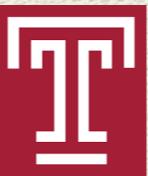


Light-cone PDFs from Lattice QCD

Martha Constantinou



Temple University



CFNS Short-range Nuclear Correlations at an Electron-Ion Collider
Brookhaven National Laboratory
September 5, 2018

OUTLINE

A. Introduction to quasi-PDFs

B. quasi-PDFs in Lattice QCD

C. Unpolarized & Polarized PDFs

- 1. Lattice Matrix Elements**
- 2. Renormalization**
- 3. Towards light-cone PDFs**

D. Transversity PDFs

E. Discussion

In Collaboration with

- ▶ C. Alexandrou ^{1,2}
- ▶ K. Cichy ³
- ▶ K. Hadjyiannakou ²
- ▶ K. Jansen ⁴
- ▶ H. Panagopoulos ¹
- ▶ A. Scapellato ¹
- ▶ F. Steffens ⁵

- 1. University of Cyprus
- 2. The Cyprus Institute
- 3. Adam Mickiewicz University
- 4. DESY, Zeuthen
- 5. Bonn University

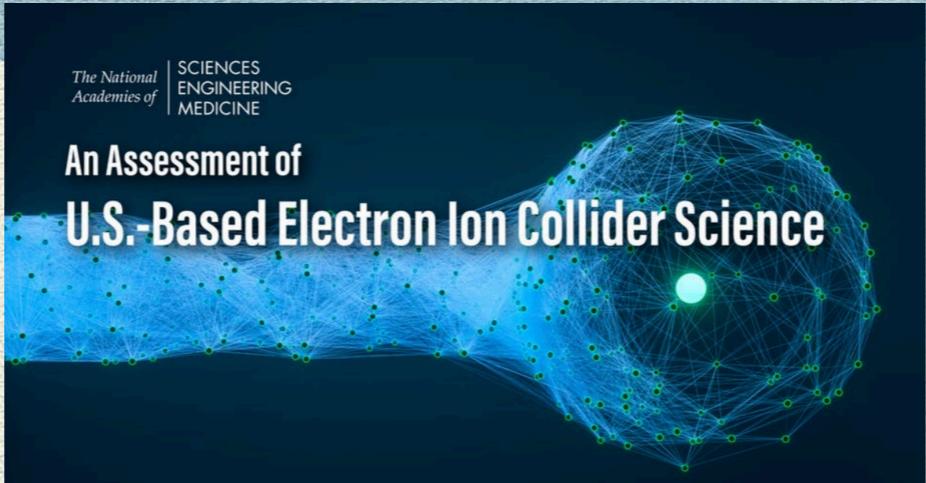
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Based on:

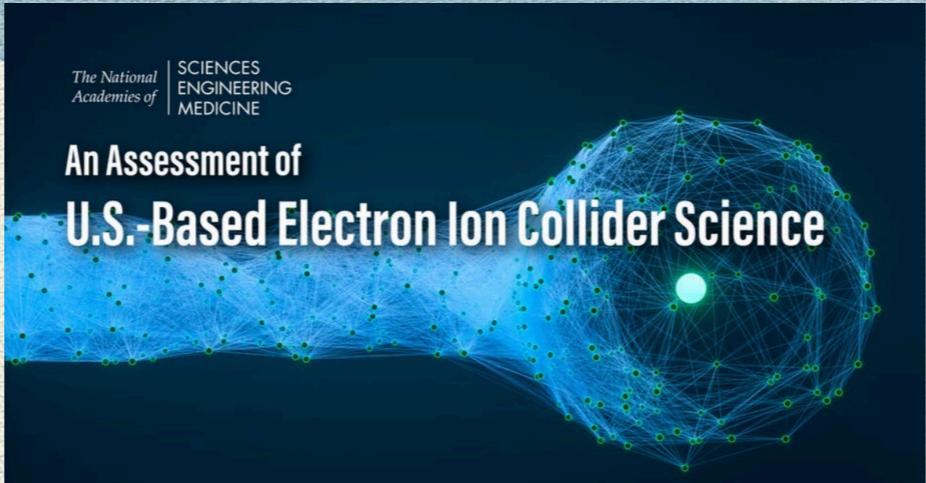
- M. Constantinou, H. Panagopoulos,
[Phys. Rev. D 96 \(2017\) 054506](#), [arXiv:1705.11193]
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[Nucl. Phys. B 923 \(2017\) 394 \(Frontiers Article\)](#), [arXiv:1706.00265]
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[Phys. Rev. Lett, in Press](#), [arXiv:1803.02685]
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The EIC will address crucial questions in hadron structure:

- * How does the mass of the nucleon arise?
- * How does the spin of the nucleon arise?
... and measure PDFs in high accuracy

These questions can be addressed in Lattice QCD



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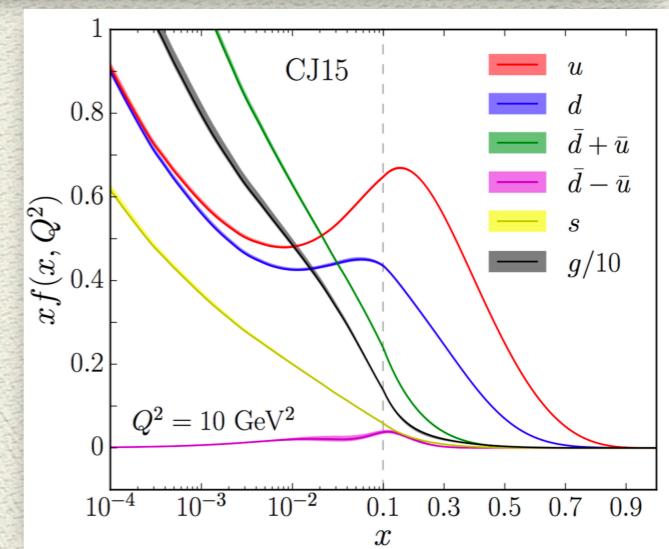
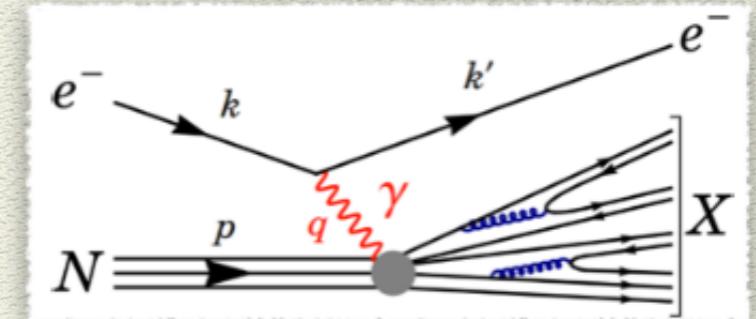
EIC will come at a time when lattice QCD:

- * will be well into the exa-scale computing era
- * will reliably compute sea quark and gluon contributions
- * is expected to reliably compute x-dependence quantities

Lattice QCD already showing promising results

Parton Distribution Functions

- Probe of hadron structure (1-D)
- Ideal for description of non-perturbative nature
- Probability densities for a given parton to carry a fraction- x of the hadron momentum
- Well-studied both experimentally and theoretically
- Necessary for analysis of DIS data
- Phenomenological input needed for data interpretation



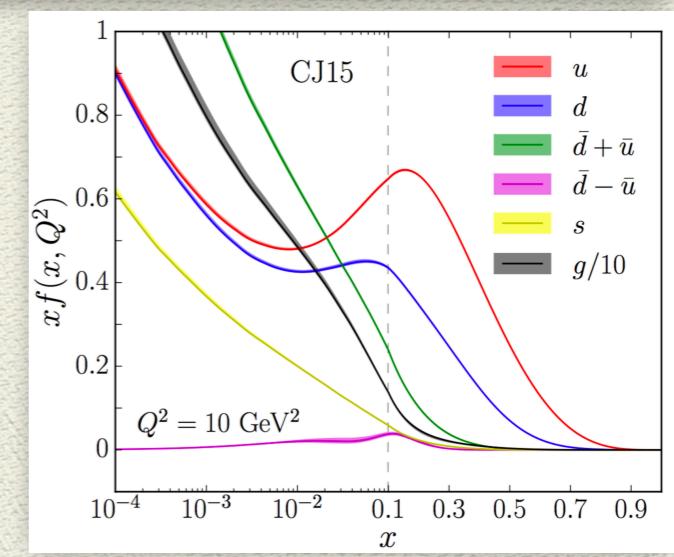
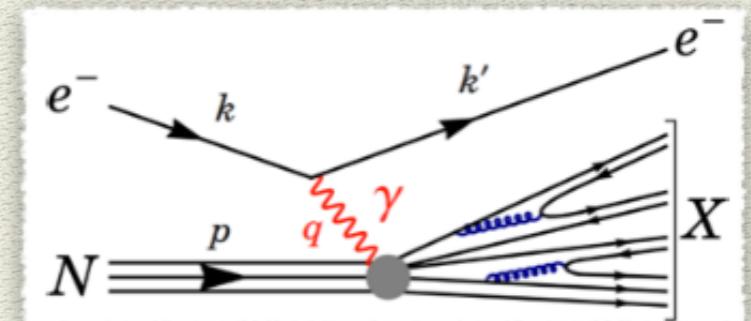
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Calculation from first principle imperative



A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- Lattice QCD is ideal ab initio formulation
- PDFs parameterized in terms of off-forward matrix elements of light-cone operators.

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Not accessible in Euclidean lattice

- On lattice: moments of PDFs (reconstructed via OPE)
- Reconstruction unfeasible:
 - ★ $n > 3$: operator mixing
 - ★ Statistical noise increases with high moments

Parton Distribution Functions

- Alternative approaches to access PDFs

- ★ Hadronic tensor [K.F. Liu, Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
- ★ Compton amplitude and OPE [A. Chambers et al. (QCDSF), [arXiv:1703.01153]]

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All methods have been investigated on the lattice

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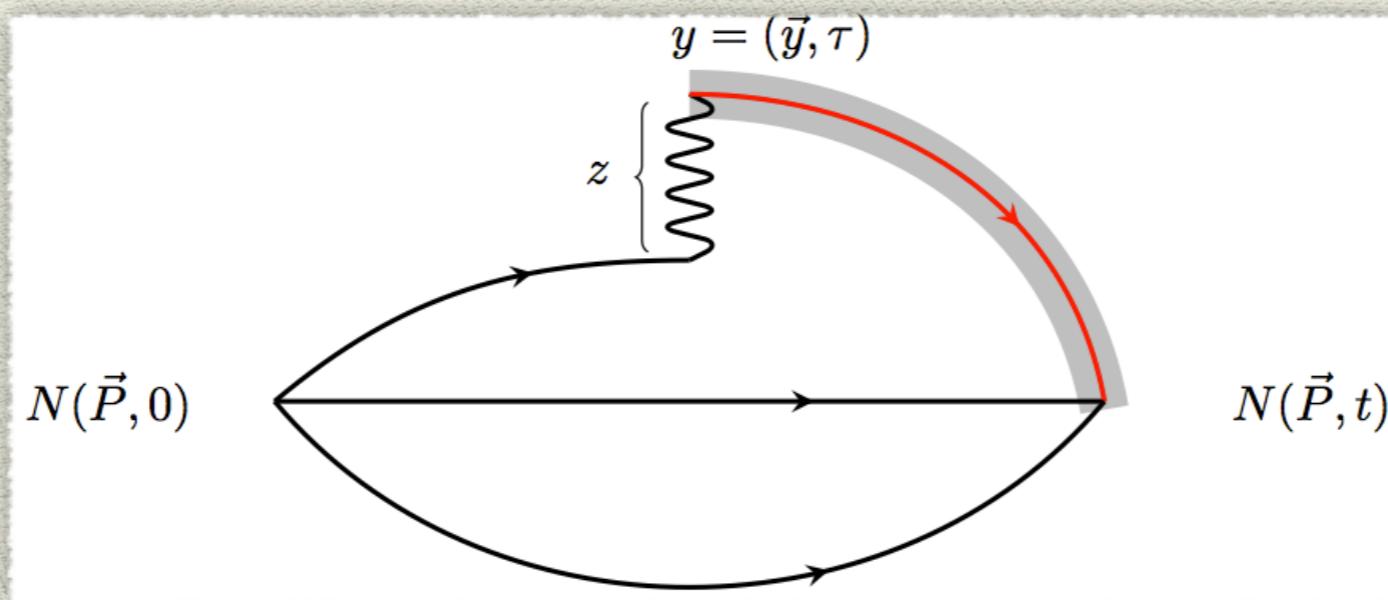
Access of PDFs on a Euclidean Lattice

quasi-PDFs

- * Based on matrix elements of spatial operators

$$\tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \langle N(P_3) | \bar{\Psi}(z) \gamma^z A(z, 0) \Psi(0) | N(P_3) \rangle_{\mu^2}$$

$A(z, 0)$: Wilson Line of length z



- * Hadron boosted with momentum in spatial direction

Reconstruction of light-cone PDFs

Contact with light-cone PDFs feasible:

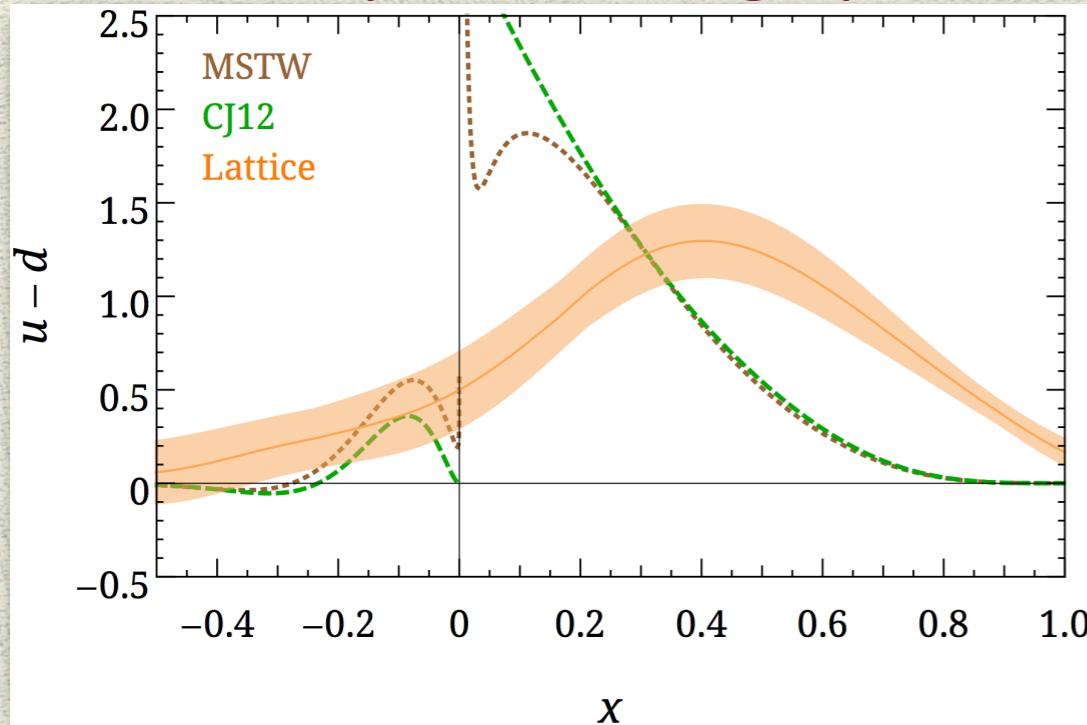
- * Difference reduced as P increases $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$
- * Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF

