Inclusive prompt photon production in e+A DIS at small x as a probe of gluon saturation*

Kaushik Roy

2018 JETSCAPE WINTER SCHOOL AND WORKSHOP

LBNL, Berkeley

Jan 3-7, 2018

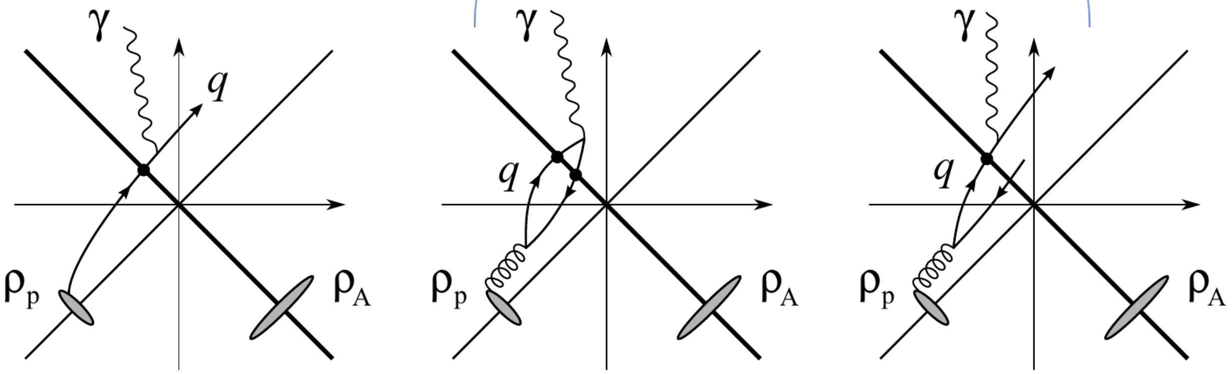
---

* Kaushik Roy and Raju Venugopalan, in preparation

Prelude: Motivation from proton-nucleus (pA) computations at small x in CGC framework

NLO in CGC power counting#

LO in CGC power counting*



- pA computation: Analytical expressions obtained in the dilute-dense limit, $\rho_p \ll 1\,, \rho_A \sim O\left(\frac{1}{g}\right) \gg 1$.
- Present work along the lines of the NLO computation of inclusive γ-production by Benic et. al.
- Shares many qualitative similarities with the pA calculations.

*Gelis, Jalilian-Marian, hep-ph/0205037, Dominguez *et. al.* ,arXiv:1101.0715

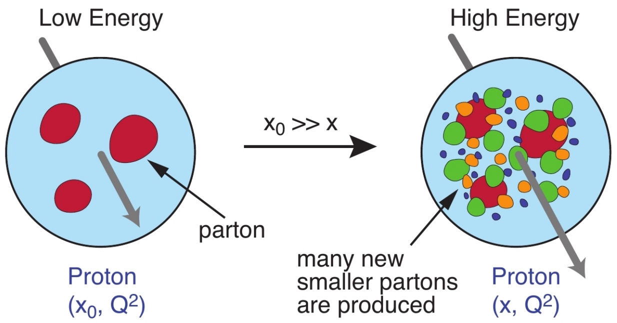
#Benic, Fukushima, arXiv:1602.01989, Benic *et. al.* , arXiv: 1609.09424

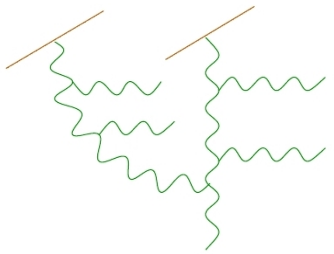
Fig. from Kovchegov, Levin: QCD at high energy (2012)

Saturation physics: Brief overview

The nuclear landscape at small x (high energies)

- At small x, a large number of gluons populate the transverse extent of the nucleus leading to a highly dense state of matter called the Color Glass Condensate (CGC).





- Non-linear effects like recombination competes with gluon proliferation to generate a resolution scale, $Q_S^2{\sim}A^{1/3}s^\lambda$ that grows with energy and nucleon number.
- For high enough energy, $Q_s^2(x) \gg \Lambda_{QCD}^2$ and weak coupling techniques are applied.
- Physics is non-perturbative due to large occupation number of gluons.

Core idea: System is weakly-coupled allowing Feynman diagram techniques but inherently non-perturbative!

McLerran, Venugopalan: hep-ph/9309289
Iancu, Venugopalan: hep-ph/0303204

CGC essentials

CGC = **classical** effective field theory in the **non-linear** regime of QCD describing dynamical gluon **fields** (small x partons) effected by static color **sources** (large x partons).

- Most small x gluons are near the saturation scale and their mutual interactions can be treated in the *classical* approximation.

$$D_\nu F^{\nu\mu,a}(x) = \delta^{\mu+}\rho^a_A(x^-,\mathbf{x}_\perp)$$

→ Source

Background classical field

x^+-independence follows from $D_+J^+ = 0$ and A^-=0

- Constructed in the infinite momentum frame (IMF) with nuclear momentum, $P^+ \to \infty$ with suitable gauge choices.

$A^{-,a} = 0$, $F^a_{ij} = 0$ with $A^{+,a}, A^{i,a}$ static (independent of x^+) (General attributes)

Lorenz gauge, $\partial_\mu A^\mu = 0$

$$A^{-,a} = A^{i,a} = 0$$

$$A^{+,a} = \int \mathrm{d}^2\mathbf{y}_\perp \langle \mathbf{x}_\perp | \frac{1}{-\nabla^2_\perp} | \mathbf{y}_\perp \rangle \rho^a(x^-,\mathbf{x}_\perp)$$

Light cone gauge, $A^+ = 0$

$$A^{+,a} = A^{-,a} = 0$$

$$A^{i,a} = \theta(x^-)\frac{i}{g}U(\mathbf{x}_\perp)(\partial^i U^\dagger(\mathbf{x}_\perp))$$

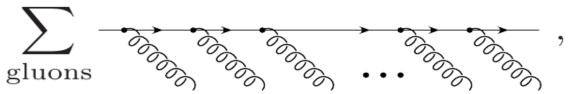
Adjoint Wilson line: $U(\mathbf{x}_\perp) = \mathcal{P}\_exp\Big[-ig\int_{-\infty}^{+\infty} \mathrm{d}z^- \frac{1}{\nabla^2_\perp}\rho^a_A(z^-,\mathbf{x}_\perp)T^a\Big]$

Gelis *et. al.* arXiv:0708.0047

CGC power counting

- In the saturation regime, occupation number of gluons $\propto \langle \rho_A \rho_A \rangle \sim O(\frac{1}{\alpha_s})$ which means we have strong sources, $\rho_A \sim O(\frac{1}{g})$.
- Attaching new sources to a diagram doesn't change the order in g, $\rho_A \; g \sim 1$ (eikonal coupling J.A)
- Independent powers arise only when a vertex is not connected to the source.
- So, in general for an observable, the following perturbative expansion holds.

