# Short-range interactions and multiparticle correlations in collisions of small systems

Mark Mace
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Short-range nuclear correlations at an Electron-Ion Collider
09/07/2018

Stony Brook University

Center for Frontiers in Nuclear Science

BROOKHAVEN NATIONAL LABORATORY

# Outline

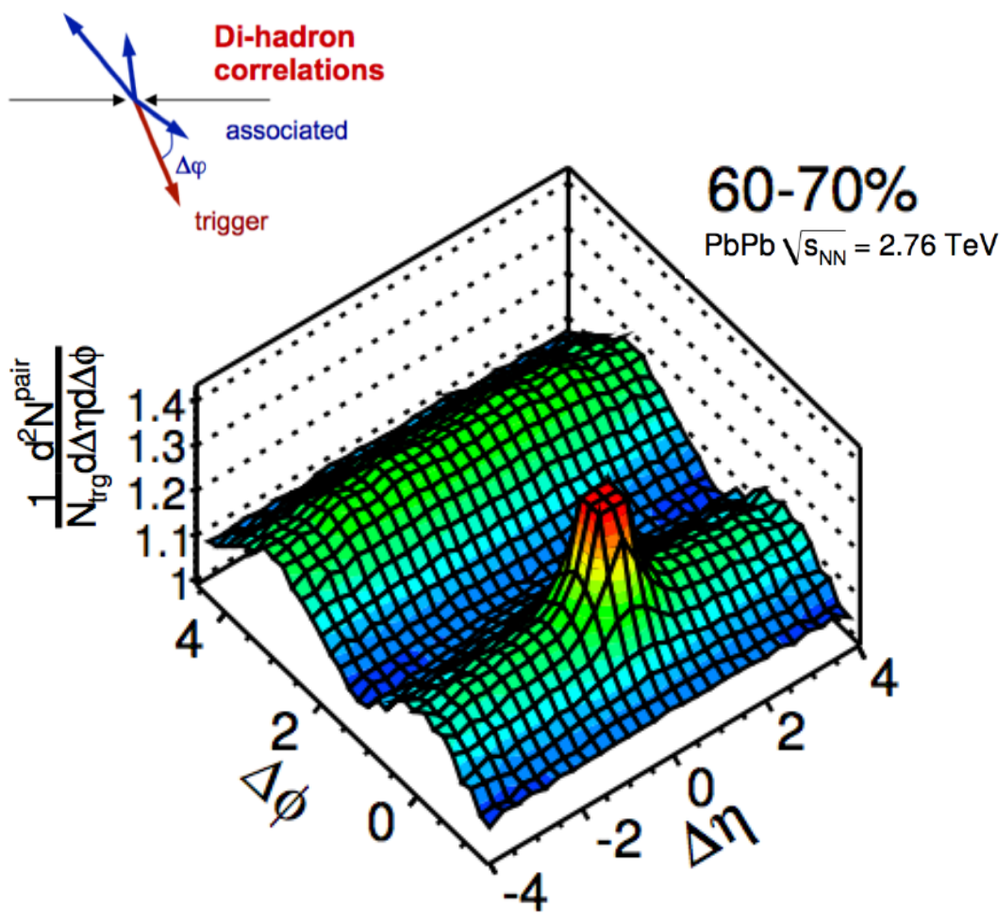
1. Multiparticle collectivity: status and challenges
   **See talk by B. Schenke**

2. Correlations from the Color Glass Condensate
   *MM, V. Skokov, P. Tribedy, R. Venugopalan PRL121 (2018); arXiv:1807.00825*
   *K. Dusling, MM, R. Venugopalan PRL120 (2018), PRD97 (2018)*

3. Opportunities to study SRC in smalls ystems

# The Ridge in A-A

Two-particle correlations as a function of Δη, Δϕ




CMS EPC 72 (2012) 10052

The Ridge: Long range correlations in Δη
Double ridge seen at near side (Δϕ≈0) and away side (Δϕ≈π)

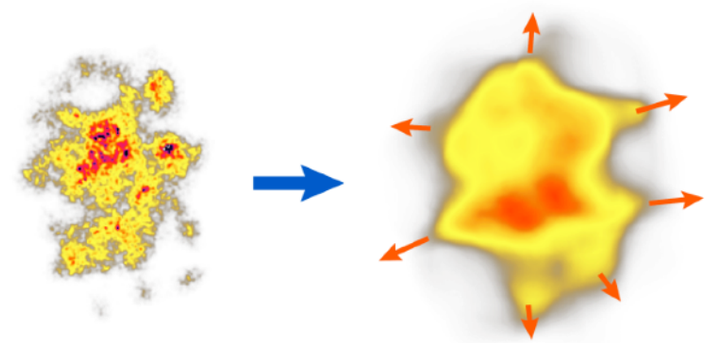
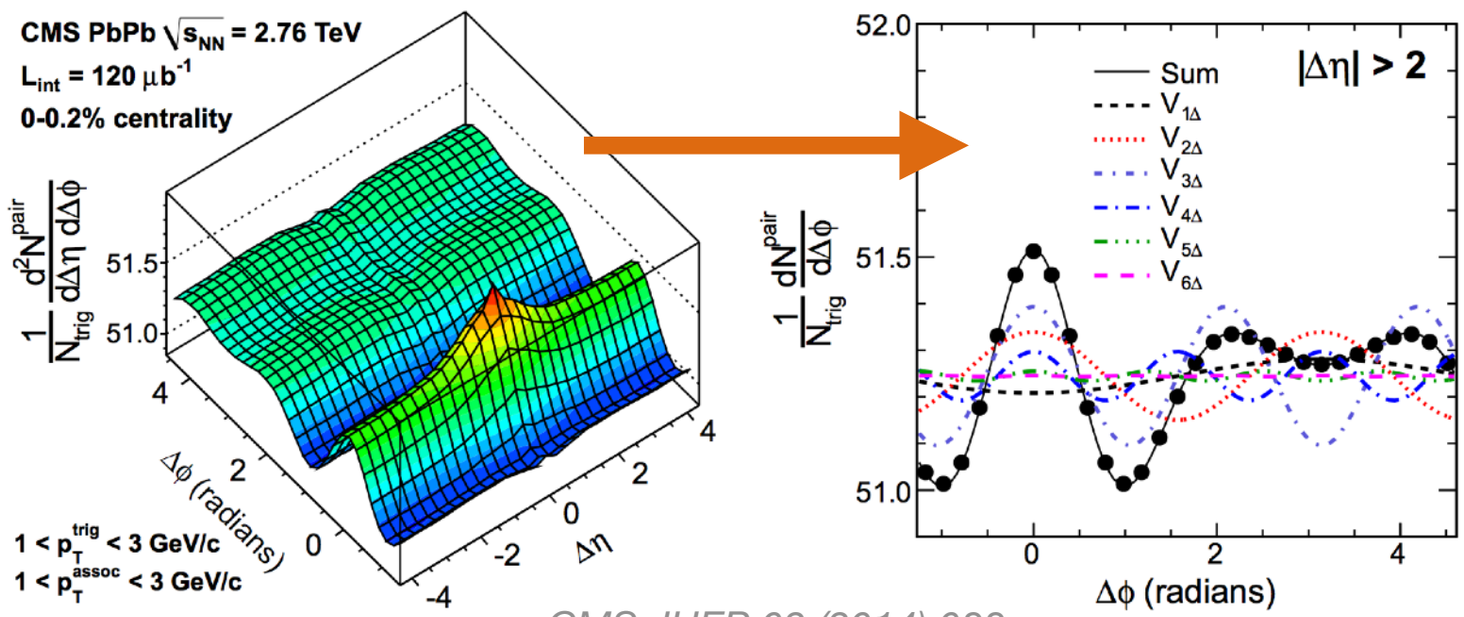


Image: B. Schenke

Event-by-event "eccentricity" fluctuations of the initial transverse geometry transported via hydrodynamics, resulting in a final state momentum correlations

# Multi-particle correlations

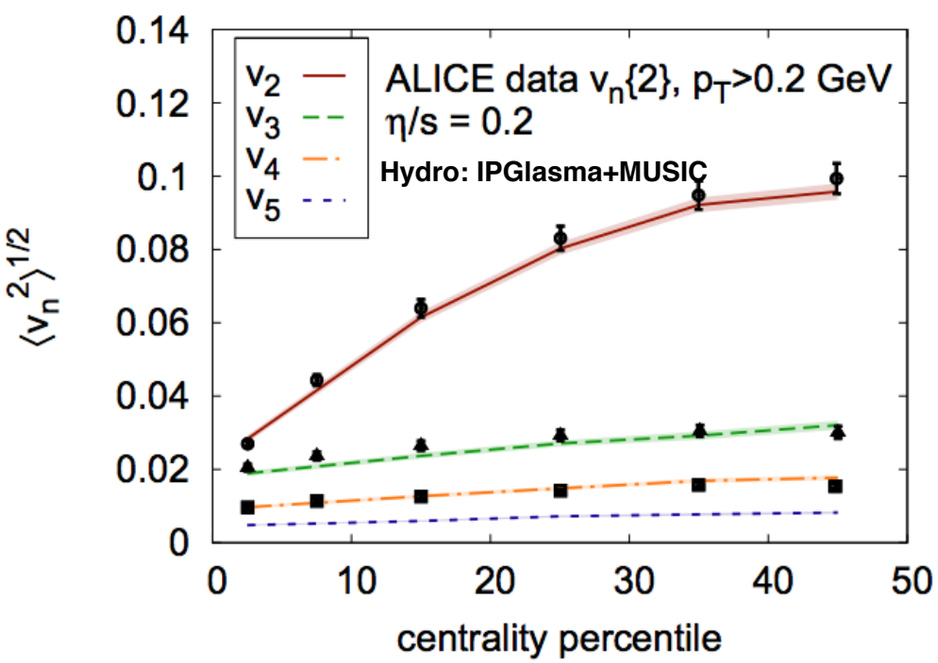



*CMS JHEP 02 (2014) 088*

$$\frac{1}{N_{trig}} \frac{dN^{\text{pair}}}{d\Delta\phi} \sim 1 + 2 \sum_{n=1}^{n=\infty} V_{n\Delta}(\mathbf{p}_T^{\text{trig}}, \mathbf{p}_T^{\text{assoc}}) \cos(n\Delta\phi)$$

Harmonics: $v_n = \sqrt{V_{n\Delta}}$

# Multi-particle correlations




*Gale, Jeon, Schenke, Tribedy, Venugopalan PRL 110 (2013)*

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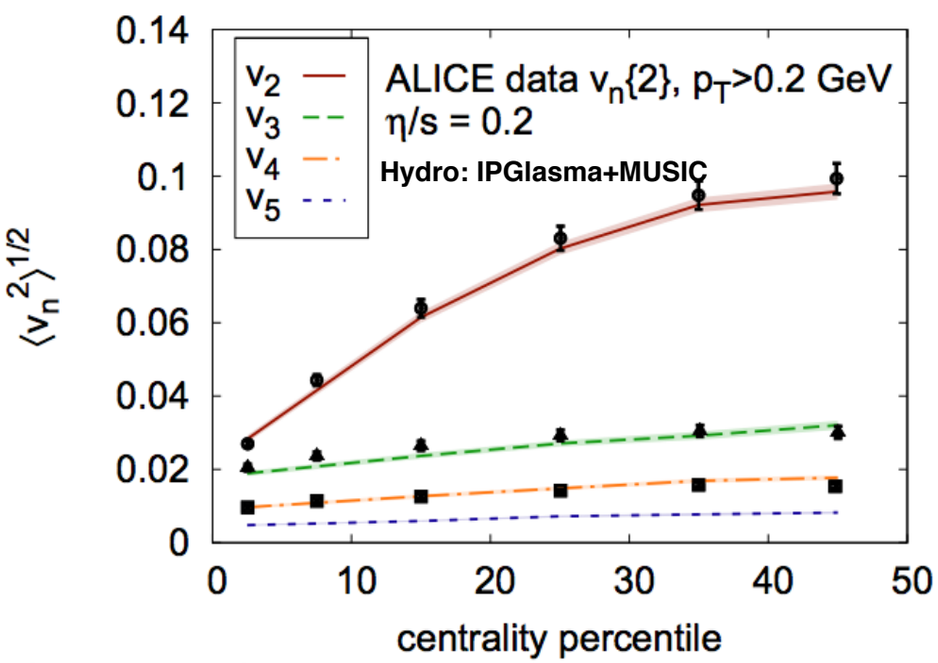
*HIC Interpretation:* From single collective fluid source, multi-particle distribution factorizes into product of single particle distributions — naturally embedded in a hydrodynamic description

