

# Nuclear Light Front Wave Functions, the Correlations Within and the EIC

Gerald A Miller, U. of Washington

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At CUA meeting X-D Ji said ``you could never do nuclear structure at the EIC because one can't get the nuclear wave function on the light front''

I said: can too

He said: cannot

I visited Maryland a month later and had fruitful discussions

J R Cooke nucl-th/0112029 , Cooke & Miller PRC66, 034002  
Miller & Machleidt PRC60, 035202 Miller, Prog. Nuc. Part. Phys. 45, 83  
Tiburzi & Miller PRC81,035201  
Smith, Miller PRC 65,015211,66,044903,PRC 65, 055206

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Do I 00 I=1, $\infty$

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I 00 continue

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# Outline

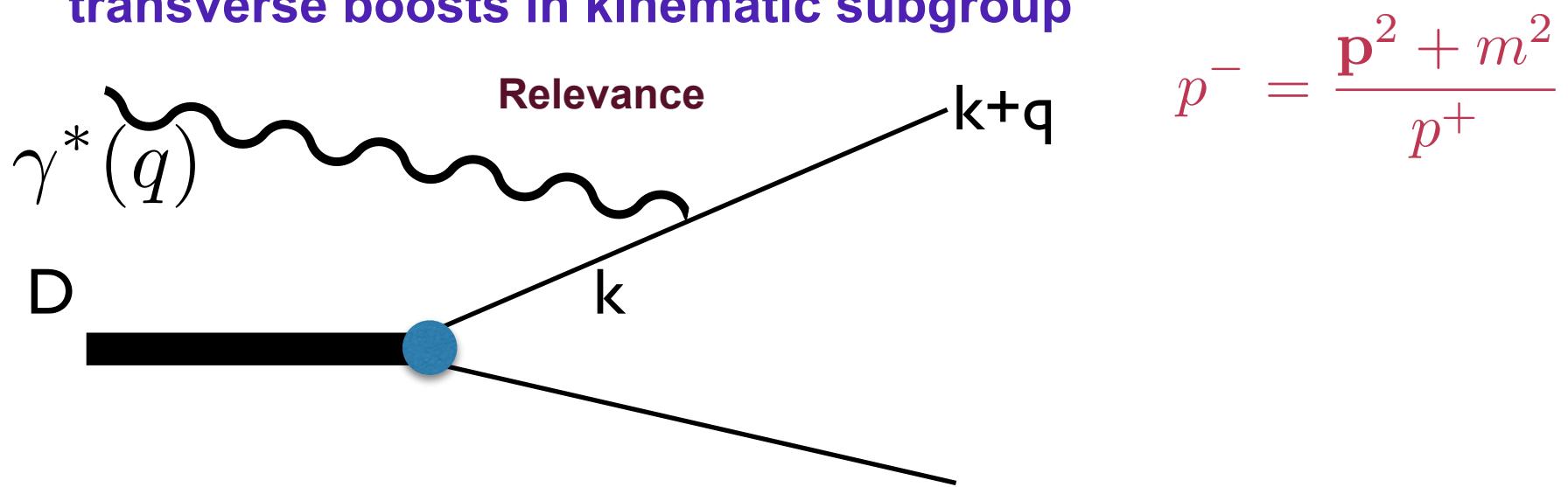
- Show how to get the nucleon-nucleon interaction and the nuclear wave function on the light front
- Demonstrate an easier procedure to get the nuclear wave function qualitatively as long as nucleon momenta not too high
- Show how to see 2,3,4 baryon correlations at the EIC based on Miller, Sievert, Venugopalan PRC 93,045202

## Light front quantization, Infinite momentum frame

“Time”,  $x^+ = x^0 + x^3$ , “Evolve”,  $p^- = p^0 - p^3$

“Space”,  $x^- = x^0 - x^3$ , “Momentum”,  $p^+$  (Bjorken)

Transverse position, momentum  $\mathbf{b}, \mathbf{p}$   
**transverse boosts in kinematic subgroup**



$$p^- = \frac{\mathbf{p}^2 + m^2}{p^+}$$

If Photon energy  $>> m$  and struck nucleon  $\approx$  on mass – shell :

$$(k + q)^2 = m^2 \rightarrow k^+ q^- \approx Q^2, \quad \frac{Q^2}{\nu^2} \ll 1$$

Integrate over  $k^-$       disappears

$$d\sigma \sim \Psi_D^2(\mathbf{k}, \frac{k^+}{P_D^+} \equiv \alpha)$$

FS '81

## Light front quantization, Infinite momentum frame

$P^-$  is LF Hamiltonian, get from Lagrangian.

LF Schroedinger eq.  $P^-|\Psi_D\rangle = M_D|\Psi_D\rangle$  Rest frame

One boson exchange

$$V = \frac{g^2}{k^+(k_1^- + k_2^- - k_3^- - k_2^- - k^-)} = \frac{g^2}{k^2 - \mu^2} \text{ Yukawa}$$