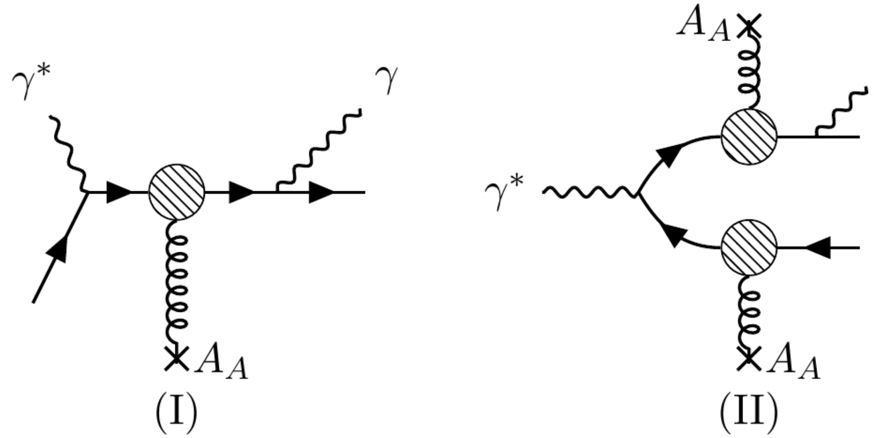
$$\mathbf{O} = c_0 + \alpha_s c_1 + \alpha_s^2 c_2 + \cdots \qquad \text{where} \quad c_n = \sum_{j=1}^{\infty} d_{nj} (g\rho_A)^j$$



- Thus we are *summing to all orders* in a twist-expansion at each order in α_S.

Back to topic: Components of the amplitude computation

A. LO processes in the CGC power counting



- Both classes of processes are leading order, O($\alpha^0{}_S$) in the CGC power counting.
- Class-I processes are suppressed at small x. Cross-section accompanied by valence quark distribution and $xG(x,Q^2) >> xf(x,Q^2)$ at small x.
- Will consider only Class-II processes in this work.

McLerran, Venugopalan: hep-ph/9402335
McLerran, Venugopalan: hep-ph/9809427

B. Fermion propagator in the background classical field

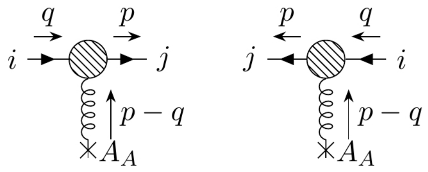
- Solve Dirac equation in the background field (with components in LC gauge) on either side of the source (x^-= 0).
- Match the solutions across the discontinuity at x^-= 0.
- Resulting Green's function is connected to the expression in which background field is in Lorenz gauge by a gauge transformation.

$$S_{LC}(x,y) = G(x^-,\mathbf{x}_\perp)S_{Lor.}(x,y)G^\dagger(x^-,\mathbf{x}_\perp)$$

where $G(x^-,\mathbf{x}_\perp) = \theta(-x^-) + \theta(x^-)\tilde{U}(\mathbf{x}_\perp)$ and $\tilde{U}(\mathbf{x}_\perp) = \mathcal{P}_-\exp\left[-ig\int_{-\infty}^{+\infty} \mathrm{d}z^- \frac{1}{\nabla_\perp^2}\rho_A^a(z^-,\mathbf{x}_\perp)t^a\right]$

Use this expression: $S_{Lor.}(q,p) = (2\pi)^4\delta^{(4)}(q-p) + S_0(q)\mathcal{T}(q,p)S_0(p)$

Effective vertex

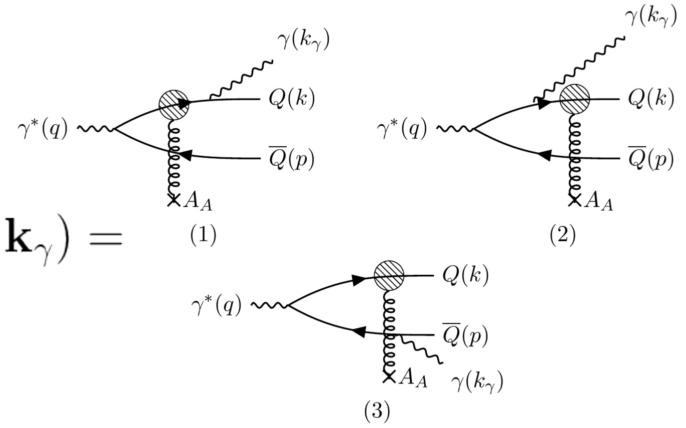


Vertex factor : $\mathcal{T}(q,p) = \pm(2\pi)\delta(p^- - q^-)\gamma^- \int \mathrm{d}^2\mathbf{x}_\perp e^{i(\mathbf{q}_\perp - \mathbf{p}_\perp).\mathbf{x}_\perp}\left[\tilde{U}^{(\pm)}(\mathbf{x}_\perp) - 1\right]_{ji}$

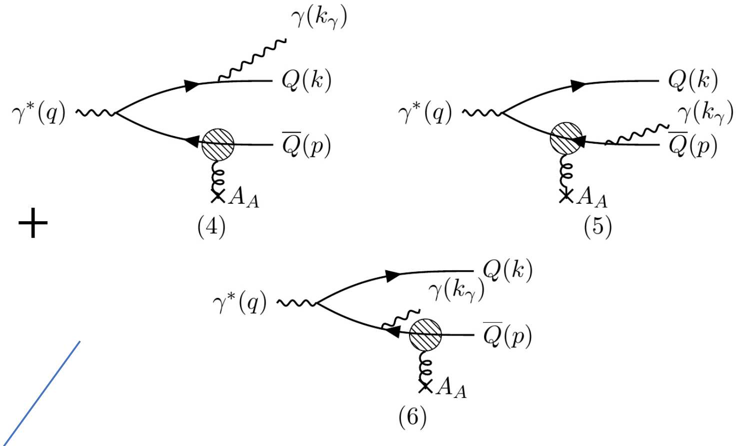
C. Contributing processes of Class II

Choice of gauge: Lorenz gauge, $\partial_\mu A^\mu = 0$

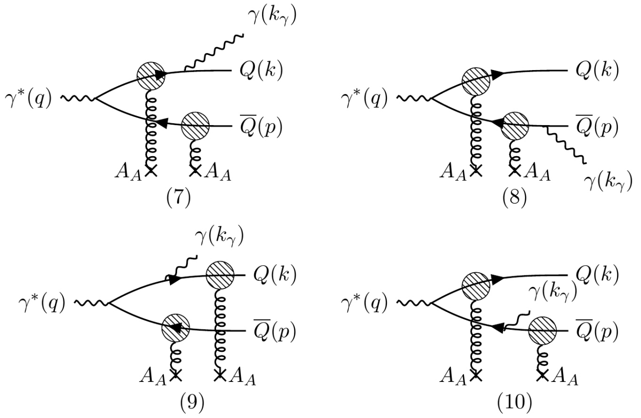
$$\mathcal{M}_{\mu\alpha}(\mathbf{q},\mathbf{k},\mathbf{p},\mathbf{k}_\gamma) =$$