Reconstruction of light-cone PDFs

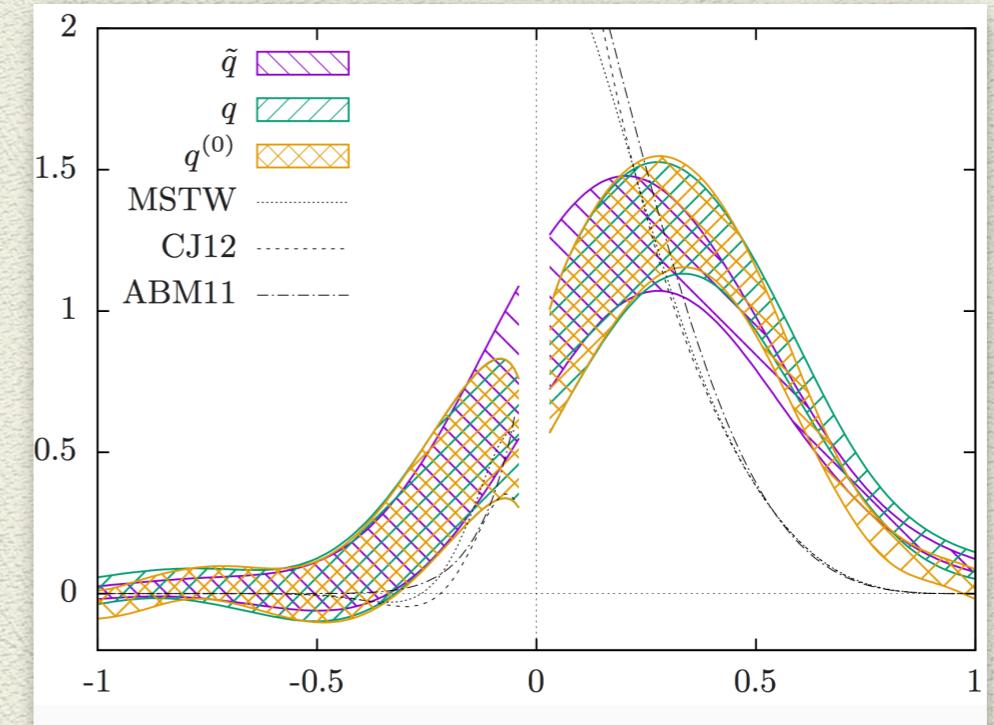
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First exploratory (nucleon) studies feasible:

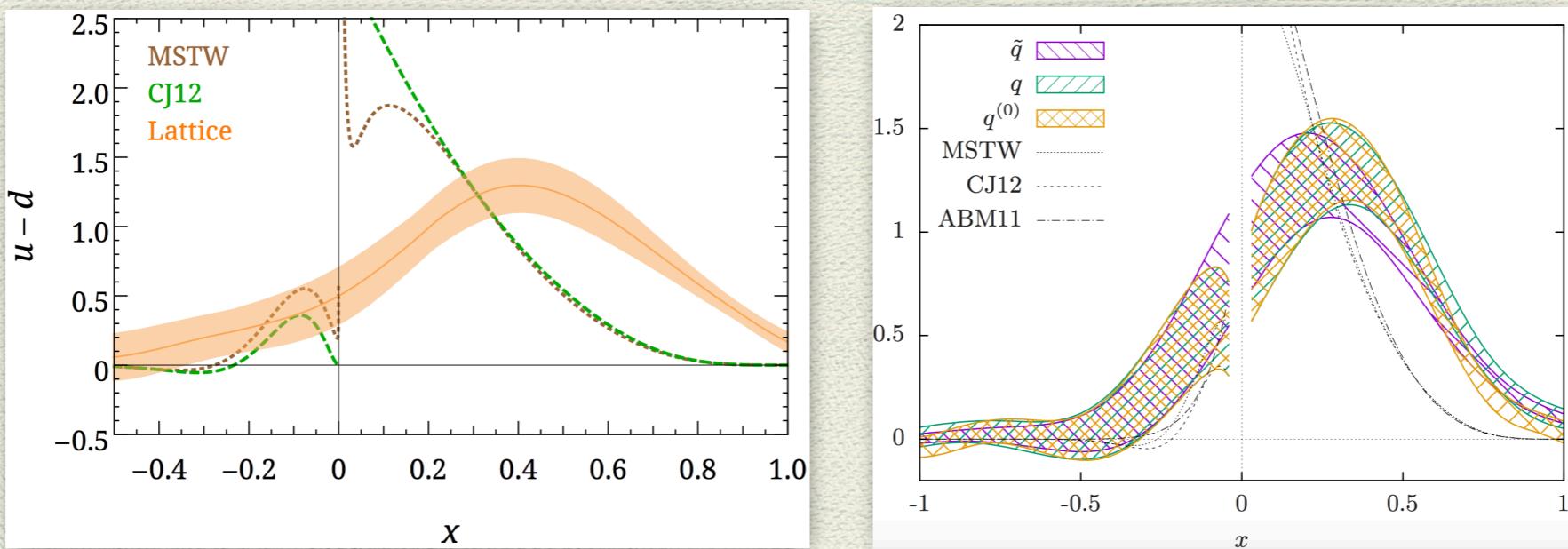


[H.W. Lin et al., Phys. Rev. D 91, 054510 (2015), arXiv:1402.1462]

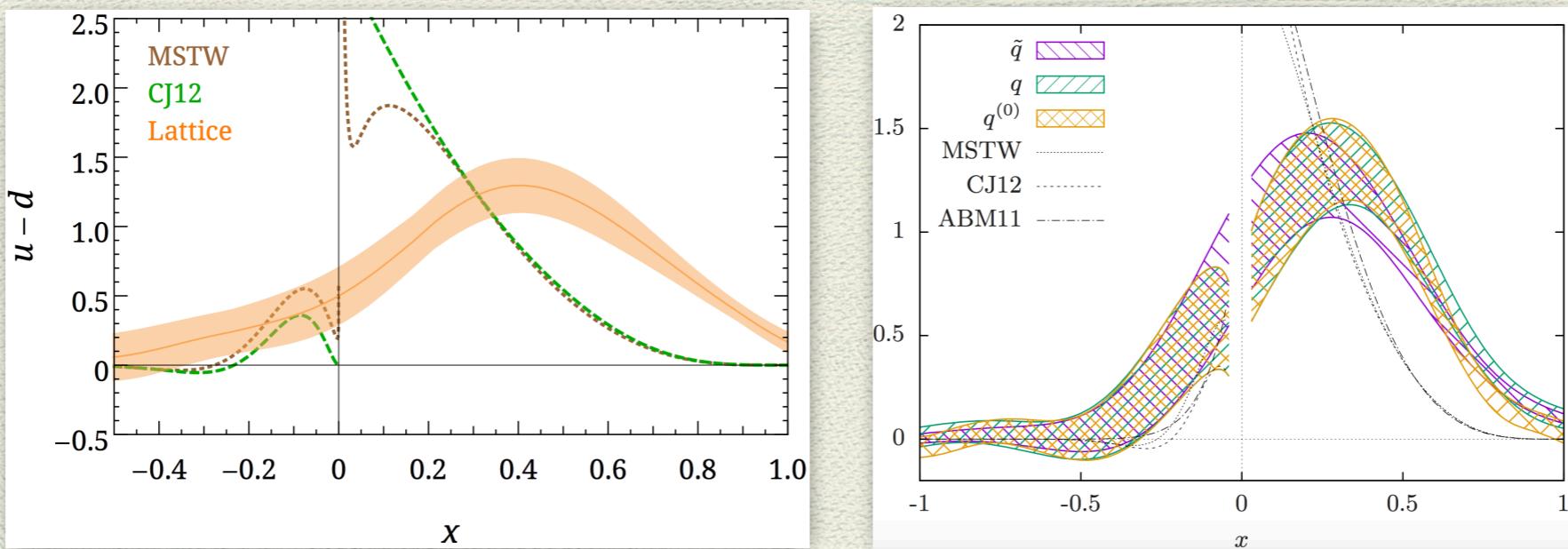


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Lattice studies of quasi-PDFs

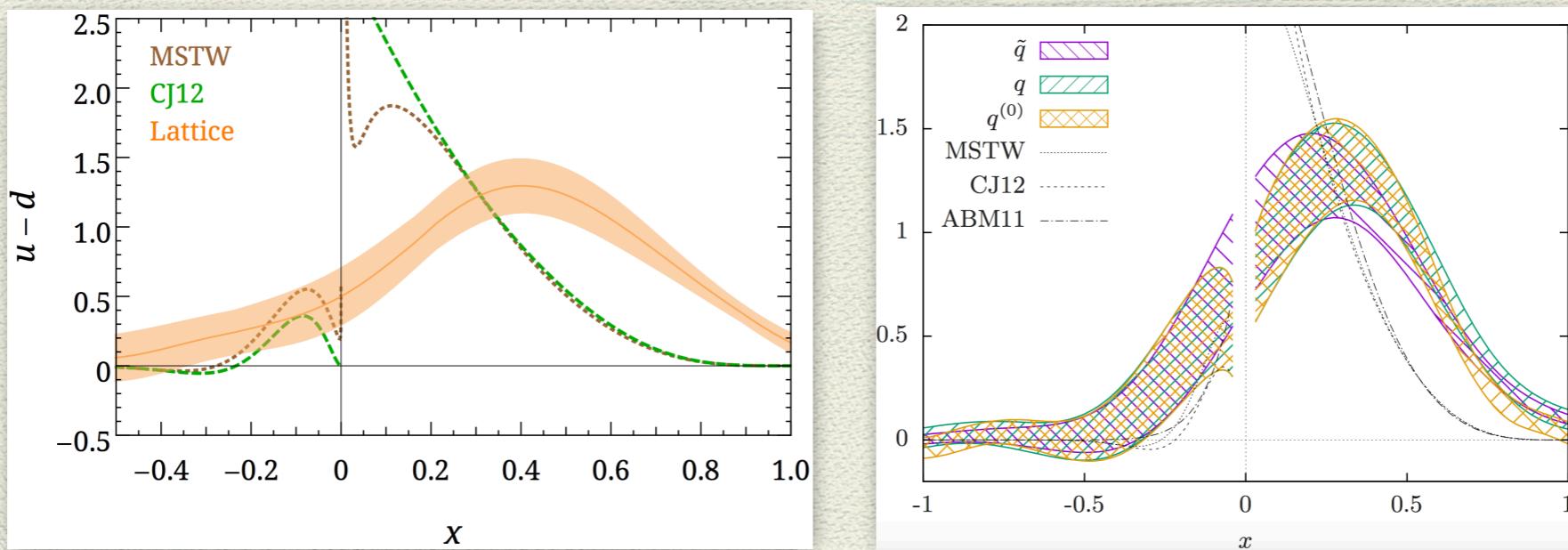


Lattice studies of quasi-PDFs



* Calculations significantly improved...

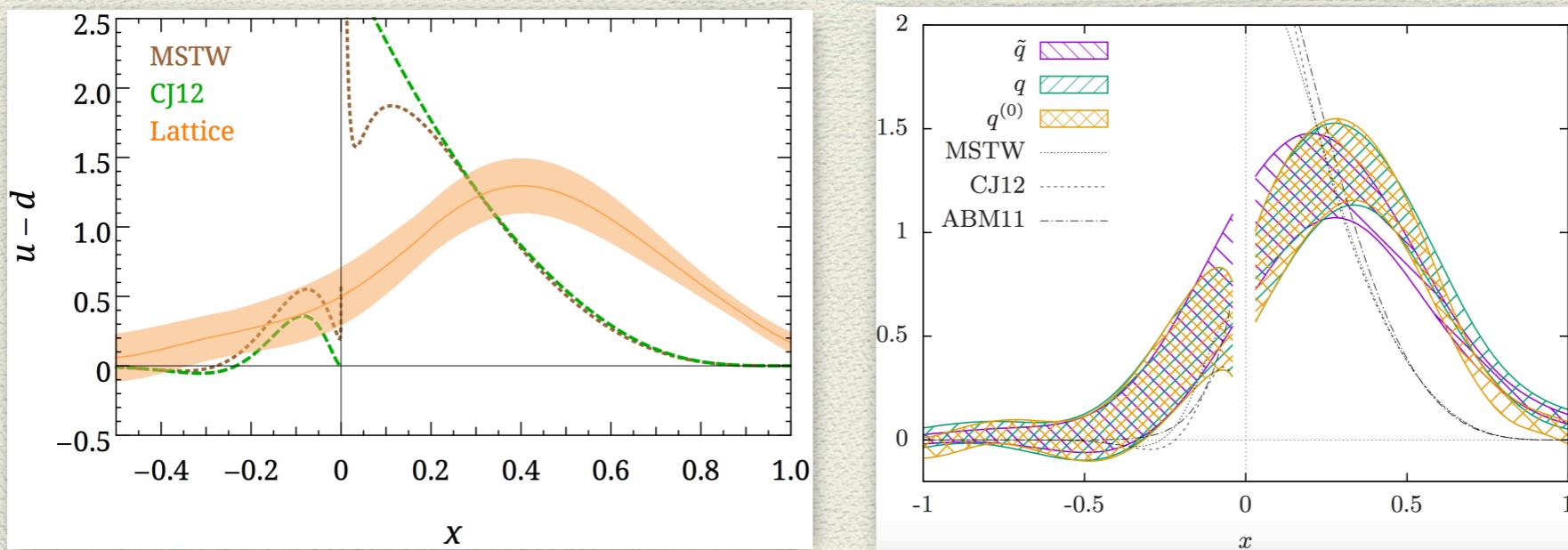
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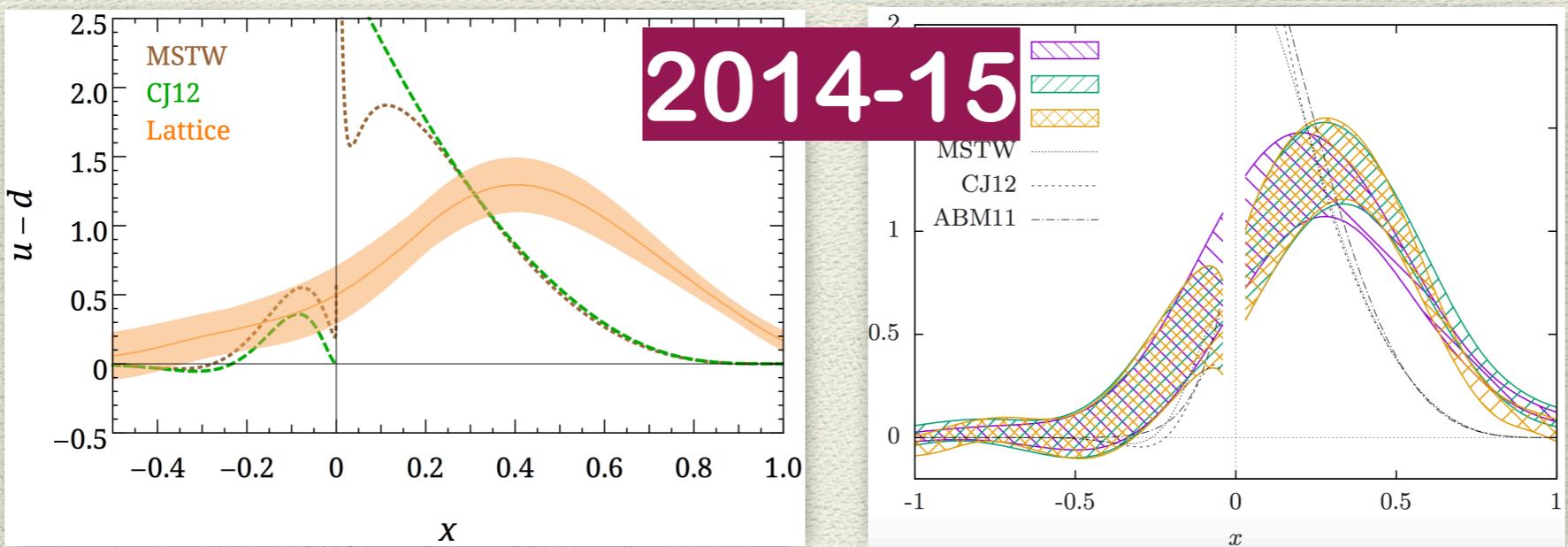
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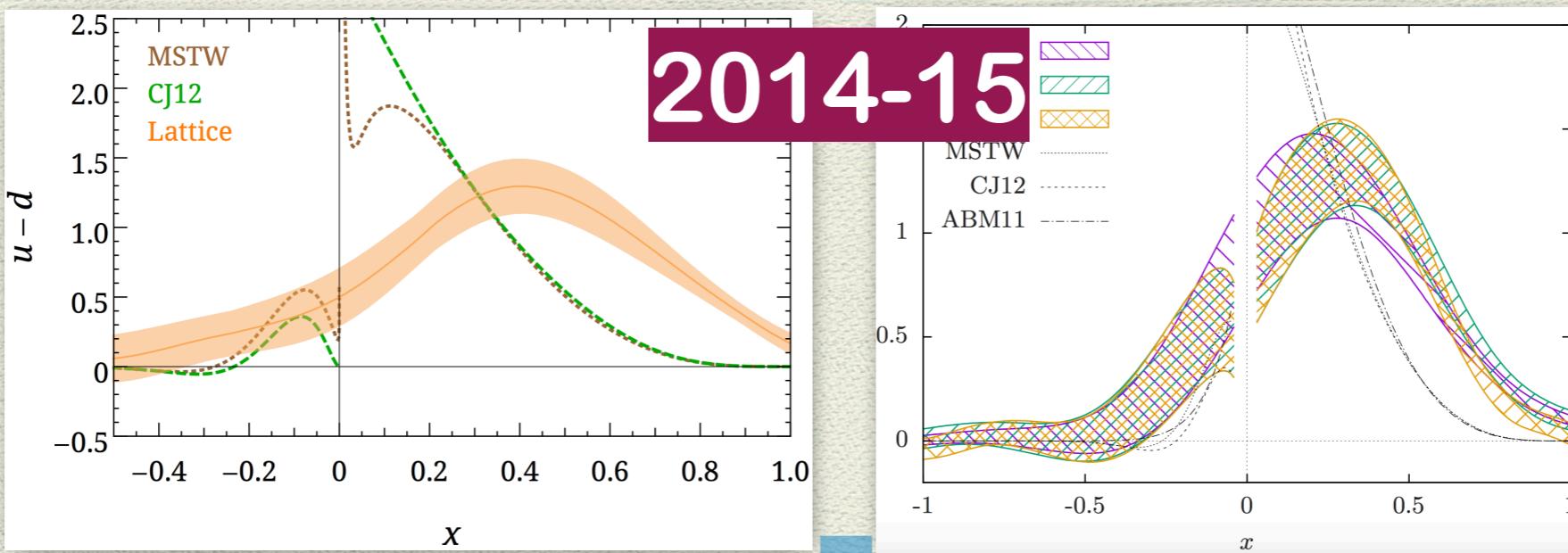
* ... and extended to other hadrons

