Ab initio and empirical approaches to quantify short-range correlations in nuclei and nuclear matter

BNL
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Washington University in St. Louis

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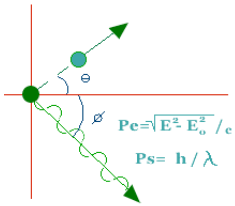


- Green's function/ propagator method for ab initio calculations and empirical analysis of data
- Ab initio calculations of SRC in matter
- Some results of calculations of SRC in finite nuclei
- Dispersive optical model <—> data <—> LRC vs SRC
- (e,e'p) results from NIKHEF <—> distorted wave impulse approximation <—> DOM ingredients
- Conclusions

Propagator / Green's function

- Lehmann representation

$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle \Psi_0^A | a_{k\ell j} | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a^\dagger_{k'\ell j} | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} + \sum_n \frac{\langle \Psi_0^A | a^\dagger_{k'\ell j} | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

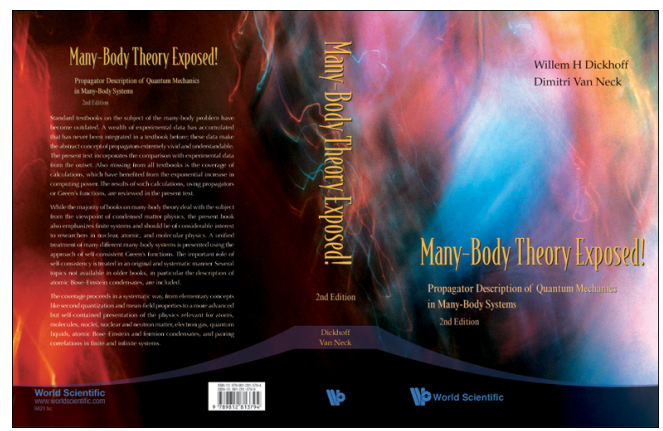
- Any other single-particle basis can be used & continuum integrals implied
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function

$$S_{\ell j}(k;E) = \frac{1}{\pi} \mathrm{Im}\, G_{\ell j}(k,k;E) = \sum_n \left| \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1})) \qquad E \le \varepsilon_F^-$$

- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk\, k^2\, S_{\ell j}(k;E)$$

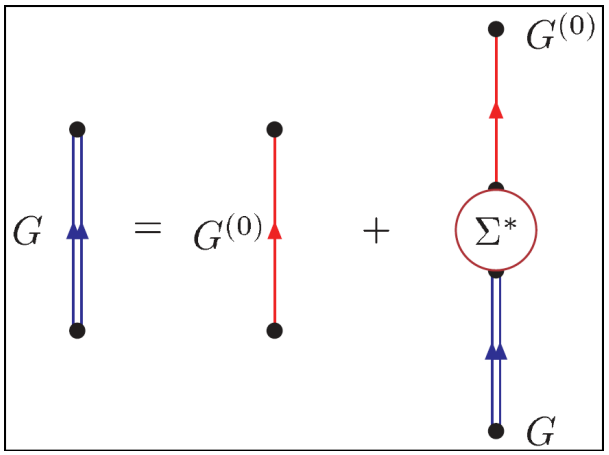
- Discrete transitions $\sqrt{S^n_{\ell j}}\, \phi^n_{\ell j}(k) = \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle$
- Positive energy —> see later



Propagator in principle generates

- Elastic scattering cross sections for p and n
- Including all polarization observables
- Total cross sections for n
- Reaction cross sections for p and n
- Overlap functions for adding p or n to bound states in Z+1 or N+1
- Plus normalization --> spectroscopic factor
- Overlap function for removing p or n with normalization
- Hole spectral function including high-momentum description
- One-body density matrix; occupation numbers; natural orbits
- Charge density
- Neutron distribution
- p and n distorted waves
- Contribution to the energy of the ground state from V_{NN}

Propagator from Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

⇒ as extracted from (e,e'p) reaction

$$\frac{k^2}{2m}\phi^n_{\ell j}(k) + \int dq\; q^2\; \Sigma^*_{\ell j}(k,q;E_n^-)\; \phi^n_{\ell j}(q) = E_n^-\; \phi^n_{\ell j}(k)$$

Spectroscopic factor $\mathcal{Z}^n_{\ell j} = \int dk\; k^2\; \left|\langle \Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle\right|^2 < 1$

Dyson equation also yields $\left[\chi^{elE}_{\ell j}(r)\right]^* = \langle \Psi^{A+1}_{elE}|\, a^\dagger_{r\ell j}\, |\Psi_0^A\rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

Role of Short-range and Tensor Correlations

- A few comments on two-body interactions

  - Lattice QCD (work in progress) —> Ishii, Aoki,…

  - Chiral perturbation theory (now many versions) —> soft —> Entem, Machleidt, Epelbaum, Meissner,……., Piarulli (now WashU), …

  - Phenomenology with mesons CDBonn —> harder —> Machleidt and history

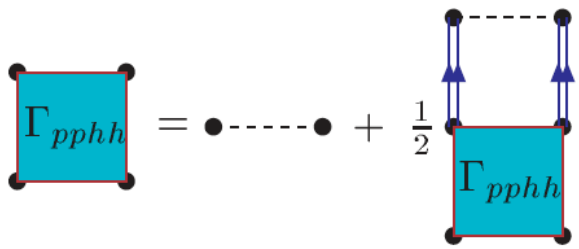
  - Phenomenology AV18 & Reid93 —> harder still

  - Old phenomenology Reid soft core —> even harder

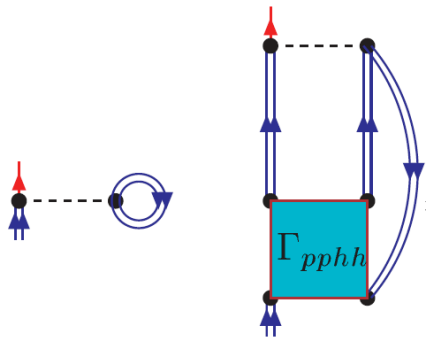
  - Old, old phenomenology —> hard cores

Full off-shell propagation in infinite matter

SCGF:
self-consistent
Green's functions
for SRC and tensor
effects

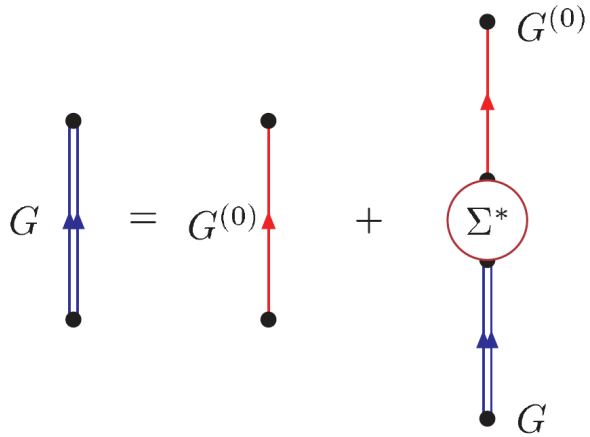


Interaction in the medium properly treating short-range and tensor correlations



Self-energy = complex potential in nuclear matter

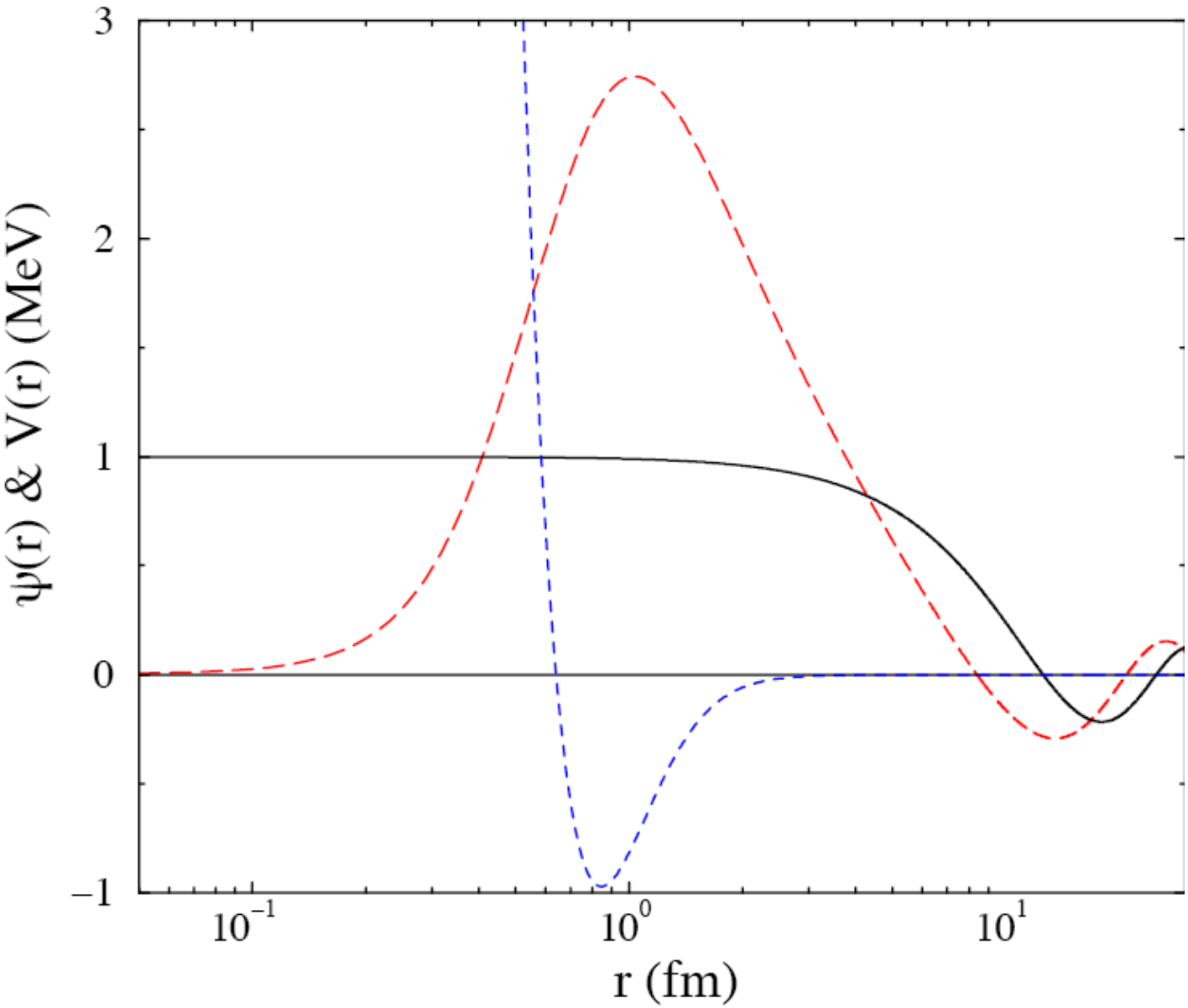
self-consistency
=> thermodynamically consistent



Dyson equation ⇒
Schrödinger equation
for dressed nucleons

Arnau Rios
Arturo Polls
W.D.
finite T avoids pairing
standard for AV18, CDBonn, N3LO, etc

Relative wave function and potential



Reid $^1S_0$

$k_0 = 0.25\ fm^{-1}$

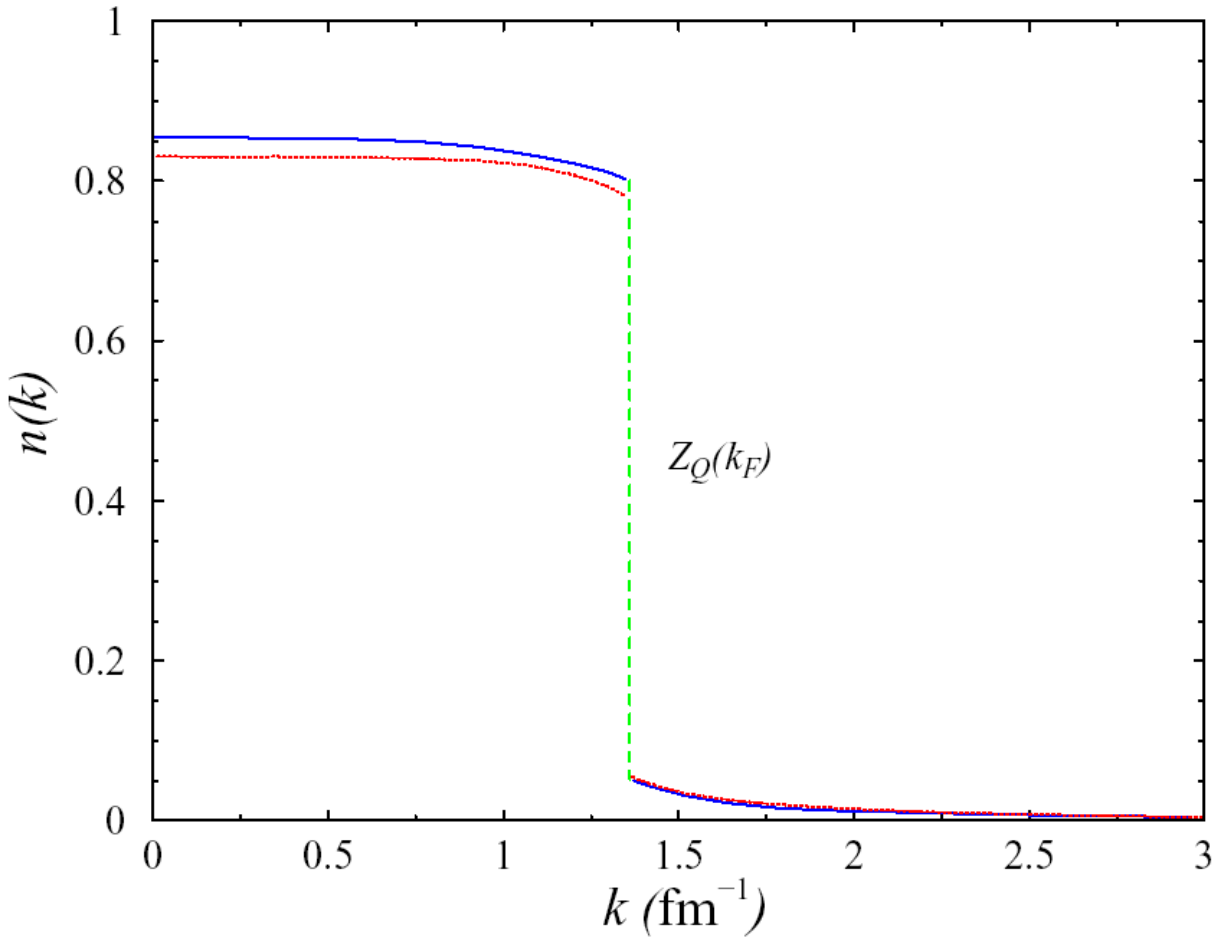
V(r)/100

$\sin(k_0 r)/k_0 r$

correlated wf

Short-range correlations in nuclear matter and n(k)

n(k = 0) = 0.83 / 0.85 ⇒ finite nuclei



Reid soft core
k_F = 1.36 fm^{-1}

Old prediction!
New result

$Z_Q(k_F)$=0.72

$Z_Q(k_F)$=0.75

B.E.Vonderfecht et al. Nucl. Phys. A555, 1 (1993)
E.R.Stoddard, thesis (self-consistent ladders)

Some results infinite matter

- Effect of temperature vs. SRC & tensor correlations
- Effect of density
- Choice of interaction: CDBonn & Argonne v18
- Symmetric nuclear matter vs. neutron matter
- Depletion vs. high-momentum components
- Asymmetric nuclear matter
- Temperature, Interaction
- Tensor, tensor, tensor ⇒ pion, pion, pion
- Recent results also for N3LO