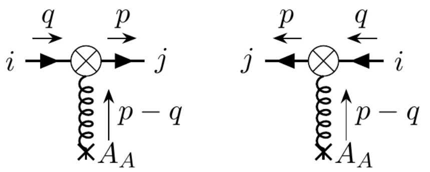
$+$



$+$

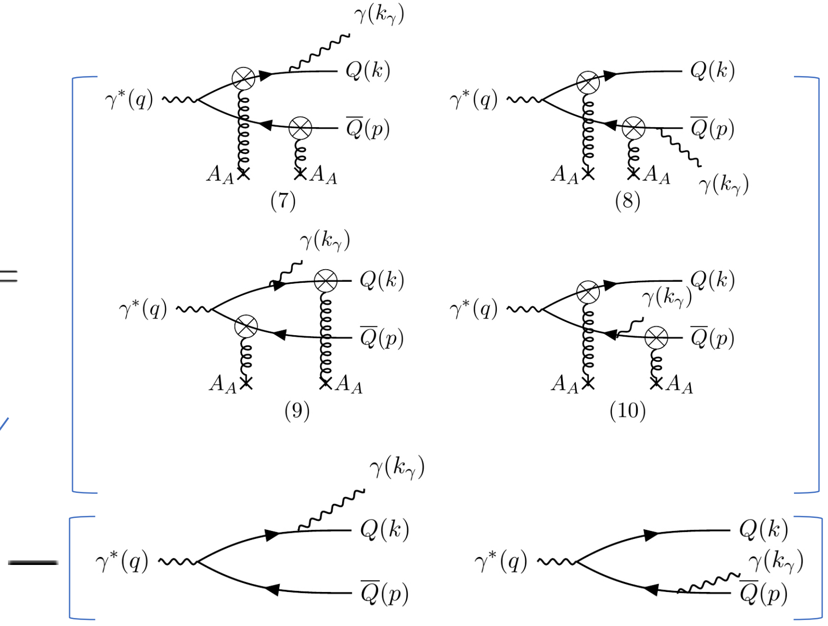


Tweak Feynman rules for effective propagator



$$\mathcal{T}_{\text{new}}(q,p) = \pm(2\pi)\delta(p^- - q^-)\gamma^- \int \mathrm{d}^2\mathbf{x}_\perp e^{i(\mathbf{q}_\perp - \mathbf{p}_\perp).\mathbf{x}_\perp} \tilde{U}^{(\pm)}(\mathbf{x}_\perp)_{ji}$$

$$\mathcal{M}_{\mu\alpha}(\mathbf{q},\mathbf{k},\mathbf{p},\mathbf{k}_\gamma) =$$



All kinematically allowed scatterings of quark & antiquark minus no scatterings – simplifies NLO computation.

D. LO calculation: Key analytical results

Single differential cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^2\mathbf{k}_{\gamma\perp}\mathrm{d}\eta_{k_\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2}\frac{1}{2q^-}\int_0^{+\infty}\frac{\mathrm{d}k^-}{k^-}\int_0^{+\infty}\frac{\mathrm{d}p^-}{p^-}\int_{\mathbf{k}_\perp,\mathbf{p}_\perp} L^{\mu\nu}\widetilde{X}_{\mu\nu}(2\pi)\delta(P^- - q^-)$$

Leptonic tensor: $$L^{\mu\nu} = \frac{2e^2}{Q^4}\left[(\widetilde{l^\mu}\widetilde{l'^\nu} + \widetilde{l^\nu}\widetilde{l'^\mu}) - \frac{Q^2}{2}g^{\mu\nu}\right]$$

Hadronic tensor: $$\widetilde{X}_{\mu\nu} = \int_{\mathbf{x}_\perp,\mathbf{y}_\perp,\mathbf{x}'_\perp,\mathbf{y}'_\perp}\int_{\mathbf{l}_\perp,\mathbf{l}'_\perp} e^{-i\mathbf{P}_\perp.(\mathbf{x}_\perp-\mathbf{x}'_\perp)}e^{i(\mathbf{l}_\perp.\mathbf{x}_\perp-\mathbf{l}'_\perp.\mathbf{x}'_\perp)}e^{i(\mathbf{l}'_\perp.\mathbf{y}'_\perp-\mathbf{l}_\perp.\mathbf{y}_\perp)}\ \tau_{\mu\nu}{}^{q\bar{q},q\bar{q}}(\mathbf{l}_\perp,\mathbf{l}'_\perp|\mathbf{P}_\perp)\ \boldsymbol{\sigma}_{dipole}$$

Complicated Dirac trace (actual form in Appendix)

Dipole cross-section:

$$\boldsymbol{\sigma}_{dipole} = 1 - \frac{1}{N_c}\mathrm{tr}_c\left(\left\langle\tilde{U}(\mathbf{x}_\perp)\tilde{U}^\dagger(\mathbf{y}_\perp)\right\rangle\right) - \frac{1}{N_c}\mathrm{tr}_c\left(\left\langle\tilde{U}(\mathbf{y}'_\perp)\tilde{U}^\dagger(\mathbf{x}'_\perp)\right\rangle\right) + \frac{1}{N_c}\mathrm{tr}_c\left(\left\langle\tilde{U}(\mathbf{y}'_\perp)\tilde{U}^\dagger(\mathbf{x}'_\perp)\tilde{U}(\mathbf{x}_\perp)\tilde{U}^\dagger(\mathbf{y}_\perp)\right\rangle\right)$$

Contains all relevant nuclear information.

