High-resolution probes of low-resolution nuclei: An OPE-RG-EFT perspective

Dick Furnstahl SRNC at an EIC workshop, September, 2018



The Ohio State University

Collaborators:

Scott Bogner (MSU)

Eric Anderson (OSU)

Sushant More (OSU→MSU)

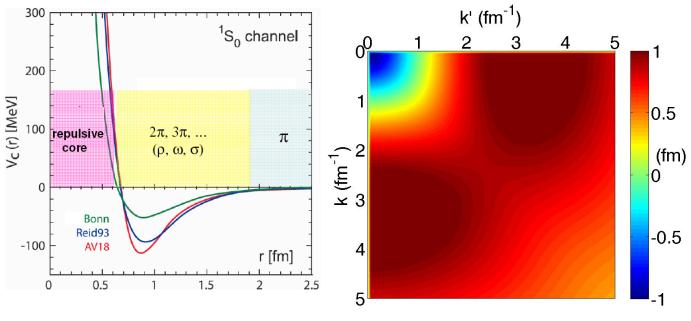
Nathan Parzuchowski (MSU→OSU)

plus many others on related topics



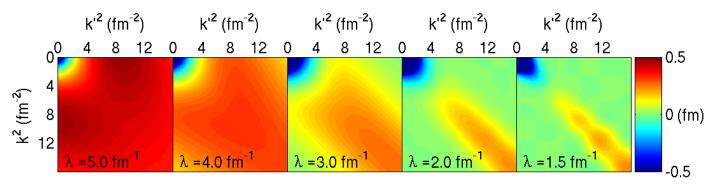




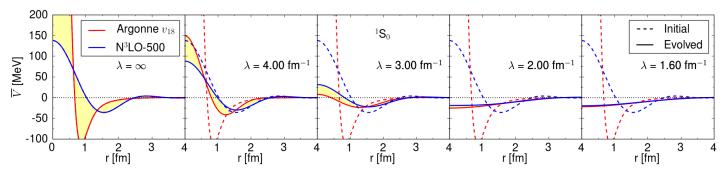


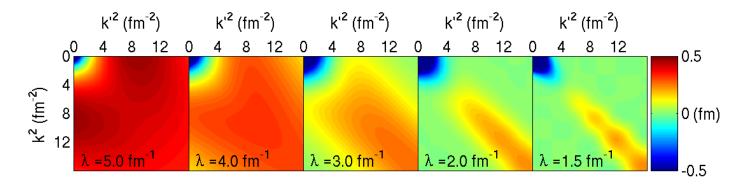
$$V_{L=0}(k,k') = \int d^3r j_0(kr) V(r) j_0(k'r) = \langle k|V_{L=0}|k'\rangle \implies V_{kk'} \text{ matrix}$$

In natural units for a large nucleus, typical relative momentum is about 1 fm⁻¹ but repulsive core (cf. tensor) couples to high-k components \rightarrow nonperturbative. Use renormalization group (RG) to evolve to lower resolution \rightarrow more perturbative.

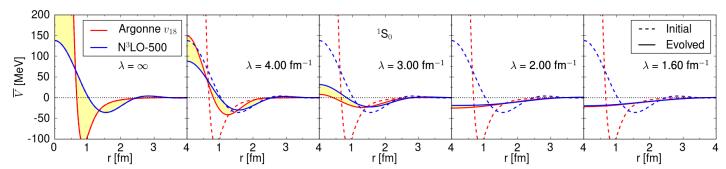


Flow equation (similarity) RG decouples in momentum space; melts core in r-space!



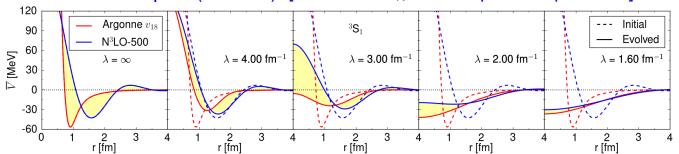


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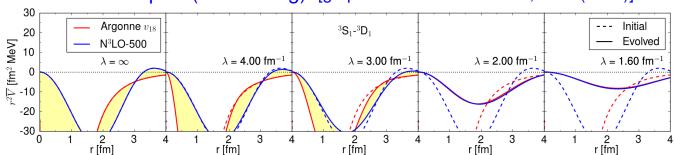


- Similarity RG is just *one way* to lower resolution in free space; technically nice.
- Different initial interactions look different (scheme) but flow to universal!
- Note: all operators change with RG evolution (not just Hamiltonian).

• Central part (S-wave) [Note: The V_{λ} 's are all phase equivalent!]



• Tensor part (S-D mixing) [graphs from K. Wendt et al., PRC (2012)]

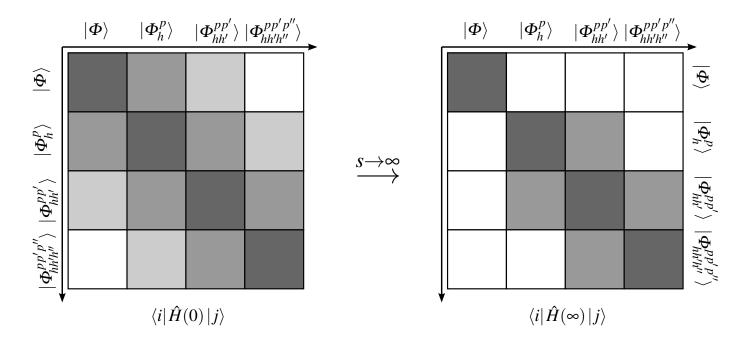


Local projection:

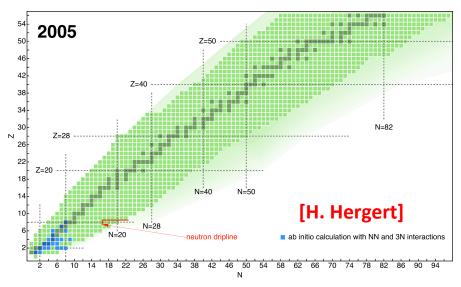
$$\overline{V}_{\lambda}(r) = \int d^3r' \ V_{\lambda}(r,r')$$

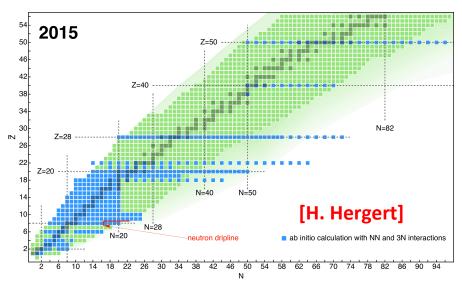
action of potential on low-k nucleons

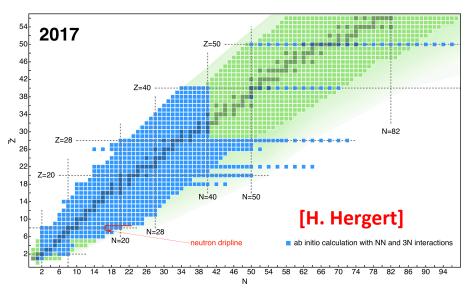
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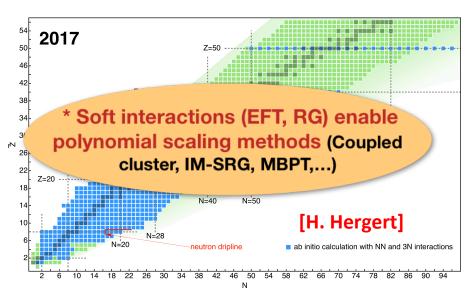


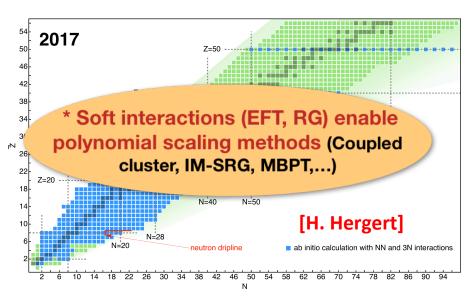
- In-medium SRG (IM-SRG) decouples excitations in-medium wrt reference state.
- Keeps leading induced many-body forces [see Hergert et al., arXiv:1612.08315].
- Note: all operators change with RG evolution (not just Hamiltonian).