A. Rios, A. Polls, and W. H. Dickhoff
Depletion of the nuclear Fermi sea.
Phys. Rev. C79, 064308 (2009).

Emphasis on depletion —> low momenta

PHYSICAL REVIEW C **79**, 064308 (2009)

Depletion of the nuclear Fermi sea

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(Received 14 April 2009; published 10 June 2009)

Emphasis on high momenta

PHYSICAL REVIEW C **89**, 044303 (2014)

Density and isospin-asymmetry dependence of high-momentum components

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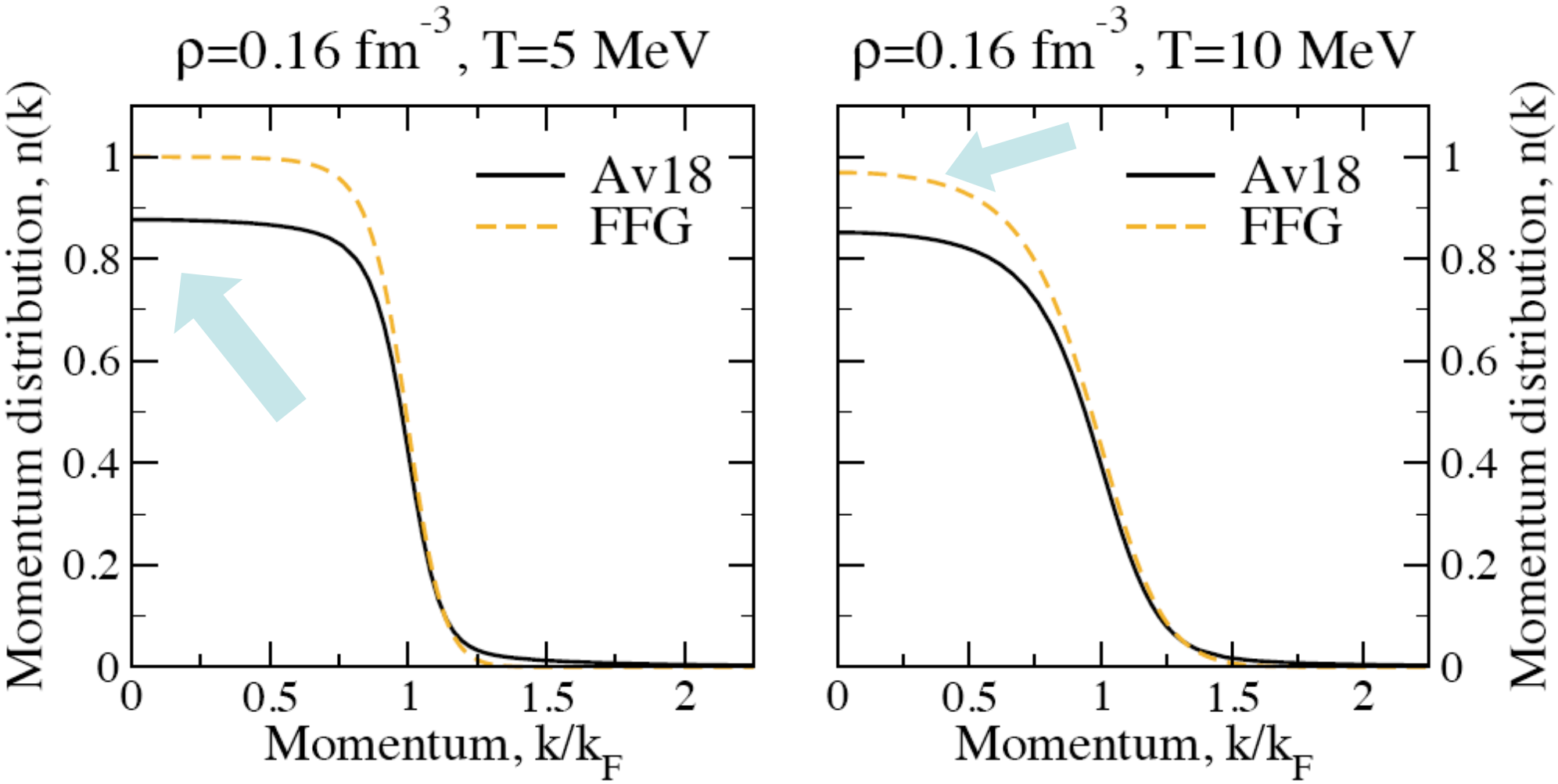
We study the one-body momentum distribution at different densities in nuclear matter, with special emphasis on its components at high momentum. Explicit calculations for finite neutron-proton asymmetry, based on the ladder self-consistent Green's function approach, allow us to access the isospin dependence of momentum distributions and elucidate their role in neutron-rich systems. Comparisons with the deuteron momentum distribution indicate that a substantial proportion of high-momentum components are dominated by tensor correlations. We identify the density dependence of these tensor correlations in the momentum distributions. Further, we find that high-momentum components are determined by the density of each subspecies and we provide a new isospin-asymmetry scaling of these components. We use different realistic nucleon-nucleon interactions to quantify the model dependence of our results.

