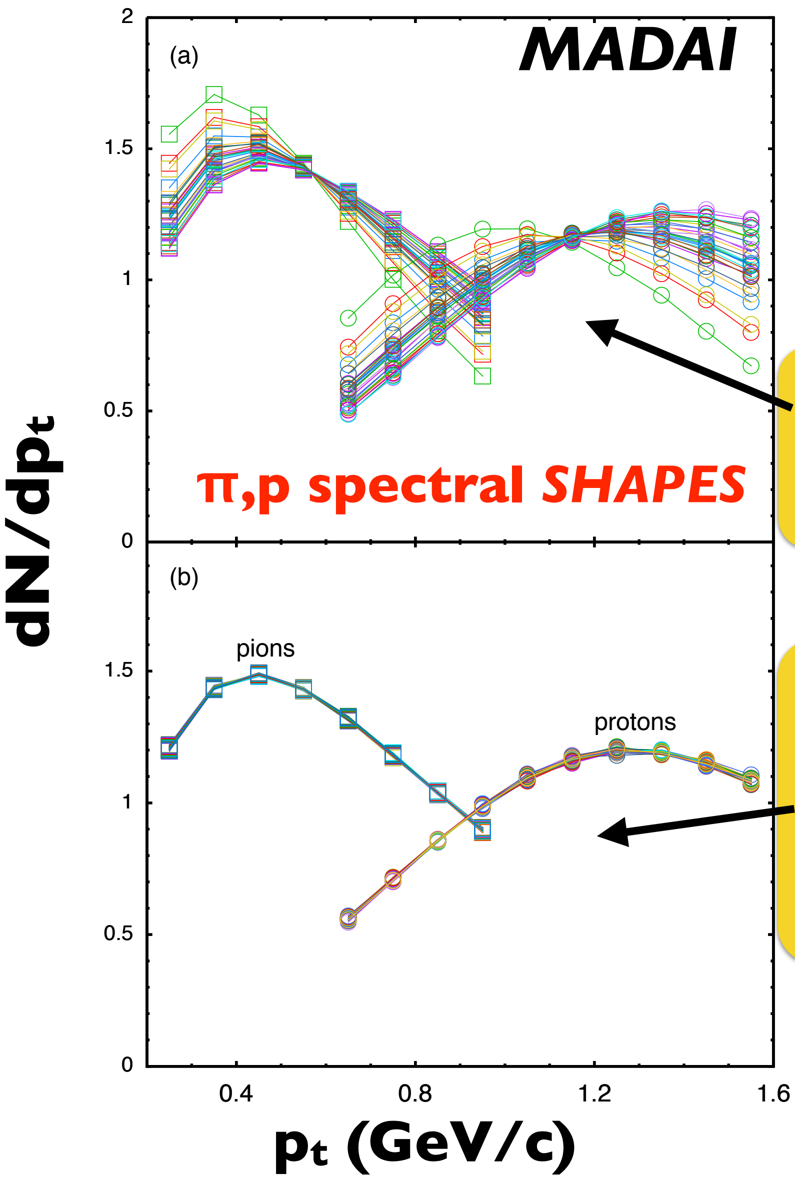
You can ignore off-diagonal elements, but set

$$\Sigma_{ab} \rightarrow n\Sigma_{aa}\delta_{ab}$$

If you think strong correlation extends ~ *n* points, just increase σ by $n^{1/2}$

III. Data Distillation

Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$



model spectra from 30 random points in parameter prior

74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV

44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV

IV. Nuisance Parameters

Systematic error has known form:

$$\delta y_a = X^{(n)} f_a$$

known form, e.g. $\exp(-p_t/\tau)$

Nuisance parameter

$$\Sigma_{ab} = (X^{(\mathrm{n})})^2 f_a f_b$$

a.k.a common-mode errors

- $X^{(n)}$ has prior distribution (Gaussian)
- f_a extends over correlated range of a
- popular in HEP to account for detector response
- could be applied to model mixing

IV. Nuisance Parameters

Example: Uncertain normalization

$$\frac{dN}{dp} = X\frac{dN^{(\mathrm{mod})}(x_1, x_2, \cdots x_n)}{dp},$$

$$\mathrm{Prob(X)} \sim e^{-(X-1)^2/2\sigma_X^2}$$

Stating Model Uncertainties

- Mainly from missing/wrong physics
- Involves soul searching
- Community discussion