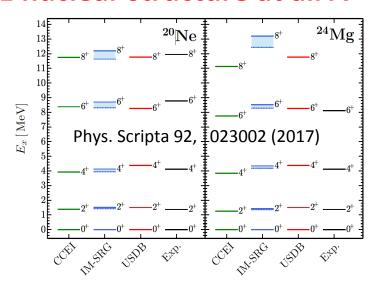


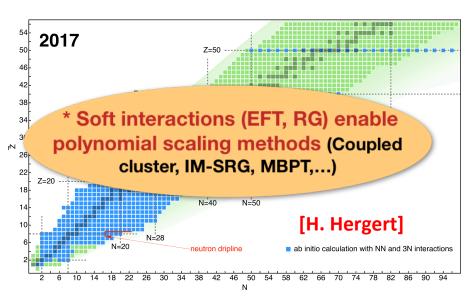


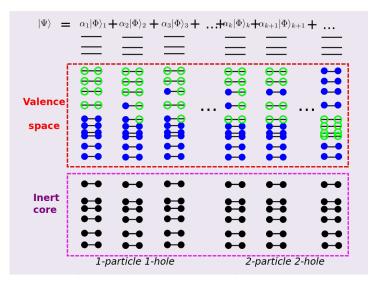


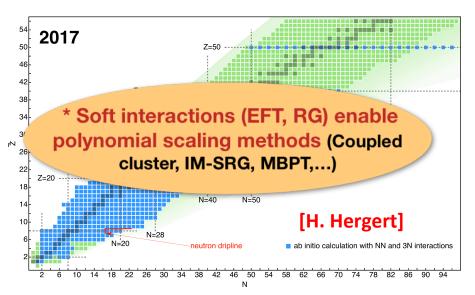




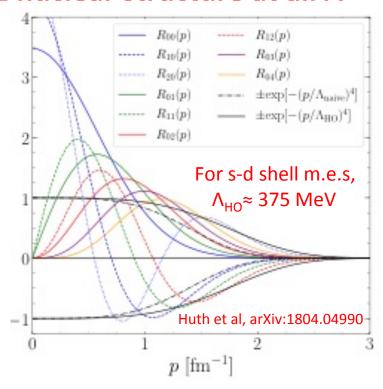


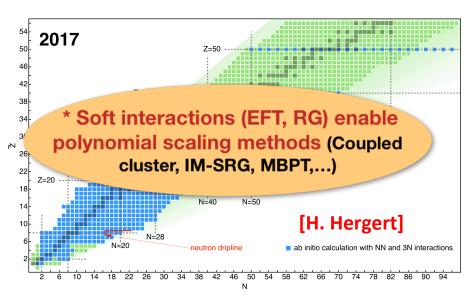




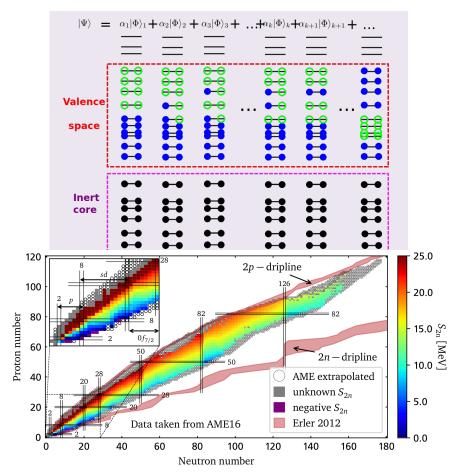


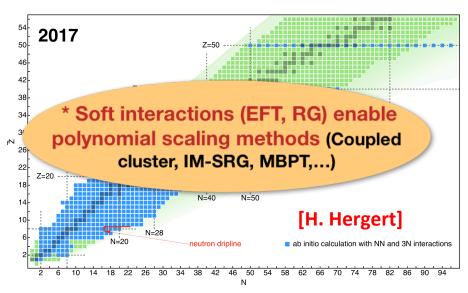
- Progress for "ab initio" methods that use basis expansions at low resolution (cf. QMC)
- In shell model, "P space" sets low resolution



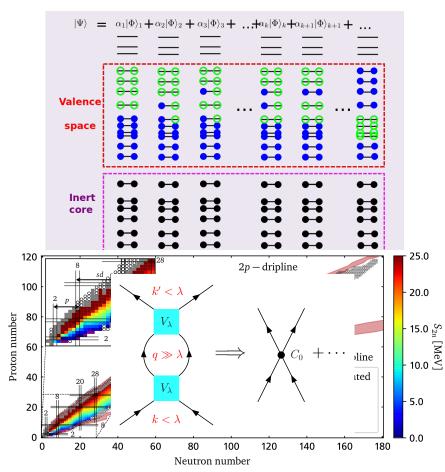


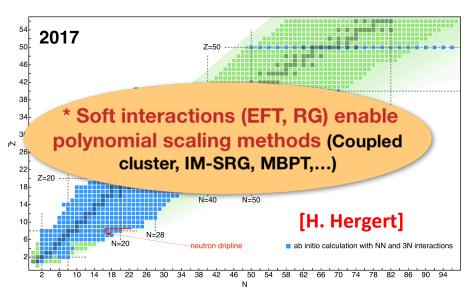
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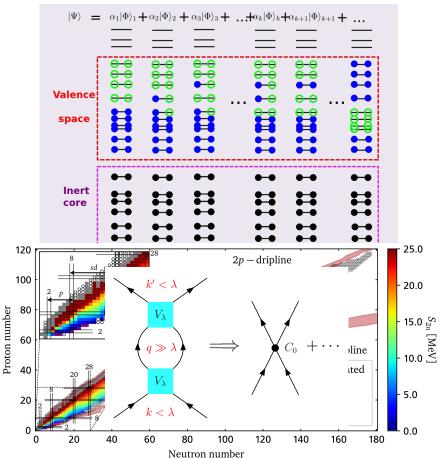


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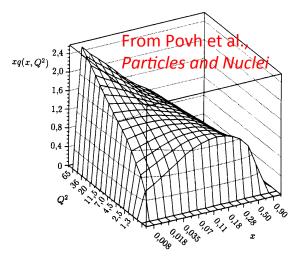


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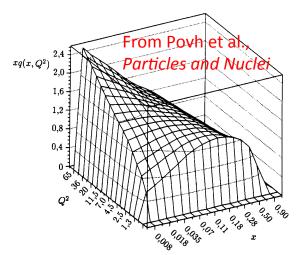
All have correlations: a Slater determinant reference state ≠ independent particle model!

Scale dependence of momentum distributions from RG

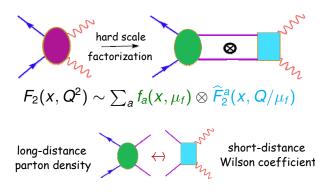


- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x \ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

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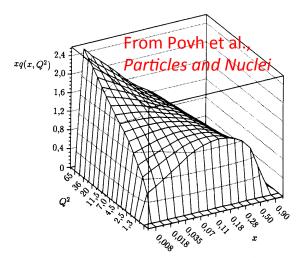
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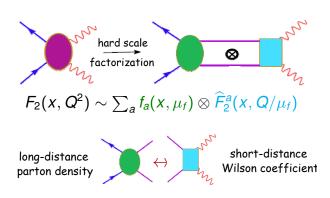
Separation between long- and short-distance physics is not unique!

Observable (e.g. form factor) is independent of factorization scale, but pieces are not.