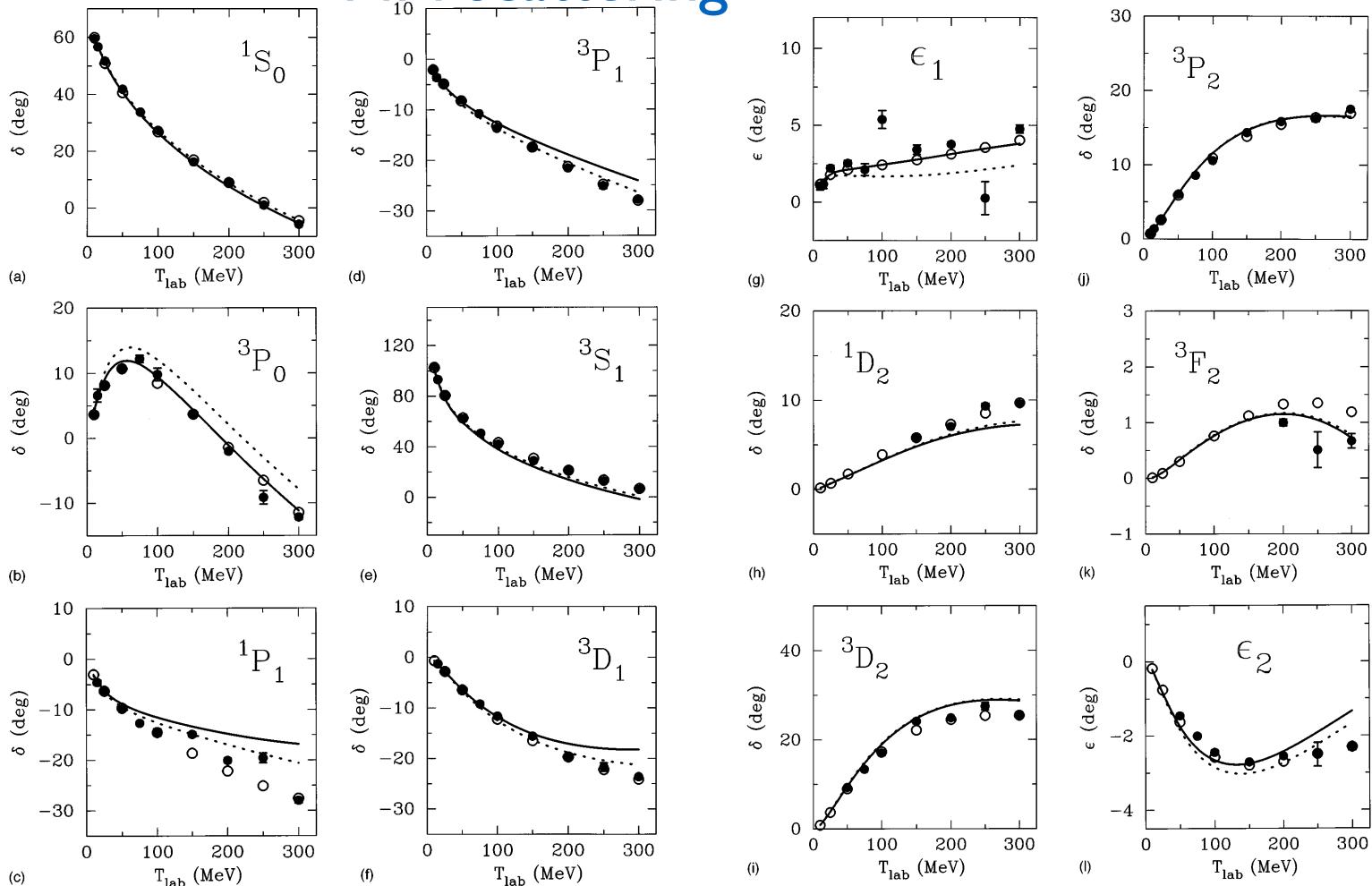
Solve

Lippmann-Schwinger eq with  $p^- = \frac{p_\perp^2 + M^2}{p^+}$  as kinetic energy operator

# Miller & Machleidt PRC 60,035202

$\pi, \eta, \rho, \omega, a_0, \sigma$  exchange with extra factor in G

## NN scattering

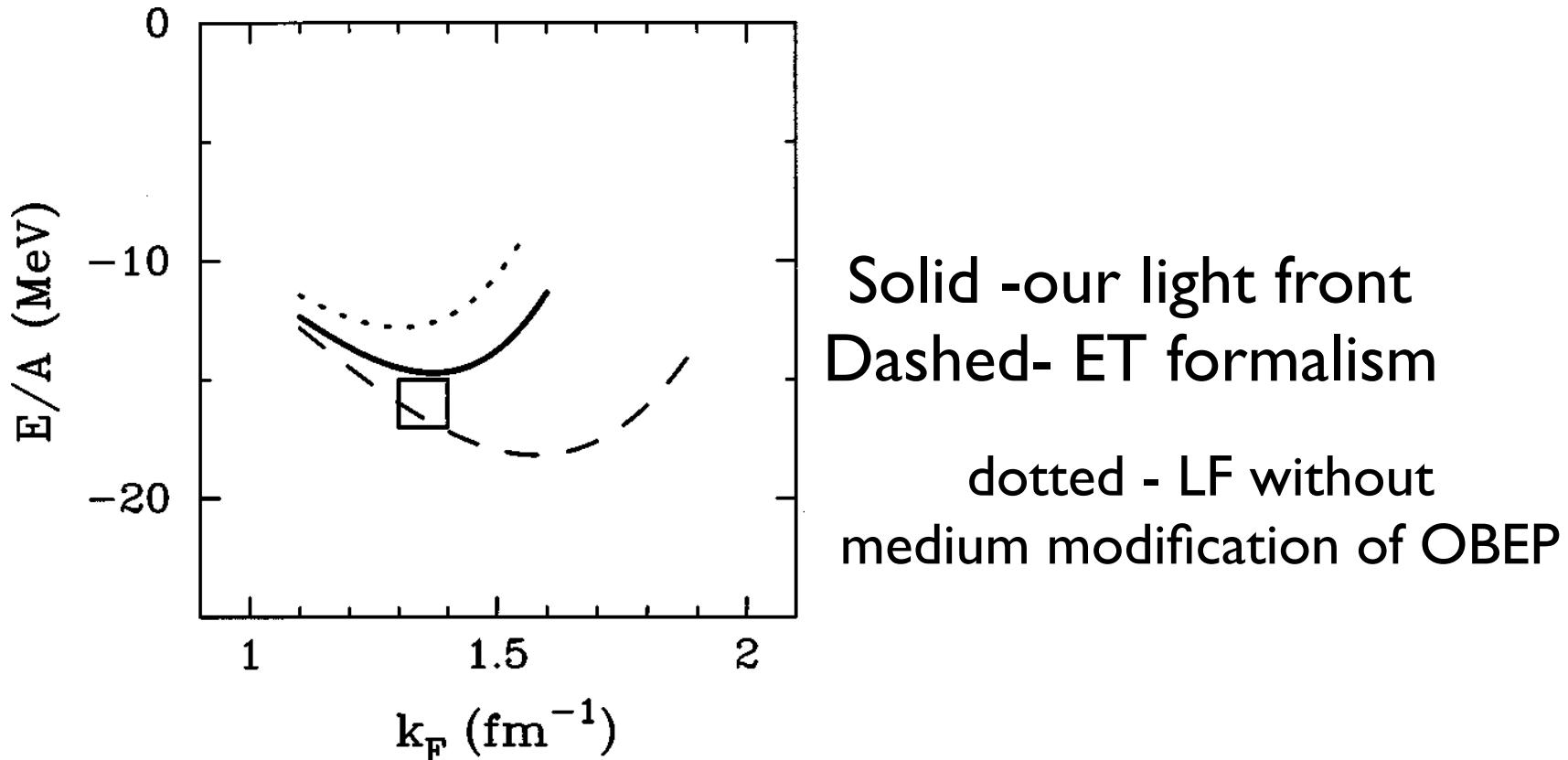


Solid LF, dashed OBEP B

$B_D = 2.245 \text{ MeV}$

# Miller & Machleidt PRC 60,035202

## Nuclear Matter Saturation in light front version of Bruckner theory



We also did shell model on the light front  
Blunden, Burkardt Miller PRC60, 055211

# Deuteron-Jason Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

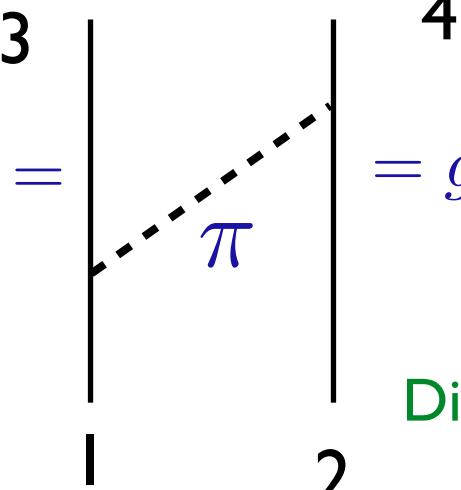
Solves LF Schroedinger eq (LFSSE) Wigner-Brillouin

$$[P_0^- + V(P^-)] |\Psi_D\rangle = P^- |\Psi_D\rangle \quad P^- = 2m - B \quad \text{rest frame}$$

$$V(P^-) = \frac{1}{P^- - k_3^- - k_2^- - k_\pi^-} = g^2 \frac{1}{P^- - k_3^- - k_2^- - k_\pi^-}$$

Manifest rotational invar. broken

Different meson propagator than Machleidt Miller



Calculations maintained rotational invariance in deuteron wave function and elastic electromagnetic form factors

# first part summary

- These projects demonstrated that standard approaches to nuclear structure could be done on the light front
- There was no particular reason do it, but as nuclear calculation become more precise including relativistic effects becomes more important. The only way to have a relativistic nuclear wave function is to use the light front formalism, maybe now possible
- Now need to estimate potential nuclear effects at the EIC - discuss an easier way to begin

# Relation between equal-time and light-front wave functions

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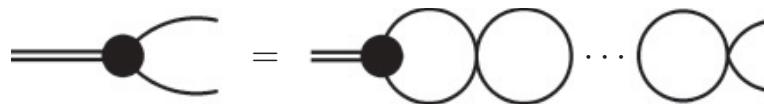
Brian C. Tiburzi<sup>†</sup>

*Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*

(Received 29 November 2009; published 4 March 2010)

The relation between equal-time and light-front wave functions is studied using models for which the four-dimensional solution of the Bethe-Salpeter wave function can be obtained. The popular prescription of defining the longitudinal momentum fraction using the instant-form free kinetic energy and third component of momentum is found to be incorrect except in the nonrelativistic limit. One may obtain light-front wave functions from rest-frame, instant-form wave functions by boosting the latter wave functions to the infinite momentum frame.