*Alver, Roland, PRC 81 (2010), Alver, Gombeaud, Luzum, Ollitrault, PRC 82 (2010)*

# Multi-particle correlations

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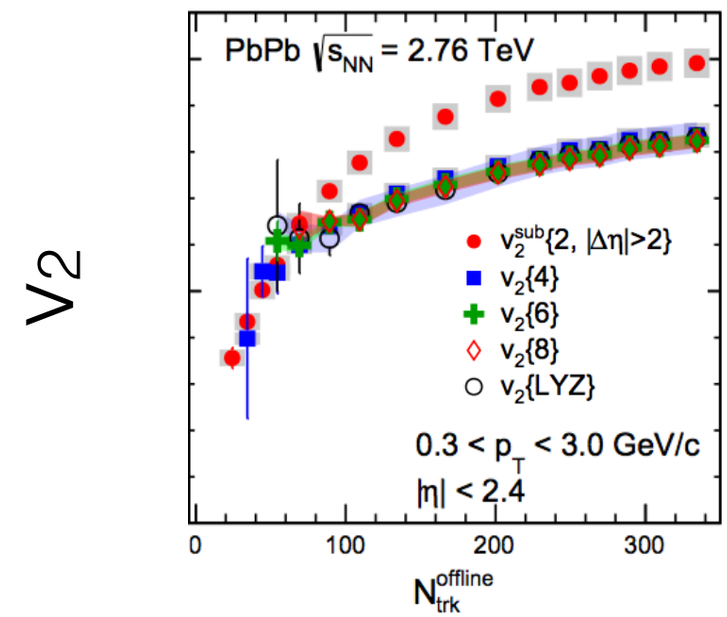


CMS PRL 115 (2015) 012301

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*Alver, Roland, PRC 81 (2010), Alver, Gombeaud, Luzum, Ollitrault, PRC 82 (2010)*

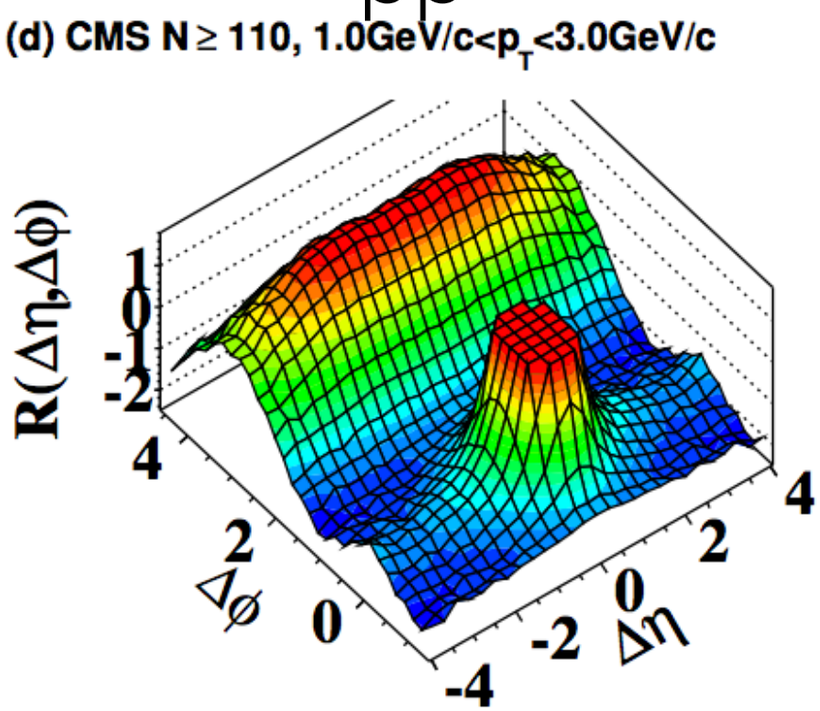
Assuming single collective source, large numbers of particles should not change harmonics

*Yan, Ollitrault PRL 112 (2014) 082301, Bzdak, Skokov NPA 943 (2015)*

# Similarity in all systems

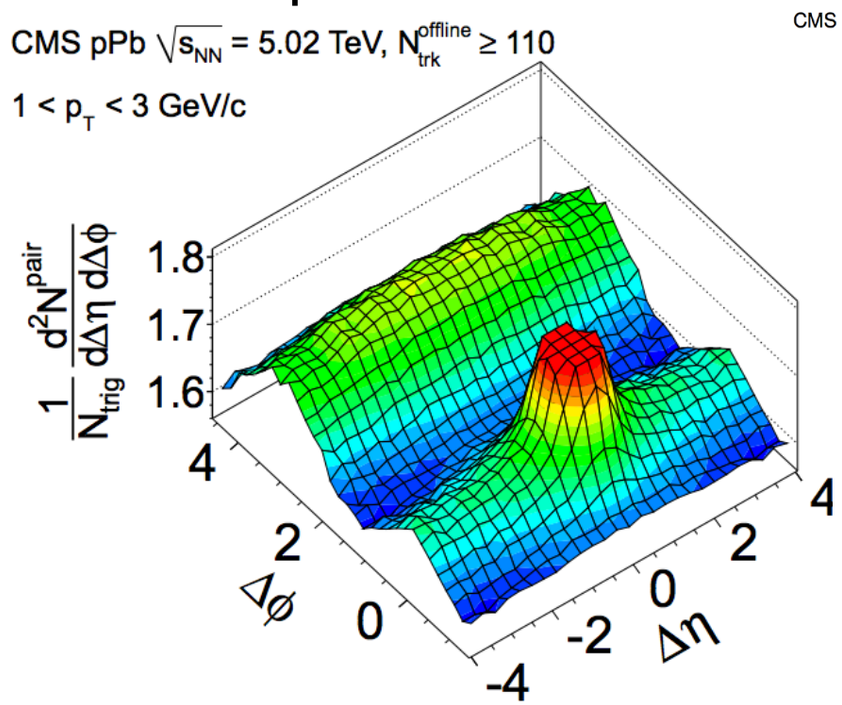
Strikingly, two particle correlations look similar across all systems

pp



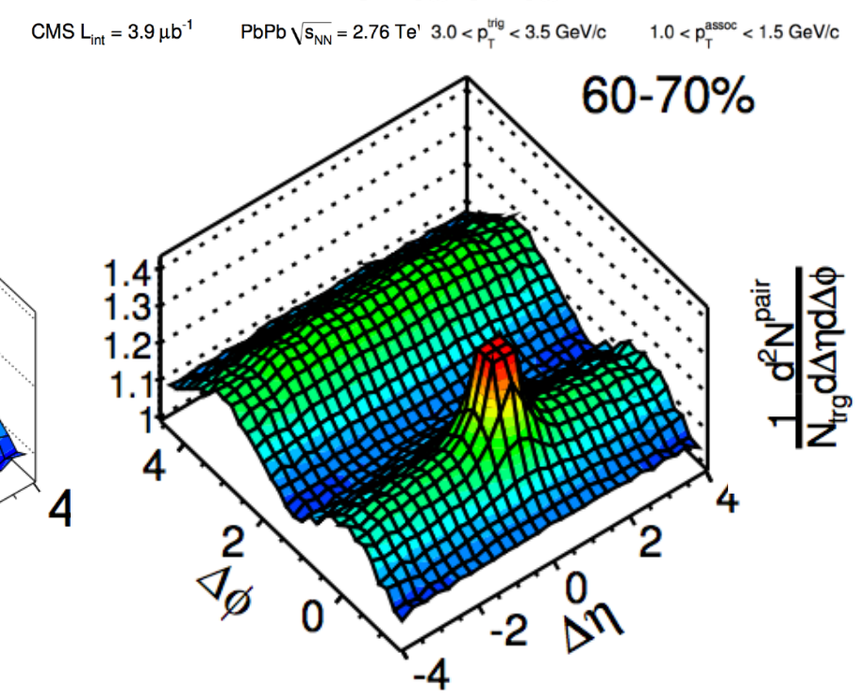

CMS JHEP 1009:091 (2010)

pPb




CMS PLB 718 (2013) 795

PbPb



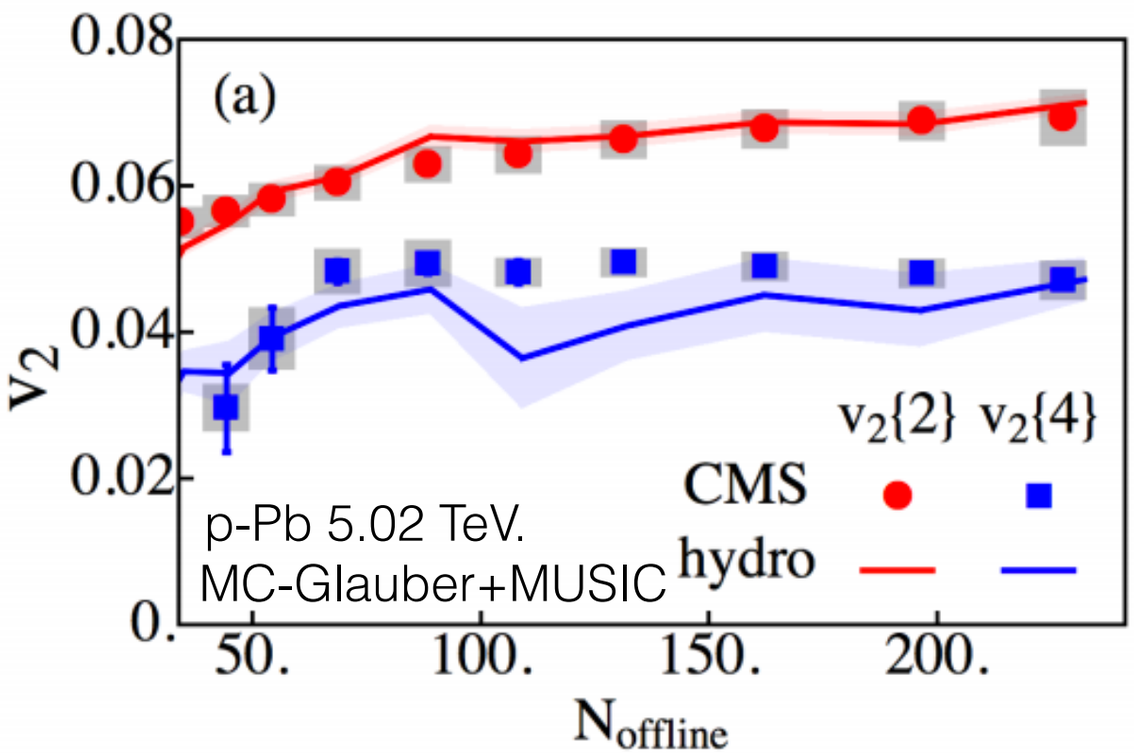

CMS EPC 72 (2012) 10052

Large systems believed to be near-perfect *collective* fluid

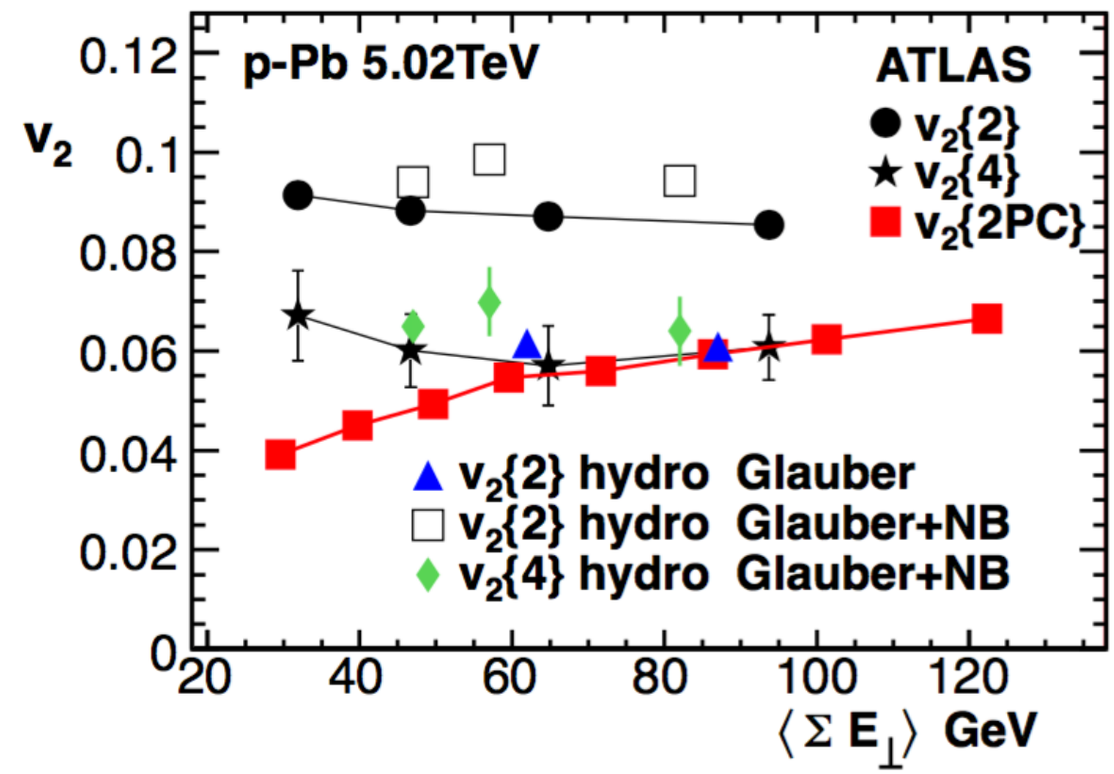
But what about in small systems?