Scale dependence of momentum distributions from RG

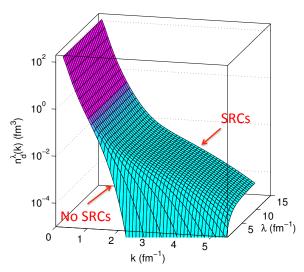


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- Deuteron momentum distribution is scale and scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5 \, \text{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

$$\langle \Psi | \mathcal{O}_{\mathrm{QCD}} | \Psi' \rangle = \langle \Psi | \sum_{i} \mathcal{O}_{\mathrm{EFT}}^{(i)} | \Psi' \rangle$$

- Operators not forbidden are compulsory
- Symmetries limit what is allowed
- Complete set of operators order-by-order
- Power counting based on naturalness

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- Power counting based on *naturalness*
- For chiral EFT, apply NDA (naïve dimensional analysis) to estimate coefficients:

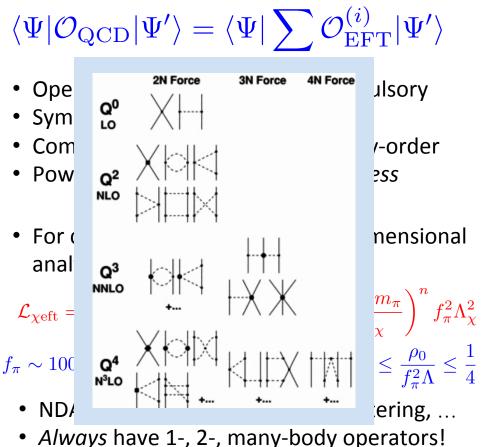
$$\mathcal{L}_{\chi \text{eft}} = c_{lmn} \left(\frac{N^{\dagger}(\cdots)N}{f_{\pi}^{2} \Lambda_{\chi}} \right)^{l} \left(\frac{\pi}{f_{\pi}} \right)^{m} \left(\frac{\partial^{\mu}, m_{\pi}}{\Lambda_{\chi}} \right)^{n} f_{\pi}^{2} \Lambda_{\chi}^{2}$$
$$f_{\pi} \sim 100 \,\text{MeV}, \qquad 1000 \geq \Lambda_{\chi} \geq 500 \Longrightarrow \frac{1}{7} \leq \frac{\rho_{0}}{f_{2}^{2} \Lambda} \leq \frac{1}{4}$$

- NDA works for fits to χPT, NN scattering, ...
- Always have 1-, 2-, many-body operators!

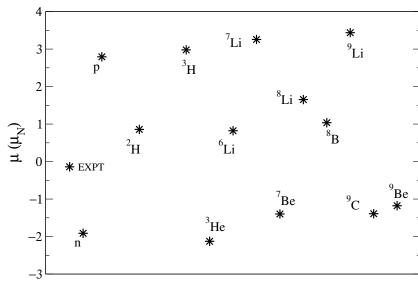
$$\langle\Psi|\mathcal{O}_{\mathrm{QCD}}|\Psi'\rangle = \langle\Psi|\sum\mathcal{O}_{\mathrm{EFT}}^{(i)}|\Psi'\rangle$$
• Ope
• Sym
• Com
• Pow
• For canal
$$\mathcal{L}_{\chi\mathrm{eft}} = f_{\pi} \sim 100$$
• NDA
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$$2^{\mathrm{NForce}} \quad 3^{\mathrm{NForce}} \quad 4^{\mathrm{NForce}} \quad 2^{\mathrm{NForce}} \quad 2^{\mathrm{NForce}} \quad 3^{\mathrm{NForce}} \quad 4^{\mathrm{NForce}} \quad 2^{\mathrm{NForce}} \quad 2^{\mathrm{$$

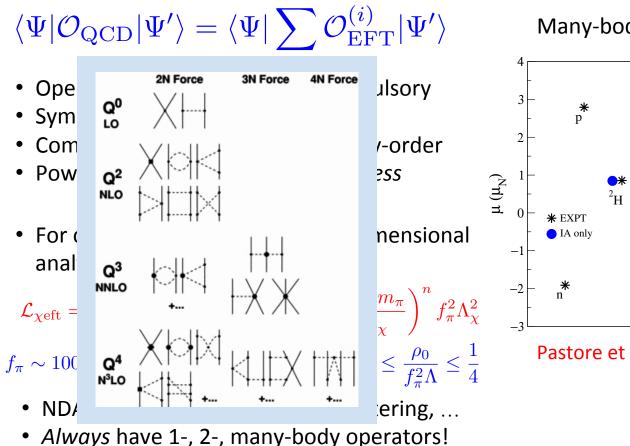
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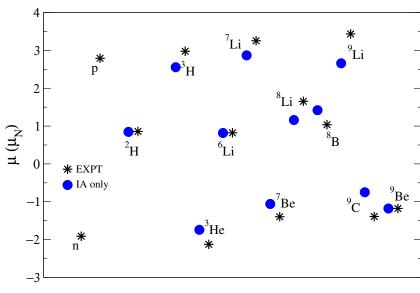
Many-body currents are inevitable!



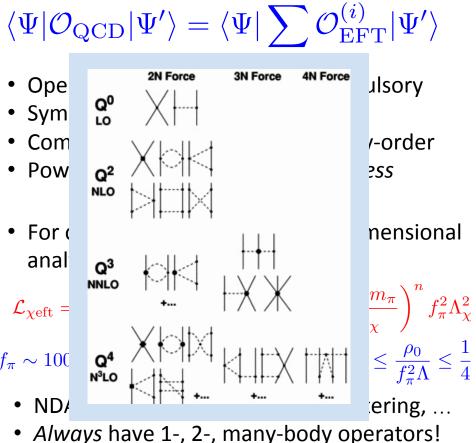
Pastore et al., AV18 wfs with χEFT currents



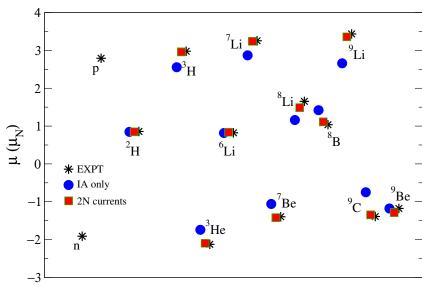
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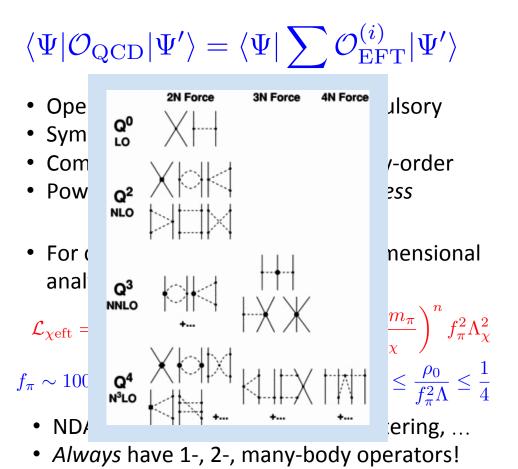
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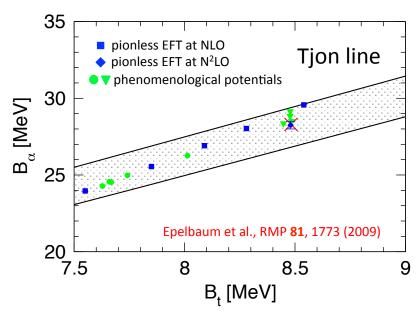


Pastore et al., AV18 wfs with χΕFT currents



If the same operator dominates

→ explains correlated quantities



E. Braaten et al., arXiv:1008.2922 + ...

System of fermions with short-range interactions with large scattering length a.

E. Braaten et al., arXiv:1008.2922 + ...

System of fermions with short-range interactions with large scattering length a.

Described by QFT formulation of Zero-Range Model → ``pionless EFT'':

$$\mathcal{H} = \sum_{\sigma} \frac{1}{m} \nabla \psi_{\sigma}^{\dagger} \cdot \nabla \psi_{\sigma}^{(\Lambda)} + \frac{g(\Lambda)}{m} \psi_{1}^{\dagger} \psi_{2}^{\dagger} \psi_{2} \psi_{1}^{(\Lambda)} + \mathcal{V}_{\text{external}} \qquad g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}$$

UV cutoff Λ is required to make matrix elements of these operators well defined.

