Probing the QCD Wigner distribution in diffractive di-jet production

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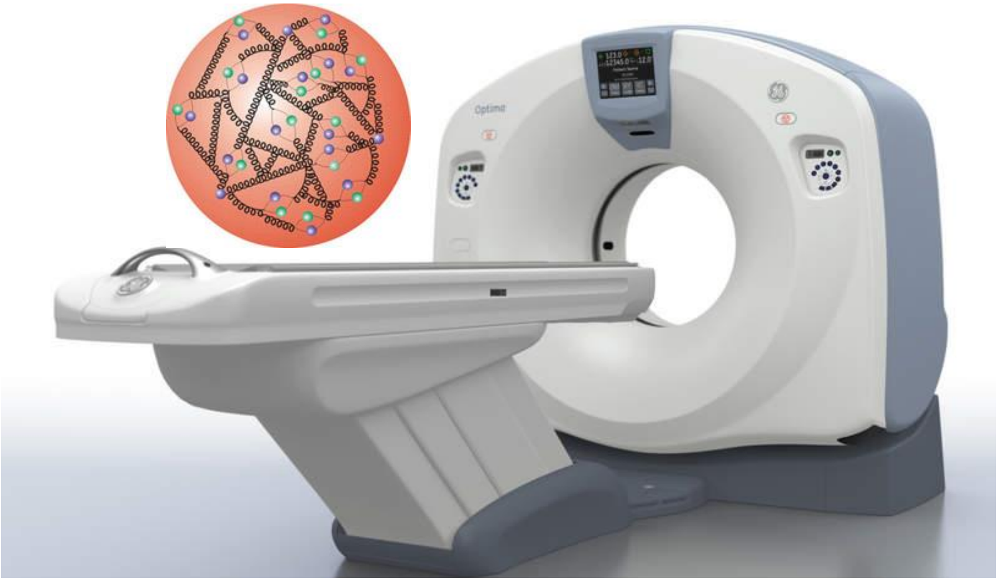
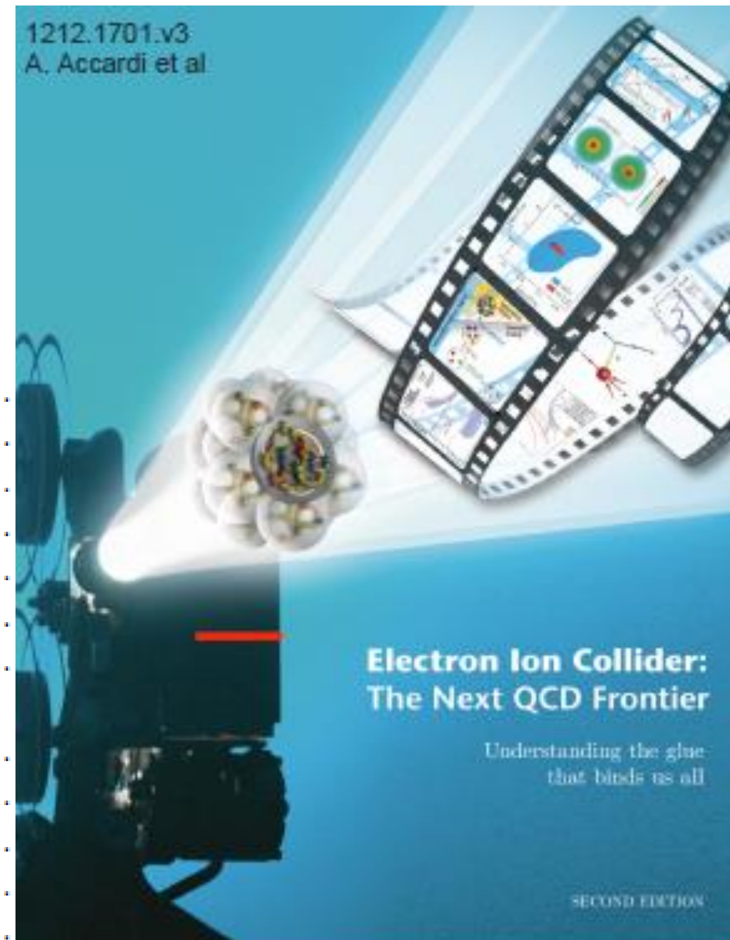
Outline

- Nucleon tomography
- Phase space distributions in QCD
- Connection to experimental observables

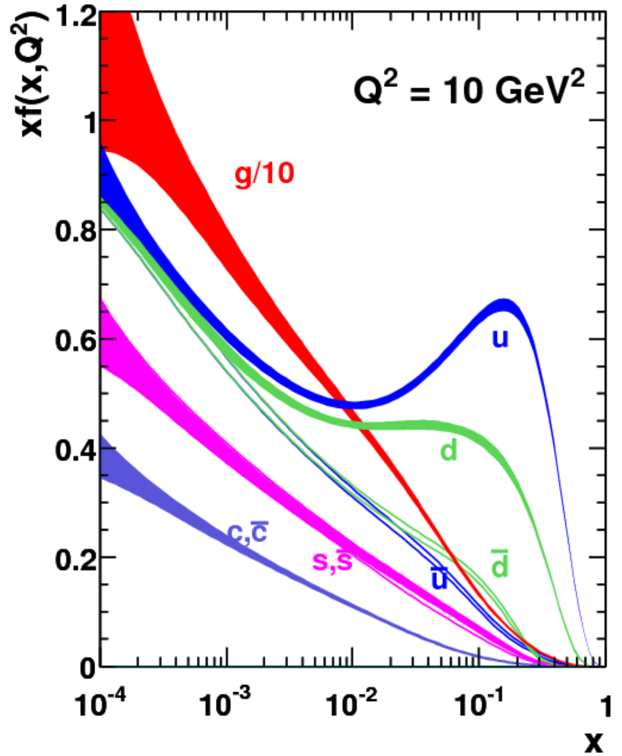
Nucleon tomography



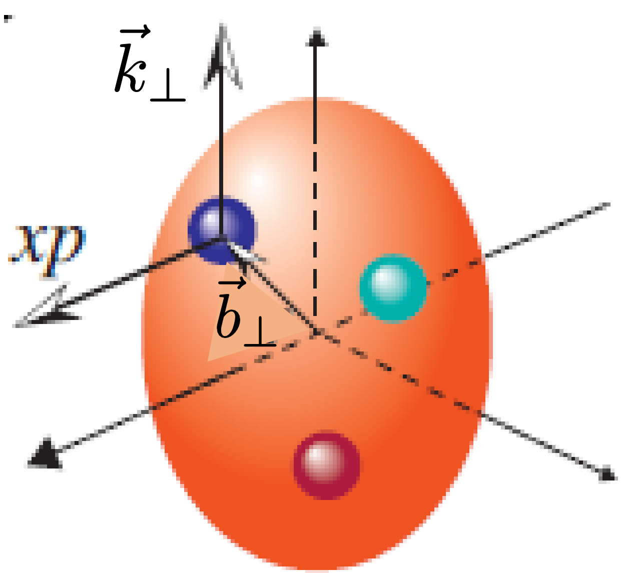


1D tomography: Parton distribution function (PDF)

$$f(x)=\int\frac{dz^-}{4\pi}e^{ixP^+z^-}\langle P|\bar{q}(-\frac{z^-}{2})\gamma^+q(\frac{z^-}{2})|P\rangle$$