- How bad is the problem?
- Is D non-relativistic?
- Is  ${}^3\text{He}$  non-relativistic?
- Answer by using solutions of Bethe-S eqn.

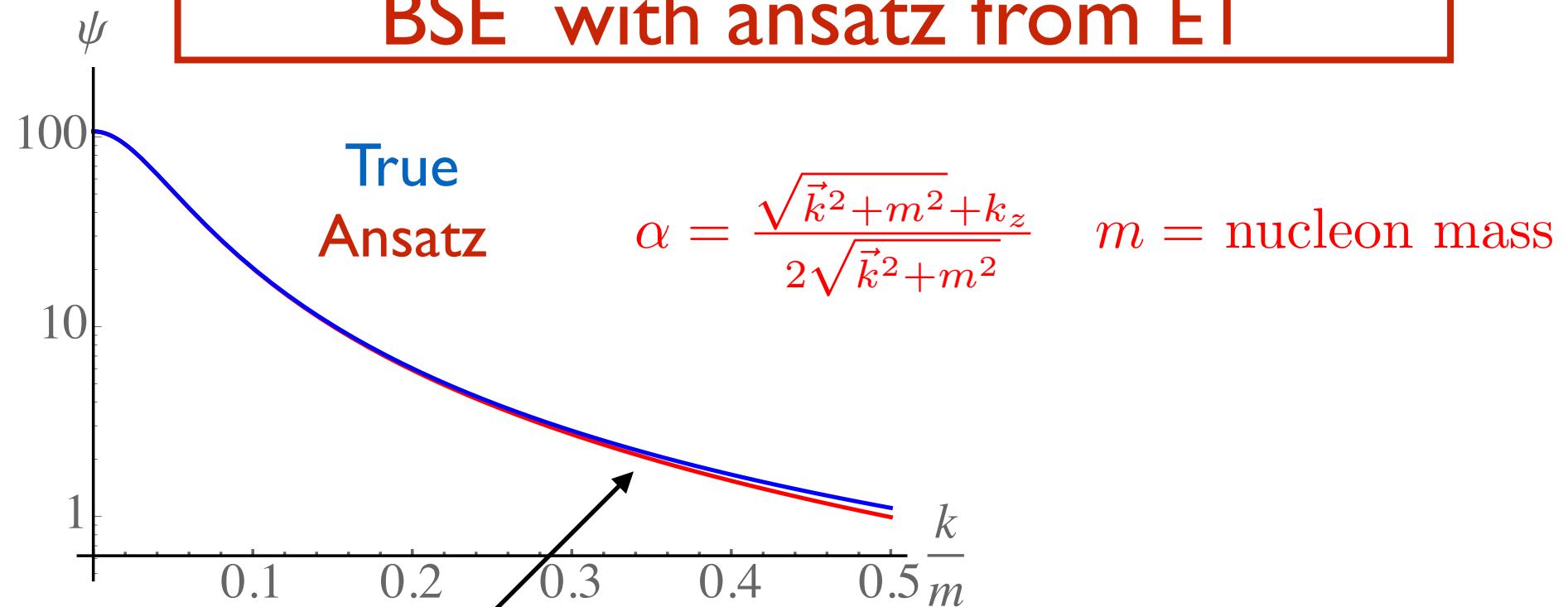


0'th order CH PT

Model

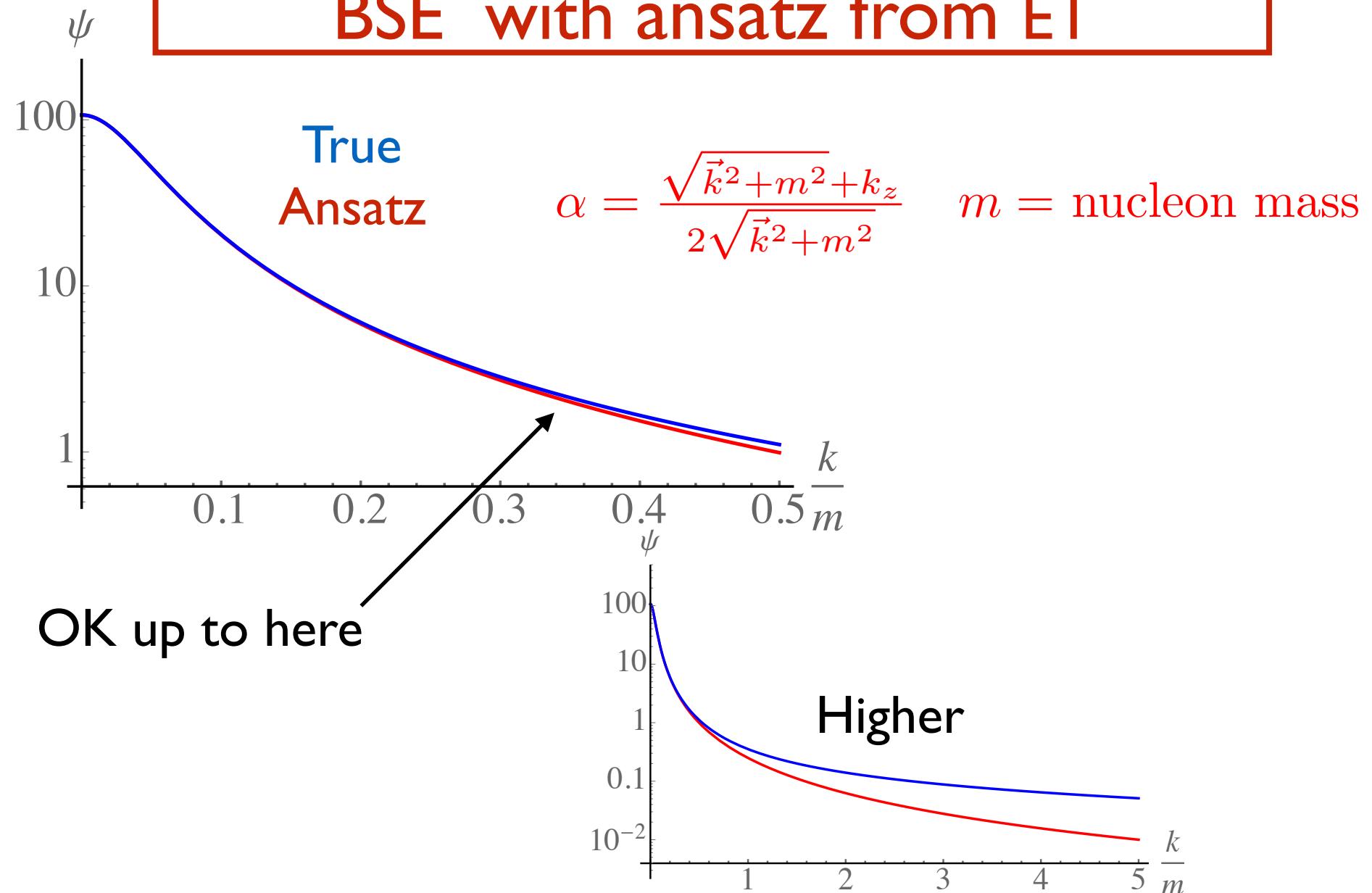
FIG. 2. Bethe-Salpeter equation for a point interaction. The state is bound by the infinite chain of bubbles.

