Sudakov suppression of jets in QCD media

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JETSCAPE winter school and workshop
@ LBNL January 6, 2018

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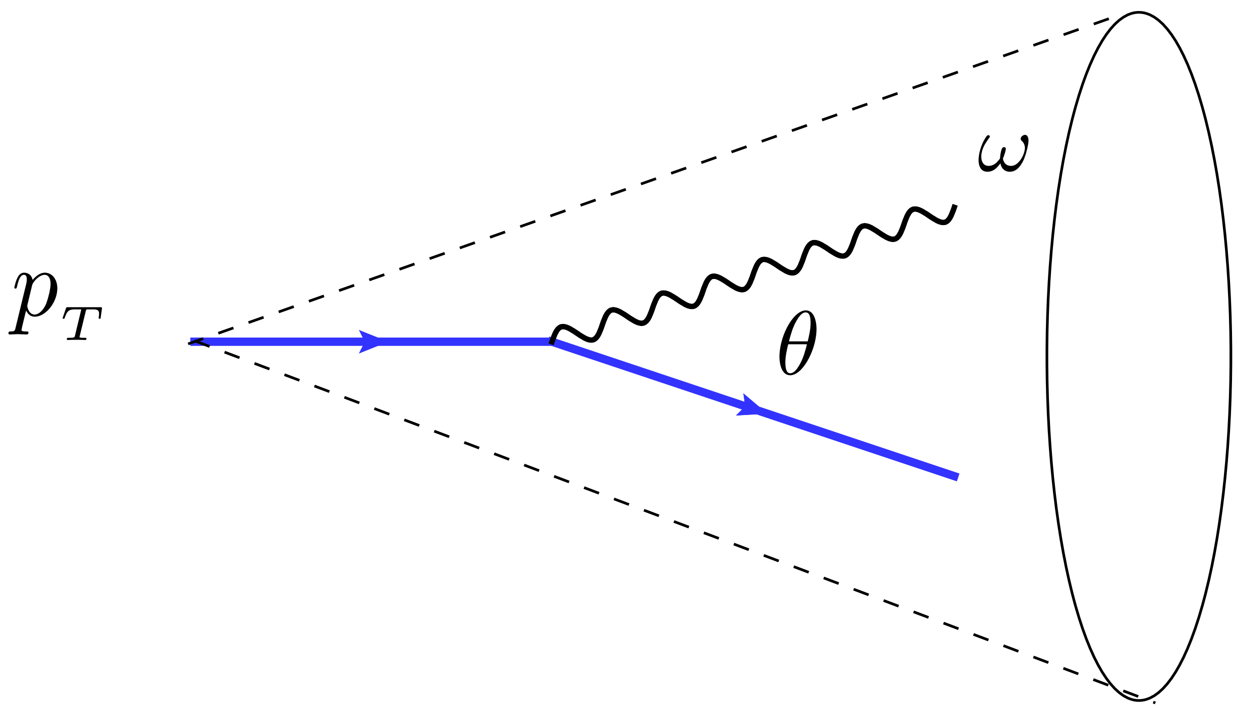
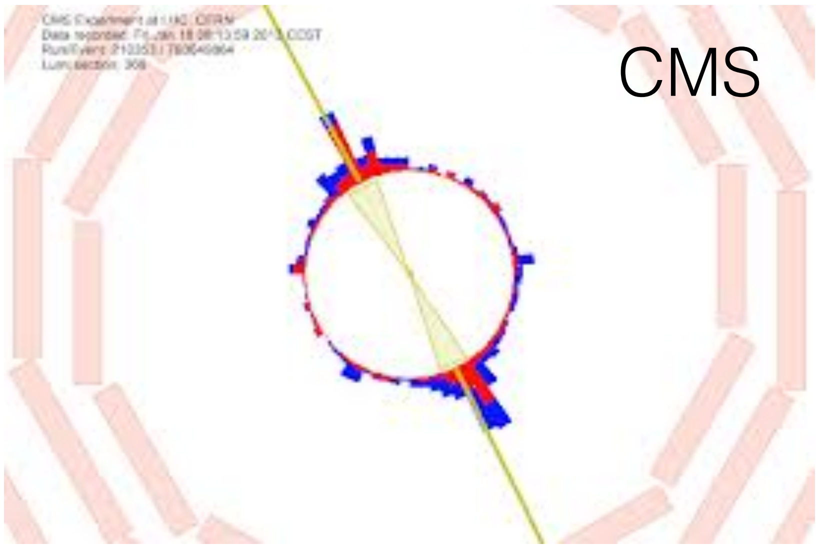
arXiv:1706.06047 [hep-ph], arXiv:1707.07361 [hep-ph]

QCD jets

Building block probability for parton cascades in vacuum





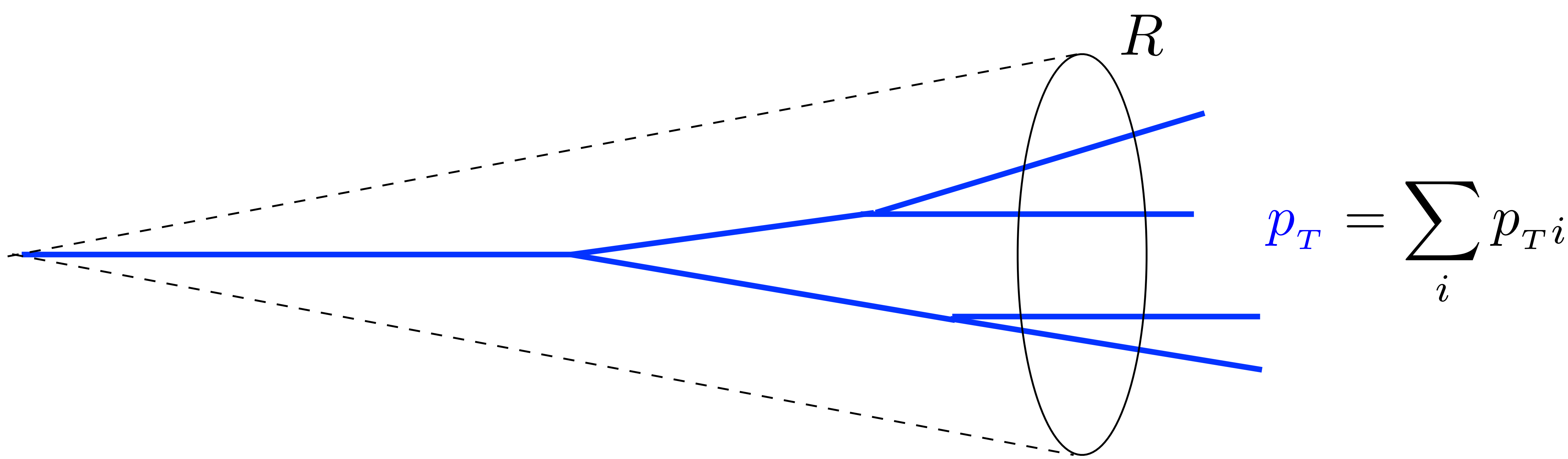
$$\mathrm{d}P \sim \alpha_s C_R \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}\omega}{\omega}$$

$\Longrightarrow$ collimation of jets

Large phase-space for multiple branching: many particles produced (implemented in Event Generators such as PYTHIA, HERWIG, SHERPA, etc.)

QCD jets

- Soft & Collinear divergences (resummation)
- **Color coherence:** angular ordered shower, interjet activity
- Not uniquely defined: cone size **R**, reconst. algo, …



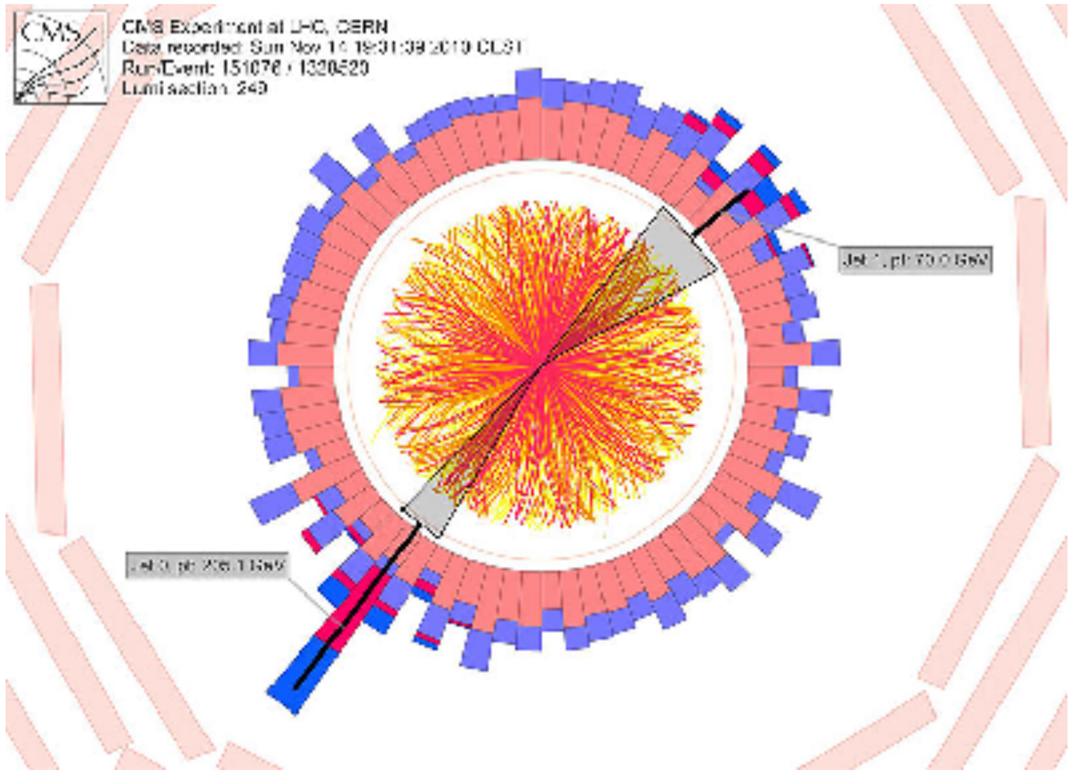
$$\frac{1}{E} \quad \ll \quad t_{\text{form}} \sim \frac{k_{||}}{k_\perp^2} \quad \ll \quad \frac{E}{\Lambda_{\text{QCD}}} \qquad\qquad p_T \equiv E$$

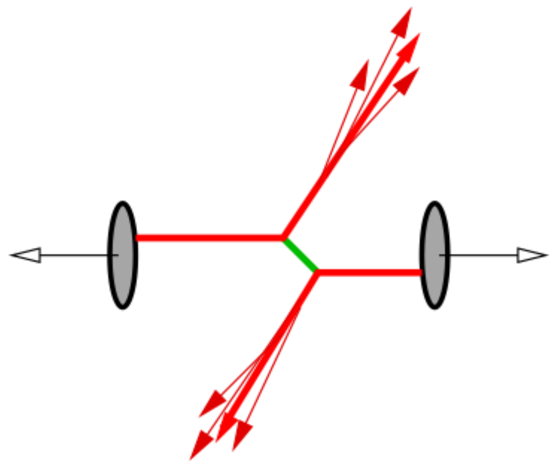
Large separation of time scales

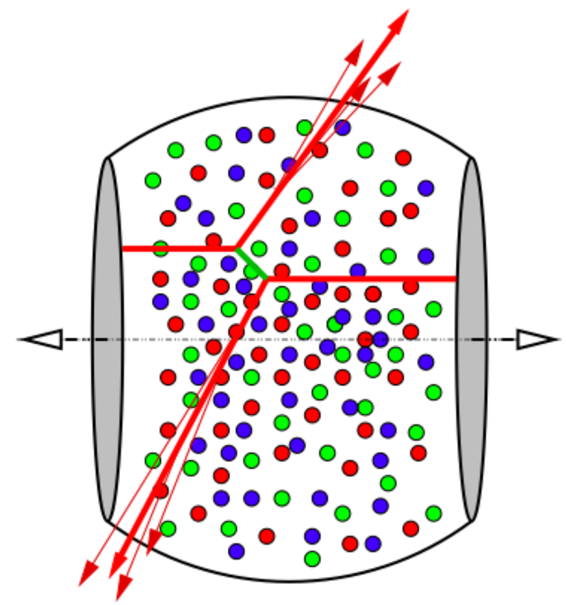
Jet observables of two types

- Infrared-Collinear (IRC) safe observables: sum over final state hadrons $\longrightarrow$ cancellations of divergences. Ex: event shape: thrust, jet mass, jet spectra, etc. Resummation of large logs, e.g. log R, log Q/M, can be necessary
- Collinear sensitive observables: pQCD still predictive (factorization theorems). Ex: Fragmentation Functions

