Jet quenching - theory

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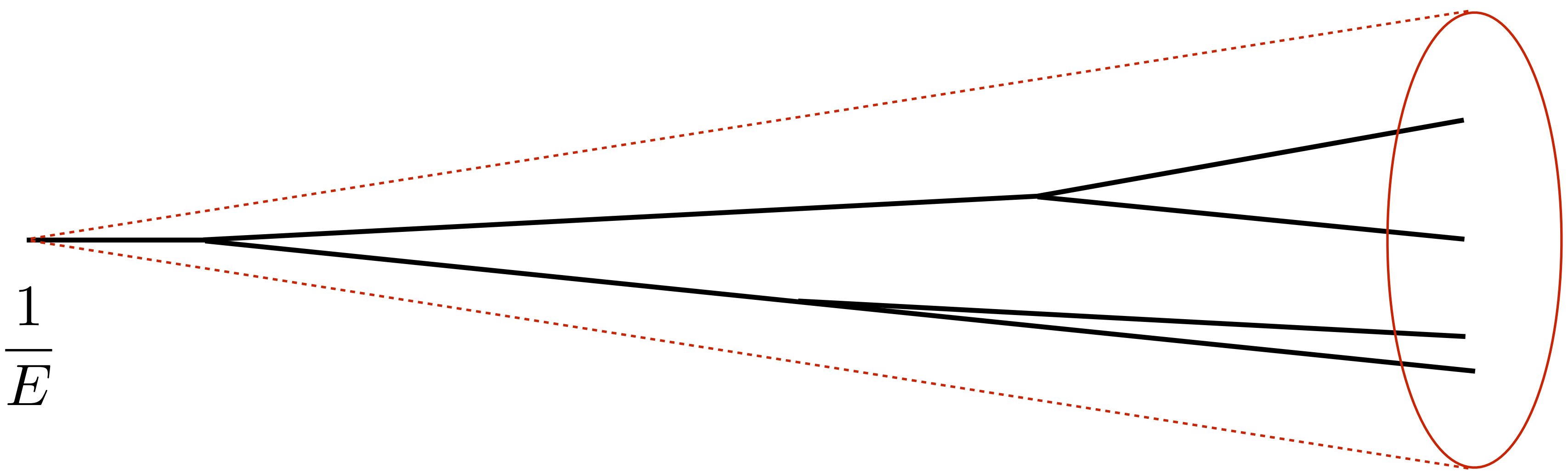
Motivation: QCD jets

- soft & collinear divergences: resummation
- color coherence: angular ordering
- strong separation of time scales (semi-classical)

$$t_f \sim \frac{k_\parallel}{k_\perp^2} \sim \frac{1}{E}$$



$$t_{\rm had} \sim \frac{E}{\Lambda^2_{\rm QCD}}$$

$\sim$ several fm

Dokshitzer, Khoze, Mueller, Troyan “Basics of Perturbative QCD” (1991)

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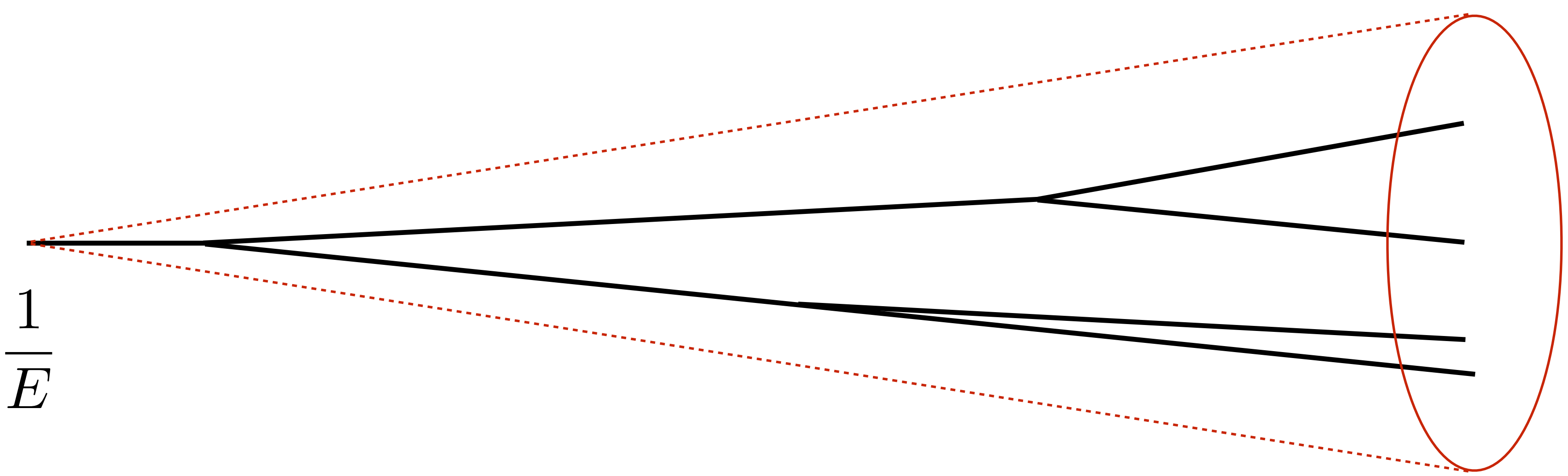

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Goal of the lectures:

how does this object lose energy in the medium?

what is the effect of **jet multi-parton fluctuations**?

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QUENCHING OF JETS

$$\frac{\mathrm{d}\sigma_{\mathrm{med}}}{\mathrm{d}p_T^2\,\mathrm{d}y} = \int_0^\infty \mathrm{d}\epsilon\, P(\epsilon)\, \frac{\mathrm{d}\sigma_{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}p_T^2\,\mathrm{d}y}$$

quenching weight: probability distribution of radiating energy out-of-cone

$$Q(p_T) = \left(\frac{\mathrm{d}\sigma_{\mathrm{med}}}{\mathrm{d}p_T^2\,\mathrm{d}y}\right) \Big/ \left(\frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}p_T^2\,\mathrm{d}y}\right)$$

quenching factor = nuclear modification factor

permits an expansion in the strong coupling constant (with hard scale)

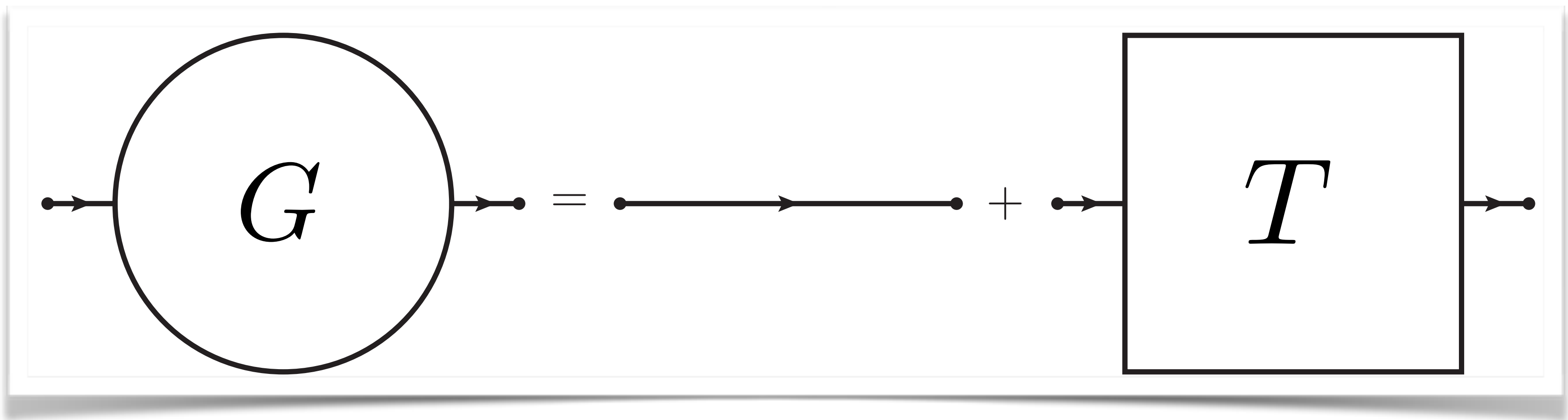
$$\frac{\mathrm{d}\sigma_{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}p_T^2\,\mathrm{d}y} \simeq \frac{\mathrm{d}\sigma_{\mathrm{vac}}(p_T)}{\mathrm{d}p_T^2\,\mathrm{d}y}\exp\left(-\frac{n\epsilon}{p_T}\right) \quad \Rightarrow \quad Q(p_T) = \tilde{P}\left(\frac{n}{p_T}\right)$$

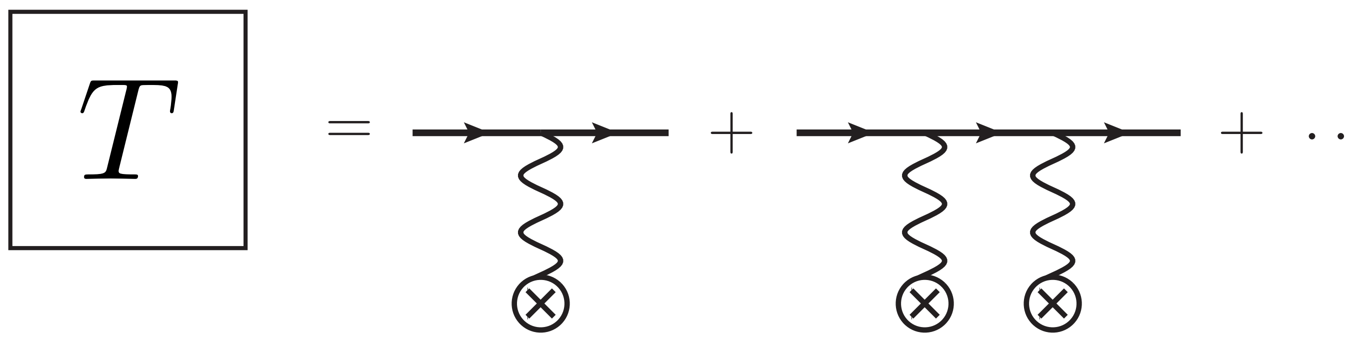
- related by Laplace transform…

OUTLINE

- in-medium parton propagation
- medium-induced radiation
- one- & multi-parton energy loss