Probability distribution of quarks and gluons with longitudinal momentum fraction $x=\frac{p^+_{parton}}{P^+_{proton}}$

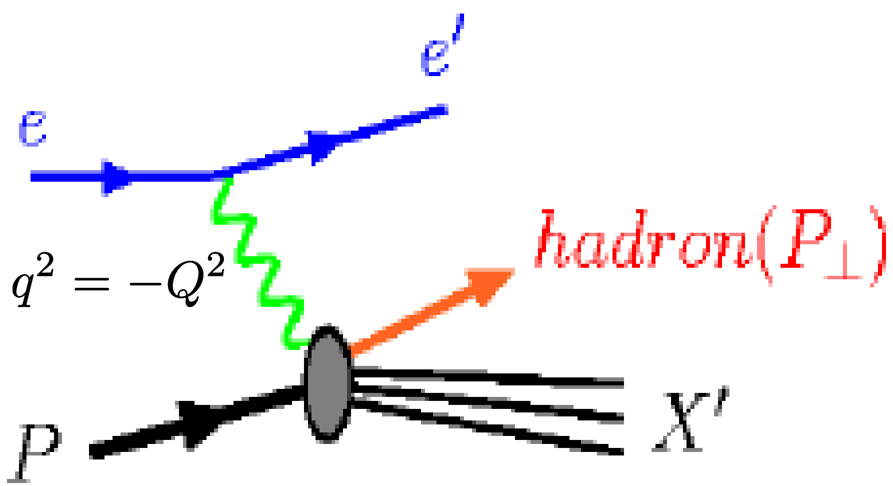


The nucleon is much more complicated!
Partons also have transverse momentum $\vec{k}_\perp$
and are spread in impact parameter space $\vec{b}_\perp$

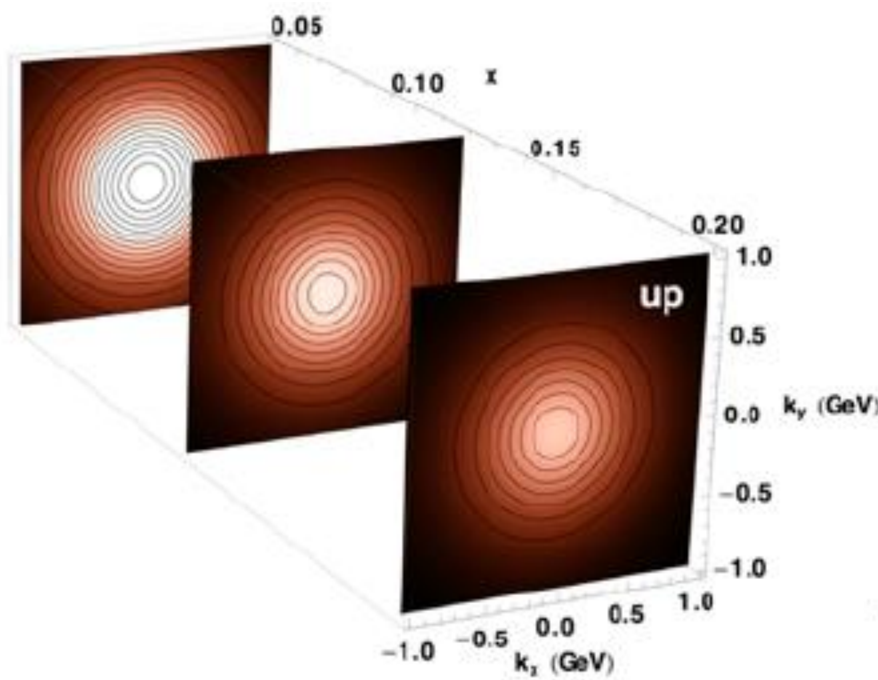
3D tomography:
Transverse momentum dependent distributions (TMD)

$$f(x,\vec{k}_\perp) = \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P|\bar{q}(-z/2)\gamma^+ W q(z/2)|P\rangle$$

Relevant in semi-inclusive DIS (SIDIS), etc.



$$Q \gg P_\perp \gtrsim \Lambda_{QCD}$$



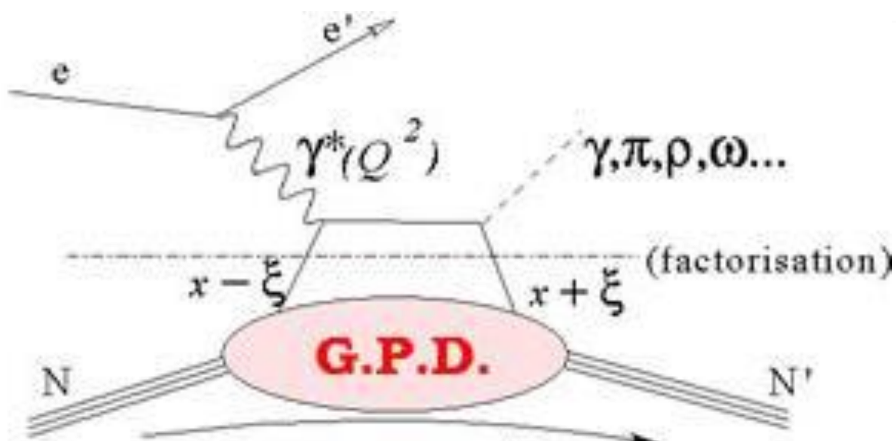
3D tomography:
Generalized parton distributions (GPD)

$$f(x,\vec{\Delta}_\perp) \sim \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2)\gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$