# Collectivity from hydro

Two and four particle correlations in p+A in hydro




*Kozlov, Denicol, Luzum, Jeon, Gale*
*NPA 931 (2014) 1045-1050*




*Bozek, Broniowski*
*Phys. Rev. C 88, 014903 (2013)*

Theory issues linger however about applicability of hydrodynamics in small systems (size of gradients,…)

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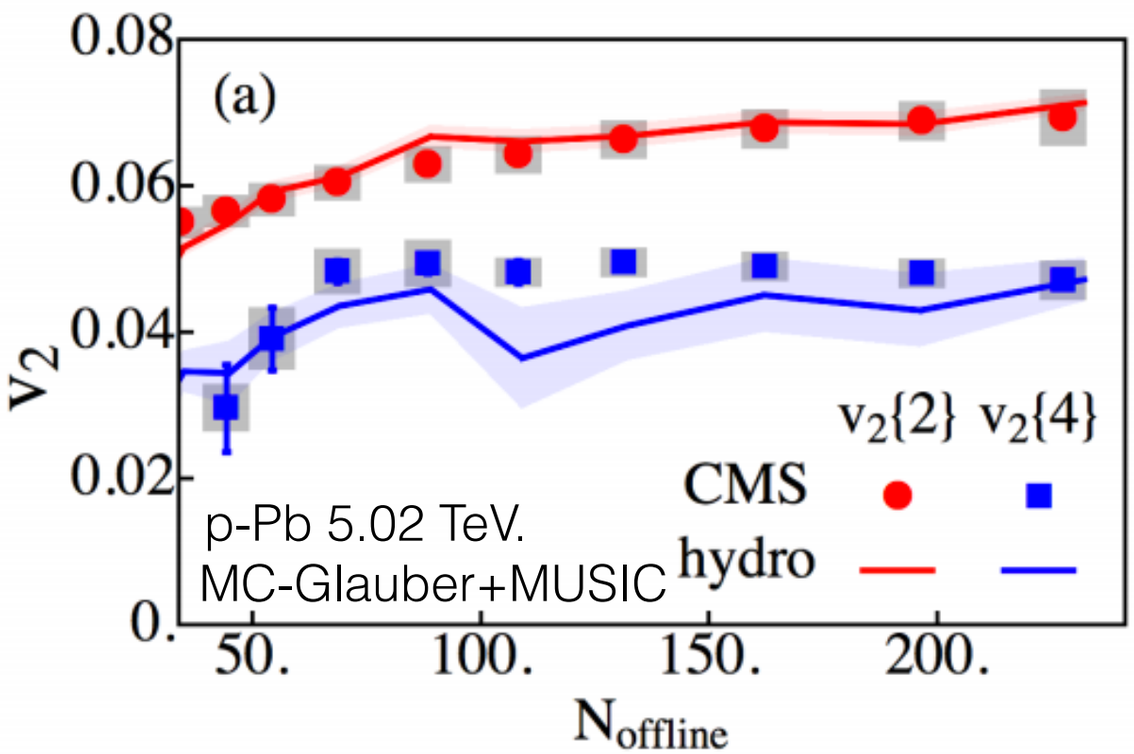


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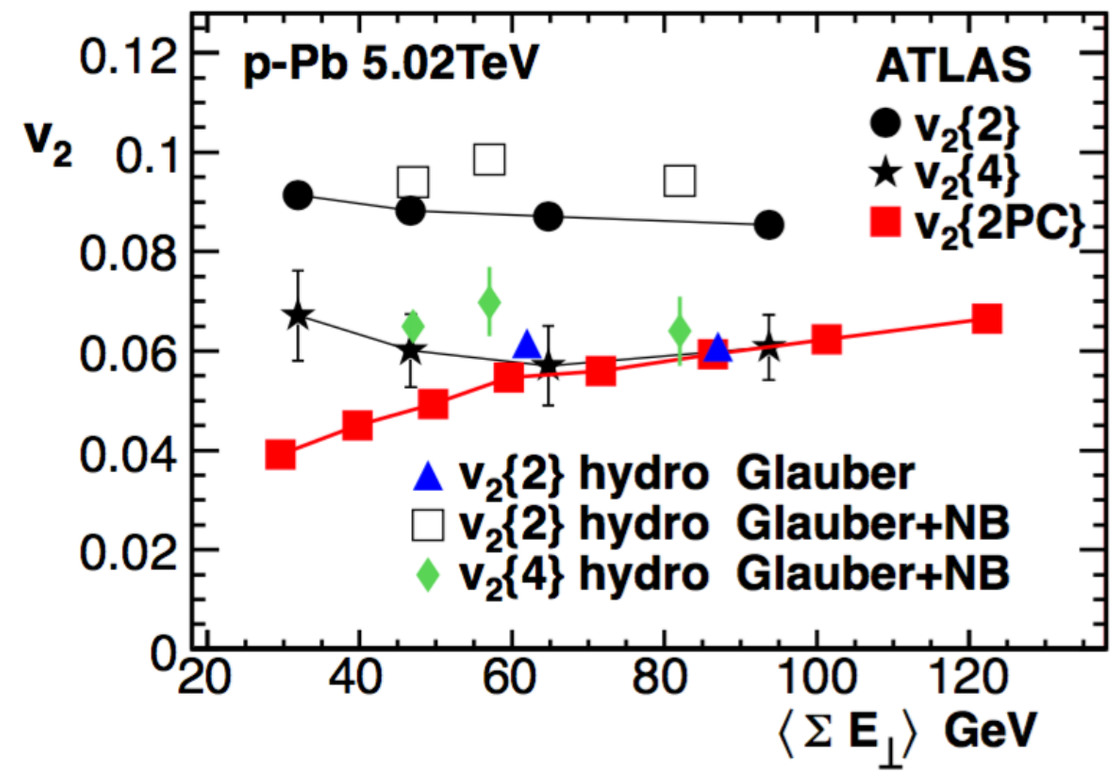


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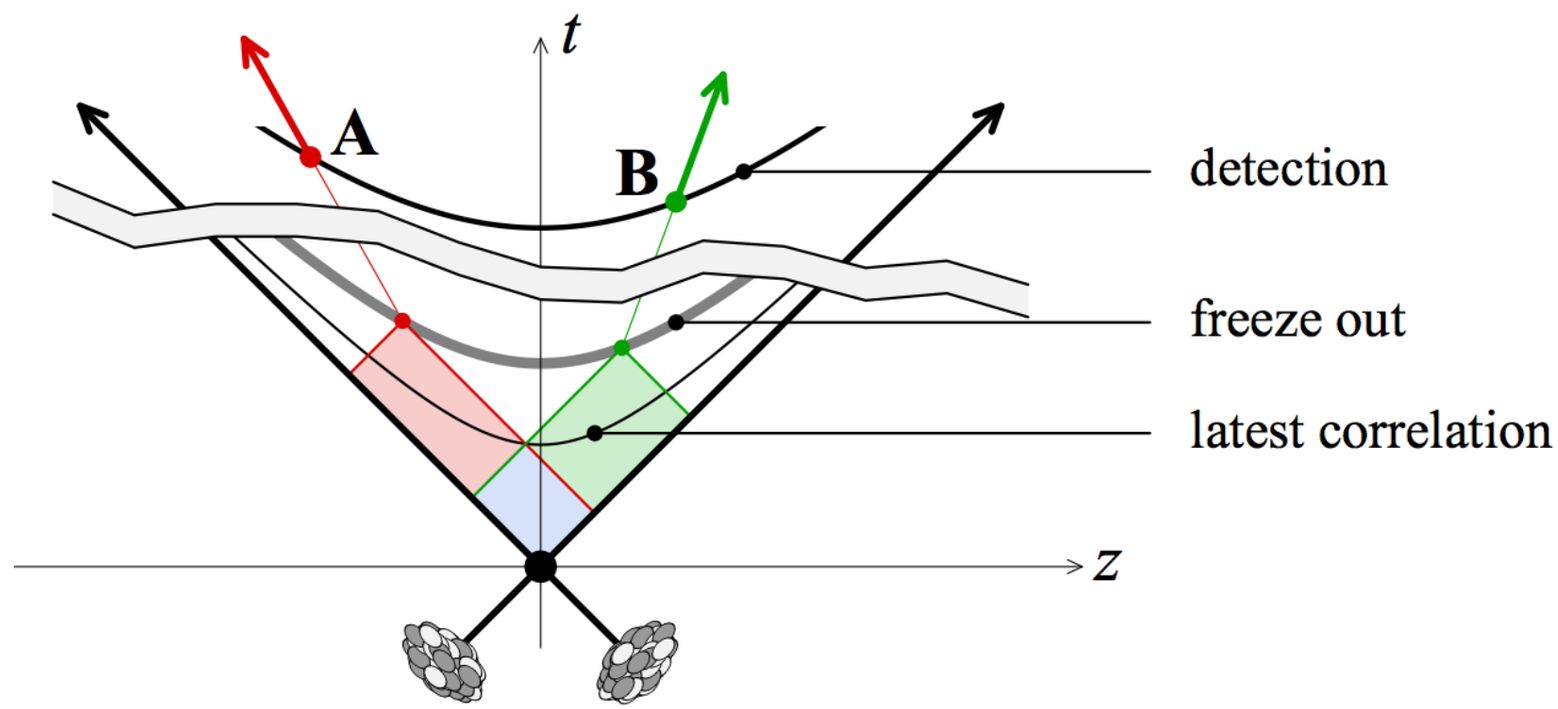
More in talk by B. Schenke Wednesday

Can there be an alternative explanation to the observed phenomena?

# Long range rapidity correlations as a chronometer




*Dumitru, Gelis, McLerran, Venugopalan*
*NPA 810 (2008) 91-108*

$$\tau_0 \leq \tau_{\mathrm{f.o.}} \exp\left(-\frac{1}{2}|y_a - y_b|\right)$$

Long range rapidity correlations sensitive to very early time dynamics (0.1 fm/c) in collision

# Color Glass Condensate

CGC is an effective field theory in the non-linear regime of QCD ($Q_s^2(x) \gg \Lambda_{QCD}^2$) describing dynamical gluon *fields* (small-x partons) effected by static color *sources* (large-x partons)

*McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204*

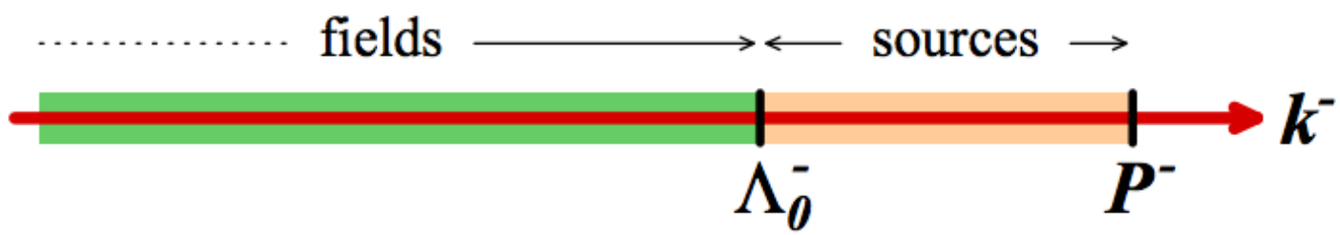
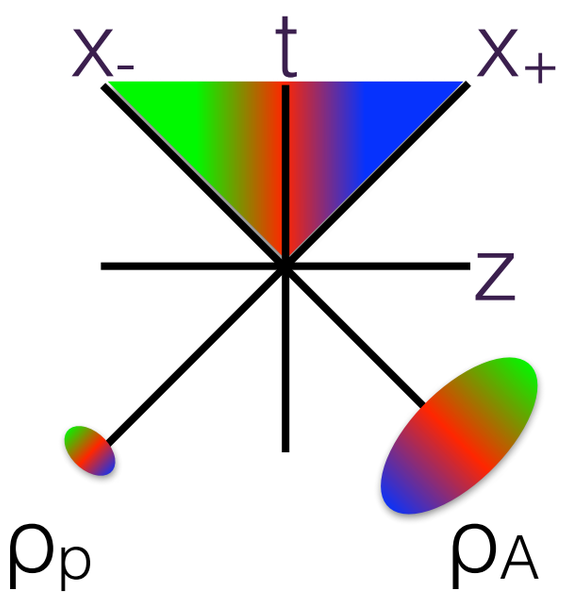