E. LO amplitude: Interesting properties

Property 1: Soft-photon factorization

In the limit, k_γ → 0, the amplitude satisfies the Low-Burnett-Kroll soft photon theorem

$$\mathcal{M}_\mu(\mathbf{q},\mathbf{k},\mathbf{p},\mathbf{k}_\gamma) \to -(eq_f)\epsilon^*_\alpha(\mathbf{k}_\gamma,\lambda)\left(\frac{p^\alpha}{p.k_\gamma} - \frac{k^\alpha}{k.k_\gamma}\right)\mathcal{M}^{NR}_\mu(\mathbf{q},\mathbf{k},\mathbf{p})$$

Polarization vector × Vectorial structure depending only on momenta of emitted particles × Non-radiative DIS amplitude

Using the non-radiative DIS amplitude, we obtain the inclusive differential cross-section

$$\frac{\mathrm{d}\sigma^{L,T}}{\mathrm{d}^3k\mathrm{d}^3p} = N_c\alpha q_f^2\delta(q^- - p^- - k^-)\int \frac{\mathrm{d}^2\mathbf{x}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{x'}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{y}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{y'}_\perp}{(2\pi)^2}\; e^{-i\mathbf{k}_\perp.(\mathbf{x}_\perp-\mathbf{x'}_\perp)}e^{-i\mathbf{p}_\perp.(\mathbf{y}_\perp-\mathbf{y'}_\perp)}$$
$$\times\sum_{\alpha,\beta}\psi^{L,T}_{\alpha\beta}(q^-,z,|\mathbf{x}_\perp-\mathbf{y}_\perp|)\;\psi^{L,T*}_{\alpha\beta}(q^-,z,|\mathbf{x'}_\perp-\mathbf{y'}_\perp|)\times\boldsymbol{\sigma}_{dipole}$$

- Familiar dipole factorized form. Matches with existing literature (Dominguez *et. al.* arXiv:1101.071).
- Both dipole and WW- gluon distributions can be studied in inclusive dijet production in DIS.
- In the collinear limit, this result allows us to measure the gluon distribution `cleanly' in a way independent from traditional approaches, like determining F_L.

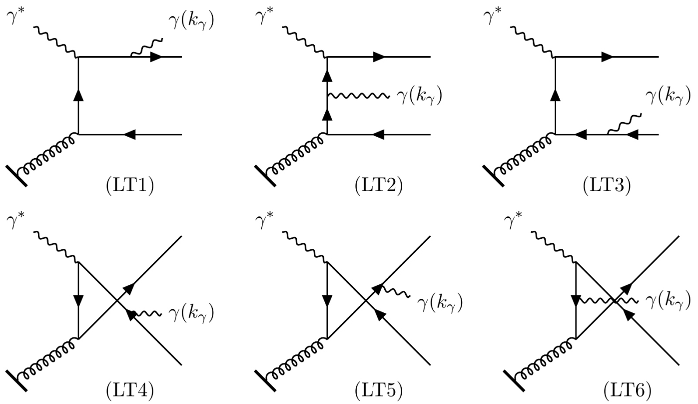
E. LO amplitude: Interesting properties

Property 2: Recover $k_\perp$-factorization and collinear factorization limits

- High-$k_\perp$ expansion ≡ expanding fundamental Wilson lines in $\tilde{X}^{\mu\nu}$ to lowest non-trivial order in $\rho_A/\nabla_\perp^2$ and use

$$\langle \rho_A^a(x^-, \mathbf{x}_\perp)\rho_A^b(y^-, \mathbf{y}_\perp)\rangle = \delta^{ab}\delta(x^- - y^-)\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)\lambda_A(x^-) \qquad \int \mathrm{d}x^- \lambda_A(x^-) = \mu_A^2 = \frac{A}{2\pi R^2}$$

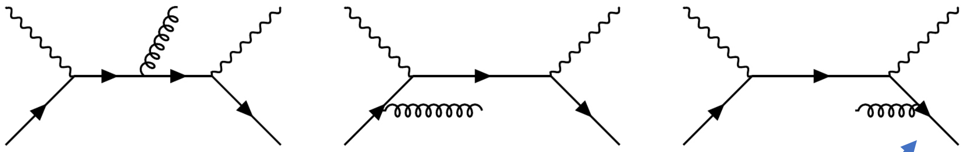
- Physics: ρ_A is large enough to be classical, but only one of the color charges in the classical color distribution is resolved by the high-$k_\perp$ probe.
- Resulting expression matches exactly to perturbative results obtained at leading twist, $\mathcal{O}(\rho_A)$

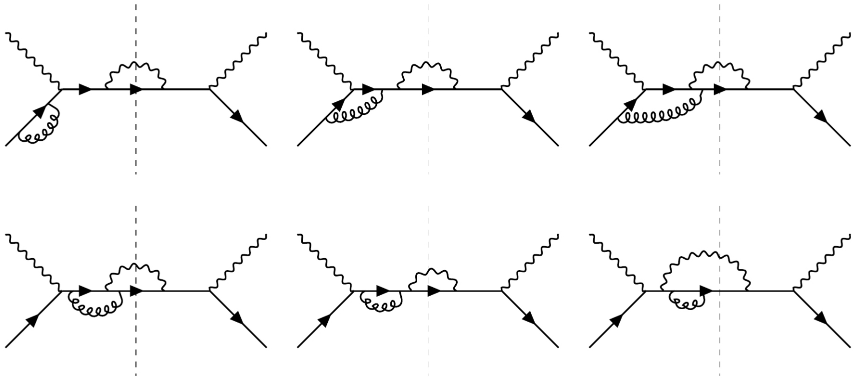


- Collinear limit, $k_\perp \to 0$ of the same can be taken and we recover the conventional collinear factorized expression.

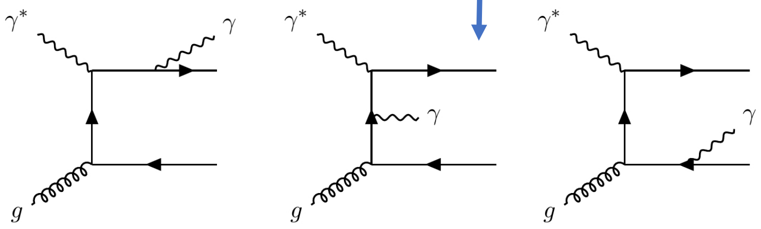
Why interesting? CGC vs pQCD power counting

- Under $k_\perp$ and collinear factorization limits: What we call LO in CGC power counting corresponds to dominant NLO processes in leading twist pQCD power counting at small x .





$$E_2\frac{d\sigma}{d\mathbf{k}_2}=\sum_q\int dx_b\Big\{G_{q/p}(x_b)\Big(\frac{\hat{s}}{\pi}\frac{d\sigma}{d\hat{t}}(\gamma+q\to\gamma+q)\cdot\delta(\hat{s}+\hat{t}+\hat{u})+\frac{\alpha_s}{2\pi}\theta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\pi}\frac{k(\hat{s},\hat{t},\hat{u})}{\hat{s}}\Big)+G_{g/p}(x_b)\frac{\alpha_s}{2\pi}\theta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\pi}\frac{k'(\hat{s},\hat{t},\hat{u})}{\hat{s}}\Big\}.$$



At small x, the above processes are suppressed relative to the photon-gluon fusion process.