# Deuteron Compares true LFD from BSE with ansatz from ET

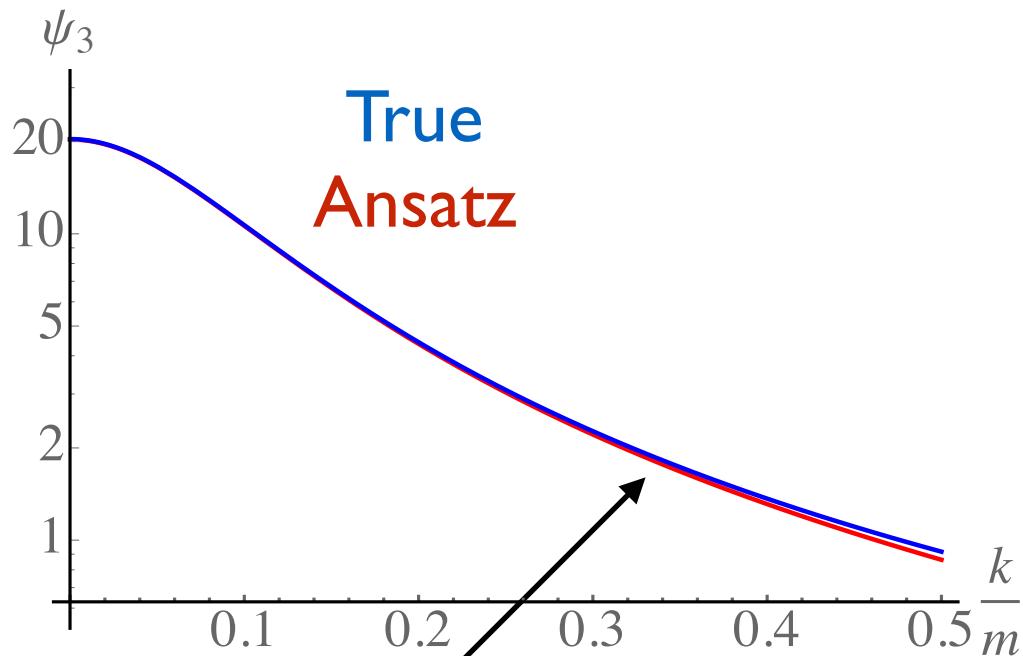


OK up to here

# Deuteron Compares true LFD from BSE with ansatz from ET



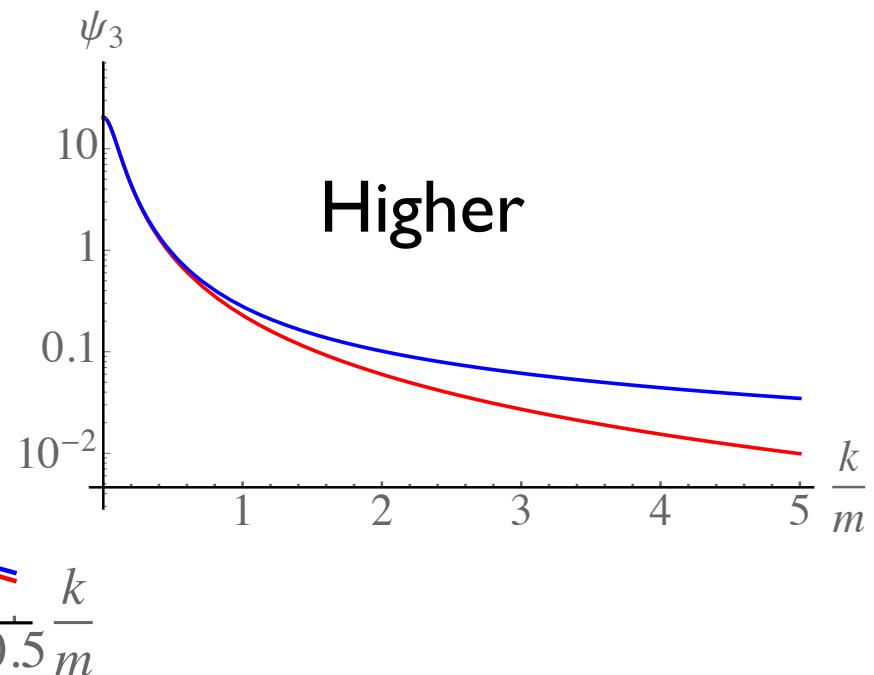
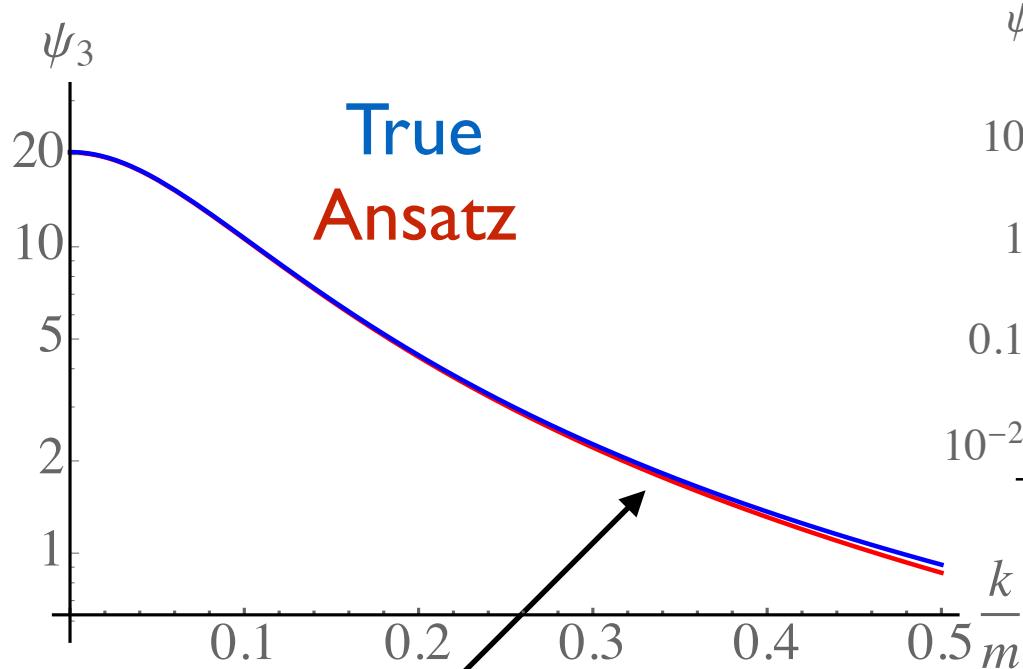
## $^3\text{He}$ Compare true LFD from BSE with ansatz from ET



$$\alpha = \frac{\sqrt{\vec{k}^2 + m^2} + k_z}{\sqrt{\vec{k}^2 + m^2} + \sqrt{\vec{k}^2 + (2m)^2}}$$

Examples with spin in papers with Smith

## $^3\text{He}$ Compare true LFD from BSE with ansatz from ET

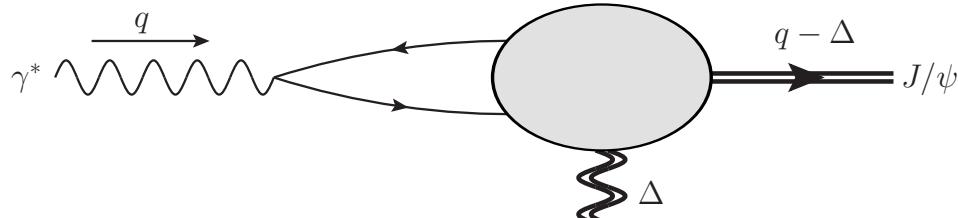


$$\alpha = \frac{\sqrt{\vec{k}^2 + m^2} + k_z}{\sqrt{\vec{k}^2 + m^2} + \sqrt{\vec{k}^2 + (2m)^2}}$$

