

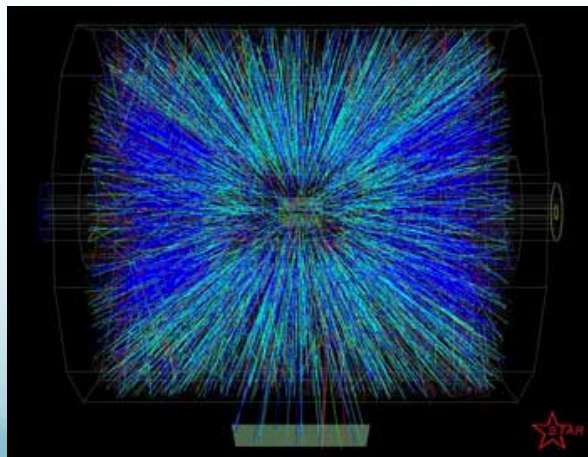
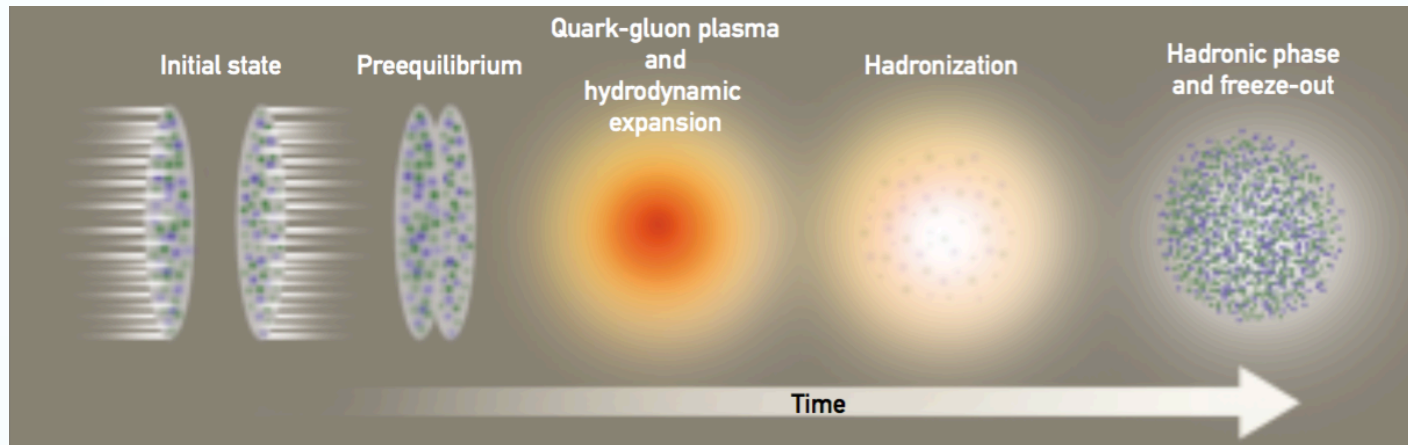
# Jet functions

Zhongbo Kang  
UCLA

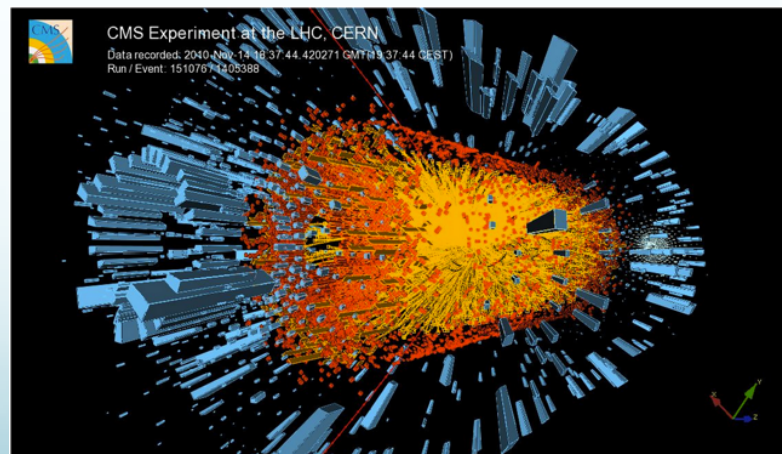
2018 JETSCAPE Winter School and Workshop  
January 3 – 7, 2018

# Heavy ion physics

- Recreate a new state of matter, the quark gluon plasma (QGP), which prevailed in early Universe in first few micro-seconds, study its properties



RHIC: Au+Au

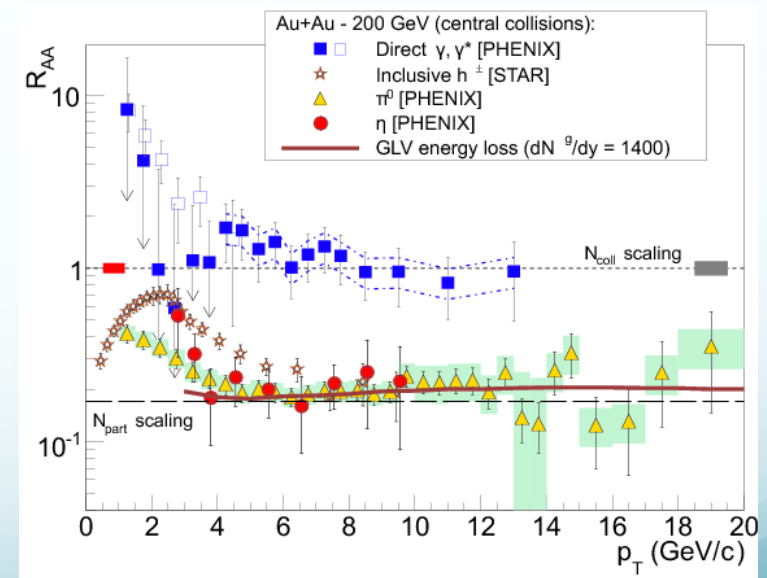
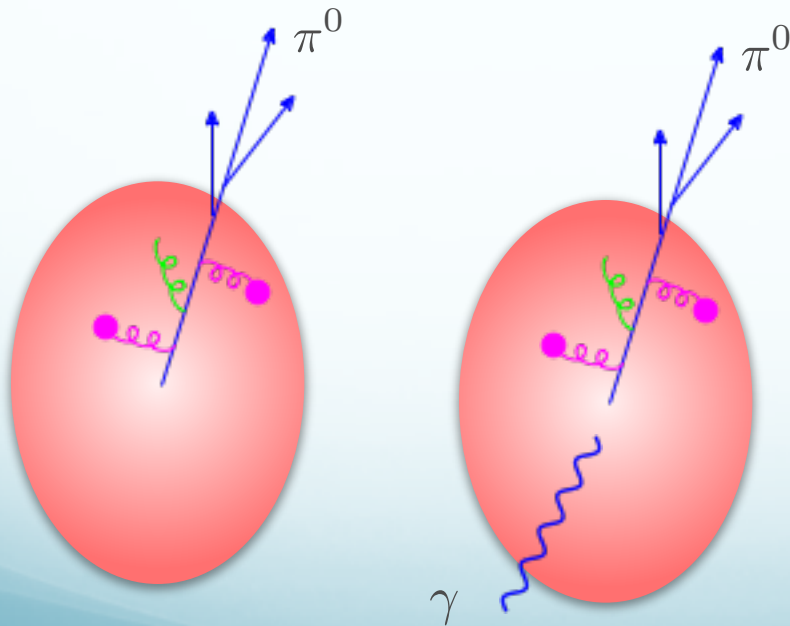


LHC: Pb+Pb

# “Jet” tomography: really hadrons

- Early jet tomography is really using “inclusive hadron production”
  - (modified) fragmentation function in p+p (A+A)

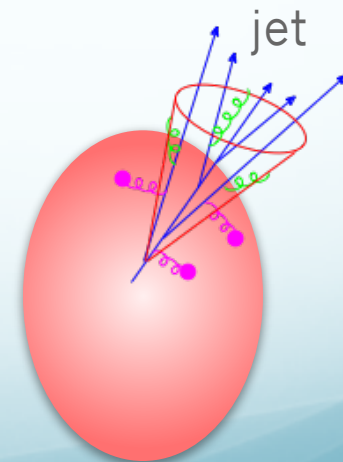
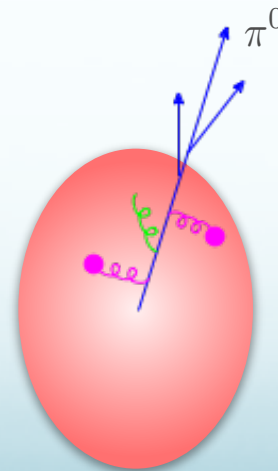
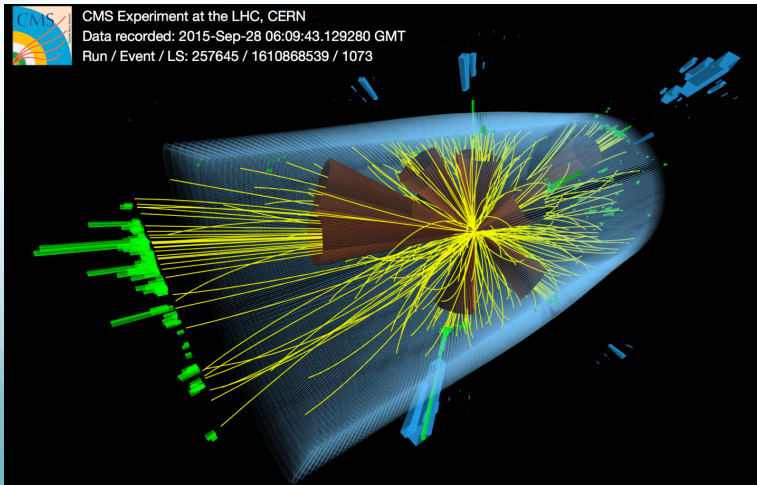
**Hadrons**  $\longleftrightarrow$  **Fragmentation functions**



# Jet tomography: true jets

- New opportunities: jets are abundantly produced at LHC (also future RHIC)
  - To describe jet production, we need jet functions

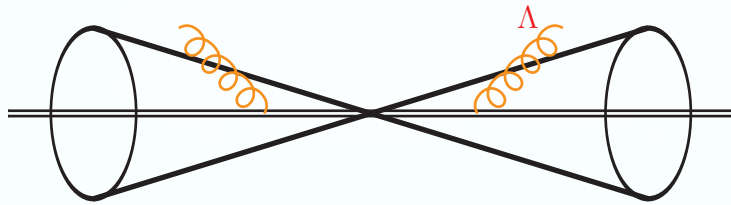
**Jets** ↔ **Jet functions**



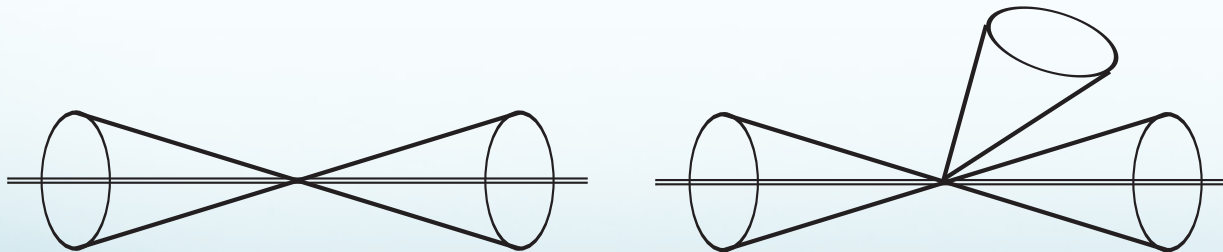


# Jets: exclusive and inclusive measurements

- Exclusive jet production
  - Make sure one has fixed-number of jets (e.g., dijet), and veto any additional jets (e.g., through energy or  $p_T$  cut)