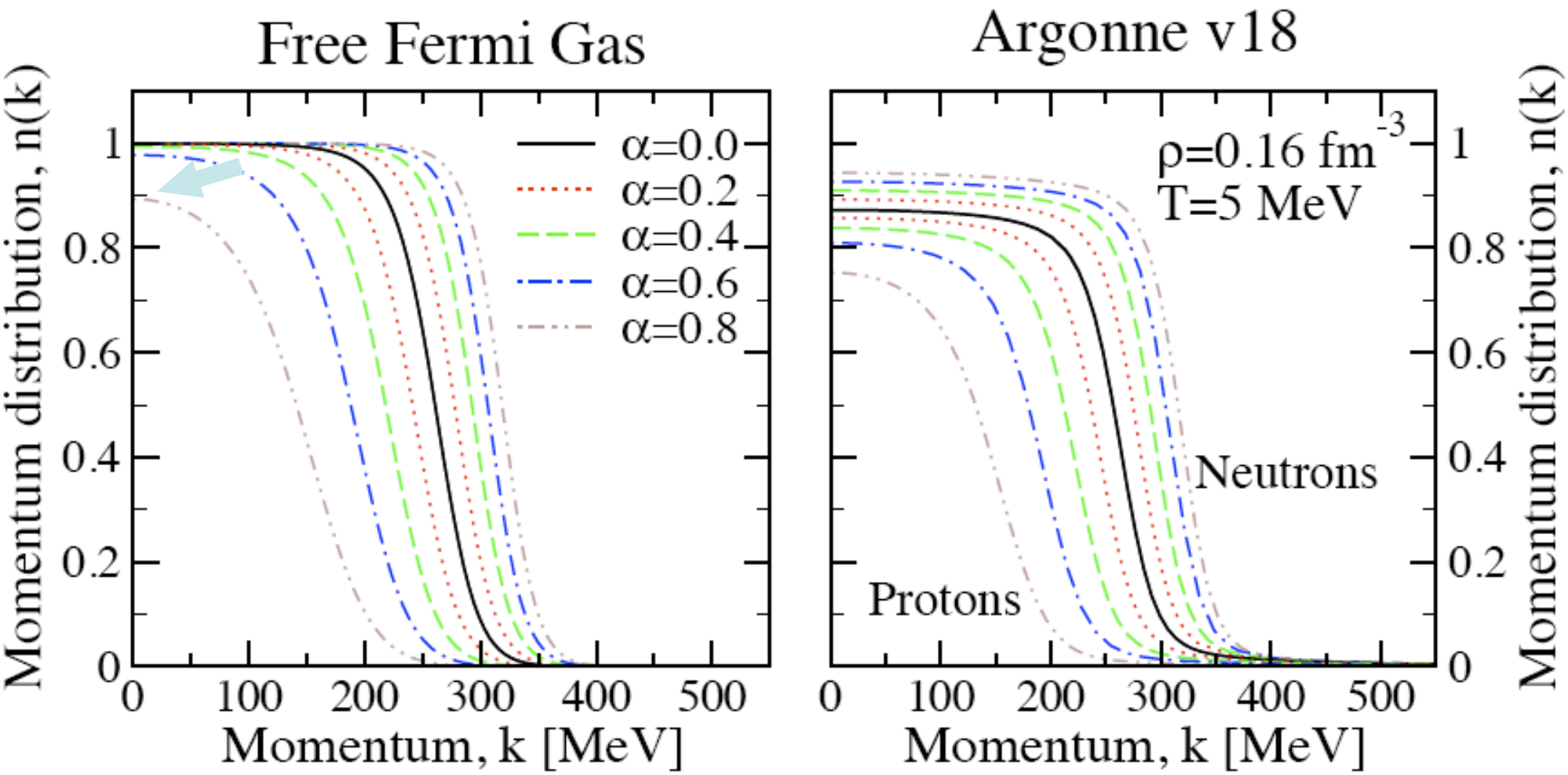
High-momentum components

Temperature vs. correlations



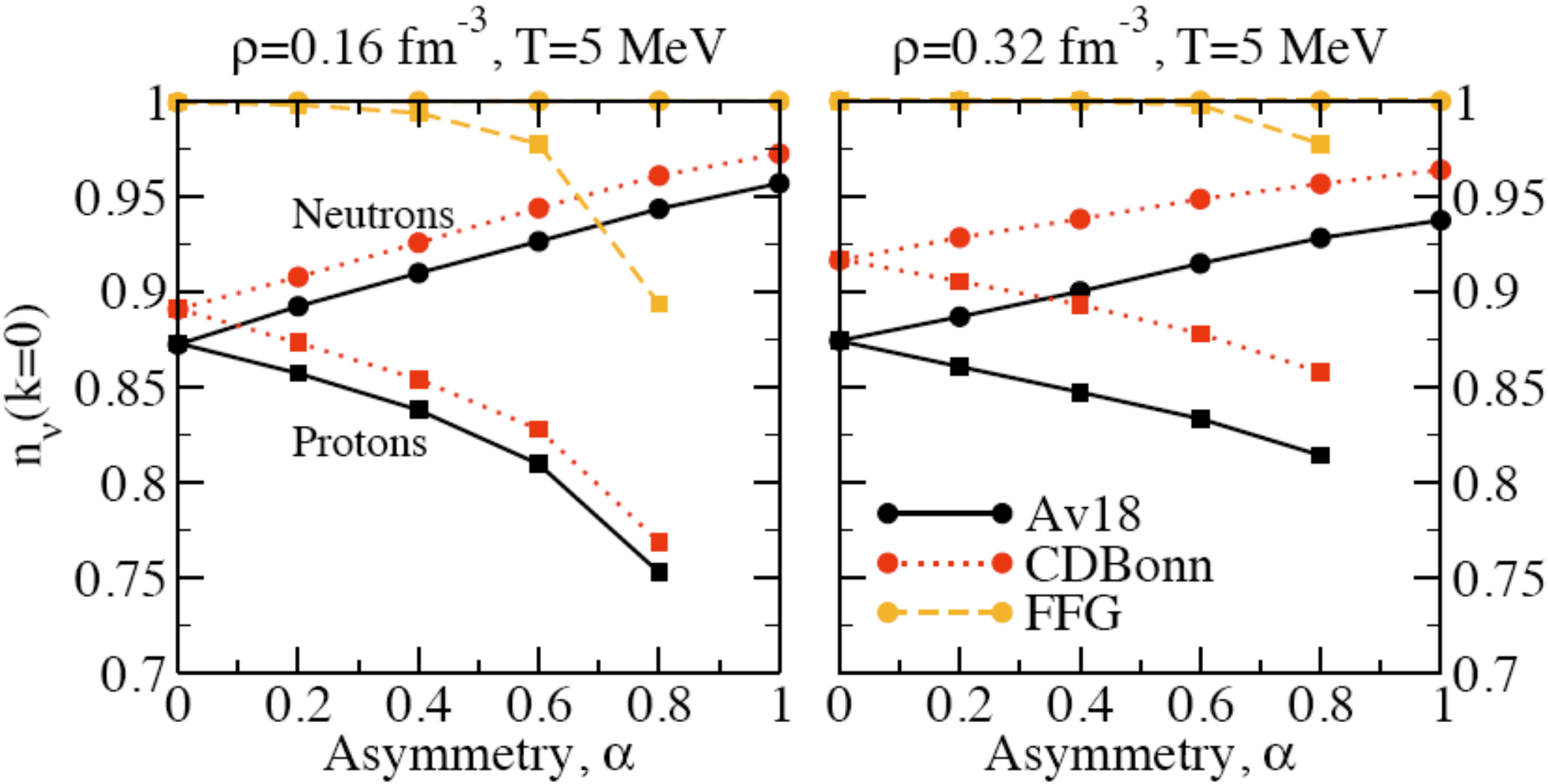
Symmetric nuclear matter

Asymmetric nuclear matter



$$\alpha = \frac{N - Z}{N + Z}$$

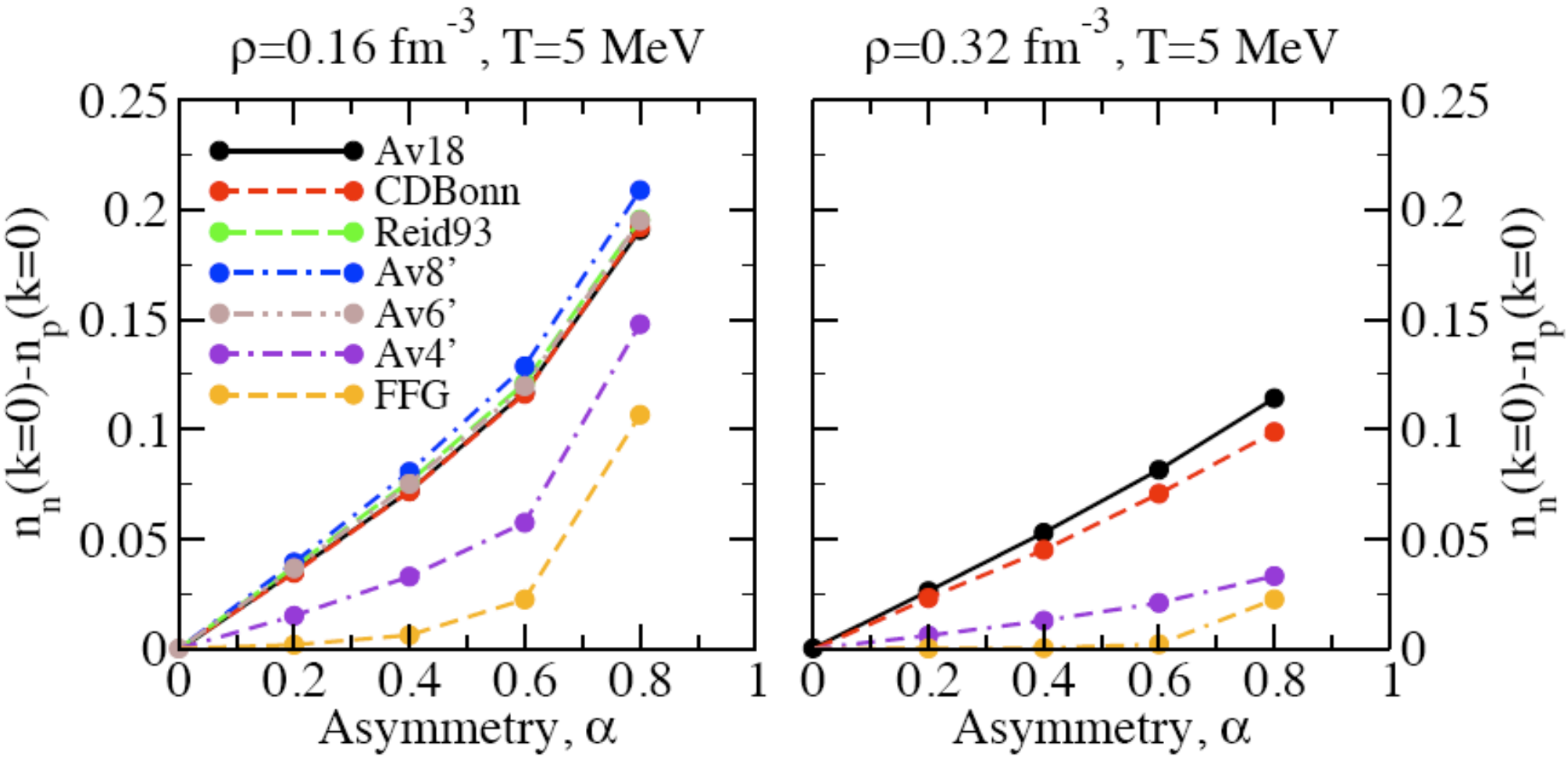
Depletion as a function of asymmetry



A. Rios, A. Polls, and W. H. Dickhoff
Depletion of the nuclear Fermi sea.
Phys. Rev. C79, 064308 (2009).

reactions and structure

Difference $n_n(k=0)-n_p(k=0)$



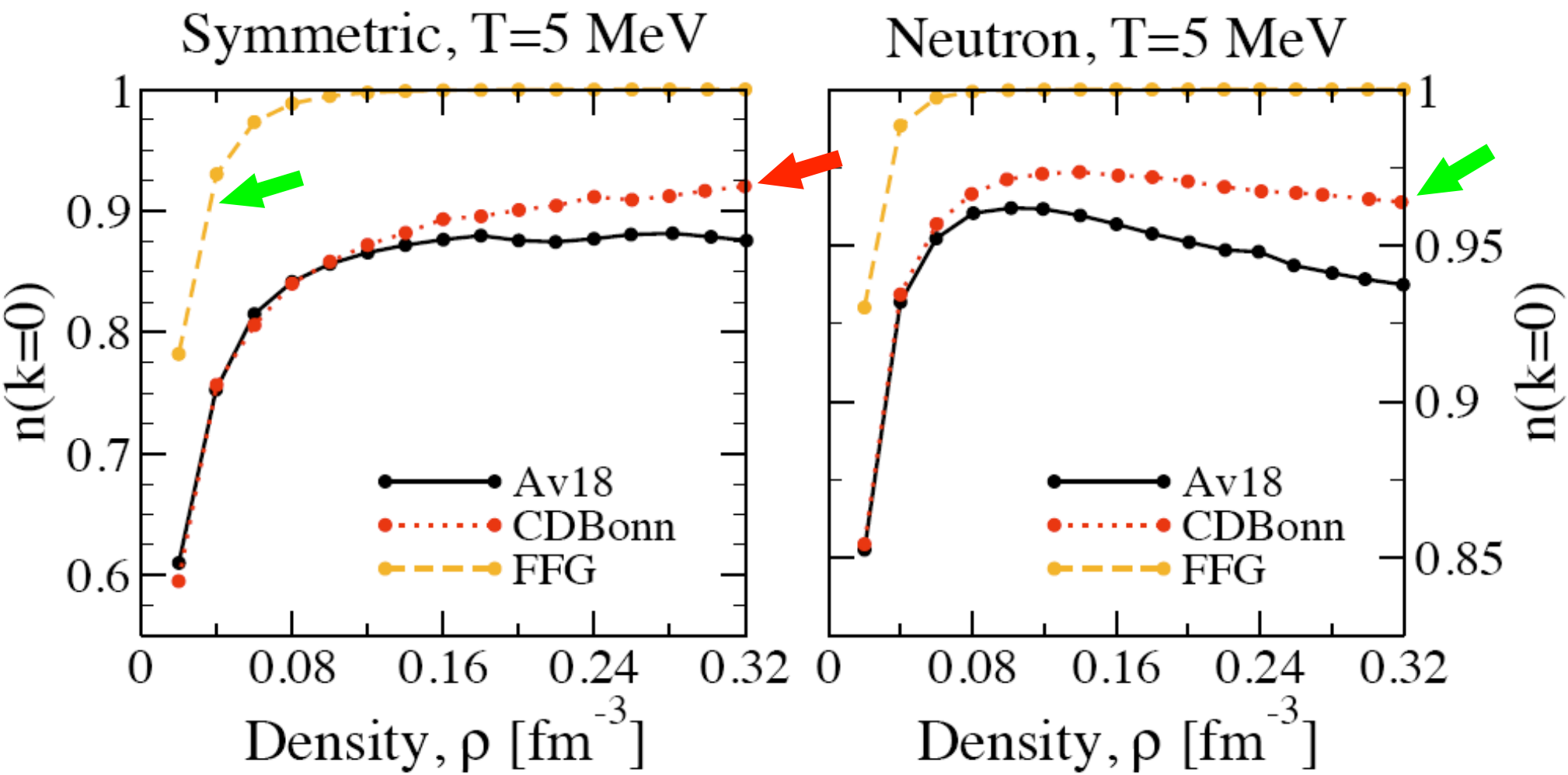
Apparently determined by phase shifts!

Tensor! Note Av4'

A. Rios, A. Polls, and W. H. Dickhoff
Depletion of the nuclear Fermi sea.
Phys. Rev. C79, 064308 (2009).

reactions and structure

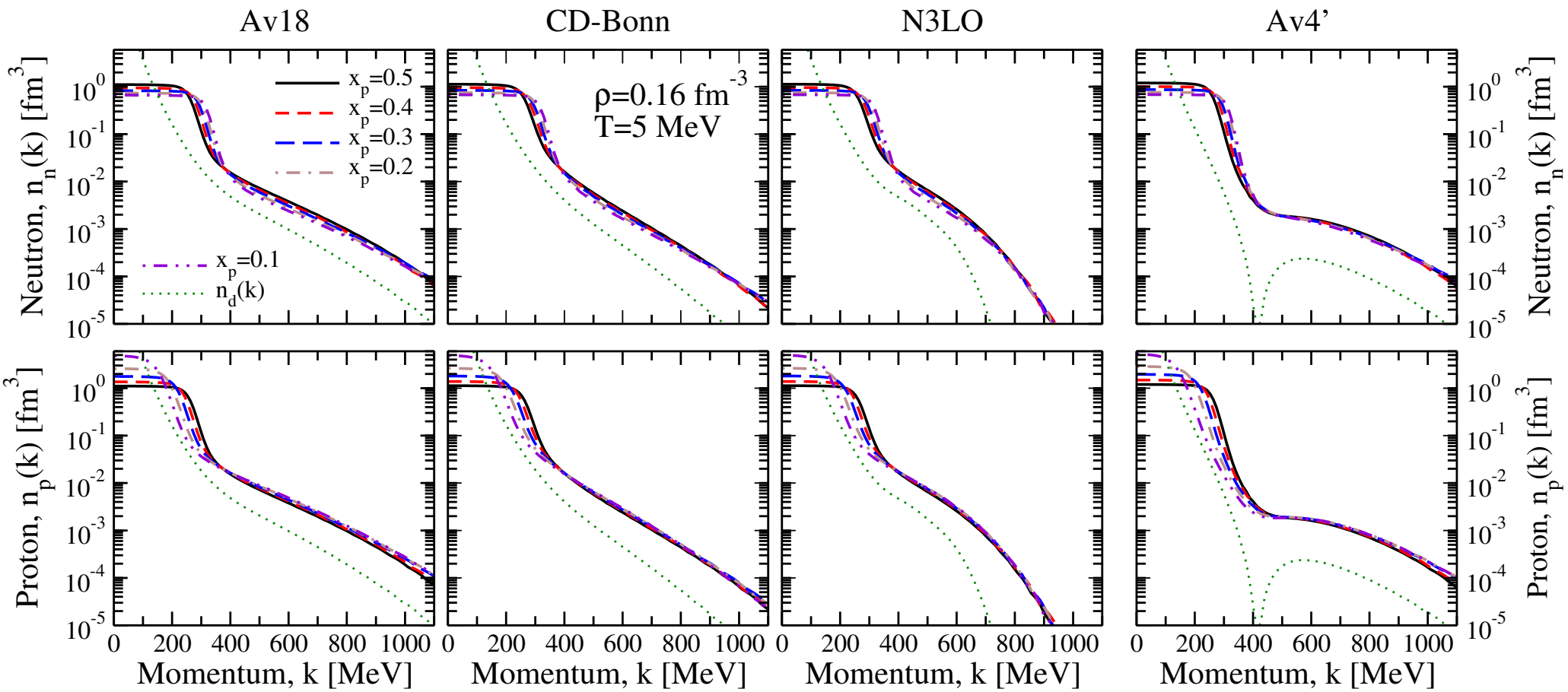
Density dependence n(0) for SNM & PNM



Different behavior for different interactions

Momentum distribution normalized to 1

hard softer very soft no tensor



a_2 as a function of density

$$a_2 = \left\langle \frac{n(k)}{n_d(k)} \right\rangle_{k=400-550\ \mathrm{MeV}}$$

- Data extraction not unambiguous
  - from cross section ratios deep inelastic scattering
  - density…
  - PRC86, 065204 (2012)

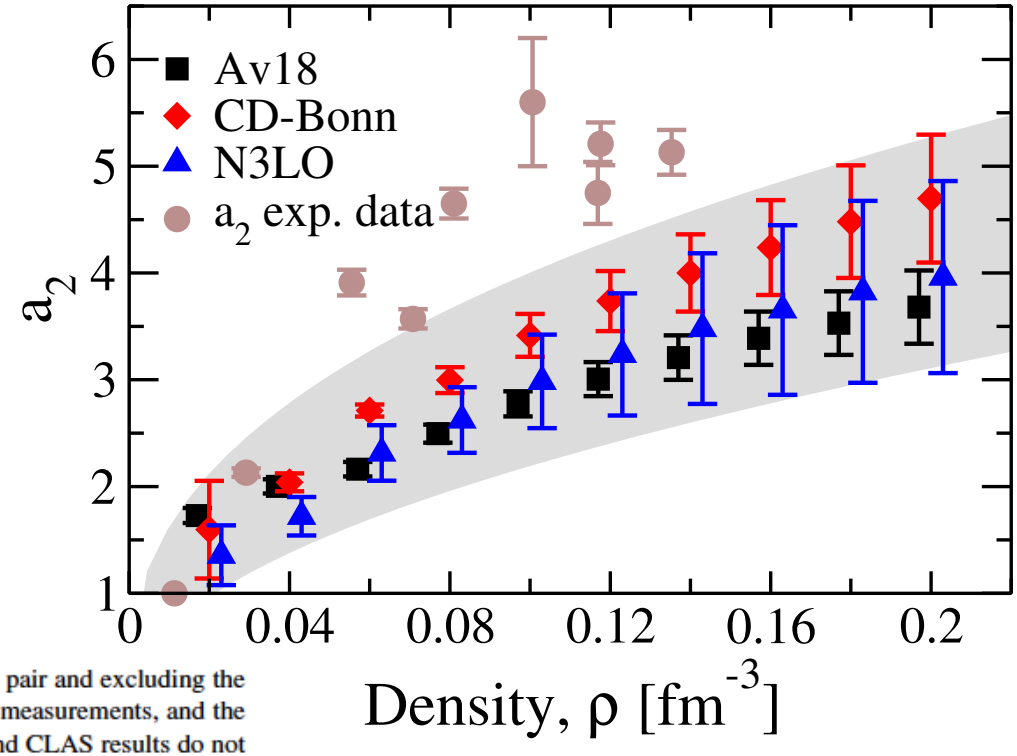


TABLE II. Existing measurements of SRC ratios, R_{2N}, all corrected for c.m. motion of the pair and excluding the isoscalar correction applied to earlier extractions. The second-to-last column combines all the measurements, and the last column shows the ratio a_2, obtained without applying the c.m. motion correction. SLAC and CLAS results do not have Coulomb corrections applied, which would raise the CLAS Fe ratio by ~5% and the SLAC Au data by ~10% (since the correction is kinematic dependent).

	E02-019	SLAC	CLAS	R_{2N}-ALL	a_2-ALL
^{3}He	1.93 ± 0.10	1.8 ± 0.3	–	1.92 ± 0.09	2.13 ± 0.04
^{4}He	3.02 ± 0.17	2.8 ± 0.4	2.80 ± 0.28	2.94 ± 0.14	3.57 ± 0.09
Be	3.37 ± 0.17	–	–	3.37 ± 0.17	3.91 ± 0.12
C	4.00 ± 0.24	4.2 ± 0.5	3.50 ± 0.35	3.89 ± 0.18	4.65 ± 0.14
Al	–	4.4 ± 0.6	–	4.40 ± 0.60	5.30 ± 0.60
Fe	–	4.3 ± 0.8	3.90 ± 0.37	3.97 ± 0.34	4.75 ± 0.29
Cu	4.33 ± 0.28	–	–	4.33 ± 0.28	5.21 ± 0.20
Au	4.26 ± 0.29	4.0 ± 0.6	–	4.21 ± 0.26	5.13 ± 0.21

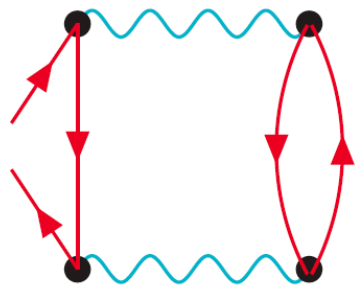
- CD-Bonn "harder" than Av18 …

Two effects associated with short-range correlations

- Depletion of the Fermi sea (see later)
- Admixture of high-momentum components to replace depleted strength (but where?)

Location of high-momentum components

high momenta



require specific intermediate states

External line **k** (large).

Intermediate holes < k_F, say total momentum ~ 0.