*Fig: Gelis, Iancu, Jalilian-Marian, Venugopalan ARNPS. 60 (2010)*




Classical background field: $[D_\mu, F^{\mu\nu}] = J^\nu$

Static color sources:

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

McLerran-Venugopalan (MV) Model: interactions between nucleons is a Gaussian random walk in color space

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = g^2\delta^{ab}\mu^2\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

# Initial State Flow

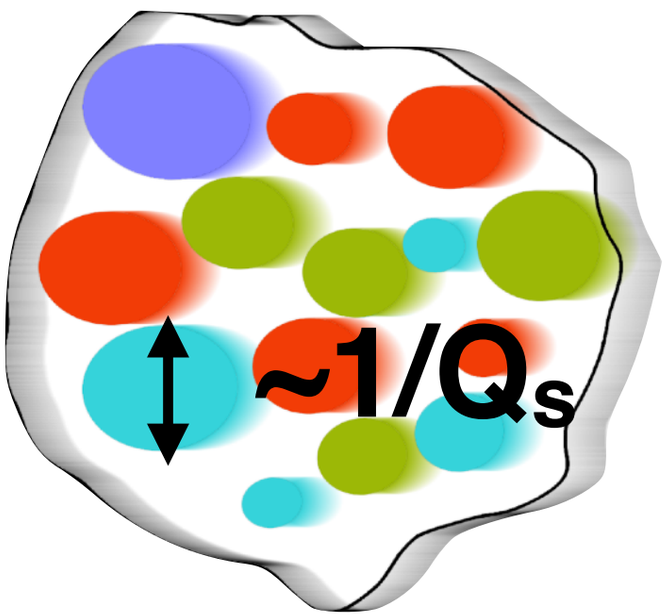
At high energy → high density gluon matter described by the Color Glass Condensate Effective Field Theory

McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204

High gluon density in QCD generates dynamical saturation scale, $Q_s$

$$Q_s^2 \sim A^{1/3} s^{\lambda}$$

Intuitive picture of CGC:
Nucleus becomes saturated with high occupancy gluons for $k_T < Q_s$
For $k_T \gg Q_s$ smooth matching of framework to pQCD




Note: Very strongly correlated system. Dependence on coupling drops out

**Can the initial state also generate multiparticle flow?**

# A parton model

Consider eikonal quark scattering off dense nuclear target with color domains of size ~$1/Q_s$

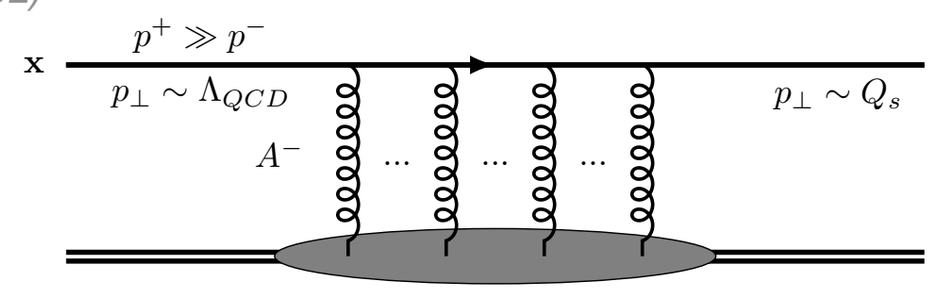
Work in dilute-dense limit: $Q_s$(target) ≫ $Q_s$(projectile)

*Lappi, PLB 744, 315 (2015); Lappi, Schenke, Schlichting, Venugopalan, JHEP 1601 (2016) 061; Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018)*

Quark coherent multiple scattering off target represented by Wilson line phase

*Bjorken, Kogut, Soper, PRD (1971), Dumitru, Jalilian-Marian,PRL 89 (2002)*

$$U(\mathbf{x}) = \mathcal{P}\exp\Big( -ig \int dz^+ A^{a-}(\mathbf{x}, z^+)\, t^a \Big)$$




Single quark inclusive distribution

$$\left\langle \frac{dN_q}{d^2\mathbf{p}} \right\rangle \simeq \int_{\mathbf{b},\mathbf{r},\mathbf{k}} e^{-|\mathbf{b}|^2/B_p} e^{-|\mathbf{k}|^2 B_p}\; e^{i(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}} \left\langle \frac{1}{N_c} \mathrm{Tr}\left( U(\mathbf{b}+\frac{\mathbf{r}}{2}) U^\dagger(\mathbf{b}-\frac{\mathbf{r}}{2}) \right) \right\rangle$$

Projectile: Wigner function

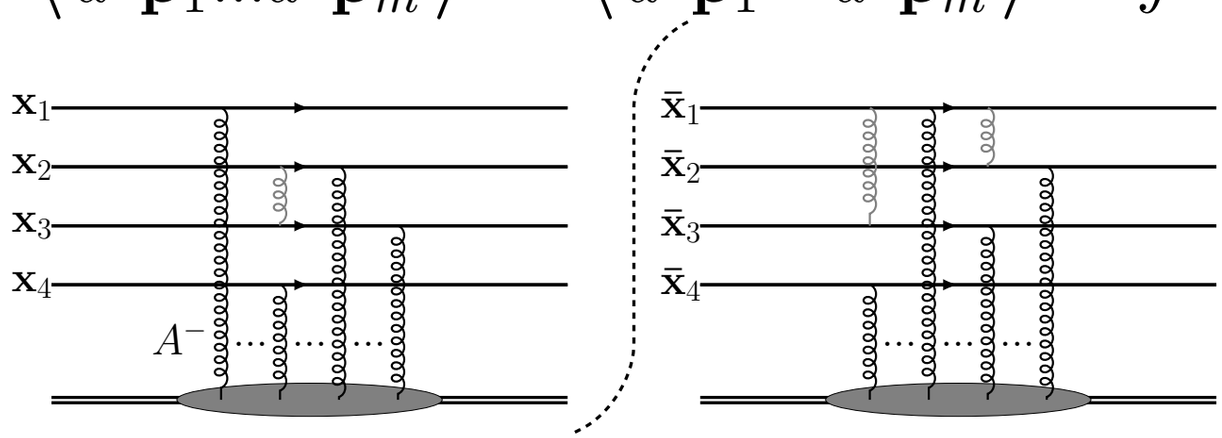
Target scattering: Dipole operator D(x,y)

*Single scale to defines projectile $B_p = 4\ \mathrm{GeV}^{-2}$ from HERA DIS fits

# A parton model

Generalizing for multiple particle correlations for *simple* model of multi particle correlations

$$\left\langle \frac{d^m N}{d^2\mathbf{p}_1 ... d^2\mathbf{p}_m} \right\rangle = \left\langle \frac{dN}{d^2\mathbf{p}_1} ... \frac{dN}{d^2\mathbf{p}_m} \right\rangle \sim \int \langle D ... D \rangle$$




**Introduced novel method to compute arbitrary Wilson line correlators in MV - arXiv:1706.06260**

$dN/d^2\mathbf{p}$ itself is not well defined. Average over classical configurations and over all events using MV model

*McLerran, Venugopalan, PRD 49, 3352, 2233 (1994)*

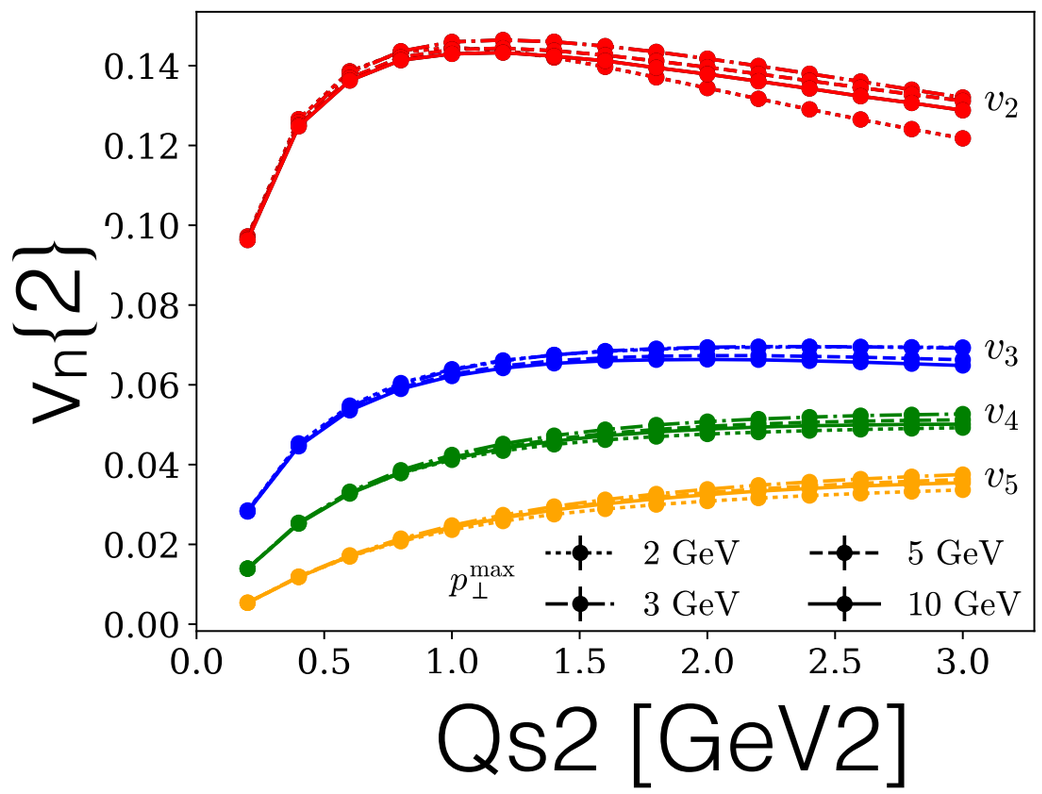
Generate cumulants, integrate to scale $p_\perp^{max}$

$$\kappa_n\{m\} = \int_{\mathbf{p}_1 ... \mathbf{p}_m} \cos\left(n\left(\phi_1^p + ... - \phi_m^p\right)\right) \left\langle \frac{d^m N}{d^2\mathbf{p}_1 ... d^2\mathbf{p}_m} \right\rangle$$

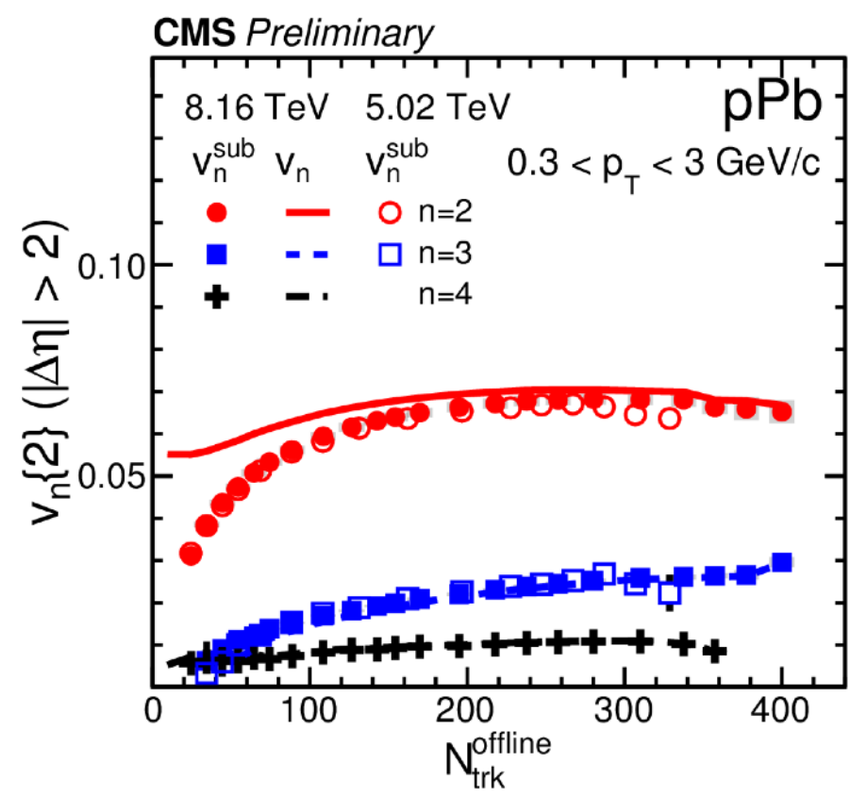
$$c_2\{2\} = \frac{\kappa_2\{2\}}{\kappa_0\{2\}}, \quad c_2\{4\} = \frac{\kappa_2\{4\}}{\kappa_0\{4\}} - 2\left(\frac{\kappa_2\{2\}}{\kappa_0\{2\}}\right)^2, \ \ldots$$