OK up to here

Examples with spin in papers with Smith

# Seeing multi-baryon correlations at the EIC



Basic idea: use  $\gamma^* + A \rightarrow J/\Psi + X$  reaction  
Two-gluon probe of nuclear structure

- Starts with Miller, Sievert, Venugopalan PRC 93,045202
- That paper, Deuteron initial state, p n back to back in **final state** -cross section large enough to observe
- Here - aim at **initial state** physics -inclusive at specific kinematics - start with (e,e')

# $A(e,e')$ at $x>1$ shows dominance of 2N SRC

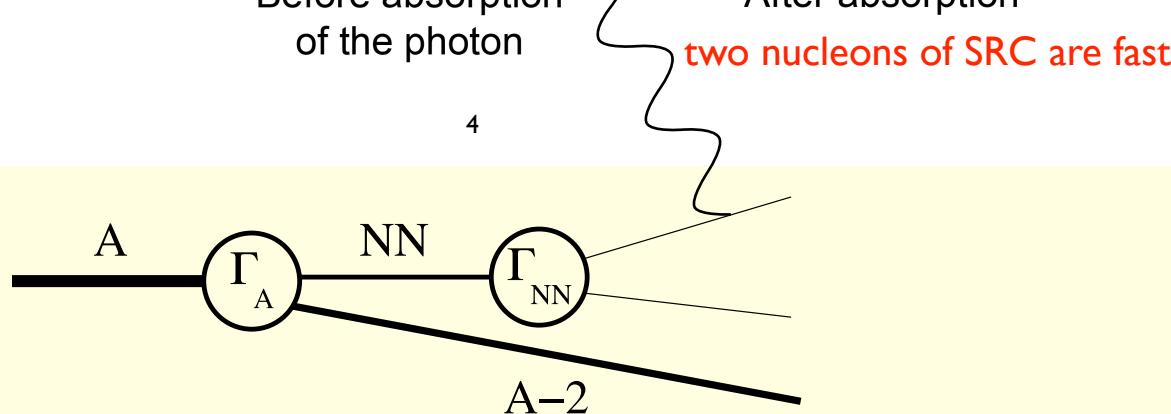
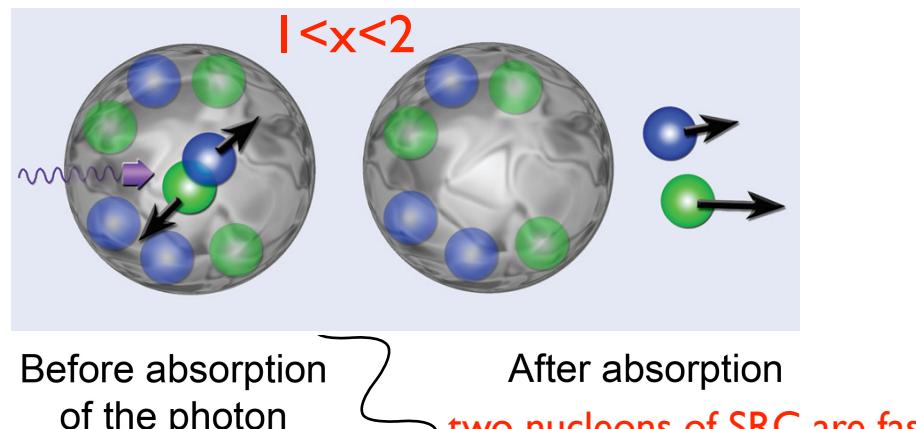
$$x = \frac{Q^2}{2M\nu}$$

$x$  goes from 1 to A

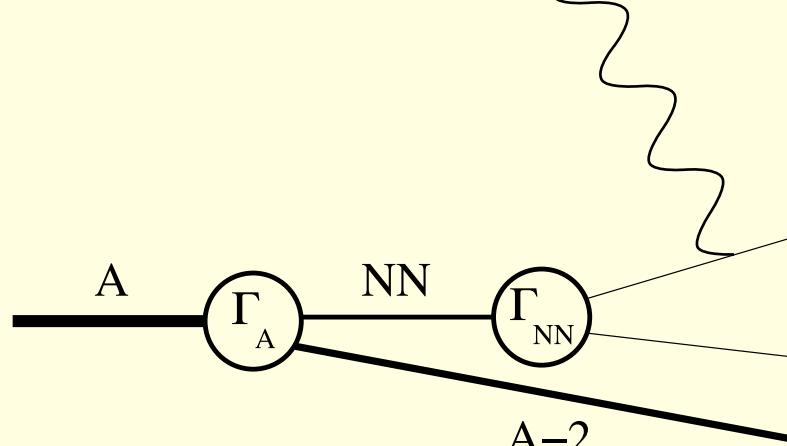
$x=1$  is **exact** kinematic limit **for all  $Q^2$**  for the scattering off a free nucleon;  
 $x=2$  ( $x=3$ ) is **exact** kinematic limit **for all  $Q^2$**  for the scattering off a  $A=2$ ( $A=3$ ) system (up to <1% correction due to nuclear binding)

M Strikman  
picture

Two nucleons cluster



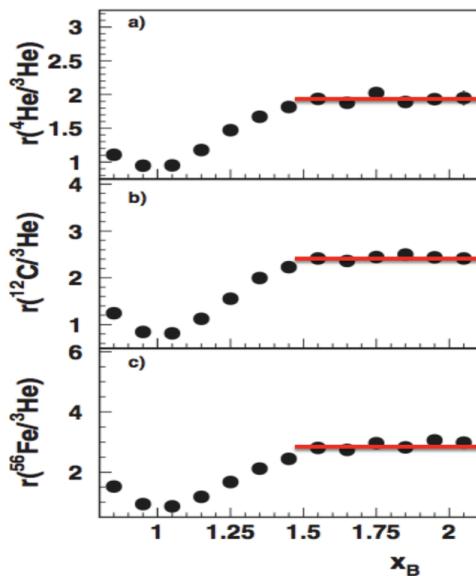
# (e,e') at high x



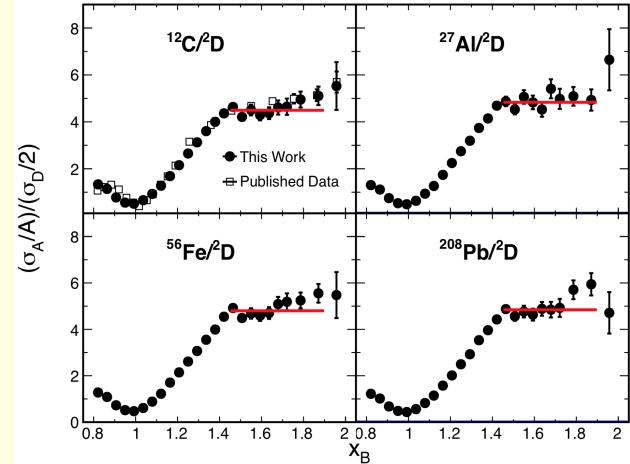
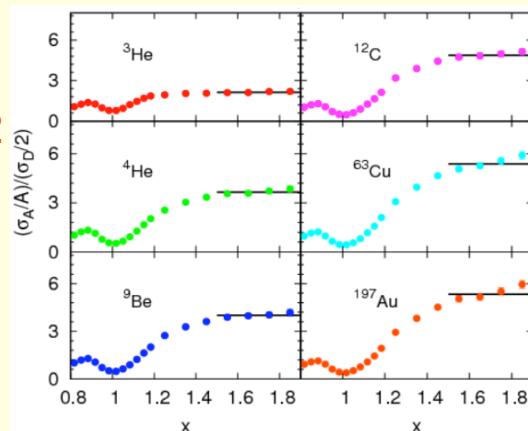
$1 < x < 2$  leading term:

$$\frac{2}{A} \sigma(x, Q^2) \approx a_2(A) \sigma_2(x, Q^2) \approx a_2(A) \sigma_D(x, Q^2)$$