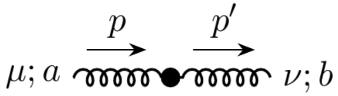
$$xG(x,Q^2)\gg xf(x,Q^2),\text{ for }x\ll 1$$

Aurenche et. al. Z.Phys. C24 (1984) 309

McLerran, Venugopalan (1994)
Ayala *et. al.* (1995)
Balitsky, Belitsky (2001)

Towards NLO: Highlights

- Apt choice of gauge is useful when gluon propagators appear in the picture.
- We work in A^{-}= 0 gauge (`Wrong' LC for our problem kinematics) with background field in the `correct' LC-gauge, A^{+}=0.
- The `dressed' momentum space gluon propagator in this gauge has a simple form as the fermion propagator.

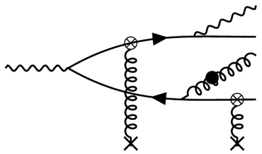


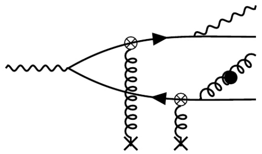
$$\mathcal{T}_{\mu\nu;ab}(p,p') = -2\pi\delta(p^- - p'^-) \times (2p^-) g_{\mu\nu}\ \ \mathrm{sign}(p^-) \int \mathrm{d}^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p'}_\perp).\mathbf{z}_\perp} U_{ab}^{\mathrm{sign}(p^-)}(\mathbf{z}_\perp)$$

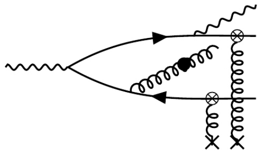
Memory refresher: For fermions, $\mathcal{T}(q,p) = (2\pi)\delta(p^- - p'^-)\gamma^- \mathrm{sign}(p^-) \int \mathrm{d}^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p'}_\perp).\mathbf{z}_\perp} \tilde{U}^{(\mathrm{sign}(p^-)}(\mathbf{z}_\perp)$

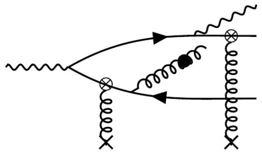
- Offers **significant clarity over existing photon impact factor calculations for fully inclusive DIS**, Balitsky, Chirilli arXiv:1207.3844 (coordinate space), Beuf arXiv: 1606.00777, arXiv: 1112.4501, arXiv:1708.06557 (LCPT)
- **Soft-photon limit of our NLO expressions will allow us to recover the above results** thus providing a consistency check.

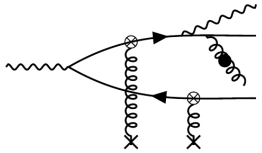
NLO machinery: Contributing processes → Real Graphs

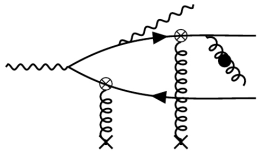

(A1)

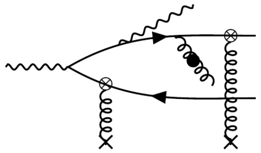

(A2)

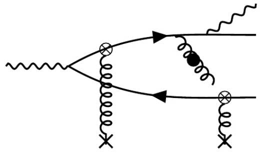

(A3)

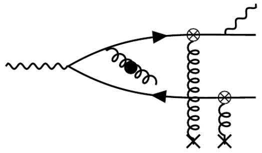

(A4)

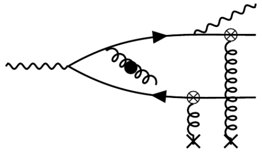

(A5)

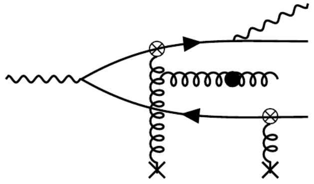

(A6)

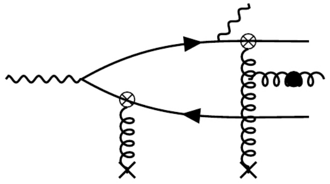

(A7)

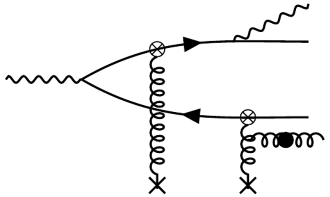

(A8)

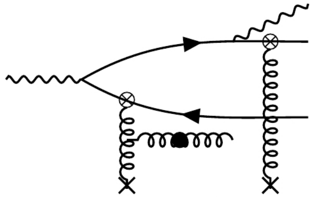

(A9)


(A10)


(B1)


(B2)

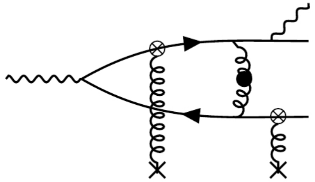

(B3)

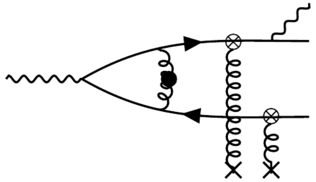

(B4)

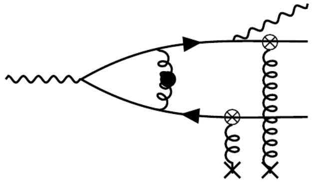
These processes contribute to the leading-log (LL) evolution of the LO dipole cross-section.

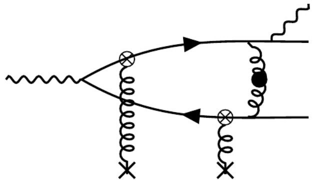
Other set of graphs obtained by quark-antiquark interchange

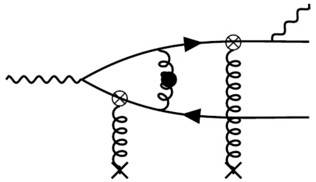
NLO machinery: Contributing processes → Virtual graphs

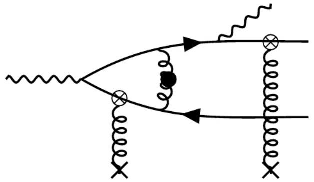

(C1)

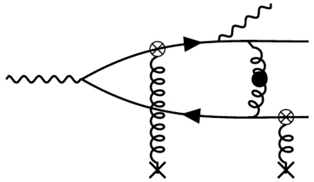

(C2)

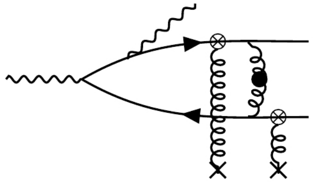

(C3)