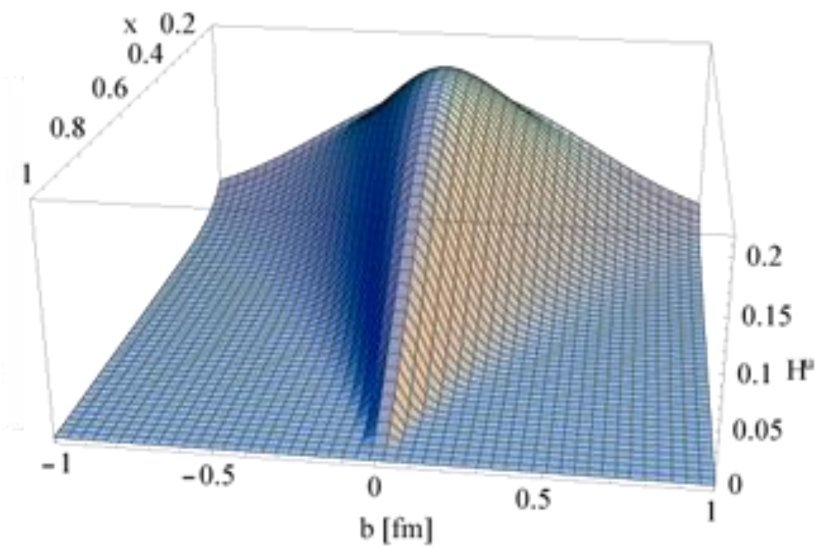
$\Longleftrightarrow$ $f(x,\vec{b}_\perp)$ distribution of partons in impact parameter space

Fourier transform

Deeply Virtual Compton Scattering (DVCS)



$$t = -\vec{\Delta}_\perp^2$$



5D tomography:
Wigner distribution— the "mother distribution"

Belitsky, Ji, Yuan (2003);
Lorce, Pasquini (2011)

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \frac{\Delta}{2}|\bar{q}(-z/2)\gamma^+ q(z/2)|P + \frac{\Delta}{2}\rangle$$

$$\begin{tikzcd}
 & W(x, \vec{k}_\perp, \vec{b}_\perp) \arrow[dl, "\int d\vec{b}_\perp"'] \arrow[dr, "\int d\vec{k}_\perp"] & & \\
\text{TMD}\quad f(x, \vec{k}_\perp) \arrow[dr, "\int d\vec{k}_\perp"'] & & \text{GPD}\quad f(x, \vec{b}_\perp) \arrow[dl, "\int d\vec{b}_\perp"] \arrow[dr, "\int dx"] & \\
 & \underset{\text{PDF}}{f(x)} \arrow[dr, "\int dx"'] & & F(\vec{b}_\perp)\quad \text{Form factor} \arrow[dl, "\int d\vec{b}_\perp"] \\
 & & \underset{\text{charge}}{Q} &
\end{tikzcd}$$

5D tomography: GTMD and Husimi

GTMD Meissner, Metz, Schlegel (2009)

$$G(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$

Husimi Hagiwara, YH (2015)

$$H(x, \vec{k}_\perp, \vec{b}_\perp)$$

Gaussian smearing in k, b

Wigner

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$

$\int d\vec{b}_\perp$

$\int d\vec{k}_\perp$

TMD $f(x, \vec{k}_\perp)$

GPD $f(x, \vec{b}_\perp)$

$\int d\vec{k}_\perp$

$\int d\vec{b}_\perp$

$\int dx$

$f(x)$

PDF

$F(\vec{b}_\perp)$ Form factor

$\int dx$

Q

charge

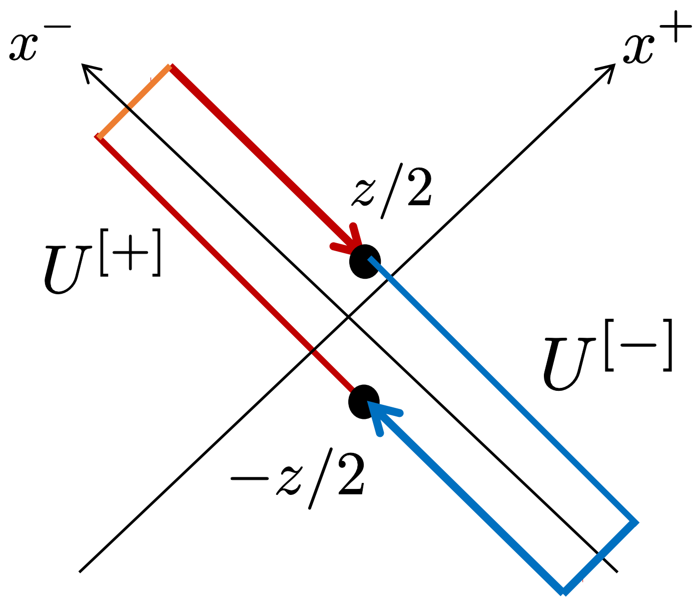
$\int d\vec{b}_\perp$

Gluon Wigner distribution—there are two of them!

$$xW(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\Delta/2|F^{+i}(-z/2)F^{+}_{i}(z/2)|P+\Delta/2\rangle$$

There are two ways to make it gauge invariant

Bomhof, Mulders (2008)
Dominguez, Marquet, Xiao, Yuan (2011)



Weizsacker-Williams (WW) distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[+]}]$$