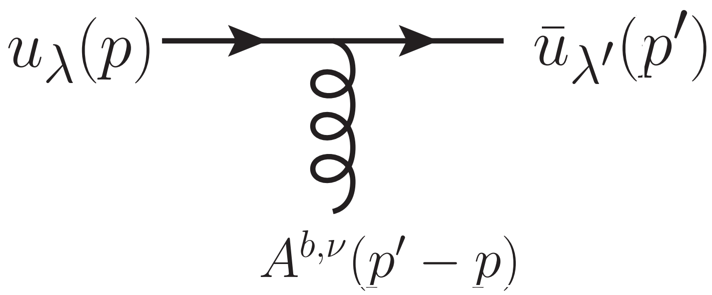
In-medium propagation

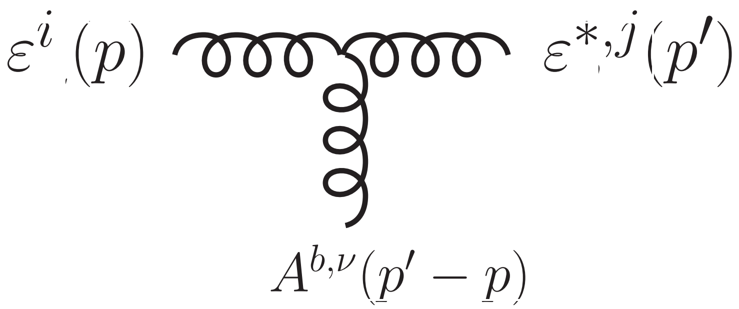




- dressed propagators: Dyson-Schwinger expansion
- high-energy approximation: only transverse momentum is exchanged with medium
  - elastic energy loss suppressed

INTERACTION WITH THE MEDIUM





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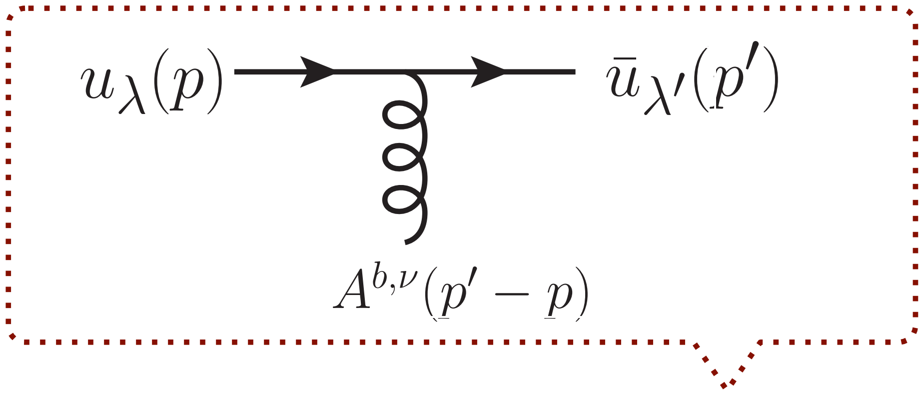

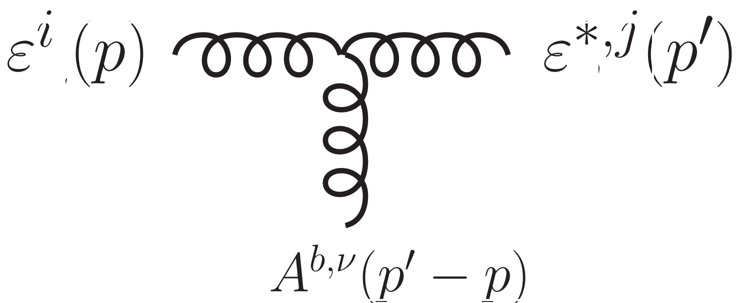

$$\bar{u}_{\lambda'}(p')\left[-ig\not{A}(p-p')\right]u_\lambda(p) \simeq -ig(2p^+)\,\delta_{\lambda,\lambda'}\mathbf{t}^b A^{-,b}(q)$$

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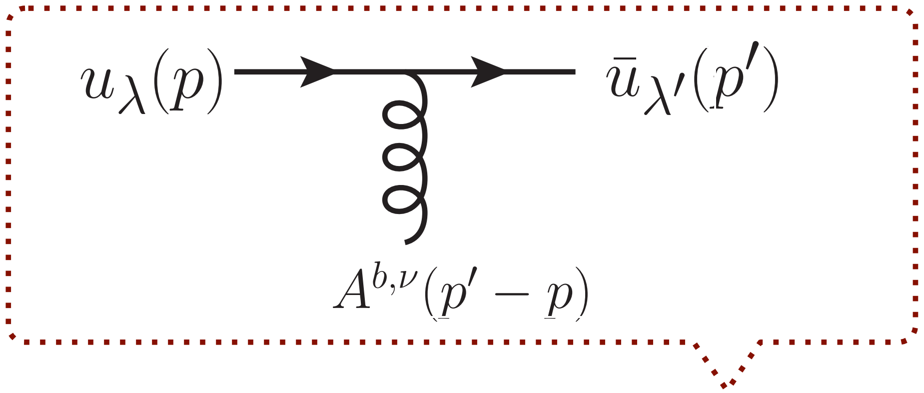

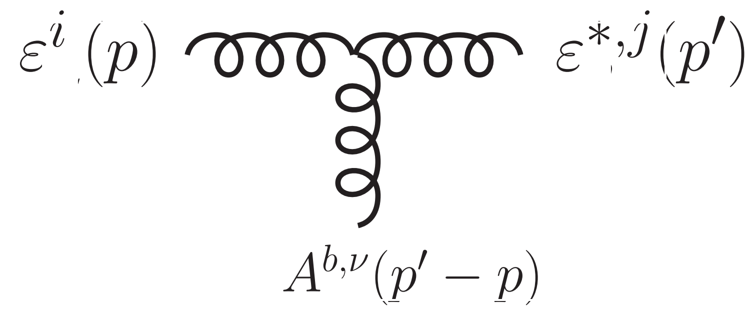

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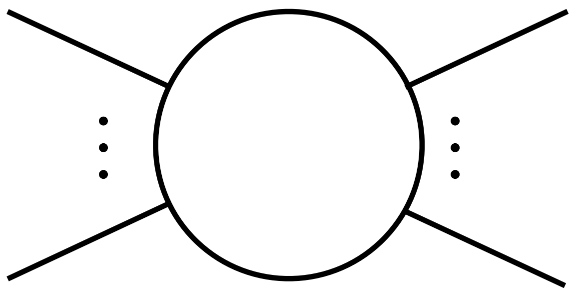
The same follows for the gluon vertex. We define interaction vertex

$$u^{ij}(p',p) = ig(2p^+)\left[\mathbf{T}\cdot A^-\right]\delta^{ij}$$

Notation for gluons: $[\mathbf{T}^b]^{ac} \equiv i f^{abc}$

This compact notation allows us to treat the propagation of quarks and gluons on an equal footing - differentiating only the color structure (high-energy approximation). We will only treat triple vertices for now.

LC DECOMPOSITION



In all vertices:

$$(2\pi)^4\delta(p_0 - p_1) = (2\pi)^3\delta(P_0 - P_1)\int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{-i(p_0^- - p_1^-)t}$$

$$P \equiv (p^+, \boldsymbol{p}) \equiv (E, \boldsymbol{p})$$

$$X \equiv (x^+, \boldsymbol{x}) \equiv (t, \boldsymbol{x})$$

Explicit introduction of "time" on behalf of 4-momentum conservation; now **"3(LC)"-momentum conserved**!

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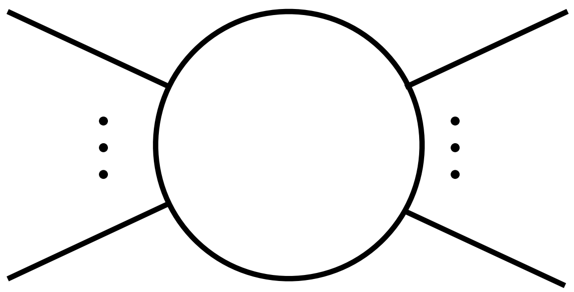

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$$A^-(q) = 2\pi\delta(q^+)\int_{-\infty}^{\infty} \mathrm{d}t\,\mathrm{e}^{-iq^- t} A^-(t, \boldsymbol{q})$$

Assumption: no longitudinal momentum exchange with the medium!
(medium is boosted in the opposite direction; has no extension in x^- coordinate)

Propagator: tensorial structure

Fundamental propagator(s) in vacuum:

$$S_0(k) = \sum_s u^s(k)\bar{u}^s(k)\, D_0(k) \qquad \Box_x D_0(x,y) = i\delta(x-y)$$

$$G_0(k) = d^{\mu\nu}(k)\, D_0(k) \qquad D_0(k) = -i/(k^2 + i\epsilon)$$

In LC gauge:

$n \cdot A \equiv A^+ = 0$

$$d^{\mu\nu}(k) = \sum_{\lambda=\pm 1,0} (-1)^{\lambda+1} \varepsilon^{\mu}_{\lambda}(k)\varepsilon^{*\nu}_{\lambda}(k) = g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n}$$

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Dyson-Schwinger construction

$$G^{\mu\nu}(p',p) = (2\pi)^4\delta(p-p')G_0^{\mu\nu}(p) + G_0^{\mu\alpha}(p')T_{\alpha\beta}(p',p)G_0^{\beta\nu}(p)$$

$$T_{\mu\nu}(p',p) = u_{\mu\nu}(p',p) + u_{\mu\alpha}(p',p'')G_0^{\alpha\beta}(p'')u_{\beta\nu}(p'',p) + \ldots$$

DRESSED PROPAGATOR

Since the only components are: $T^{ij} = \delta^{ij} T \equiv \delta^{ij} \left(T^{kk}/2 \right)$

$$G^{\mu\nu}(p',p) = d^{\mu i}(p') d^{j\nu}(p) \left[-\delta^{ij} G(p',p) \right] + \text{instantaneous term}$$

Only transverse (physical) degrees of freedom are propagated. Problem reduces to finding the **dressed scalar propagator** that also follows from a DS equation. For "stitching together" Feynman diagrams, the mixed representation is very useful:

$$\begin{aligned} \mathcal{G}(t',E',\boldsymbol{p}';t,E,\boldsymbol{p}) &= -2E \int \frac{\mathrm{d}p'^-}{2\pi} \int \frac{\mathrm{d}p^-}{2\pi} \mathrm{e}^{+ip^- t - ip'^- t'} G(p',p) \\ &= 2\pi\delta(E-E') \mathcal{G}(t',\boldsymbol{p}';t,\boldsymbol{p}|E) \end{aligned}$$

where $\mathcal{G}_0(t,\boldsymbol{p}) \equiv -2E \int \frac{\mathrm{d}p^-}{2\pi} \mathrm{e}^{-p^- t} D_0(p) = \Theta(t) \mathrm{e}^{i \frac{\boldsymbol{p}^2}{2E} t}$ is the retarded part ($E>0$).