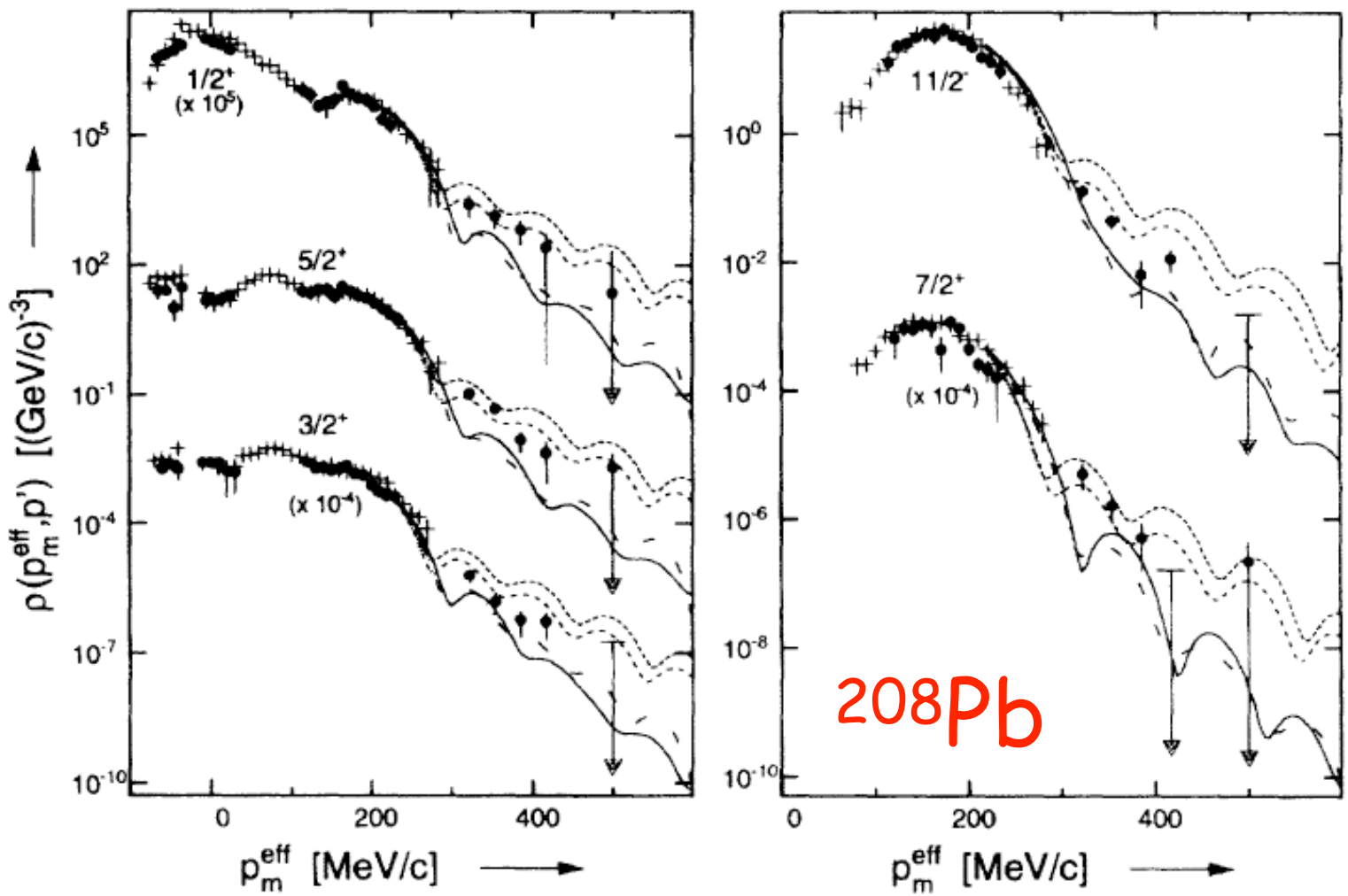
Momentum conservation: intermediate particle -**k**

⇒ Energy intermediate state ~ $\langle\varepsilon_{2h}\rangle - \varepsilon(\mathbf{k})$

⇒ the higher k the more negative the location of its strength

⇒ no high-momentum components near ε_F

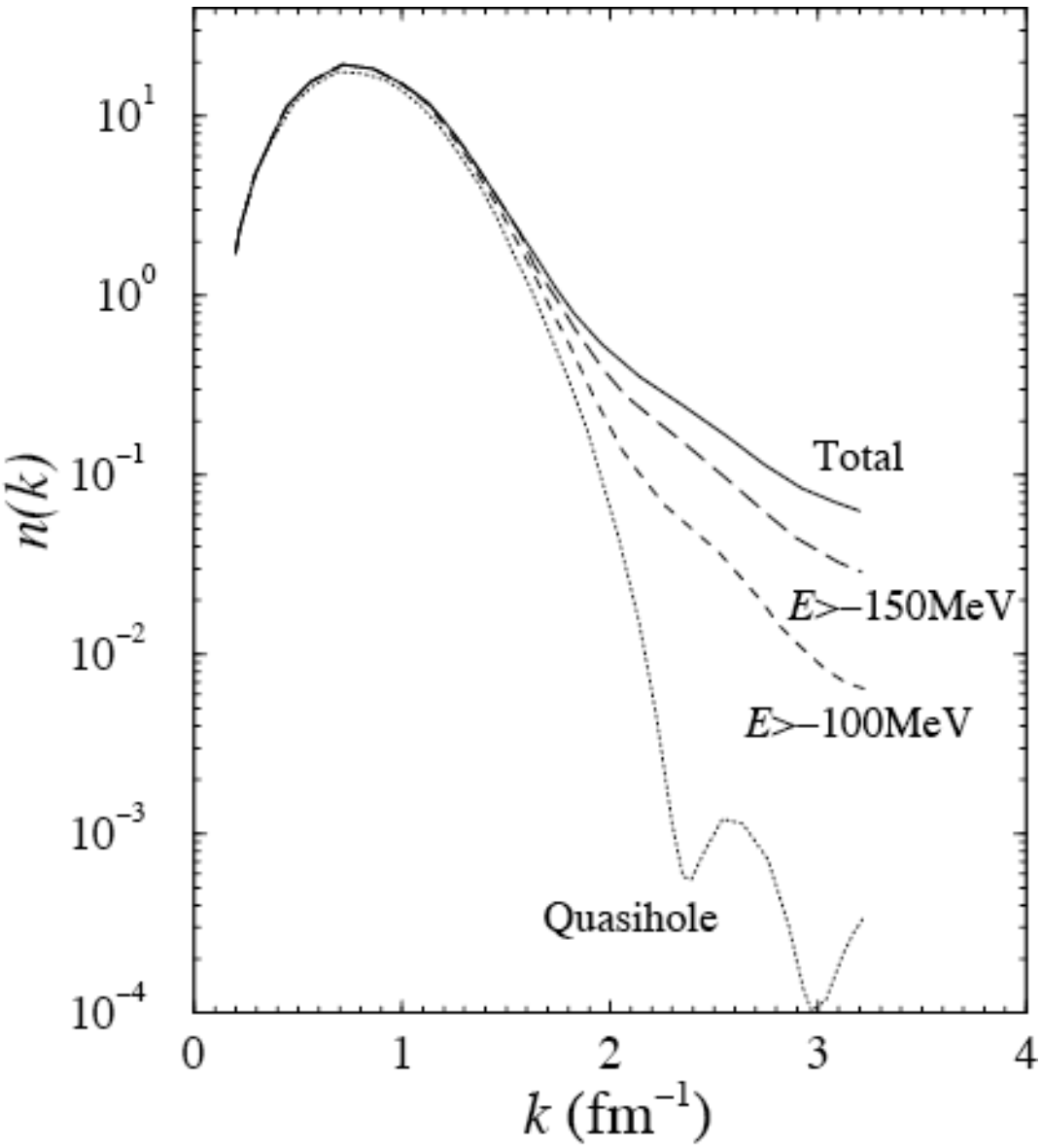
High-momenta near ε_F?



NO!

I. Bobeldijk et al., Phys. Rev. Lett. **73**, 2684 (1994)

Momentum distribution ^{16}O

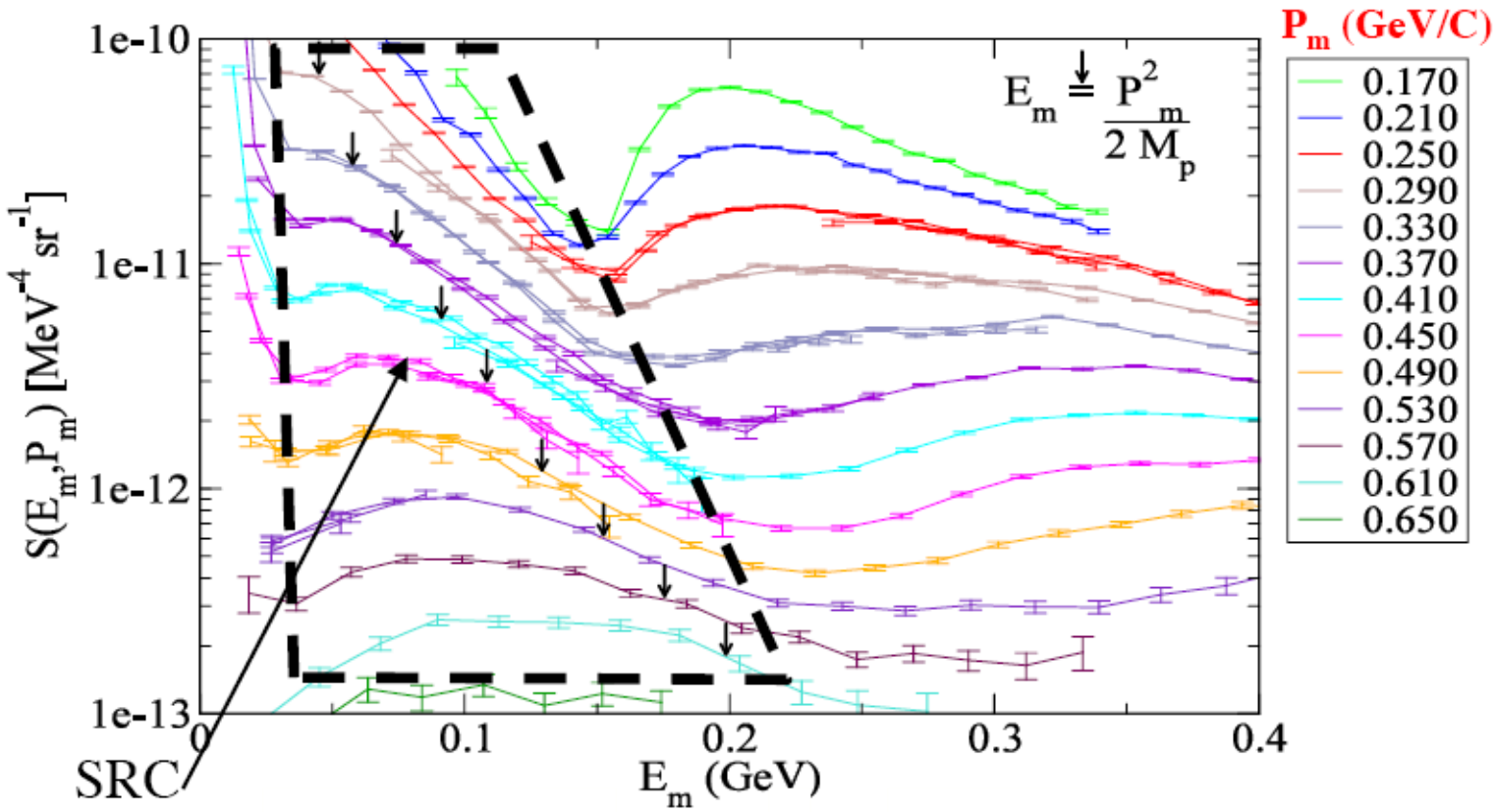


Confirms expectation:

High momentum nucleons can be found at large negative energies

What are the rest of the protons doing?

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



^{12}C

- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

  $\Rightarrow$ 0.6 protons for ^{12}C

Some theoretical results for ^{16}O

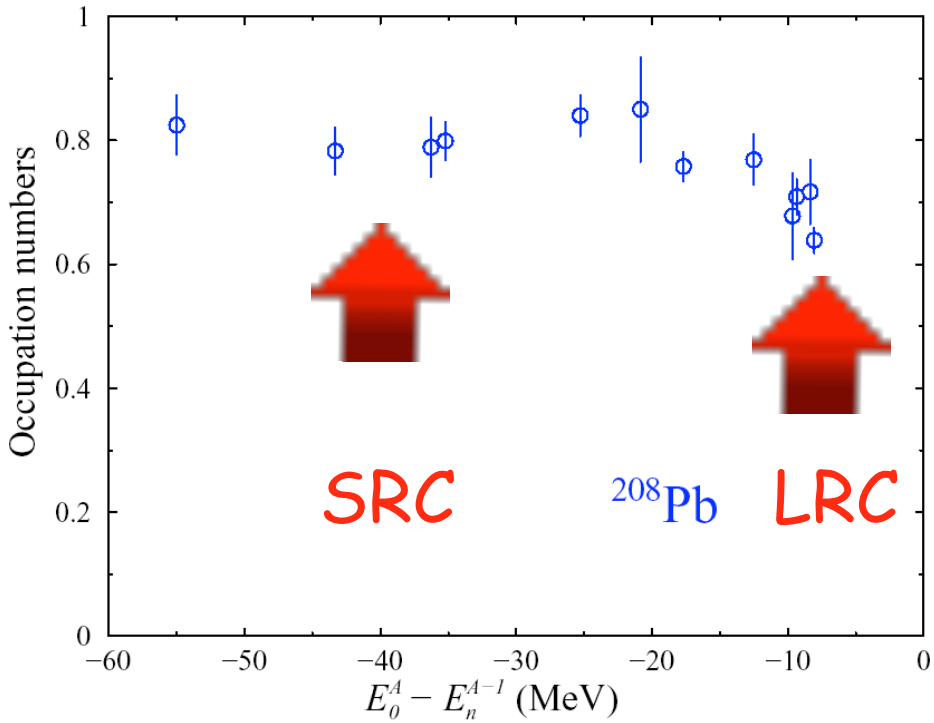
	$S_{p1/2}$	$S_{p3/2}$		
• Experiment	0.63	0.67	±0.05	NIKHEF '94
• Short-range oriented methods				
VMC [Argonne, '94]	0.90			SRC 10%
GF(SRC) [St.Louis-Tübingen '95]	0.91	0.89		SRC 10%
FHNC/SOC [Pisa '00]	0.90			SRC 10%
• Including particle-phonon couplings				
GF(Faddeev) [St.Louis '01]	0.77	0.72		LRC rest
• → relevance of collective motion				

LRC ≈ particle-phonon (GR) coupling

M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´ p) ^{207}Tl

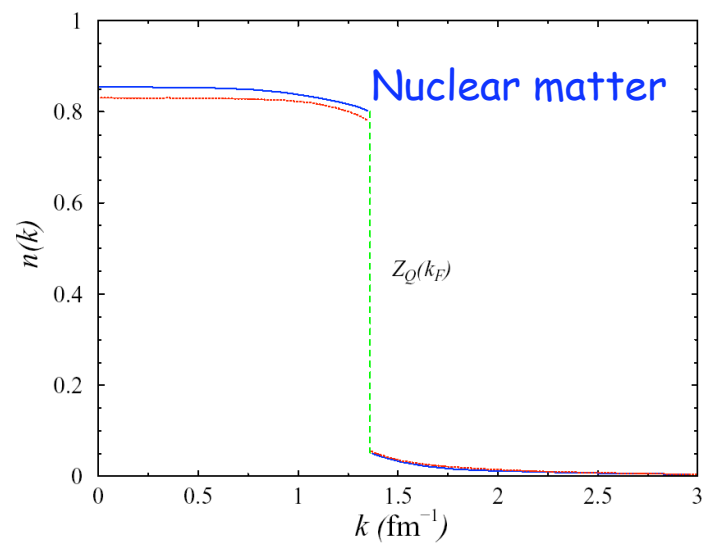
NIKHEF 2001 data (one of the last experiments)

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the **whole** mean-field domain!!