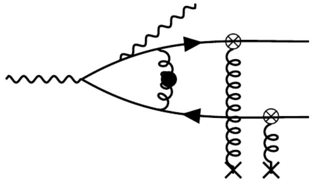

(C4)

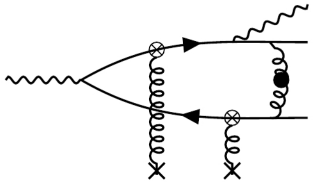

(C5)

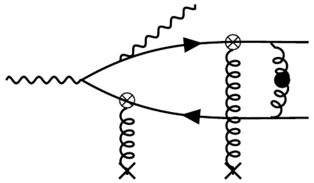

(C6)

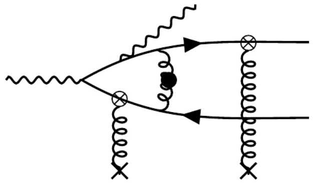

(C7)

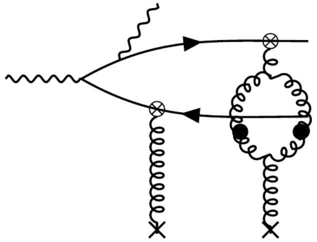

(C8)

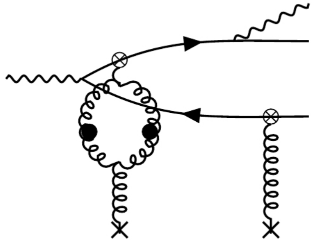

(C9)

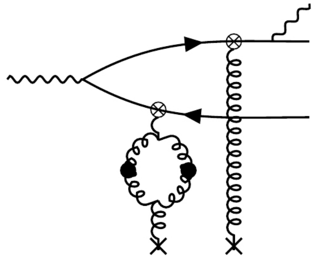

(C10)

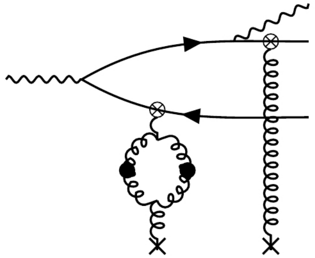

(C11)


(C12)


(G1)

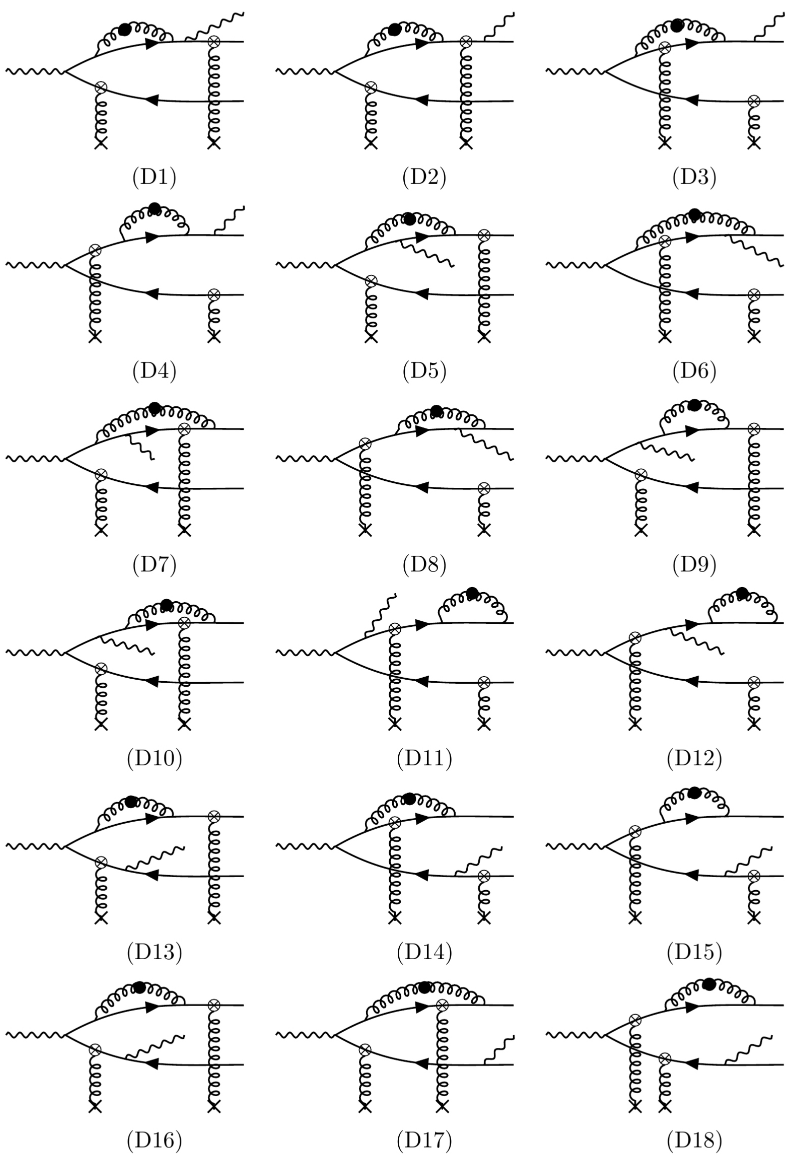

(G2)