Dressed propagator: final form

In mixed representation ($\int_{\boldsymbol{q}} \equiv \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}$):

$$\mathcal{G}(t',\boldsymbol{p}';t,\boldsymbol{p}) = (2\pi)^2\delta(\boldsymbol{p}-\boldsymbol{p}')\mathcal{G}_0(t'-t,\boldsymbol{p})$$

$$+ (ig)\int_t^{t'} \mathrm{d}s \int_{\boldsymbol{q}} \mathcal{G}_0(t'-s,\boldsymbol{p}')[\mathbf{T}\cdot A^-](s,\boldsymbol{q})\mathcal{G}(s,\boldsymbol{p}'+\boldsymbol{q};t,\boldsymbol{p})$$

In coordinate representation ($\int_X \equiv \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^2\boldsymbol{x}$):

$$\mathcal{G}(X;Y) = \mathcal{G}_0(X;Y) + (ig)\int_Z \mathcal{G}_0(X;Z)[\mathbf{T}\cdot A^-](Z)\mathcal{G}(Z;Y)$$

DRESSED PROPAGATOR: FINAL FORM

In mixed representation ($\int_{\boldsymbol{q}} \equiv \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}$):

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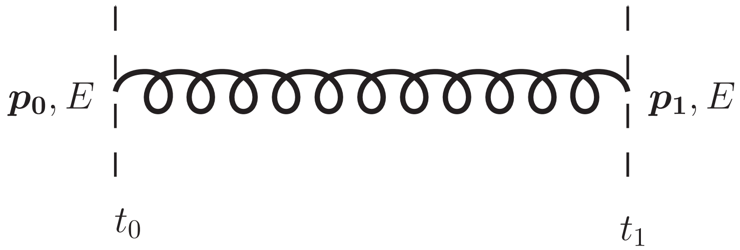
$$\mathcal{G}(X;Y) = \int_{\boldsymbol{r}(t)=\boldsymbol{y}}^{\boldsymbol{r}(t')=\boldsymbol{x}} \mathcal{D}\boldsymbol{r} \exp\left[i\frac{E}{2}\int_t^{t'} \mathrm{d}s\, \dot{\boldsymbol{r}}^2\right] U(t',t;[\boldsymbol{r}(s)])$$

$$U(t',t;[\boldsymbol{r}(s)]) = \mathcal{P}_+ \exp\left[ig\int_t^{t'} \mathrm{d}s\, \mathbf{T}\cdot A^-(s,\boldsymbol{r})\right]$$

Propagator involves a Wilson line.

IN-MEDIUM FEYNMAN RULES

Propagator



$$= \frac{1}{2E}\mathcal{G}(t',\boldsymbol{p}';t,\boldsymbol{p}|E)$$

[Propagators going out of the medium are easily found.]

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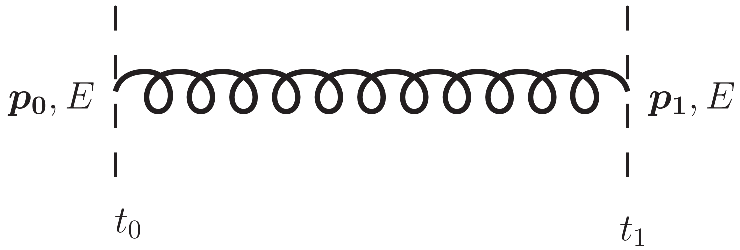

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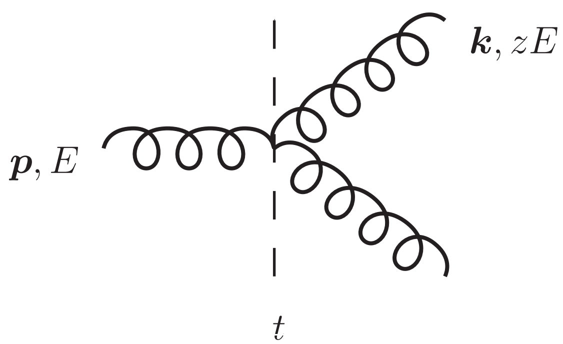
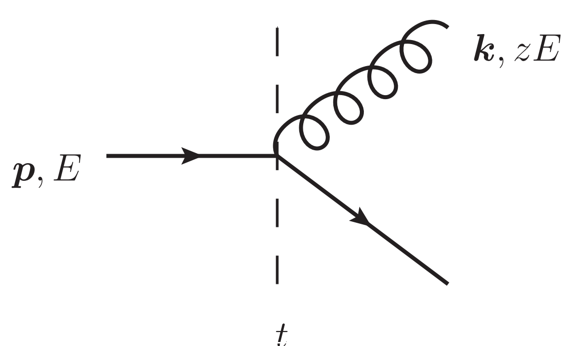
Vertices: conservation of 3-mom & integrate out time



$$= 2g f^{abc}\left[\frac{1}{1-z}\boldsymbol{\kappa}^k\delta^{ij} + \frac{1}{z}\boldsymbol{\kappa}^j\delta^{ik} - \boldsymbol{\kappa}^i\delta^{jk}\right]$$



$$= -2g\,\mathbf{t}^a\frac{1}{z\sqrt{1-z}}\delta_{ss'}\left(\delta_{\lambda,s} + (1-z)\delta_{\lambda,-s}\right)\boldsymbol{\kappa}^i$$

$$\boldsymbol{\kappa} = \boldsymbol{k} - z\boldsymbol{p}$$

[Only transverse degrees of freedom propagating = transverse vertices]

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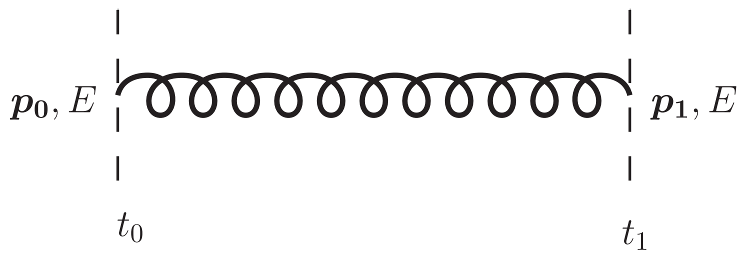

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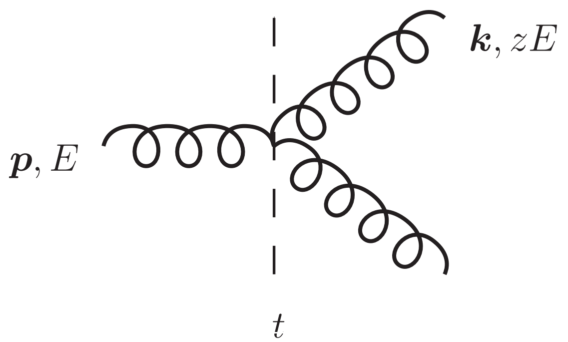

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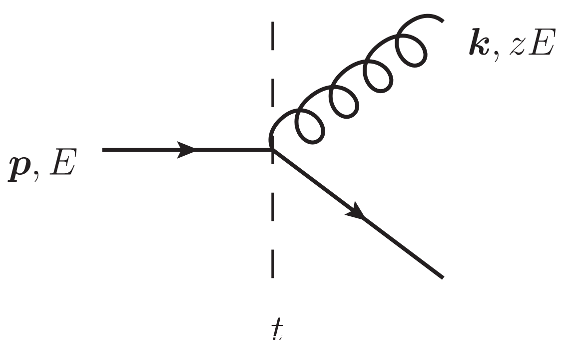

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High-energy expansion

The classical path between the endpoints is simply $\boldsymbol{x}^{\rm cl}(s) = \boldsymbol{y} + \frac{s-t}{t'-t}(\boldsymbol{x}-\boldsymbol{y})$

Expanding around this trajectory, the zeroth term reads:

$$\mathcal{G}(X,Y) \approx \mathcal{G}_0(X,Y)U(t',t;[\boldsymbol{x}^{\rm cl}(s)])$$

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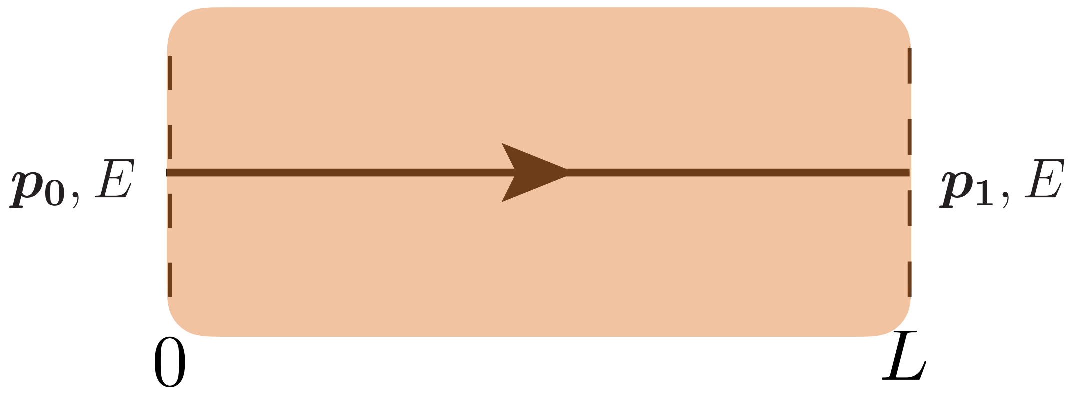
In mixed representation, E≫1/(t'-t) ("localization"):

$$\mathcal{G}(t',\boldsymbol{p}';t,\boldsymbol{p}) = (2\pi)^2\delta(\boldsymbol{p}-\boldsymbol{p}')\mathcal{G}_0(t'-t,\boldsymbol{p})U(t',t;[\boldsymbol{n}s])$$

where $\boldsymbol{n} \equiv \boldsymbol{p}/E$

We call this the **"tilted" Wilson line**. We use these propagators to describe hard (energetic, "vacuum-like") particles that act as sources for medium-induced radiation.