- Inclusive jet production
  - Sum over all particles in the final state besides the observed jets
  - Example: single inclusive jet production



# Soft-Collinear Effective Theory (SCET)

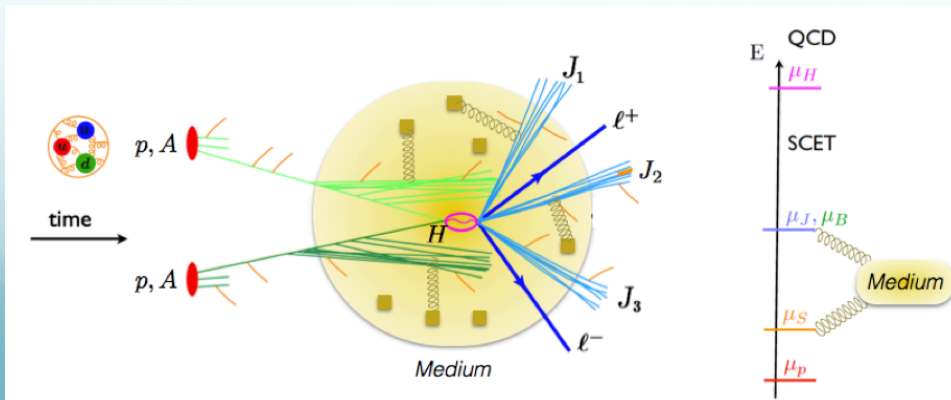
- SCET: an effective field theory of QCD Bauer et al. 01, Pirjol et al. 04
  - Suitable for processes where there are energetic, nearly light-like (**collinear**) degrees of freedom interacting with one another via **soft** radiation

## Modes in SCET

Collinear quarks, antiquarks	$\xi_n, \bar{\xi}_n$
Collinear gluons, soft gluons	$A_n, A_s$

modes	$p^\mu = (+, -, \perp)$	$p^2$	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n, A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s, A_s^\mu$

- Especially suited for jet physics: QCD factorization of modes



$$\sigma = H \otimes S \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^N J_j$$

# Comments

- Most of SCET calculations are dealing with “exclusive” jet production
- Most of heavy ion measurements are performed for “inclusive” jet samples
- Jet functions involved in “exclusive” and “inclusive” cases are different, even follow different renormalization group equations
- What are the jet functions for inclusive jet production?

# Tradition tools: QCD factorization

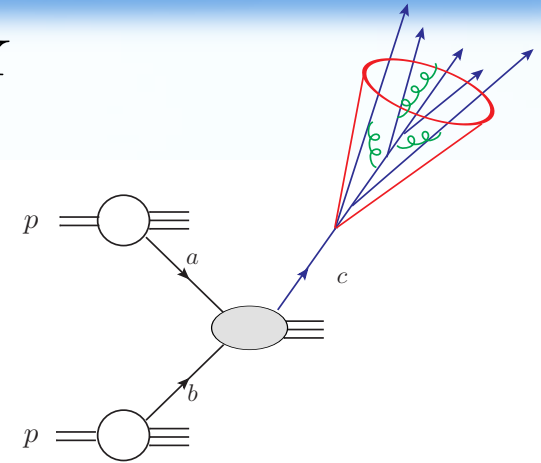
- Single inclusive jet production:  $p + p \rightarrow \text{jet} + X$

$$\frac{d\sigma^{pp \rightarrow \text{jet} X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}$$



partonic hard-scattering cross section

$$H_{ab} = \alpha_s^2 \left( H_{ab}^{(0)} + \alpha_s H_{ab}^{(1)} + \alpha_s^2 H_{ab}^{(2)} + \dots \right)$$

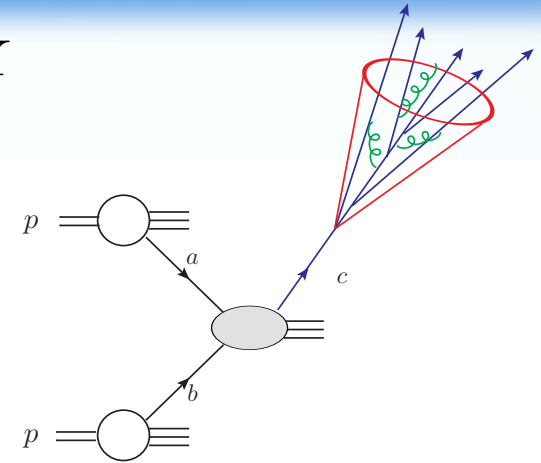


- The idea is simple: dynamics which happen in very different scales do not interfere with each other:  $\Lambda_{\text{QCD}}$  vs  $P_T$

# Tradition tools: QCD factorization

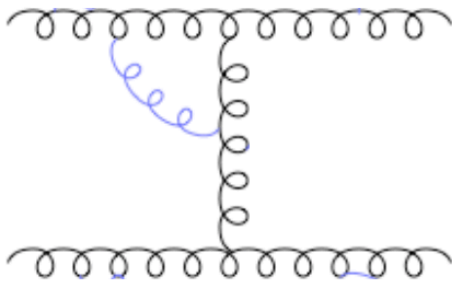
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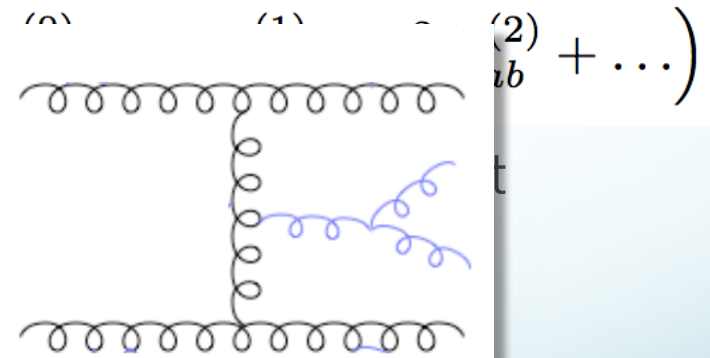
partonic hard-scattering cross section

- The scal



NLO 1990

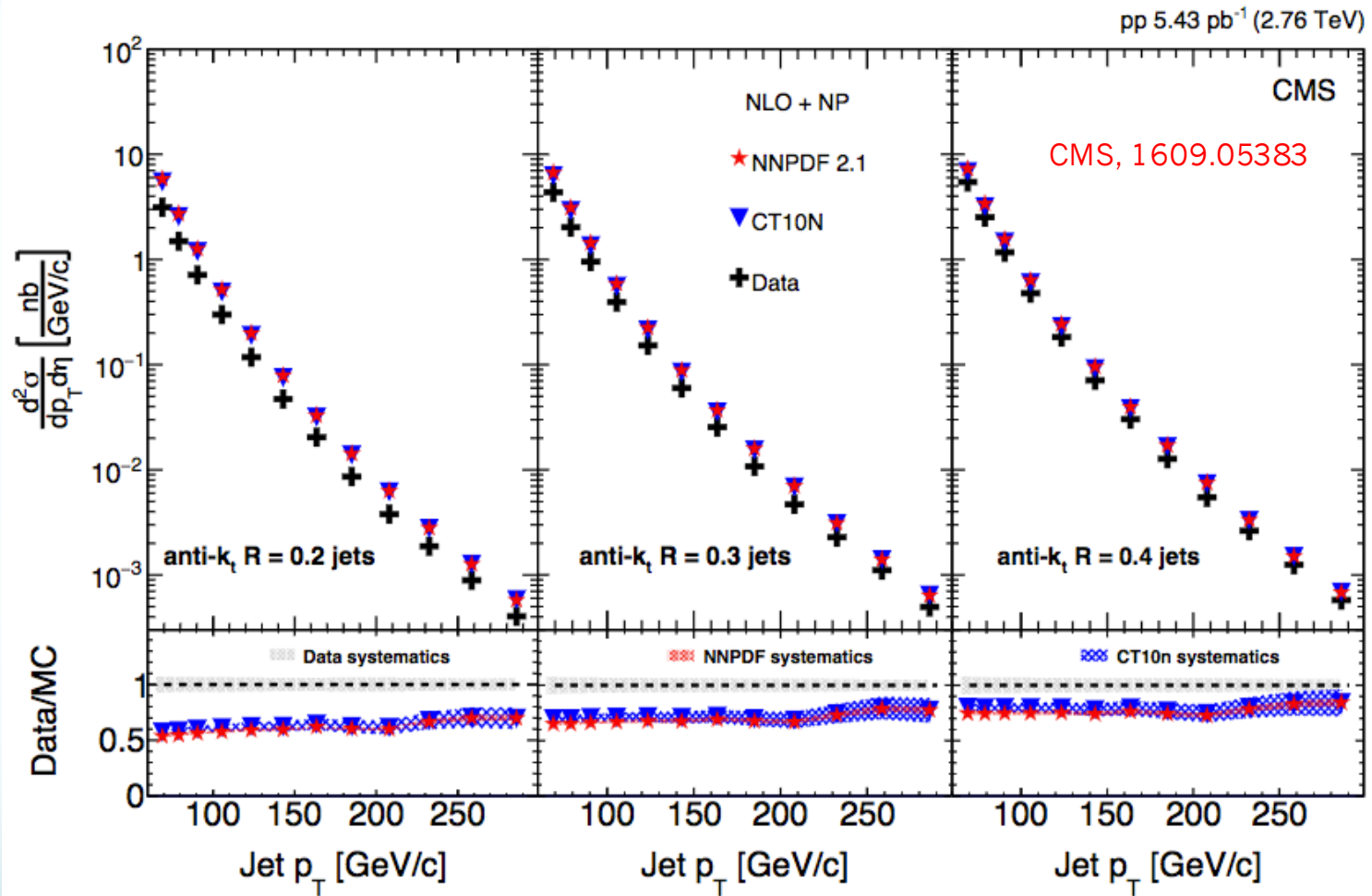
Ellis, Kunszt, Soper '90



NNLO 2016 ...