Confirms predictions for depletion

n(0) ⇒ 0.85 Reid

0.87 Argonne V18

0.89 CDBonn/N3LO

Reviewed in Prog. Part. Nucl. Phys. 52 (2004) 377-496

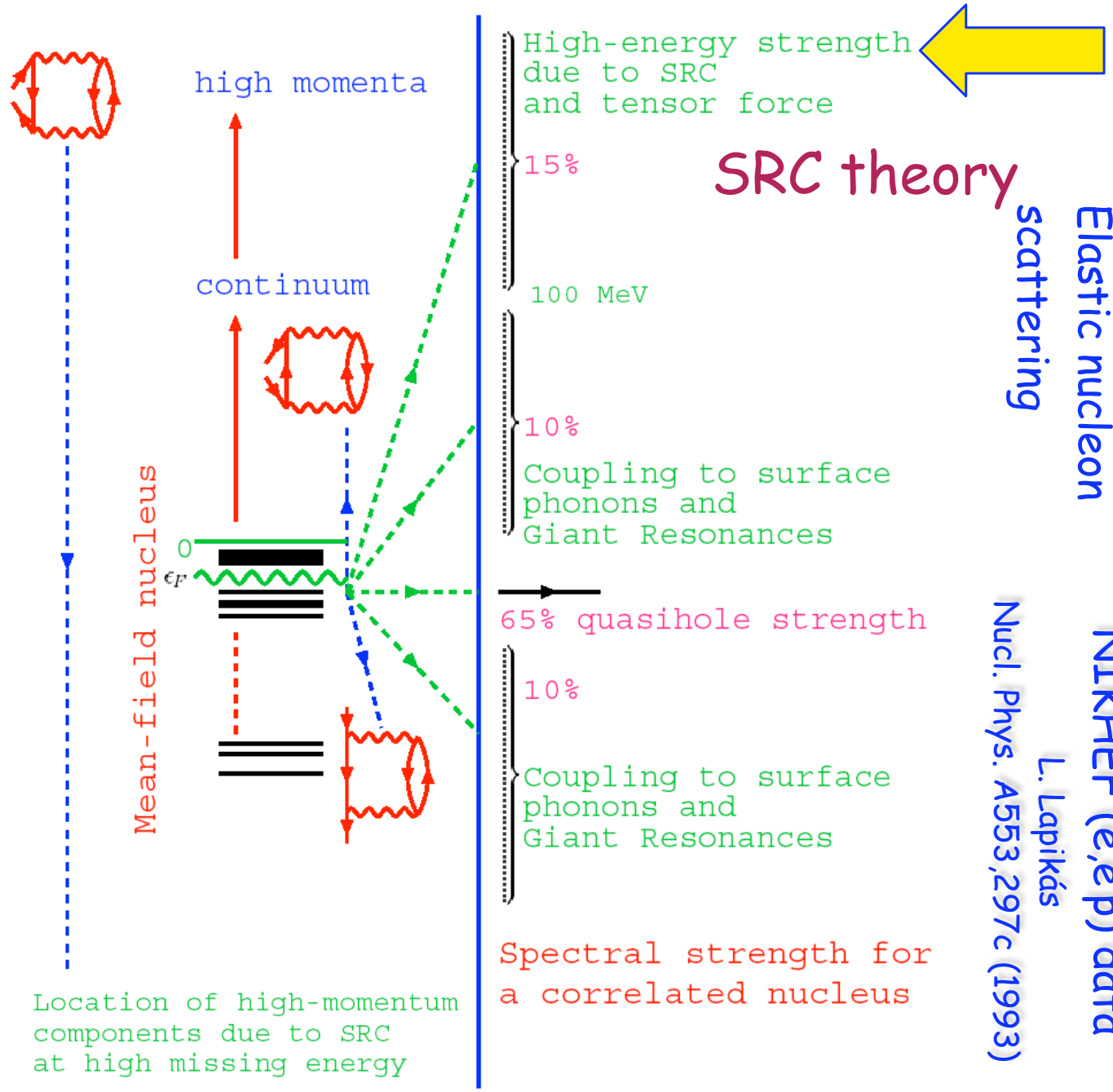
Location of single-particle strength in closed-shell (stable) nuclei

For example: protons in ^{208}Pb

SRC

JLab E97-006

Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



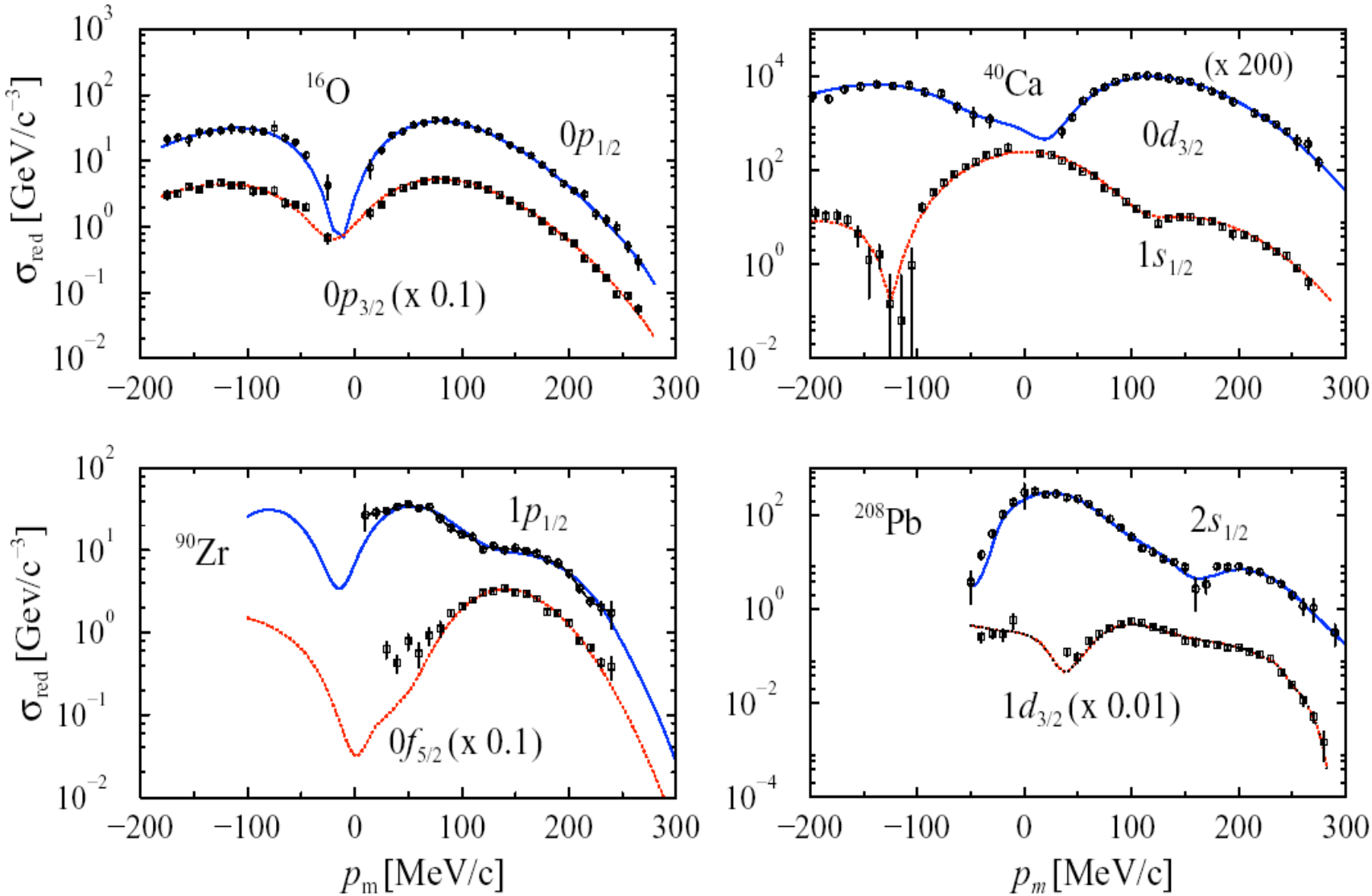
reactions and structure

(e,e'p) data for nuclei —> strength near ε_F

- Requires DWIA
- Distorted waves required to describe elastic proton scattering at the energy of the ejected proton
- Consistent description requires that cross section at different energies for the outgoing proton is changed accordingly
- Momentum dependence of cross section dominated by the corresponding overlap function of the nucleon that is removed
- Parallel kinematics favors the one-body (charge) excitation operator

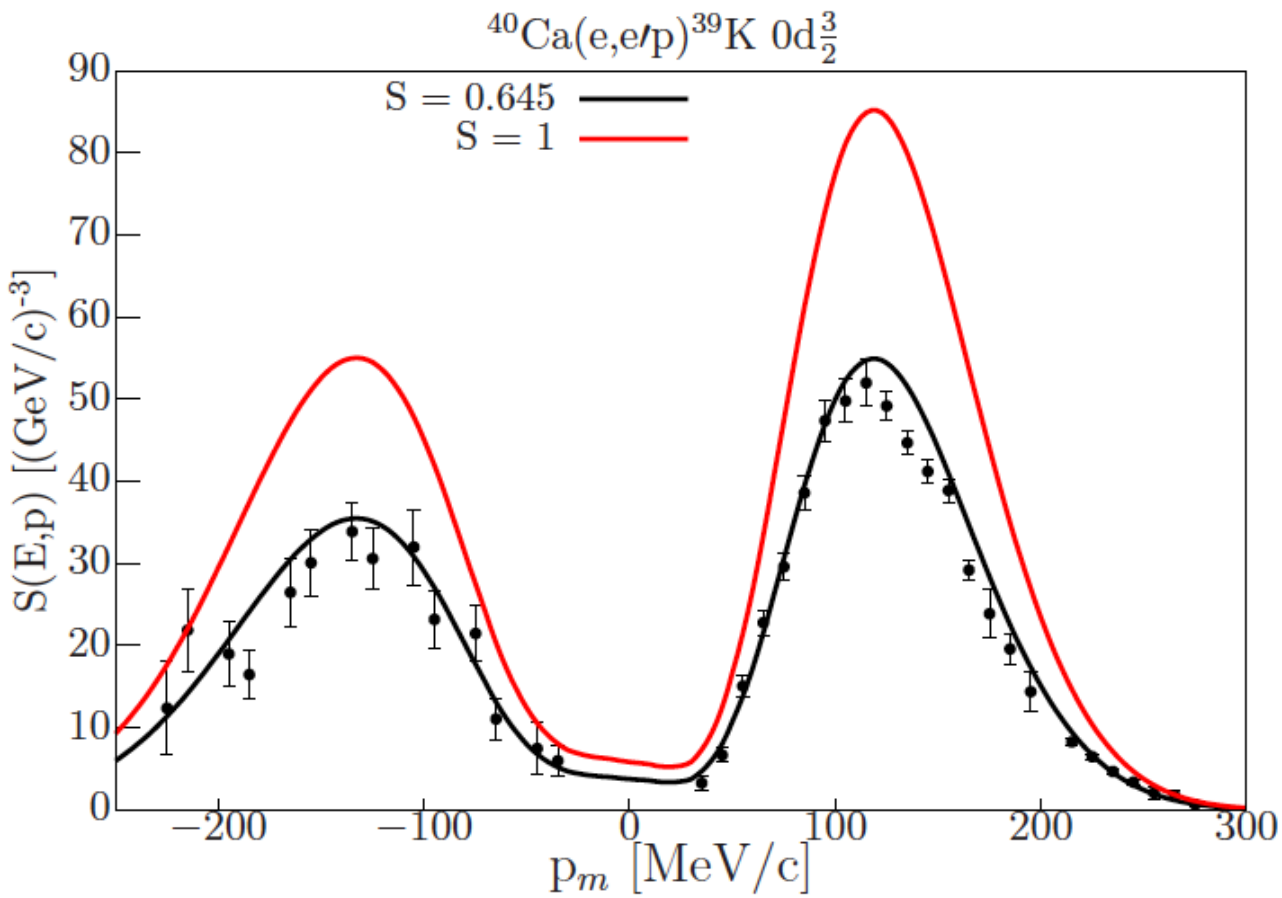
Momentum profiles for nucleon removal

- Closed-shell nuclei
- NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



NIKHEF analysis PLB227,199(1989) 40Ca(e,e'p)

- Schwandt et al. (1981) optical potential
- BSW from adjusted WS
  - radius —> shape
  - depth —> separation energy
  - Perey-type corrections applied (angular momentum independent)



How about calculating DWIA cross section?

- What can we do ab initio?
- Is it accurate? NO
- Can we ever get accurate spectral distributions? May take time
- Can we ever get accurate optical potentials? Probably not
- Consider the dispersive optical model (DOM) as an interface between ab initio theory and experiment

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- e.g. Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation
  - contributions from the hole (structure) and particle (reaction) domain

General dispersion relation for self-energy:

$$\mathrm{Re}\ \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\mathrm{Re}\ \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\mathrm{Re}\ \Sigma(E) = \mathrm{Re}\ \widetilde{\Sigma^{HF}}(\varepsilon_F)$$

$$- \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

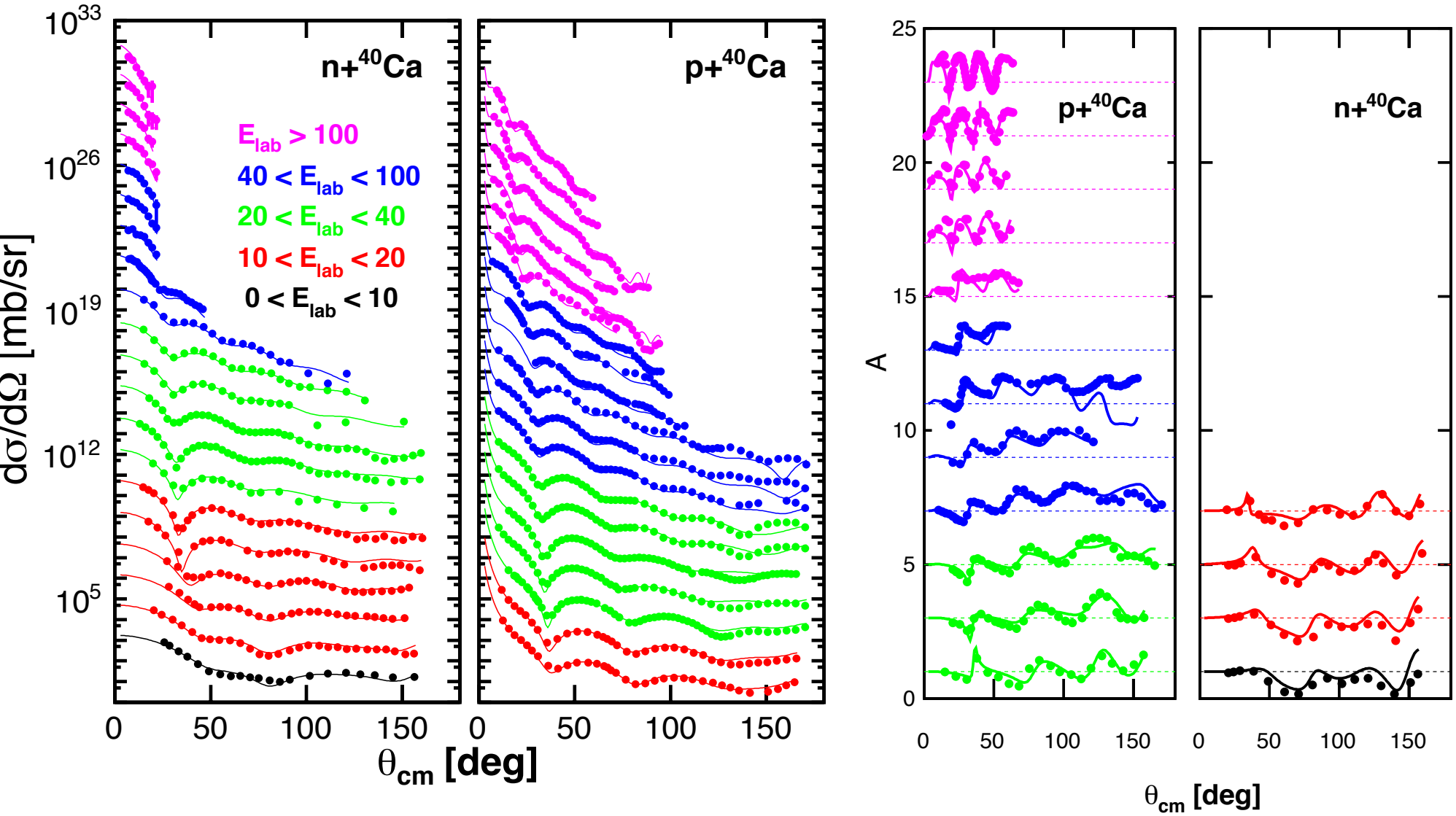
Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Ab initio FRPA & SRC --> different nonlocal properties above and below the Fermi energy Phys. Rev. C84, 034616 (2011) & Phys. Rev.C84, 044319 (2011)
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e'p) data from JLab

Implications

- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of interpretation (e,e'p) reaction (see later)