# Multi-particle quark correlations

Ordering in two particle Fourier harmonics similar to data



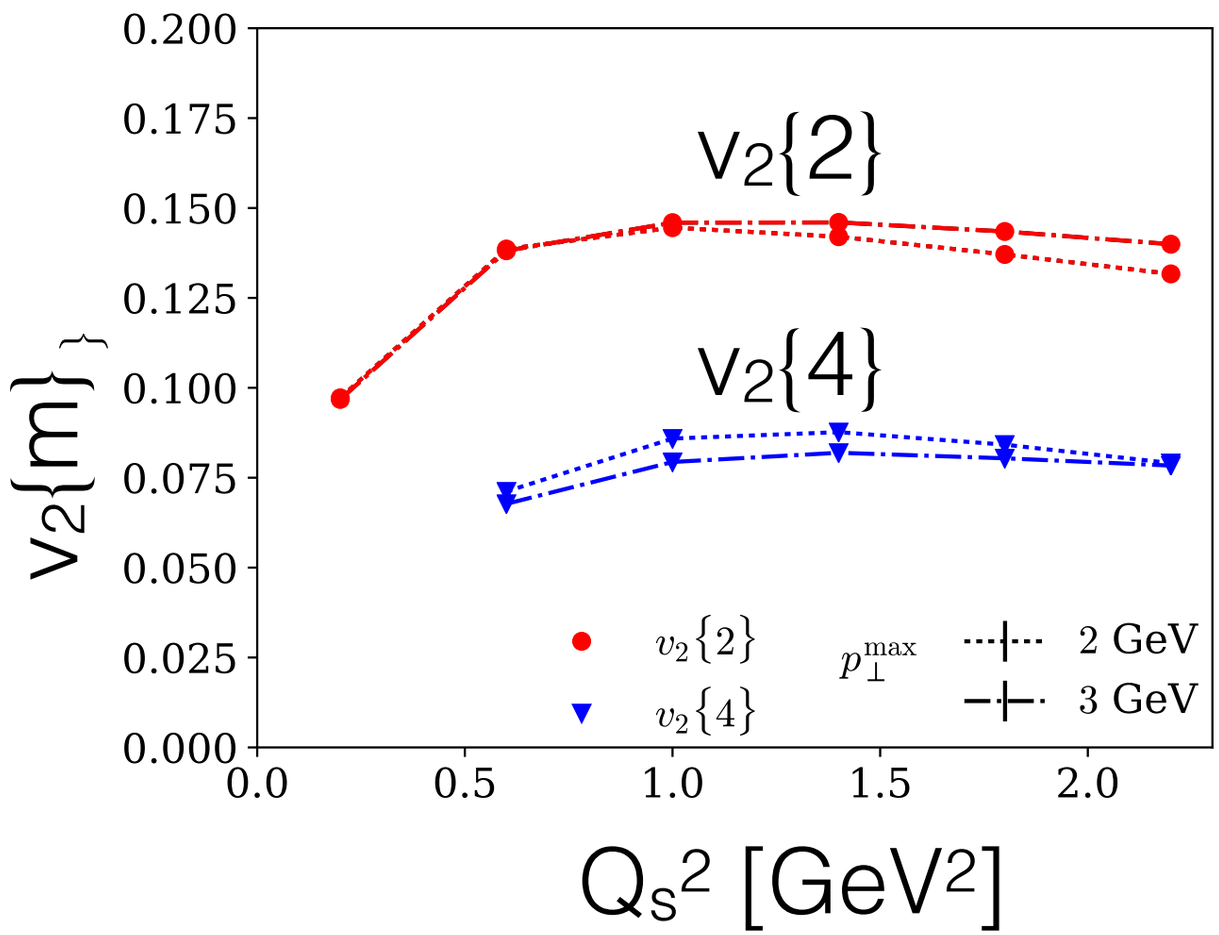

*Dusling, MM, Venugopalan PRL 120 (2018)*



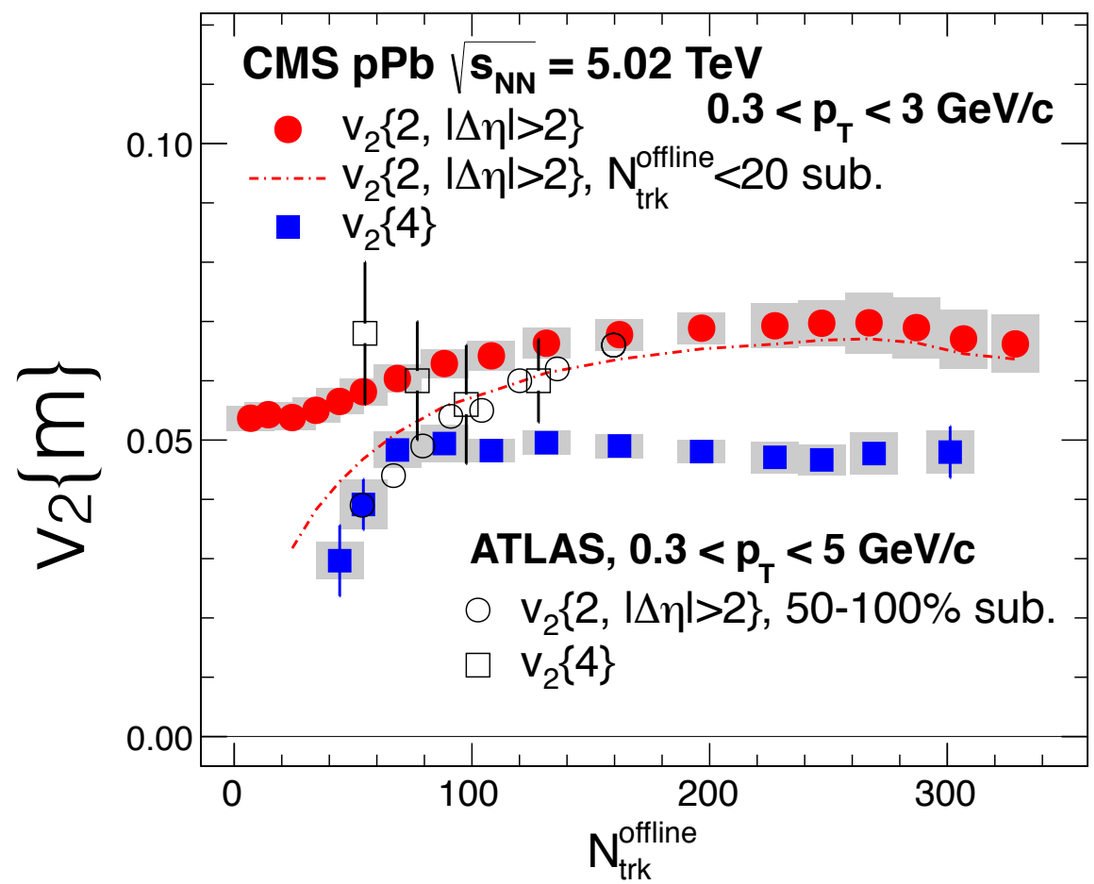

*CMS-PAS-HIN-16-022*

# Multi-particle quark correlations

$c_2\{4\}$ becomes negative for increasing $Q_s \longmapsto$ real $v_2\{4\}$




*Dusling, MM, Venugopalan PRL 120 (2018)*

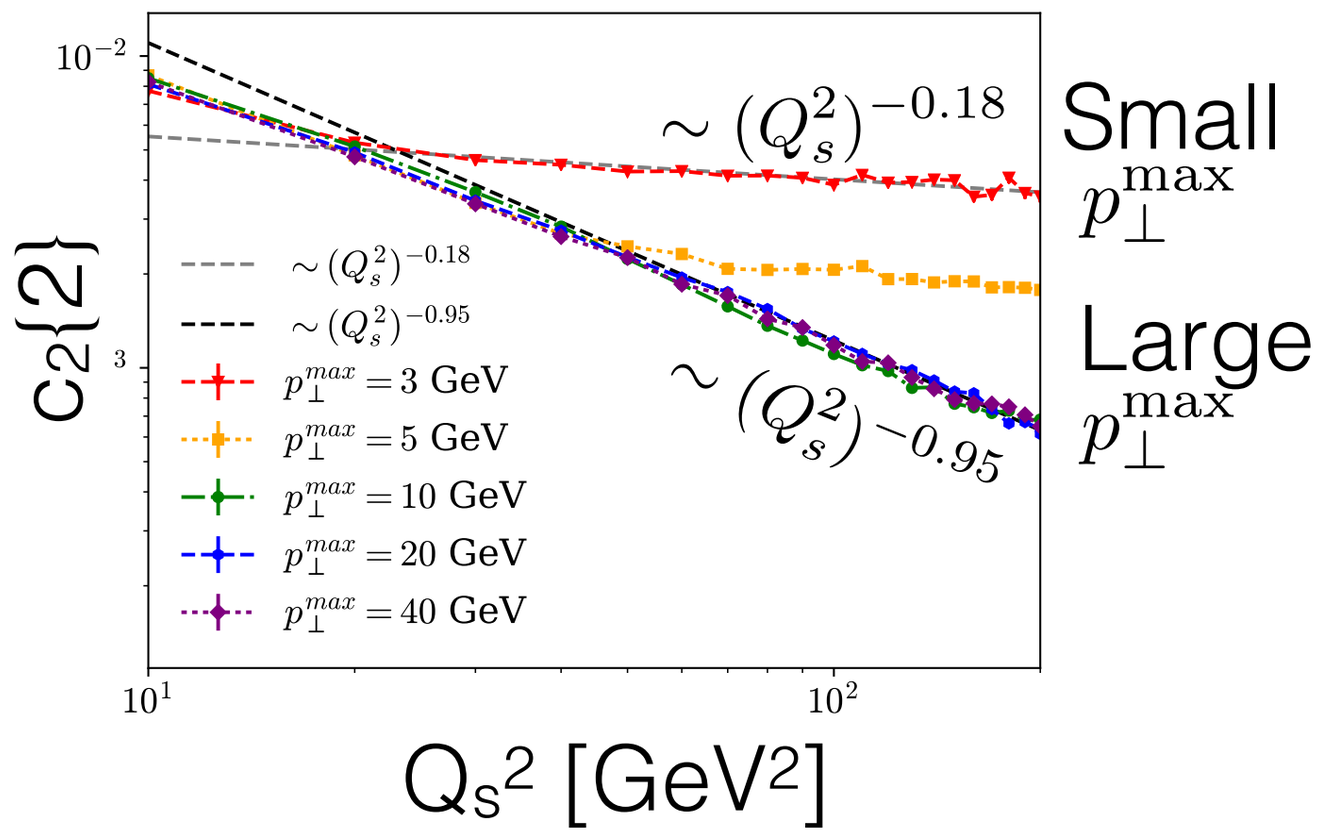



*CMS PLB 724 (2013) 213*

No inverse scaling by number of domains in CGC and data

# Scale dependence

Two dimensionless scales: $Q_s^2 B_p$, the number of domains, and the ratio of resolution scales, $Q_s^2/(p_\perp^{\max})^2$.