Dipole distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[-]}]$$

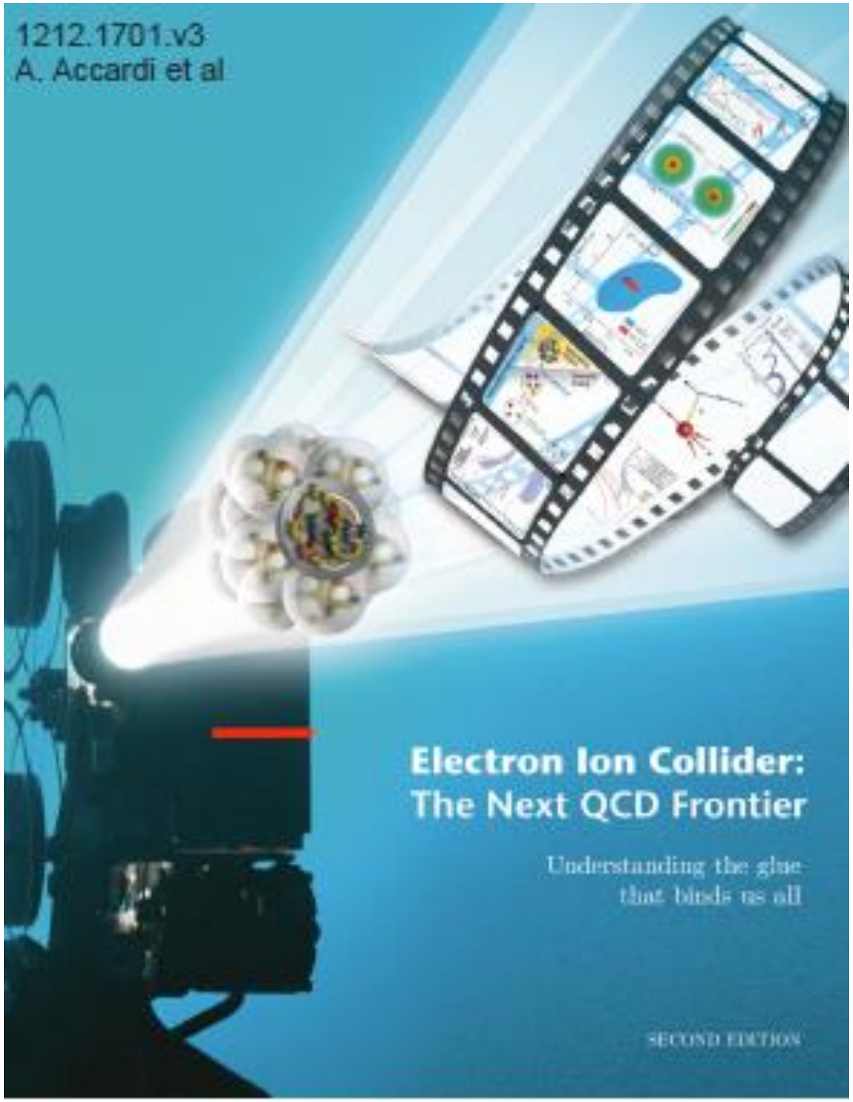
Wigner in 2012 EIC white paper?

Almost no account.
Only briefly mentioned in two places.

Although there is no known way to measure Wigner distributions for quarks and gluons, they provide a unifying theoretical framework for the different aspects of hadron structure.

A lot of progress since then!

Wigner $\neq$ TMD+GPD



Wigner distribution and orbital angular momentum

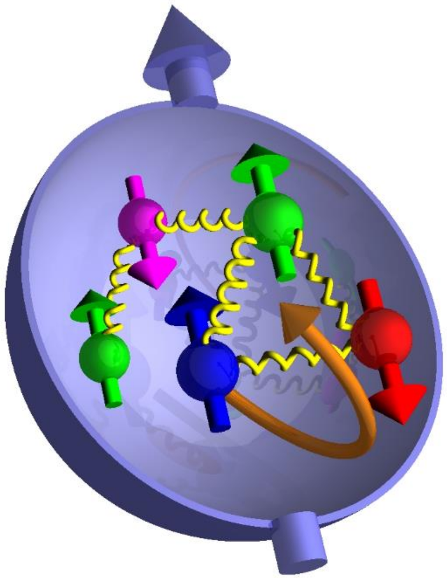
Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Canonical Orbital angular momentum (OAM)



$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Lorce, Pasquini, (2011);
YH (2011)
Lorce, Pasquini, Xiong, Yuan, (2011)

For the gluon OAM, dipole and WW Wigner give the same result.

`Entropy' of partons

Hagiwara, YH, Xiao, Yuan (2018)

Phase space distribution naturally defines an entropy.
Use the QCD Husimi distribution

$$S(x) \equiv - \int d^2b_\perp d^2k_\perp xH(x,b_\perp,k_\perp) \ln xH(x,b_\perp,k_\perp)$$

$$S(x) \sim \frac{N_c}{\alpha_s} Q_s^2(x) S_\perp \propto A \left(\frac{1}{x}\right)^{\#\alpha_s}$$

$x \to 0$

cf. Kutak, Kovner-Lublinsky

Measure of `complexity' of the multiparton system.
Proportional to the final state multiplicity?
Saturation of entropy due to the Pomeron loop effect?

Wigner distribution: Is it measurable?

In quantum optics, yes!

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Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Raymer
Department of Physics and Chemical Physics Institute, U

A. Faridani
Department of Mathematics, Oregon State Uni
(Received 16 Novembe

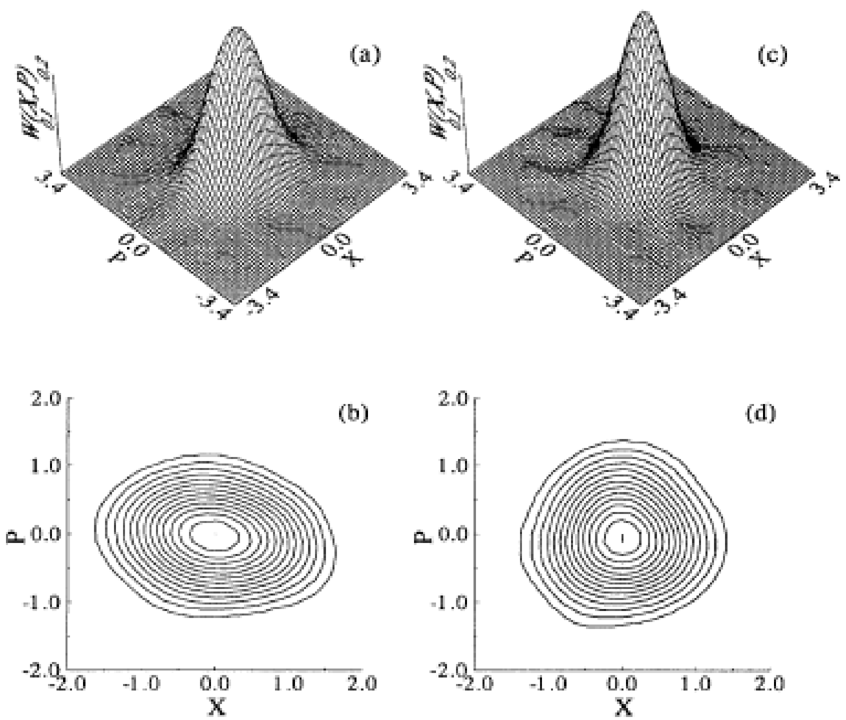


FIG. 1. Measured Wigner distributions for (a),(b) a squeezed state and (c),(d) a vacuum state, viewed in 3D and as contour plots, with equal numbers of constant-height contours. Squeezing of the noise distribution is clearly seen in (b).