Recent review: C. Monahan @ Lattice 2018

Lattice studies of quasi-PDFs



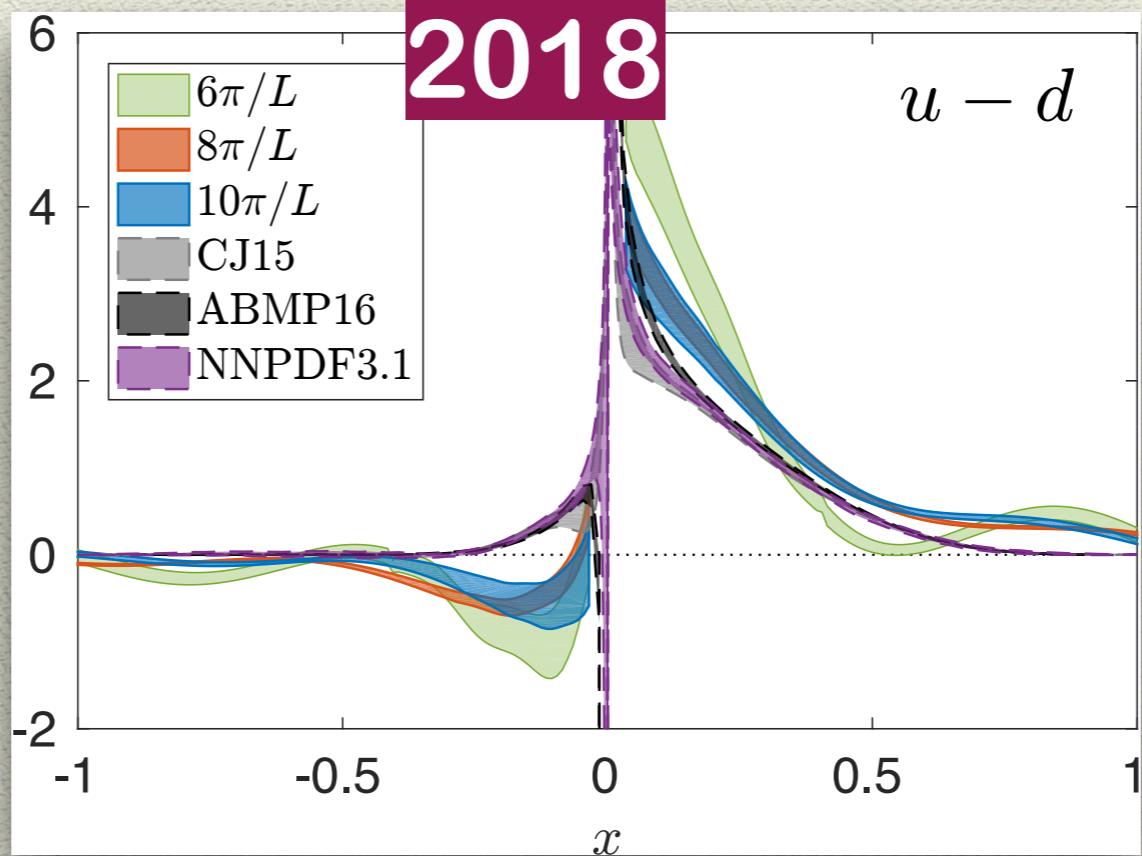
Lattice studies of quasi-PDFs



2014-15

2018

- * Simulations at physical point
- * Renormalization
- * Matching



Calculation of nucleon matrix elements

Multi-component task:

Calculation of nucleon matrix elements

Multi-component task:

* Calculation of 2pt- and 3pt-correlators

$$\langle N | N \rangle, \quad \langle N | \bar{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle$$

dependence on : length of Wilson line z and nucleon momentum P

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- * Target mass corrections

elimination of residual m_N / P_3 dependence

Parameters of Calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

- * Nf=2 twisted mass fermions & clover term
- * Ensemble parameters:

$\beta = 2.10$,	$c_{SW} = 1.57751$,	$a = 0.0938(3)(2) \text{ fm}$
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4) \text{ GeV}$
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Ins.	N_{conf}	N_{meas}	Ins.	N_{conf}	N_{meas}	Ins.	N_{conf}	N_{meas}
γ_3	100	9600	γ_3	425	38250	γ_3	811	72990
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* Excited states investigation:

$$T_{\text{sink}} = 8a, 9a, 10a, 12a \quad (T_{\text{sink}} = 0.75, 0.84, 0.94, 1.13 \text{ fm})$$

Systematic Uncertainties

Laborious effort to eliminate uncertainties

- * Cut-off Effects due to finite lattice spacing
- * Finite Volume Effects
- * Contamination from other hadron states
- * Chiral extrapolation for unphysical pion mass
- * Renormalization and mixing

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Discussed in this work

Challenges of calculation

Noise-to-signal ratio increases with:

- Hadron momentum boost
- Simulations at the physical point
- Source-sink separation

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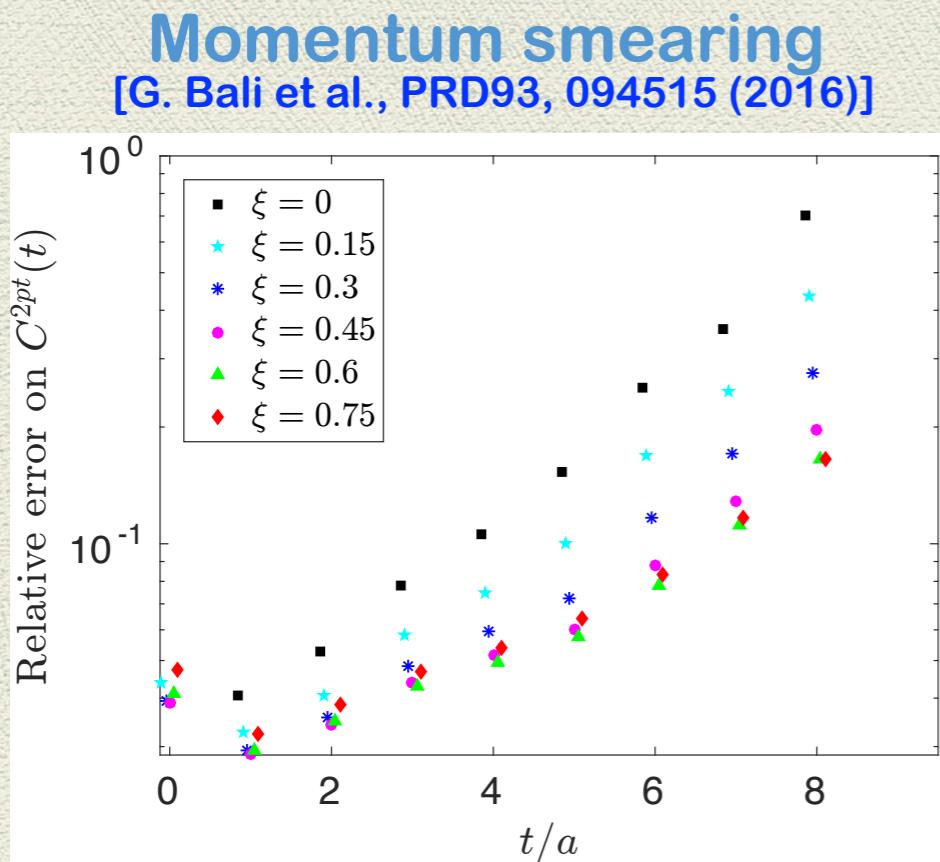
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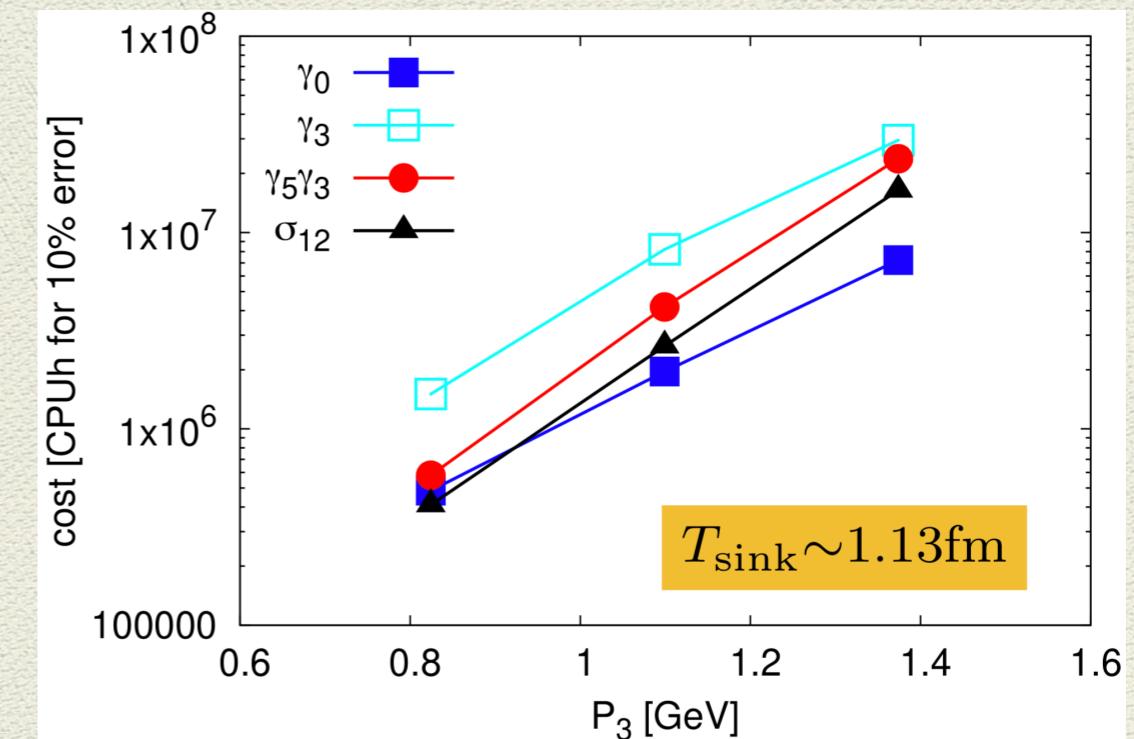
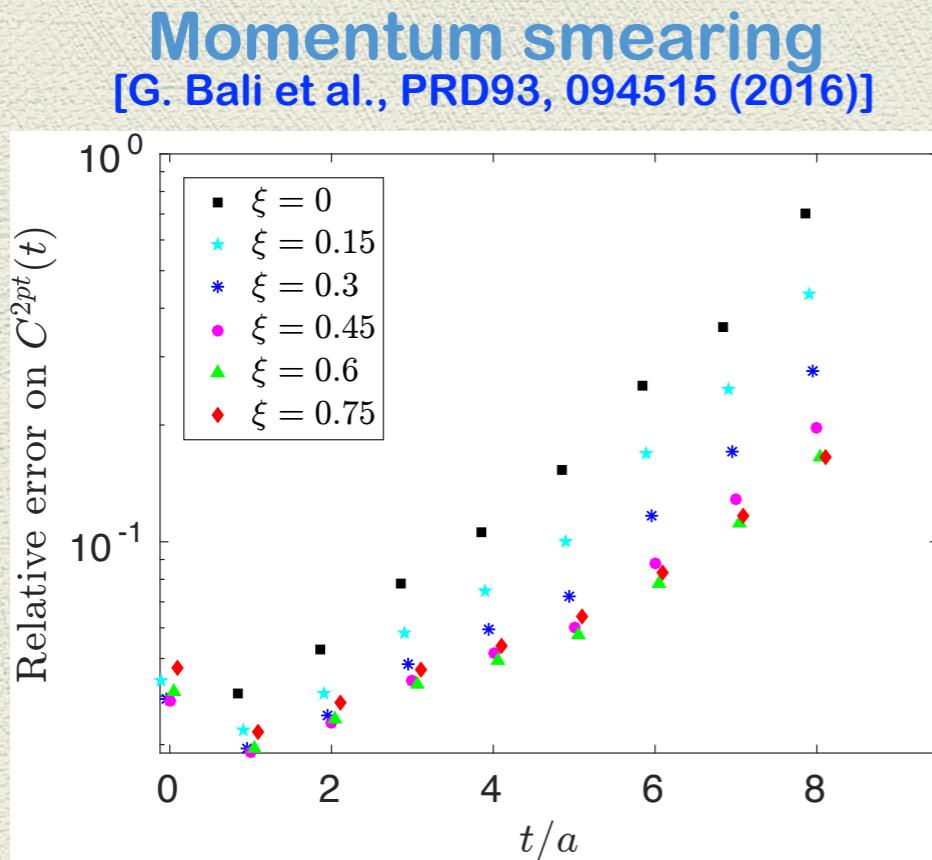
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- Momentum smearing helps reach higher momenta
- But limitations in max momentum due to comput. cost

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Bare matrix elements

Unpolarized:

- * Initial studies used $\gamma\mu$ in same direction with Wilson line
- * Mixing with higher twist revealed perturbatively

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Bare matrix elements

Unpolarized:

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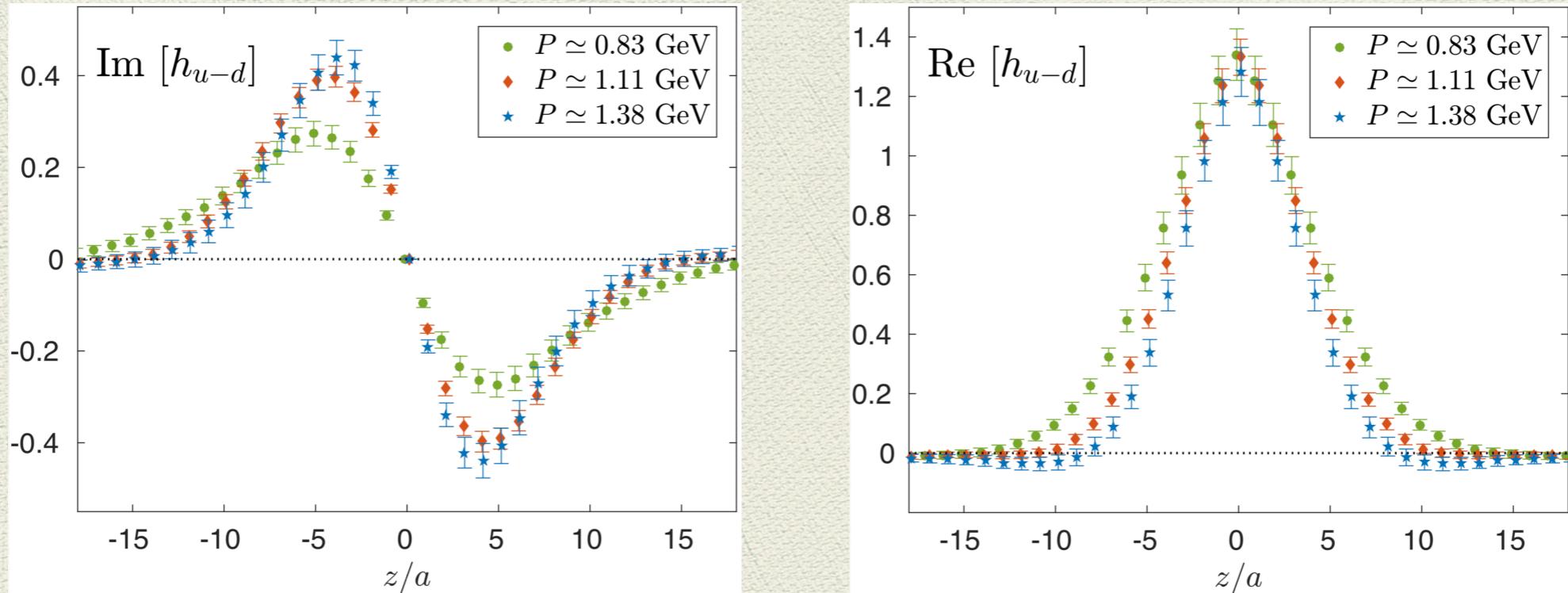
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- * Similar general features for polarized and transversity
- * Highest priority: deliver reliable results

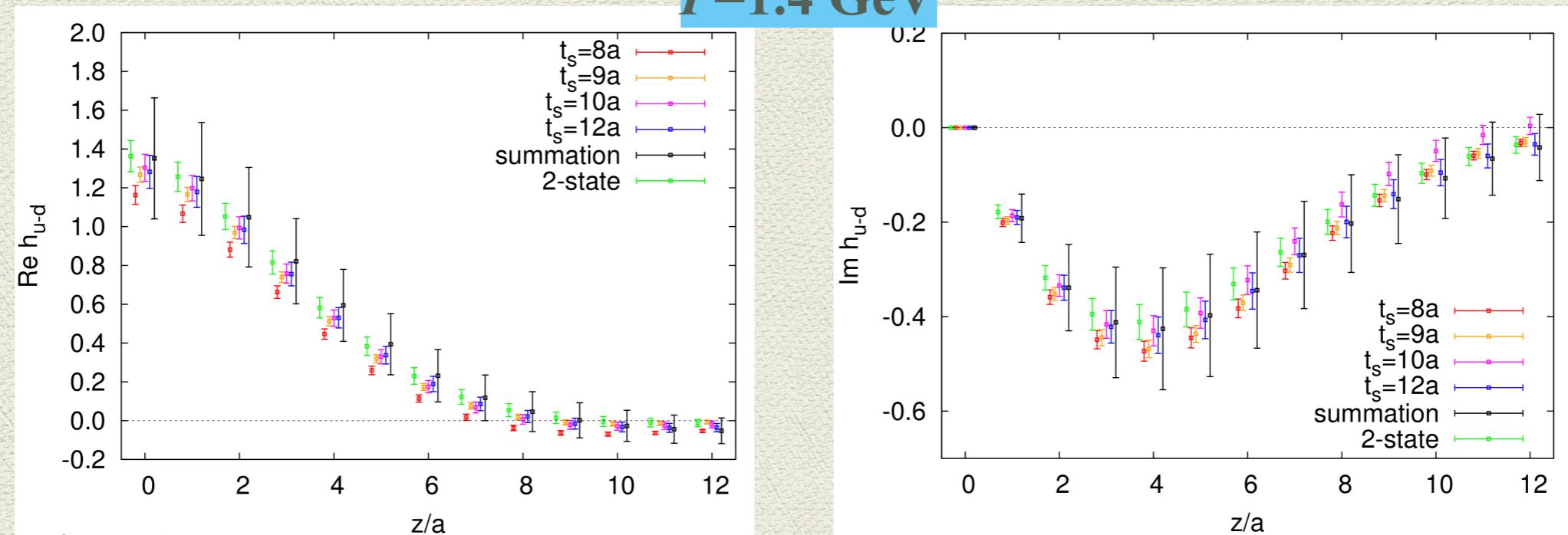
Excited states contamination

Analyses techniques:

* Single-state fit,

Two-state fit,

Summation method



Conclusions:

- * $T_{\text{sink}}=8a$ heavily contaminated by excited states
- * $T_{\text{sink}}=9a-10a$ not consistent within uncertainties

Excited states contamination

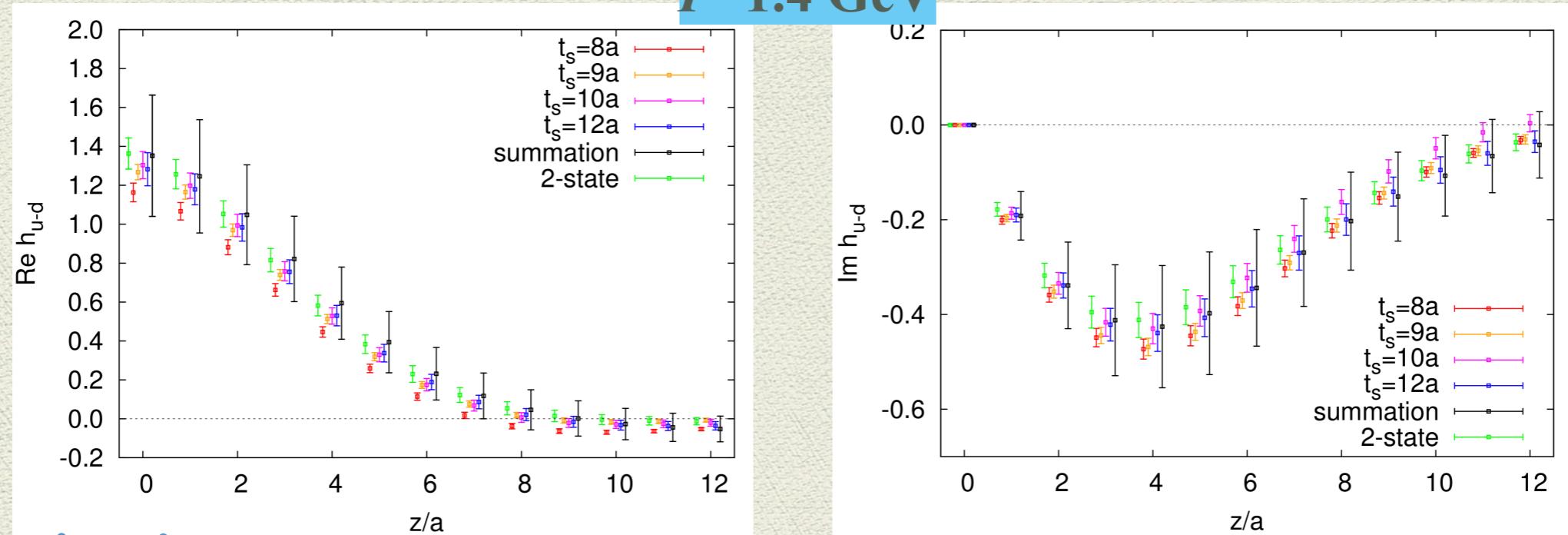
Analyses techniques:

* Single-state fit,

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$P=1.4 \text{ GeV}$

Summation method

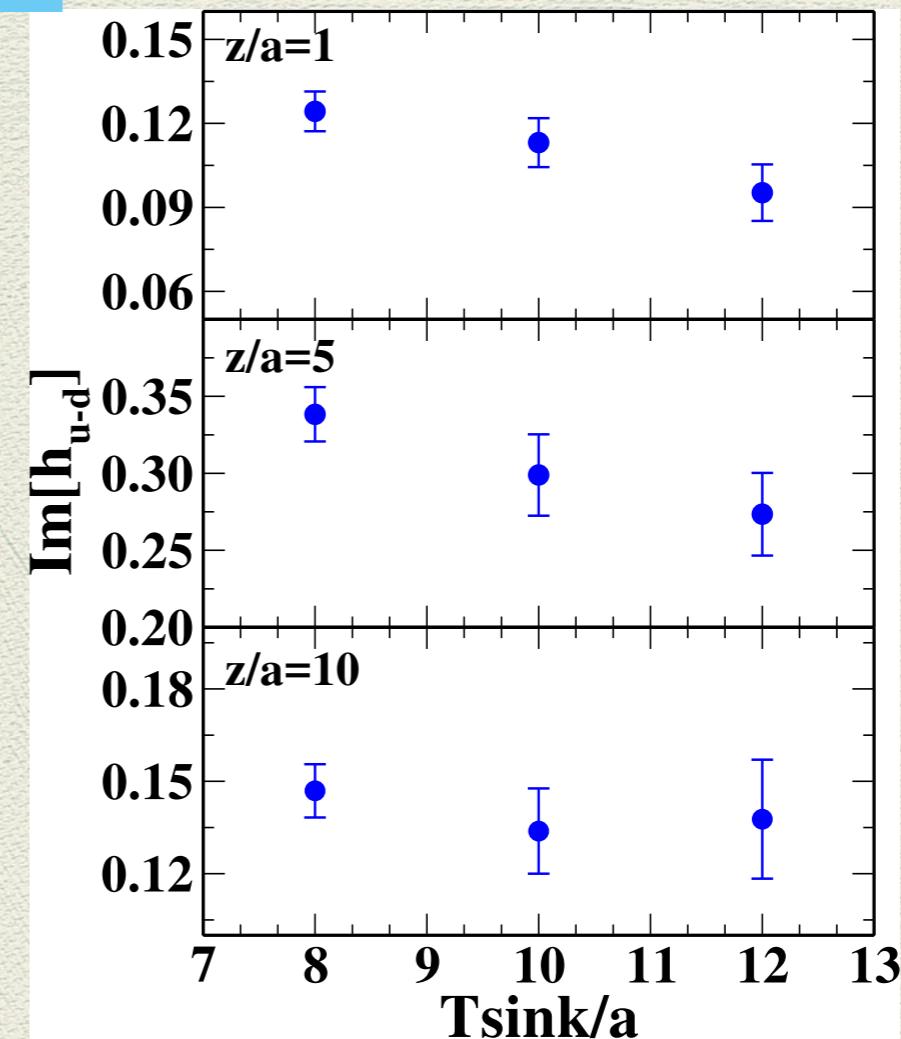
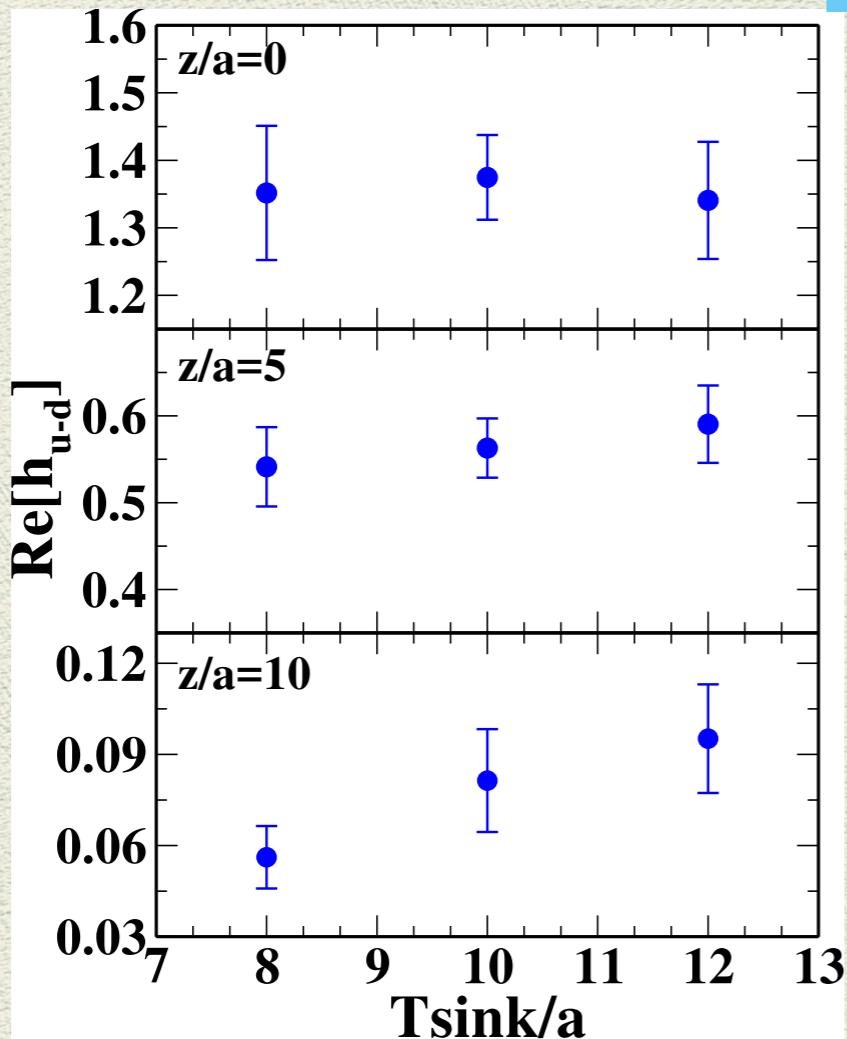


Conclusions:

- * $T_{\text{sink}}=8a$ heavily contaminated by excited states
- * $T_{\text{sink}}=9a-10a$ not consistent within uncertainties
- ! Crucial to have same error for reliable 2-state fit
- ! Excited states worsen as momentum P increases
- ! For momenta in this work, $T_{\text{sink}}=1\text{fm}$ is safe

Excited states contamination

$P=0.83 \text{ GeV}$



- * Non-predictable behavior (depends in z value)
- * Real and imaginary part affected differently

Conclusions:

- * Excited states uncontrolled for $T\text{sink} < 1\text{fm}$
- * Multi-sink analysis **demands** same accuracy for all data

OUTLINE

A. Introduction to quasi-PDFs

B. quasi-PDFs in Lattice QCD

C. Unpolarized & Polarized PDFs

1. Lattice Matrix Elements

2. Renormalization

3. Towards light-cone PDFs

D. Transversity PDFs

E. Discussion

Renormalization

Critical part of calculation:

- * Elimination of power and logarithmic divergences and dependence on regulator
- * Identification and elimination of mixing
- * Comparison with phenomenology becomes a real possibility

[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, arXiv:1705.11193]

[C.Alexandrou et al., Nucl. Phys. B 923 (2017) 394 (Frontiers Article), arXiv:1706.00265]

- * Proposed scheme now used in other studies

Renormalization scheme:

- * RI'-type, employed non-perturbatively
- * Applicable for cases of mixing
- * Pert. theory used for conversion to MSbar scheme