$(p_\perp^{\max})^2 \lesssim Q_s^2$ : probe coarse graining over multiple domains

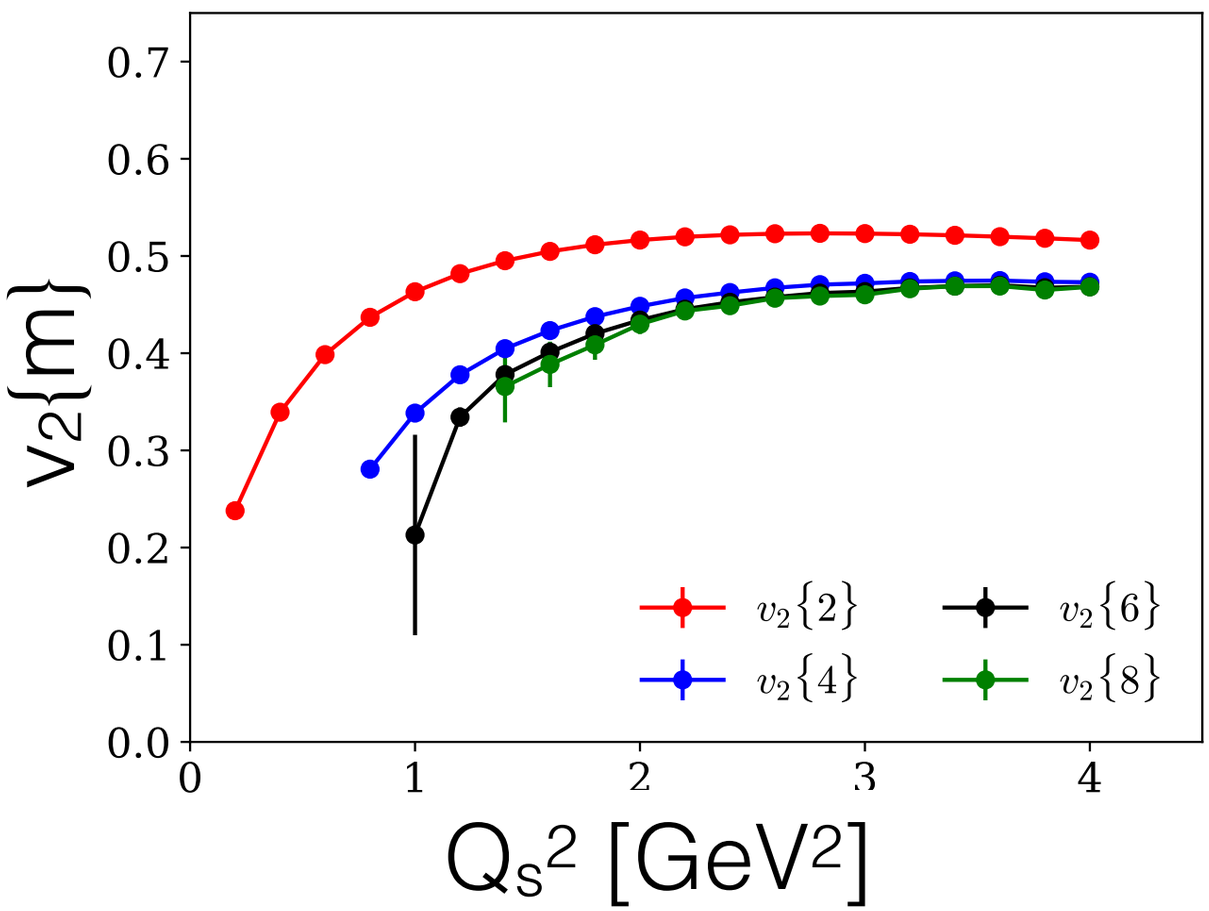
$(p_\perp^{\max})^2 \gtrsim Q_s^2$ : probe resolves area less than domain size

Scaling with inverse number of domains seen only for large $p_\perp^{\max}$

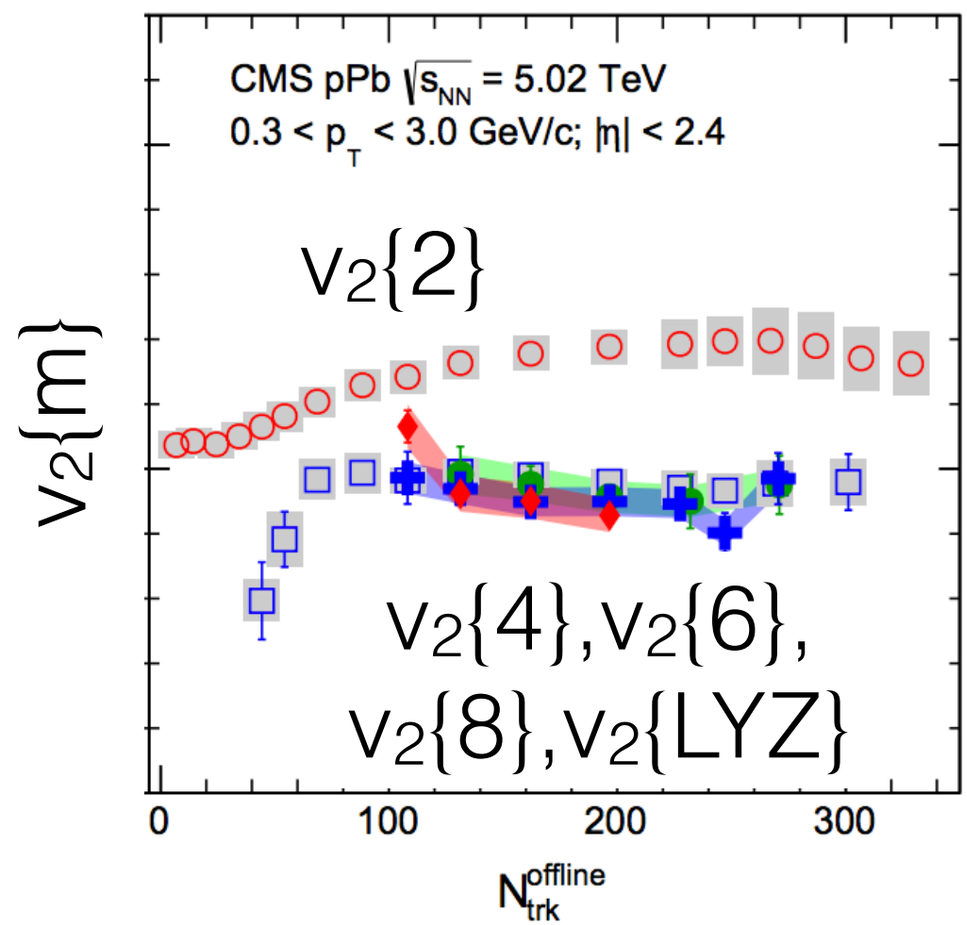
*Dusling, MM, Venugopalan PRD 97 (2018)*

# Collectivity from parton model

For computational reduction, consider Abelian version




*Dusling, MM, Venugopalan PRL 120 (2018)*

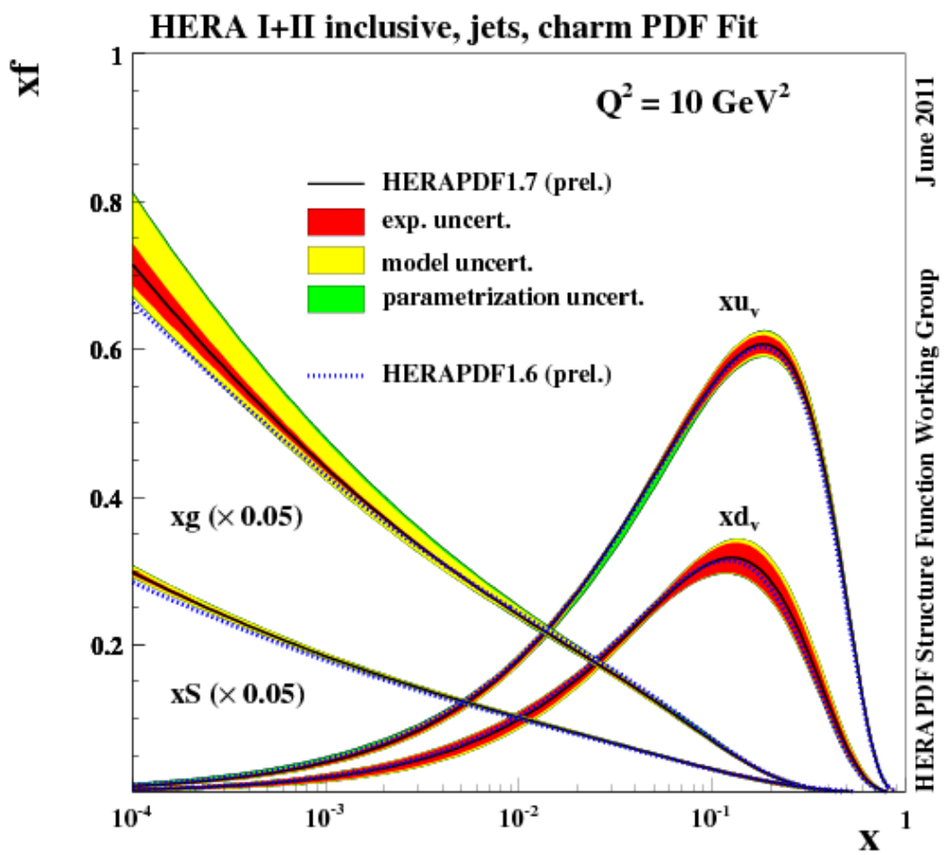



*CMS PRL 115 (2015) 012301*

Clear demonstration that $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$
collectivity not unique to hydrodynamics

# The role of glue?

Previous discussion only included quarks scattering off CGC…




*Zeus and H1 - arXiv:1112.2107*

What about gluons, which are dominant at small x or high energies?

# Dilute-dense CGC EFT

Determine initial gluon densities with nuclear position sampling+IP-Sat model

*Kowalski, Teaney, Phys.Rev. D68 (2003),*
*Schenke, Tribedy, Venugopalan PRL 108 (2012)*

Compute scattering of gluons off saturated nuclear target in dilute-dense CGC

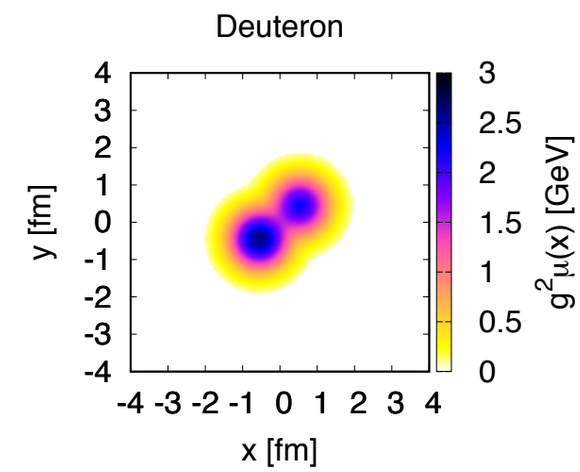
*Kovner, Wiedemann PRD 64 (2001)*
*Dumitru, McLerran NPA 700 (2002),*
*Blaizot, Gelis, Venugopalan NPA 743 (2004)*
*McLerran, Skokov NPA 959 (2017)*

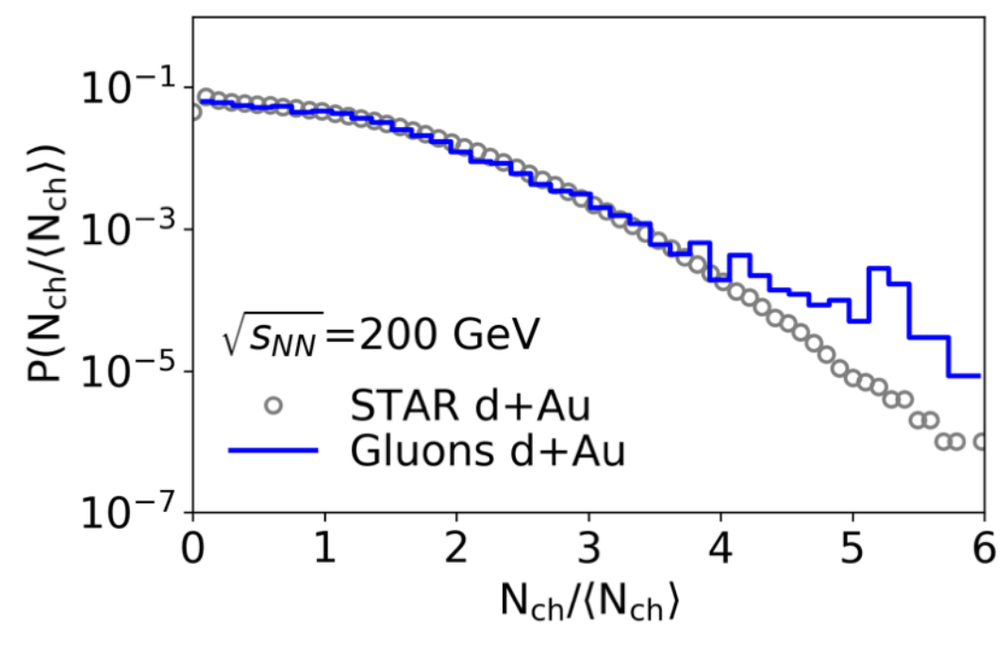
Generates negative binomial distributions from first principles, not an input!