Measuring Wigner/GTMD in experiments

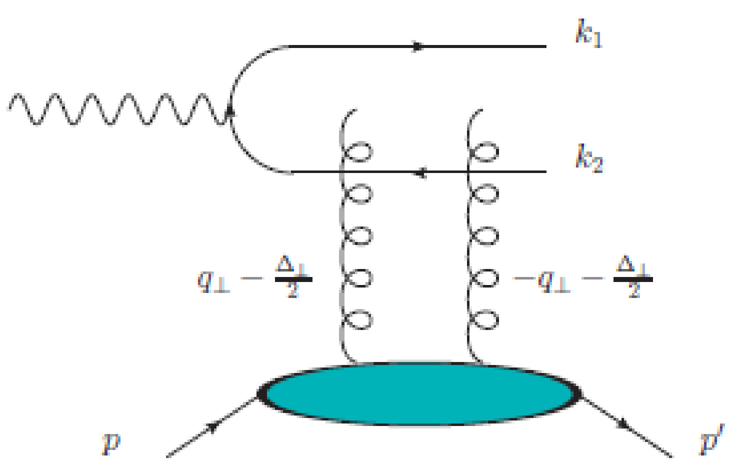
① Tag two hadrons (jets) in the final state, together with the recoiling proton

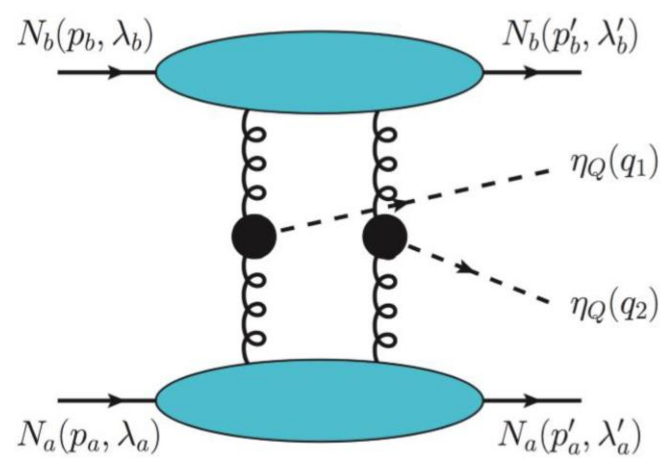
YH, Xiao, Yuan (2016); Bhattacharya, Metz, Zhou (2017)

Relative momentum between the two hadrons

$$G(x, k_\perp, \Delta_\perp)$$

Recoiling proton momentum





Measuring Wigner/GTMD in experiments

② Go to small-x (forward particle production)

Approximate $e^{ixP^+z^-} \approx 1$

$$\boxed{xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp,\vec{r}_\perp)}$$

``Dipole S-matrix"

$$S_x(\vec{b}_\perp,\vec{r}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$

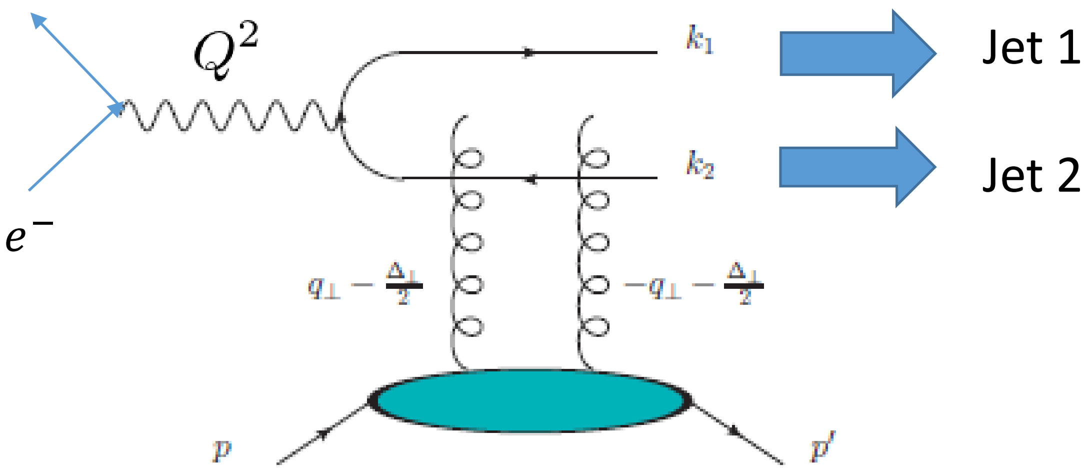
$\cos 2\phi$ correlation expected

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = W_0(x,k_\perp,b_\perp) + 2\cos 2(\phi_k - \phi_b) W_1(x,k_\perp,b_\perp) + \cdots$$

\`Elliptic Wigner' distribution

Probing dipole Wigner (GTMD) in diffractive dijet production

YH, Xiao, Yuan (2016), see also Altinoluk, Armesto, Beuf, Rezaeian (2015)



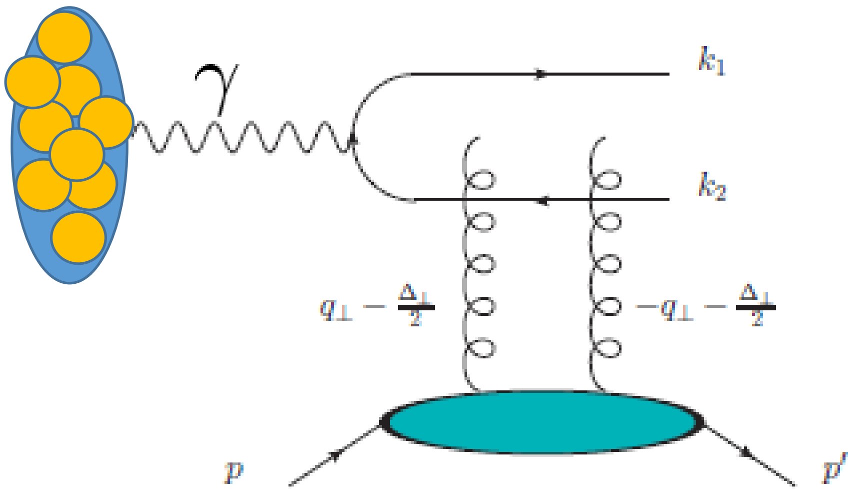
$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$
$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

GTMD

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \propto z(1-z)[z^2+(1-z)^2]\int d^2q_\perp d^2q'_\perp S(q_\perp,\Delta_\perp)S(q'_\perp,\Delta_\perp)$$
$$\times\left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2+\epsilon^2}\right]\cdot\left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2+\epsilon^2}\right]$$
$$\sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta)d\tilde{\sigma}$$

$$\epsilon^2 = z(1-z)Q^2$$

In ultra-peripheral collisions, too!



Hagiwara, YH, Pasechnik, Tasevsky, Teryaev (2017)

Q^2 preferably small

Use the Weiszacker-Williams photons in UPC!