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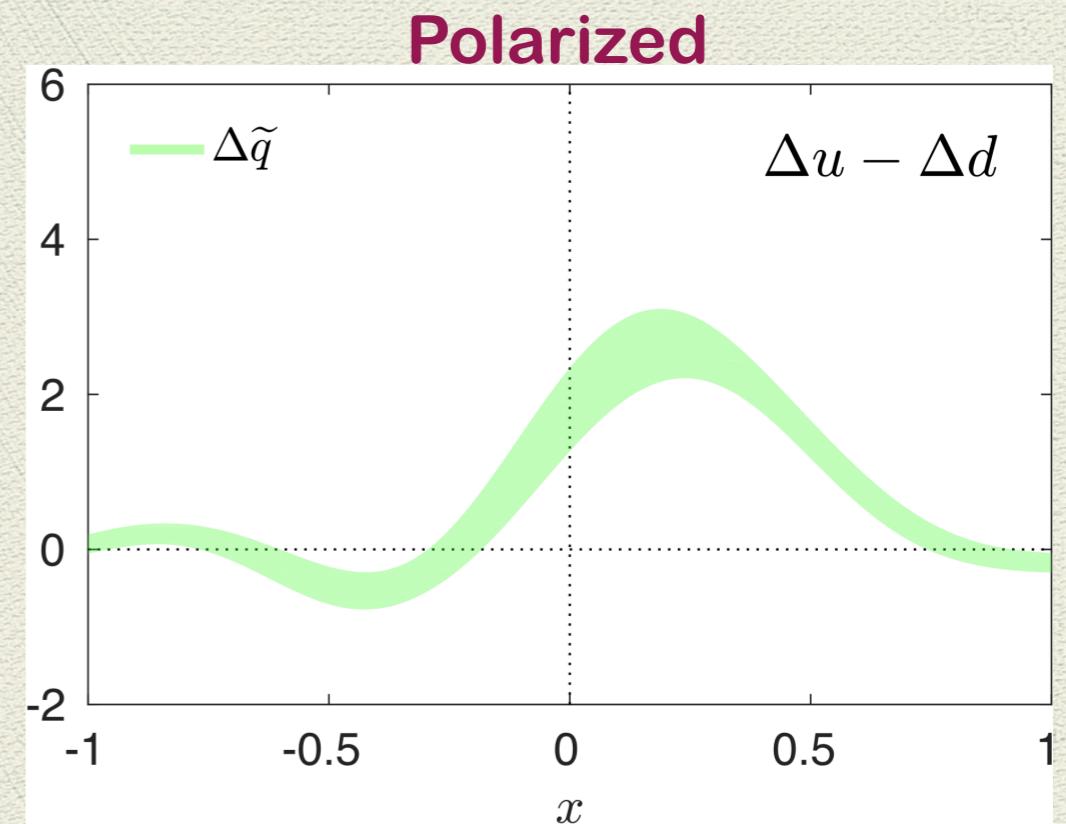
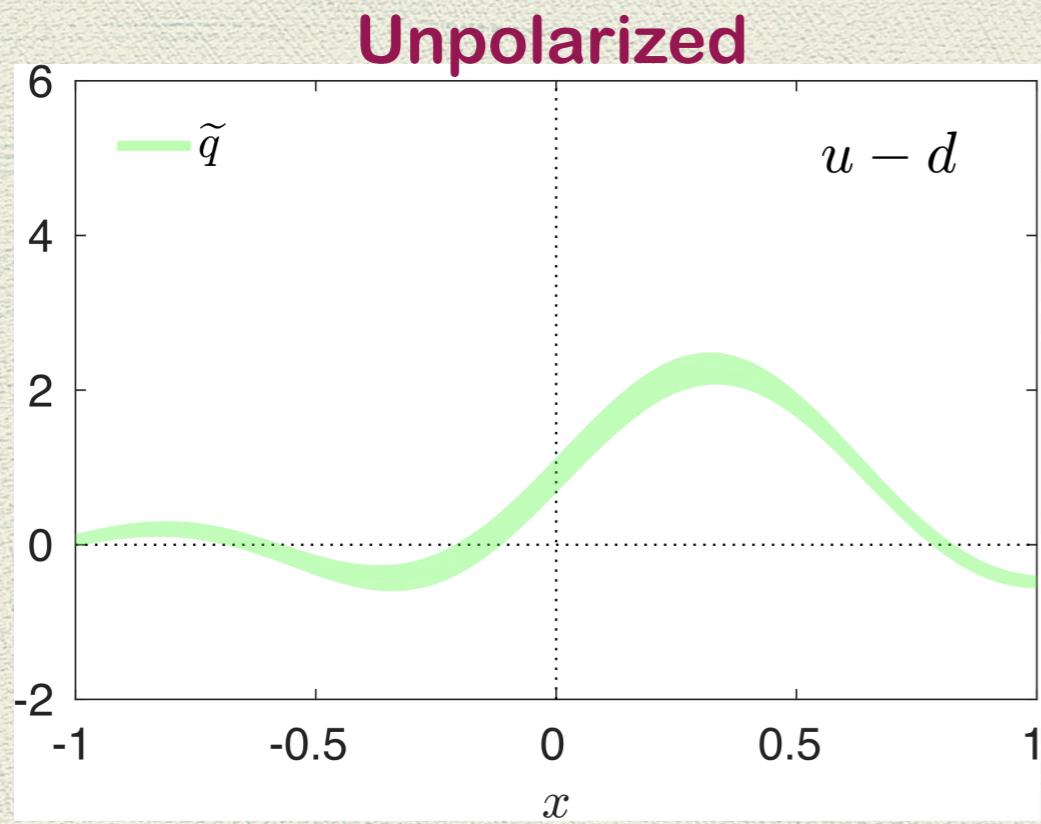
Towards light-cone PDFs

Evolution of lattice data ($P=1.4\text{GeV}$):

Towards light-cone PDFs

Evolution of lattice data ($P=1.4\text{GeV}$):

- * Fourier Transform of renormalized matrix elements

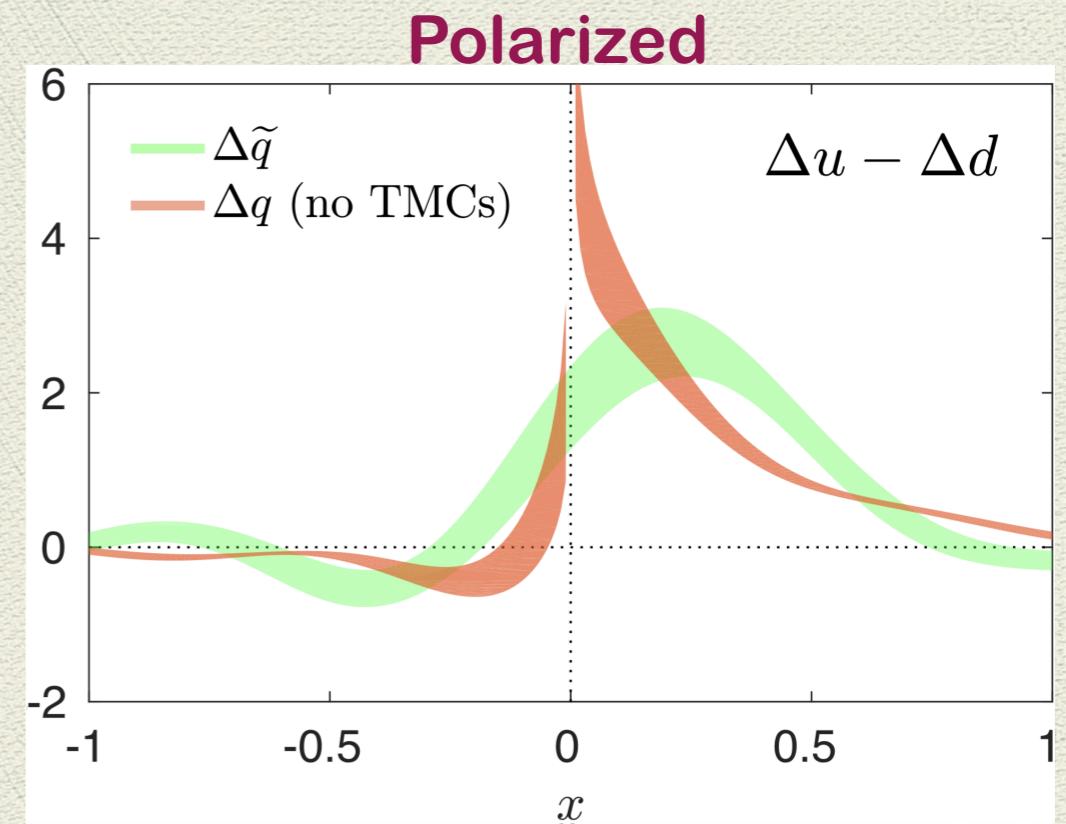
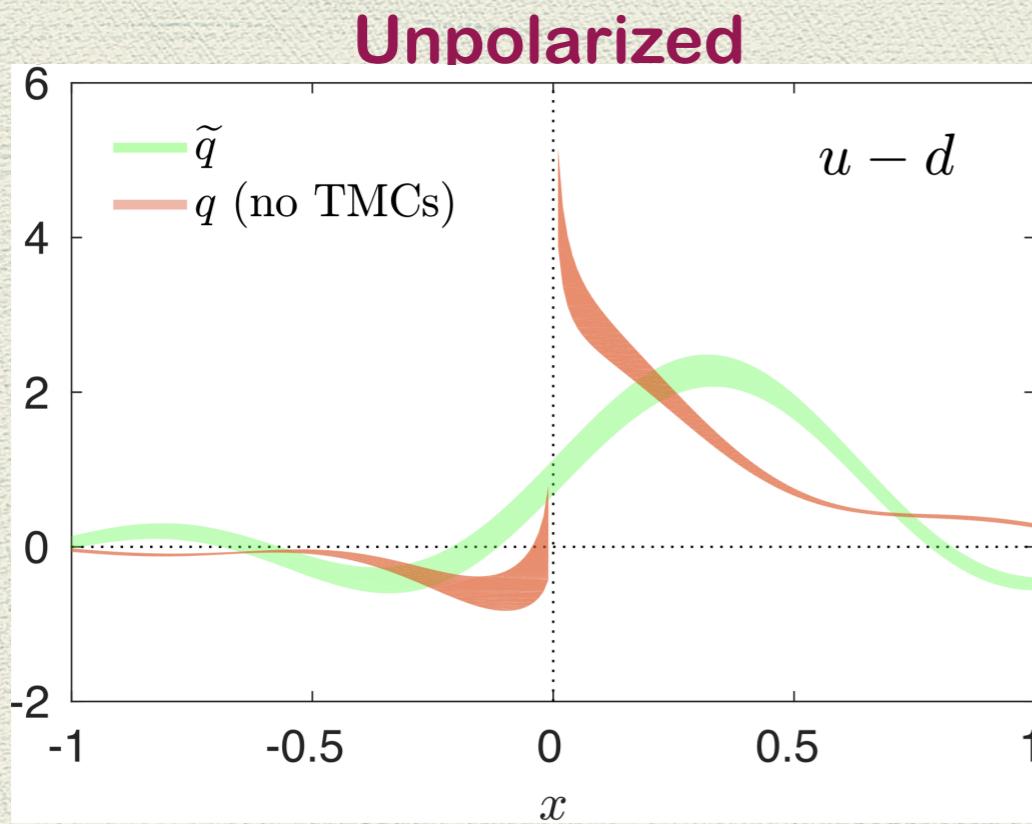


Towards light-cone PDFs

Evolution of lattice data ($P=1.4\text{GeV}$):

- * Fourier Transform of renormarmalized matrix elements
- * Matching of quasi-PDFs (LaMET)

[C. Alexandrou et al., (PRL), arXiv:1803.02685]



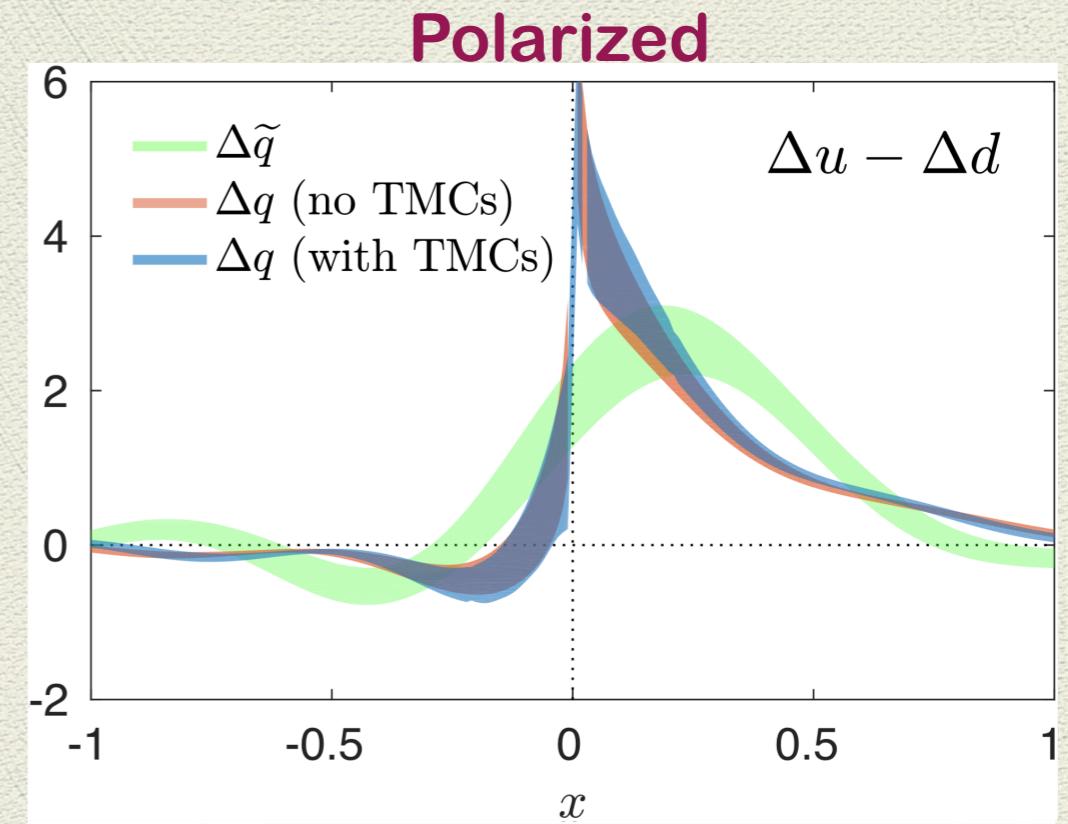
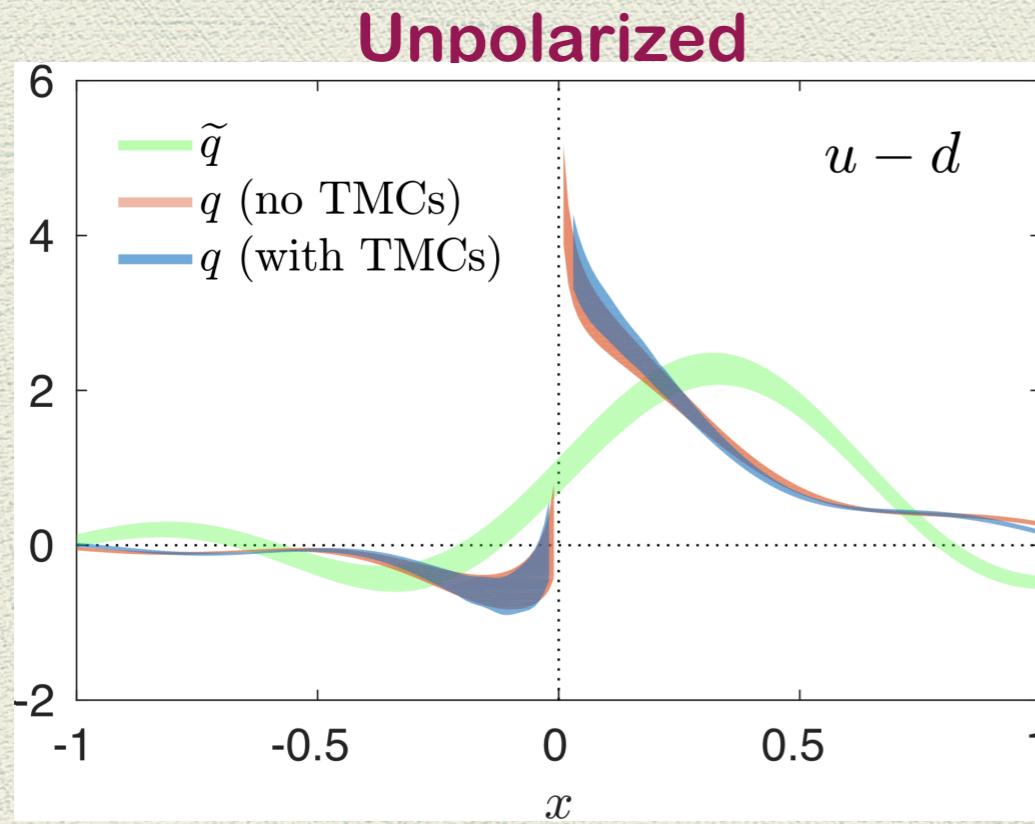
Towards light-cone PDFs