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The short-distance OPE is an operator identity with $|r| \rightarrow 0$; for example:

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E.g., momentum density at large **k** from small $|\mathbf{r}|: n_{\sigma}(\mathbf{k}) \to C/k^4$ [from non-analytic r!]

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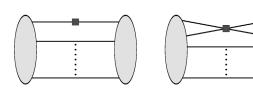
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Note that the *ratio* of $\psi_1^{\dagger} \psi_2^{\dagger} \psi_2 \psi_1^{(\Lambda)}(\mathbf{R})$ in different states will be finite from cancellation!

Chen and Detmold, Phys. Lett. B 625 (2005) Chen et al., Phys. Rev. Lett. 119 (2017)

Q: How can we describe hard partonic processes using EFT for low-energy QCD?

A: OPE factors hard/soft parts and gives moments of PDFs as matrix elements of local twist-2 QCD operators → match to leading nucleon operators in EFT

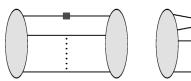


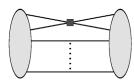
nucleon m.e.: $v_{\mu_1} \dots v_{\mu_n} \langle N | \mathcal{O}^{\mu_1 \cdots \mu_n} | N \rangle = \langle x^n \rangle_N$ nuclear m.e.: $\langle x^n \rangle_A = \langle x^n \rangle_N \langle A | \mathcal{N}^\dagger N + \alpha_n (N^\dagger N)^2 + \dots | A \rangle_\Lambda$

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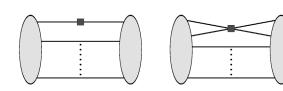
After inverse Mellin transform and some manipulations, the structure function factorizes:

$$F_2^A(x,Q^2)/A \approx F_2^N(x,Q^2) + \underbrace{g_2(A,\Lambda)}_{A,\Lambda \text{ only universal}} \underbrace{f_2(x,Q^2,\Lambda)}_{\text{universal}} \quad \text{where} \quad g_2(A,\Lambda) = \frac{1}{2A} \langle A | (N^\dagger N)^2 | A \rangle_{\Lambda}$$

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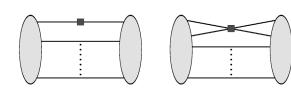
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Each term is scale/scheme dependent, but cancels in ratio: $a_2(A,x) \equiv \left. \frac{2\sigma_A}{A\sigma_d} \right|_{1.5 < x < 2} pprox \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)}$

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nuclear m.e.:
$$\langle x^n \rangle_A = \langle x^n \rangle_N \langle A | \mathcal{N}^\dagger N + \frac{\alpha_n (N^\dagger N)^2}{2} + \dots | A \rangle_\Lambda$$

After inverse Mellin transform and some manipulations, the structure function factorizes:

$$F_2^A(x,Q^2)/A pprox F_2^N(x,Q^2) + \underbrace{g_2(A,\Lambda)}_{A,\Lambda \text{ only universal}} \underbrace{f_2(x,Q^2,\Lambda)}_{\text{universal}} \quad ext{where} \quad g_2(A,\Lambda) = \frac{1}{2A} \langle A | (N^\dagger N)^2 | A
angle_\Lambda$$

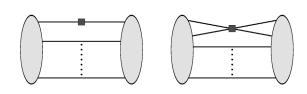
Each term is scale/scheme dependent, but *cancels* in ratio:
$$a_2(A,x) \equiv \frac{2\sigma_A}{A\sigma_d}\bigg|_{1.5 < x < 2} pprox \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)}$$

EMC slope also \approx factorizes into a_2 and x-dependence \Rightarrow explains correlation at \approx 20%!

Chen and Detmold, Phys. Lett. B 625 (2005) Chen et al., Phys. Rev. Lett. 119 (2017)

Q: How can we describe hard partonic processes using EFT for low-energy QCD?

A: OPE factors hard/soft parts and gives moments of PDFs as matrix elements of local twist-2 QCD operators → match to leading nucleon operators in EFT



nucleon m.e.:
$$v_{\mu_1} \dots v_{\mu_n} \langle N | \mathcal{O}^{\mu_1 \dots \mu_n} | N \rangle = \langle x^n \rangle_N$$

nuclear m.e.:
$$\langle x^n \rangle_A = \langle x^n \rangle_N \langle A | \mathcal{N}^\dagger N + \frac{\alpha_n (N^\dagger N)^2}{\Lambda} + \dots | A \rangle_\Lambda$$

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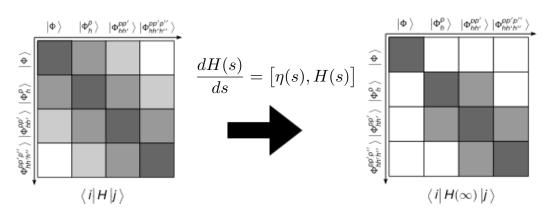
$$F_2^A(x,Q^2)/A pprox F_2^N(x,Q^2) + \underbrace{g_2(A,\Lambda)}_{A,\Lambda \text{ only}} \underbrace{f_2(x,Q^2,\Lambda)}_{\text{universal}} \quad \text{where} \quad g_2(A,\Lambda) = \frac{1}{2A} \langle A | (N^\dagger N)^2 | A \rangle_{\Lambda}$$

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EMC slope also \approx factorizes into a_2 and x-dependence \Rightarrow explains correlation at \approx 20%!

We see a_2 is from physics below scale Λ ; can we calculate it in low-resolution nuclei?

IM-SRG(2) calculations of a_2 [in progress by Parzuchowski, Bogner, rjf]



$$\hat{\rho}_{21}(s=0) = \frac{1}{2} \sum_{\alpha,\beta} \int d\mathbf{R} \, N_{\alpha}^{\dagger} N_{\beta}^{\dagger} N_{\beta} N_{\alpha}$$
$$\frac{d\hat{\rho}_{21}(s)}{ds} = \left[\eta(s), \hat{\rho}_{21}(s) \right]$$

$$\langle \Psi_0 | \hat{\rho}_{21} | \Psi_0 \rangle = \langle \Phi | \hat{\rho}_{21}(\infty) | \Phi \rangle$$

Opportunities

access heavier nuclei (here up to A = 40)
access wider range of interactions**
(not limited to local interactions)

Challenges

QMC cleanly extrapolates to r = 0 (vs. implicit smearing due to truncated HO basis) impact of IM-SRG(2) truncation errors?

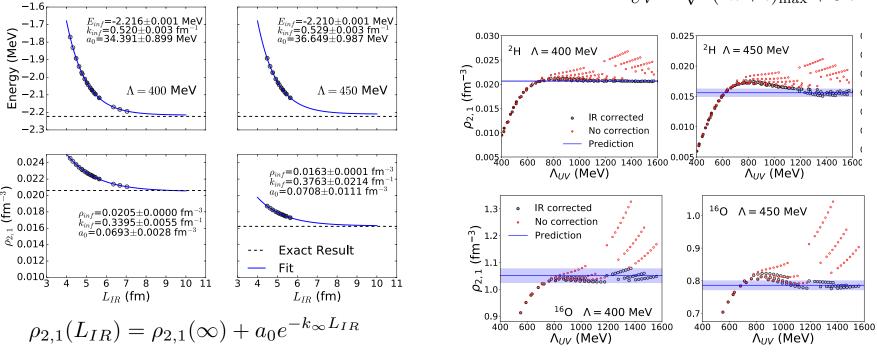
** here we use the semi-local n4lo NN interaction of Reinert, Krebs, and Epelbaum arXiv:1711.08821

IR and UV extrapolations of $\rho_{2,1}$

Truncated HO basis ==> IR cutoff (box size L_{IR}) and UV cutoff:

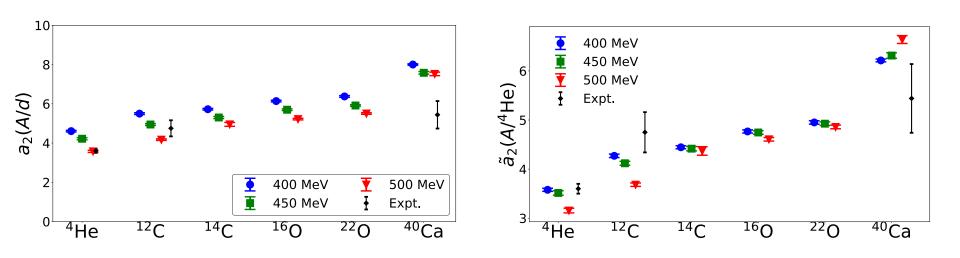
$$L_{IR} \sim \sqrt{2(2n+l)_{\text{max}} + 3} \ b$$

 $\Lambda_{UV} \sim \sqrt{2(2n+l)_{\text{max}} + 3} \ b^{-1}$



Motivated by energy/radii formulas in More et al. PRC87 (2013), but needs further work

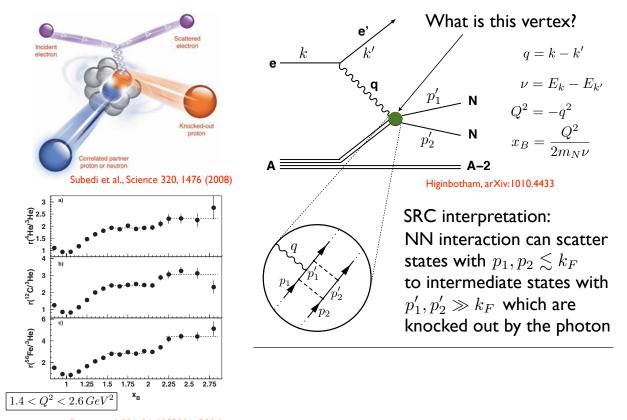
Preliminary results for a₂



Systematic Λ dependence, but in part due to treatment of deuteron; ratio to He better

Status: Need to better control UV/IR convergence and IM-SRG(2) truncation errors (and estimate EFT truncation error) before concluding if a_2 is scale-independent.

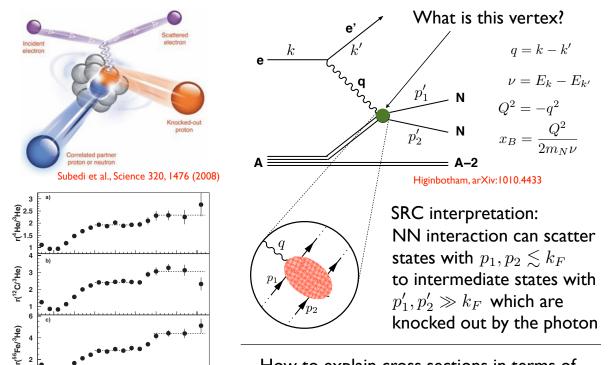
Large Q^2 scattering at different RG decoupling scales



Egiyan et al. PRL 96, 1082501 (2006)

SRC explanation relies on high-momentum nucleons in structure

Large Q^2 scattering at different RG decoupling scales



How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

RG evolution changes physics interpretation but not cross section!

 $1.4 < Q^2 < 2.6 \, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

SRG changes resolution with unitary transformations

[Note: the following discussion is more general than the SRG.]

• The cross section is *guaranteed* to be the same from $\widehat{U}^{\dagger}\widehat{U}=1$

$$d\sigma \sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2$$

$$= \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | (\widehat{\boldsymbol{U}}^{\dagger} \widehat{\boldsymbol{U}}) \widehat{\rho}(\mathbf{q}) (\widehat{\boldsymbol{U}}^{\dagger} \widehat{\boldsymbol{U}}) | \Psi_i \rangle|^2$$

$$= \sum \delta(\omega + E_i - E_f) |(\langle \Psi_f | \widehat{\boldsymbol{U}}^{\dagger}) (\widehat{\boldsymbol{U}} \widehat{\rho}(\mathbf{q}) \widehat{\boldsymbol{U}}^{\dagger}) (\widehat{\boldsymbol{U}} | \Psi_i \rangle)|^2$$

but the pieces are different now.

- Schematically, the SRG has $\hat{U} = 1 + \frac{1}{2}(U 1)a^{\dagger}a^{\dagger}aa + \cdots$
 - U is found by solving for the unitary transformation in the A=2 system (this is the easy part!)
 - The · · · 's represent higher-body operators
 - One-body operators (∝ a[†] a) gain many-body pieces
 (EFT: there are always many-body pieces at some level!)
 - Both initial and final states are modified (and therefore FSI)

U-factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_{\lambda}(k,q) \to K_{\lambda}(k)Q_{\lambda}(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_{\alpha}^{\infty}(q) \approx \gamma^{\lambda}(q) \int_{0}^{\lambda} p^{2} dp \ Z(\lambda) \Psi_{\alpha}^{\lambda}(p) + \eta^{\lambda}(q) \int_{0}^{\lambda} p^{2} dp \ p^{2} \ Z(\lambda) \Psi_{\alpha}^{\lambda}(p) + \cdots$$

• Construct unitary transformation to get $U_{\lambda}(k,q) \approx K_{\lambda}(k)Q_{\lambda}(q)$

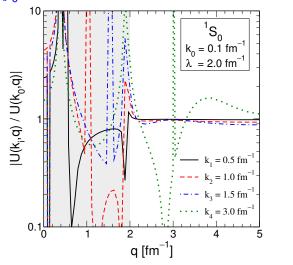
$$U_{\lambda}(k,q) = \sum_{\alpha} \langle k | \psi_{\alpha}^{\lambda} \rangle \langle \psi_{\alpha}^{\infty} | q \rangle \rightarrow \left[\sum_{\alpha}^{\alpha_{low}} \langle k | \psi_{\alpha}^{\lambda} \rangle \int_{0}^{\lambda} p^{2} dp \ Z(\lambda) \Psi_{\alpha}^{\lambda}(p) \right] \gamma^{\lambda}(q) + \cdots$$

Test of factorization of U:

$$\frac{U_{\lambda}(k_i,q)}{U_{\lambda}(k_0,q)} \to \frac{K_{\lambda}(k_i)Q_{\lambda}(q)}{K_{\lambda}(k_0)Q_{\lambda}(q)},$$

so for $q\gg\lambda\Rightarrowrac{K_{\lambda}(k_{i})}{K_{\lambda}(k_{0})}\stackrel{\mathrm{LO}}{\longrightarrow}1$

- Look for plateaus: $k_i \lesssim 2 \, \text{fm}^{-1} \lesssim q$ \implies it works!
- Leading order ⇒ contact term!



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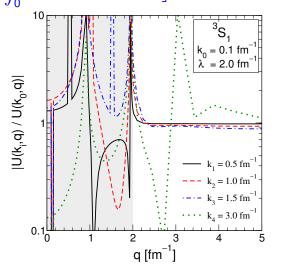
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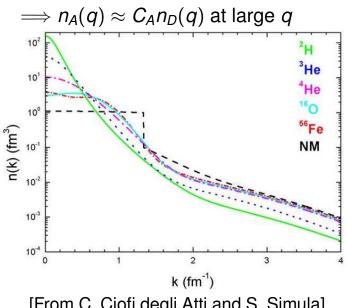
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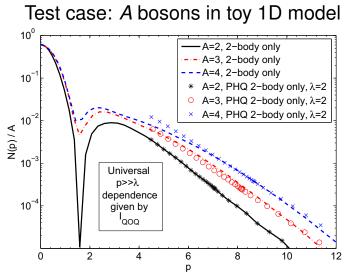