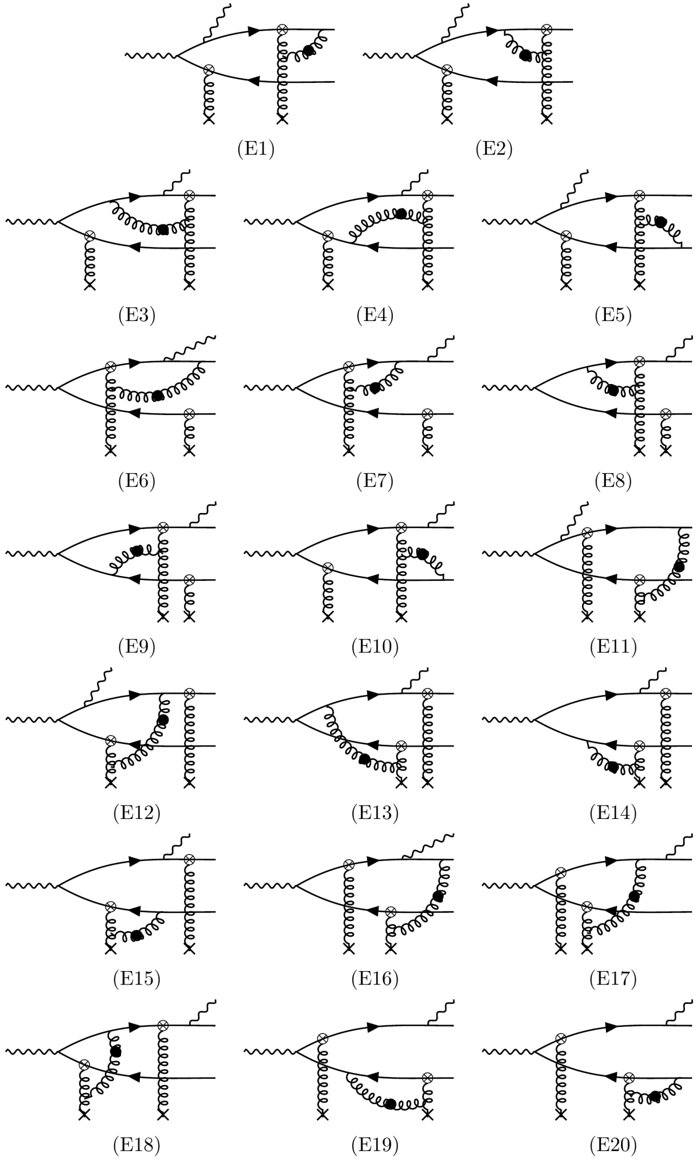

(G3)


(G4)

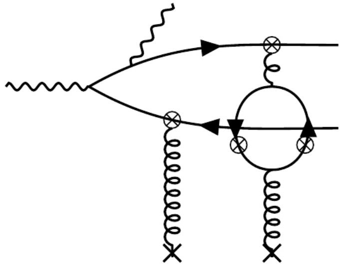
These processes contribute to the leading-log (LL) evolution of the LO dipole cross-section.

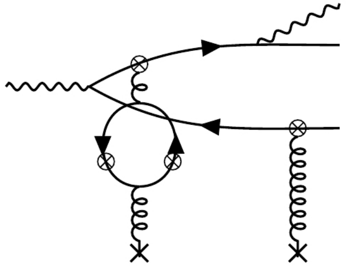
Other set of graphs obtained by quark-antiquark interchange

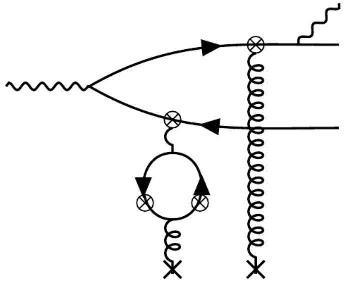

(D1) (D2) (D3)
(D4) (D5) (D6)
(D7) (D8) (D9)
(D10) (D11) (D12)
(D13) (D14) (D15)
(D16) (D17) (D18)

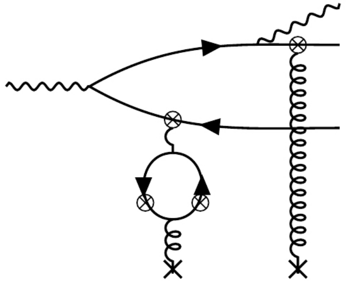

(E1) (E2)
(E3) (E4) (E5)
(E6) (E7) (E8)
(E9) (E10) (E11)
(E12) (E13) (E14)
(E15) (E16) (E17)
(E18) (E19) (E20)

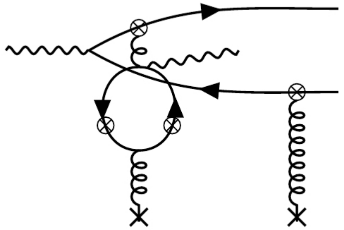
Other set of graphs obtained by quark-antiquark interchange

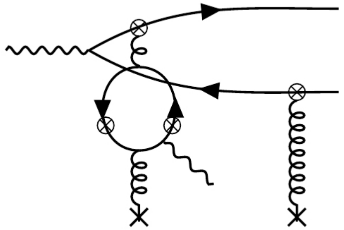

(F1)

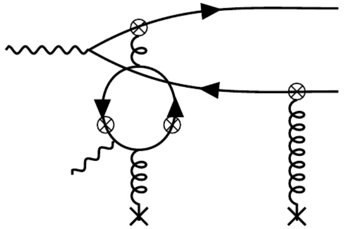

(F2)

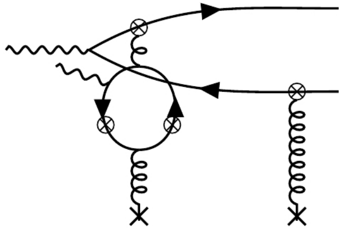

(F3)


(F4)

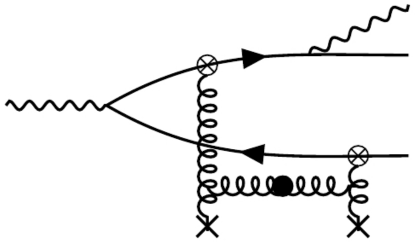

(F5)

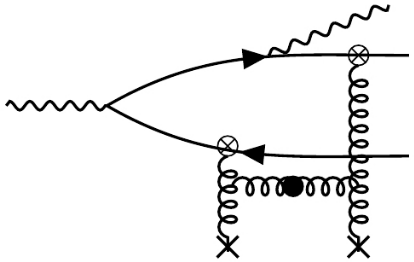

(F6)


(F7)


(F8)

These processes contribute to the leading-log (LL) evolution of the LO dipole cross-section.


(H1)


(H2)

These processes contribute to the next-to-leading-log (NLL) evolution of the LO dipole cross-section.

Other set of graphs obtained by quark-antiquark interchange

Summary and prospects:

- DIS at small x allows us to study a new emergent regime in QCD with matter having aspects of both weak and strong interactions.
- Inclusive photon production in CGC is a very clean process to study saturation physics. This can be observed and studied at the upcoming Electron Ion Collider (EIC) facility at BNL/Jlab.
- EIC will use proliferated parton densities $\sim A^{1/3}$ ($\simeq$ 6 for A=200) in a nuclear environment and Q^2 >> Λ^2_{QCD} to access an uncharted small x $(10^{-4} - 10^{-3})$ regime of QCD, where abundant gluons saturate in density and dominate its behavior.
- Understanding QCD in this regime will provide us insights into many existing fundamental questions such as the behavior of high-energy cross-section, nature of multi-particle production, universality of hadron properties at small x etc.

