Deuteron GPDs in a covariant framework

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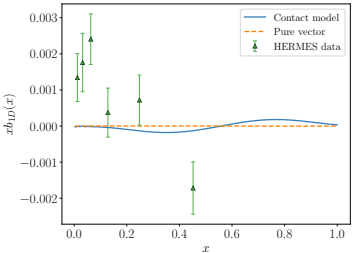
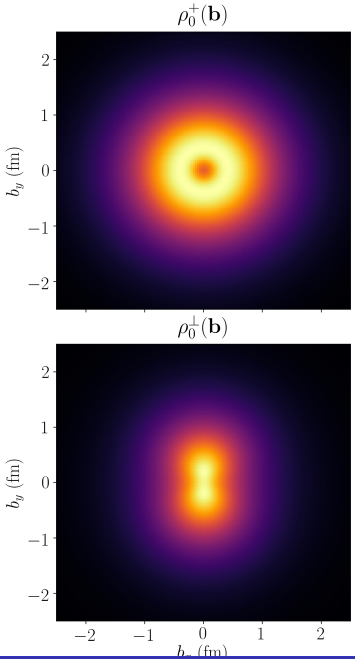
Why the deuteron?

- The deuteron is a spin-1 system: has **more structure** than the proton or neutron.
- The deuteron has an electric quadrupole moment—and a huge one, 0.286 fm^2.
- The deuteron also has a tensor polarization, which is sensitive to exotic components like hidden color.
- JLab experiment **E12-13-011** will measure tensor-polarized DIS of the deuteron.



Left: Tensor-polarized deuteron DIS

Right: Light cone transverse charge density of the deuteron. (top) longitudinally polarized (bottom) transversely polarized.



Deuteron polarization may be relevant to EIC design!

What are GPDs?

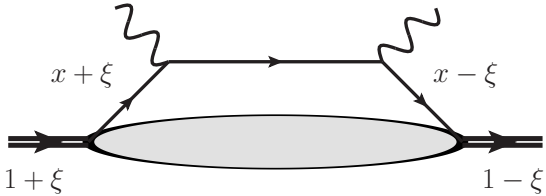
Generalized parton distributions (GPDs) are defined using the same operators (light cone correlators) as PDFs.

- A familiar example: vector quark correlator for the nucleon.

$$\frac{1}{2}\int \frac{dz}{2\pi} e^{-iP\cdot nzx} \langle p'|\bar{q}\left(\frac{nz}{2}\right) \not{n} q\left(-\frac{nz}{2}\right)|p\rangle = \bar{u}(p')\left[\not{n} H_N(x,\xi,t) + \frac{i\sigma^{n\Delta}}{2m_N} E_N(x,\xi,t)\right] u(p)$$

...in the light cone gauge. Other gauges require a Wilson line.

- GPDs are defined using *different* momenta in the initial and final states.
- The limit $p' \to p$ gives us traditional PDFs.



- x is the *average* light cone momentum fraction between initial and final states.
- 2ξ is the light cone momentum fraction *lost* by the target.
- t is the invariant momentum transfer.

Going up in spin

The deuteron (as a spin-1 system) has **more GPDs** than the proton.

- A spin-0 system (π, ^{4}He) has **one** vector GPD.
- A spin-$\frac{1}{2}$ system (p, n, ^{3}H, ^{3}He) has **two** vector GPDs.
- A spin-1 system (**deuteron**, ρ) has **five** vector GPDs.

This increase in the number of GPDs is analogous to the increasing number of form factors, or of DIS structure functions, as spin increases.

$$\langle \not{n} \rangle = -(\varepsilon \cdot \varepsilon'^*) H_1 + \frac{(n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon) - (n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*)}{2P \cdot n} H_2 + \frac{(\varepsilon \cdot \Delta)(\varepsilon'^* \cdot \Delta)}{2M_D^2} H_3$$
$$- \frac{(n \cdot \varepsilon)(\Delta \cdot \varepsilon'^*) + (n \cdot \varepsilon'^*)(\Delta \cdot \varepsilon)}{2P \cdot n} H_4 + \left[\frac{(n \cdot \varepsilon)(n \cdot \varepsilon'^*) M_D^2}{(P \cdot n)^2} + \frac{1}{3}(\varepsilon \cdot \varepsilon'^*) \right] H_5$$

This big equation tells us how the five vector GPDs are defined.