- RG unitary transformation with scale separation: $\hat{U} \to U_{\lambda}(k, q)$
- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_{\lambda}(k,q) \to K_{\lambda}(k)Q_{\lambda}(q)$

$$\frac{n_{\mathsf{A}}(q)}{n_{\mathsf{d}}(q)} = \frac{\langle A|a_{\mathsf{q}}^{\dagger}a_{\mathsf{q}}|A\rangle}{\langle d|a_{\mathsf{q}}^{\dagger}a_{\mathsf{q}}|d\rangle} \quad \overset{\mathsf{RG}}{\underset{\widehat{U}^{\dagger}\widehat{U}=1}{\Longrightarrow}} \quad \widehat{U}|d\rangle \rightarrow |\widetilde{d}\rangle \;, \; \widehat{U}|A\rangle \rightarrow |\widetilde{A}\rangle \;, \; \widehat{U}a_{\mathsf{q}}^{\dagger}a_{\mathsf{q}}\widehat{U}^{\dagger}$$



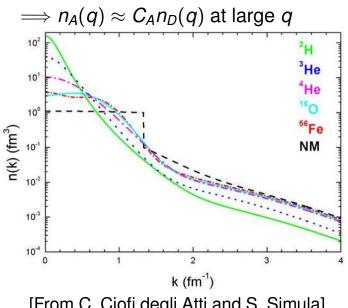
[From C. Ciofi degli Atti and S. Simula]



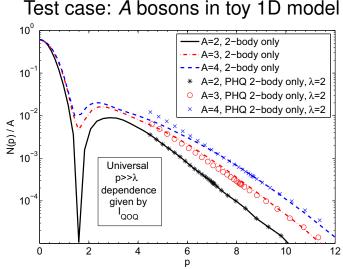
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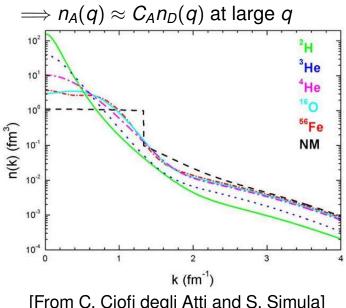
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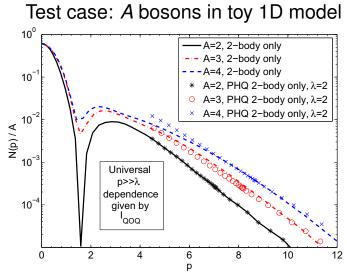
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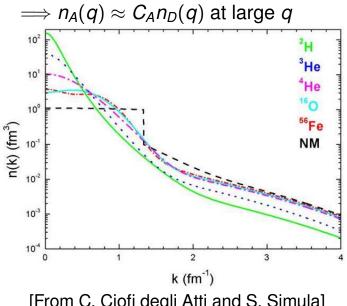
[From C. Ciofi degli Atti and S. Simula]



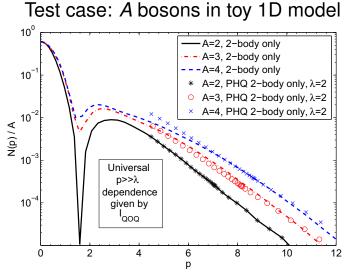
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[From C. Ciofi degli Atti and S. Simula]



[Anderson et al., arXiv:1008.1569] [also Bogner, Roscher, arXiv:1208.1734]

Test ground for scale dependence: ²H(e,e'p)n

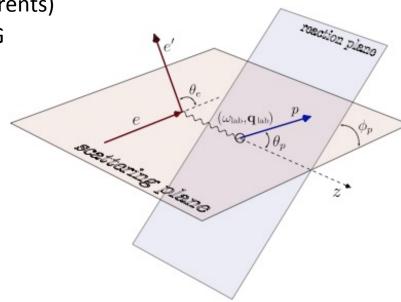
More, Bogner, rjf, PRC96, 2017

Simplest knockout process (no induced 3N forces/currents)

- Use AV18 as "hard" interaction, evolve with SRG
- Treat non-relativistically for all kinematics

Focus on longitudinal structure function f_L

$$f_L \sim \sum_{m_s, m_I} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$$



Test ground for scale dependence: ²H(e,e'p)n

More, Bogner, rjf, PRC96, 2017

reaction plane

Simplest knockout process (no induced 3N forces/currents)

- Use AV18 as "hard" interaction, evolve with SRG
- Treat non-relativistically for all kinematics

Focus on longitudinal structure function f_L

$$f_L \sim \sum_{m_s,m_I} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$$

$$f_L^{\lambda} \sim \left| \langle \underbrace{\psi_f | U_{\lambda}^{\dagger}}_{\psi_f^{\lambda}} \underbrace{U_{\lambda} J_0 U_{\lambda}^{\dagger}}_{J_0^{\lambda}} \underbrace{U_{\lambda} | \psi_i \rangle}_{\psi_i^{\lambda}} \right|^2; \quad U_{\lambda}^{\dagger} U_{\lambda} = I; \quad f_L^{\lambda} = f_L$$

 $f_L^{\lambda} = f_L$

Components (deuteron wf, transition operator, FSI) are scale-dependent; total is not!

Test ground for scale dependence: ²H(e,e'p)n

More, Bogner, rjf, PRC96, 2017

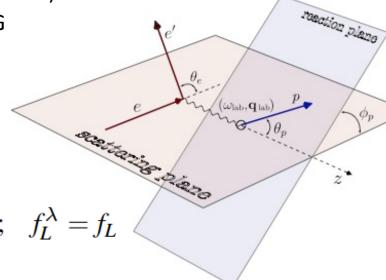
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Focus on longitudinal structure function f

$$f_L \sim \sum_{m_s,m_I} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$$

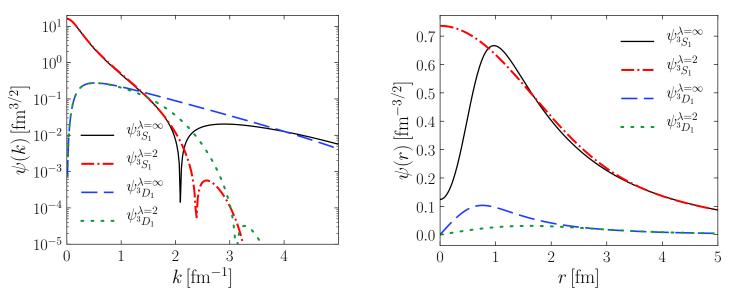
$$f_L^{\lambda} \sim \left| \langle \underline{\psi_f} | \underline{U_{\lambda}^{\dagger}} \underline{U_{\lambda}} \underline{J_0} \underline{U_{\lambda}^{\dagger}} \underline{U_{\lambda}} | \underline{\psi_i} \rangle \right|^2; \quad U_{\lambda}^{\dagger} U_{\lambda} = I; \quad f_L^{\lambda} = f_L$$



Components (deuteron wf, transition operator, FSI) are scale-dependent; total is not!

Are some resolutions "better" than others? E.g., in a given kinematics, can FSI be minimized with different choices of λ ??

Deuteron wave function evolution

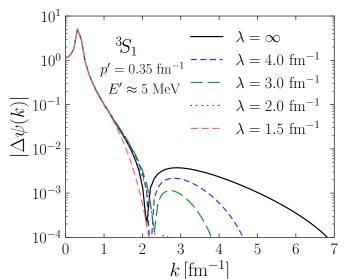


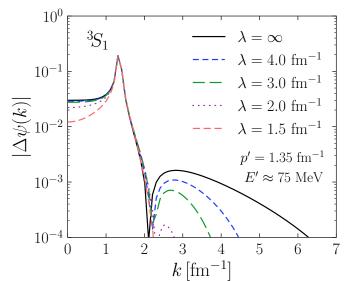
 $k < \lambda$ components invariant <==> RG preserves long-distance physics

 $k > \lambda$ components suppressed <==> short-range correlations blurred out

Folklore: Simple wave functions at low $\lambda <==>$ more complicated operators? especially for high q processes?