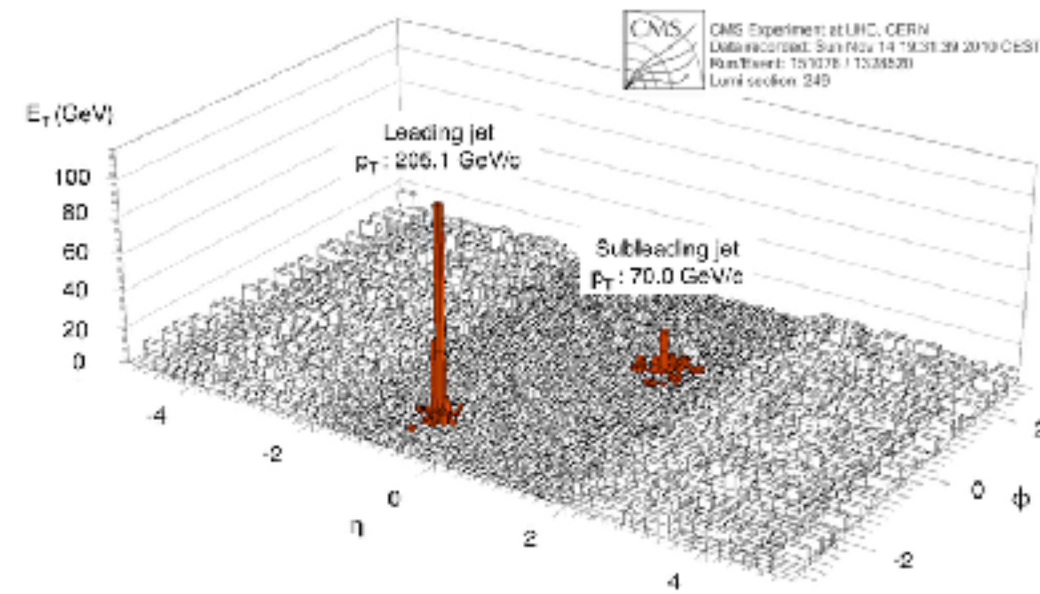
Jets in Heavy Ion Collisions







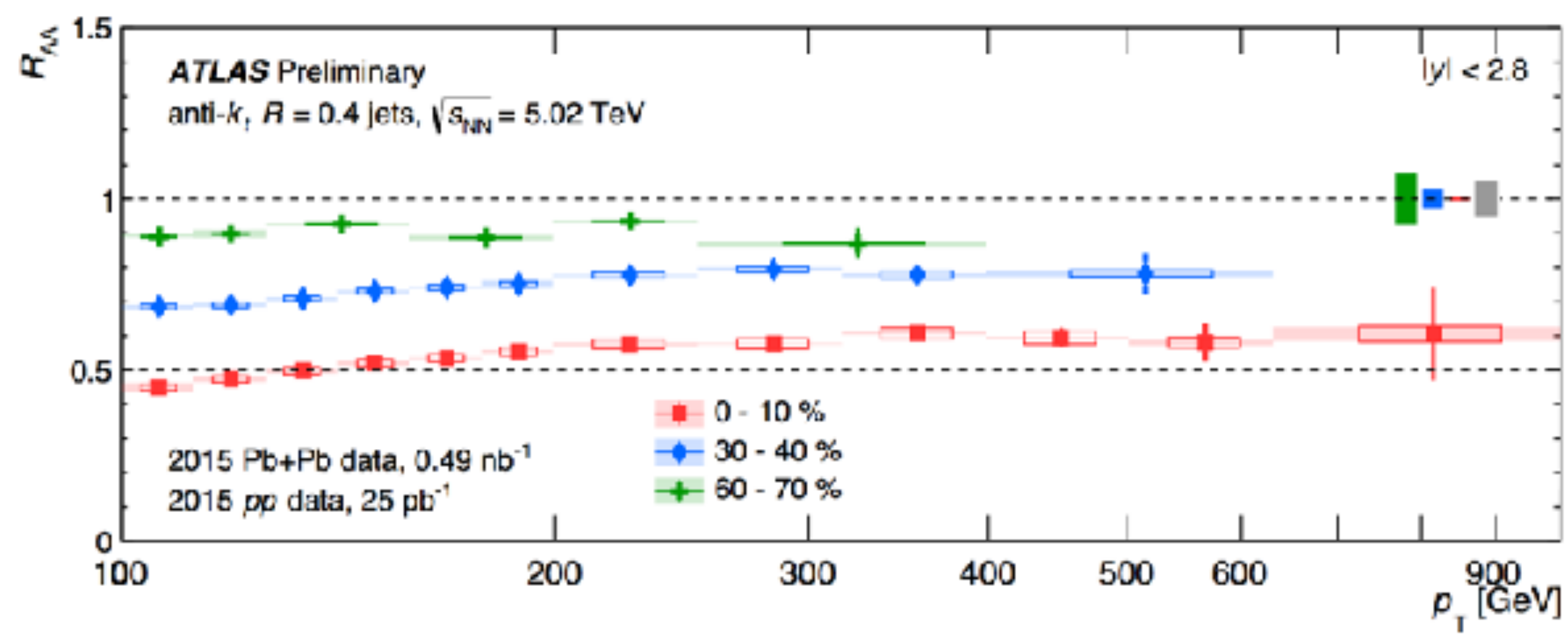
quark-gluon plasma



Strong jet suppression (**up to 1 TeV!**) observed in ultra-relativistic heavy ion collisions at LHC

Inclusive jet spectra ratio

$$R_{\mathrm{AA}} = \frac{\mathrm{d}N_{\mathrm{AA}}/\mathrm{d}^2 p_T}{N_{\mathrm{coll}} \times \mathrm{d}N_{\mathrm{pp}}/\mathrm{d}^2 p_T}$$



How much energy is lost?

- **A rough estimate:** consider a constant energy loss ϵ
- using a power spectrum $$\frac{\mathrm{d}N}{\mathrm{d}^2 p_T} \sim p_T^{-n}$$

we have

$$R_{\mathrm{AA}} \sim \frac{p_T^n}{(p_T + \epsilon)^n} \simeq 1 - \frac{n\epsilon}{p_T}$$

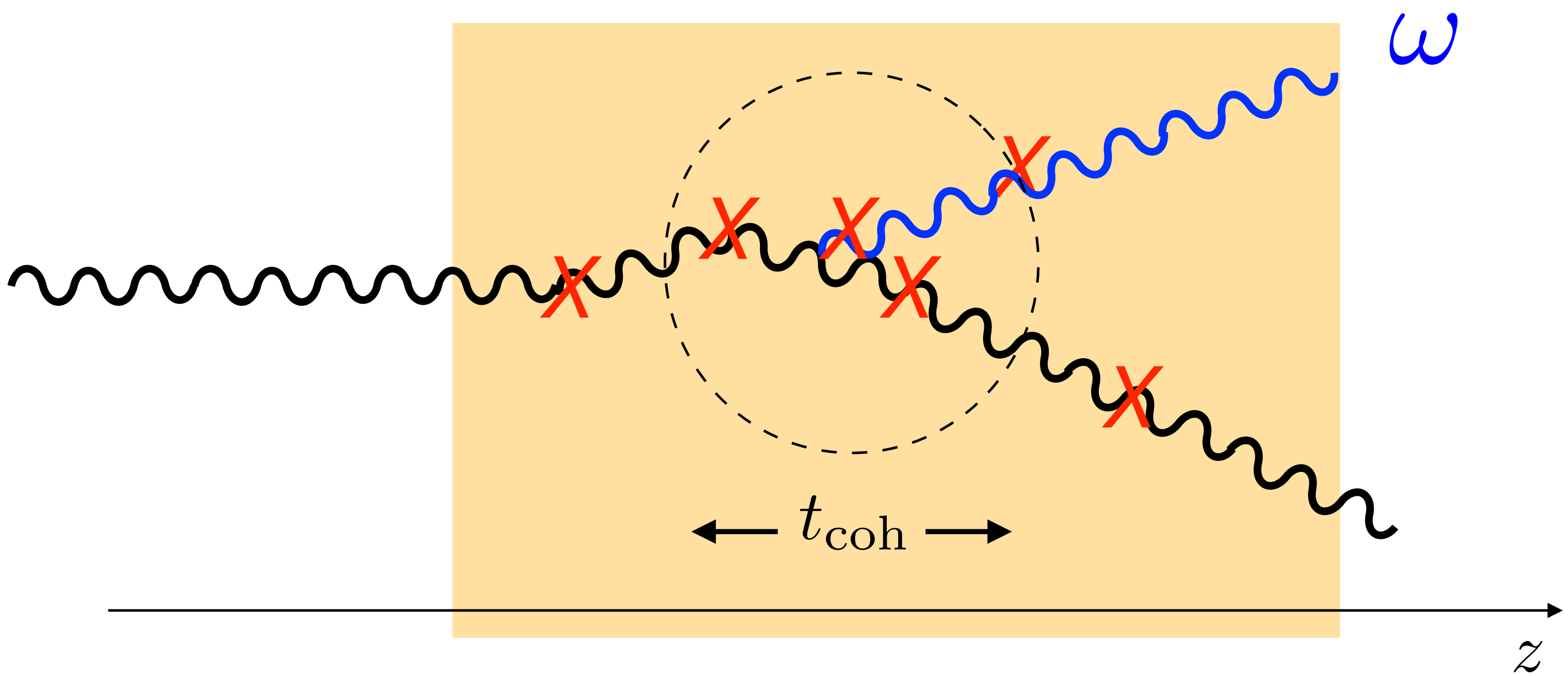
Hence, for $R_{AA} \sim 0.5$ and **n=6**, one finds that jets with **$p_T \sim$ 300 GeV** lose typically about **$\epsilon \sim$ 25 GeV**

Radiative energy loss

Medium-induced splittings

- Multiple scattering can trigger gluon radiation
- **Laudau-Pomeranchuk-Migdal effect:** during the splitting time many scattering centers act coherently as a single one and thus, suppressing the radiation rate ($k_\perp^2 \sim \hat{q}\, t$)

$$t_{\rm coh} = \frac{\omega}{k_\perp^2} \sim \frac{\omega}{\hat{q}\, t_{\rm coh}} \quad \Rightarrow \quad t_{\rm coh} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



Radiative spectrum

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q} L^2}{\omega}}$$

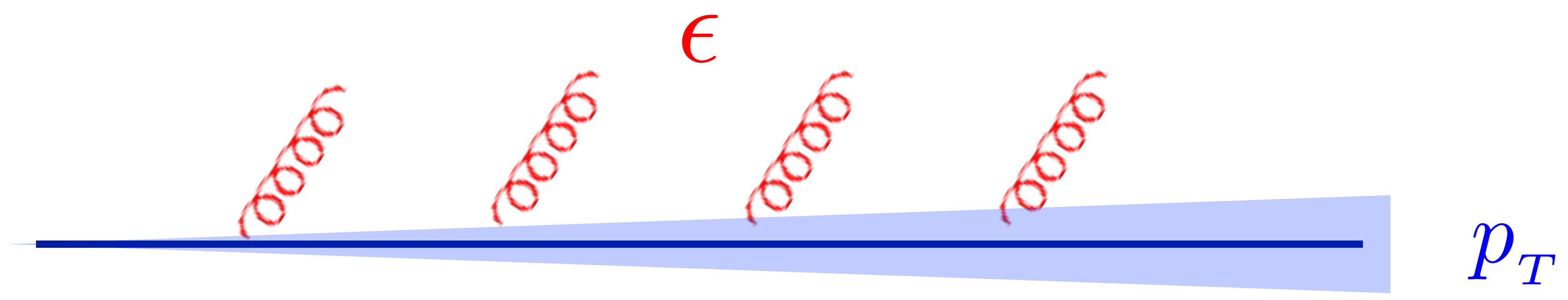
[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Wiedemann (2000) Arnold, Moore, Yaffe (2002)]

[Dilute medium limit. N=1 opacity expansion: Gyulassy, Levai, Vitev (2000) Guo, Wang (2000)]