Scaling



$a_2$



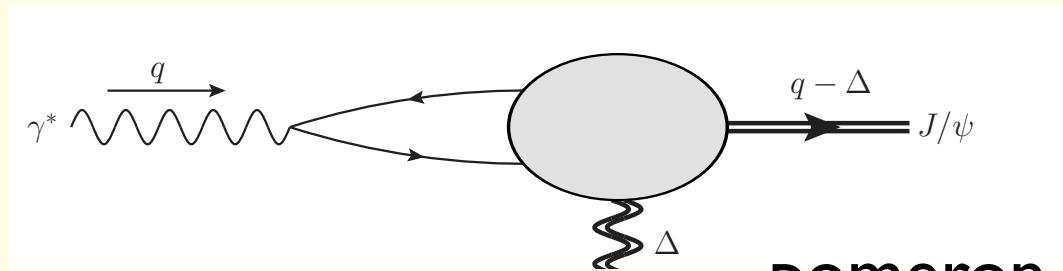
Schmookler et al

'18  
14

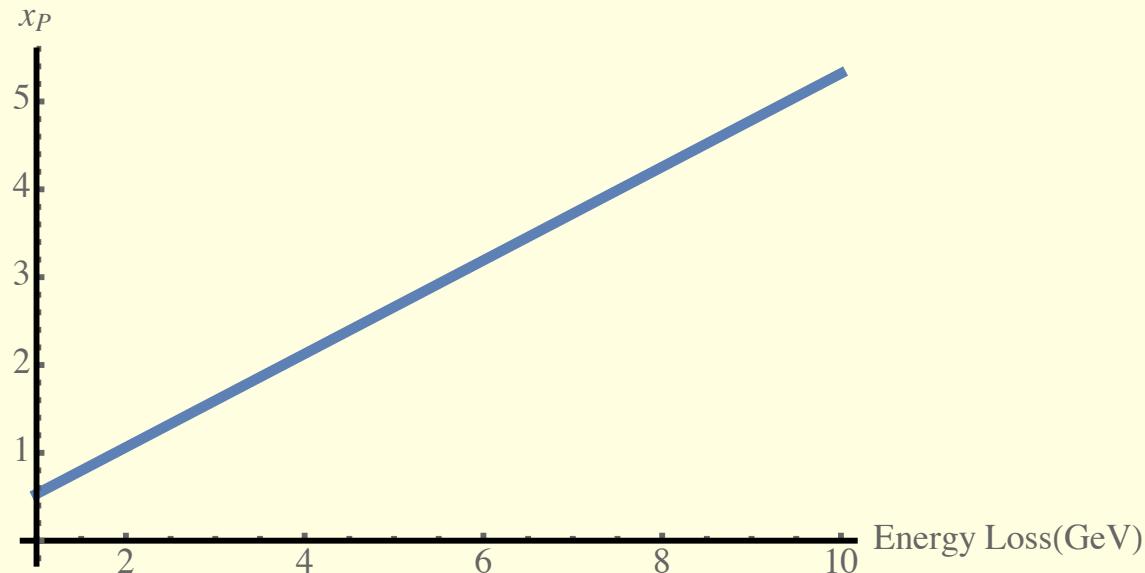
Egiyan '06

Fomin et al 11

How to go to higher x - see 3,4 N core's



pomeron two gluons



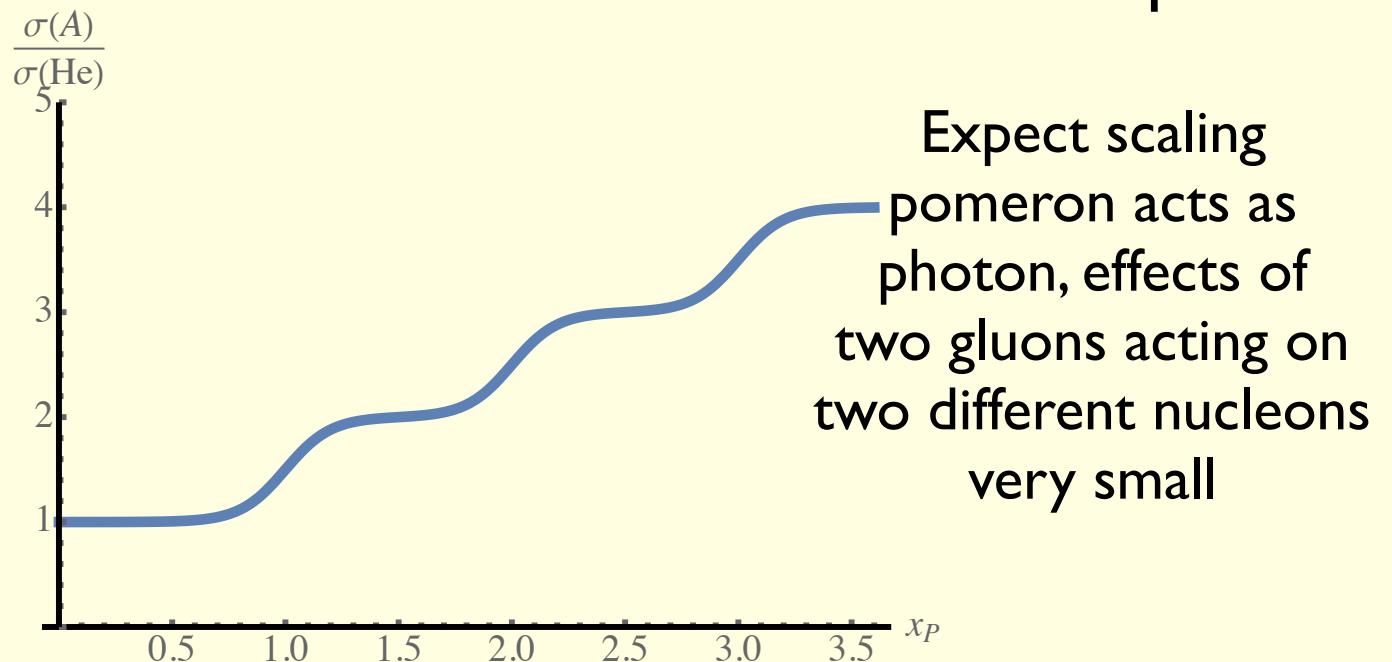
Energy loss is photon energy - vector energy  
(lab)

