Nuclear Level Density Rotational Enhancement Factor

S. M. Grimes
Ohio University

CNR*18 Sept. 24-28, 2018 Berkeley, California
Rotational Enhancement Factor: by how much does the level density for a deformed nucleus exceed that for neighboring spherical nuclei at the same excitation energy.

$$R = \frac{\rho(E)_{\text{deformed}}}{\rho(E)_{\text{spherical}}}$$
The formula $R(J) = \sigma_1^2$ was proposed.

This gives a factor of 50 for rare earth nuclei.

Level densities often inferred from s-wave resonances counting to find the density of $\frac{1}{2}^+$ states using the Bethe $J$ distribution.

$$s(J) = \frac{(J + 1/2)}{2\sigma^2} \exp\left(-\frac{(J + 1/2)^2}{2\sigma^2}\right)$$

$$\rho(E) = \rho(E, 1/2) \frac{\sum s(J)}{s(1/2)}$$

H.A. Bethe, Phys. Rev. 50 332 (1936).
Previous Investigations

  • Compared level densities for spherical nuclei with deformed nuclei.
  • Found $R \sim 1$ not $\sigma_\perp^2$

  • Also found $R \sim 1$

  • Derived Level Density parameters. Found no enhancement

• In each case the Bethe Formula was used
Recent Results

More recently, Grimes (Phys. Rev. C 88, 024613 (2013)).

Showed that $R$ was not $R(E)$ but $R(E, J, K)$

$$R(E, J, K) = \frac{(J + 1)^2 - K^2}{2J + 1} \exp \left( -K^2 \left( \frac{1}{2\sigma^2_\parallel} - \frac{1}{2\sigma^2_\perp} \right) \right)$$

$K$ is normally mixed above 3 MeV which gives:

$$R_1(E, J) = \sum_K R(E, J, K)$$
Behavior of $R_1(E,J)$

For deformed nuclei the $R_1(E,J)$ depends explicitly on $J$.

Dependence on $E$ is through:

$$
\sigma_{||}^2 = \frac{I_{||} \theta}{\hbar^2}
$$

$$
\sigma_{\perp}^2 = \frac{I_{\perp} \theta}{\hbar^2}
$$

$$
\theta = \sqrt{E/a}
$$

$a$ — is the level density parameter, $I_{||}$ and $I_{\perp}$ are the moments of inertia around the axis of symmetry and a perpendicular axis respectively.
• Note: 

\[ R_1(E, 0) = R_1(E, 1/2) = 1 \]

• Even A: \( J = 0 \) does not split when nucleus deforms

• Odd A: \( J = \frac{1}{2} \) does not split when nucleus deforms

• Other \( J \) values - give rotational enhancement of \( J + 1 \) (even) or \( J + \frac{1}{2} \) (odd).

• Build bands but no new band member have the lowest \( J \).

• Explains small enhancement 10% of the maximum

• Note: Bethe (1936) says his spin formula is only valid for spherical nuclei.
Iljinov et al.

- Resonances in their analysis
- They used $\rho(E) = \frac{2\rho(E,1/2) \sum J s(J)}{s(1/2)}$
- **ODD A** - $\rho(E) = \frac{2\rho(E,1/2) \sum J s(J)}{s(1/2)}$
- **EVEN A** - $\rho(E) = \frac{2(\rho(E,2)+\rho(E,3)) \sum J s(J)}{(s(2)+s(3))}$
- The factor of 2 is for parity
- New analysis $s(J) \rightarrow s(J)R_1(E,J)$
- Use of $R_1(E,J)$ shows that s-wave analysis does yield $R \neq 1$
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Nucleus & J$^\pi$ & $S_n$ (MeV) & D (ev) & $a_1$ & $a_2$ & $a_3$ & $a_4$ & R & LDx10$^7$ \\
\hline
$^{159}$Dy & $\frac{1}{2}^+$ & 6.83 & 30 & 20.81 & 13.05 & 22.0 & 29.1 & 27.5 & 8.42 \\
$^{161}$Dy & $\frac{1}{2}^+$ & 6.45 & 27.3 & 22.12 & 14.06 & 23.5 & 31.1 & 27 & 8.97 \\
$^{162}$Dy & $2^+,3^+$ & 8.197 & 2 & 21.34 & 13.38 & 19.8 & 25.8 & 29.0 & 3.36 \\
$^{163}$Dy & $\frac{1}{2}^+$ & 6.27 & 69 & 21.08 & 13.01 & 22.2 & 29.0 & 26.9 & 3.48 \\
$^{165}$Dy & $\frac{1}{2}^+$ & 5.71 & 170 & 21.05 & 12.67 & 22.4 & 30.1 & 25.8 & 1.31 \\
\hline
\end{tabular}
\end{table}

$a_1$ – parameter derived by Iljinov et al.

$a_2$ – parameter derived by Iljinov et al. from level density divided by R

$a_3$ - parameter derived by calculating level density with $R_1(E,J)$ removed (intrinsic level density)

$a_4$ – parameter derived with $R_1(E,J)$ included (total level density)

R – derived rotational enhancement averaged over $J$ ($\sim 0.6 \sigma_1^2$)
LD – actual total level densities

use of $R_1(E,J)$ formula with appropriate $\sigma_\parallel$ and $\sigma_\perp$ gives:

$\langle R_1(E,J) \rangle_J \sim 5$ for $A \sim 24$

$\langle R_1(E,J) \rangle_J \sim 44$ for $A \sim 235$
Microscopic Level Density Calculations

- Calculations include two body force
- Included $^{162}$Dy among the nuclei modeled
- Important because deformation emerged naturally
- Deformation can change with energy
- Nucleus could become spherical
Microscopic Calculations (cont.)

- Rotational bands -- levels are **not** created spontaneously – quadrupole force brings them down from high energy

- Direct comparison shows that present $^{162}$Dy level density is about 15 times larger than the al hassid et al. result at the same energy

- Why??  Al Hassid et al. assume spherical degeneracies in converting state densities to level densities.

\[
\rho_L(u) = \frac{\rho_S(u)}{\sqrt{2\pi} \sigma}
\]

- Actually degeneracy is not $(2j+1)$ but is 2 for a deformed nucleus puts Al Hassid into agreement
Reaction Studies


• They have looked for a change in $R$ caused by transition to spherical shape at some energy.

• Evaporation spectra are proportional to the state density where $R$ are on the order of $\sigma_\perp$. Rather than level density where $R$ is proportional to $\sigma^2_\perp$.

• Present results give $\sim 10$ enhancement rather than $\sim 30$.

• These changes should be properly modeled if deformed Hauser-Feshbach code is used.
Conclusions

• Long standing indications from resonance analysis that $R \sim 1$ are found to be due to ignoring the J dependence of $R$.

• Bethe spin formula is not correct for deformed nuclei.

• Use of the new form for $R_1(E,J)$ brings much better agreement between experiment and microscopic calculations.