- Helpful mnemonic: H_1-H_3 are defined by same Lorentz structures as EM form factors F_1-F_3.

Polynomiality rules for the nucleon

Nucleon GPDs are known to obey **polynomiality sum rules** [X. Ji, J.Phys. G24 (1998) 1181]:

$$\int_{-1}^{1} x^s H_N(x,\xi,t)dx = \sum_{\substack{l=0\\2|l}}^{s} A_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s E_N(x,\xi,t)dx = \sum_{\substack{l=0\\2|l}}^{s} B_{s+1,l}(t)(2\xi)^l - \mathrm{mod}(s,2)C_N(t)(2\xi)^{s+1}$$

- A, B, and C are called **generalized form factors**.
- These rules are a **result of Lorentz covariance**.
- They are **violated for models that break covariance** (*e.g.*, models with Fock space truncations or which use non-relativistic nuclear wave functions).

Spin-1 systems will have polynomiality rules too (due to Lorentz symmetry).

Polynomiality sum rules for the deuteron

I have derived the following sum rules for spin-1 systems (with $x \in [-1, 1]$ convention):

$$\int_{-1}^{1} x^s H_1(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{A}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{F}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_2(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{B}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_3(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s} \mathcal{C}_{s+1,l}(t)(2\xi)^l + \mathrm{mod}(s,2)\mathcal{G}_{s+1}(t)(2\xi)^{s+1}$$

$$\int_{-1}^{1} x^s H_4(x,\xi,t)dx = \sum_{\substack{l=1 \\ 2\nmid l}}^{s} \mathcal{D}_{s+1,l}(t)(2\xi)^l$$

$$\int_{-1}^{1} x^s H_5(x,\xi,t)dx = \sum_{\substack{l=0 \\ 2|l}}^{s-1} \mathcal{E}_{s+1,l+1}(t)(2\xi)^l$$

Only H_1 and H_3 (related to **electric distribution**, not magnetic) have the $(2\xi)^{s+1}$ term. **Two D-terms???**

Special cases of generalized form factors

The first Mellin moments ($s = 0$) give **electromagnetic form factors**:

$$\int_{-1}^{1} H_1(x,\xi,t)dx = F_1(t) \qquad \int_{-1}^{1} H_2(x,\xi,t)dx = F_2(t)$$

$$\int_{-1}^{1} H_3(x,\xi,t)dx = F_3(t) \qquad \int_{-1}^{1} H_4(x,\xi,t)dx = \int_{-1}^{1} H_5(x,\xi,t)dx = 0$$

The second Mellin moments ($s = 1$) give **gravitational form factors**:

$$\int_{-1}^{1} xH_1(x,\xi,t)dx = \mathcal{G}_1(t) + (2\xi)^2\mathcal{G}_3(t) \qquad \int_{-1}^{1} xH_2(x,\xi,t)dx = \mathcal{G}_5(t)$$

$$\int_{-1}^{1} xH_3(x,\xi,t)dx = \mathcal{G}_2(t) + (2\xi)^2\mathcal{G}_4(t)$$

$$\int_{-1}^{1} xH_4(x,\xi,t)dx = (2\xi)\mathcal{G}_6(t) \qquad \int_{-1}^{1} xH_5(x,\xi,t)dx = \mathcal{G}_7(t)$$

Information contained in GFFs

The GFFs contain extra information that electromagnetic FFs don't.

- Can construct a **Newtonian form factor** (monopole gravitational) and define a **gravitational radius**:

$$\mathcal{G}_N(t) = \left(1 + \frac{2}{3}\tau\right)\mathcal{G}_1(t) - \frac{2}{3}\tau\mathcal{G}_5(t) + \frac{2}{3}\tau(1+\tau)\mathcal{G}_2(t)$$

  where $\tau = -t/(4M_D^2)$.

$$\langle r_G^2 \rangle = 6\frac{d}{dt}\left[\mathcal{G}_N(t)\right]$$

- Taneja *et al.* (Phys.Rev. **D86** (2012) 036008) tell us that

$$J(t) = \frac{1}{2}\mathcal{G}_5(t)$$

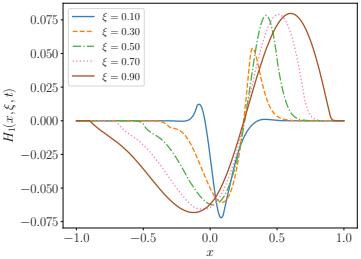
- To unambiguously extract this information requires GPD calculations to obey polynomiality. **Lorentz covariance in GPD calculations is a necessity.**