*Schenke, Tribedy, Venugopalan PRC 86 (2012)*
*McLerran, Tribedy NPA 945 (2016)*

Good agreement found with STAR d+Au multiplicity distribution

Color charge density



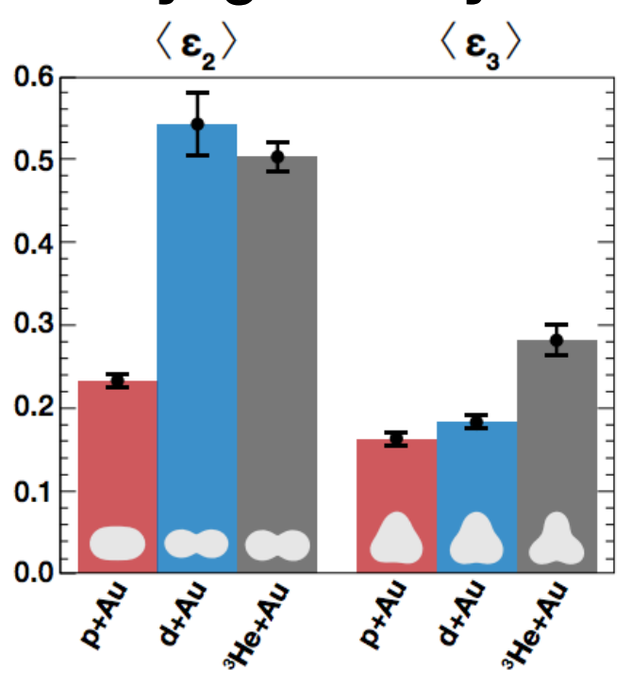

*MM, Skokov, Tribedy, Venugopalan arXiv:1805.09342*
*STAR PRC 79 (2009)*

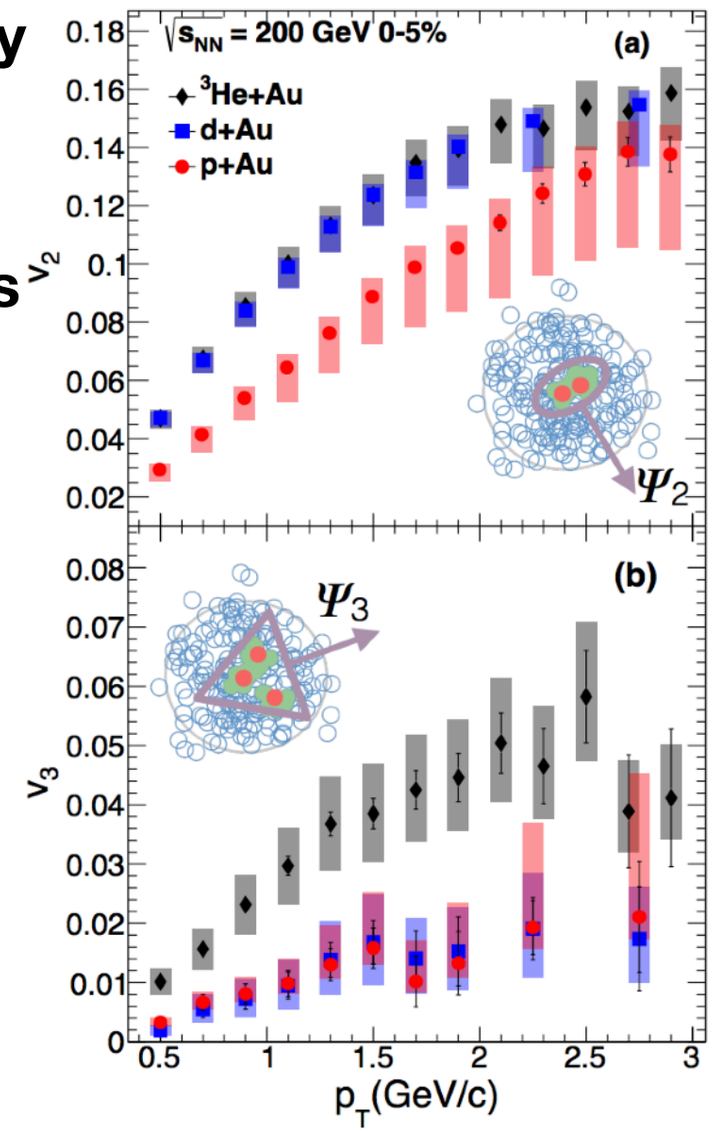
# Small system scan

- Recent PHENIX results for proton/deuteron/$^3$He+Au

**Unique opportunity to simultaneously study different small systems**

**Motivated by 'geometry'-driven models**

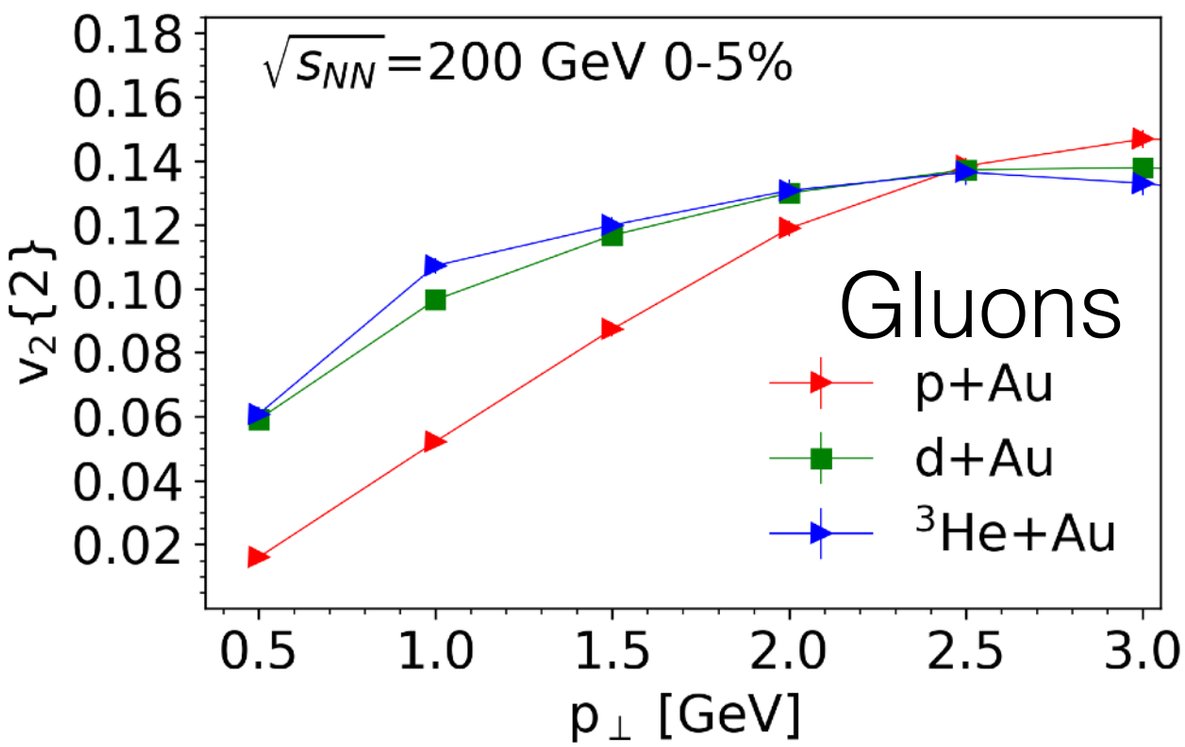


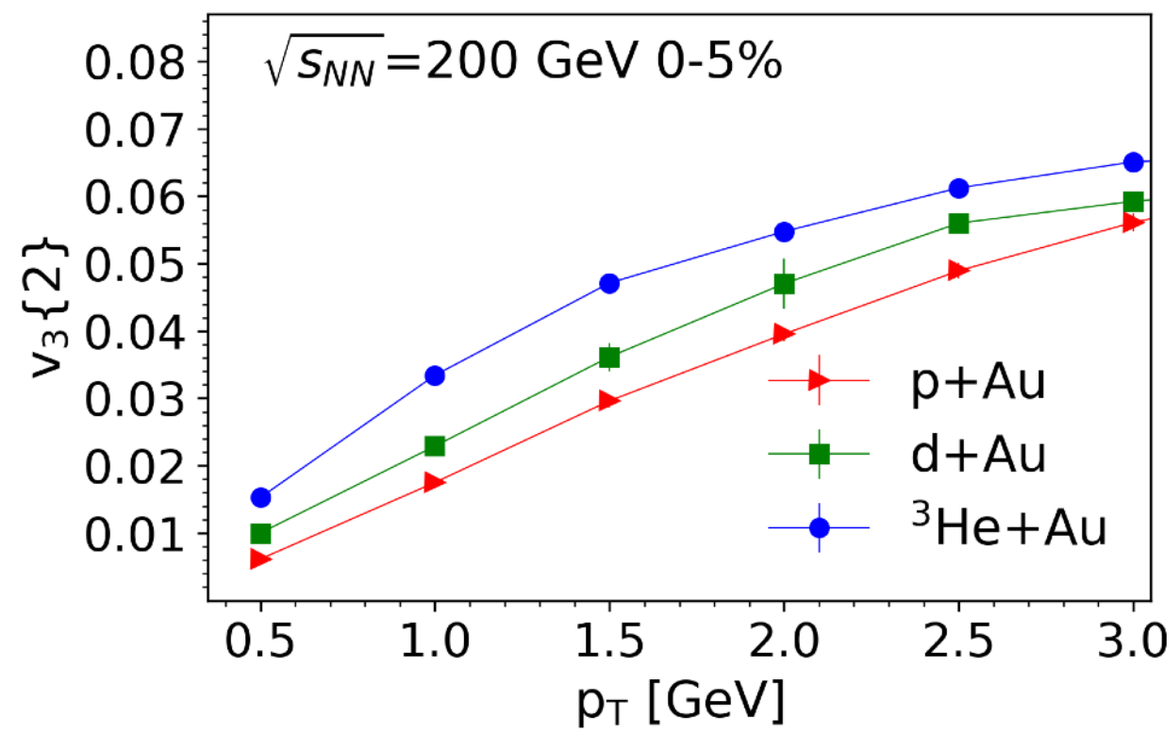




*PHENIX arXiv:1805.02973*

# Hierarchy of anisotropies across systems

System size dependence at RHIC captured by CGC initial state gluon correlations




*MM, Skokov, Tribedy, Venugopalan arXiv:1805.09342*

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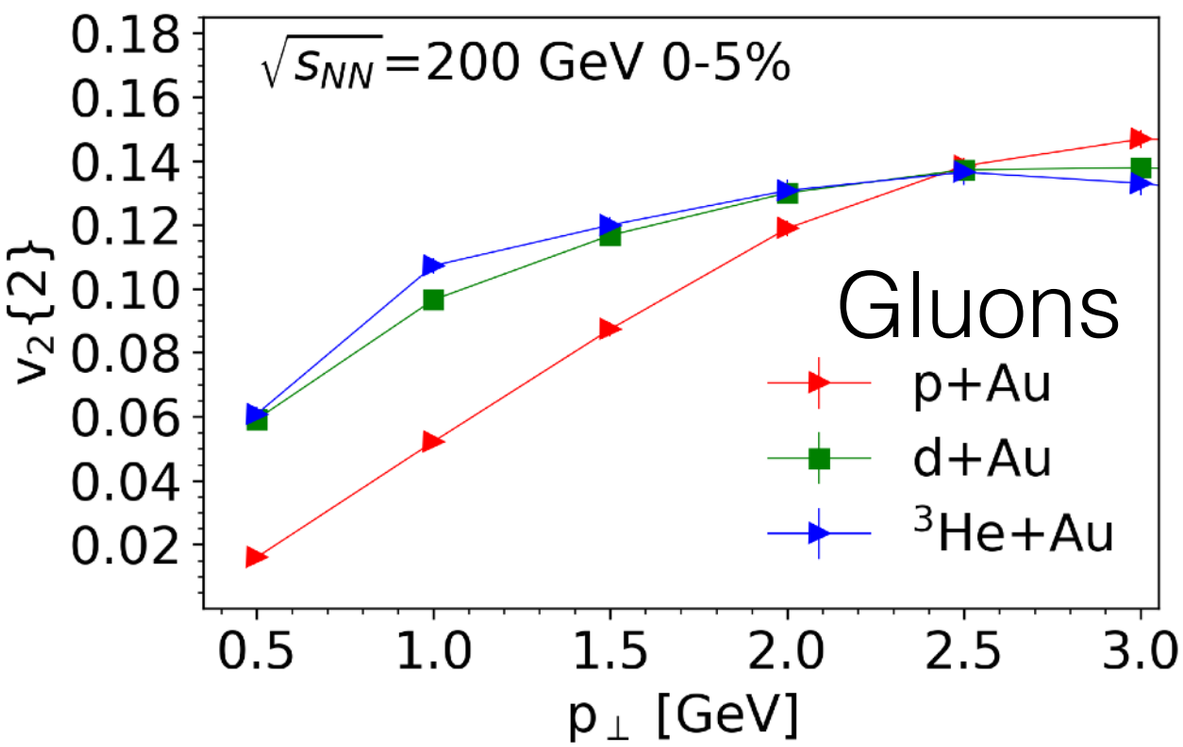
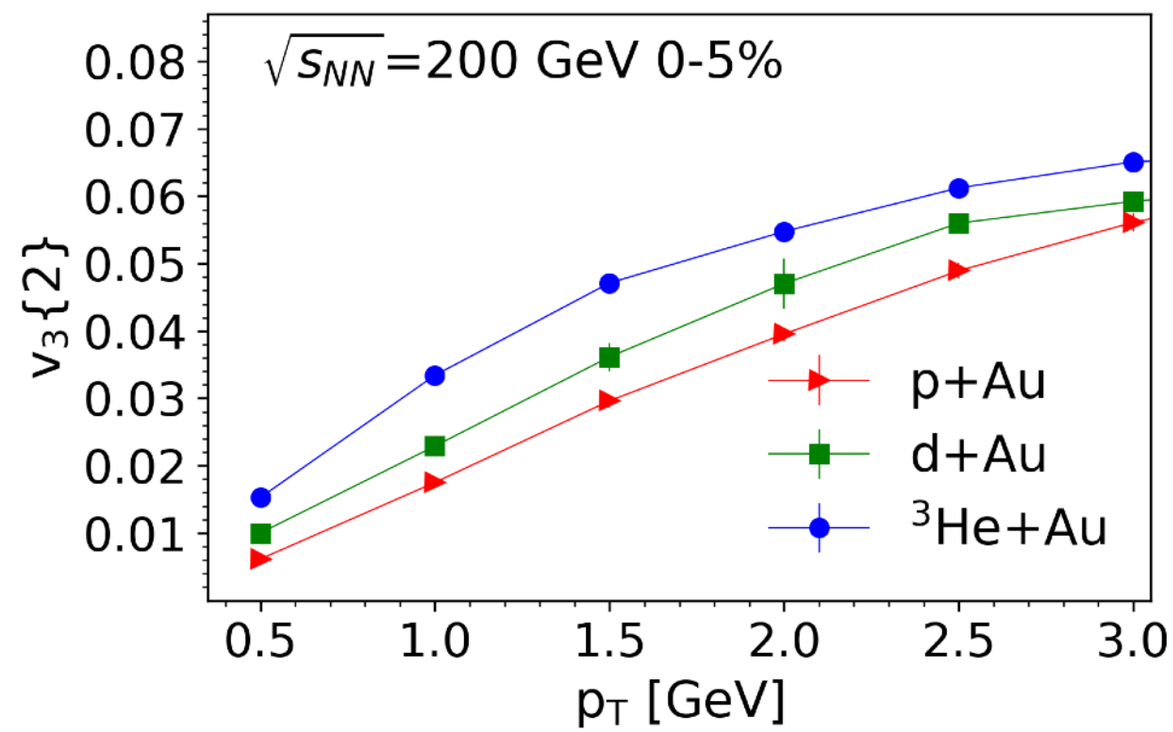


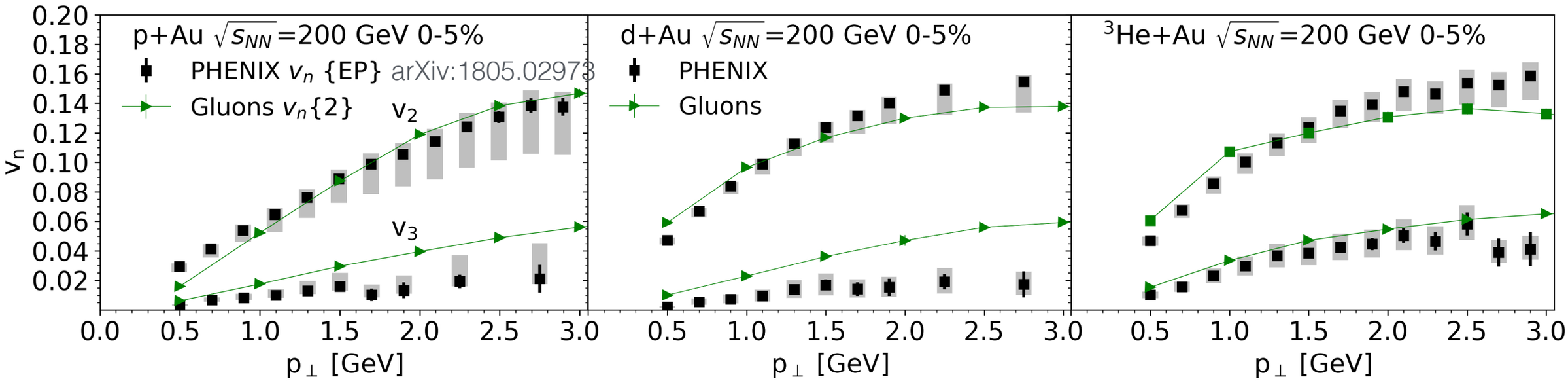




*MM, Skokov, Tribedy, Venugopalan arXiv:1805.09342*

Fixed centrality bin $\longmapsto$ larger average $N_{ch}$ for larger systems

$\longmapsto$ larger average $Q_s$ $\longmapsto$ more correlations

# Gluon correlations vs RHIC data for small systems




*MM, Skokov, Tribedy, Venugopalan, PRL121 (2018) arXiv:1805.09342*

Key features of system dependence captured by initial state gluon correlations

$v_3$ known to be fluctuation dominated — mismatch on high multiplicity tail needs to be better understood

*Alver, Roland PRC 81 (2010)*