# Production cross section

- Much larger than estimated in PRC93, 045202

$$\sigma(x_P, \Delta^2) = \sum_{j=2}^A \frac{A}{j} a_j \sigma_j(x_p, \Delta^2)$$

FS cluster decomposition



accessible with EIC

# Summary: Multi-nucleon Correlations at EIC

Basic idea: use  $\gamma^* + A \rightarrow J/\Psi + X$  reaction

Two-gluon probe of nuclear structure

Pomeron acts as high x photon, to probe multi-nucleon kinematics

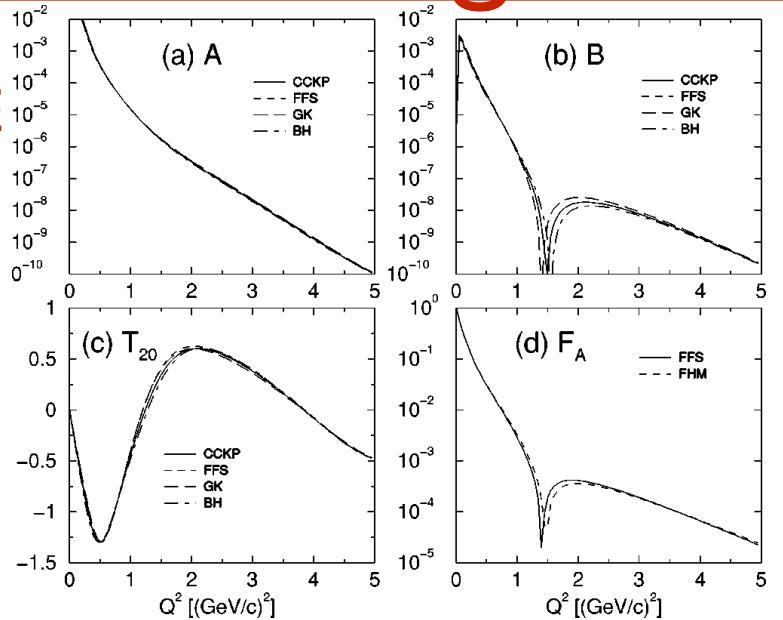
Relatively high production cross sections

EIC has ideal kinematics to observe multi-nucleon correlations

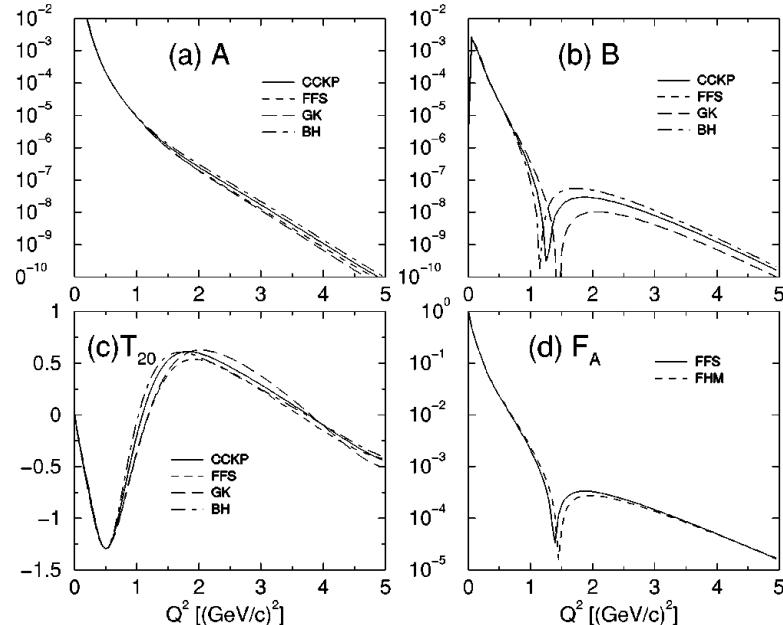
# Spares follow

# Restoring? RI in form factors

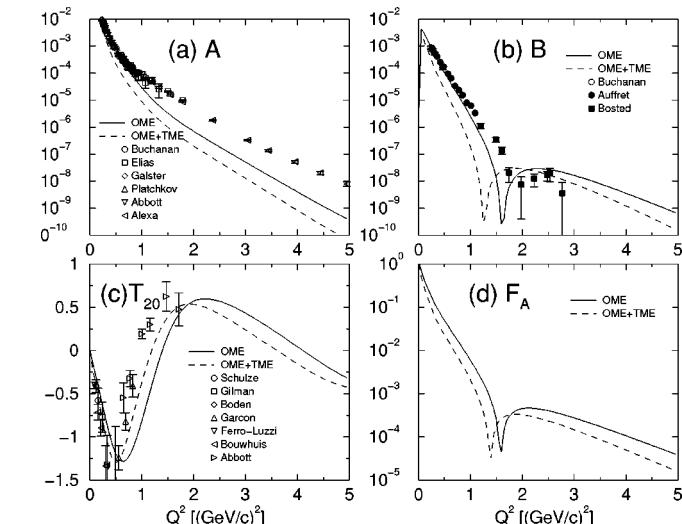
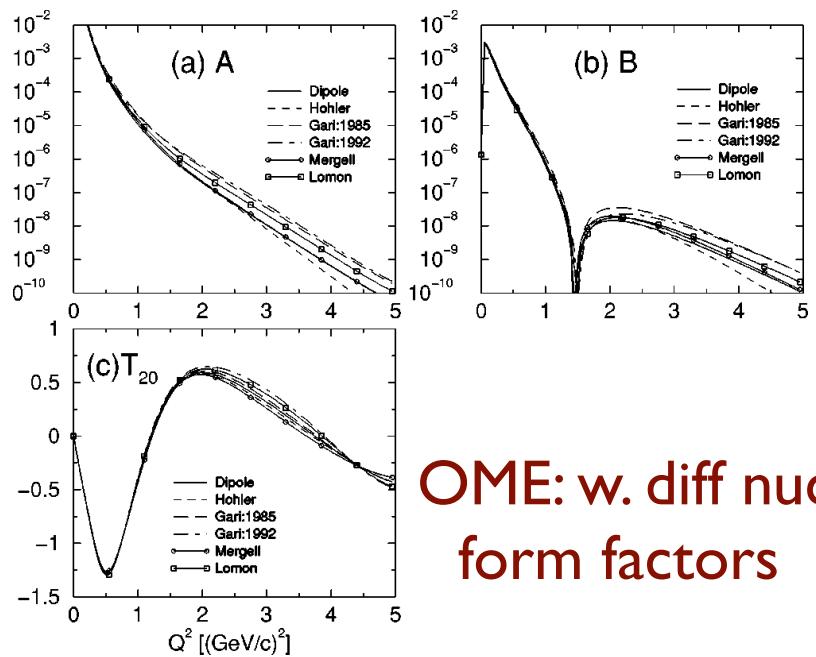
OME



TME



OME: w. diff nucleon  
form factors



# The real problem- Bethe Salpeter Eq. (BSE)

$$T = \text{Diagram} = K + G \text{Diagram}$$

K is sum of irreducible diagrams

$G$  is 4 dimensional -product of two Feynman propagators.

Intermediate state 4 dimensional integral  $d^4 k = dk^0 d^3 k = dk^+ dk^- d^2 k_\perp$

Reduce to 3 dimensions:

ET: integrate over  $k^0$ . Ignore  $k^0$  except in  $G$ . Sets relative time to 0.