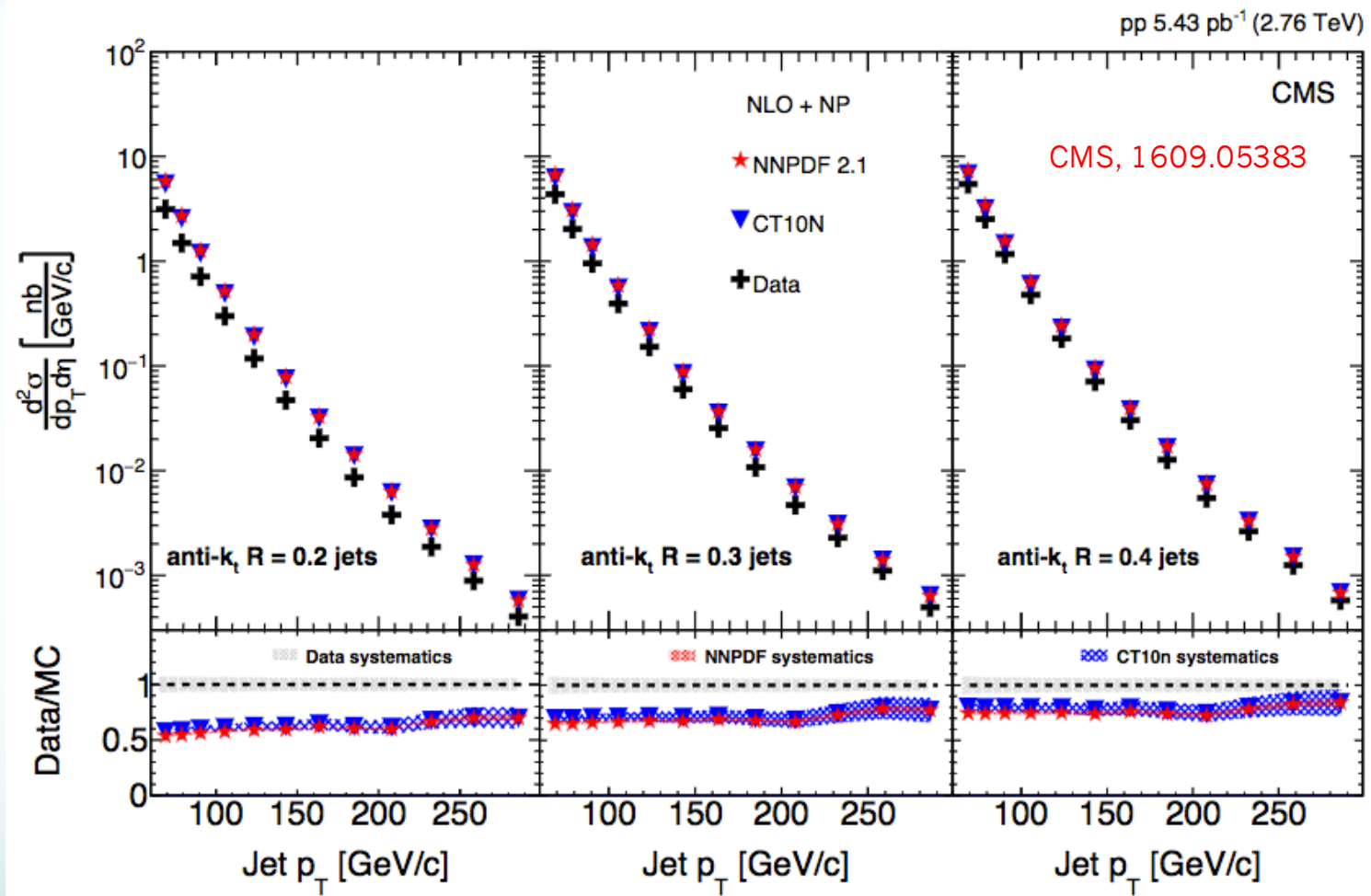
Currie, Glover, Pires '16

# Most recent jet measurements



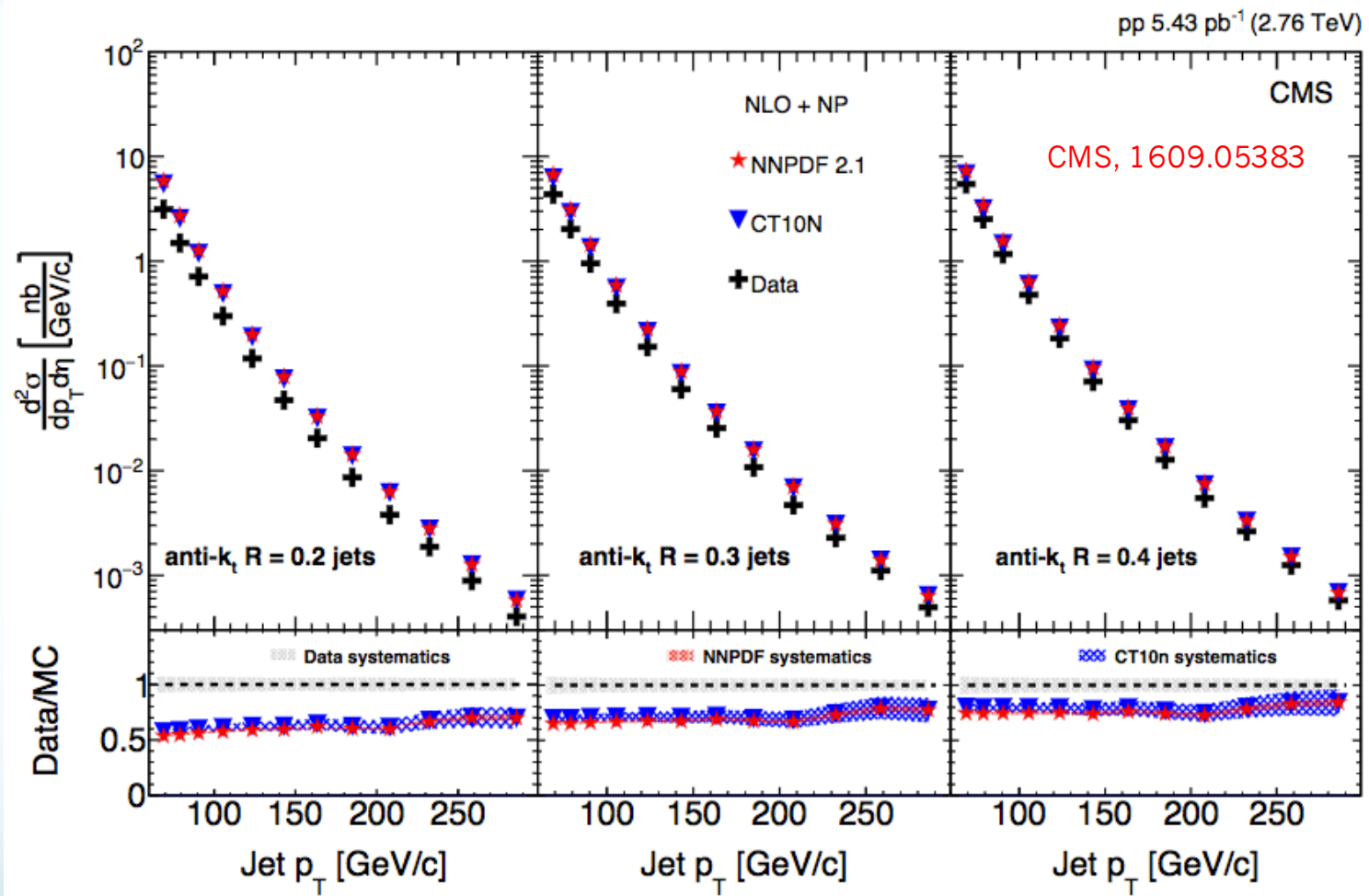


# Most recent jet measurements



✓ NNLO does not help: further increase, even worse

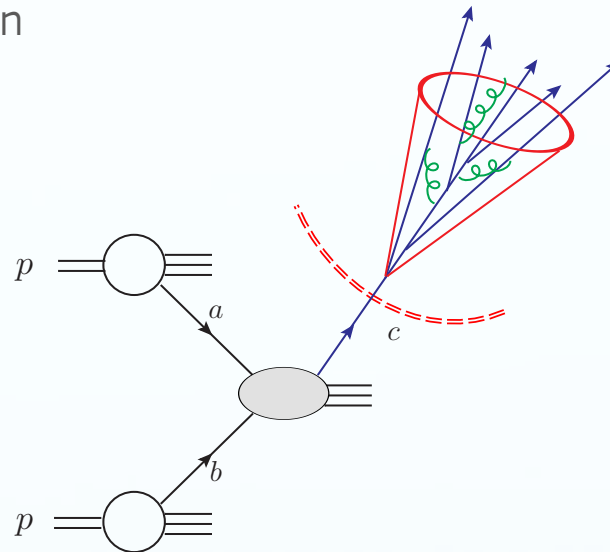
# Most recent jet measurements



- ✓ NNLO does not help: further increase, even worse
- ✓ Jet radius  $R$  is small: 0.2 – 0.4,  $[\alpha_s \ln(R)]^n$  resummation?

# Refactorization: semi-inclusive jet function

- When  $R \ll 1$ , the relevant scales for single jet production
  - Two momenta: (1) hard collision:  $p_T$  (2) jet radius can build one:  $p_T R$
  - A further factorization



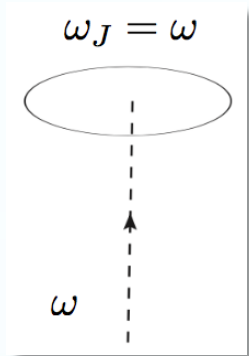
$$\frac{d\sigma^{pp \rightarrow \text{jet } X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes J_c(z, \mu \sim p_T R)$$

- Good thing: semi-inclusive jet function  $J_{q,g}(z, R)$  are purely perturbative

Kang, Ringer, Vitev, arXiv:1606.06732, Dai, Kim, Leibovich, 1606.07411,  
see also, Kaufmann, Mukherjee, Vogelsang, 1506.01415

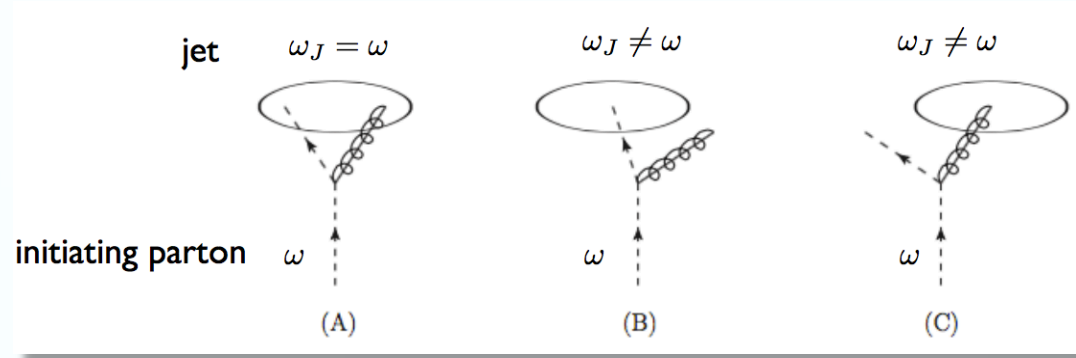
# Semi-inclusive jet functions

- Describe how a parton (q or g) is transformed into a jet (with a jet radius R) and energy fraction z



**LO**

$$J_q^{(0)}(z, \omega_J) = \delta(1 - z)$$



**NLO**

$$z = \omega_J / \omega$$

- Semi-inclusive quark/gluon jet functions follow DGLAP evolution equation, just like hadron fragmentation function

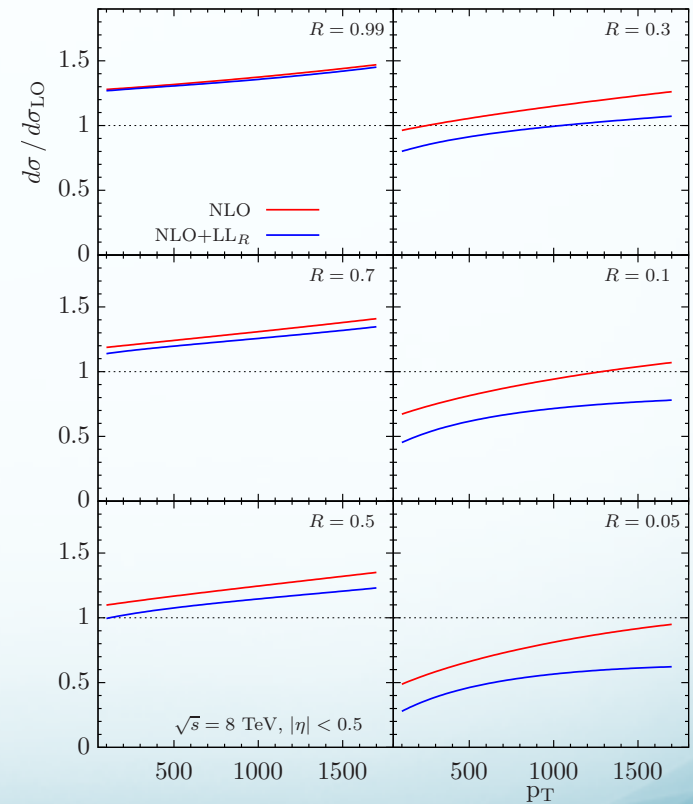
$$\mu \frac{d}{d\mu} J_i(z, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji} \left( \frac{z}{z'}, \mu \right) J_j(z', \omega_J, \mu)$$