Single quark
energy loss

Single quark energy loss

- **Standard energy loss picture:** medium-induced radiation off a single parton [Baier, Dokshitzer, Mueller, Schiff, JHEP (2001)]



- **Jet spectrum:** convolution of the energy loss probability with the spectrum in vacuum

$$\frac{\mathrm{d}\sigma(p_T)}{\mathrm{d}^2p_T\mathrm{d}y} = \int_0^\infty \mathrm{d}\epsilon\,\mathcal{P}(\epsilon)\,\frac{\mathrm{d}\sigma^{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}^2p_T\mathrm{d}y}$$

Single quark energy loss

Because the **jet spectrum** is **steeply falling (n >> 1),** one can make the following approximation

$$\frac{\mathrm{d}\sigma^{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}^2 p_T \mathrm{d}y} \sim \frac{1}{(p_T+\epsilon)^n} \simeq \frac{\mathrm{e}^{-\frac{n\epsilon}{p_T}}}{{p_T}^n}$$

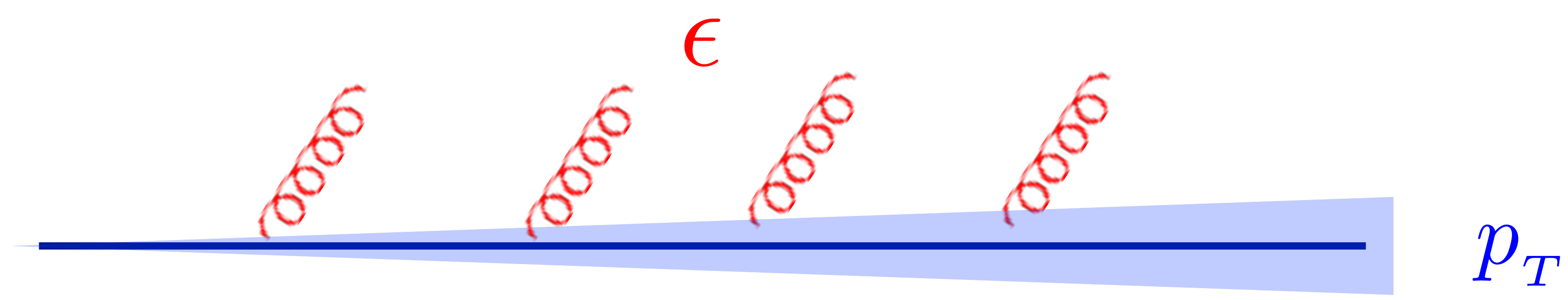
This allows to relate the **jet spectrum** to the **Laplace Transform** of the quenching probability

$$R_{\mathrm{AA}} \sim Q(p_T) \equiv \tilde{\mathcal{P}}(\nu = n/p_T)$$

where

$$\tilde{\mathcal{P}}(\nu) = \int \mathrm{d}\epsilon\, \mathcal{P}(\epsilon)\ \mathrm{e}^{-\nu\epsilon}$$

Single quark energy loss



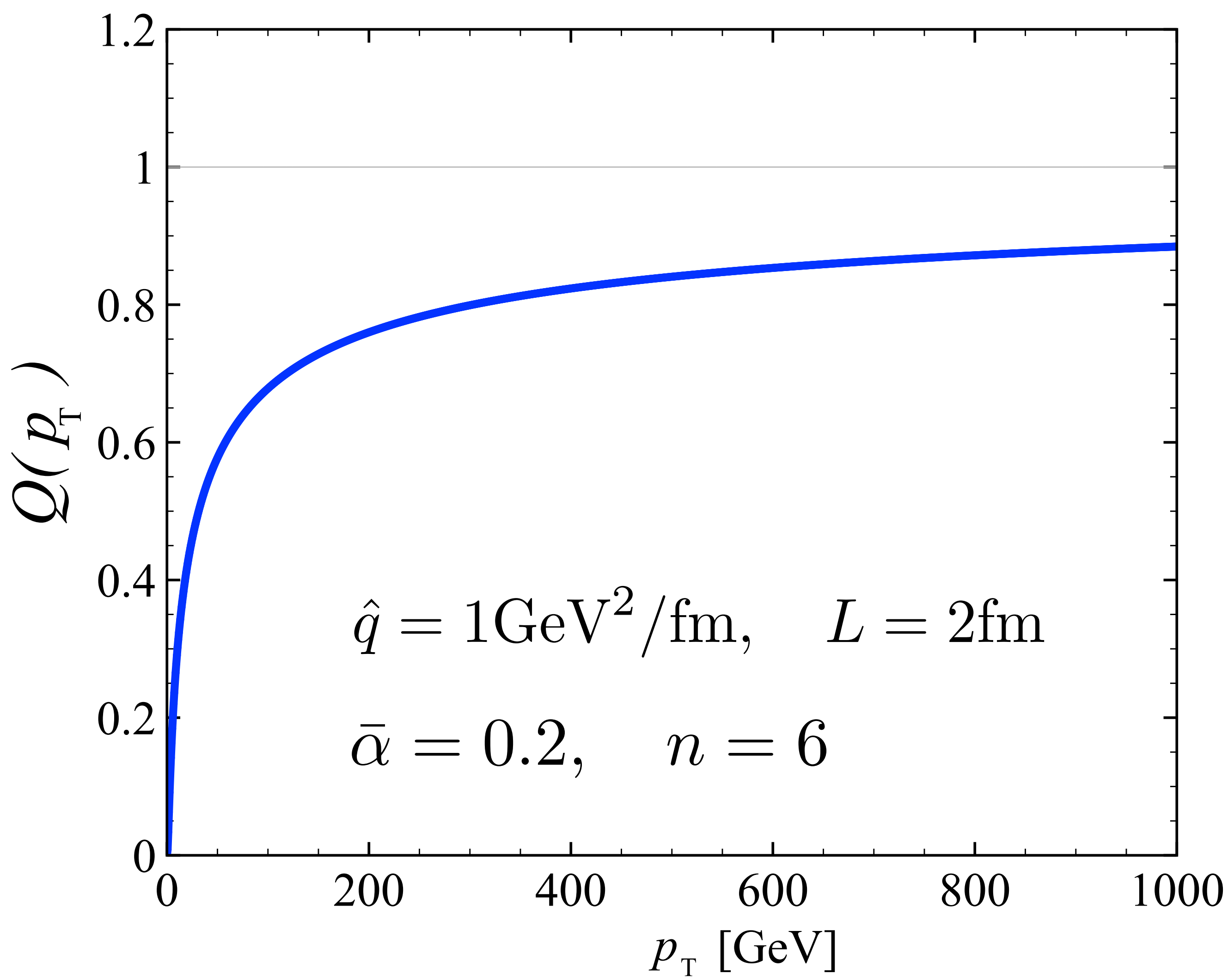
In the **short formation time** approximation soft radiations can be treaded as independent and exponentiate in Laplace space

$$\tilde{\mathcal{P}}(\nu) = \exp\left[-\int \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\left(1 - \mathrm{e}^{-\nu\omega}\right)\right]$$

Single quark energy loss

- Neglecting **finite size effect** one obtains a simple analytic formula for the quenching factor

$$\mathcal{Q}(p_T) \simeq \exp\left(-\bar{\alpha}\, L \sqrt{\frac{\pi \hat{q}\, n}{p_T}}\right)$$



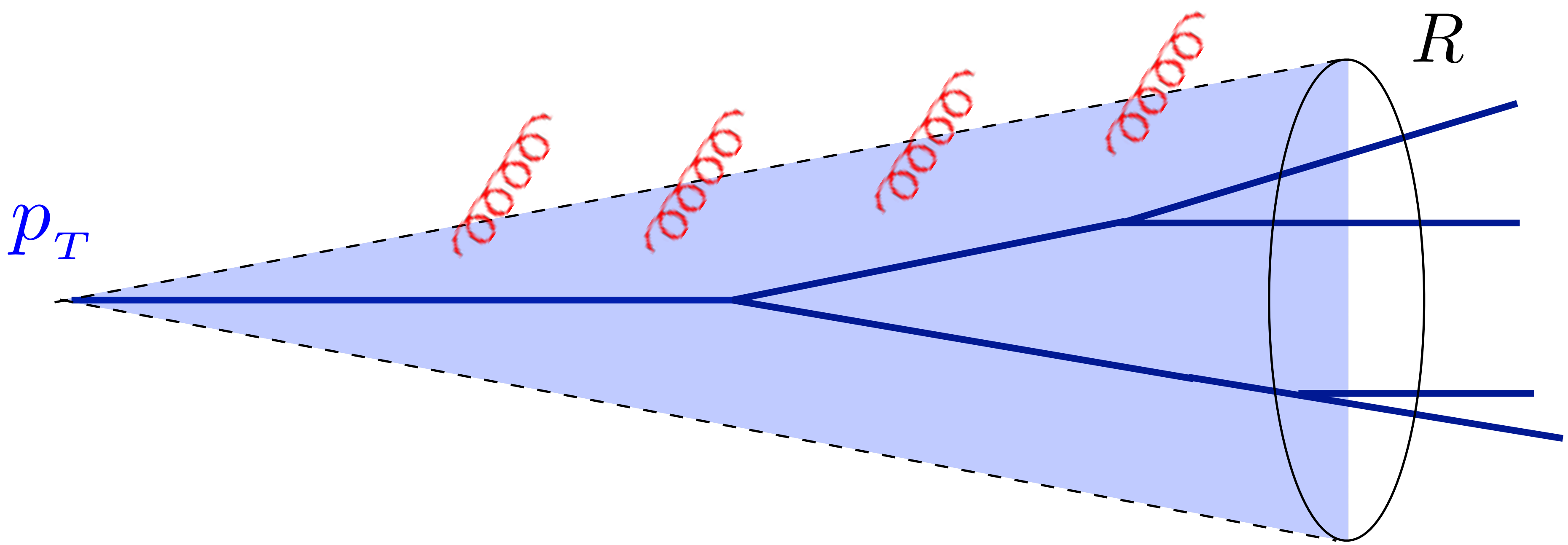
Strong quenching

$$p_T \ll \pi\, n\, \bar{\alpha}^2 \hat{q} L^2$$

$$Q(p_T) \ll 1$$

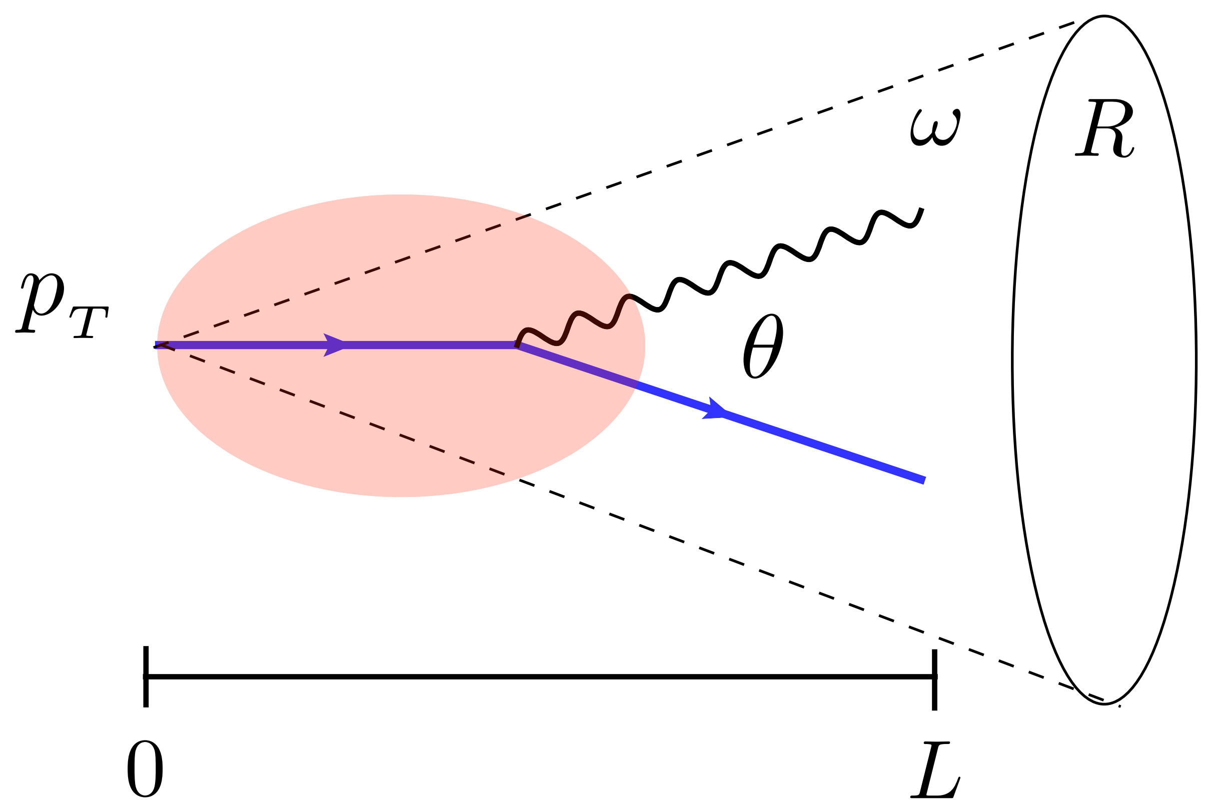
Jet quenching and fluctuations

- Energy is lost mainly via radiation **but how does a jet as a muti-parton system lose energy to the medium?**
- Does one need to account for fluctuations of energy loss due to **fluctuations of the jet substructure**?