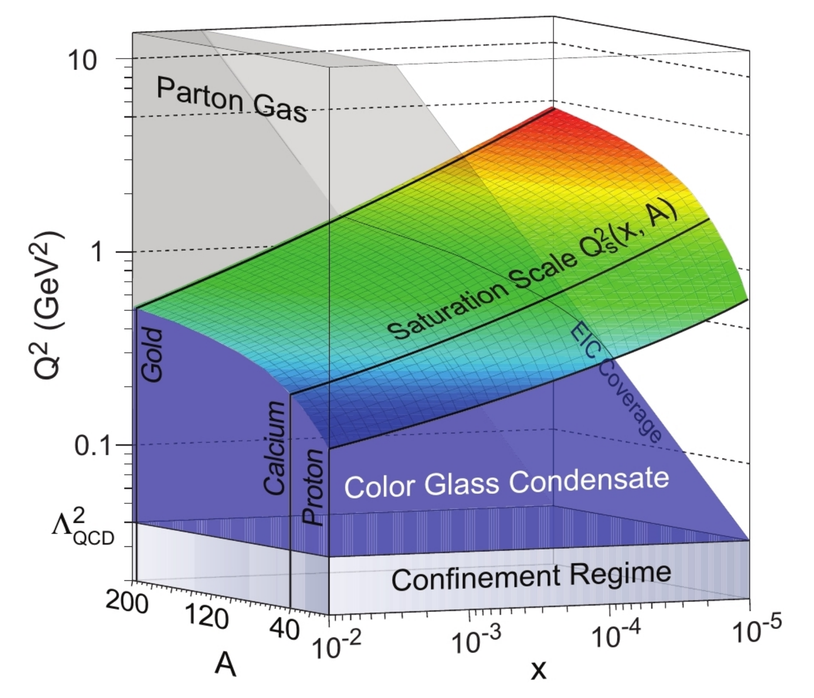


Fig. taken from EIC white paper

Thank you for listening….

Appendix

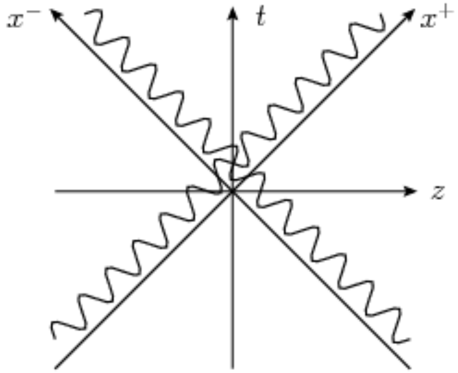


Fig. Light cone/front frame or Infinite Momentum frame (IMF)

Light-cone coordinates:

The proper language to describe high energy kinematics

$$a^+ = \frac{a^0 + a^3}{\sqrt{2}}, \qquad a^- = \frac{a^0 - a^3}{\sqrt{2}}$$

$$p.x = p^+ x^- + p^- x^+ - \mathbf{p}_\perp . \mathbf{x}_\perp$$

Complicated Dirac trace:

$$\tau_{\mu\nu}{}^{q\bar{q},q\bar{q}}(\mathbf{l}_\perp, \mathbf{l'}_\perp | \mathbf{P}_\perp) = \mathrm{tr}_d \Big[(\not{k} + m) T_\nu^{(q\bar{q})^\alpha}(\mathbf{l}_\perp, \mathbf{P}_\perp)(m - \not{p})\hat{\gamma}^0 T_{\mu\alpha}^{(q\bar{q})^\dagger}(\mathbf{l'}_\perp, \mathbf{P}_\perp)\hat{\gamma}^0 \Big]$$

$$T_{\mu\alpha}^{(q\bar{q})}(\mathbf{l}_\perp, \mathbf{P}_\perp) = \sum_{\beta=7}^{10} R_{\mu\alpha}^{\beta}(\mathbf{l}_\perp, \mathbf{P}_\perp)$$

$$\mathbf{P}_\perp = \mathbf{k}_\perp + \mathbf{p}_\perp + \mathbf{k}_{\gamma\perp}$$

LO amplitude:

$$\mathcal{M}_{\mu\alpha}(\mathbf{q}, \mathbf{k}, \mathbf{p}, \mathbf{k}_\gamma) = \sum_{\beta=1}^{10} \mathcal{M}_{\mu\alpha}^{\beta}(\mathbf{q}, \mathbf{k}, \mathbf{p}, \mathbf{k}_\gamma) = 2\pi(eq_f)^2 \delta(P^- - q^-) \int_{\mathbf{x}_\perp} \int_{\mathbf{y}_\perp} \int_{\mathbf{l}_\perp} e^{-i\mathbf{P}_\perp . \mathbf{x}_\perp + i\mathbf{l}_\perp . \mathbf{x}_\perp} e^{-i\mathbf{l}_\perp . \mathbf{y}_\perp}$$

$$\times \overline{u(\mathbf{k})} \Big[T_{\mu\alpha}^{(q\bar{q})}(\mathbf{l}_\perp, \mathbf{P}_\perp) \big[\tilde{U}(\mathbf{x}_\perp) \tilde{U}^\dagger(\mathbf{y}_\perp) - 1 \big] \Big] v(\mathbf{p})$$

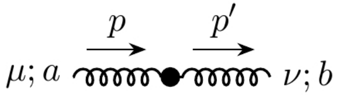
NLO machinery: Feynman rules

In addition to the machinery of LO calculation, we need the propagator for small-fluctuation gluon field, b(x).

$$A^\mu = B^\mu + b^\mu$$

- Assume b¯(x¯= 0) to prevent x+ -evolution of charge density.
- Solve small-fluctuation equations of motion. Calculate Green’s function.

$$\langle b^{\mu,a}(x) b^{\nu,b}(y)\rangle = iG^{\mu\nu;ab}(x,y)$$



Blob represents multiple scatterings with the nucleus

In momentum space, $iG^{ij;ab}(p,p') = (2\pi)^4\delta^{(4)}(p-p')\Big(iG_0^{ij;ab}(p)\Big) + \Big(iG_0^{ik;ac}(p)\Big)\mathcal{T}_{kl;cd}(p,p')\Big(iG_0^{lj;db}(p')\Big)$

Free propagator: $iG_0^{\mu\nu;ab}(p) = \dfrac{i}{p^2 + i\varepsilon}\Big(-g^{\mu\nu} + \dfrac{p^\mu n^\nu + p^\nu n^\mu}{n.p}\Big)\delta^{ab}, \qquad n^\mu = \delta^{\mu +}$

Vertex factor: $\mathcal{T}_{\mu\nu;ab}(p,p') = -2\pi\delta(p^- - p'^-) \times (2p^-)g_{\mu\nu}\ \mathrm{sign}(p^-)\int \mathrm{d}^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp).\mathbf{z}_\perp} U_{ab}^{\mathrm{sign}(p^-)}(\mathbf{z}_\perp)$

Memory refresher: For fermions, $\mathcal{T}(q,p) = (2\pi)\delta(p^- - p'^-)\gamma^-\mathrm{sign}(p^-)\int \mathrm{d}^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp).\mathbf{z}_\perp}\tilde{U}^{(\mathrm{sign}(p^-))}(\mathbf{z}_\perp)$