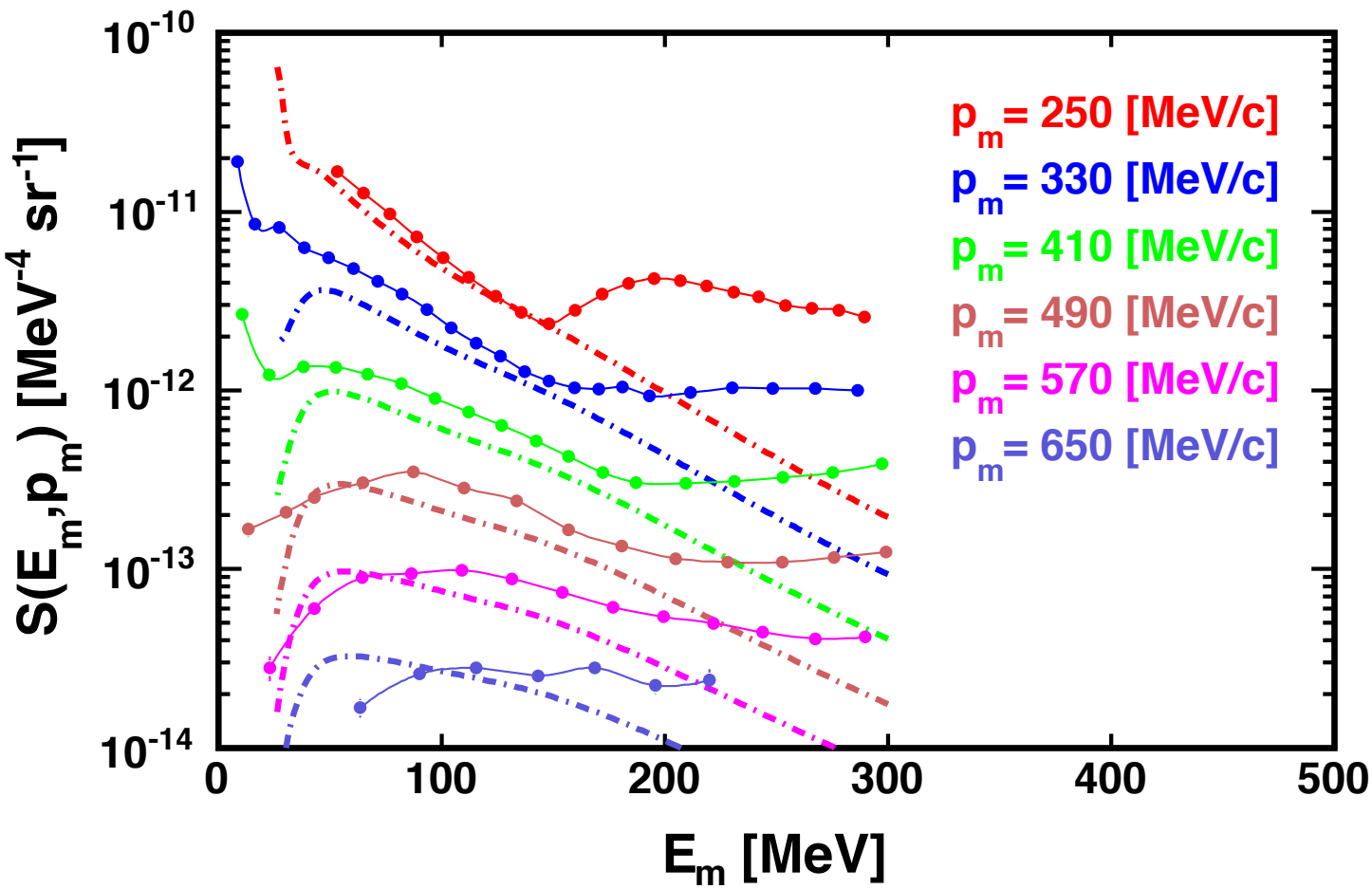
Differential cross sections and analyzing powers



Jefferson Lab data per proton

- Pion/isobar contributions cannot be described
- Rescattering contributes some cross section

C. Barbieri and L. Lapikás Phys. Rev. C 70, 054612 (2004)



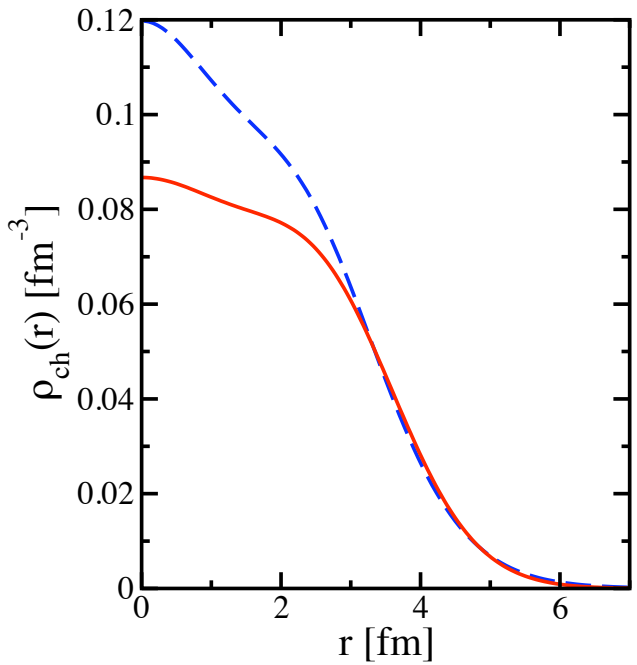
Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

Critical experimental data—> charge density

Local version

radius correct…

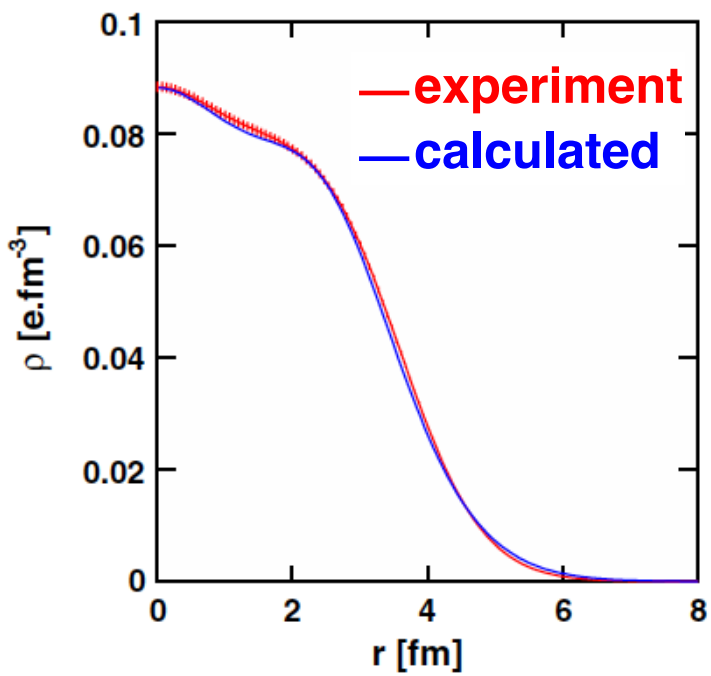
PRC82, 054306 (2010)



Charge density ^{40}Ca

Non-locality essential

PRL 112,162503(2014)

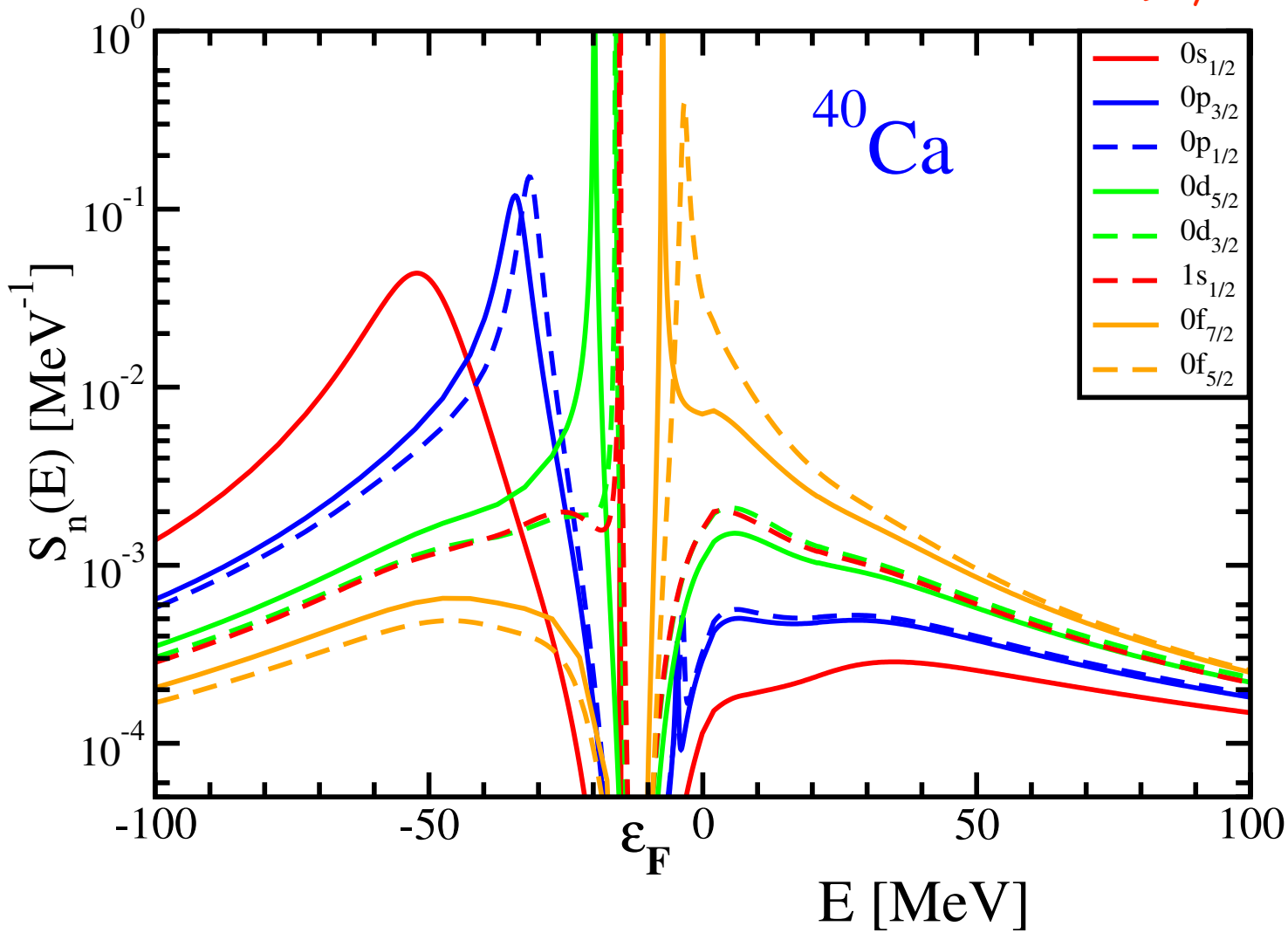


High-momentum nucleons —> JLab can also be described —> E/A

Spectral function for bound states

- [0,200] MeV —> constrained by elastic scattering data

PRC90, 061603(R) (2014)



Emptiness constrained but amount depends on orbit's distance to the continuum!

Quantitatively

- Orbit closer to the continuum —> more strength in the continuum
- Note "particle" orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca. $d_{nlj}[0,200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$. Last column $d_{nlj}[0,200]$ depletion numbers for the CDBonn calculation.

orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0,200]$ DOM	$n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$ DOM	$d_{n\ell j}[0,200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

PRC90, 061603(R) (2014)

DOM and (e,e'p)

- Assess DWIA description of (e,e'p) reaction
- When is it valid?
- DOM provides all necessary distorted waves
- plus overlap function with its normalization
- all constrained by other data

- DWIA and DOM consistent for (e,e'p)?
- ^{40}Ca(e,e'p) data published at T_p = 100 MeV
- also available but not yet published at 70 and 135 MeV

Preprint

Validity of the distorted-wave impulse-approximation description of ^{40}Ca$(e, e'p)$ data using only ingredients from a nonlocal dispersive optical model

M. C. Atkinson[1], H.P. Blok[2,3], L. Lapikás[2], R. J. Charity[4], and W. H. Dickhoff[1]

arXiv:1808.08895v1 [nucl-th] 27 Aug 2018

reactions and structure

Another look at (e,e'p) data

- collaboration with Louk Lapikás and Henk Blok
- Data published at E_p = 100 MeV Kramer thesis NIKHEF for $^{40}Ca(e,e'p)^{39}K$
  Phys.Lett.B227(1989)199
  Results: $S(d_{3/2})$=0.65 and $S(s_{1/2})$=0.51
- More data at 70 and 135 MeV (only in a conference paper)
- What do these spectroscopic factor numbers really represent?
  - Assume DWIA for the reaction description
  - Use kinematics (momentum transfer parallel to initial proton momentum) favoring simplest part of the excitation operator (no two-body current)
  - Overlap function:
    - WS with radius adjusted to shape of cross section
    - Depth adjusted to separation energy
  - Distorted proton wave from standard "global optical potential"
  - Fit normalization of overlap function to data -> spectroscopic factor

Why go back there?

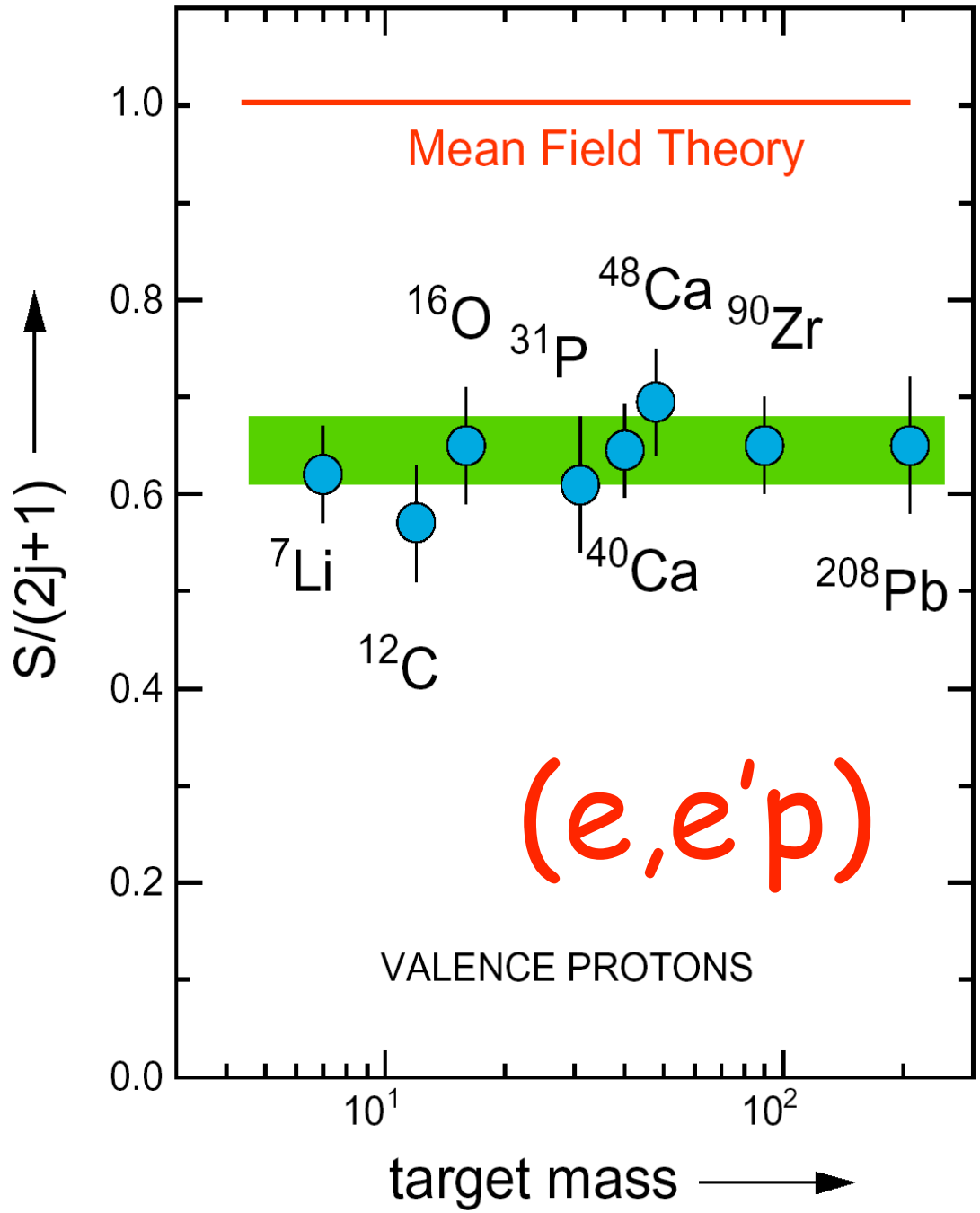
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