Evolution of lattice data ($P=1.4\text{GeV}$):

- * Fourier Transform of renormalized matrix elements
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- * Target Mass Corrections (m_N/P : finite)

[C. Alexandrou et al., (PRL), arXiv:1803.02685]

[J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]



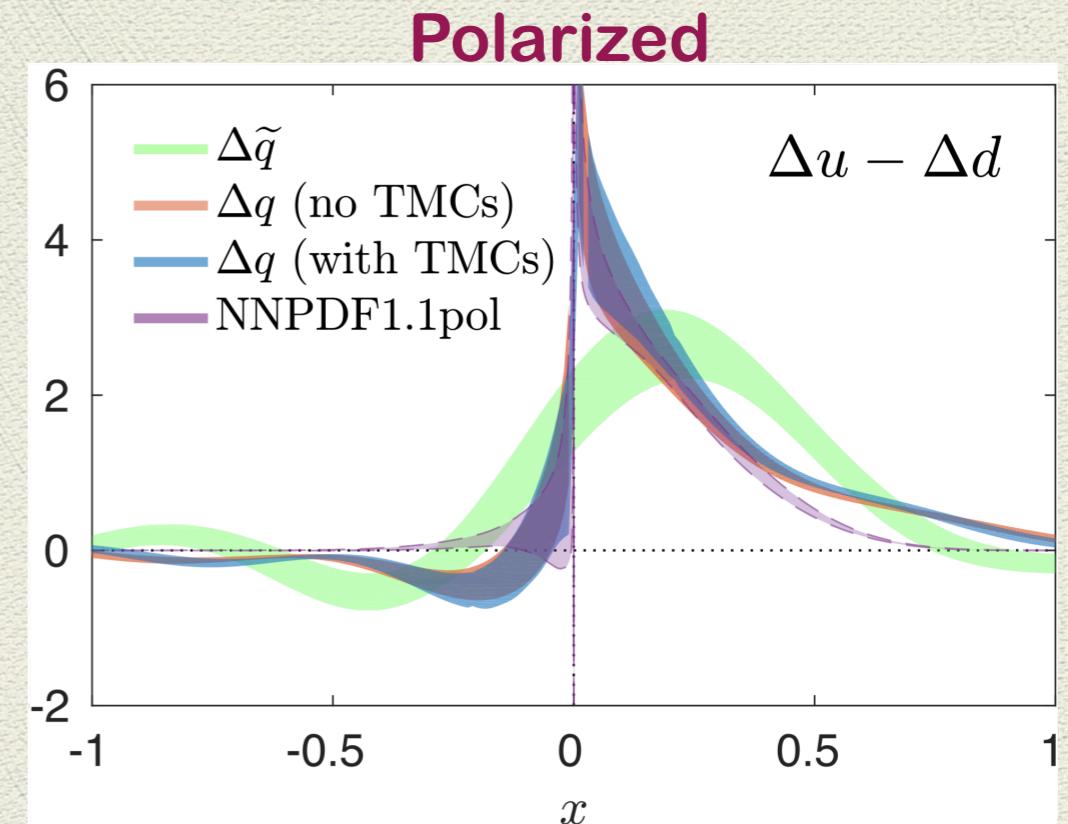
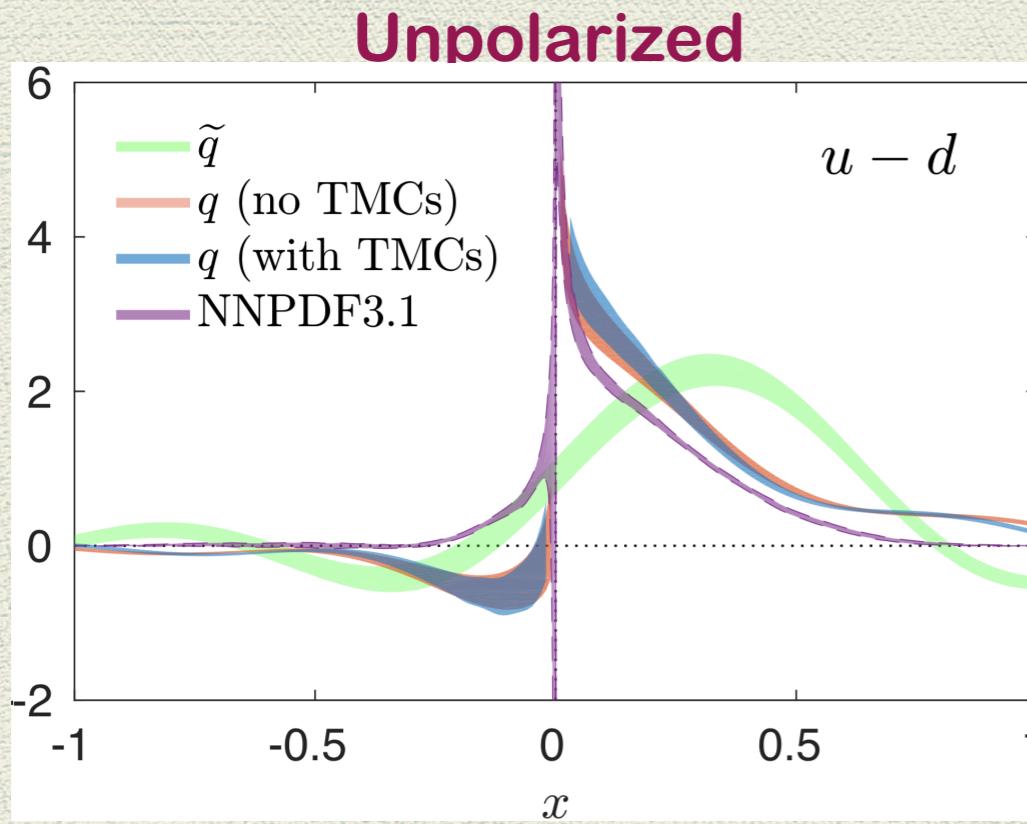
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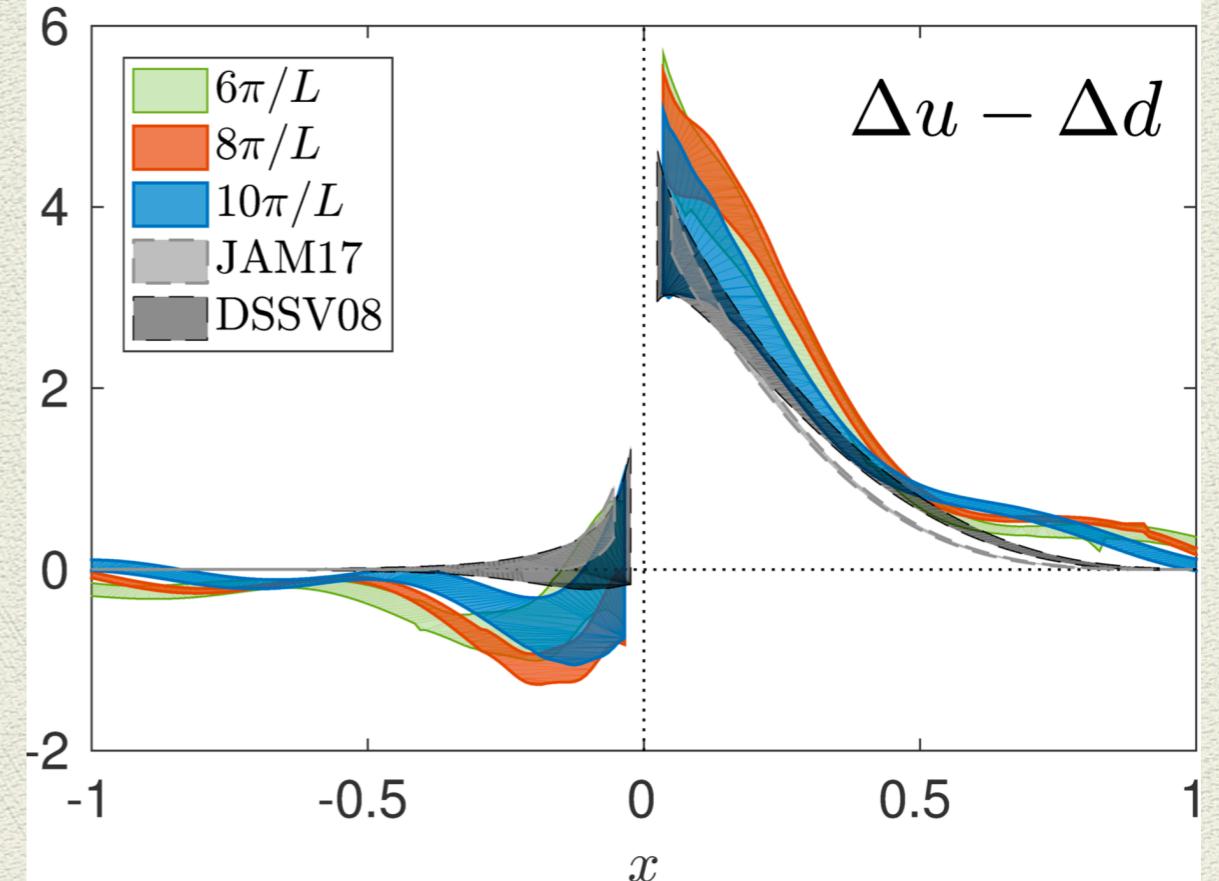
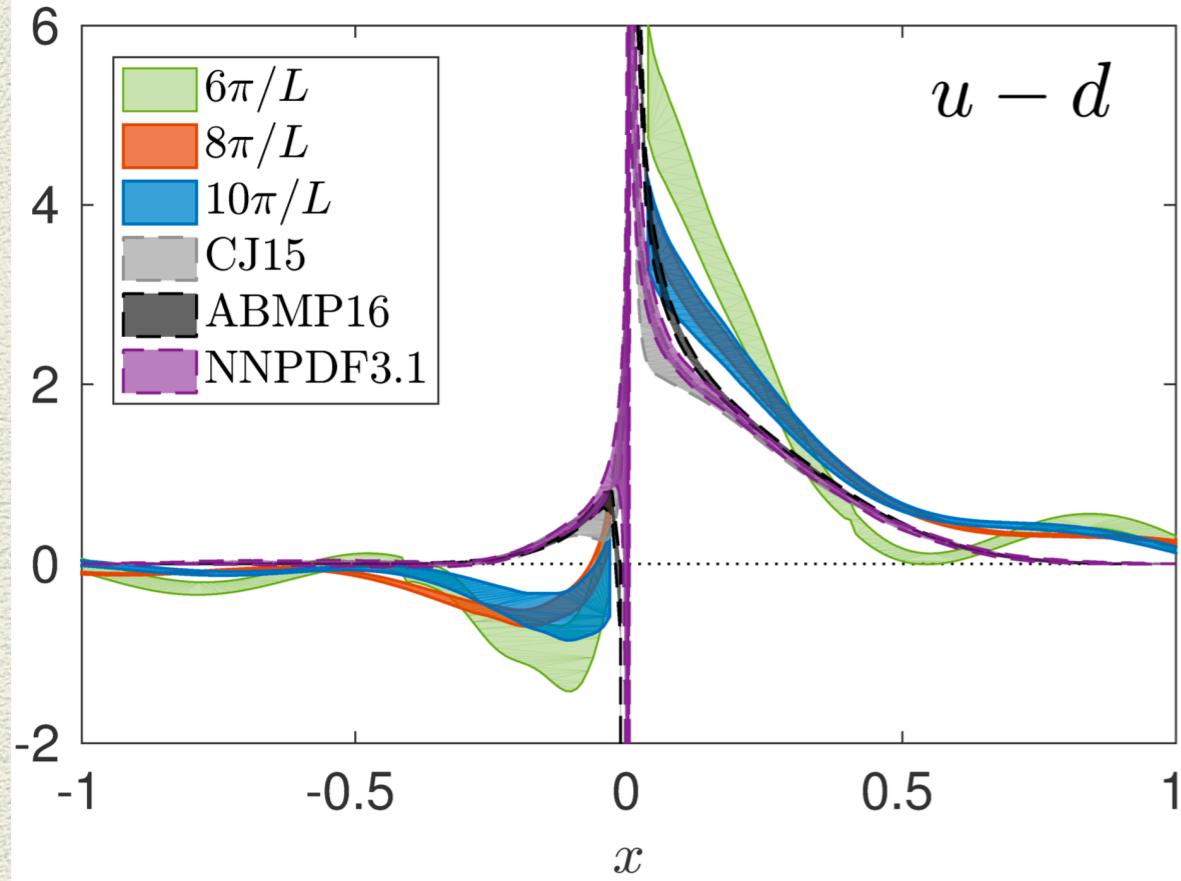
[J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]