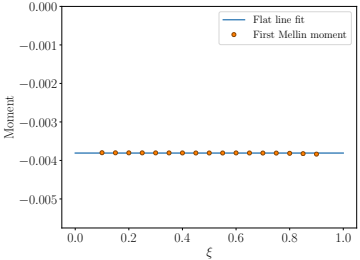
Importance of covariance

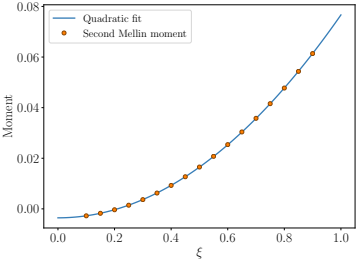
Let's say we want to compute the deuteron D-term(s).

- One can compute $H_1(x, \xi, t)$ and $H_3(x, \xi, t)$ in a covariant framework.
- Take the second Mellin moments of these at multiple ξ values and fit to $A(t) + B(t)\xi^2$ per t value.
- $B(t)$ is proportional to the desired form factor.

Example: look at first and second moments of H_1, at $t = -0.5$ GeV^2:







This doesn't work for light cone approaches with a truncated Fock space.

Motivation for a contact model

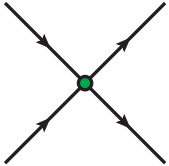
For computing the GPDs themselves, covariance is of the utmost importance.

- Can be difficult to maintain covariance while solving a bound state equation for fermions.
  - Impressive headway is being made for realistic BSE kernels by W. de Paula, *et al.* [PRD94 (2016), 071901], and Carbonell and Karmanov [EPJA46 (2010), 387].
  - However we want a simpler approach that can be immediately generalized to 3+ body systems.
- Covariantly solving a four-Fermi contact interaction is tractable.
- Success of the Nambu-Jona-Lasinio (NJL) model suggests this approach has promise.
- Thus we consider **a contact model of nucleon-nucleon interactions**.

Lagrangian

Construct most general possible NN Lagrangian that:

- Has four-fermi contact interactions.
- Has no derivatives in interaction terms.
- Obeys $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_A$ isospin symmetry.

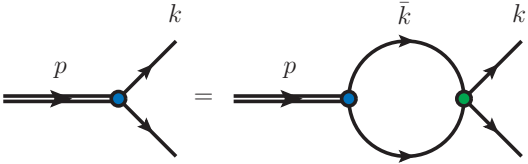


$$
\begin{aligned}
\mathcal{L}_{NN} = \bar{\psi}(i\partial\!\!\!/ - m)\psi \\
&- G_S \left[\left(\bar{\psi}\tau_j C \tau_2 \bar{\psi}^T\right) \left(\psi^T C^{-1} \tau_2 \tau_j \psi\right) - \left(\bar{\psi}\tau_j \gamma^5 C \tau_2 \bar{\psi}^T\right) \left(\psi^T C^{-1} \tau_2 \gamma^5 \tau_j \psi\right) \right] \\
&- G_V \left[\left(\bar{\psi}\tau_j \gamma^5 \gamma^\mu C \tau_2 \bar{\psi}^T\right) \left(\psi^T C^{-1} \tau_2 \gamma^5 \gamma_\mu \tau_j \psi\right) + \left(\bar{\psi}\gamma^\mu C \tau_2 \bar{\psi}^T\right) \left(\psi^T C^{-1} \tau_2 \gamma_\mu \psi\right) \right] \\
&- \frac{1}{2} G_T \left[\left(\bar{\psi} i\sigma^{\mu\nu} C \tau_2 \bar{\psi}^T\right) \left(\psi^T C^{-1} \tau_2 i\sigma_{\mu\nu} \psi\right) \right]
\end{aligned}
$$

- Neglect charge-symmetry violation (assume $m_p = m_n \equiv m_N$).
- Interactions decouple into separate isoscalar and isovector sectors.
- Vector and tensor interactions both contribute to the deuteron.