# Ln(R) resummation

- Natural scale for jet functions:  $p_T * R$
- Jet radius resummation:  $(\alpha_s \ln R)^n$ 
  - Note:  $\ln(R) < 0$  when  $R < 1$

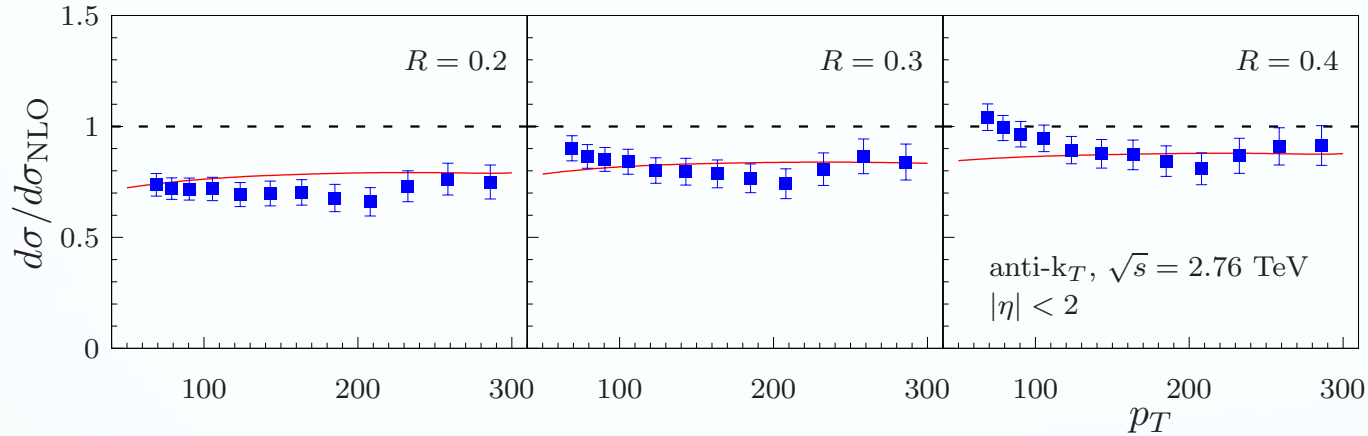


Kang, Ringer, Vitev, arXiv:1606.06732, JHEP 16



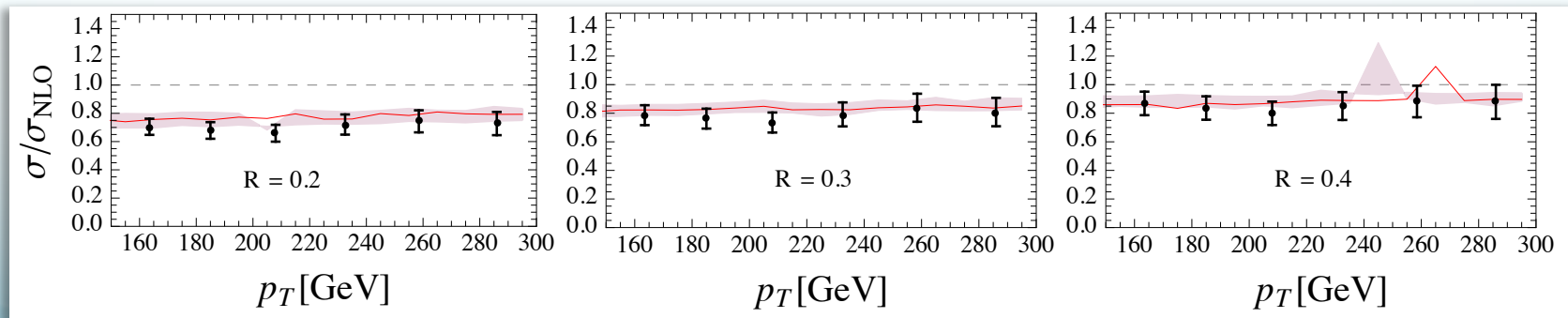
# Effect of $\ln(R)$ resummation

- The  $\ln(R)$  is the main source for the discrepancy



Kang, Ringer, Vitev, 1707.00913, PLB 17

- Threshold resummation further improve the agreement

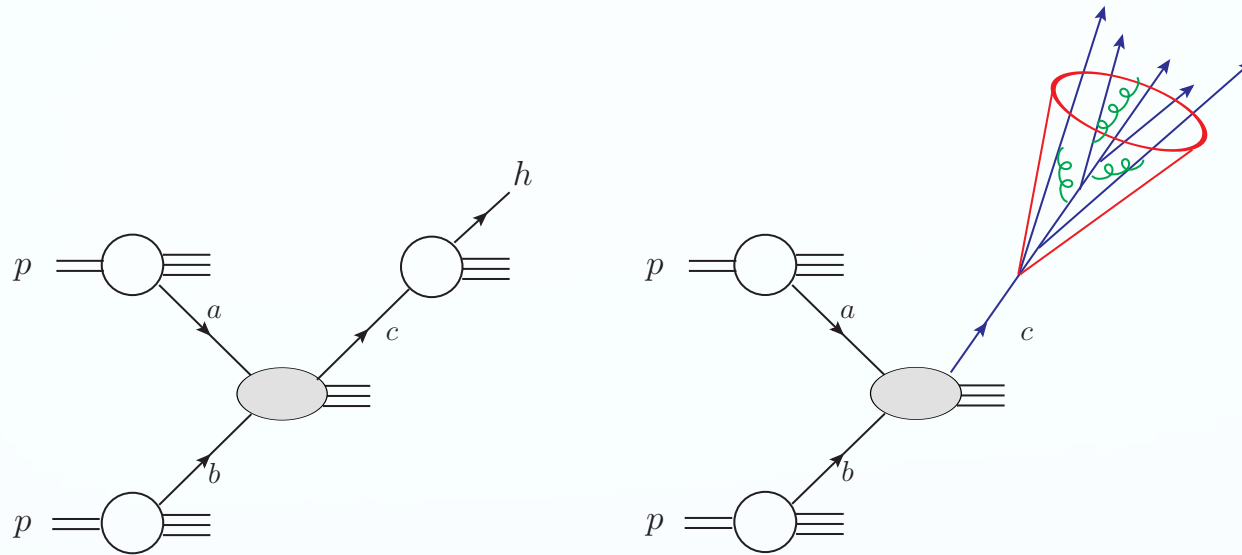


Liu, Moch, Ringer, PRL 2017



# Features: unified formalism

- Unified factorization formalism for hadron and jet production



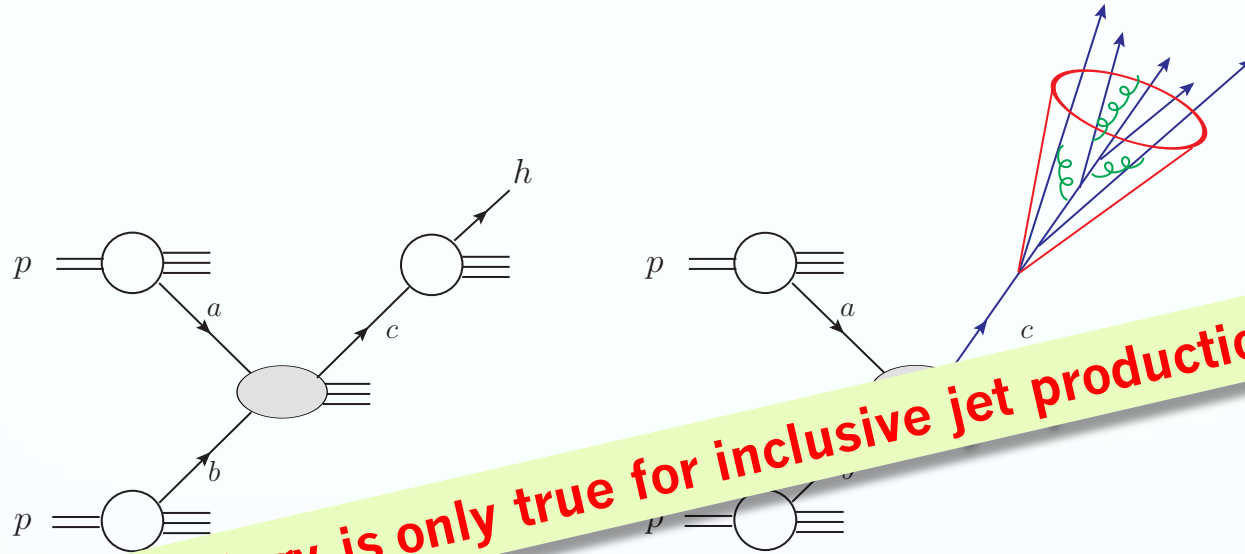
$$\frac{d\sigma^{pp \rightarrow hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes D_c^h(z, \mu)$$

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- Consistent definition of what are called quark/gluon jets at NLO
- Even though derived for small R,  $R = 0.7$ , the difference between small R approximation and full result is less than 5%