- * Lattice PDFs similar behavior as phenomenological fits

Towards light-cone PDFs

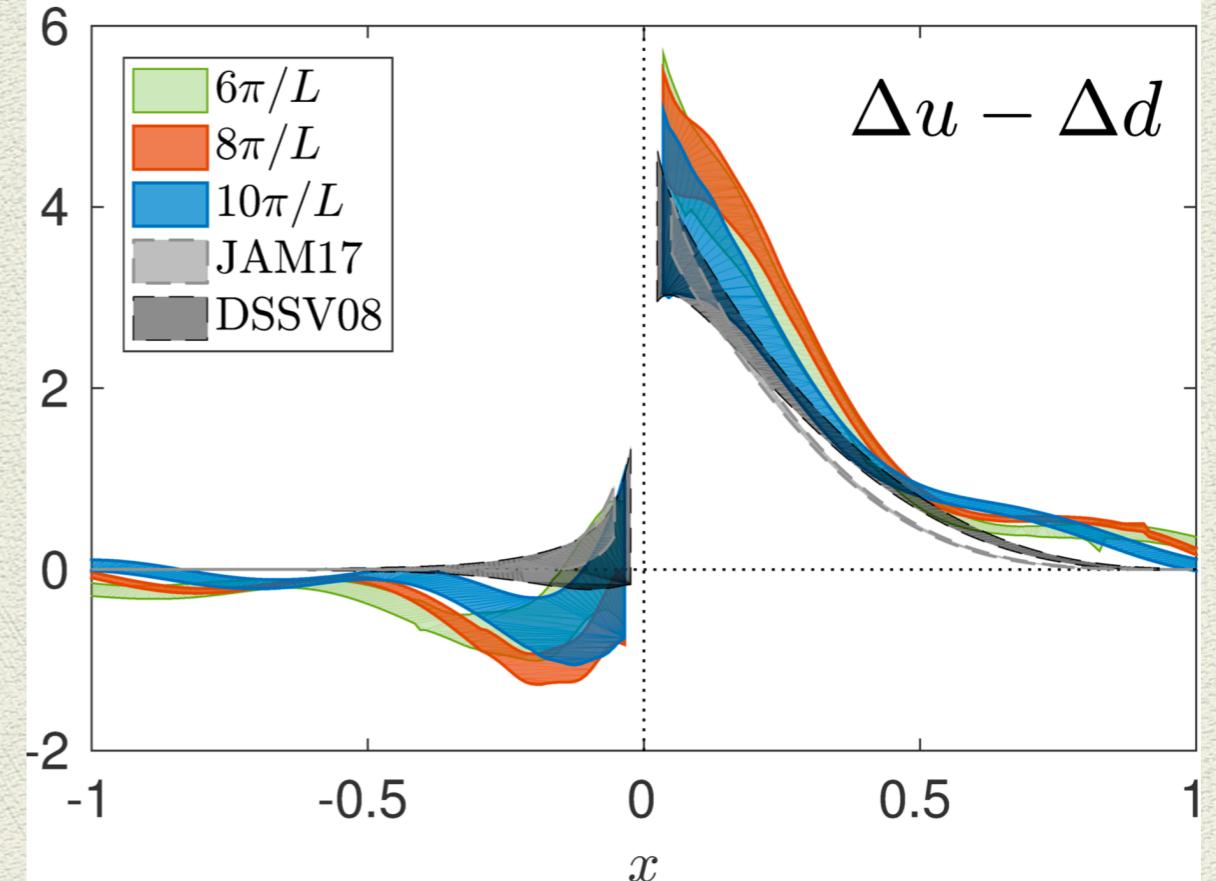
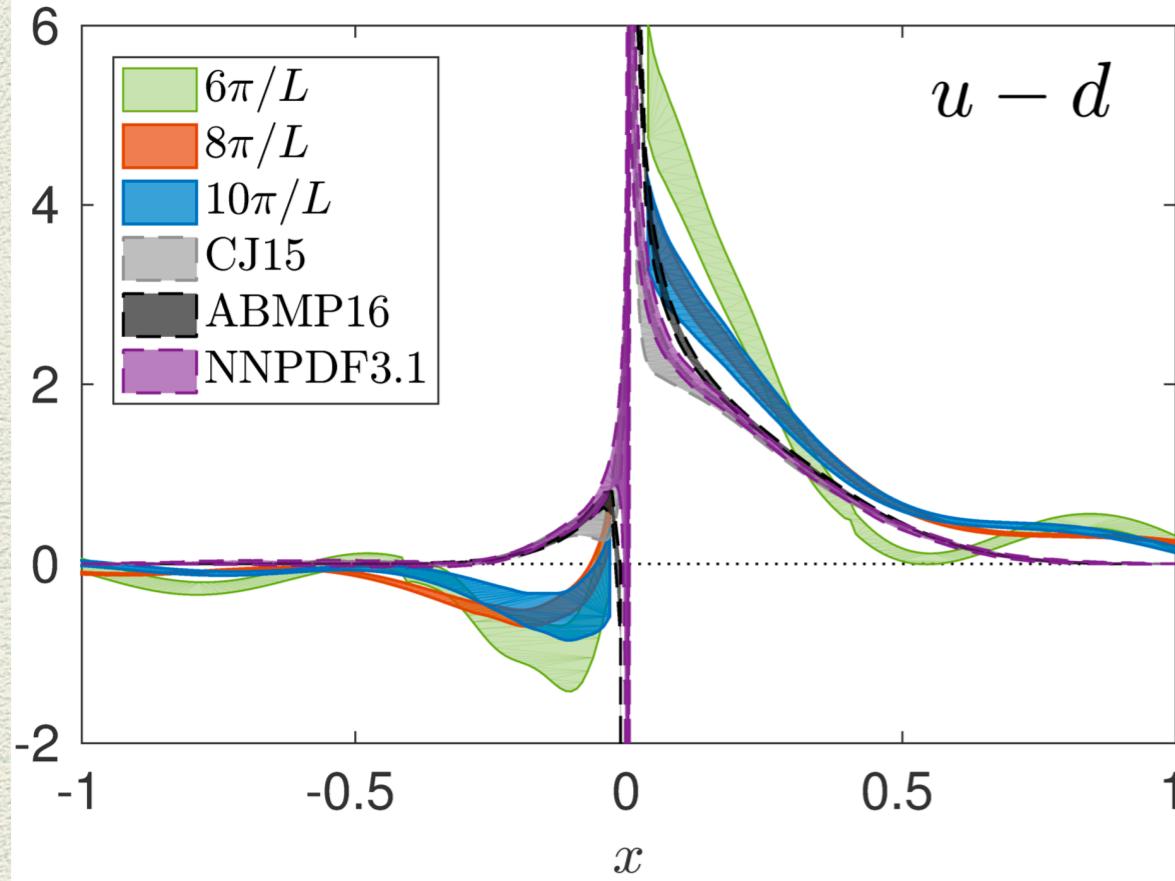
Nucleon boost dependence: Unpolarized



- * Increasing momentum approaches the phenom. fits a saturation of PDFs for $p=8\pi/L$ and $p=10\pi/L$
- * $0 < x < 0.5$: Lattice polarized PDF overlap with phenomenology
- * Negative x region: anti-quark contribution
- * $x \sim 1$: affected by finite nucleon momentum

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Nucleon boost dependence: Unpolarized



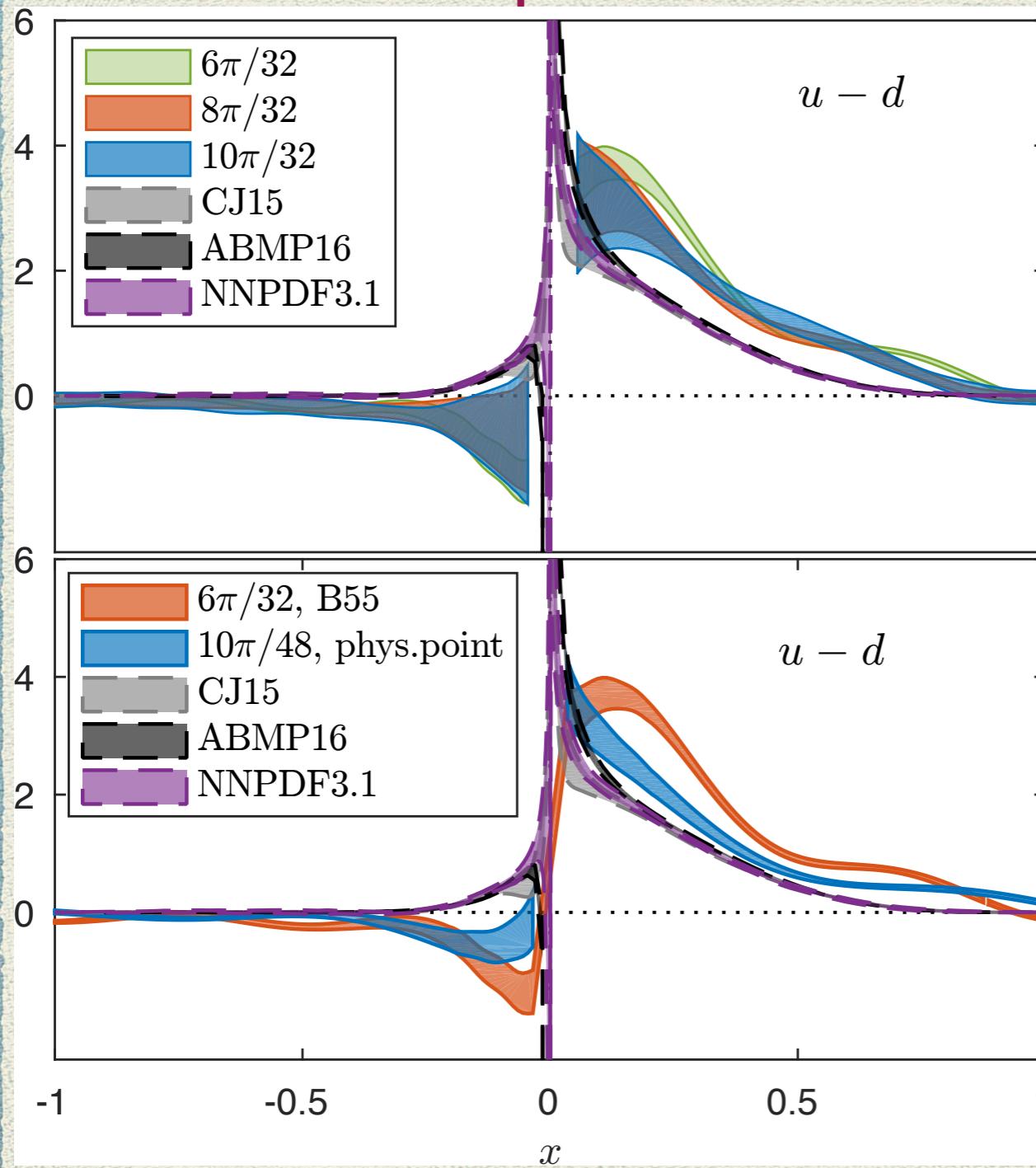
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BACKUP SLIDES

Pion mass dependence

Simulations at physical m_π crucial for above conclusions

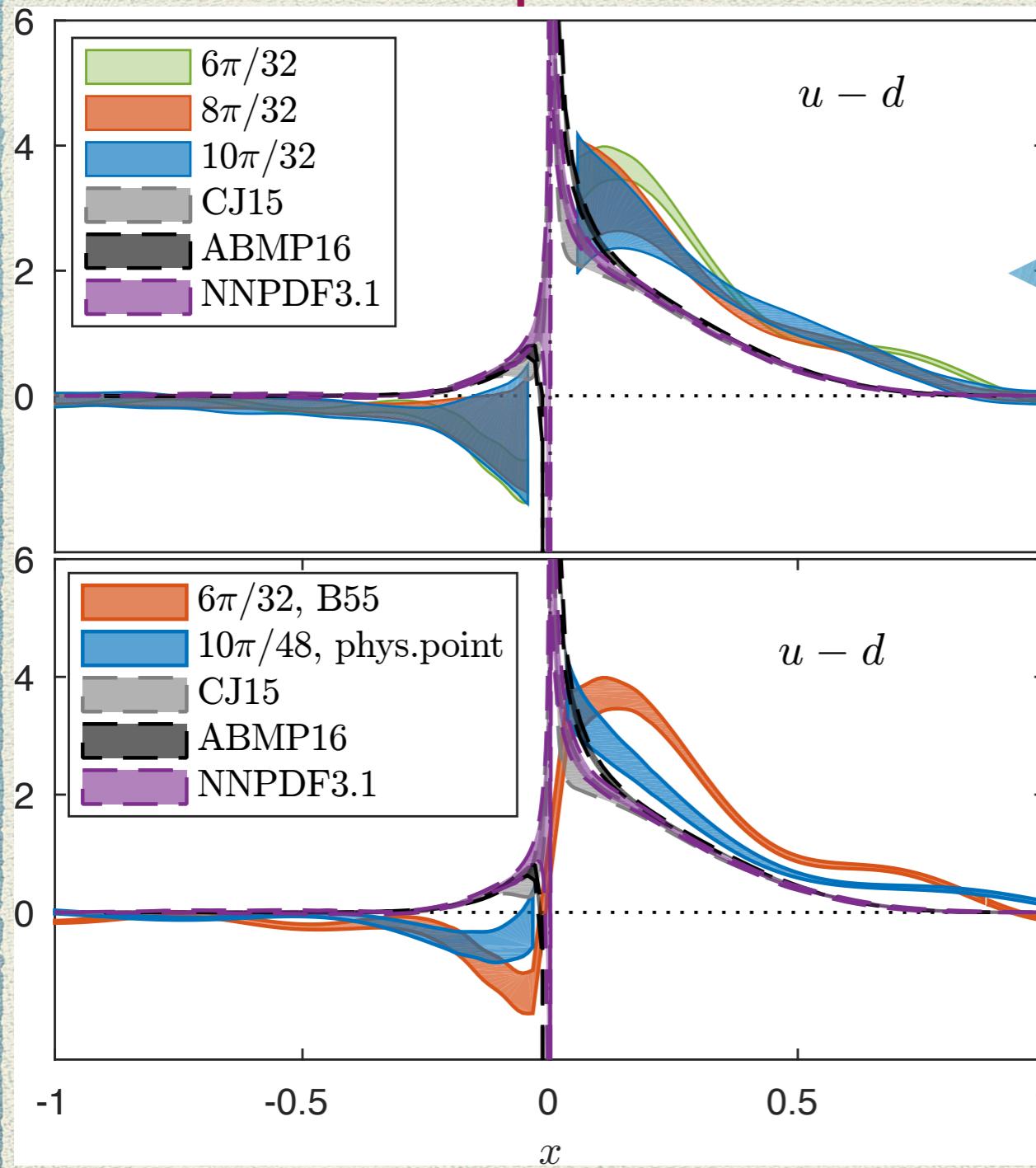
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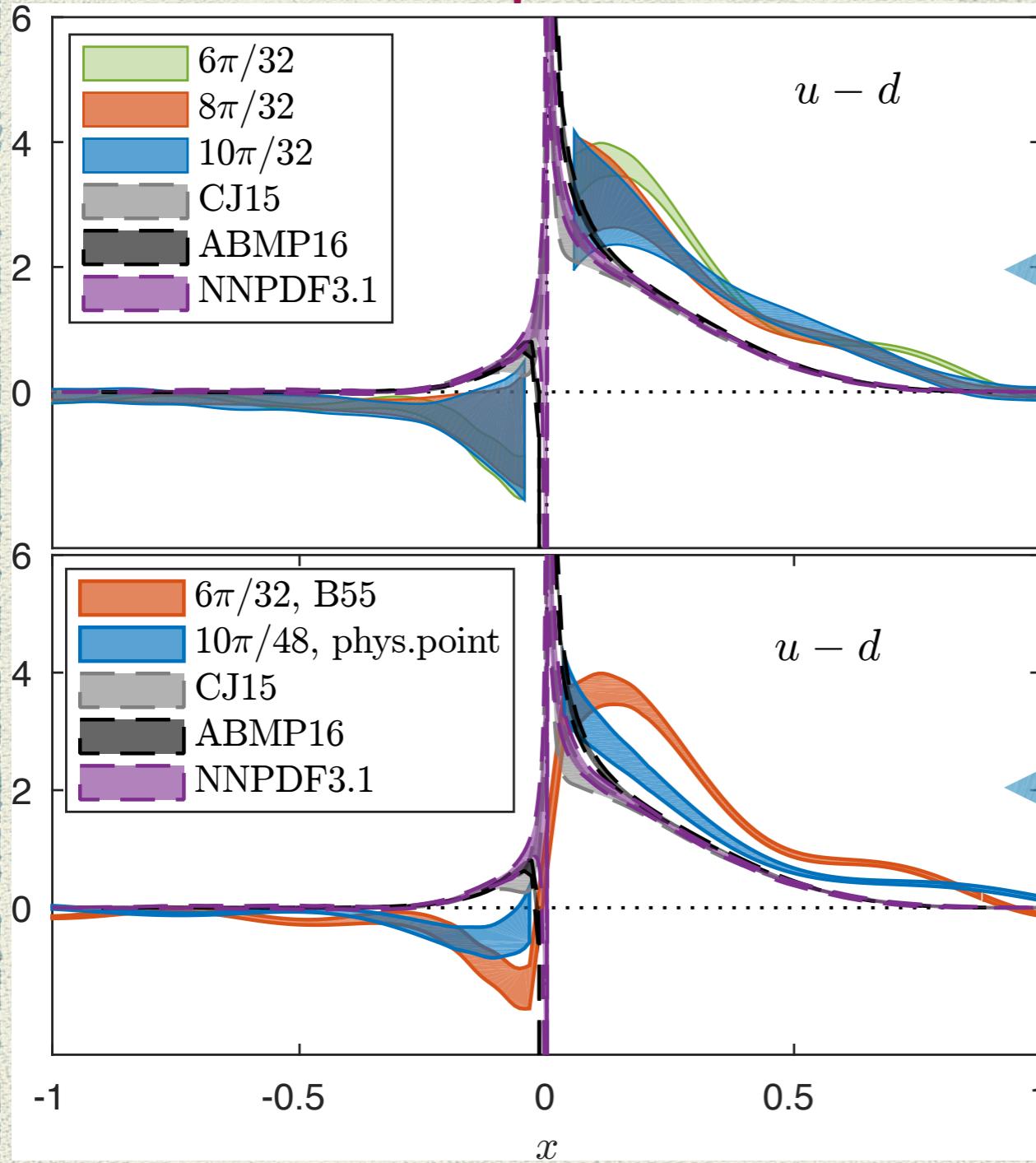


$m_\pi = 375$ MeV:
Lattice data saturate away
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Pion mass dependence

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$m_\pi = 375$ MeV:
Lattice data saturate away
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$m_\pi = 132, 375$ MeV
Significant pion mass
dependence