LF: Integrate over  $k^-$ . Ignore  $k^-$  except in  $G$ . Sets relative  $\tau = 0$

3 dimensional version of  $G$  is  $g_{ET}$  (Blankenbecler Sugar) or  $g_{LF}$  (Weinberg)

$$T = \text{Diagram} = V + g \text{Diagram}$$

Puts particles on mass shell

Either  $g: V = K + K(G - g)V$ . Same on-shell  $T$ ,  $V$ 's and wave fcns differ

No relation between wave functions in principle

# Restoring RI in form factors

- Rotational invariance gives angular condition FS
- Angular condition is upheld better when Deut is computed using only one meson exchange OME potentials than two meson exchange TME
- However, form factors do not depend much on choice of bad currents

# Restoring Rot. Inv.

PRC66, 034002

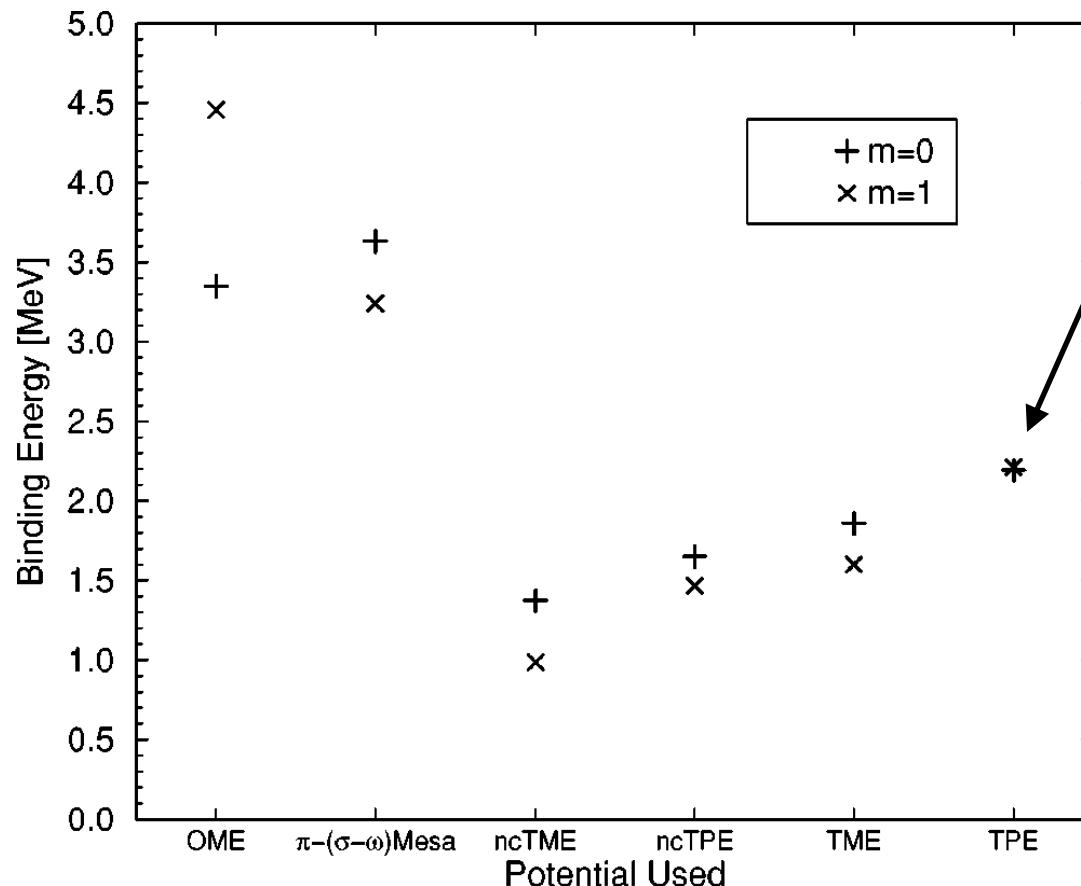


FIG. 9. The values of the binding energy for the  $m=0$  and  $m=1$  states for different nucleon-nucleon light-front potentials. The  $\sigma$

Uses only  
2 pion exch  
in 2BE

# Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

## Two Meson Dynamics

### Instantaneous terms

$$(a) \quad V_{\text{TME:M}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{1m} - k_{1f}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 2: } q_f = k_{2m} - k_{2f}, q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(b) \quad V_{\text{TME:SB}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 2: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(c) \quad V_{\text{TME:SBI}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 2: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \\ \text{Diagram 3: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 4: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(d) \quad V_{\text{TME:SBII}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 2: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \end{array} \right)$$

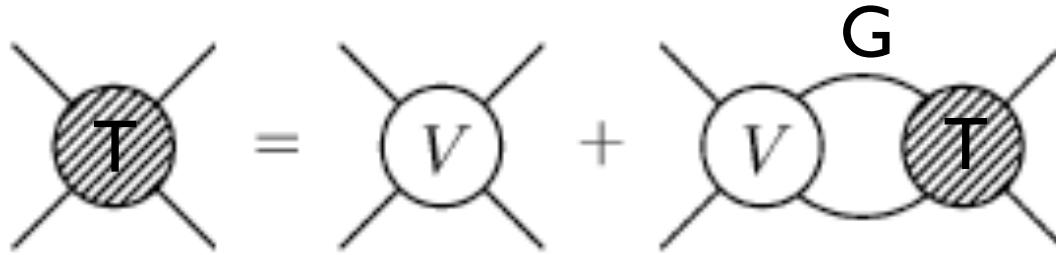
### Chiral contact terms

$$(a) \quad V_{\text{TME:C}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{2f} - k_{2m}, q_i = k_{2i} - k_{2m} \\ \text{Diagram 2: } q_f = k_{1i} - k_{1m}, q_i = k_{1i} - k_{1m} \end{array} \right)$$

$$(b) \quad V_{\text{TME:SBC}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{2m} - k_{2f}, q_i = k_{2i} - k_{2m} \\ \text{Diagram 2: } q_f = k_{1m} - k_{1f}, q_i = k_{1i} - k_{1m} \\ \text{Diagram 3: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 4: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(c) \quad V_{\text{TME:SBIC}} = \left( \begin{array}{c} \text{Diagram 1: } q_f = k_{1f} - k_{1m}, q_i = k_{1m} - k_{1i} \\ \text{Diagram 2: } q_f = k_{2f} - k_{2m}, q_i = k_{2m} - k_{2i} \\ \text{Diagram 3: } q_f = k_{2m} - k_{2f}, q_i = k_{2i} - k_{2m} \\ \text{Diagram 4: } q_f = k_{1m} - k_{1f}, q_i = k_{1i} - k_{1m} \end{array} \right)$$

$$(d) \quad V_{\text{TME:SBCC}} = \left( \begin{array}{c} \text{Diagram 1: } q_f / \text{ and } q_i / \\ \text{Diagram 2: } q_f / \text{ and } q_i / \end{array} \right)$$



$$G \sim \frac{1}{\alpha(1-\alpha)} \frac{d^2 p_\perp d\alpha}{P^2 - \frac{p_\perp^2 + m^2}{\alpha(1-\alpha)}}$$

**define**  $p_z$  :  $\frac{p_\perp^2 + m^2}{4\alpha(1-\alpha)} = p_\perp^2 + p_z^2 + m^2 = \vec{p}^2 + m^2$

$$G \sim \frac{\frac{m^2}{\sqrt{\vec{p}^2 + m^2}}}{\frac{d^3 p}{p_i^2 - p^2}} \text{ Usual propagator with extra factor}$$

Extra factor is close to unity for D wave function

# Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

## Dynamics

- Chiral Lagrangian with  $\pi, \eta, \rho, \omega, \delta, \sigma$
- Two meson exchange!
- Explicit  $P^-$  dependence

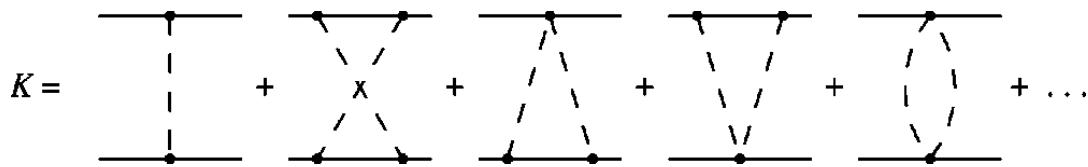


FIG. 1. The first several terms of the full kernel for the Bethe-Salpeter equation of the nuclear model with chiral symmetry.

Calculations maintained rotational invariance in deuteron wave function and elastic electromagnetic form factors