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega \frac{dN}{d\omega} \frac{N_c \alpha_{em} (2\pi)^4}{P_\perp^2} \sum_f e_f^2 2z(1-z)(z^2+(1-z)^2) \left(A^2 + 2\cos 2(\phi_P - \phi_\Delta) AB \right)$$

photon flux $\propto Z^2$

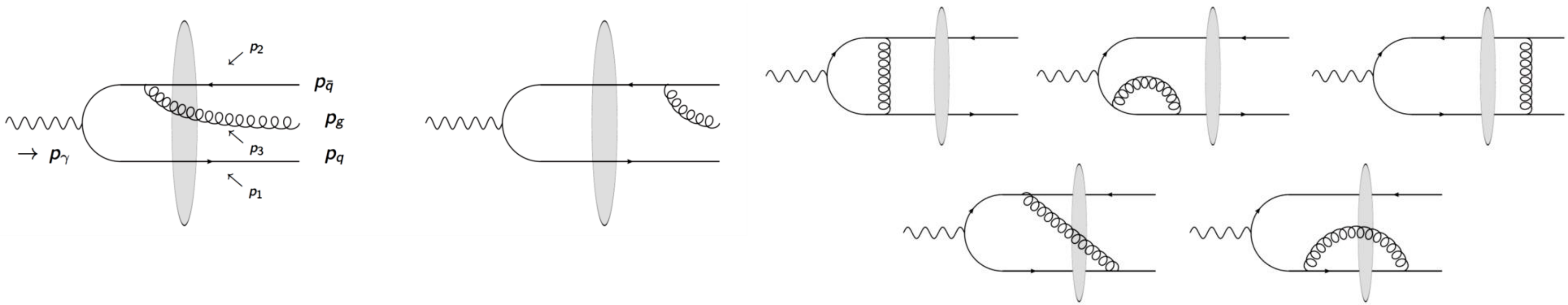
Inversion can be done analytically.

$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp} \frac{\partial}{\partial P_\perp} A(P_\perp, \Delta_\perp).$$

$$S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2} \int^{P_\perp^2} \frac{dP_\perp'^2}{P_\perp'^2} B(P_\perp', \Delta_\perp)$$

Factorization at NLO

Renaud Boussarie, talk at POETIC 8



$$\Phi_L^{(0)} = \frac{2x\bar{x}p_V^+ Q}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2},$$

$$\Phi_T^{(0)} = -\frac{(x-\bar{x})p_V^+(\bar{x}\vec{p}_{1\perp} - x\vec{p}_{2\perp})\cdot\vec{\varepsilon}_{\gamma_T}}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2}$$

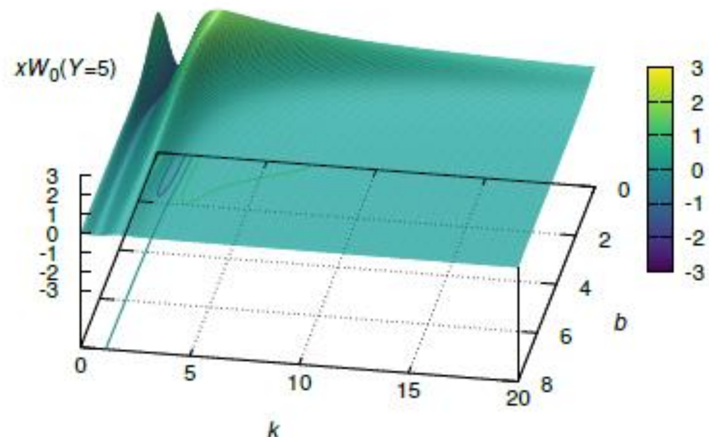
No end point singularity, even for a transverse photon and even in the photoproduction limit and even at NLO.
With null transverse momenta in the t channel, one could encounter $x \in \{0, 1\}$ end point singularities as $\frac{1}{x\bar{x}Q^2}$ thus breaking collinear factorization.

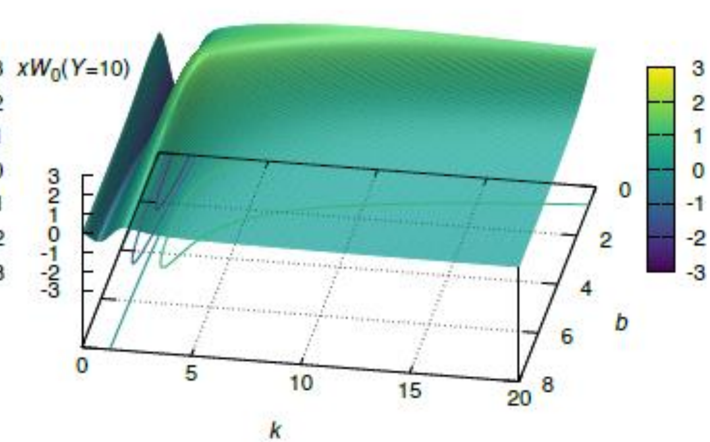
Dipole Wigner from Balitsky-Kovchegov equation

Hagiwara, YH, Ueda (2016)

Peak at the saturation momentum

$W_0(k_\perp, b_\perp)$

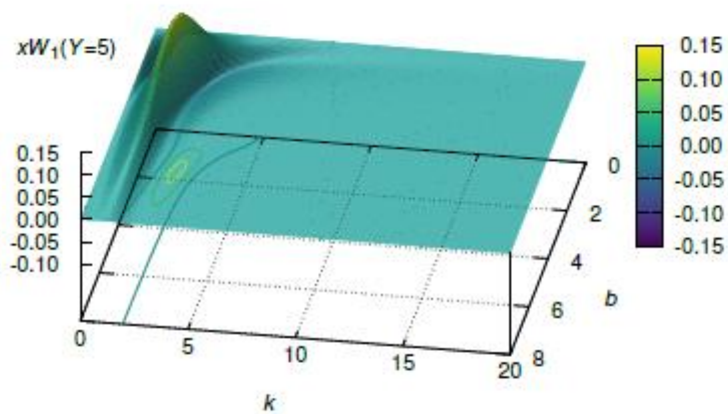


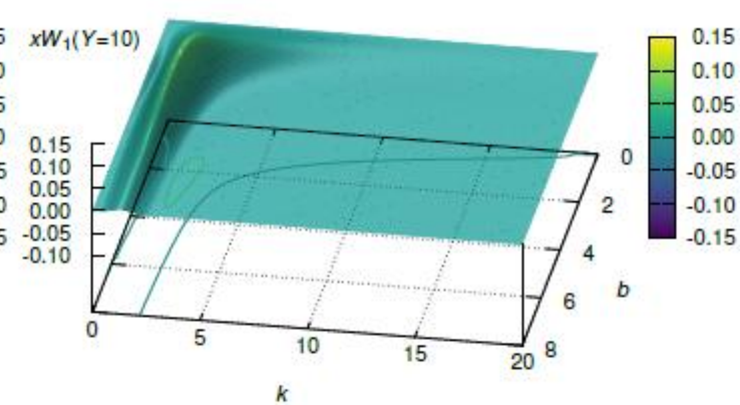


$$Y = \ln\frac{1}{x} = 5$$

$$Y = 10$$

$W_1(k_\perp, b_\perp)$





Elliptic part small in magnitude (a few percent effect). No geometric scaling.