BROADENING I



Our first Feynman diagram! 😀

$$\mathcal{M}^i(P) = \int_{\boldsymbol{p}_0} \mathcal{G}(L,\boldsymbol{p};0,\boldsymbol{p}_0)^{ij}\mathcal{M}^j(P_0)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_p} = \frac{1}{N_c} n_f \sum_s \langle|\mathcal{M}|^2\rangle$$

$$\mathrm{d}\Omega_p = \frac{1}{4\pi}\frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}^2\boldsymbol{p}}{(2\pi)^2}$$

The cross section involves averaging out medium fluctuations. Need knowledge about the 2-point correlator in the medium:

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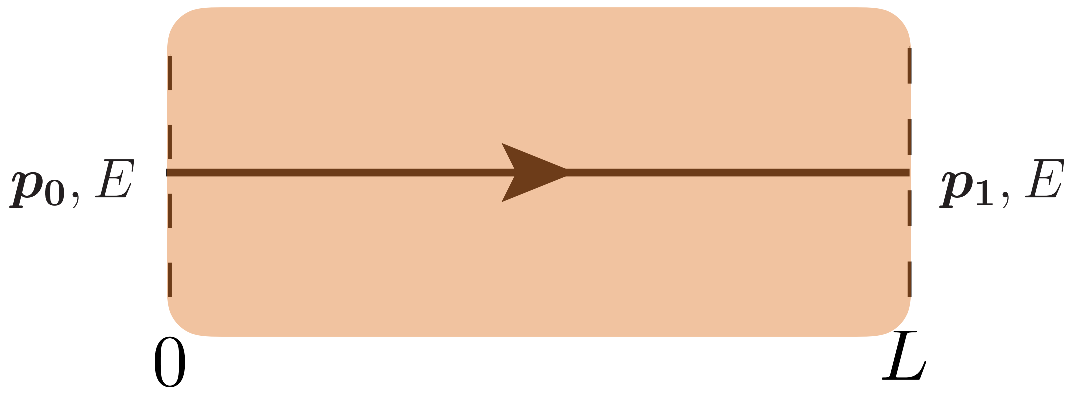

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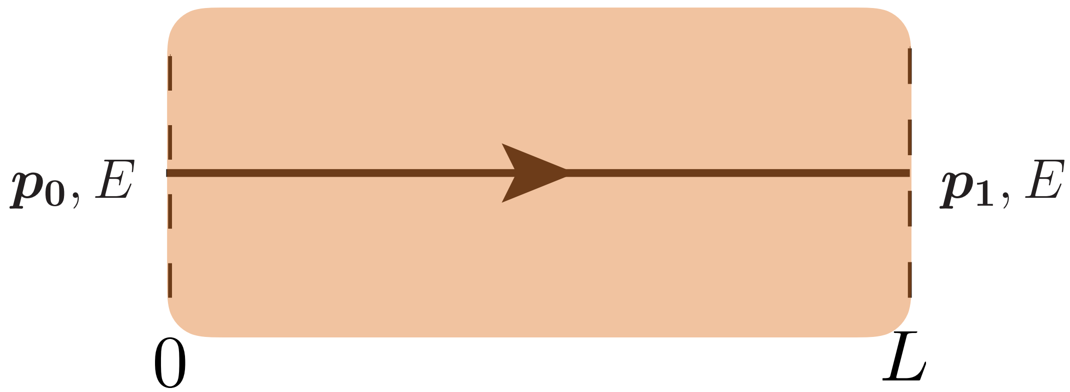
$$\langle A^{a,-}(t',\boldsymbol{q}')A^{*b,-}(t,\boldsymbol{q})\rangle = \delta^{ab} n(t)\delta(t-t')(2\pi)^2\delta(\boldsymbol{q}-\boldsymbol{q}')\gamma(\boldsymbol{q})$$

[Background field is real!]

Broadening I



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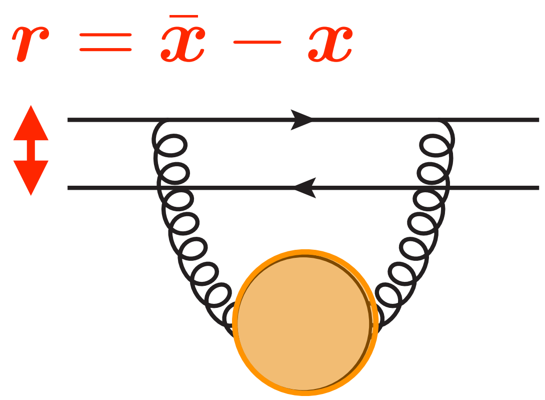
[Background field is real!]

The calculation involves the 2-point function that depends on the broadening probability distribution:

$$S^{(2)}(t',t) \equiv \frac{1}{N_c}\langle \mathrm{tr}\,\mathcal{G}(t',t)\mathcal{G}^\dagger(t',t)\rangle = (2\pi)^2\delta(\boldsymbol{p}_0-\bar{\boldsymbol{p}}_0)\mathcal{P}(\boldsymbol{p}-\boldsymbol{p}_0,t'-t)$$

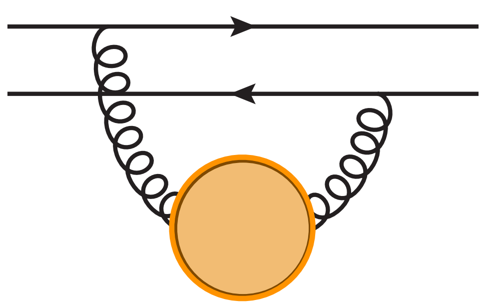
BROADENING 2

$$\mathcal{P}(\boldsymbol{p},t) = \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\, \frac{1}{N_c} \langle \mathrm{tr}\, U(\boldsymbol{x}) U^\dagger(\bar{\boldsymbol{x}}) \rangle$$



$$\sim -\frac{g^2\, \mathrm{tr}(\mathbf{t}^a \mathbf{t}^b)}{2N_c} \int \mathrm{d}s \int \mathrm{d}s' \int_{\boldsymbol{q},\boldsymbol{q}'} \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x} - i\boldsymbol{q}'\cdot\boldsymbol{x}} \langle A^a(s,\boldsymbol{q}) A^b(s',\boldsymbol{q}') \rangle$$

$$\sim -g^2 C_F \int \mathrm{d}s\, n\gamma(0)$$



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$$\sim g^2 C_F \int \mathrm{d}s\, n\gamma(\boldsymbol{r})$$

We ultimately recover the **dipole scattering rate**:

$$\frac{1}{N_c} \langle \ldots \rangle = \mathrm{e}^{-g^2 N_c \int \mathrm{d}s\, n[\gamma(0) - \gamma(\boldsymbol{r})]} = \mathrm{e}^{-\frac{1}{2} \int \mathrm{d}s\, \sigma(s,\boldsymbol{r})}$$

BROADENING AND QHAT

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_p} = \int_{\boldsymbol{p}_0} \mathcal{P}(\boldsymbol{p} - \boldsymbol{p}_0, L) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{p_0}}$$

What is the form of the broadening probability distribution?

Broadening and qhat

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What is the form of the broadening probability distribution?

$$\sigma(\boldsymbol{r}) \simeq \frac{1}{2}\hat{q}\boldsymbol{r}^2 \qquad \Rightarrow \qquad \mathcal{P}(t,\boldsymbol{p}) = \frac{4\pi}{\hat{q}t}\mathrm{e}^{-\frac{\boldsymbol{p}^2}{\hat{q}t}}$$

"harmonic oscillator"/"dipole" approximation

Medium scale: $Q_s^2 \equiv \hat{q}L$

Broadening and qhat

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_p} = \int_{\boldsymbol{p}_0} \mathcal{P}(\boldsymbol{p} - \boldsymbol{p}_0, L) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{p_0}}$$

What is the form of the broadening probability distribution?

$$\sigma(\boldsymbol{r}) \simeq \frac{1}{2}\hat{q}\boldsymbol{r}^2 \quad \Rightarrow \quad \mathcal{P}(t, \boldsymbol{p}) = \frac{4\pi}{\hat{q}t} \mathrm{e}^{-\frac{\boldsymbol{p}^2}{\hat{q}t}}$$

"harmonic oscillator"/"dipole" approximation

Medium scale: $Q_s^2 \equiv \hat{q}L$

In this scheme, q̂ is a transport coefficient describing diffusion in space transverse to the beam/projectile.