# Features: unified formalism

- Unified factorization formalism for hadron and jet production



**Such analogy is only true for inclusive jet production**

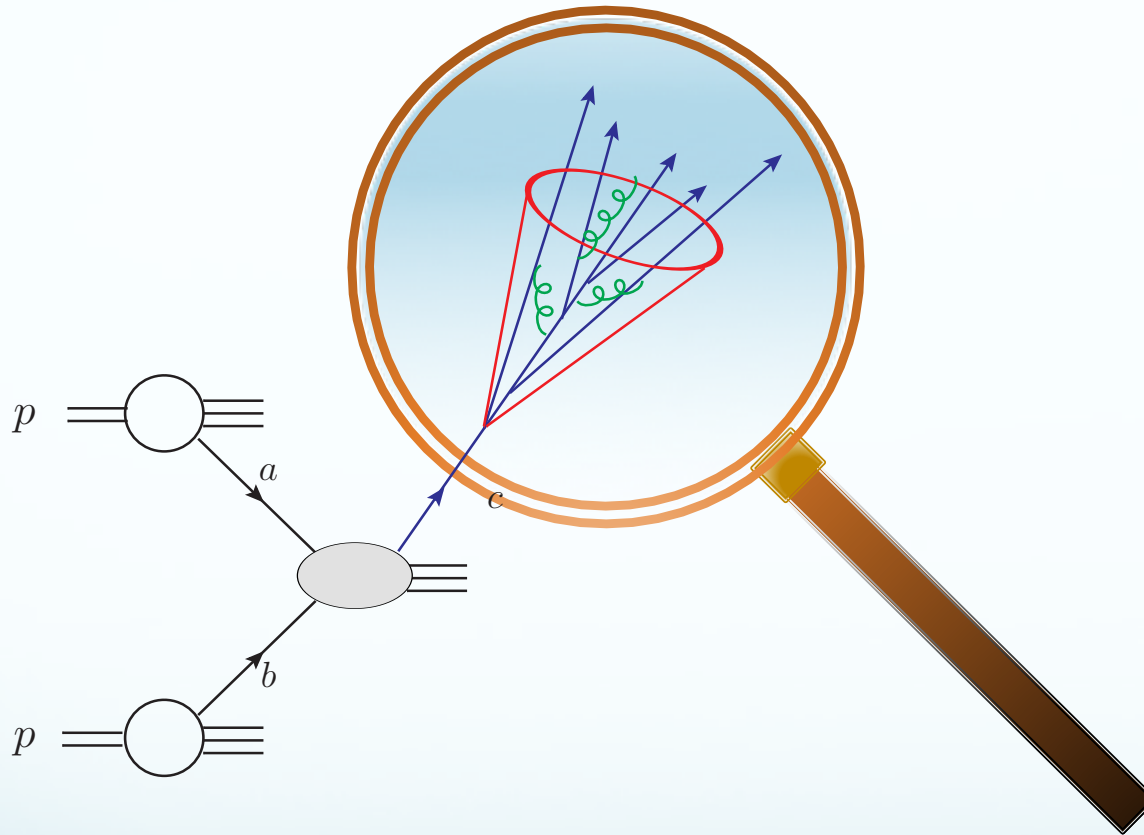
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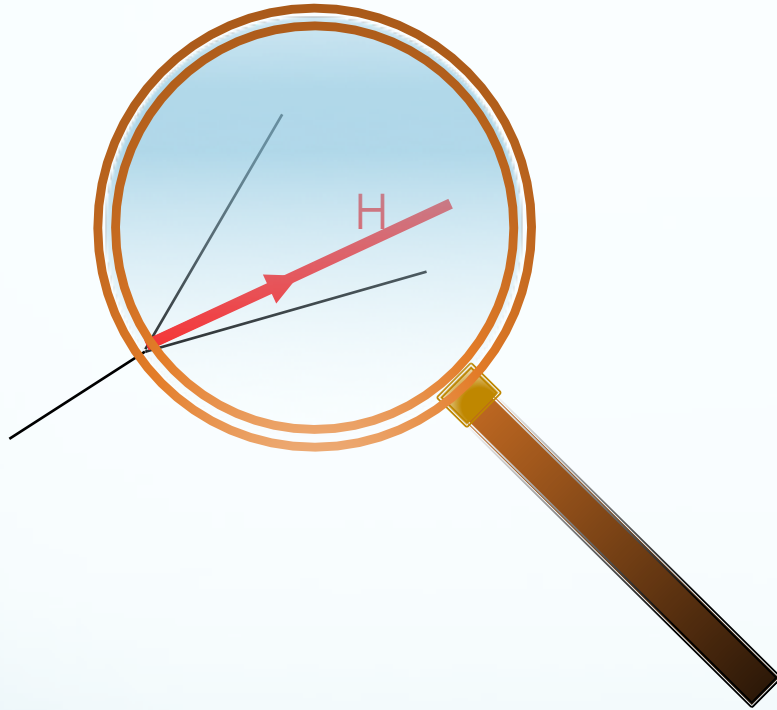
# Jet substructure

- Look inside the jet



# Simplest example: hadron inside the jet

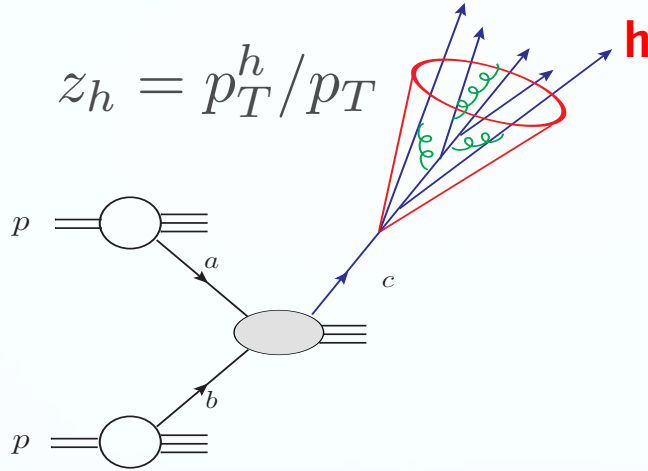
- Jet fragmentation function



# Semi-inclusive fragmenting jet function

- One needs a more complicated jet function

Kang, Ringer, Vitev, 1606.07063, JHEP 16



$$J_C(z, p_T R, \mu) \rightarrow \mathcal{G}_C^h(z, z_h, p_T R, \mu)$$

- Two DGLAPs: 
$$\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji} \left( \frac{z}{z'} \right) \mathcal{G}_j^h(z', z_h, \mu)$$

$$\mathcal{G}_i^h(z, z_h, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \mathcal{J}_{ij}(z, z'_h, \mu) D_j^h \left( \frac{z_h}{z'_h}, \mu \right)$$

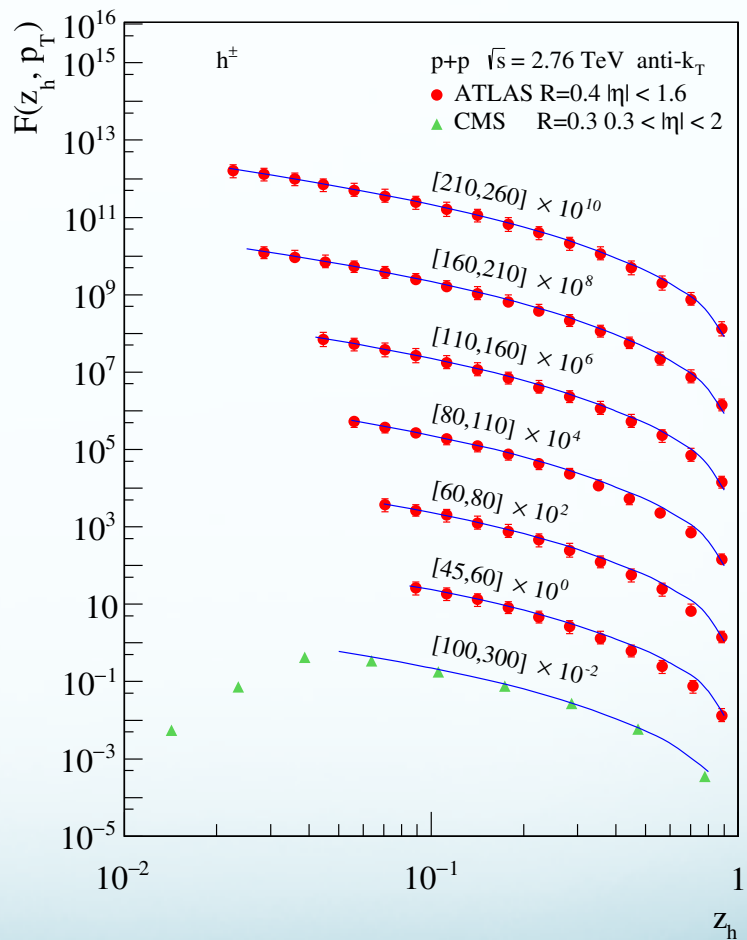
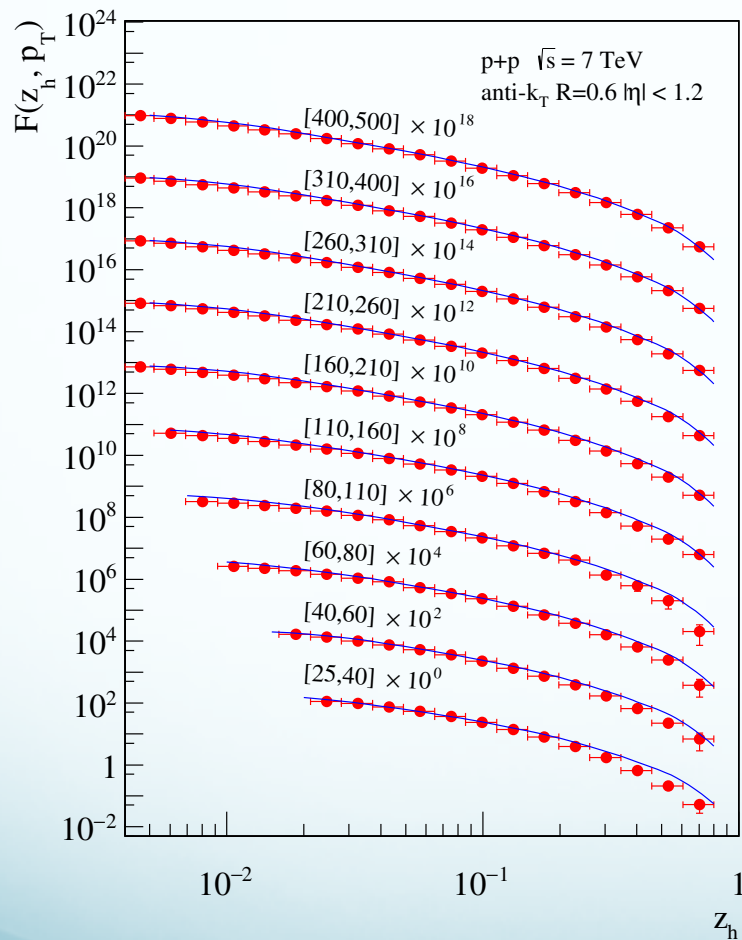
$\mu \sim p_T$