Elliptic Wigner in DVCS

YH, Xiao, Yuan (2017)

Gluon transversity GPD

$$\frac{1}{P^+}\int \frac{d\zeta^-}{2\pi} e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$= \frac{\delta^{ij}}{2} xH_g(x,\Delta_\perp) + \frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j - \frac{\delta^{ij}\Delta_\perp^2}{2}\right) + \cdots,$$

Elliptic GTMD

$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 S_1$$

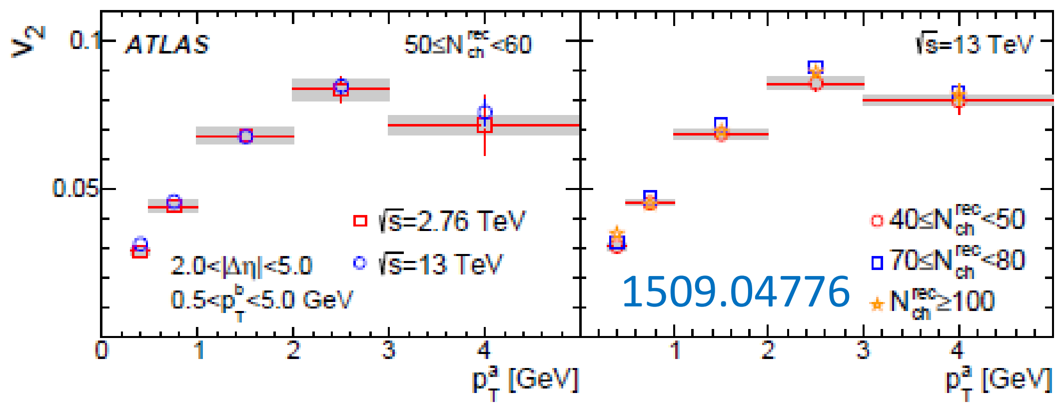
$$\frac{d\sigma(ep\to e'\gamma p')}{dx_B dQ^2 d^2\Delta_\perp} = \frac{\alpha_{em}^3}{\pi x_{Bj}Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)(\mathcal{A}_0^2+\mathcal{A}_2^2) + 2(1-y)\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l})\right.$$
$$\left. +(2-y)\sqrt{1-y}(\mathcal{A}_0+\mathcal{A}_2)\mathcal{A}_L\cos\phi_{\Delta l} + (1-y)\mathcal{A}_L^2\right\}$$

Elliptic Wigner in high-multiplicity pp and pA

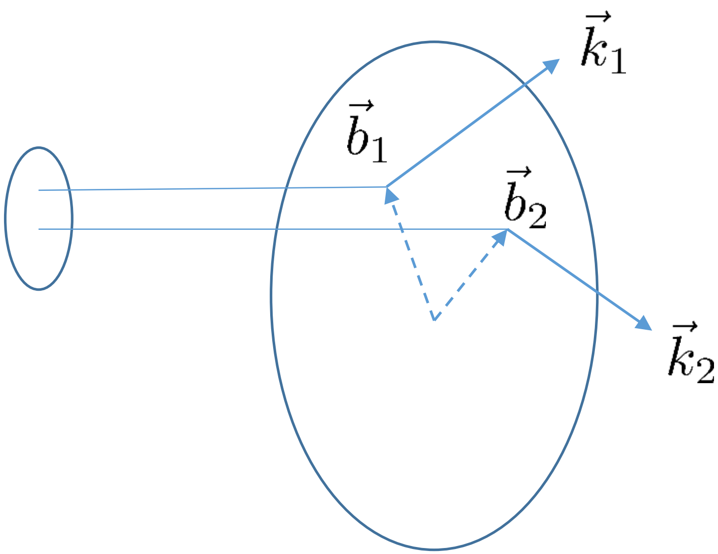
Kopeliovich et al. (2008),
Levin Rezaeian (2011),
Hagiwara, YH, Xiao, Yuan (2017)

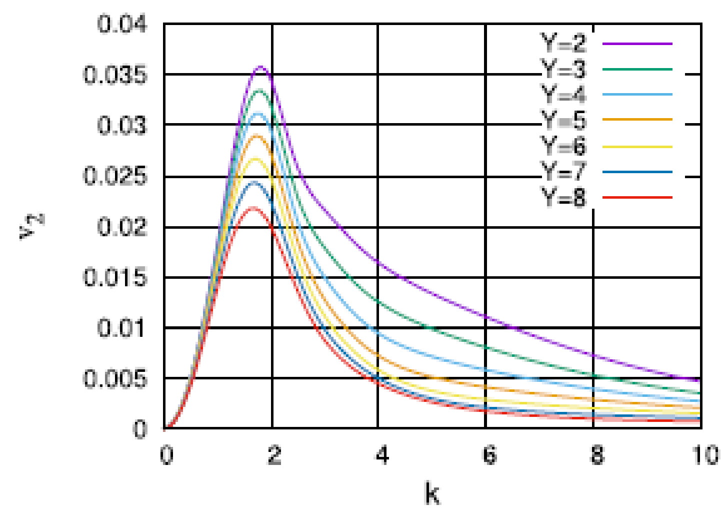
Elliptic flow v_2 observed in high-multiplicity pp and pA.

Initial state or final state effect?