Final-state wave function evolution



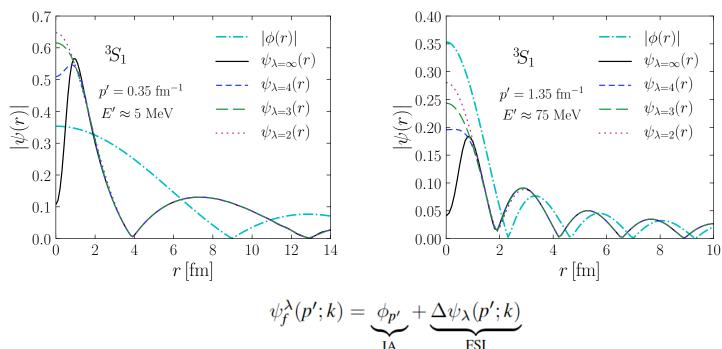


$$\psi_f^{\lambda}(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_{\lambda}(p';k)}_{\text{ESI}}$$

- High-k tails suppressed with evolution
- For $p' \gtrsim \lambda$, $\Delta \psi_f^{\lambda}(p';k)$ localized around outgoing p'

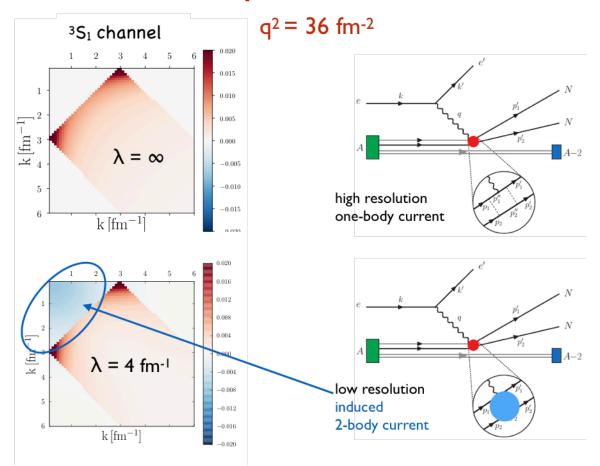
"local decoupling" Dainton et al. PRC 89 (2014)

Final-state wave function evolution

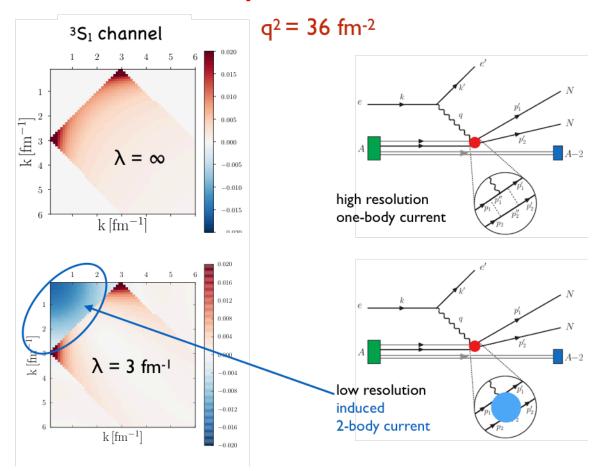


- Correlation "wound" at small r smoothed out under evolution
- Long-distance tail invariant (phase shifts preserved)

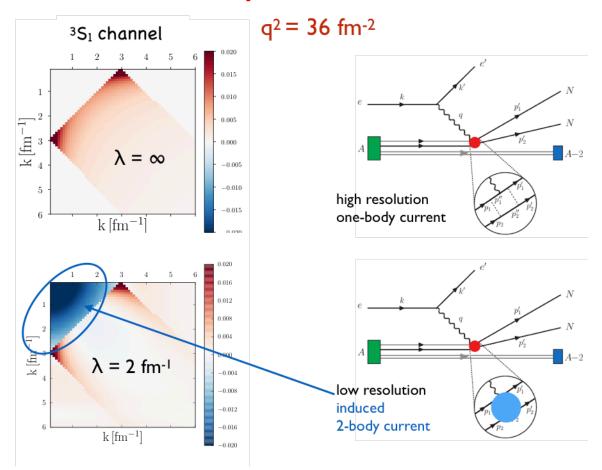
Current operator evolution



Current operator evolution

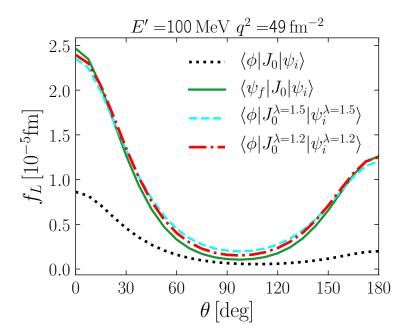


Current operator evolution



λ dependence of Final State Interactions

Look at kinematics relevant to SRC studies



 $x_{B}=1.64$, $Q^{2}=1.78$ GeV²

FSI sizable at large λ but negligible at low-resolution!

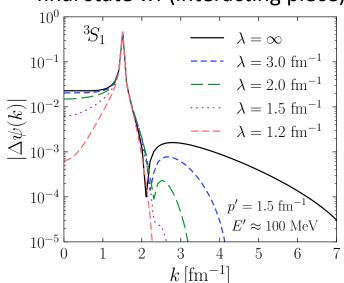
Folklore:

shouldn't hard processes be complicated in low resolution $(\lambda << q)$ pictures?

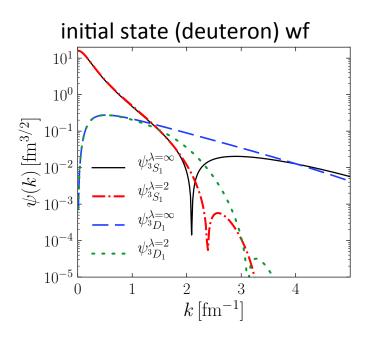
Why are FSI so small at low λ ?

λ dependence of Final State Interactions

final state wf (interacting piece)

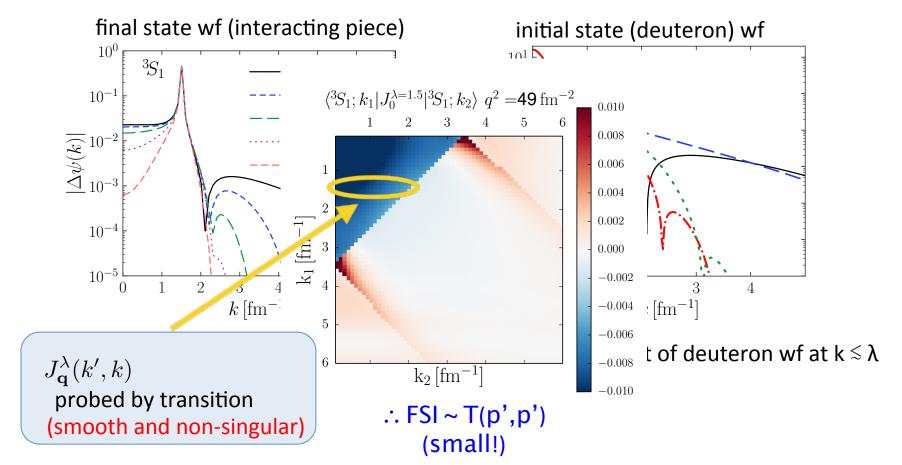


For $p' \gtrsim \lambda$, interacting part of final state wf localized at $k \approx p'$



Dominant support of deuteron wf at $k \lesssim \lambda$

λ dependence of Final State Interactions



Can we account for the cross section at *both* high and low resolution with simple pictures?