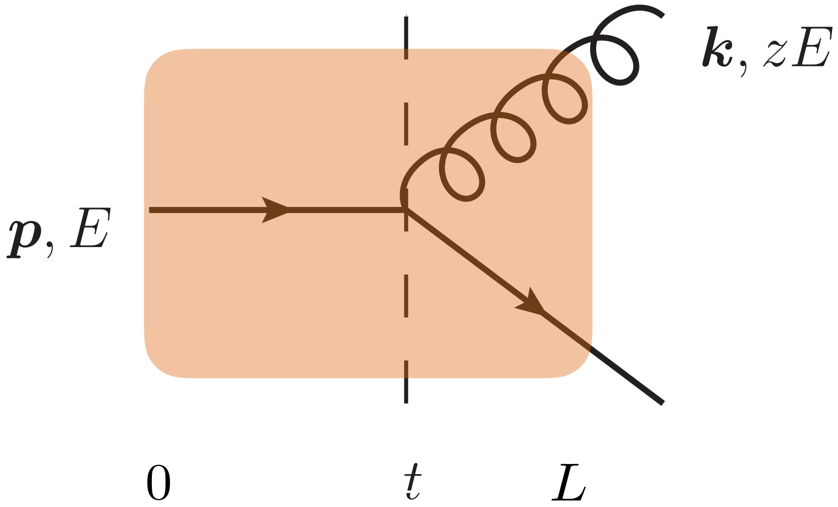
$$\hat{q} \sim g^4 C_R n \int_{\boldsymbol{q}} \boldsymbol{q}^2 \gamma(\boldsymbol{q}) \sim g^4 T^3 \ln \frac{1}{\boldsymbol{r}^2 m_{\mathrm{D}}^2}$$

No energy loss?!

In the high-energy limit, radiative processes are responsible for energy being redistributed among many fragments.

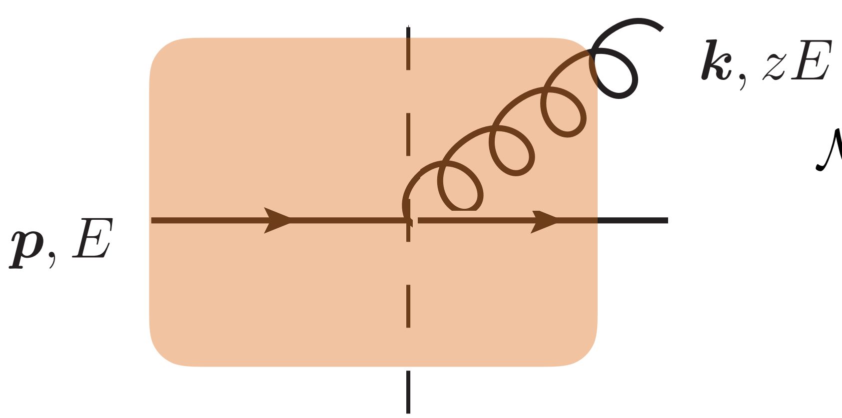
SOFT GLUON RADIATION AMPLITUDE

Expression is easy to write using the Feynman rules derived earlier:



$$\mathcal{M}^{a,i}(p,k) = \int_{\boldsymbol{k}',\boldsymbol{p}',\boldsymbol{p}_0} \int_0^\infty \mathrm{d}t\, \mathcal{G}(L,\boldsymbol{k};t,\boldsymbol{k}'|zE)^{ab} \frac{1}{2E} \times \left[\mathcal{G}(L,\boldsymbol{p};t,\boldsymbol{p}'-\boldsymbol{k}'|(1-z)E)\, V^{i,b}(\boldsymbol{k}'-z\boldsymbol{p}',z)\, \mathcal{G}(t,\boldsymbol{p}';0,\boldsymbol{p}_0|E)\right]^{ij} \times \mathcal{M}^j(p_0)\,.$$

For our purposes (energy loss), we will derive the rate of emission of soft gluons! For energetic quarks, we use the "tilted" Wilson lines, and find



$$\mathcal{M}^{(a,i)}(p,k) = \frac{g}{\omega} \int_0^L \mathrm{d}t\; \mathrm{e}^{i\frac{\omega}{2}\boldsymbol{n}^2 t}\, (\boldsymbol{\partial}_x + i\omega\boldsymbol{n}) \cdot \boldsymbol{\epsilon}^*_\lambda\; \mathcal{G}^{ab}(\boldsymbol{k},L;\boldsymbol{x},t)\Big|_{\boldsymbol{x}=\boldsymbol{n}t} \times \left[V(L,t)\mathbf{t}^b\, V(t,0)\right]^{ij} \mathcal{M}^j_0$$

We have used the vertex in the limit z≪1, and V are Wilson lines in the fundamental representation that are tracing the quark trajectory

CROSS SECTION

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_k\,\mathrm{d}\Omega_p} = n_f \left\langle \frac{1}{2N_c} \sum_{\lambda,s} |\mathcal{M}^{(a,i)}(p,k)|^2 \right\rangle$$

We will not be interested in the transverse momentum of the emitted gluon, assuming that it is sufficiently soft to be radiated at large angles. Then

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega \mathrm{d}\Omega_p} \simeq \frac{\mathrm{d}I}{\mathrm{d}\omega}\,\frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}\Omega_p}\,,$$

$$\frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{g^2}{8\pi N_c\,\omega^3} 2\mathrm{Re} \int_0^L \mathrm{d}\bar{t} \int_0^{\bar{t}} \mathrm{d}t\, \mathrm{e}^{i\frac{\omega}{2}\boldsymbol{n}^2(t-\bar{t})} \left(\boldsymbol{\partial}_{\boldsymbol{x}} + i\omega\boldsymbol{n}\right) \cdot \left(\boldsymbol{\partial}_{\bar{\boldsymbol{x}}} - i\omega\boldsymbol{n}\right) \times \left\langle \mathcal{G}^{\bar{b}b}(\bar{\boldsymbol{x}},\bar{t};\boldsymbol{x},t)\, U^{\bar{b}b}_{\boldsymbol{x}}(\bar{t},t) \right\rangle_{\boldsymbol{x}=\boldsymbol{n}t,\bar{\boldsymbol{x}}=\boldsymbol{n}\bar{t}}$$

where we used that (Fierz) $\mathrm{tr}\left(V^\dagger(\bar{t},t)\mathbf{t}^{\bar{b}} V(\bar{t},t)\mathbf{t}^b\right) = \frac{1}{2} U^{\bar{b}b}(\bar{t},t)$

This spectrum does not depend directly on the energy of the projectile (eikonal limit), and it is easy to demonstrate its independence on $\boldsymbol{n}$ as well.

TWO-POINT FUNCTION

$$S^{(2)}(\mathcal{X}_f;\mathcal{X}_i) = \frac{1}{N_c^2-1}\langle \mathrm{Tr}\, U_1^\dagger \mathcal{G}(\bar t, \boldsymbol{z}_f; t, \boldsymbol{z}_i)\rangle$$
$$= \int \mathcal{D}\boldsymbol{r}\, \exp\left\{ \frac{i\omega}{2}\int_t^{\bar t} \mathrm{d}s\, \dot{\boldsymbol{r}}^2(s) - \frac{1}{2}\int_t^{\bar t}\mathrm{d}s\, \sigma(\boldsymbol{r}-\boldsymbol{x}_1)\right\}$$
$$= \underbrace{\int \mathcal{D}\boldsymbol{r}\, \exp\left\{ \int_t^{\bar t} \mathrm{d}s\left[\frac{i\omega}{2}\dot{\boldsymbol{r}}^2(s) - \frac{1}{2}\sigma(\boldsymbol{r})\right]\right\}}_{\mathcal{K}(\boldsymbol{x},\boldsymbol{y})} \times \text{phases}$$

$$\frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_F}{\omega_3}\, 2\mathrm{Re}\int_0^L \mathrm{d}\bar t \int_0^{\bar t}\mathrm{d}t\, \boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{\partial}_{\boldsymbol{y}}\mathcal{K}(\boldsymbol{x},\boldsymbol{y})\big|_{\boldsymbol{x}=\boldsymbol{y}=0}$$

The function $\mathcal{K}$ is suppressed at a characteristic time scale, called the branching time. Assuming $t_f \sim \sqrt{(\omega/\hat q)} \ll L$, we can use the following trick do define a rate!

$$\int_t^L \mathrm{d}\bar t = \int_0^{L-t}\mathrm{d}\tau \approx \int_0^\infty \mathrm{d}\tau$$

SOME DETAILS

Explicitly, in the harmonic oscillator approximation:

$$\mathcal{K}(\boldsymbol{x}_f, \boldsymbol{x}_i) = \int \mathcal{D}\boldsymbol{x} \, \exp\left[\frac{i\omega}{2}\int_t^{t'} \mathrm{d}s \left(\dot{\boldsymbol{x}}^2 + i\frac{\boldsymbol{x}^2}{2{t_\mathrm{f}}^2}\right)\right]$$

$$= \frac{\omega\Omega}{2\pi i \, \sinh \Omega\tau} \exp\left\{\frac{i\omega\Omega}{4}\left[\tanh\frac{\Omega\tau}{2}(\boldsymbol{x}_f + \boldsymbol{x}_i)^2 + \coth\frac{\Omega\tau}{2}(\boldsymbol{x}_f - \boldsymbol{x}_i)^2\right]\right\}$$

where $t_\mathrm{f} \equiv \sqrt{\omega/\hat{q}}$ $\quad \Omega \equiv (1+i)/(2t_\mathrm{f})$

$$\boldsymbol{\partial}_x \cdot \boldsymbol{\partial}_y \, \mathcal{K}(\boldsymbol{x}, \boldsymbol{y})\Big|_{\boldsymbol{x}=\boldsymbol{y}=\boldsymbol{0}} = -\frac{1}{2\pi}\left(\frac{\omega\Omega}{\sinh\Omega\tau}\right)^2$$

and has to be regularized in $\tau \to 0$…

Can also expand in Ω to obtain the opacity expansion (N=1).