Bethe-Salpeter vertex

- We can solve the Bethe-Salpeter equation



in the contact model:

$$\Gamma_D(p,\lambda) = \left[\alpha_V \not{\varepsilon}(p,\lambda) + i\alpha_T \frac{\sigma^{\varepsilon p}}{M_D}\right] C\tau_2$$

- We consider two variants: a **pure vector** solution, and a **full** solution that includes tensor interactions.
- GPDs, form factors, *etc.* are then computed in an **impulse approximation**.

Solution and static observables

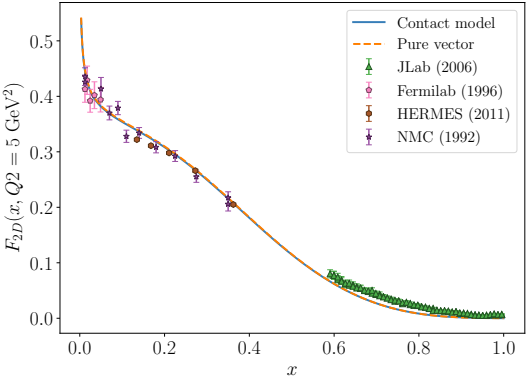
- Couplings G_V, G_T and UV regulator Λ determined by fit to static observables:
  - Deuteron binding energy
  - Deuteron electromagnetic moments
  - $^3S_1$-$^3D_1$ scattering parameters.

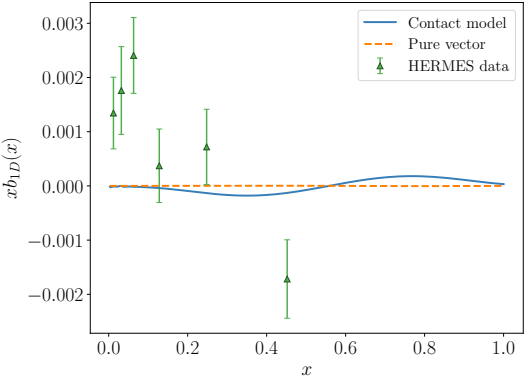
	Pure vector	Full model	Empirical
ϵ_D (MeV)	2.19	2.18	2.22
r_E (fm)	2.10	2.09	2.14
μ_D	0.882	0.879	0.857
$\mathcal{Q}_D$ (fm^2)	**-0.0074**	**0.285**	**0.286**
$^3a_1$ (fm)	5.34	5.26	5.42
$^3r_1$ (fm)	1.77	1.78	1.76
Λ (MeV)	135	139	—
G_V (GeV^{-2})	42.8	-683	—
G_T (GeV^{-2})	0	-715	—

Full model can describe the quadrupole moment.

DIS structure functions

How well can this model describe DIS structure functions?
(Use CJ15 for nucleon PDFs; **impulse approximation**.)

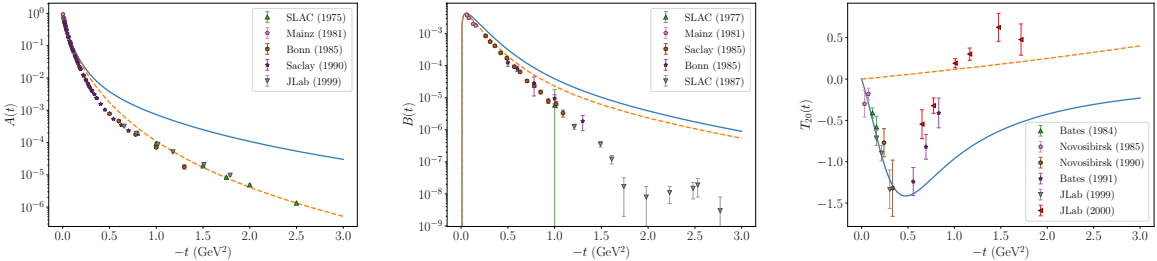




- Not bad for $F_2(x, Q^2)$ (underestimate at high x due to lack of target mass corrections).
- Doesn't describe HERMES data for $b_1(x, Q^2)$, but that's expected.

Electromagnetic form factors

What about electromagnetic form factors?
(Use Kelly-Riordan nucleon form factors.)