Double parton scattering + elliptic Wigner = elliptic flow



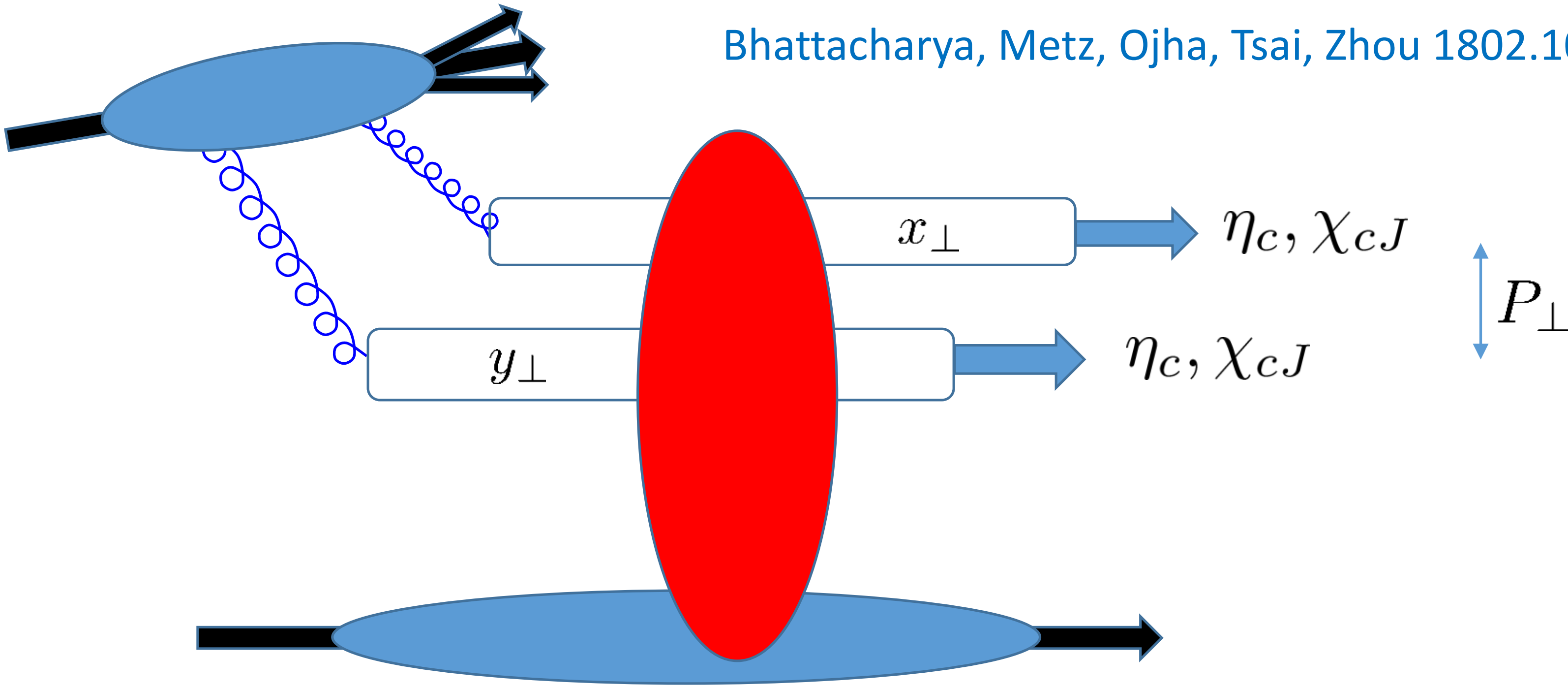


Probing the gluon WW GTMD in pp

Boussarie, YH, Xiao, Yuan, 1807.08697

See also,

Bhattacharya, Metz, Ojha, Tsai, Zhou 1802.10550



Diffractive production of a $C = +1$ quarkonium pair

Amplitude proportional to

$$\int d^2(x_\perp - y_\perp) e^{iP_\perp \cdot (x_\perp - y_\perp)} \langle P' | U_x \vec{\partial} U_x^\dagger U_y \vec{\partial} U_y^\dagger | P \rangle$$

Boussarie, YH, Xiao, Yuan, 1807.08697

Very simple result in the case of χ_{c1}, χ_{c1} production, in the limit $P_\perp \gg \Delta_\perp$

Gluon dPDF

Long distance matrix element (LDME)

$$\frac{d\sigma}{dY_1 dY_2 d^2\Delta_\perp d^2P_\perp} = \frac{x_1 x_2 F(x_1, x_2)}{64 m^{18} N_c^4 (N_c^2-1)^2} \alpha_s^4 \left\langle \mathcal{O}_{\chi_1} \right\rangle^2 P_\perp^4 \left(G(P_\perp, \Delta_\perp) + \frac{P_\perp^2}{2M^2} G_2 \right)^2$$

WW gluon GTMD

Linearly polarized WW gluon GTMD

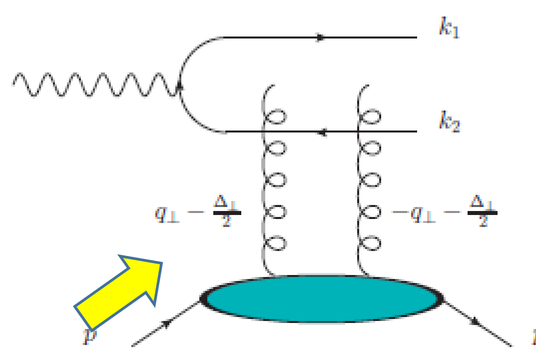
No convolution in $P_\perp$!

Caveat: only color-singlet production included

Towards measuring the orbital angular momentum

Longitudinal single spin asymmetry in dijet production

$$d\Delta\sigma \sim$$



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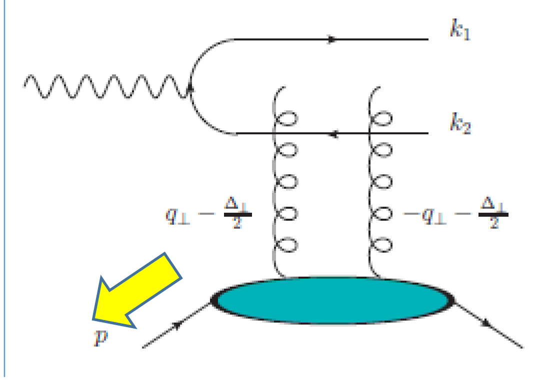

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Large-x gluon	Ji, Yuan, Zhao
Small-x gluon	YH, Nakagawa, Xiao, Yuan, Zhao
quark	Bhattacharya, Metz, Ojha, Tsai, Zhou

Sensitive to the OAM distribution

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = W_0(x,k_\perp,b_\perp) + S^+ \sin(\phi_k - \phi_b) W_{OAM}(x,k_\perp,b_\perp) + \cdots$$

Net angular momentum comes from the large-x region. At small-x, $L_g(x) \approx -\Delta G(x)$

Conclusions

- Let's get 5 dimensional. Even richer physics than TMD and GPD combined. A lot of progress since the 2012 EIC white paper.

- Wigner/GTMD measurable in ep, pp, pA, including the elliptic part and spin-dependent part (connection to OAM).

- Need more foundational works. Proper definition, evolution,…

cf. Ecchevarria, Idilbi, Kanazawa, Lorce, Metz, Pasquini, Schlegel (2016)