Phase-space analysis

- How large are next-to-leading order contributions?



$$t_{\mathrm{f}} = \frac{1}{\omega\theta^2}$$

- Probability for a virtual quark to split inside the medium:

$$\mathrm{PS} = \bar{\alpha} \int_0^{p_T} \frac{\mathrm{d}\omega}{\omega} \int_0^R \frac{\mathrm{d}\theta}{\theta} \;\Theta(t_{\mathrm{f}} < L) = \frac{\bar{\alpha}}{4} \log^2 \left(p_T R^2 L \right)$$

Phase-space analysis

- Large double-logarithmic phase-space at high pT:

$$\frac{1}{p_T R^2} \ll t_f \ll L$$

- When $\bar{\alpha} \log^2(p_T R^2 L) \gtrsim 1$ higher-orders are not negligible

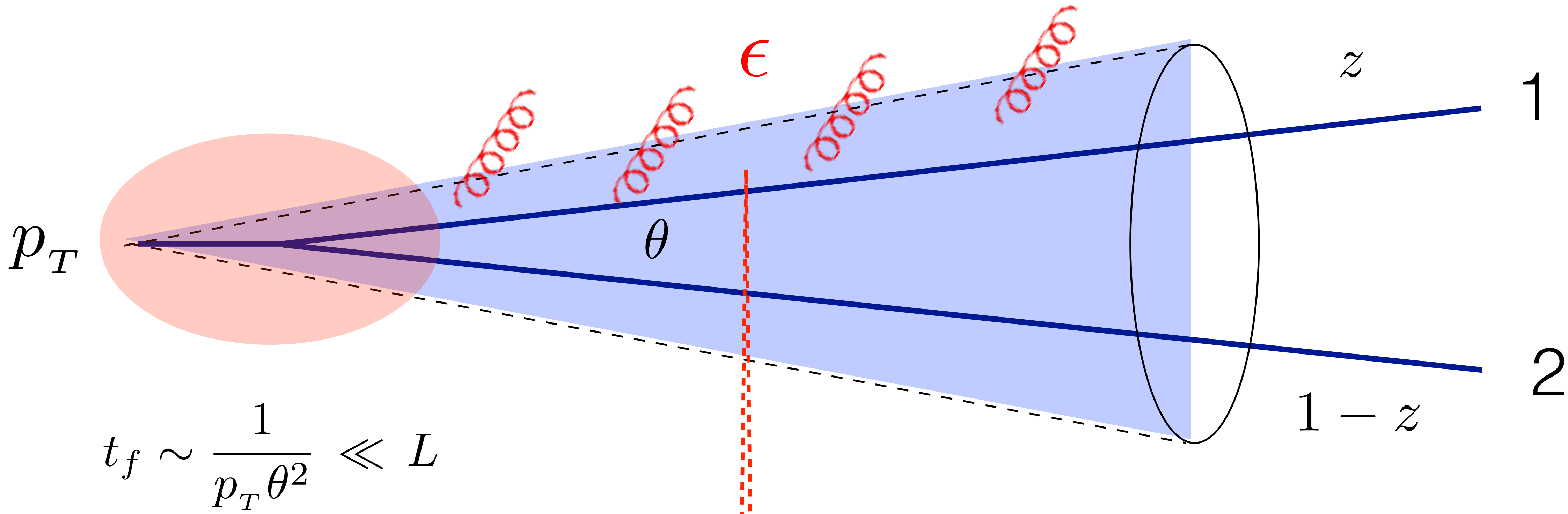
$\Rightarrow$ **double-logs (DL) need to be resummed**

- **Estimate:** for R=0.3, L=2 fm and pT=500 GeV,
one finds **Log² ~ 40**

Two-pronged energy loss

Two-pronged energy loss

- Consider a high energy parton that splits rapidly into two hard subjects within the jet cone
- At high p_T the branching time is shorter than the length of the medium ⇒ factorization

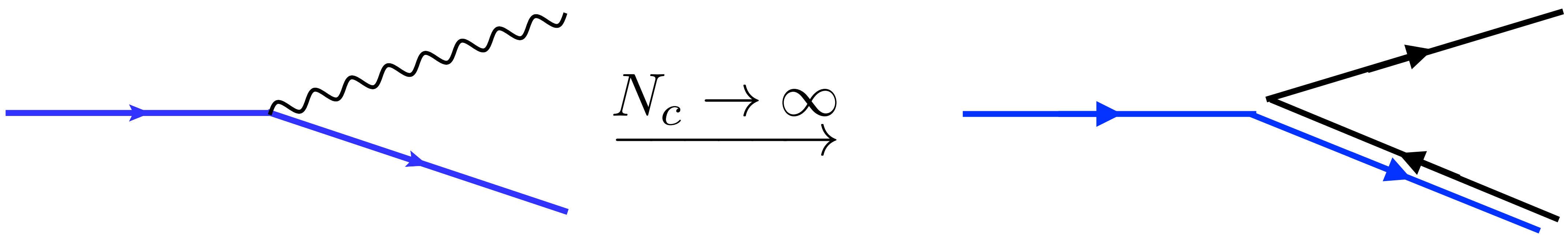


- Two pronged inclusive spectrum:

$$\theta \frac{\mathrm{d}N}{\mathrm{d}\theta \mathrm{d}z \mathrm{d}p_T} = \int_0^\infty \mathrm{d}\epsilon \, P_2(\epsilon) \, \bar{\alpha} \, P(z) \, \frac{\mathrm{d}N^{\mathrm{vac}}(p_T + \epsilon)}{\mathrm{d}p_T}$$

Two-pronged energy loss

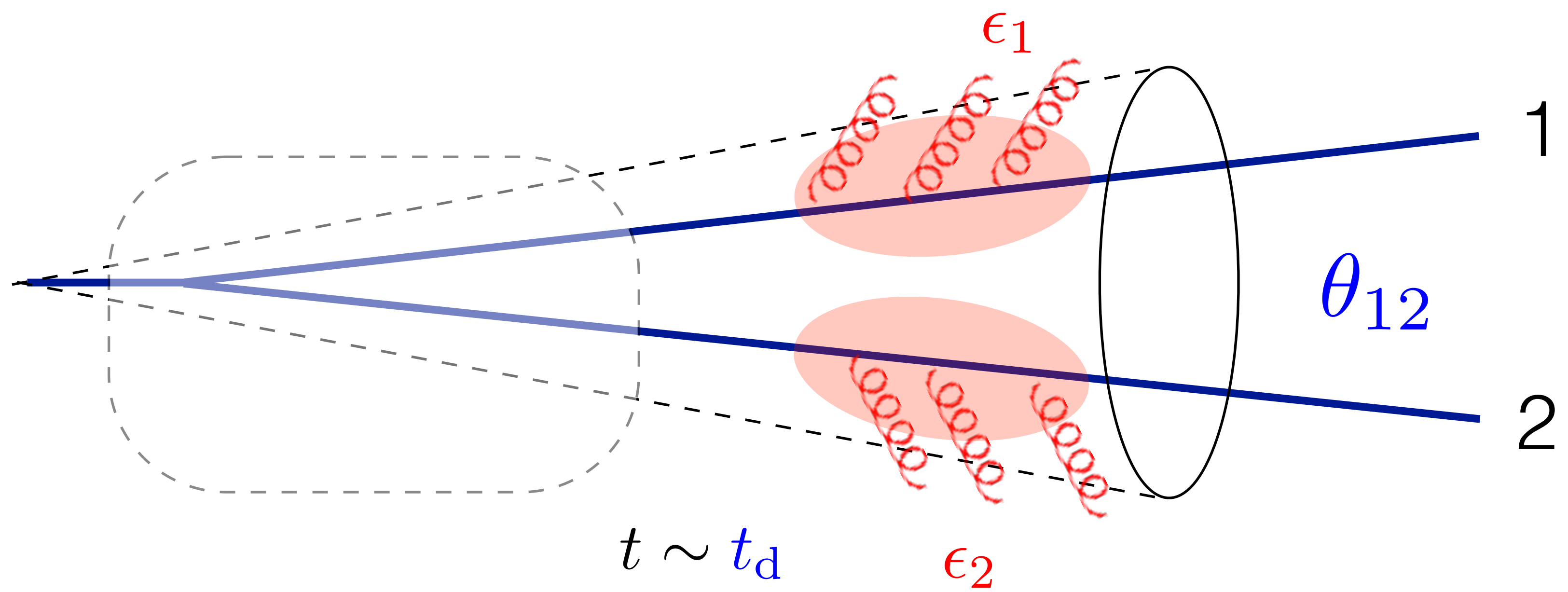
- In the large-Nc approximation



- The two-pronged energy loss probability factorizes into the total charge probability convoluted with the color singlet antenna probability distribution

$$\mathcal{P}(\epsilon) = \int_{\epsilon_1,\epsilon_2} \mathcal{P}_{\rm tot}(\epsilon_1)\,\mathcal{P}_{\rm sing}(\epsilon_2)\;\delta(\epsilon - \epsilon_1 - \epsilon_2)$$

Two-pronged energy loss



no energy loss

independent energy loss

- Propagation of two color charges at fixed angle
- Up to the decoherence time $t_d \sim (\hat{q}\, \theta_{12}^2)^{-1/3}$ radiation off the total charge
- At large angle: suppression of neighboring jets

Two-pronged energy loss

arXiv:1706.06047 [hep-ph]

- The **color singlet antenna** probability distribution reads:

$$\mathcal{P}_{\rm sing}(\epsilon, L) = \int_{\epsilon_1,\epsilon_2} \mathcal{P}_q(\epsilon_1, L)\, \mathcal{P}_q(\epsilon_2, L)\ \delta(\epsilon - \epsilon_1 - \epsilon_2)$$
$$+\, 2\int_0^L {\rm d}t \int_{\epsilon_1,\epsilon_2,\omega} \mathcal{P}_q(\epsilon_1, L-t)\mathcal{P}_q(\epsilon_2, L-t)\,\Gamma(\omega)\, S(t)\delta(\epsilon - \epsilon_1 - \epsilon_2 - \omega)$$

- with $$\Gamma(\omega) = \frac{{\rm d}I}{{\rm d}\omega{\rm d}t} - \delta(\omega)\int_0^\infty {\rm d}\omega' \frac{{\rm d}I}{{\rm d}\omega'{\rm d}t}$$

- **Decoherence** time scale

$$t_{\rm d} \equiv (\hat{q}\,\theta^2)^{1/3}$$

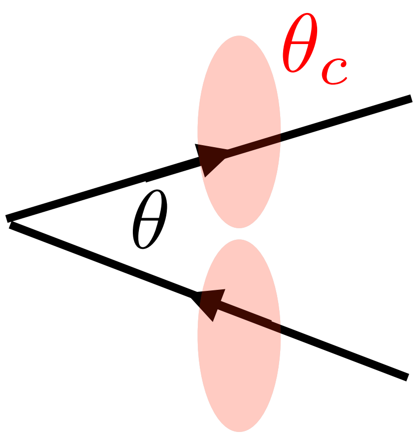
- Two terms: **independent energy loss** + **interferences**

Two-pronged energy loss

Two limiting cases:

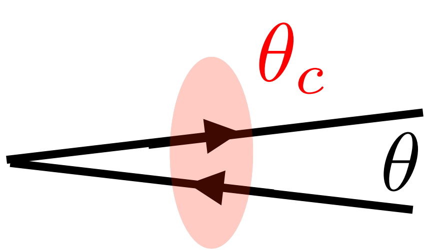
I - the medium resolves the antenna: $t_{\rm d} \gg L \quad (\theta \gg \theta_c \equiv 1/\sqrt{\hat{q}L^3})$

$$\mathcal{P}_{\rm sing}(\epsilon) \to \int_1 \mathcal{P}_q(\epsilon_1)\mathcal{P}_q(\epsilon - \epsilon_1)$$



II - the medium does not resolve the antenna: $t_{\rm d} \gg L \quad (\theta \gg \theta_c)$

$$\mathcal{P}_{\rm sing}(\epsilon) \to \delta(\epsilon)$$



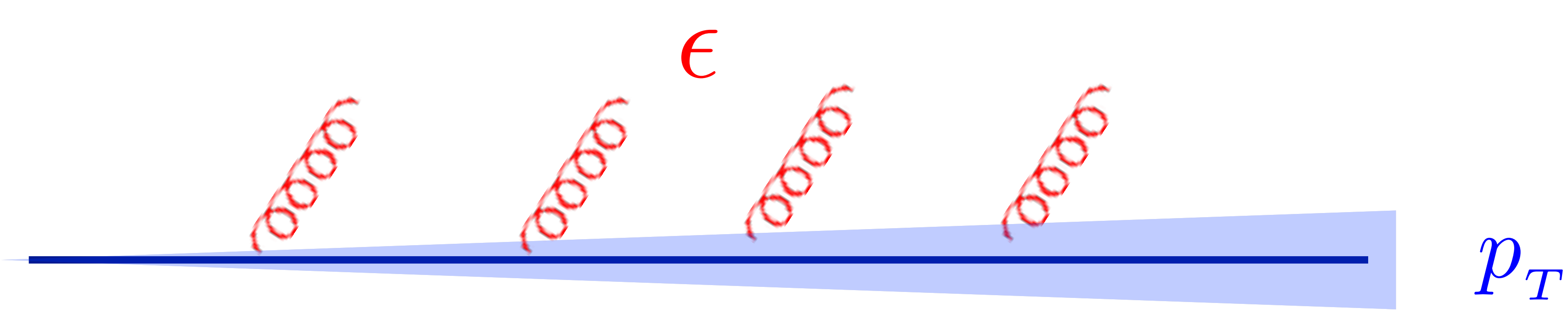
MT, Salgado, Tywoniuk PRL (2011), PLB (2012), JHEP (2011-2012)
Casalderrey-Solana, Iancu JHEP (2012)
Casalderrey-Solana, MT, Salgado, Tywoniuk PLB (2013)

Jet spectrum

First correction to the jet spectrum

- To LO the quenching factor is that of the total charge (primary quark)

$$Q^{(0)}(p_T) = Q_{\rm tot}(p_T) \equiv Q_{\rm q}(p_T)$$

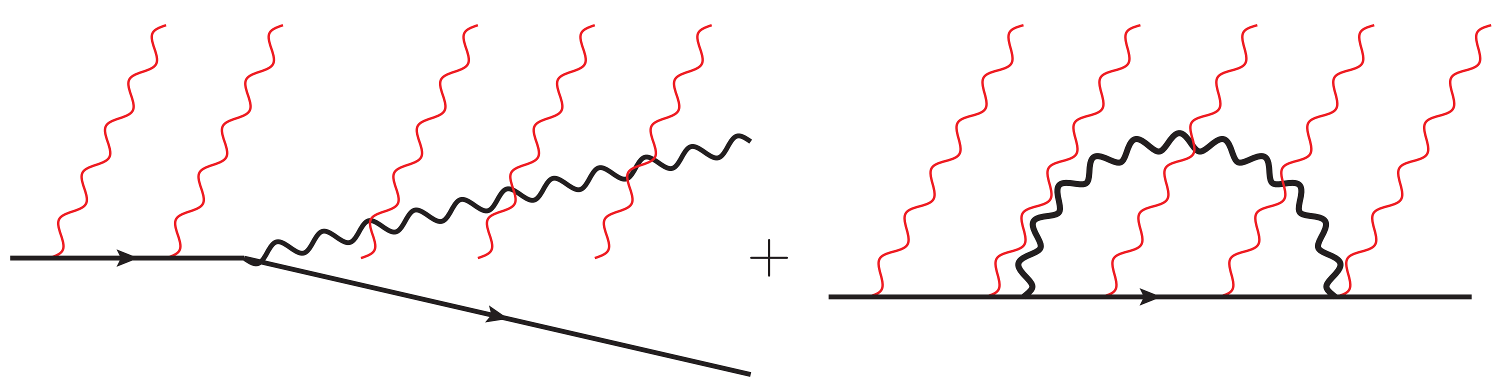


First correction to the jet spectrum

- Using the two pronged energy loss one can compute the first correction to the jet spectrum in the leading logarithmic (LL) accuracy (accounting for virtual corrections)
- There are exact cancellation between real an virtual corrections as in vacuum except when: $t_{\rm f} \ll t_{\rm d} \ll L$

$$Q^{(1)}(p_T) = \bar{\alpha} \int_{\theta_c}^{R} \frac{{\rm d}\theta}{\theta} \int_{(\hat{q}/\theta^4)^{1/3}}^{p_T} \frac{{\rm d}\omega}{\omega} \left[Q_q^2(p_T) - 1\right] Q_{\rm tot}(p_T)$$

Mismatch between real and virtual

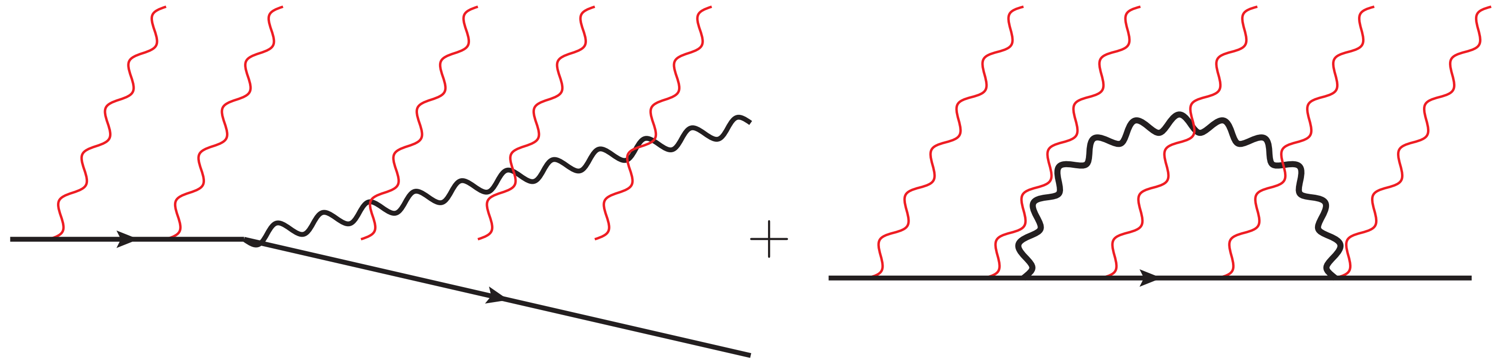


gluon+quark energy loss ≪ quark energy loss

First correction to the jet spectrum

- Using the two pronged energy loss one can compute the first correction to the jet spectrum in the leading logarithmic (LL) accuracy (accounting for virtual corrections)
- There are exact cancellation between real an virtual corrections as in vacuum except when: $t_\mathrm{f} \ll t_\mathrm{d} \ll L$

$$Q^{(1)}(p_T) = 2\bar{\alpha}\left[Q_q^2(p_T) - 1\right]\ Q_\mathrm{tot}(p_T) \times \ln\frac{R}{\theta_c}\left[\ln\frac{p_T}{\omega_c} + \frac{2}{3}\ln\frac{R}{\theta_c}\right]$$



gluon+quark energy loss $\ll$ quark energy loss

First correction to the jet spectrum

- Instructive limit: **strong quenching** $Q_{\rm tot}(p_T) \ll 1$

$$Q(p_T) = Q_{\rm tot}(p_T)\left[1 - 2\bar{\alpha}\ln\frac{R}{\theta_c}\left(\ln\frac{p_T}{\omega_c} + \frac{2}{3}\ln\frac{R}{\theta_c}\right)\right]$$

$$\omega_c \equiv \hat{q}L^2$$

$\Rightarrow$ Fluctuations of the jet substructure yield additional suppression to the jet spectrum

Exponentiation of the Double-Logs

- It can be shown that the leading logarithms exponentiate into a **Sudakov Form Factor**

$$Q(p_T) = Q_{\rm tot}(p_T) \times C(p_T)$$

- where

$$C(p_T) = \exp\left[-2\bar{\alpha}\ln\frac{R}{\theta_c}\left(\ln\frac{p_T}{\omega_c} + \frac{2}{3}\ln\frac{R}{\theta_c}\right)\right]$$

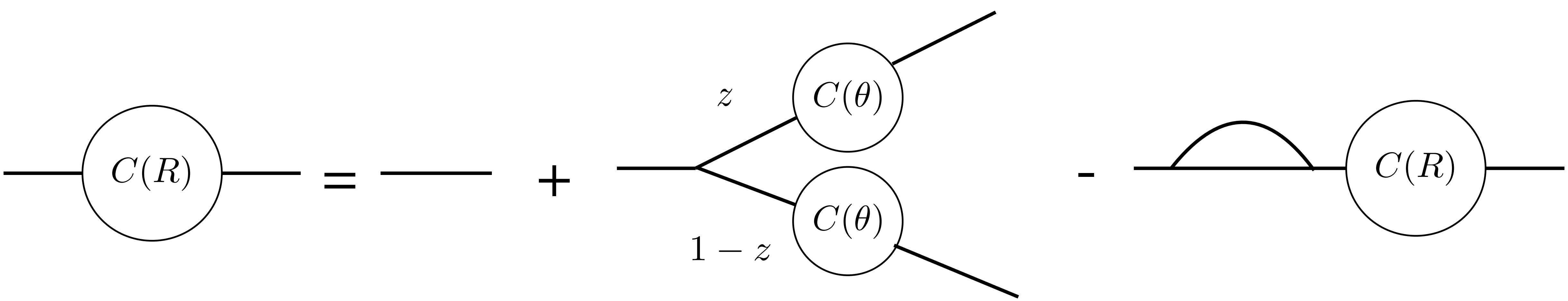
- **Coherent limit:** note that when $R \ll \theta_c$ the medium “sees” only the total charge, in this case