S ≈ 0.65 for valence protons
Reduction ⇒ both SRC and LRC

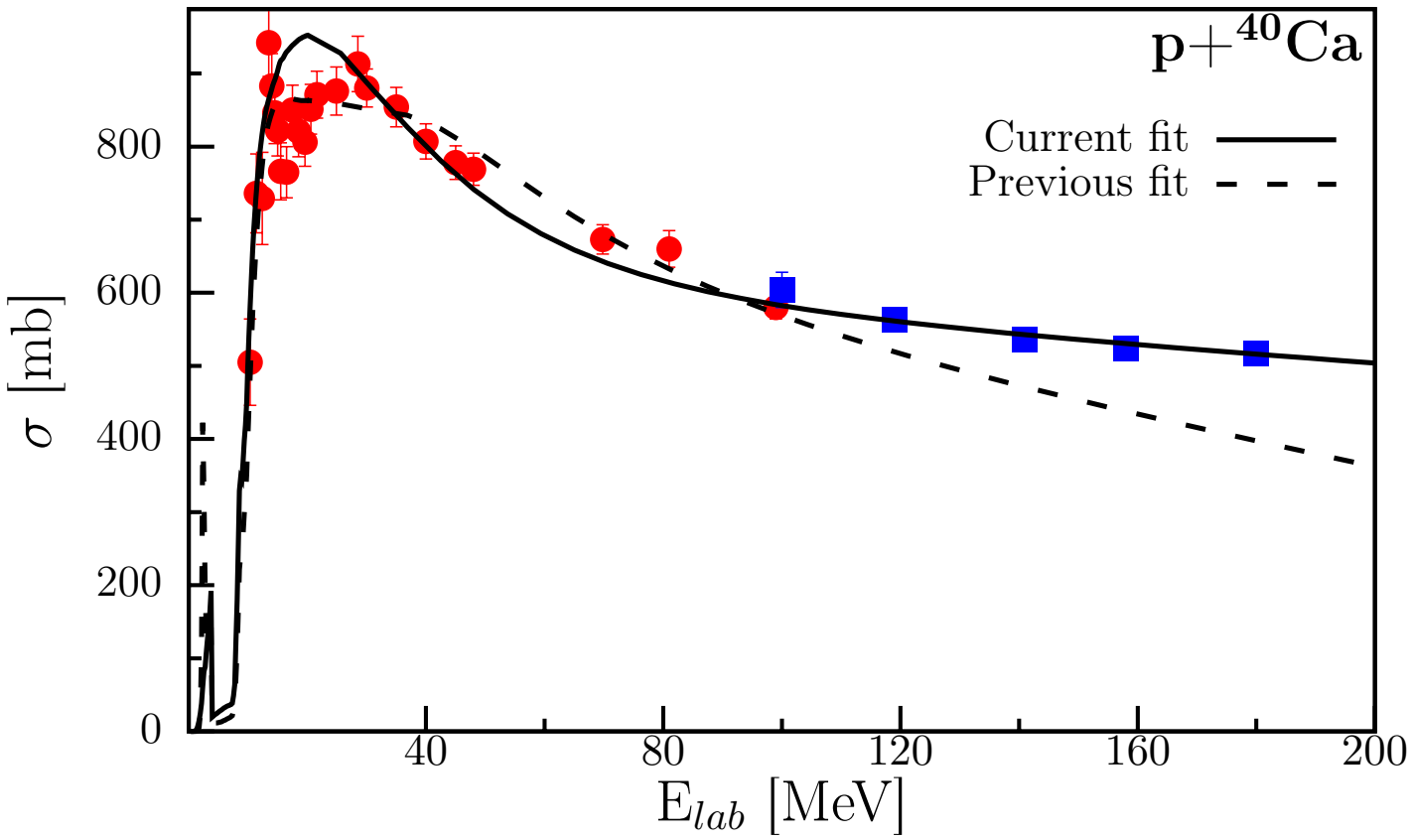
Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted wave --> associated uncertainty ~ 5-15%

Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far.....)
but now available for ^{40}Ca!



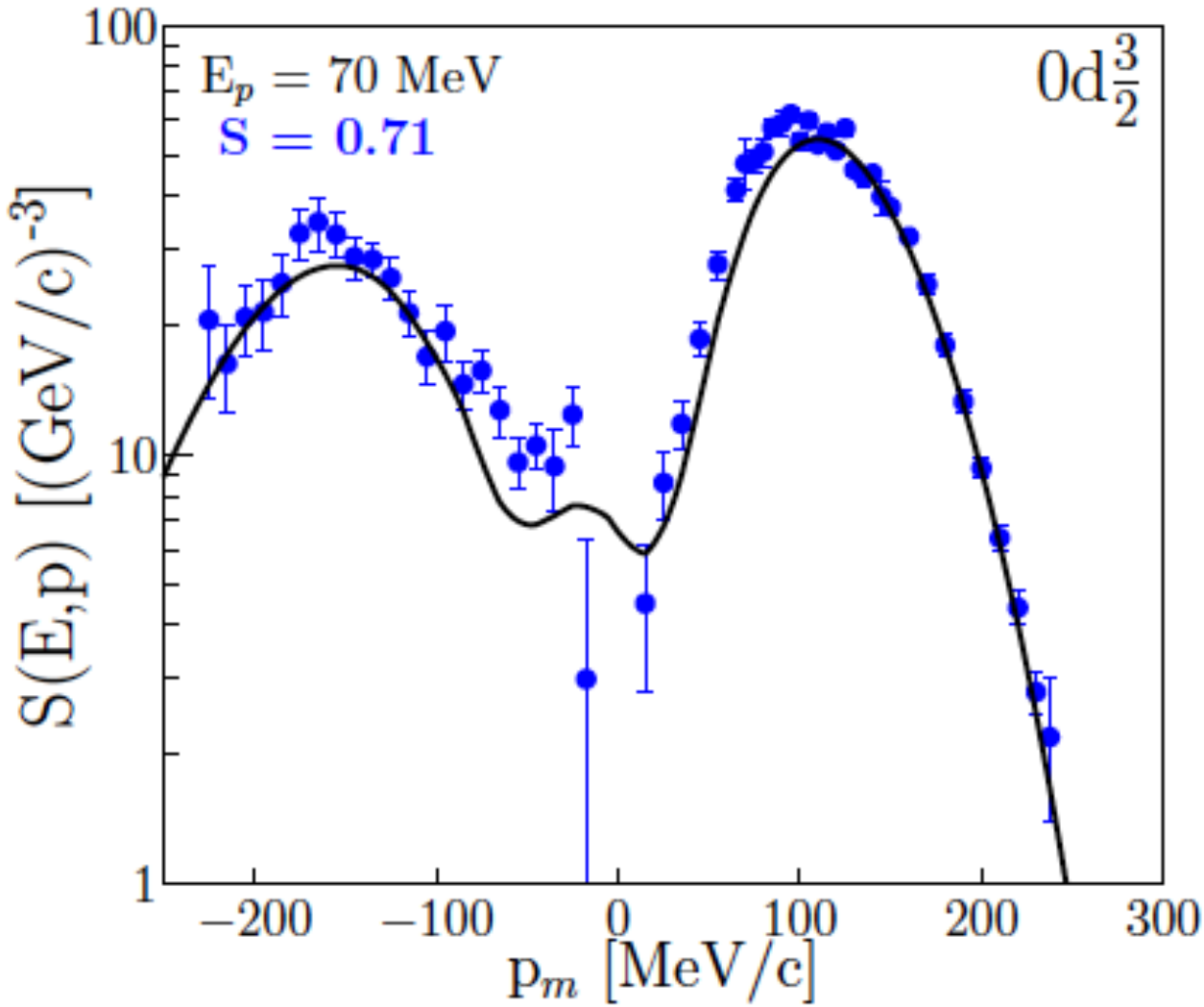
Update from 2014 PRL

- To accurately account for the higher-energy data the imaginary part of the volume absorption is larger than in the 2014 fit.
- This leads to a reduction of the spectroscopic factors near the Fermi energy of 0.05



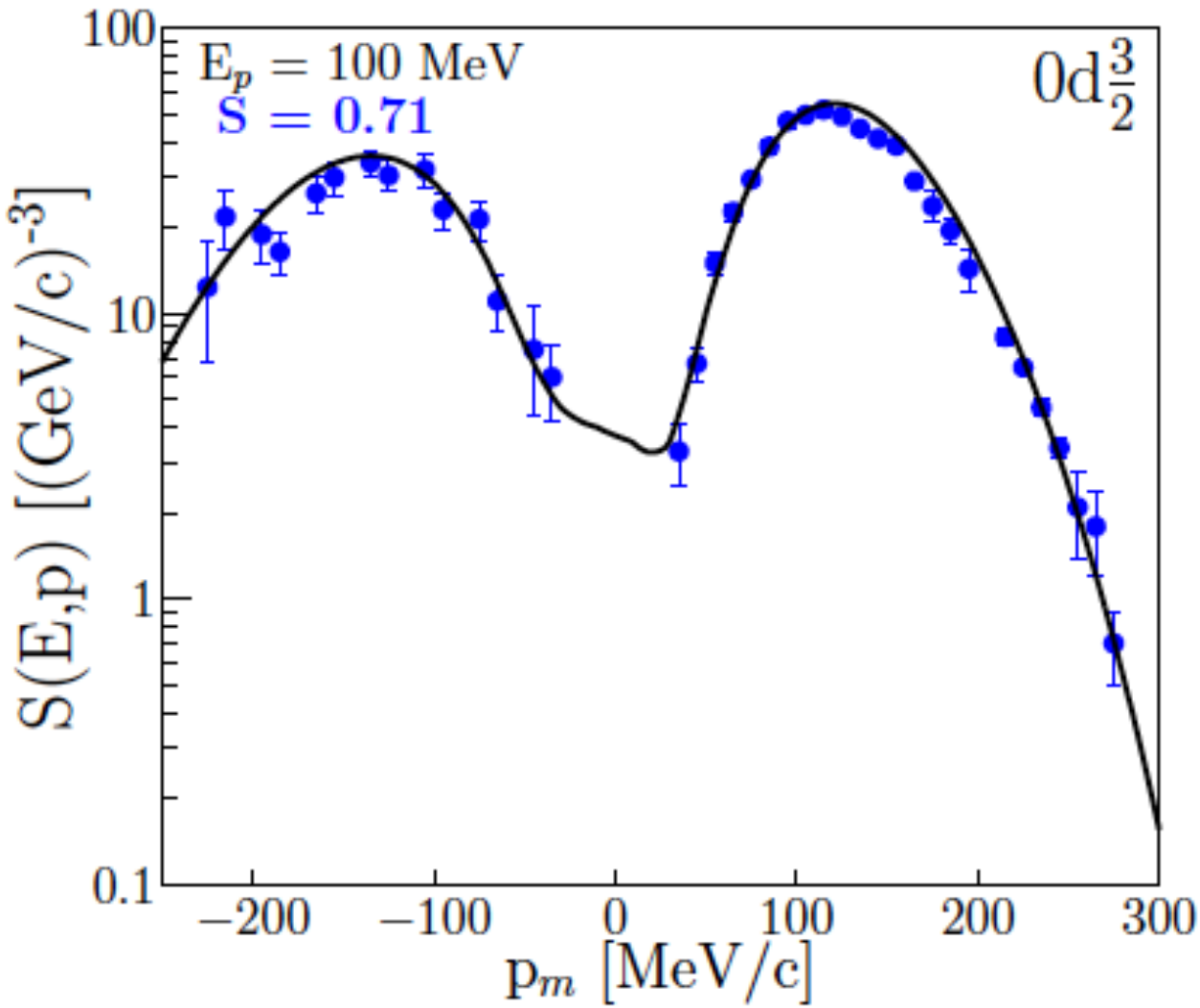
NIKHEF data unpublished

- Only DOM ingredients
- DWEEPY code C. Giusti



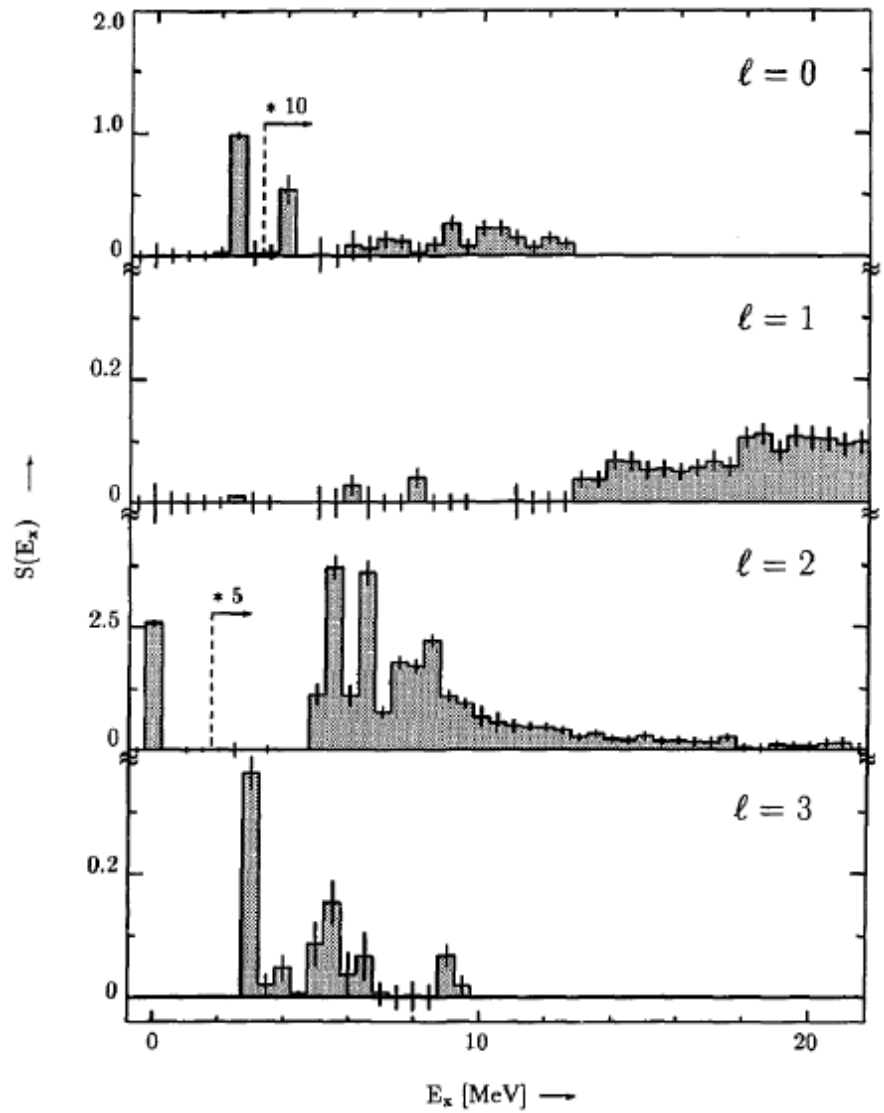
NIKHEF data PLB227,199(1989)