- **Pure vector** can describe **unpolarized** structure, but not tensor-polarized.
- **Full model** works only at small $-t$, but contains tensor polarization.
- Reminiscent of NJL model—subleading structures become too large at large $-t$.
- **Contact model limited to small** $-t$. Limited applicability of GPDs obtained here.
- We are working on adding pion exchange to the pure vector model—could add tensor polarization without overestimating $A(t)$ and $B(t)$.

Convolution formalism

To use contact model B-S vertex, need **convolution formula** (impulse approximation) for nuclear GPDs.
This is ostensibly straightforward:

- Get a model for the nucleon GPDs H_N and E_N.
- Compute the matrix element

$$\langle p', \lambda' | \left[\not{n} H_N + \frac{i\sigma^{n\Delta}}{2m_N} E_N \right] | p, \lambda \rangle$$

  assuming pointlike nucleons.
  (The factors H_N and E_N *fold in the non-pointlike structure.*)
- An **ambiguity** arises: identities like Gordon decomposition that are true for on-shell nucleons **will lead to different results for off-shell nucleons**.
- This turns out to matter for the nucleon D-terms.

The D-term and Gordon decomposition

In models such as [Goeke *et al.*, Prog. Part. Nucl. Phys. 47 (2001)], the nucleon GPD is broken into a **double distribution** and a **D-term**:

$$H_N(x,\xi,t) = H_{DD}(x,\xi,t) + D\left(\frac{x}{\xi},t\right) \qquad E_N(x,\xi,t) = E_{DD}(x,\xi,t) - D\left(\frac{x}{\xi},t\right)$$

- The D-term here contributes to the $(2\xi)^{s+1}$ GFF in the polynomiality sum rules.
- The same D-term enters both H_N and E_N with opposite sign.
- **This is due to Lorentz invariance.** [X. Ji, J.Phys. G24 (1998) 1181]

Using **Gordon decomposition**, we can write:

$$\bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_N + \frac{i\sigma^{n\Delta}}{2m_N}E_N\right]u(\mathbf{p},\sigma) = \bar{u}(\mathbf{p}',\sigma')\left[\not{n}H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N}E_{DD} + \frac{p\cdot n}{m_N}D_N\right]u(\mathbf{p},\sigma)$$

for *on-shell* spinors.

We must decide between the LHS and RHS for the "unmodified" deuteron GPD.

- **The RHS gives polynomiality.**
- Meson GPD calculations in the NJL model tell us that **the RHS is correct for off-shell quarks**.
- Off-shell nucleon derivation in progress.

The master convolution formula

Evaluating the matrix element

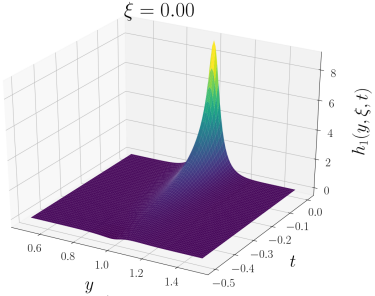
$$\langle p', \lambda' | \left[\not{n} H_{DD} + \frac{i\sigma^{n\Delta}}{2m_N} E_{DD} + \frac{p \cdot n}{m_N} D_N \right] | p, \lambda \rangle$$

gives a **master convolution formula**:

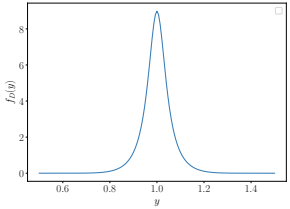
$$H_i(x, \xi, t) = \int \frac{dy}{y} \left[h_i(y, \xi, t) H_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + e_i(y, \xi, t) E_{DD}\left(\frac{x}{y}, \frac{\xi}{y}, t\right) + y d_i\left(\frac{y}{\xi}, t\right) D_N\left(\frac{x}{\xi}, t\right) \right]$$

- h_i, e_i, and d_i describe how the nucleons are distributed in the nucleus, using GPD language. Call them **generalized nucleon distributions** (GNDs).
- By construction, H_{DD}, E_{DD}, and D_N already obey polynomiality.
- We can prove that when the GNDs obey polynomiality sum rules, so do the deuteron GPDs.
- Taking Mellin moments of the master convolution formula will give **discrete convolution relations** for the GFFs. Numerically faster than computing GPDs directly.
- Also, Gegenbauer moments (linear combinations of Mellin moments) evolve multiplicatively—**easier evolution**. We plan to study their convergence for contact model.