$\mu_J \sim p_T \times R$

$\mu_D \sim 1 \text{ GeV}$

# Light hadrons: work well

- Light charged hadrons

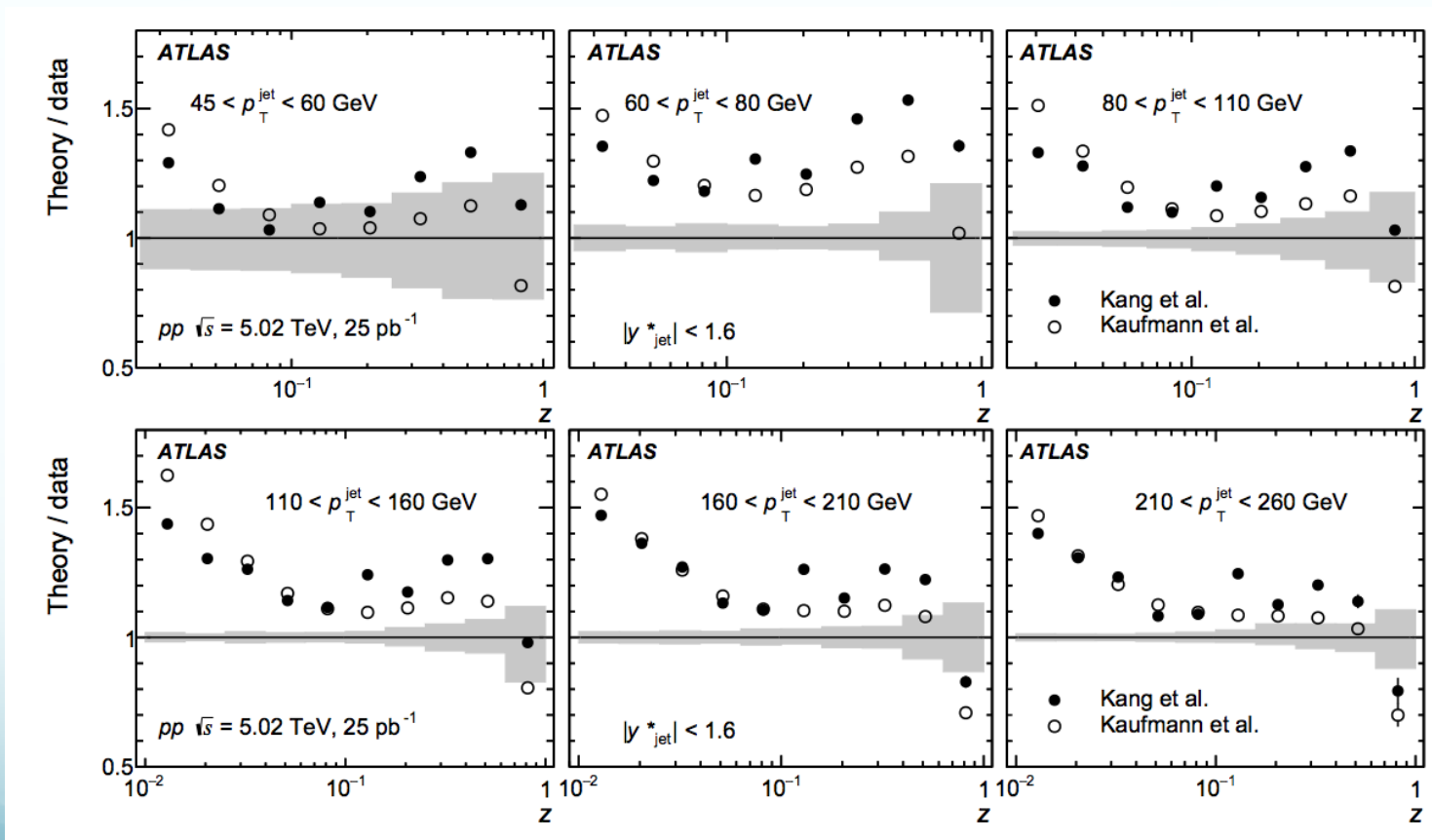


Kang, Ringer, Vitev, 1606.07063, JHEP, 16



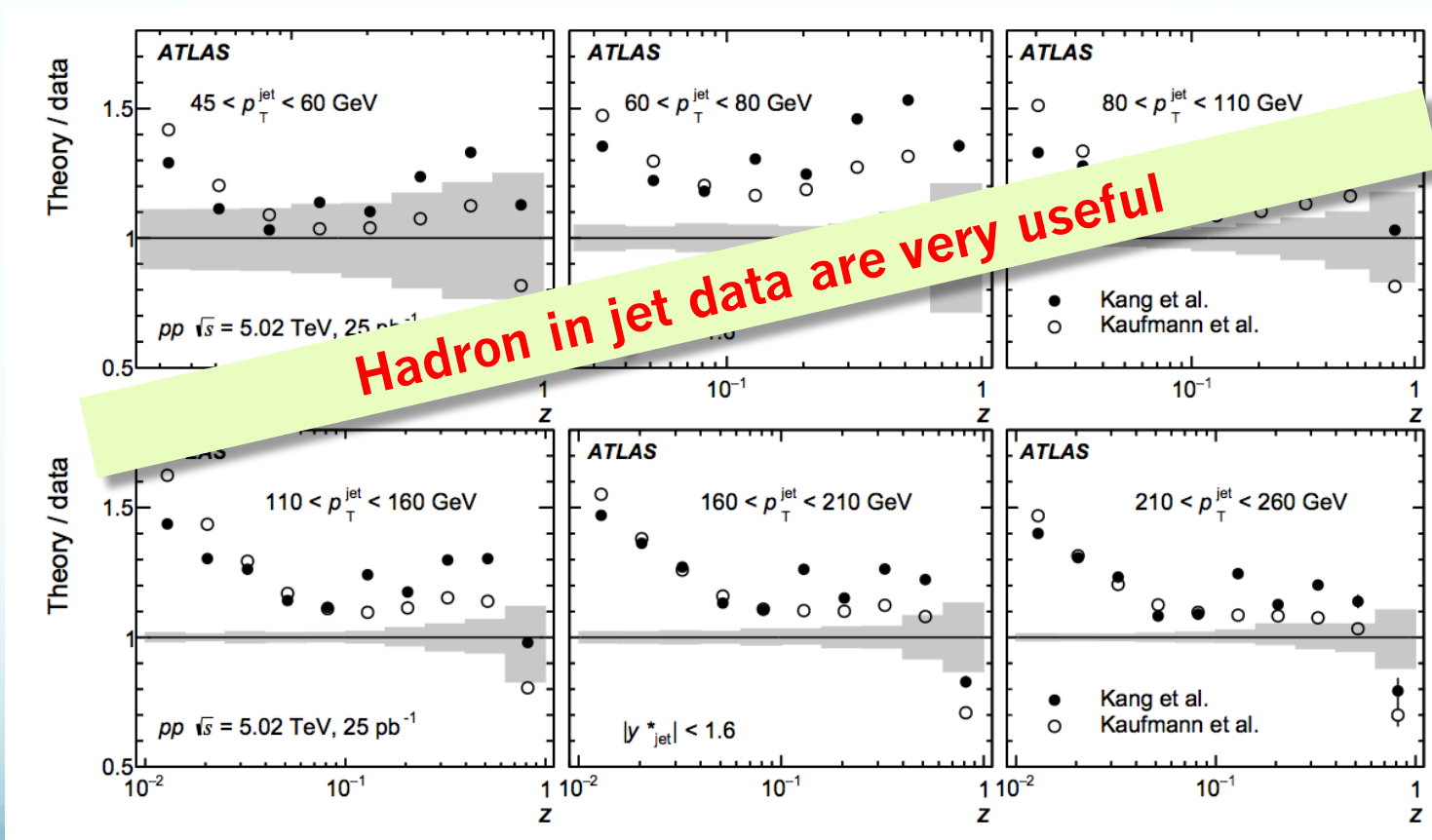
# Further improvement

- So far standard FFs is only constrain for  $z > 0.05$ 
  - These data can constrain small- $z$
  - One might need threshold resummation for large  $z$  region



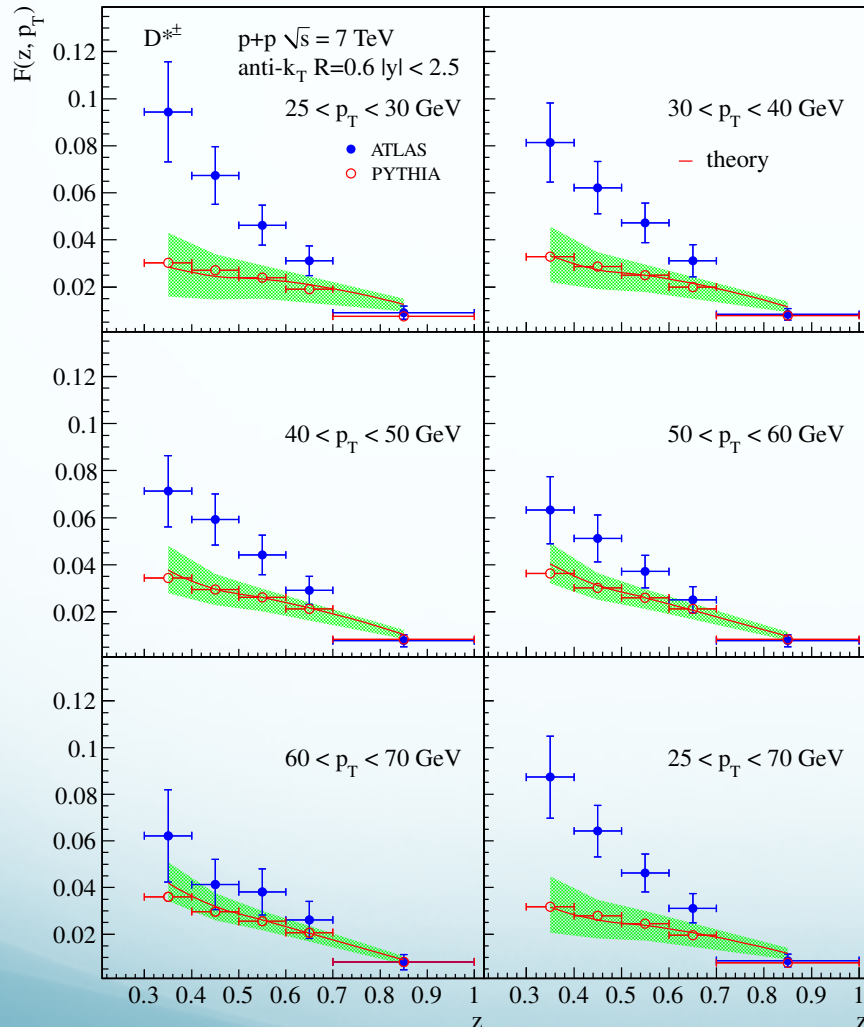
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# Jet fragmentation function for heavy meson

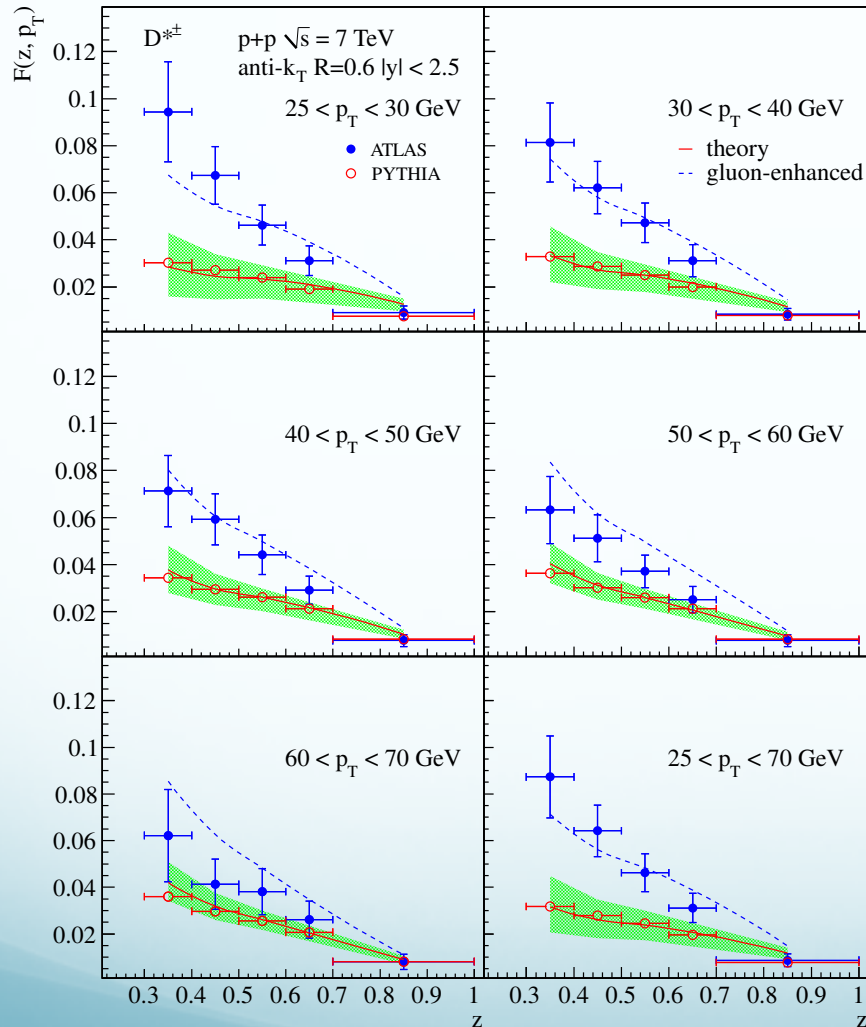
- Using D meson FFs fitted from e+e- data [Kneesch, Kniehl, Kramer, Schienbein, 08](#)