- NIKHEF: $S(d_{3/2})=0.65\pm0.06$
- Only DOM ingredients

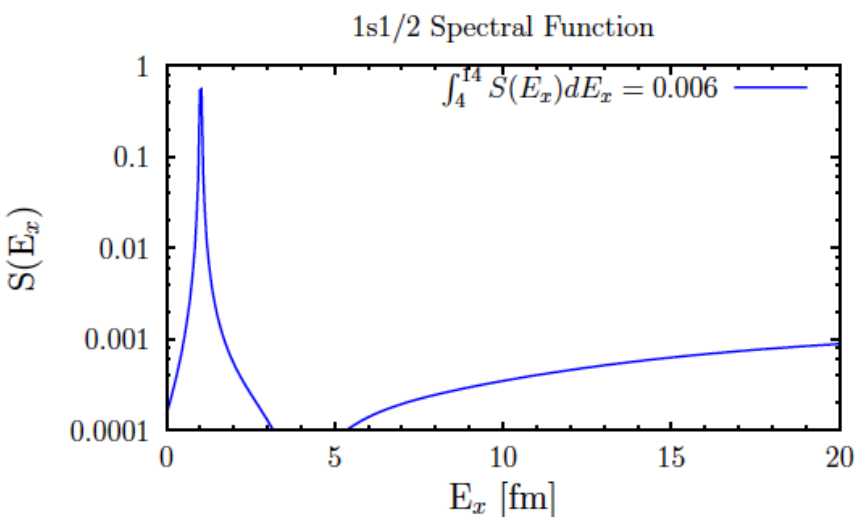


Thesis G. J. Kramer (1990)

- $s_{1/2}$ strength fragmented



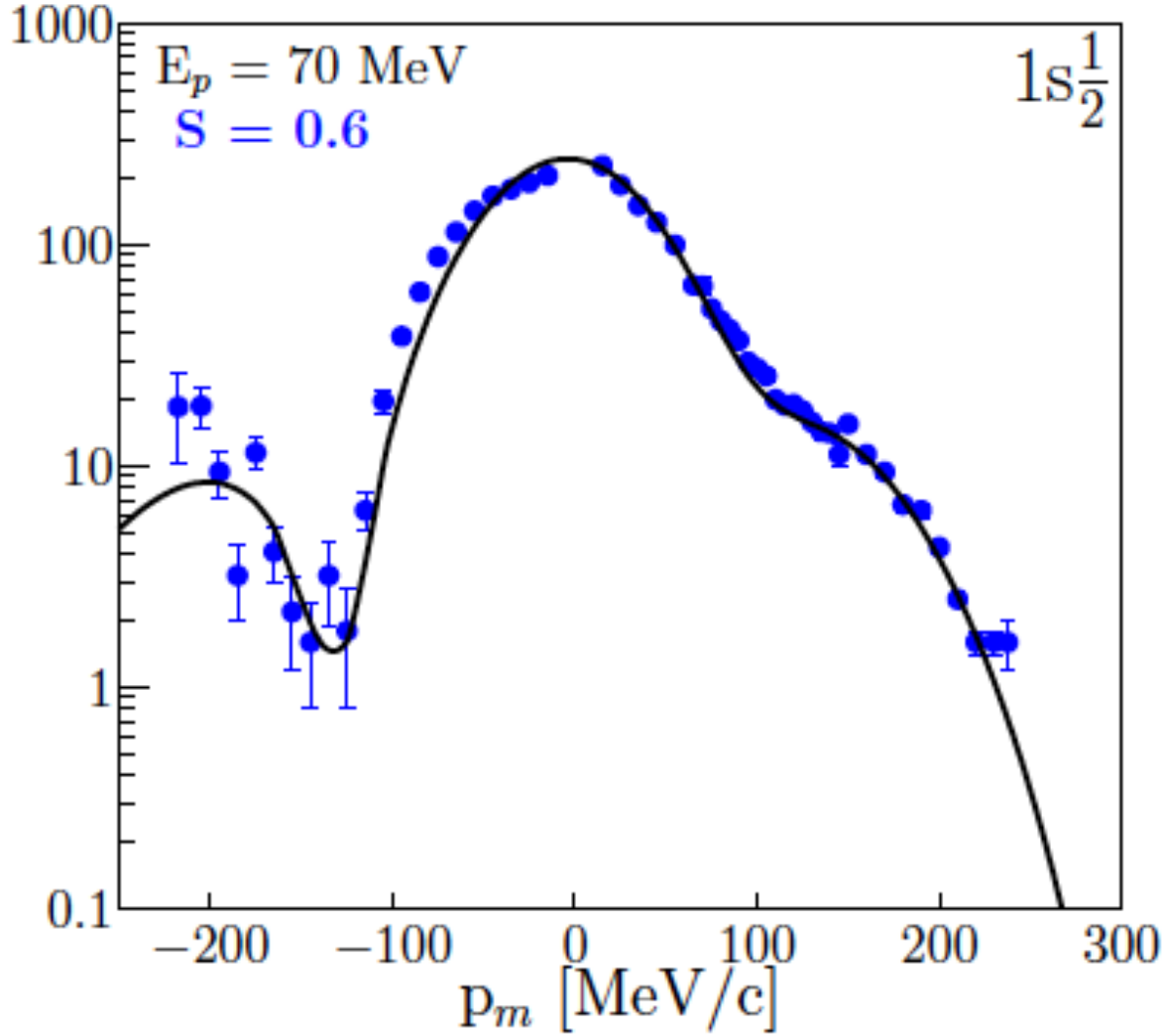
- Not yet included in DOM



- Corrects DOM spectroscopic factor from 0.74 to 0.60

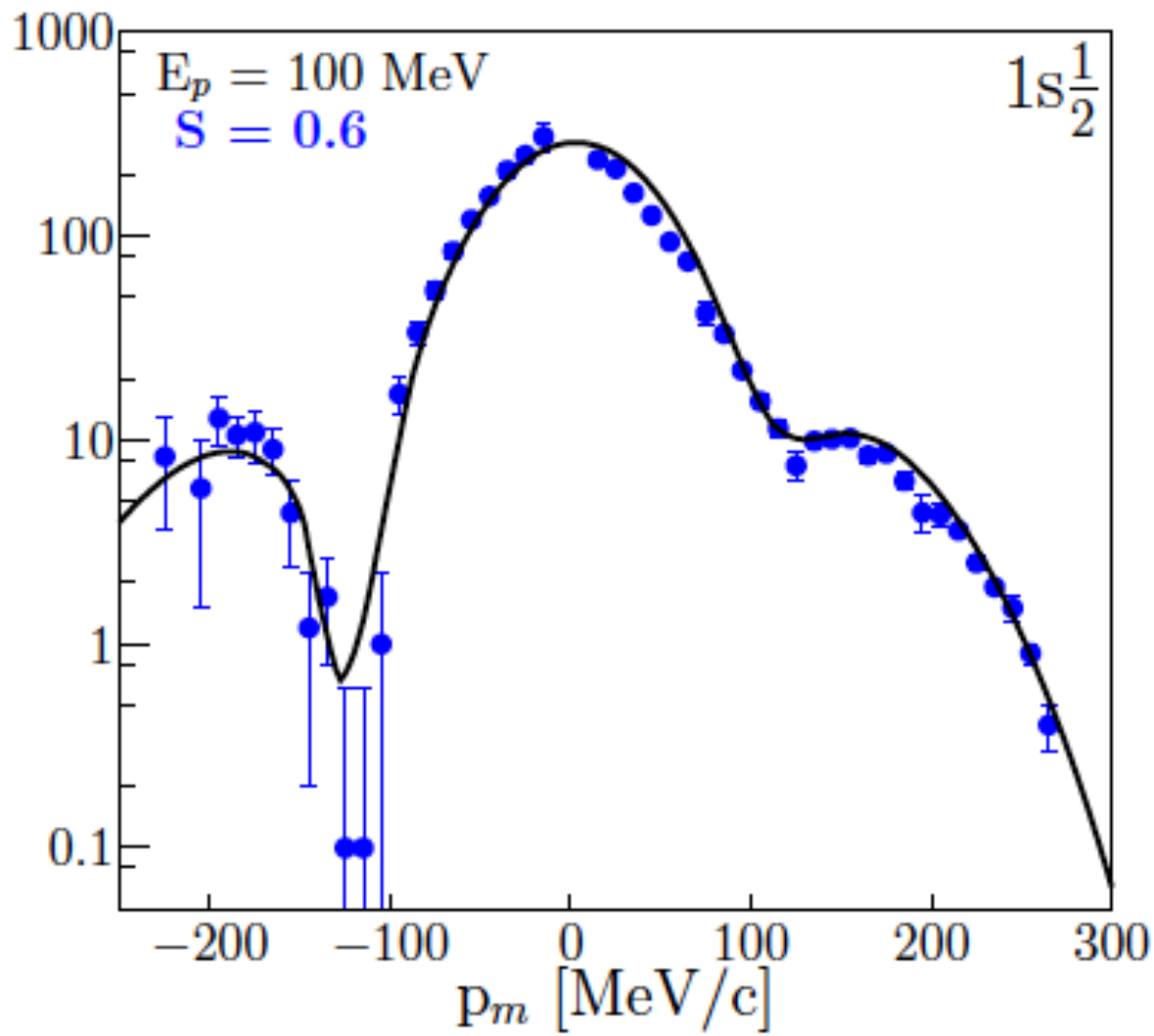
NIKHEF data unpublished

- Only DOM ingredients



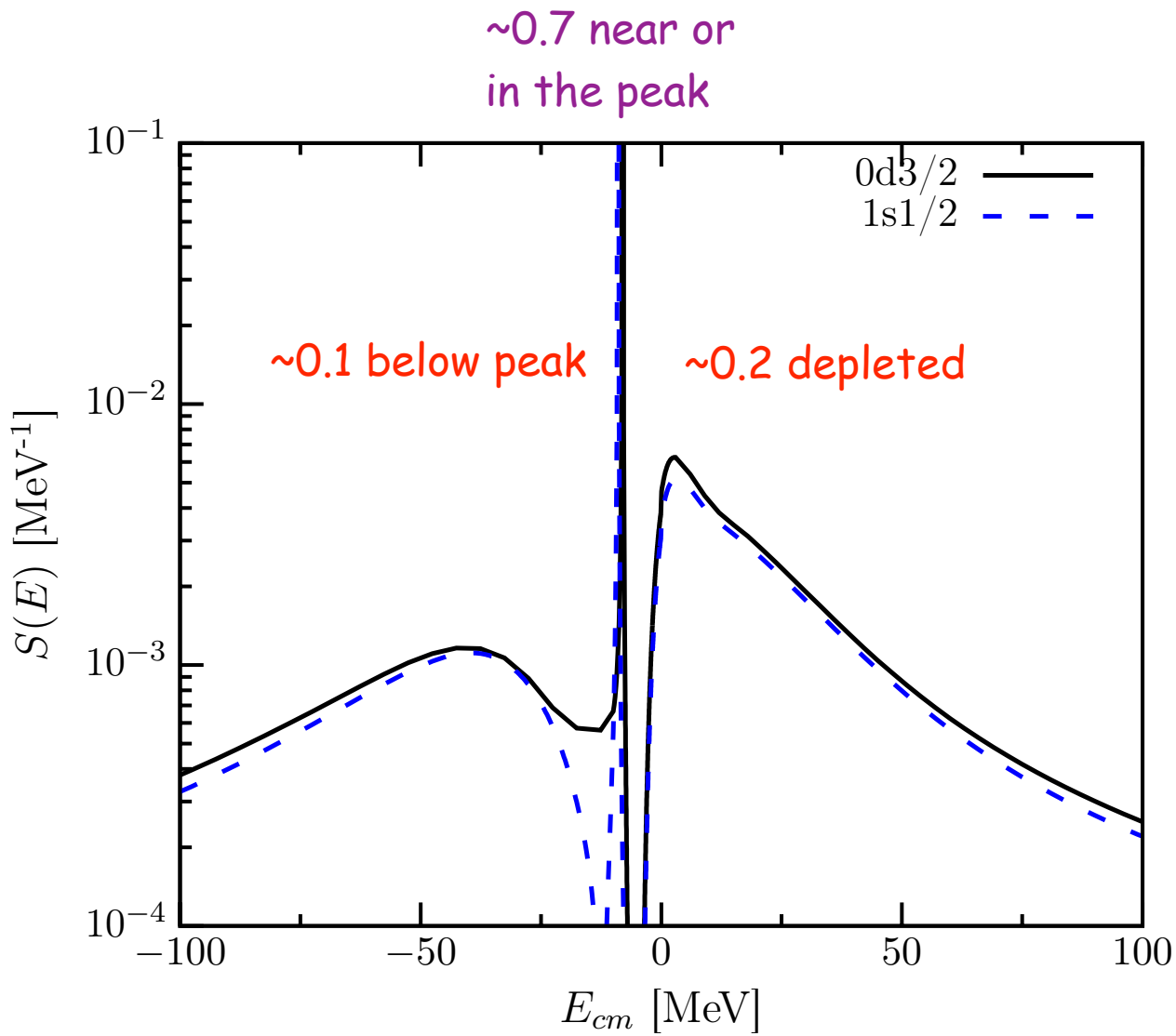
NIKHEF data PLB227,199(1989)

- NIKHEF: $S(s_{1/2})=0.51\pm0.05$



^{40}Ca spectral distribution

- $0d_{3/2}$ and $1s_{1/2}$



Message

- Nonlocal dispersive potentials yield consistent input

- Constraints from other data generate spectroscopic factors $S(d_{3/2})=0.71$ in ^{40}Ca for ground state transition
- Experimental $s_{1/2}$ strength distribution: 2.5 MeV $S(s_{1/2})=0.60$

- NIKHEF 0.65±0.06 and 0.51±0.05, respectively (local)

Conclusions

- Accurate calculations of spectroscopic factors requires LRC and SRC
- LRC somewhat more important than SRC
- SRC demonstrated in (e,e'p) at Jefferson lab
- Difficult to describe low-energy fragmentation with ab initio approaches e.g in ^{16}O and beyond
- Ab initio calculation of DWIA of (e,e'p) not in the cards right now
- DOM excellent interface between theory and experiment
- DOM ingredients provide consistent and excellent DWIA description of (e,e'p) cross section at T_p = 100 MeV
- Slightly larger spectroscopic factors compared to Nikhef —> due to nonlocal potentials
- Soft asymmetry dependence
- Can be extended to (p,pN) and (d,p) or (p,d)

Theory <—> Experiment ~Stability

- **SRC**: proper treatment of core of NN interaction
- Consequences—>
  - removal of valence sp strength (matter and finite nuclei ~ 0.1 near stability) from a lot of theoretical work
  - admixture of high-momentum components (below the bottom of mean-field potential) —> experimentally confirmed! (**not** near Fermi energy)
  - includes some tensor effects in depleting valence strength to high energy
- Jefferson Lab data
  - 10% or 20% for example in the case of ^{12}C SHOULD BE CLARIFIED
  - N-Z dependence is documented
- Ab initio asymmetric matter results unambiguous for AV18, CDBonn, and N3LO
- (e,e'p) data & DWIA using DOM ingredients well understood for ^{40}Ca —> not too much room for more than 10% effect of SRC