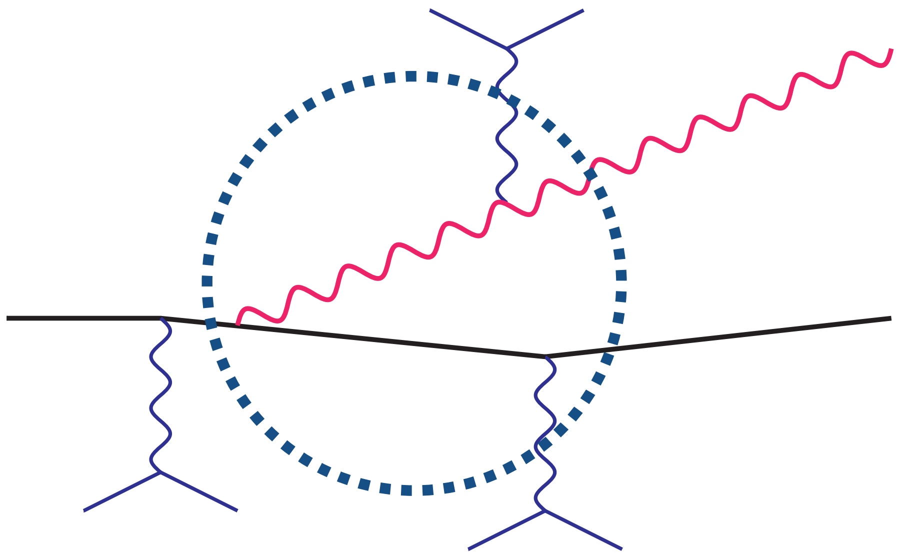
LPM EFFECT

Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000); Zakharov (1996);...

momentum broadening $\langle k_\perp^2 \rangle \sim \hat{q}t$

modified splitting kinematics $t_{\rm f} = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}}$

lack of collinear singularity!



$$\omega \frac{{\rm d}I}{{\rm d}\omega} = \frac{\alpha_s C_R}{2\pi}\sqrt{\frac{\hat{q}L^2}{\omega}}$$

[coupling sensitive to medium scale: $\alpha_s(\hat{q}L)$]

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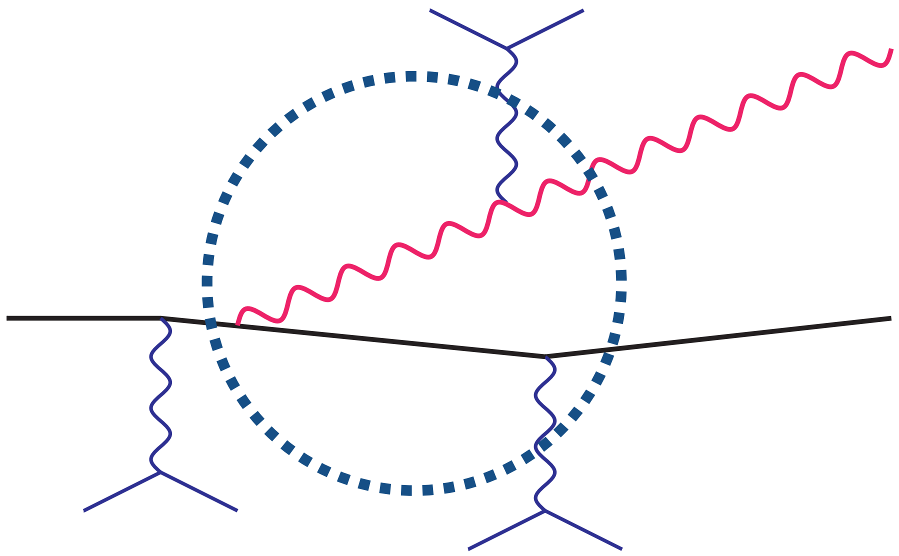

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$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{2\pi} \sqrt{\frac{\hat{q} L^2}{\omega}}$$

[coupling sensitive to medium scale: $\alpha_s(\hat{q}L)$]

Medium scales: $\omega_c \sim \hat{q} L^2$ $\omega_s \sim \bar{\alpha}^2 \hat{q} L^2$

$$N(\omega) = \int_\omega^\infty \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}}$$

multiplicity above a certain energy ω

$N(\omega_c) \sim \mathcal{O}(\bar{\alpha})$ rare emissions, hard BDMPS

$N(\omega_s) \sim \mathcal{O}(1)$ copious production, need for resummation, large fluctuations

Two regimes

$$t_{\mathrm{br}}(\omega) = \sqrt{\frac{\omega}{\hat{q}}}$$

$$t_{\mathrm{br}}(\omega_c) \sim \mathcal{O}(L)$$

takes a long time to form, emerge at *the end of the medium*

$$t_{\mathrm{br}}(\omega_s) \sim \bar{\alpha}\mathcal{O}(L)$$

produced rapidly, further branching highly probable

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$$\theta_{\rm br}(\omega) = \sqrt[4]{\frac{\hat{q}}{\omega^3}}$$

$$\theta_{\rm br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

minimal angle!

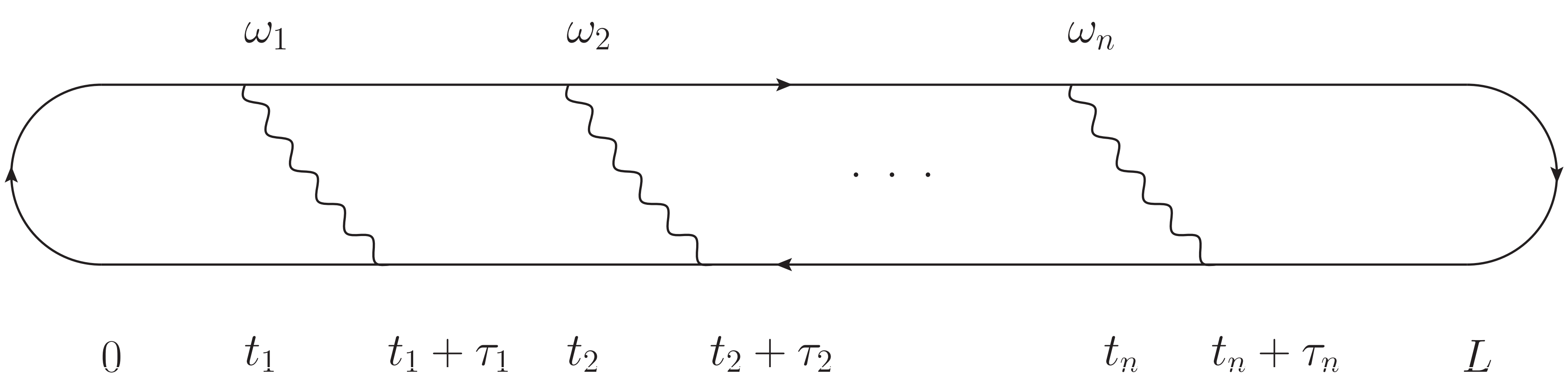
$$\theta_{\rm br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

energy transported to parametrically large angles

Resummation of quenching weight

$$\frac{\mathrm{d}\sigma_{\mathrm{med}}}{\mathrm{d}p_T^2\,\mathrm{d}y} = \int_0^\infty \mathrm{d}\epsilon\, P(\epsilon)\, \frac{\mathrm{d}\sigma_{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}p_T^2\,\mathrm{d}y}$$

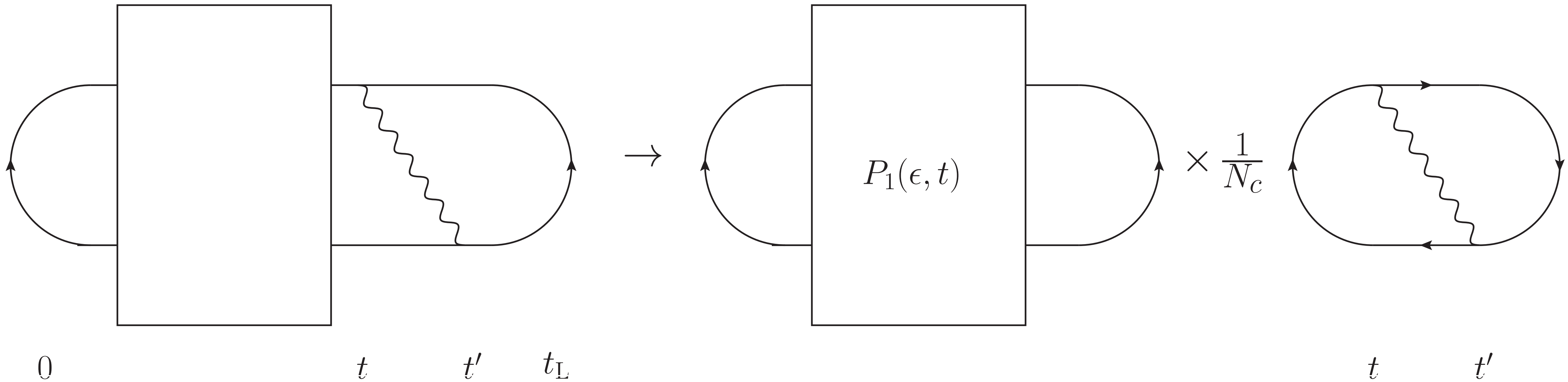
Flow of energy away from leading particle dominated by copious, soft gluon emission at large angles. Next step is to resum these emissions into a probability distribution for energy loss.



First step: jet is a single quark/gluon.

For short emission times: overlap are suppressed by L/t_f compared to independent: stick to independent emissions!

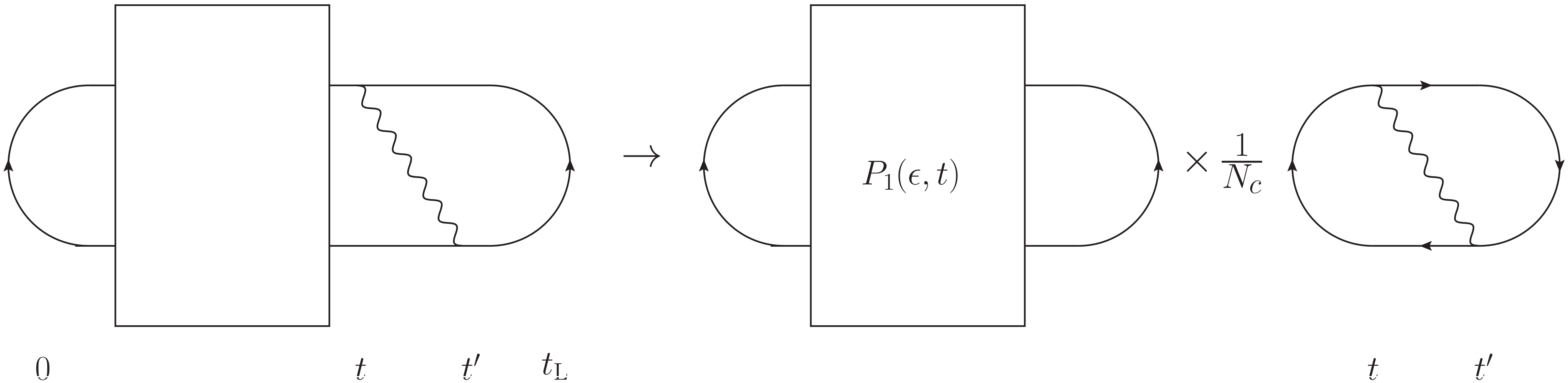
RATE EQUATION



Real emission: $\Delta P_1(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega \, \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} P_1(\epsilon - \omega, t)\,,$

One-prong energy loss prob: only one possible color structure propagates.