Using ZM-VFNS scheme:  
[Chien, Kang, Ringer, Vitev, Xing, 1512.06851, JHEP 16](#)

# Jet fragmentation function for heavy meson

- Using D meson FFs fitted from e+e- data Kneesch, Kniehl, Kramer, Schienbein, 08



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1512.06851, JHEP 16

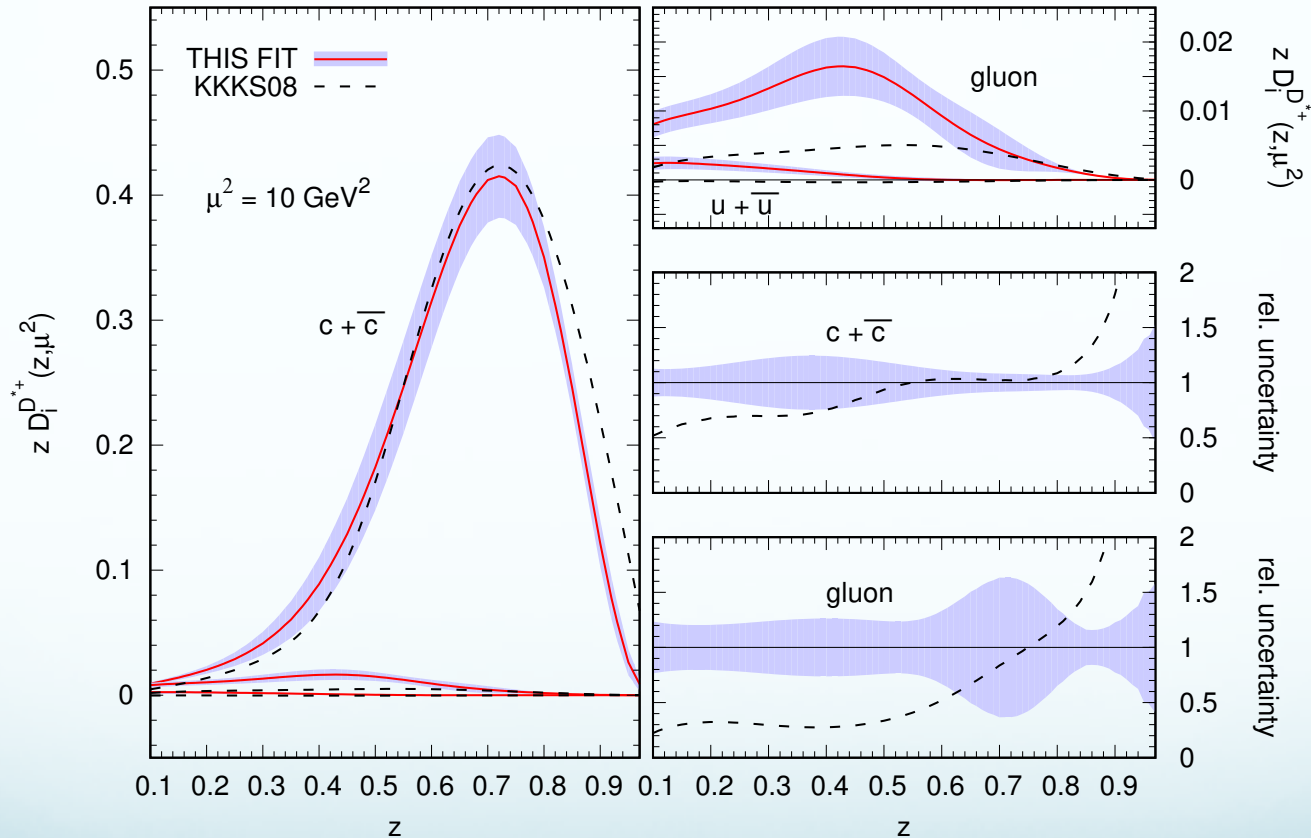
$$\text{---} D_g^D(z, \mu) \rightarrow 2D_g^D(z, \mu)$$

New fit of D-meson FFs needed

# A new global analysis of FFs

- New fit of D-meson FFs

New fit of D-meson FFs:  
Stratmann, et.al., PRD 2017



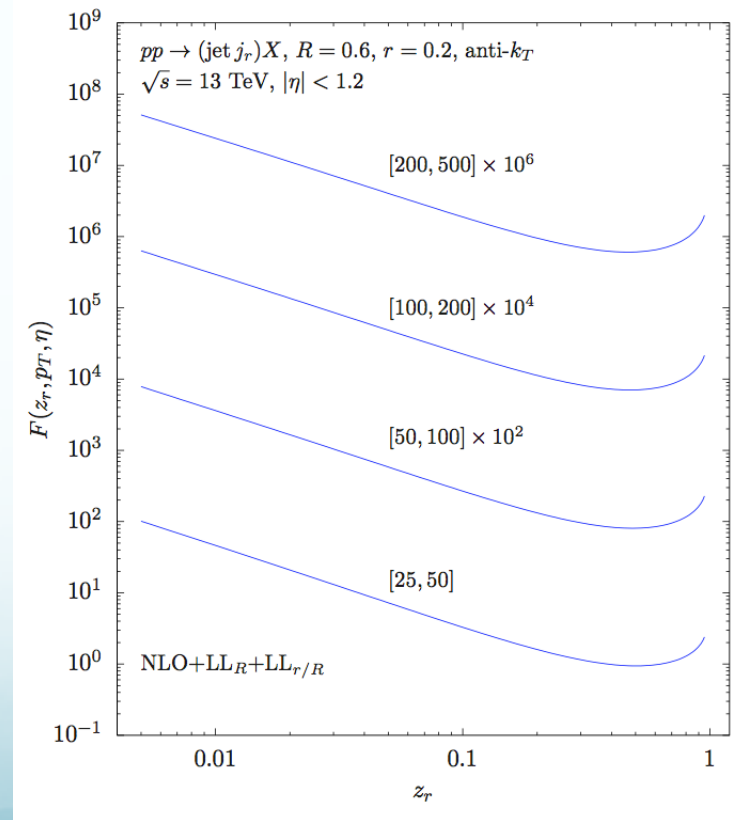
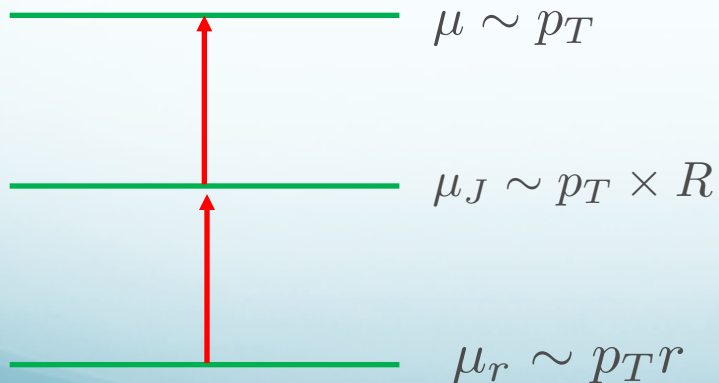
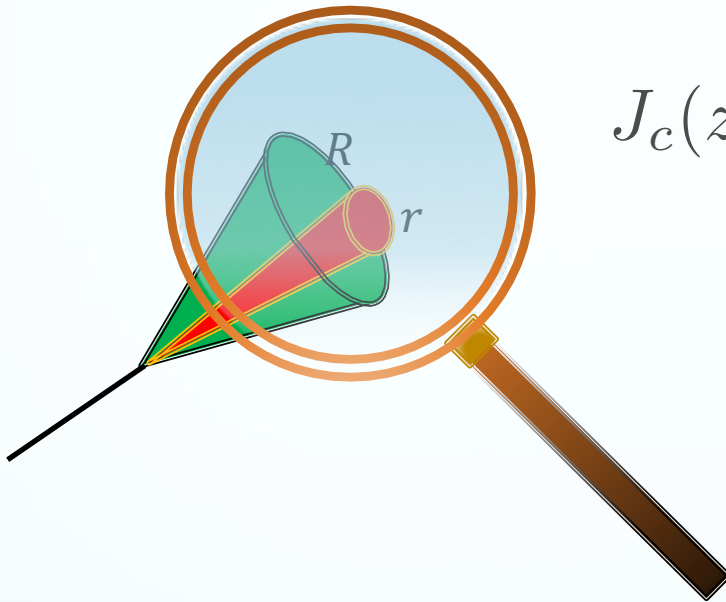
**Confirms our earlier guess**

# Subjet in jet: subjet function

- We may try to observe a subjet with radius  $r$

Kang, Ringer, Waalewijn, 1705.05375, JHEP, 17

$$J_c(z, p_T R, \mu) \rightarrow \mathcal{G}_c^{\text{jet}}(z, z_r, p_T R, \mu)$$

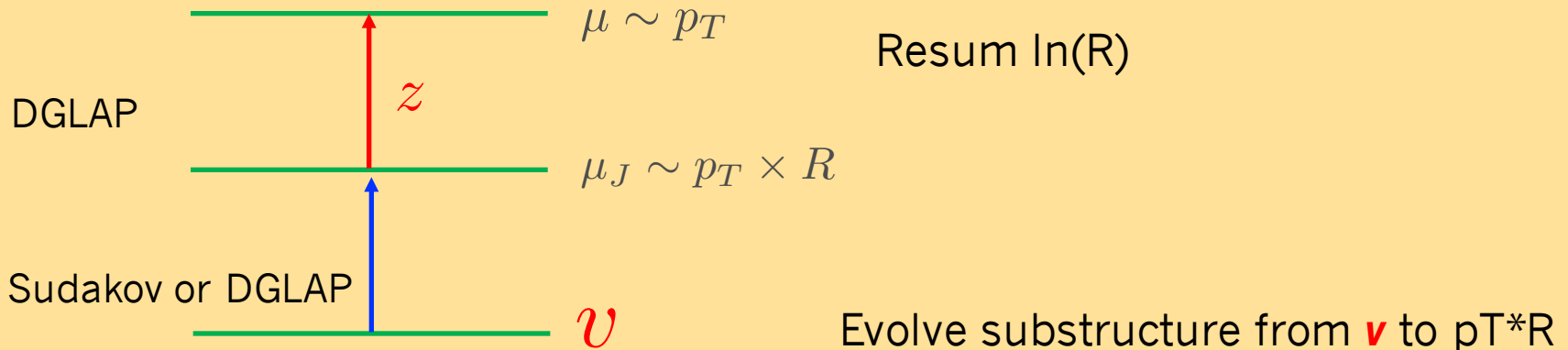
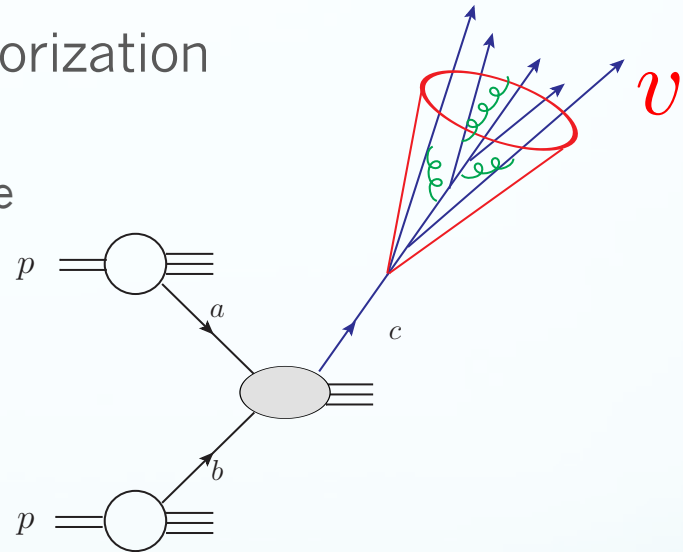