$$C(p_T) \to 1 \qquad \text{and} \qquad Q(p_T) \to Q_{\rm tot}(p_T)$$

Non-linear evolution equation

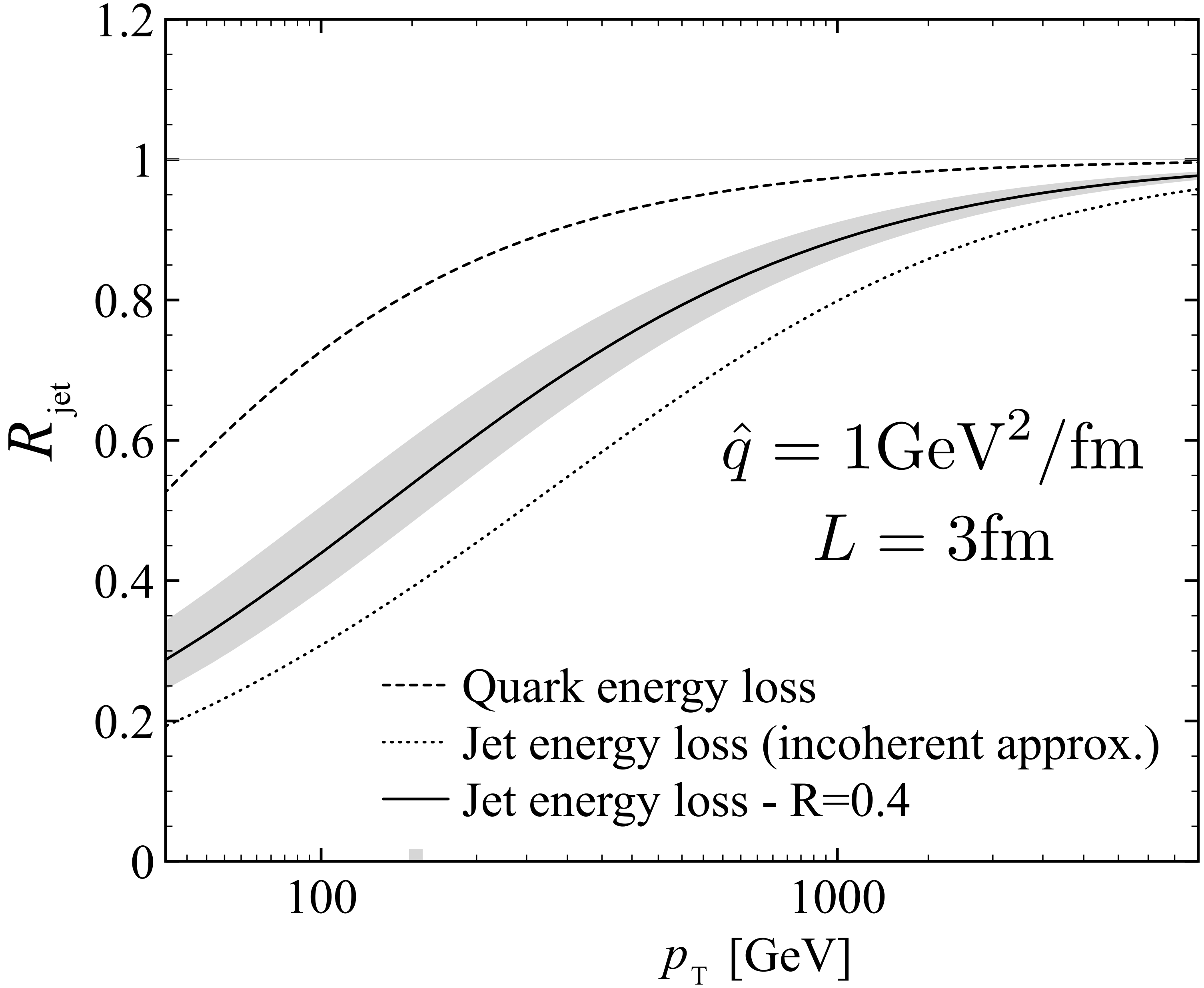
- The function C(p) obeys a **non-linear evolution equation** that resums the leading logarithms: arXiv:1707.07361 [hep-ph]

$$C_q(p_T, R) = 1 + \int_0^1 \mathrm{d}z \int_0^R \frac{\mathrm{d}\theta}{\theta} \frac{\alpha_s(k_\perp)}{\pi} P_{qg}(z)\ \Theta(t_\mathrm{f} < t_\mathrm{d} < L)$$
$$\times \left[\ C_q(zp_T, \theta)\ C_g(zp_T, \theta)\ \mathcal{Q}_q^2(p_T) -\ C_q(zp_T, \theta)\ \right]$$



Nuclear modification factor

$$R_{\rm jet} = Q_{\rm tot}(p_T) \times C(p_T)$$



Large contribution from fluctuating jet substructure

Summary

- Due to the large logarithmic phase-space for jets to branch inside a large medium higher-order corrections are found to be important ⇒ relevant for probing medium properties
- These corrections can be resumed to all orders to leading logarithmic accuracy by a non-linear evolution equation
- The effect of color coherence mitigates the importance of higher order corrections to the jet spectrum for narrow jets
- **Outlook:** Investigate infrared/collinear safe observables such as the jet mass. Develop a MC event generator.

Backup

In-medium gluon cascade

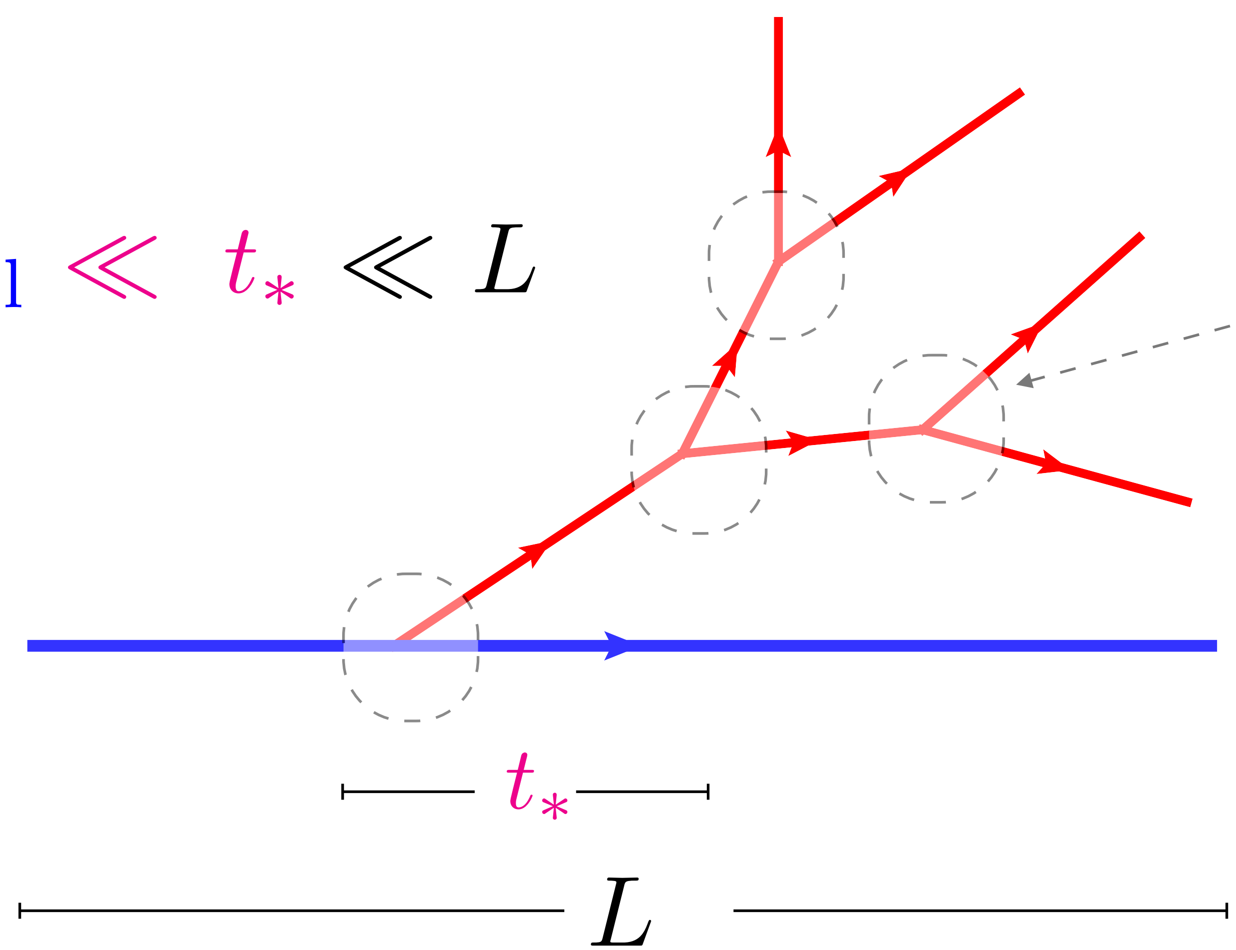
- **Probabilistic picture:** large probability for **soft, rapid and independent multiple gluon branching**

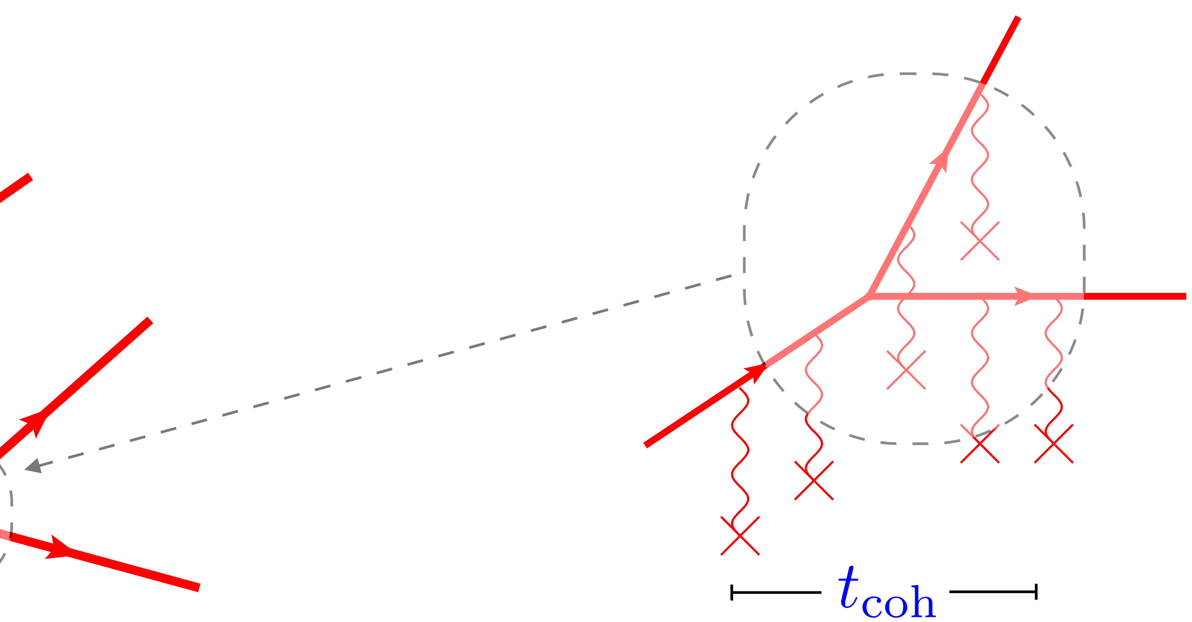
branching time:

$$t_*(\omega) = \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}$$

$$\omega \frac{\mathrm{d}P}{\mathrm{d}\omega \mathrm{d}t} \equiv \frac{\alpha_s}{t_{\mathrm{coh}}} \equiv \frac{1}{t_*}$$

$$t_{\mathrm{col}} \ll t_* \ll L$$





[Baier, Dokshitzer, Schiff, Mueller (2001)
Jeon, Moore (2003)
Blaizot, Dominguez, Iancu, MT (2013-2014)]

Energy flow at large angle

[Blaizot, Iancu, Fister, Torres, MT (2013-2014) Kurkela, Wiedemann (2014)]

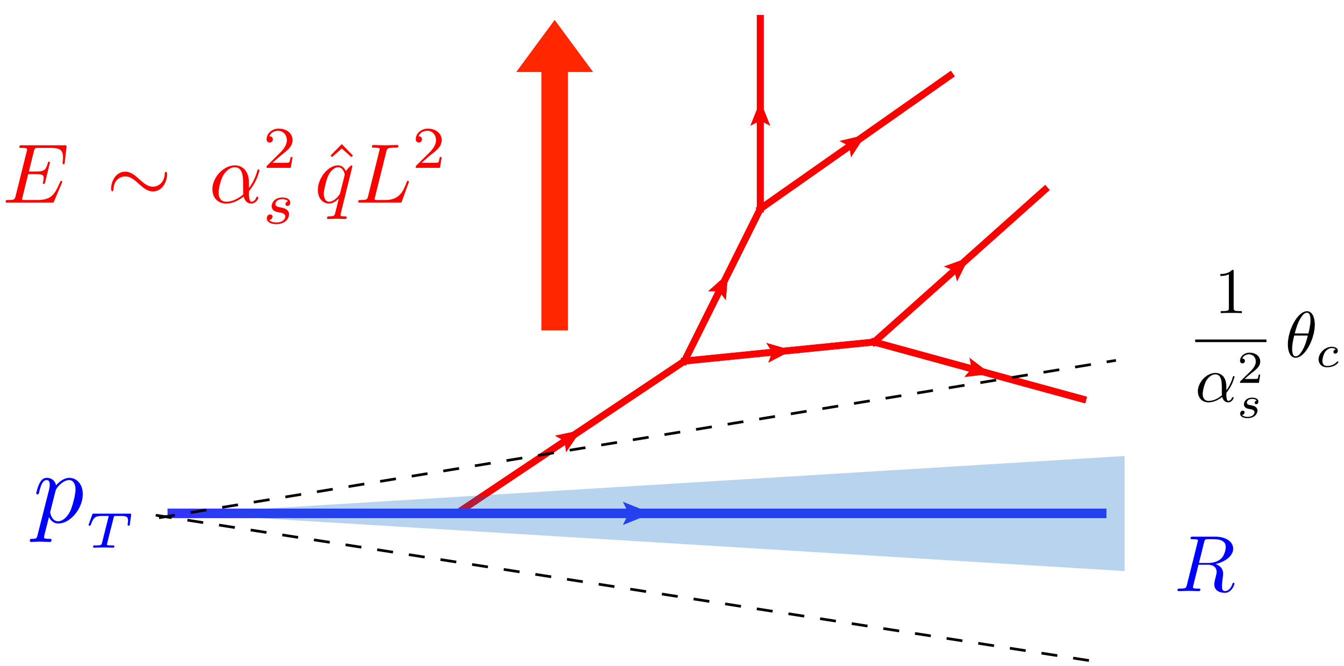
- Multiple branchings at parametrically large angle