RATE EQUATION



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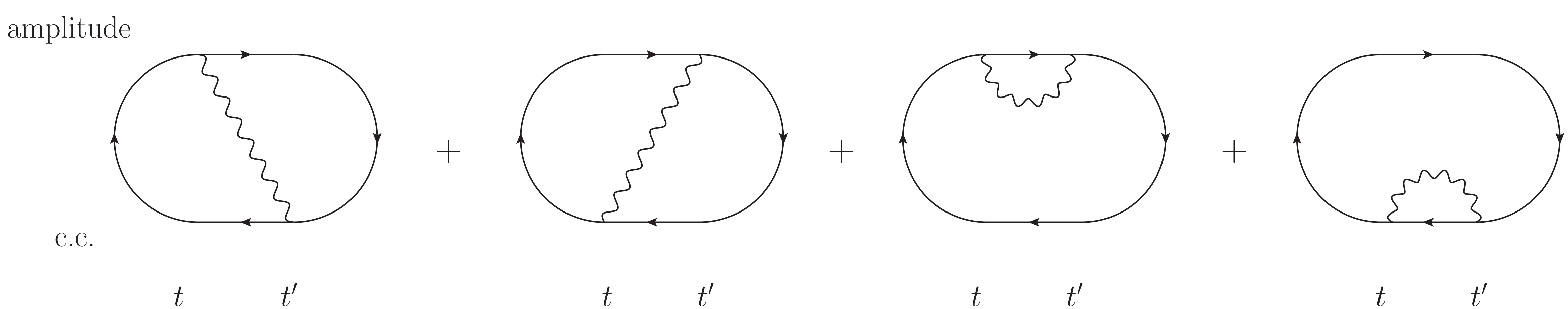
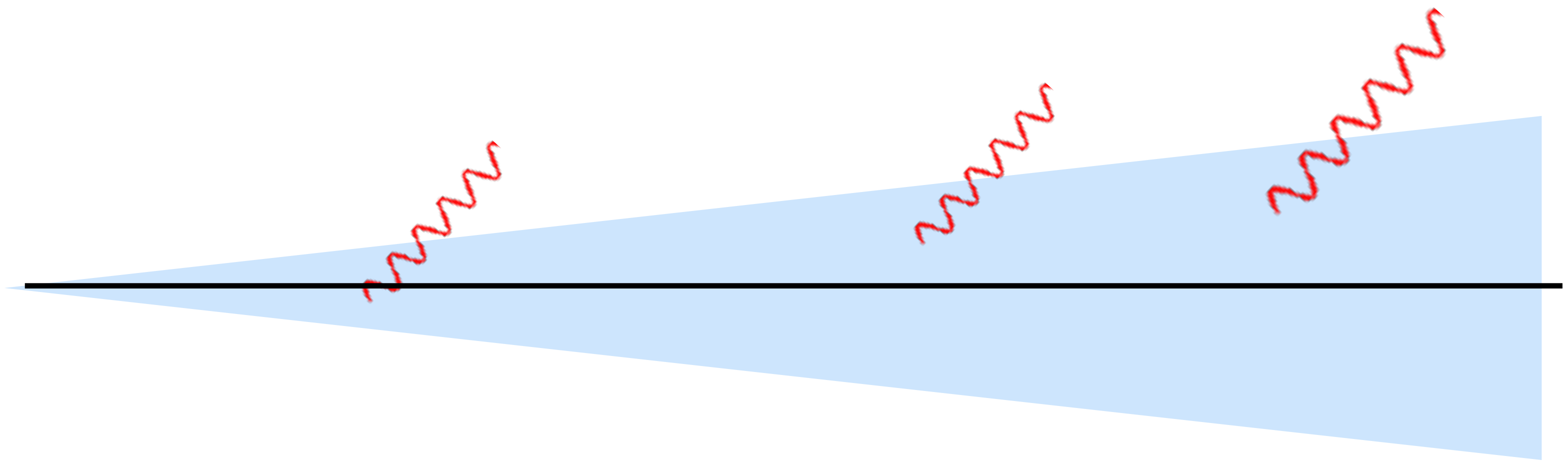
All possible time-orderings:



SINGLE-CHARGE QUENCHING



Resummation of multiple (primary) emissions = Poisson distribution

$$\frac{\partial}{\partial t} P_1(\epsilon, t) = \int_0^\infty \mathrm{d}\omega \left[\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega) \int_0^\infty \mathrm{d}\omega' \frac{\mathrm{d}I}{\mathrm{d}\omega' \mathrm{d}t} \right] P_1(\epsilon - \omega, t)$$

- single color charge + soft gluons
- modest intra-jet modification of splitting function

Energy loss dominated by *typical* emitted energy (large medium)

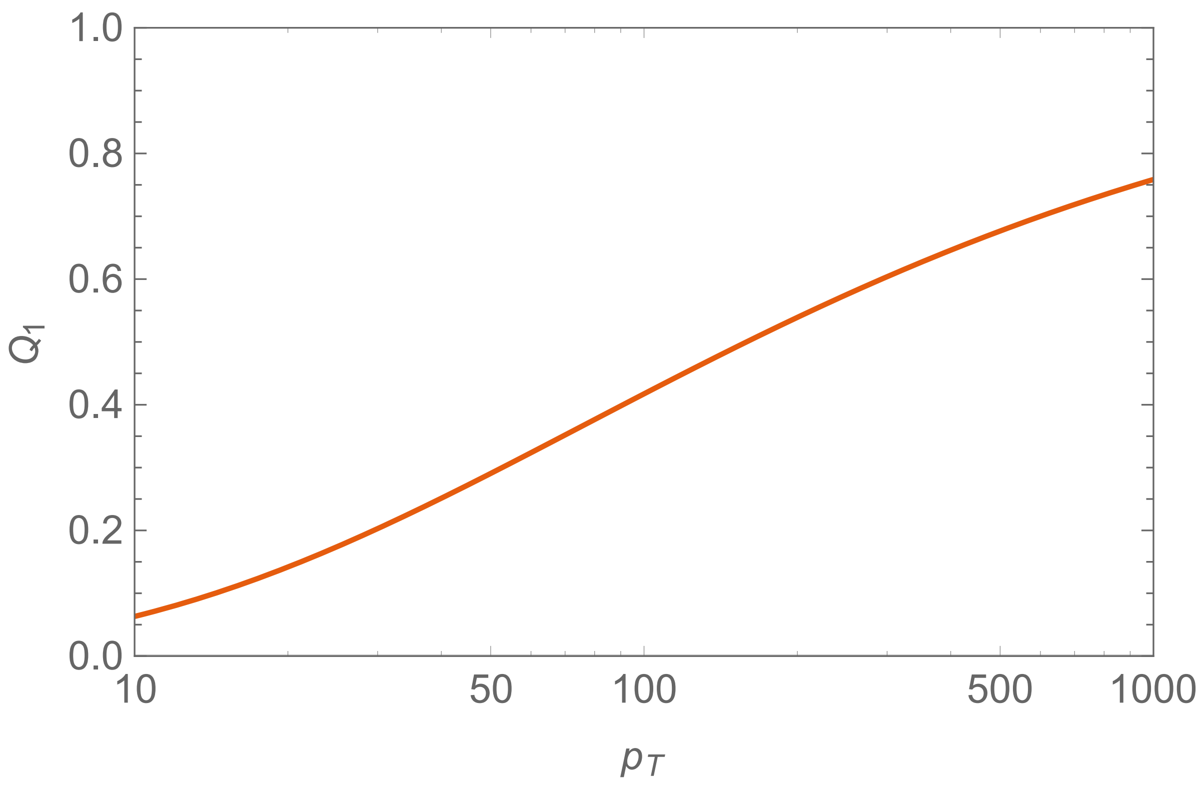
$$P_1(\epsilon, L) = \sqrt{\frac{\omega_s}{\epsilon^3}} \mathrm{e}^{-\frac{\pi \omega_s}{\epsilon}}$$

Baier, Dokshitzer, Mueller, Schiff (2001)

Coherent jet quenching

Baier, Dokshitzer, Mueller, Schiff (2001), Salgado, Wiedemann (2003)

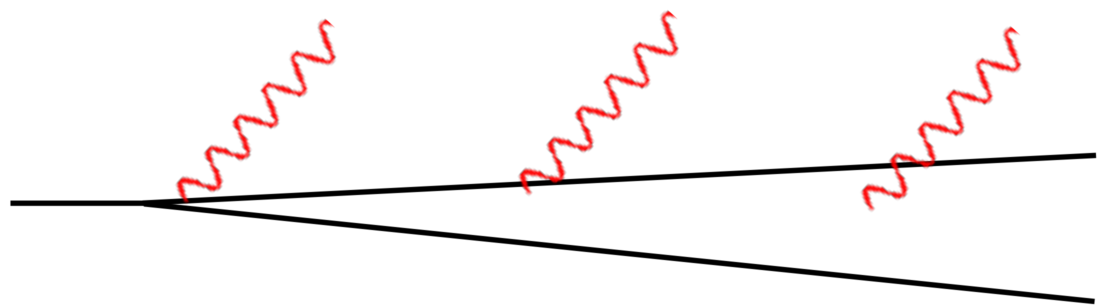
$$Q_1(p_T) \approx \exp\left[-\sqrt{\frac{\pi n \bar{\alpha}^2 \hat{q} L^2}{p_T}}\right]$$



Quenching factor of jet total charge

- bias due to steeply falling spectrum
- dying off slowly with p_T
- important improvement: secondary emissions merging onto medium-induced cascade