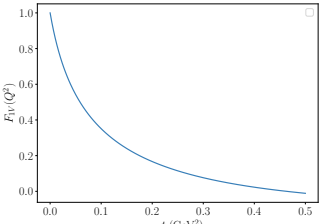
Generalized nucleon distributions (full contact model)



$\xrightarrow{t=0}$



$\downarrow \sum_q e_q \int_{-1}^{1} dx$

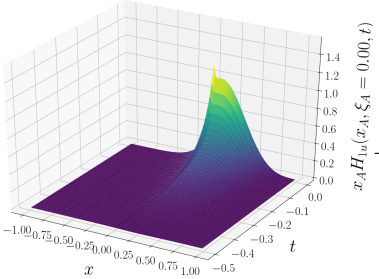


h_1 is the "typical" GND.

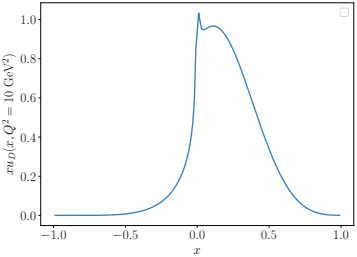
- Reduces to light cone density in forward limit.
- Gives F_{1V} "body" form factor (for pointlike nucleons) when integrated over x.

GNDs are nucleon densities in the deuteron using GPD language.

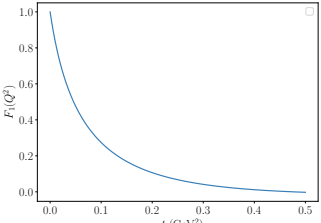
GPD results (full contact model)



$\xrightarrow{t=0}$



$\downarrow \sum_q e_q \int_{-1}^{1} dx$

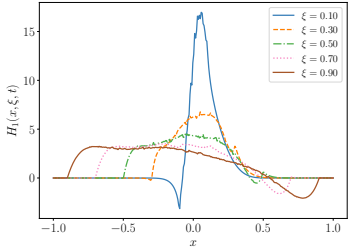


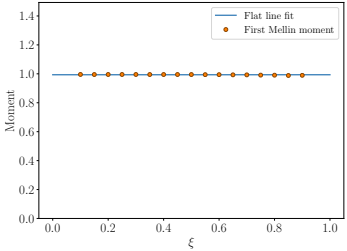
H_1 is the "typical" GPD. (Dominated by monopole.)

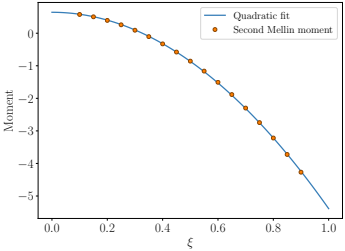
- Reduces to unpolarized PDF in the forward limit.
- Gives F_1 form factor (for real nucleons) when integrated over x.

Skewed up quark GPDs (in full contact model)

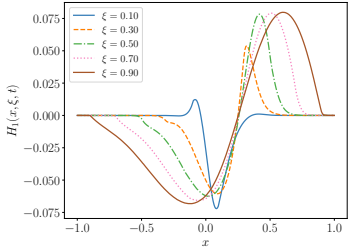
At $t = 0$ (please forgive numerical jitters! They're part of why I'm looking at Gegenbauer moments...)

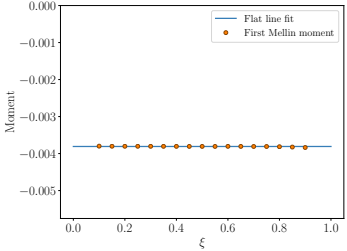


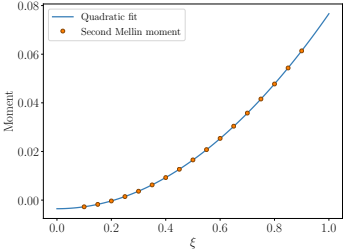




At $t = -0.5$ GeV^2 (lack of jitters)

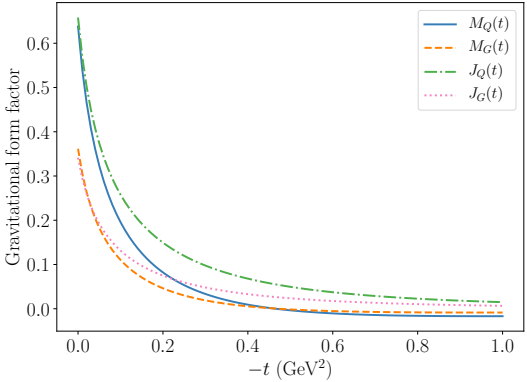






Gravitational form factors

Because of polynomiality, we can get **unambiguous gravitational form factor extractions**.