# Pattern emerging for the evolution

- When we measure any jet substructure variable “ $v$ ” from the jet, once we evolve to jet dynamics scale  $p_T^*R$ , the remaining evolution to hard scale  $p_T$  is given by DGLAP evolution
- Jet substructure: two-layer QCD factorization
  - Producing the jet
  - Concentrating on the internal substructure

$$\frac{d\sigma}{d\eta dp_T dv}$$



# Jet angularity

- Trust was defined as an event shape parameter to understand radiation pattern

$$T = \frac{1}{Q} \max_{\mathbf{t}} \sum_{i \in X} |\mathbf{t} \cdot \mathbf{p}_i| = 1 - \tau_0$$

- $\tau_0 \rightarrow 0$  is equivalent to dijet limit
- A generalized class of IR safe observables, angularity (applied to jet)

$$\tau_a^{e^+e^-} = \frac{1}{E_J} \sum_{i \in J} E_i \theta_{iJ}^{2-a}$$

$$\tau_a^{pp} = \frac{1}{p_T} \sum_{i \in J} p_{Ti} (\Delta R_{iJ})^{2-a}$$

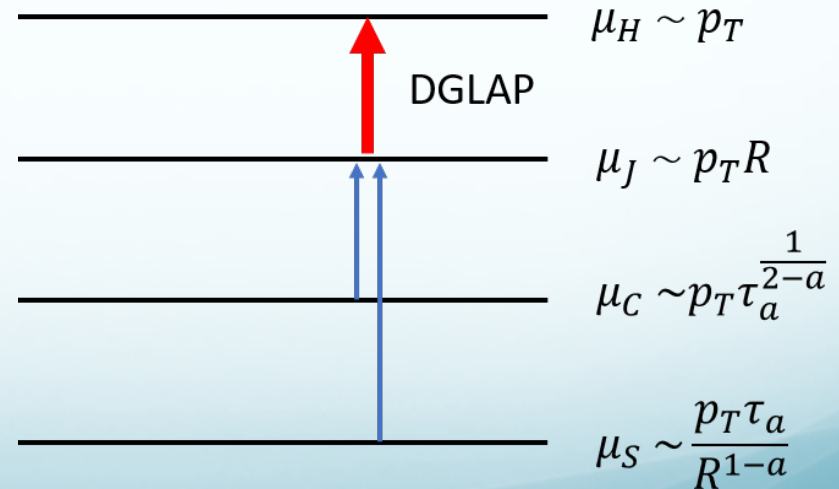
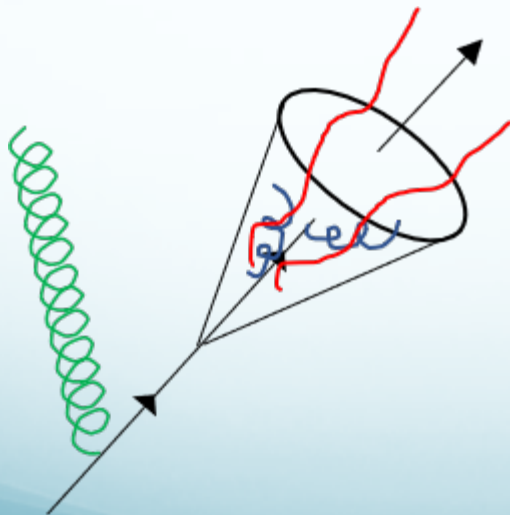
Sterman, et.al. 03, 08, C. Lee, et.al. 10  
Hornig, Makris, Mehen, 16

- $a=0$  related to thrust (jet mass)
- $a=1$  related to jet broadening

# Semi-inclusive angularity jet function

- Similar replacement:  $J_c(z, p_T R, \mu) \rightarrow \mathcal{G}_c(z, p_T R, \tau_a, \mu)$
- Refactorization

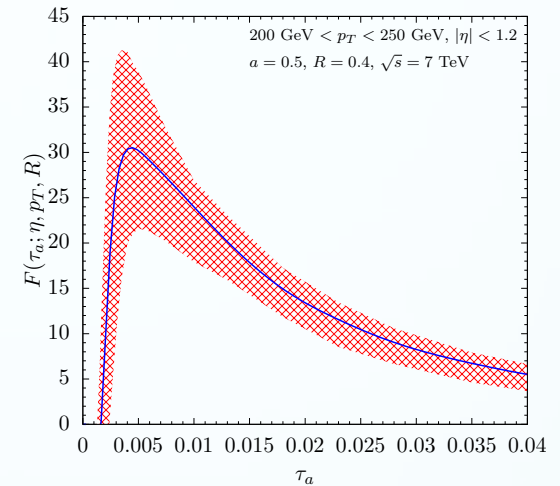
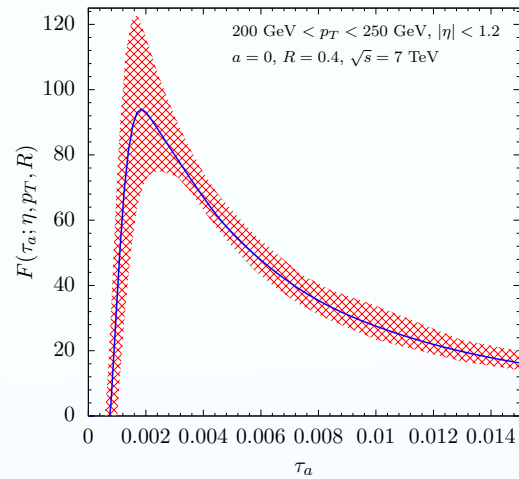
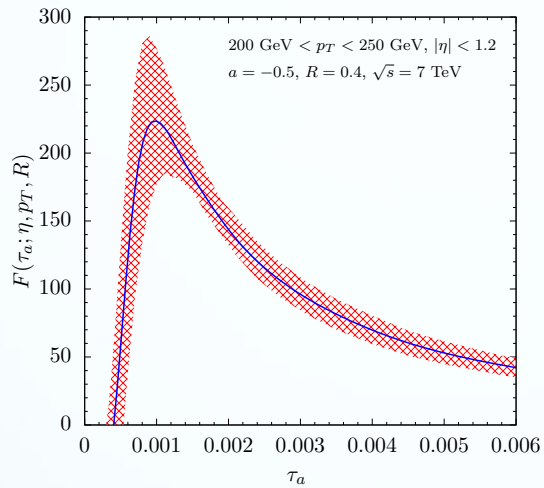
$$\mathcal{G}_c(z, p_T R, \tau_a, \mu) = \sum_i \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \times \int d\tau_a^{C_i} d\tau_a^{S_i} \delta(\tau_a - \tau_a^{C_i} - \tau_a^{S_i}) \mathcal{C}_i(\tau_a^{C_i}, p_T \tau_a^{\frac{1}{2-a}}, \mu) \mathcal{S}_i(\tau_a^{S_i}, \frac{p_T \tau_a}{R^{1-a}}, \mu)$$



# LHC phenomenology

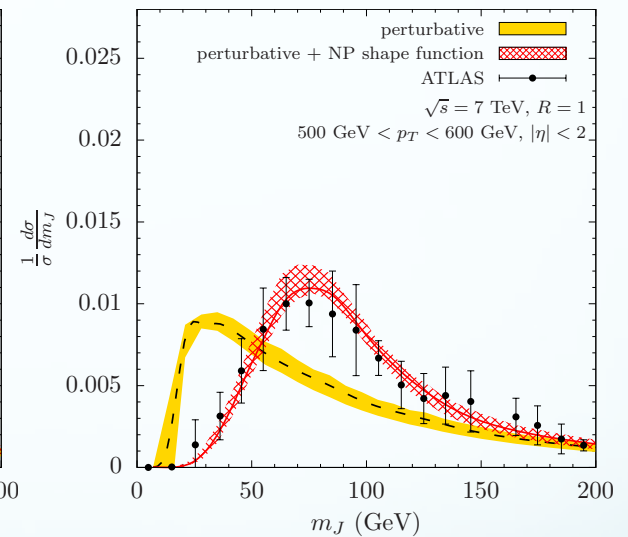
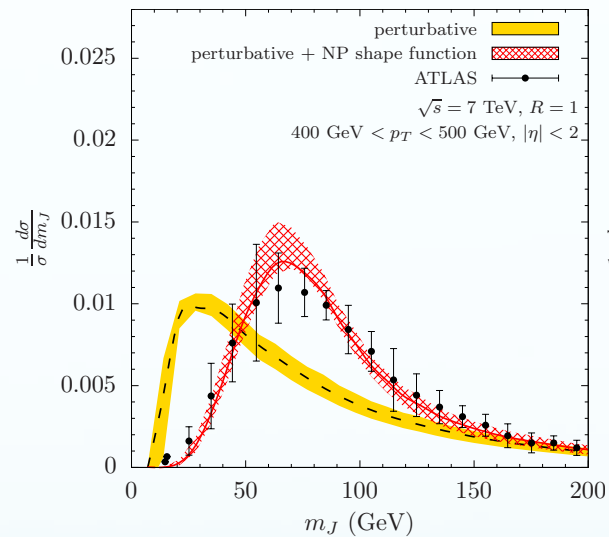
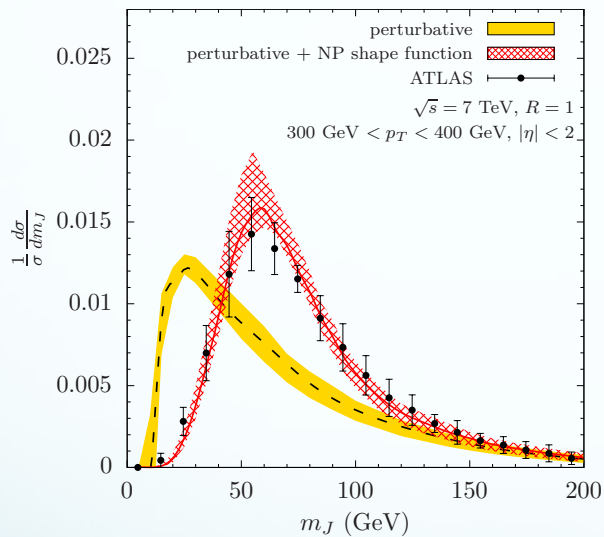
- Prediction at the LHC

Kang, Lee, Ringer, 1801.00790



# Jet mass: $a = 0$

- Comparison with jet mass measurements at the LHC



Compiled from Kang, Lee, Ringer, 1801.00790,  
See also, C.S. Lee, et al., 1412.1337, JHEP 15

# Summary

- A consistent formalism for study inclusive jets and jet substructure is introduced, through so-called semi-inclusive jet functions
- For inclusive jet cross section, these novel semi-inclusive jet functions are purely perturbative, and follow the usual DGLAP evolution equations, which can be used to perform  $\ln(R)$  resummation
- Jet substructure for inclusive jets can be computed similarly
- Exciting jet physics for inclusive measurements can be pursued

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Thank you!