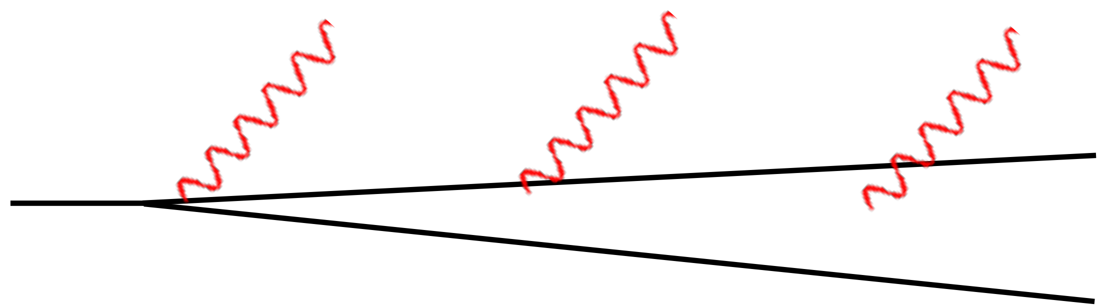
HIGHER ORDER CORRECTIONS



- higher-order jet structures demands analyzing interference terms
- simple rate equation at large-N_c for rapid splittings

$$\frac{\partial}{\partial t} P_{\rm sing}(\epsilon, t) = \int_0^\infty {\rm d}\omega \sum_i \Gamma_{ii}(\omega, t)\, P_{\rm sing}(\epsilon - \omega, t) + \int_0^\infty {\rm d}\omega \left[1 - \Delta_{\rm med}(t)\right] \sum_{i \neq j} \Gamma_{ij}(\omega, t) \delta(\epsilon - \omega)$$

Higher order corrections



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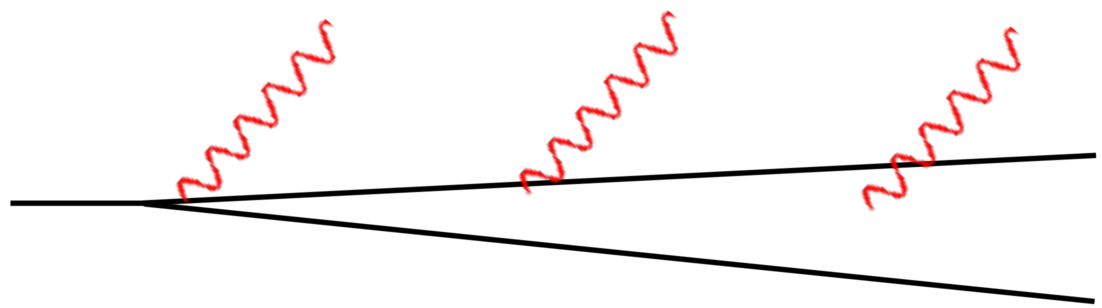

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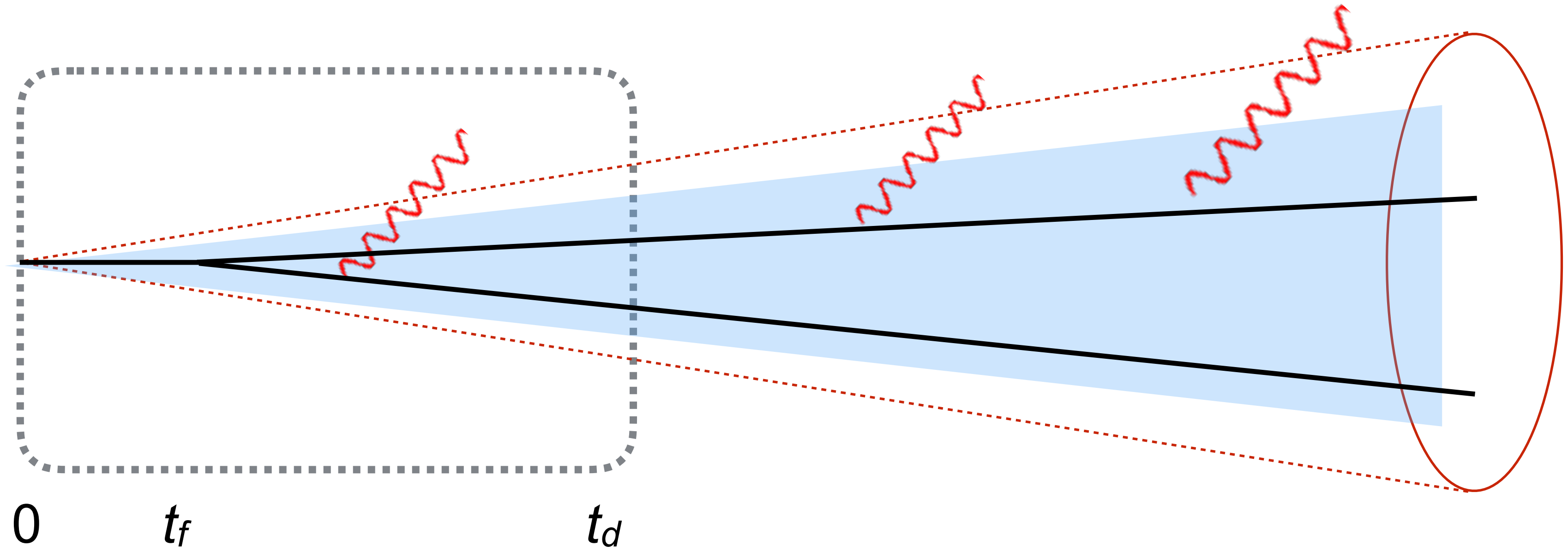
decoherence parameter

Time scale for decoherence in medium: $t_{\rm d} \sim (\hat{q}\theta^2)^{-1/3}$

Two-prong energy loss

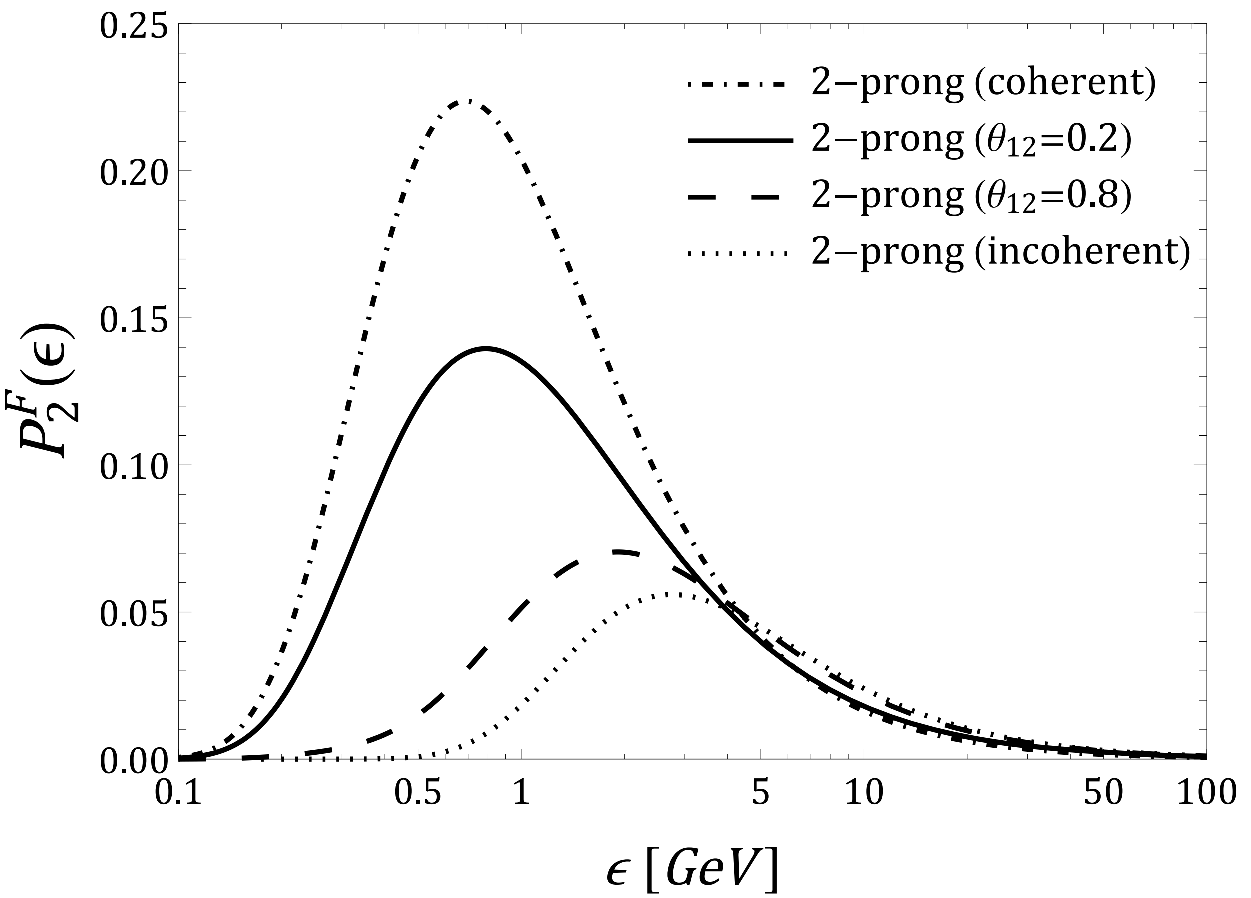
Y. Mehtar-Tani, KT arXiv:1706.06047 [hep-ph]



$$t_{\rm d} \sim (\hat{q}\theta_{12})^{-1/3}$$

$$P_2(\epsilon, L) = P_1(\epsilon_1, L) \otimes P_{\rm sing}(\epsilon_2, L)$$

- time scales: formation & decoherence
- angular dependence
  - minimal angle for resolving jet substructure



SUMMARY

- in-medium propagation
  - high-energy propagation: broadening and color precession
- medium-induced radiation
  - main source of energy-loss at high-energy
  - copious soft gluon production at large angles
- energy-loss of one- & multi-parton systems
  - importance of interference effects: **a new time-scale**!
  - putting it all together