Work in final neutron-proton rest frame at $\theta=0^\circ$ Assume photon momentum absorbed entirely by proton

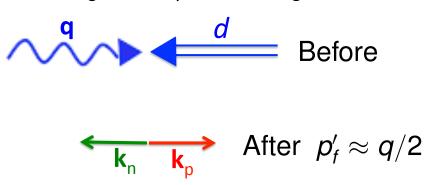
Scattering on the quasi-free ridge:



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Scattering on the quasi-free ridge:

Deuteron wave function probed at low momentum

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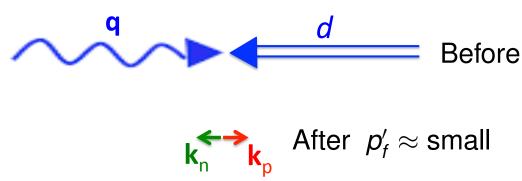
Scattering near threshold with SRC kinematics:



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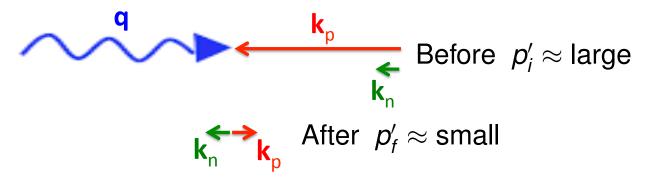
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Work in final neutron-proton rest frame at $\theta=0^\circ$ Assume photon momentum absorbed entirely by proton

Scattering near threshold with SRC kinematics:

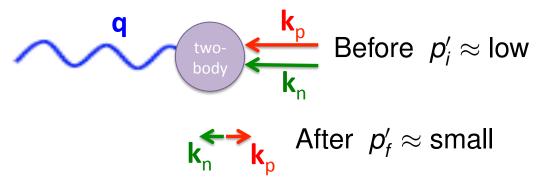


Cross section from short-range correlation

Can we account for the cross section at *both* high and low resolution with simple pictures?

Work in final neutron-proton rest frame at $\theta=0^\circ$ Assume photon momentum absorbed entirely by proton

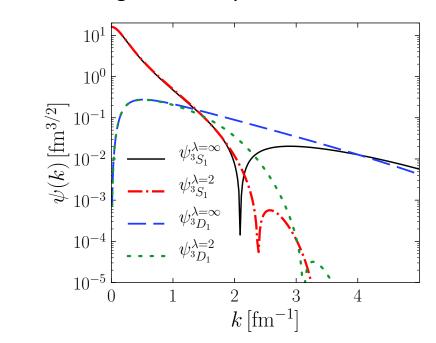
Scattering near threshold with SRC kinematics:

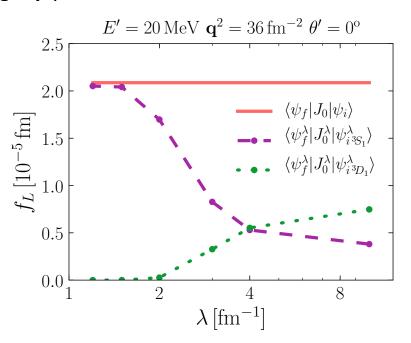


Cross section from low momentum!

Scale (λ) dependence of interpretations

- Analysis/interpretation of a reaction involves understanding which part(s) of the wave functions are probed (highly scale dependent!)
- E.g., sensitivity to D-state w.f. in large **q**² processes



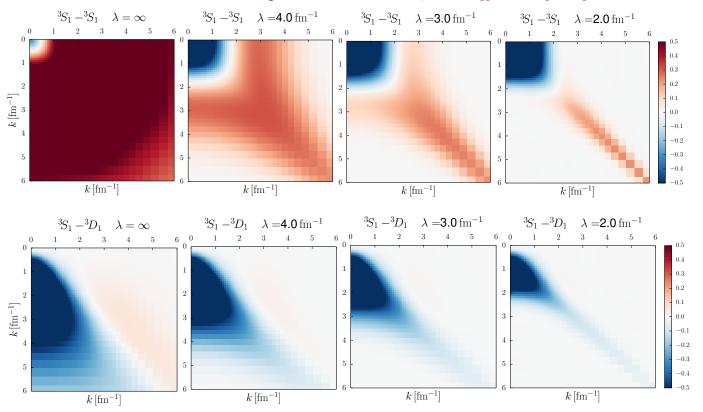


Summary: The OPE-RG-EFT perspective

- Is consistent with the full range of controlled many-body calculations
- Can connect with QCD analysis of hard processes
- Helps identify correlations (find common leading operator)
- Allows separation of A dependence and reaction dependence
- Makes it possible to estimate relative contributions of operators
- Allows for systematic corrections and uncertainty quantification
- Highlights scale and scheme dependence of quantities like the tail of the momentum distribution and of physics interpretations

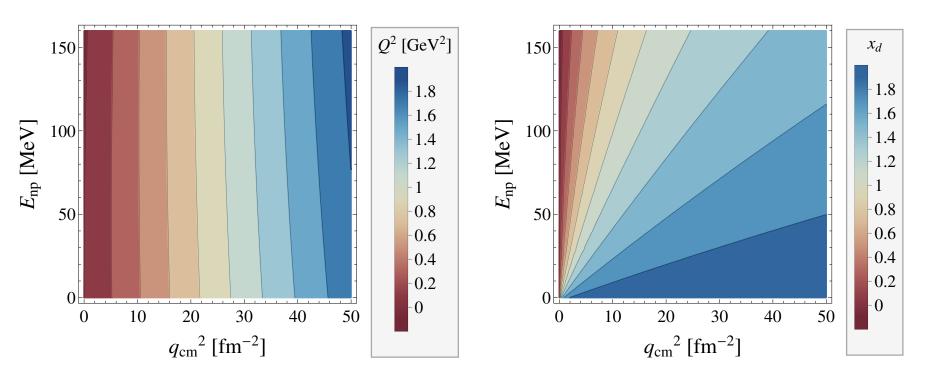
Back-up Slides

SRG evolution of AV18 potential $dH_s/ds = [[G_s, H_s], H_s], G_s = T$



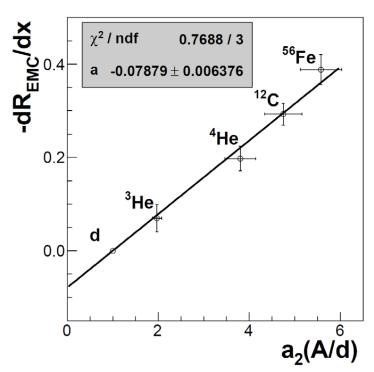
Notes: unitary transformation $\widehat{O}_{\lambda} = \widehat{U}_{\lambda} \widehat{O}_{\lambda=\infty} \widehat{U}_{\lambda}^{\dagger}$; λ sets decoupling scale: $V_{\lambda}(k,k') \approx V_{\lambda=\infty}(k,k')e^{-\left(\frac{k^2-k'^2}{\lambda^2}\right)^2}$ (nonlocality!); scheme dependence from G_s

Kinematic variables transformation



In the rest frame of outgoing nucleons, E' is their energy and $q_{\text{c.m.}}$ is the three-momentum transfer by the virtual photon.

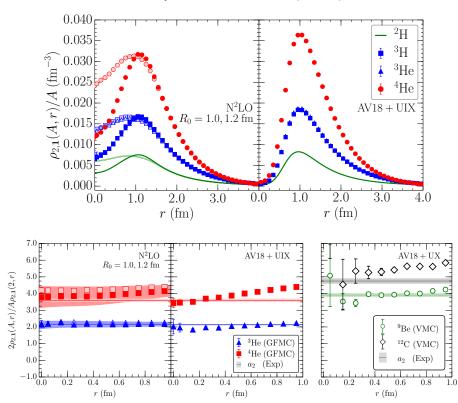
Preview: SRCs and the EMC effect in EFT



L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

$$a_2(A) = \frac{2}{A} \frac{\rho_{2,1}(A,0)}{\rho_{2,1}(2,0)}$$

Chen et al., Phys. Rev. Lett. 119 (2017)



Recent explosion of chiral EFT interactions

Interactions from 2003-4: EM(500, 600 MeV) or family of EGM potentials; used similar non-local regulators and similar fits to NN, 3N

New generation NN+3N interactions [references at end]

- Non-local: updated EMN and new soft; Ekstrom et al. sim, sat, and with Δs
- Local (for QMC): Gezerlis et al. nucleons only; Piarulli et al. with Δs
- Semi-local: Bochum-Julich group, SCS (x-space) and SMS (p-space)

Issues: power counting, regulator artifacts, EFT convergence, fitting protocols, fine-tuning, over/under-fitting, parameter redundancies, how to do UQ?

Parameter estimation issues repeat with LECs for currents and for other EFTs