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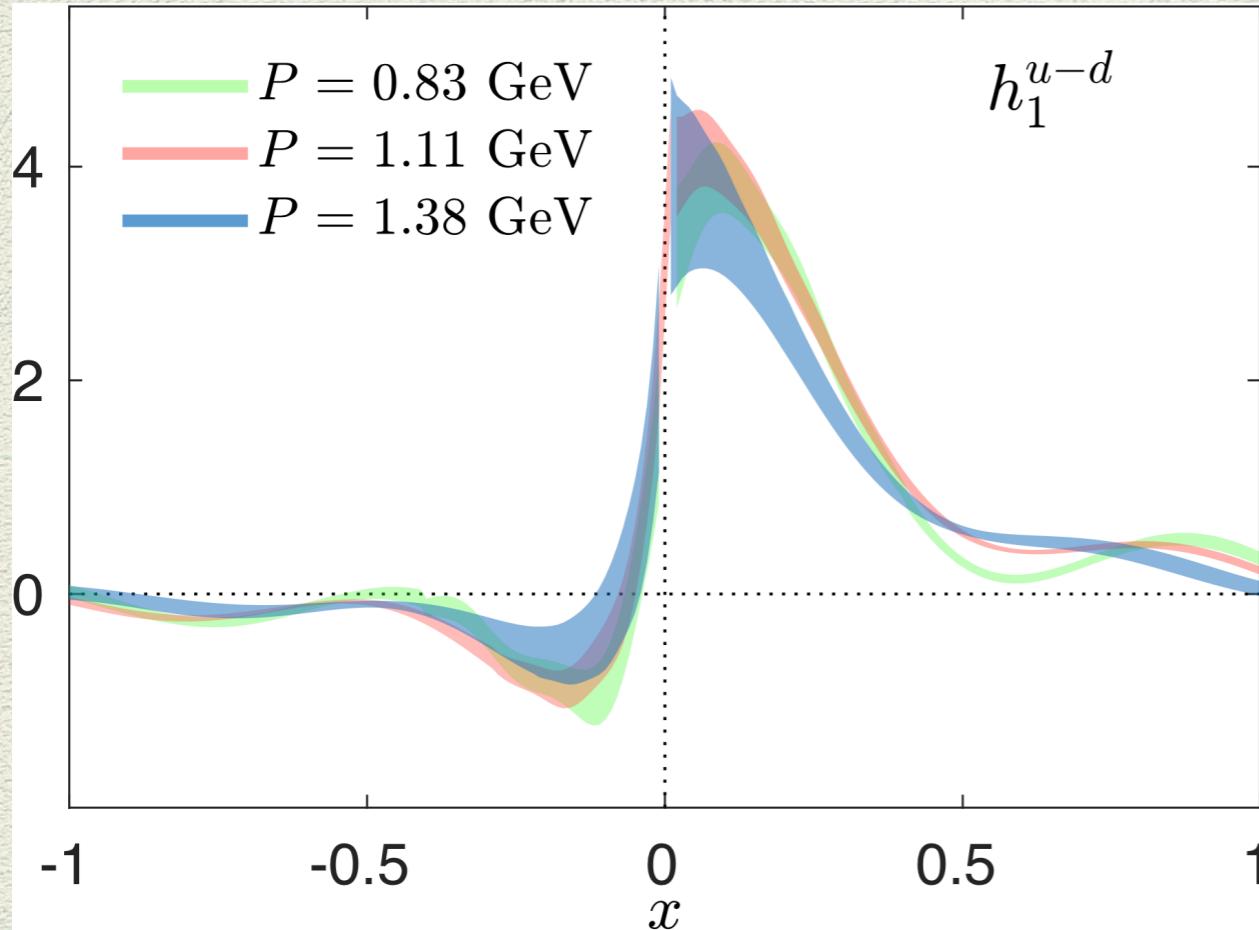
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Transversity

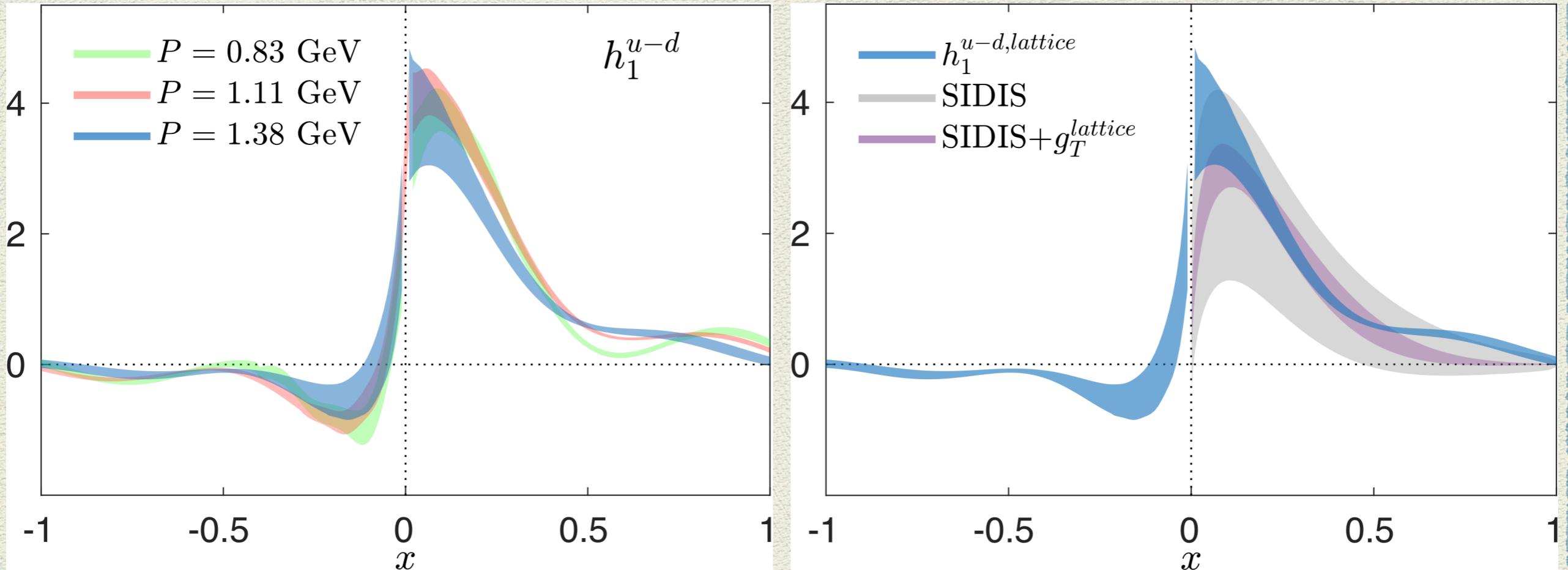
[C. Alexandrou et al., arXiv:1807.00232]



- * Mild dependence on nucleon momentum
- * Integral of PDF ($g_T=1.09(11)$) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]

Transversity

[C. Alexandrou et al., arXiv:1807.00232]



- * Mild dependence on nucleon momentum
- * Integral of PDF ($g_T=1.09(11)$) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]
- * Lattice data from quasi-PDFs more accurate than SIDIS
- * SIDIS improved with g_T^{Lat} constraints, but ab initio quasi-PDFs statistically more accurate

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Great progress over the last years:

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- * Simulations at the physical point
- * Unpolarized operator that avoid mixing (γ_0)
- * First results for transversity PDFs
- * Development of non-perturbative renormalization
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Further investigations:

Discussion

Great progress over the last years:

- * Simulations at the physical point
- * Unpolarized operator that avoid mixing (γ_0)
- * First results for transversity PDFs
- * Development of non-perturbative renormalization
- * Improving matching to light-cone PDFs

Further investigations:

- * Careful assessment of systematic uncertainties
Volume & quenching effects, continuum limit, ...
- * Increase of momentum seems a natural next step
BUT is a major challenge if reliable results are desired
- * Other directions should be pursued, with more urgent a
2-loop matching

THANK YOU



TMD Topical Collaboration

Office of
Science



Grant No. PHY-1714407

BACKUP SLIDES

Alternative Fourier

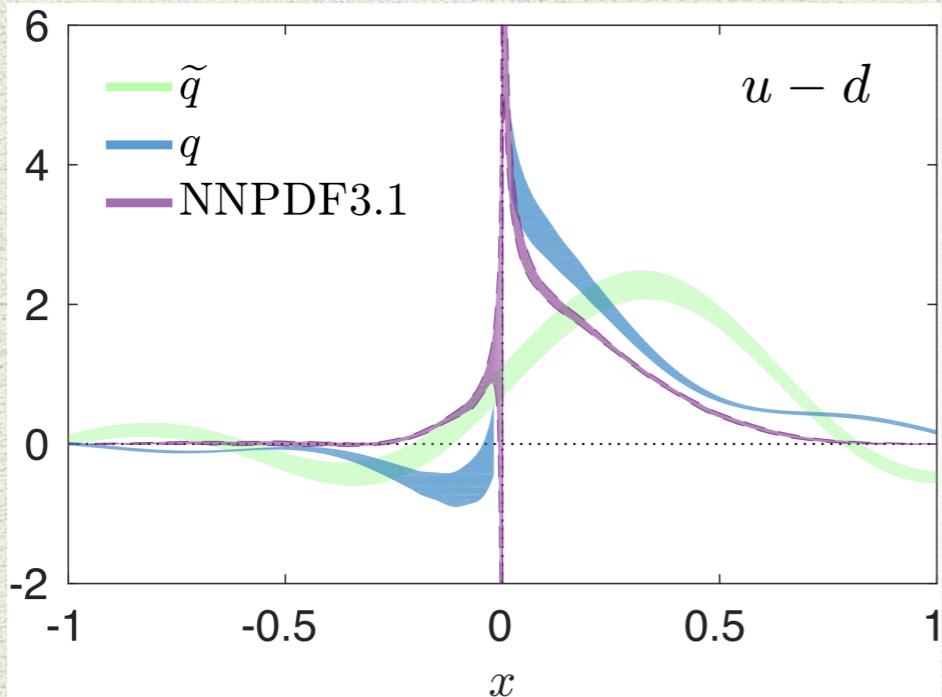
* Standard Fourier (SF) : $\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$

can be written using integration by parts (DF):

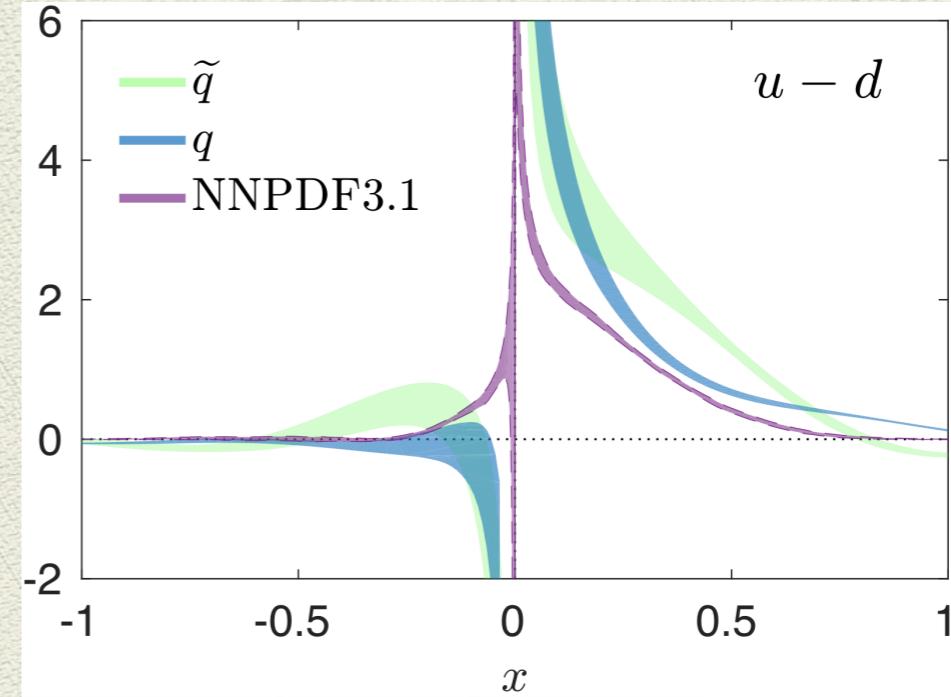
$$\tilde{q}(x) = h(z) \frac{e^{ixzP_3}}{2\pi ix} \Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP_3}}{ix} h'(z)$$

[H.W. Lin et al., arXiv:1708.05301]

Standard FT

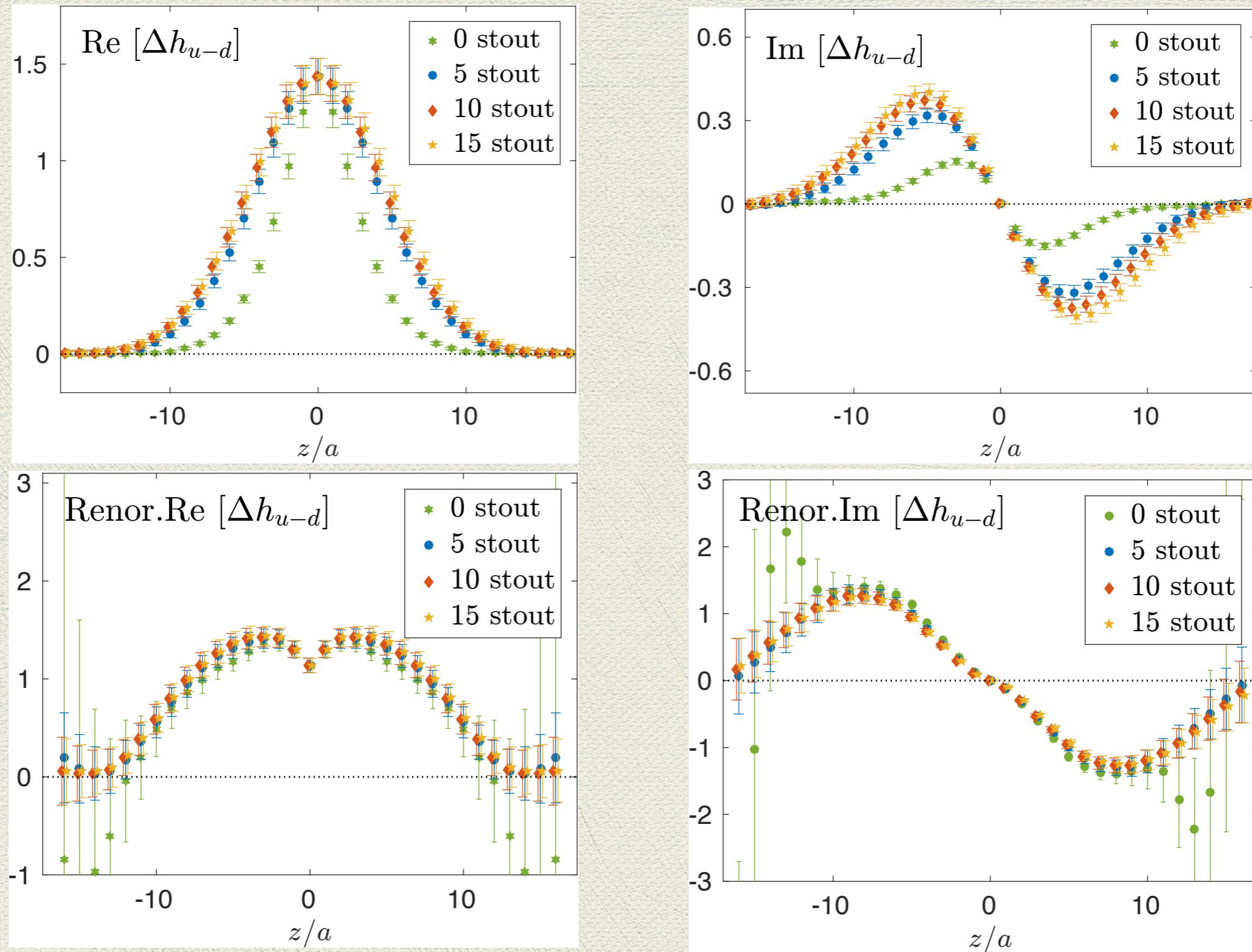


Derivative FT



- * Truncation at z_{\max} (SF) vs neglecting surface term (DF)
(latter non-negligible numerically)
- * Oscillations reduced for DF, but small- x not well-behaved
- * SF, DF different systematics, but DF uses interpolated data instead of raw ME \Rightarrow enhanced cut-off effects

Renormalized Matrix Elements



* Renormalized ME have no dependence on the stout smearing