Can estimate mass radii.

- $\langle r_{MQ} \rangle \approx \langle r_{MG} \rangle \approx 1.96$ fm
- Compare to $\langle r_E \rangle \approx 2.09$ fm in this model.

Impact parameter PDFs

Another neat application of GPDs: **impact parameter PDFs**.

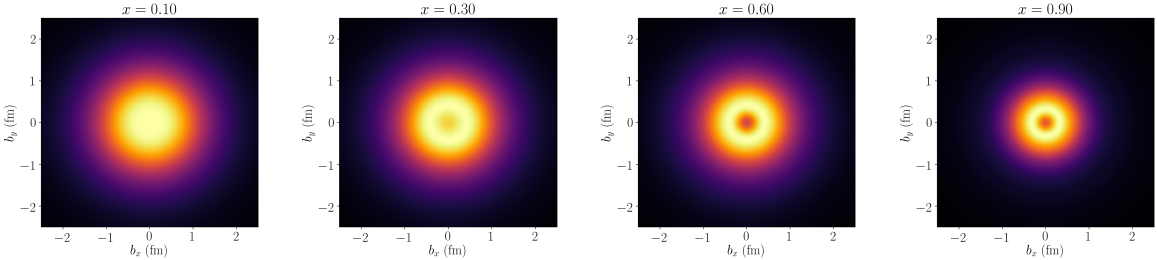
- Zero skewness ($\xi = 0$) means light cone Fock space expansion poses no problems.
- Final and initial states related by *kinematic transforms* (assuming light cone quantization).
- Can do a **two-dimensional** Fourier transform on transferred momentum:

$$\rho_q(x, \mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp V_q(x, \xi = 0, t = -\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{k}_\perp}$$

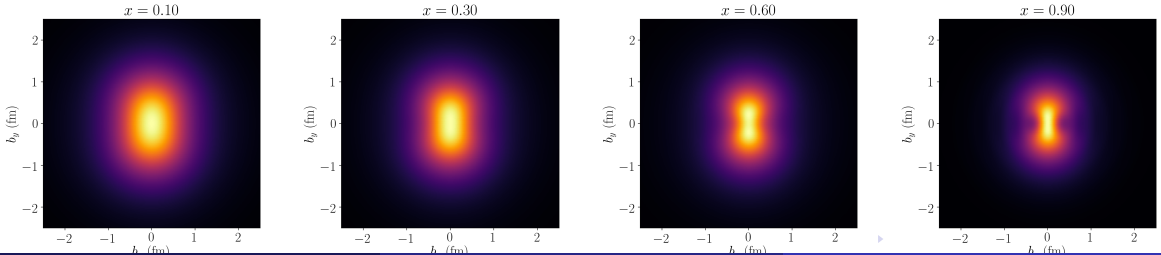
  (where V is the full light cone correlation operator).
- The result is a positive-definite density of partons over x and **transverse spatial position**.
- **It's a relativistic spatial density!** (Thanks to light cone physics.)
- See G. Miller, Ann.Rev.Nucl.Part.Sci. 60 (2010) 1-25

Impact parameter PDFs—contact model

Light cone helicity zero: donut-shaped quark density



Transversely-polarized deuteron: peanut-shaped density



Conclusions and outlook

In conclusion:

- We have calculated deuteron GPDs in a manifestly covariant contact model.
- Our GPDs obey polynomiality sum rules, and allow an unambiguous extraction of generalized form factors.

Future work to be done:

- We will use these GPDs to make predictions for cross sections and asymmetries in DVCS, for both JLab and the EIC.
- The model will be extended to other light nuclei (triton and helium).
- The NJL model can be used to compute covariant *nucleon* GPDs. (We've already computed meson GPDs!)
- We're working on how to add long-range pion exchange to the model for more accurate behavior at high $-t$.

Thanks for your time and attention!