$$\theta_{\rm br} \gg \frac{1}{\alpha_s^2}\theta_c \gg R$$

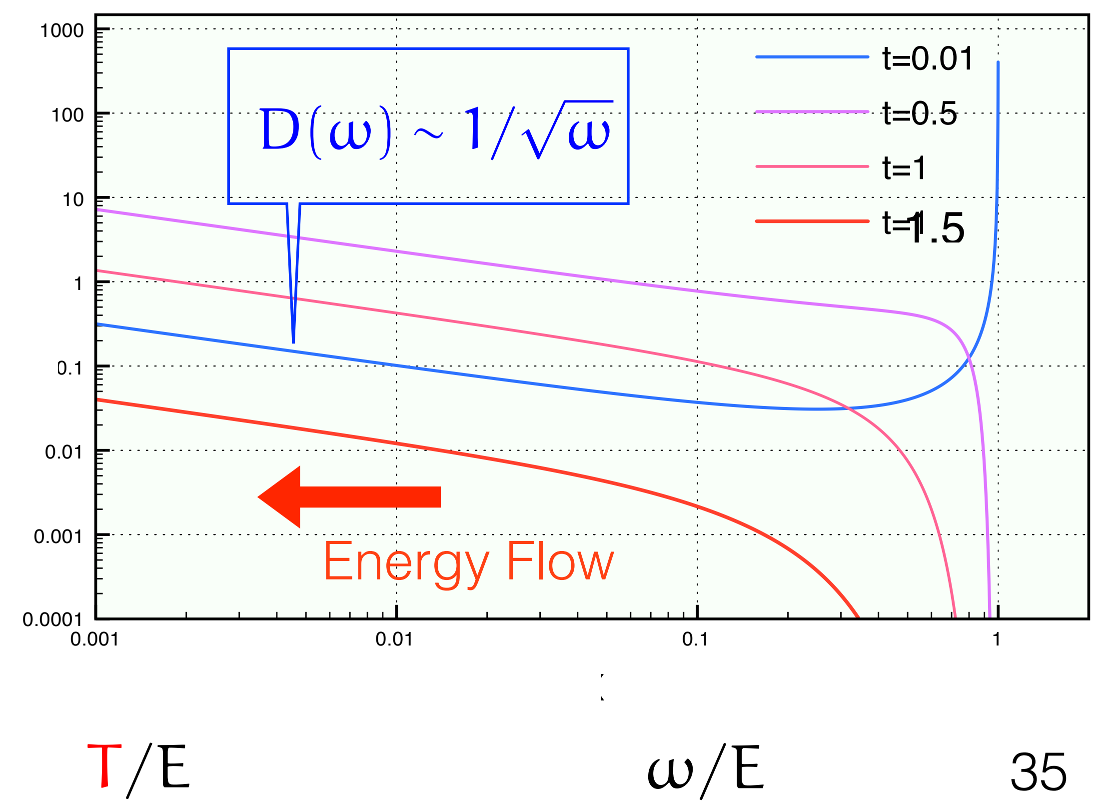
- Constant energy flow from jet energy scale p_T energy down to the medium temperature scale $\omega \sim T$ [Iancu, Wu (2015)]

Energy lost to the medium:

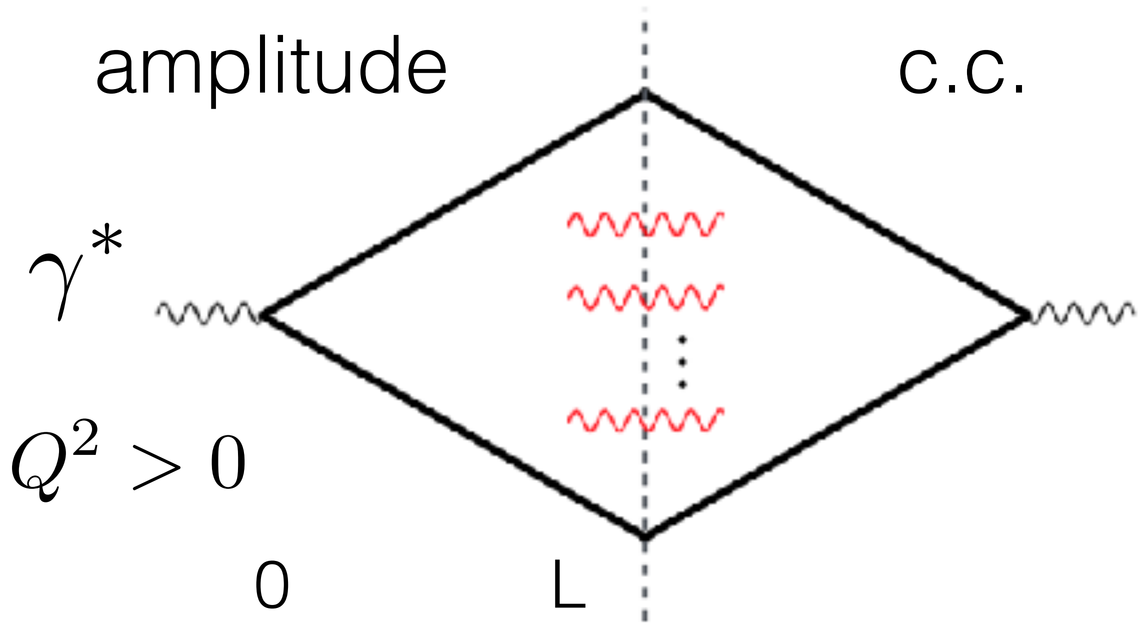
$$\Delta E \sim \alpha_s^2\, \hat{q} L^2$$

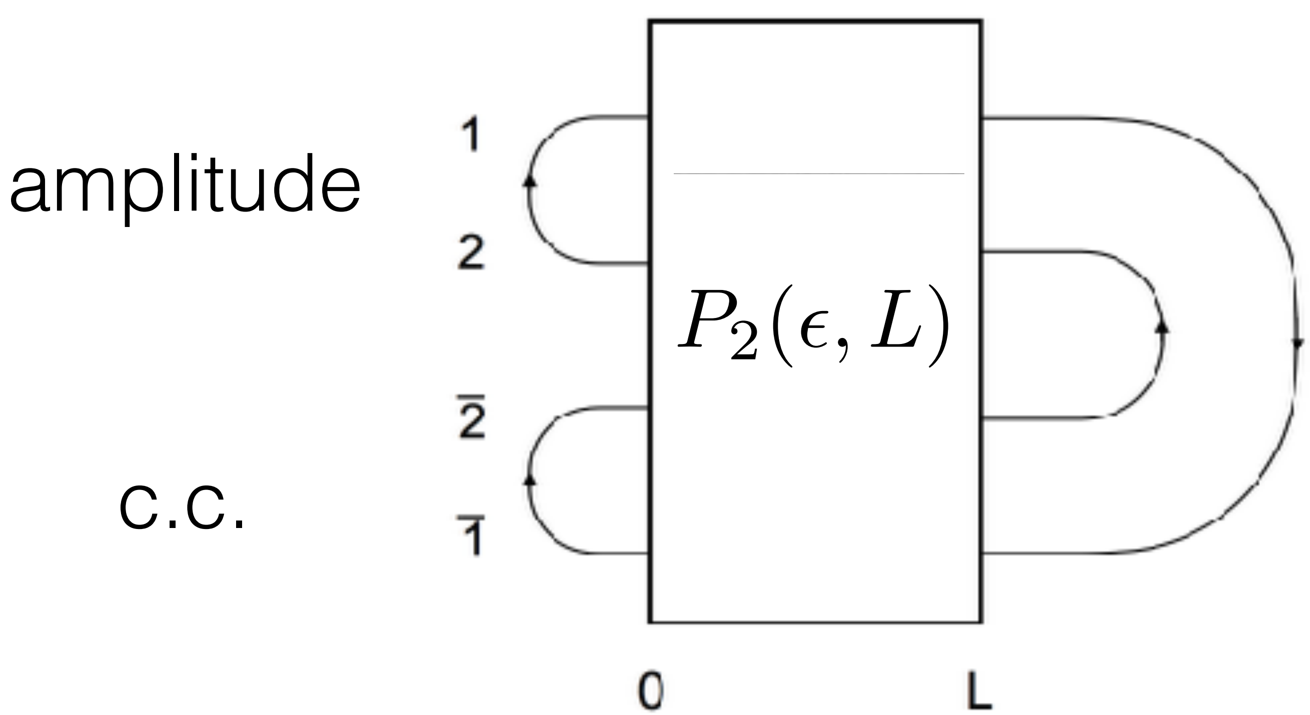


Energy distribution as function of time



Two-pronged energy loss



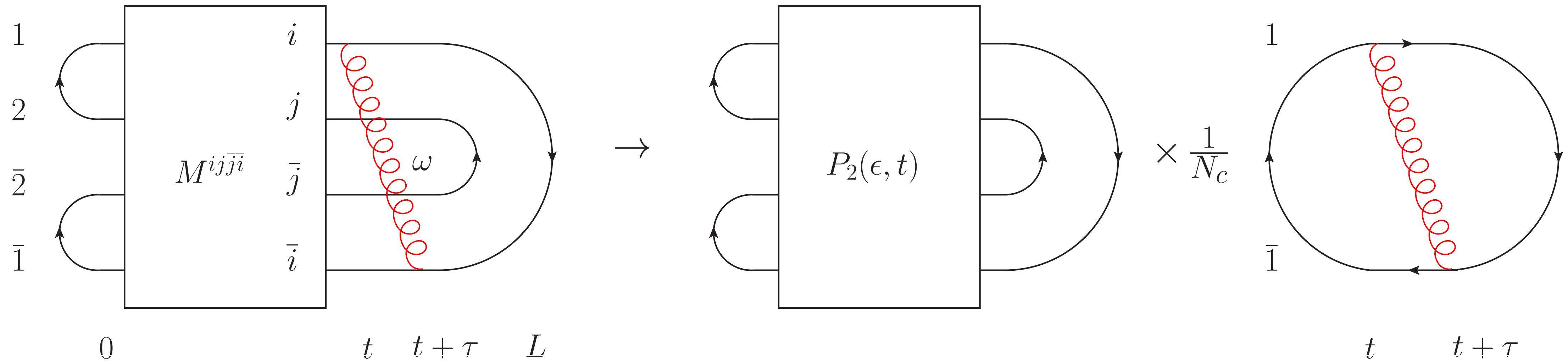


Caveats:

- large N_c
- resum medium-induced soft emissions
- short formation times
- color singlet: straightforward generalization to triplet and octet configurations

Two-pronged energy loss

- Direct emission term (diagonal contribution)



$$\Delta P_2(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega \, \Gamma_{11}(\omega, t) \, P_2(\epsilon - \omega, t)$$

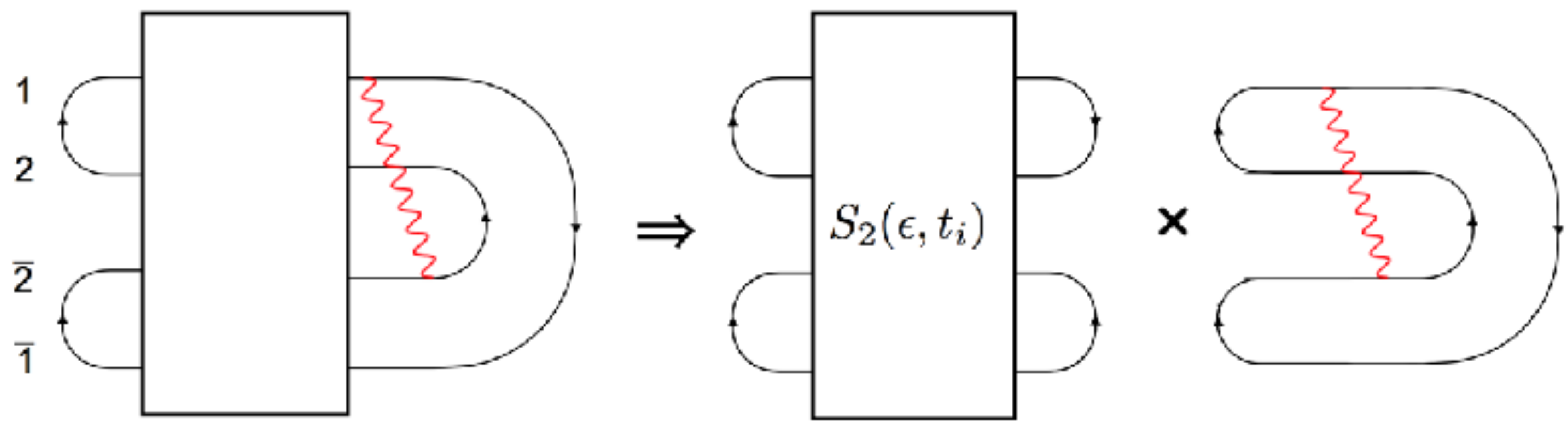
- Correction identical to single particle case:

$$\Gamma_{11}(\omega, t) \equiv \frac{\mathrm{d}I_{11}}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega) \int_0^\infty \mathrm{d}\omega' \frac{\mathrm{d}I_{11}}{\mathrm{d}\omega' \mathrm{d}t}$$

real virtual

Two-pronged energy loss

- Interferences and color flip (recall that all propagators are evaluated in the medium background field)



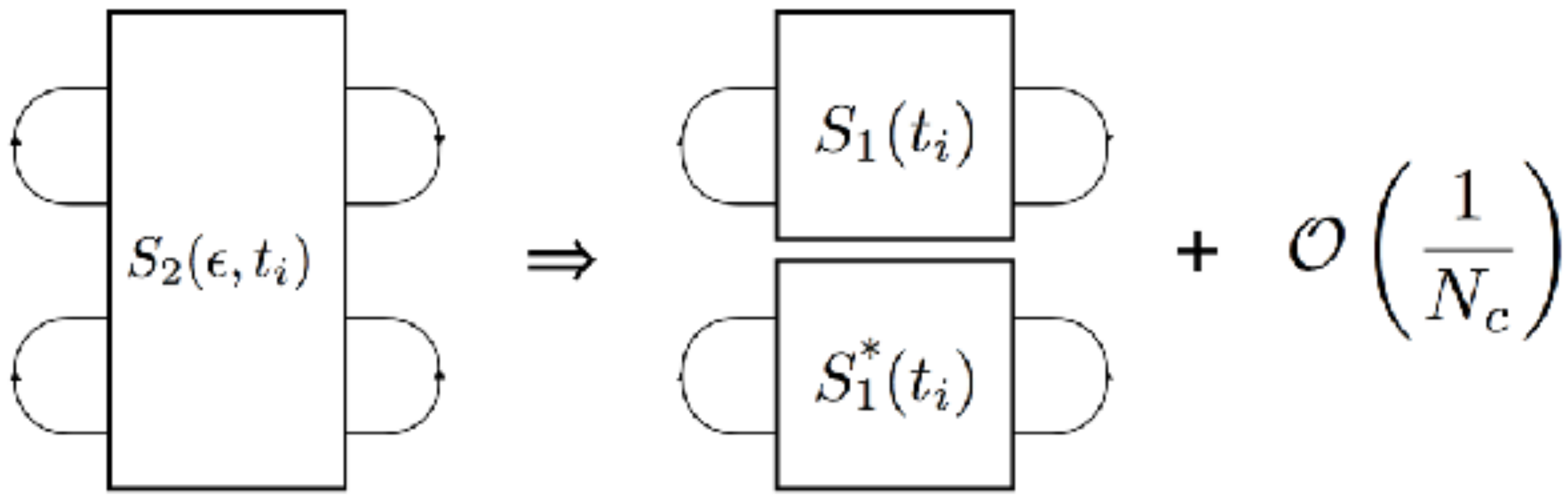
$$\Delta P_2(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega \, \Gamma_{12}(\omega, t) \; S_2(\epsilon - \omega, t)$$

- Involves new color structure

$$S_2 \sim \langle \mathrm{tr}(V_2^\dagger V_1) \, \mathrm{tr}(V_1^\dagger V_2) \rangle$$

Two-pronged energy loss

- In the Large N_c approximation



- Amplitude and c.c. are disconnected ⇒ only virtual emissions contribute

- In the absence of radiation we recover the decoherence parameter: $\Delta_{\rm med} \equiv 1 - S_1^2$

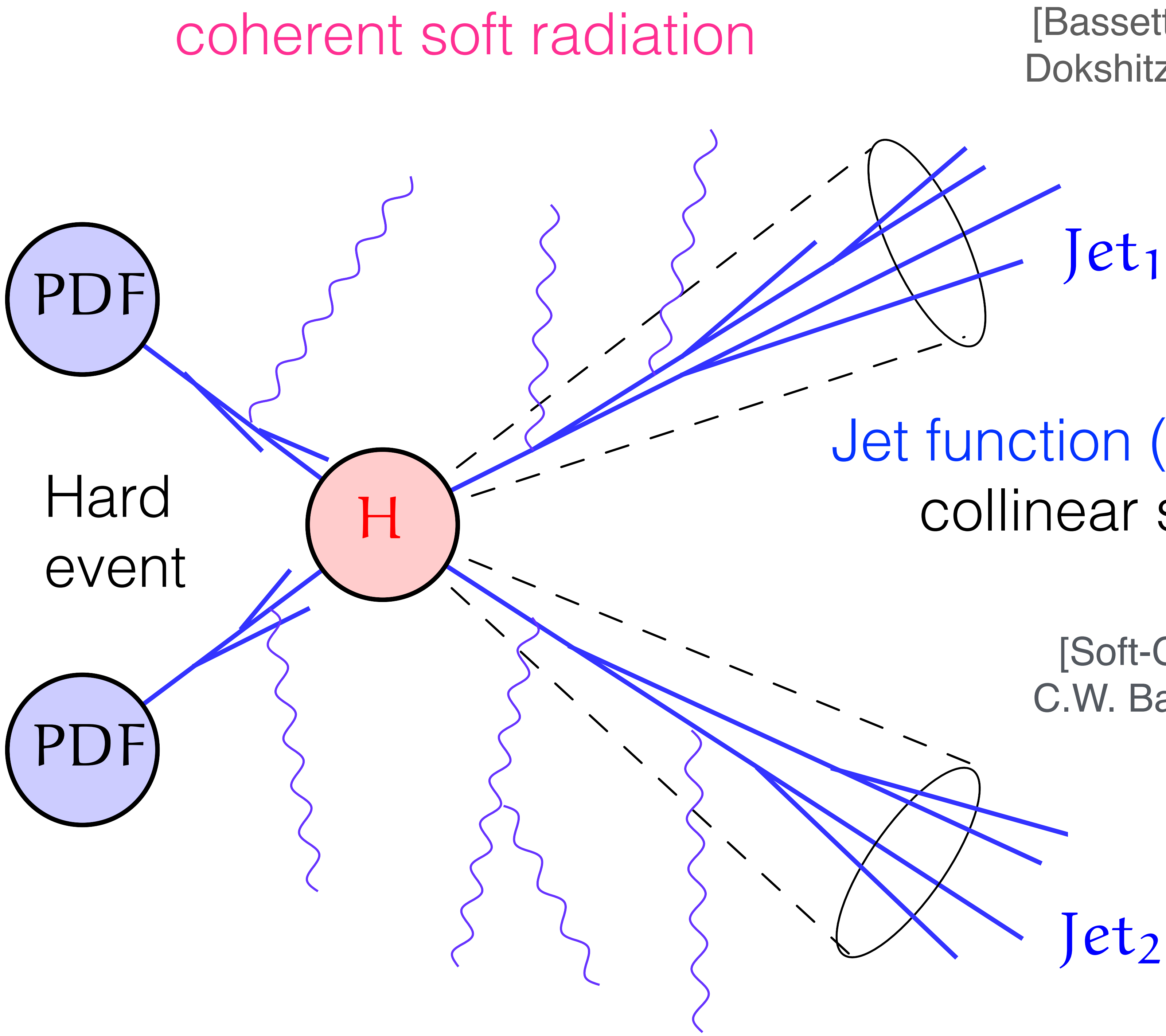
antenna transverse size

$$S_1(t) \equiv \langle {\rm tr} V_2^\dagger V_1 \rangle_{\rm med} \sim \exp\left[-\frac{1}{4}\,\hat{q} \int_0^t {\rm d}t'\, \boldsymbol{x}_{12}^2(t')\right]$$

[MT, Salgado, Tywoniuk, arXiv:1105.1346, MT, Salgado, Tywoniuk arXiv:1205.5739, Casalderrey-Solana, Iancu arXiv:1105:1760]

Jets in pQCD and color coherence

[Bassetto, Ciafaloni, Marchesini, Mueller, Dokshitzer, Khoze, Toyan, Collins, Soper, Sterman … 1980's]



[Soft-Collinear-Efffective Theory (SCET): C.W. Bauer, S. Fleming, D. Pirjol, and I.W. Stewart 2000-2002]

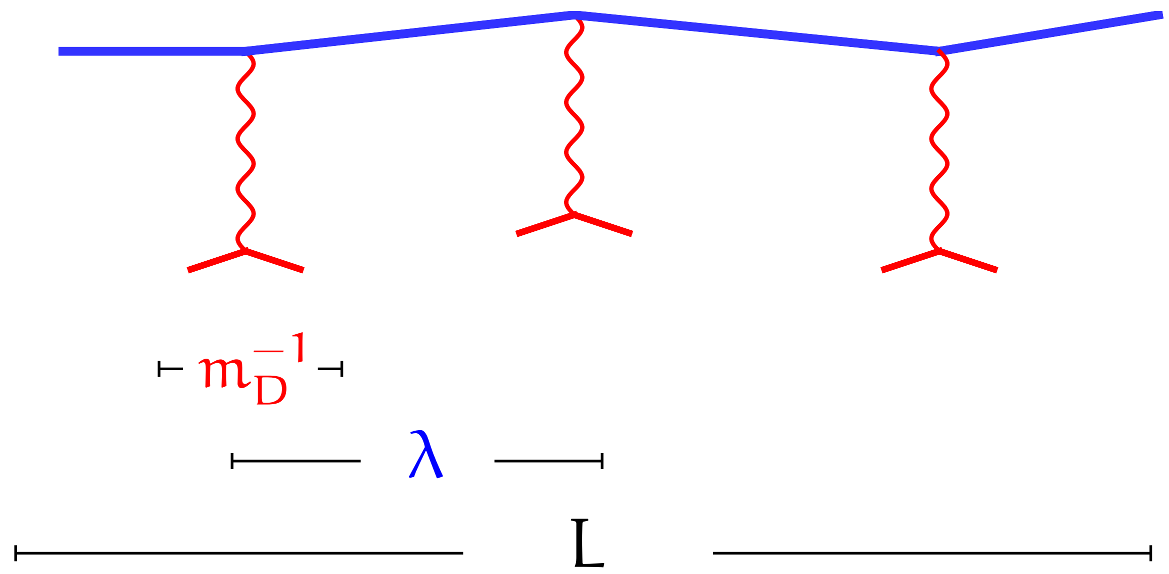
Factorization of the cross-section

$$\sigma \sim \mathrm{PDF} \times H \times J_1 \times J_2 \times S$$

The jet-quenching parameter

correlation length ≪ mean-free-path ≪ L

Momentum broadening
(diffusion in transverse
momentum space):



$$\langle k_\perp^2 \rangle \equiv \hat{q} L$$

- the jet-quenching $\hat{q}$ parameter encodes **medium properties** (LO: 2 to 2 elastic scattering):

$$\hat{q} \equiv n \int_{q_\perp} q_\perp^2 \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}q_\perp} \sim \alpha_s^2 \, C_R \, n \, \ln \frac{Q^2}{m_D^2}$$

estimate: $Q^2 \sim \hat{q} L \sim 10\,\mathrm{GeV}^2$