# Quantifying systematic uncertainties

Dilute-dense approximation: high density effects need to be quantified

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Fragmentation: CGC+ Lund string model phenomenologically successful for mass ordering, can be applied here

*e.g. Schenke, Schlichting, Tribedy, Venugopalan, PRL 117 (2016) no.16, 162301*

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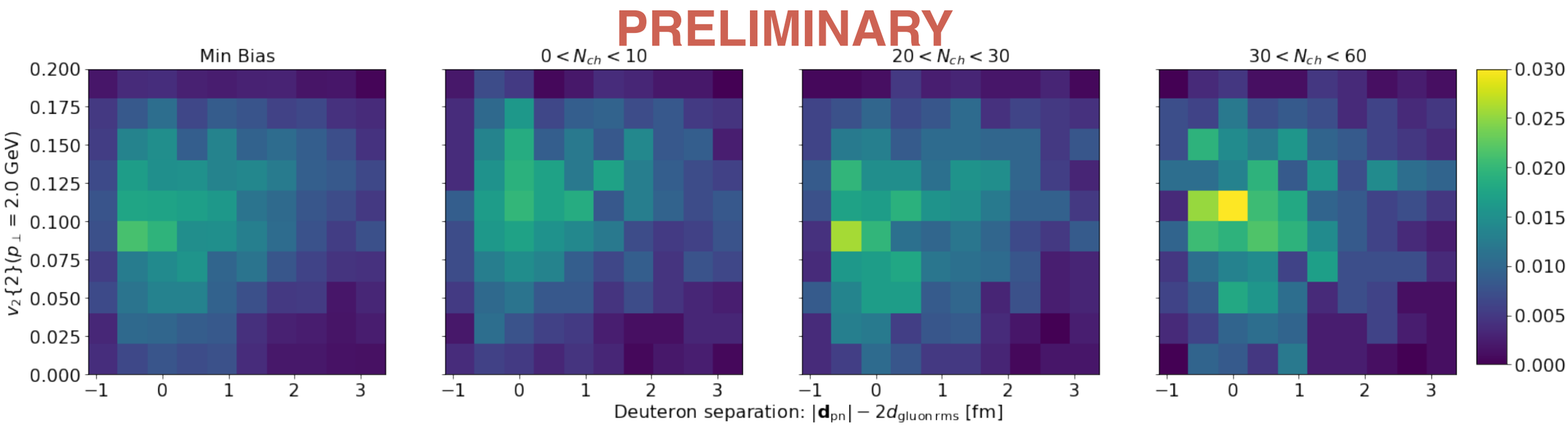
**Nuclear wave function:** strong short-range correlations! Exciting prospect to quantify influence on high multiplicity events in small systems — probes *rare* configurations

*Hen, Miller, Piasetzky, Weinstein Rev.Mod.Phys. 89 (2017)*

*Hen, MM, Schmidt, Venugopalan, in progress.*

# What do deuteron configurations look like?




Close configurations of nucleons contribute most to high multiplicity events and to generating $v_2$

Very important to have accurate sampling of ‘close’ nucleons

*MM, Skokov, Tribedy, Venugopalan, in preparation*

# How to include SRC

Current nucleon modeling for HICs assumes uncorrelated wavefunctions (Hulten for *d*, Wood-Saxon for large A, etc)

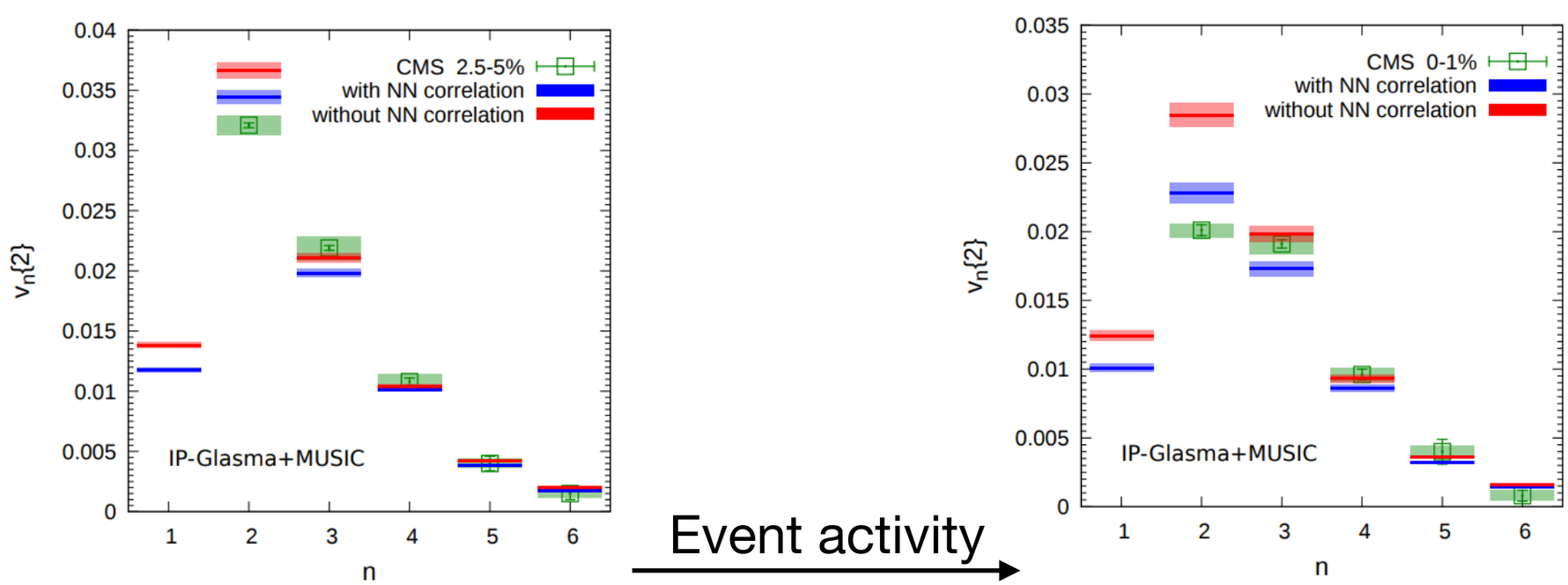
Previous attempts to include NN correlations use one parameter Gaussian model $\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \rho^{(1)}(\mathbf{r}_1)\rho^{(1)}(\mathbf{r}_2)\left(1 - C\left(\mathbf{r}_1, \mathbf{r}_2\right)\right)$

*Avioli, Drescher, Strikman PLB680 (2009)*

*Avioli, Holopainen, Eskola, Strikman PRC85 (2012)*

Applied to Pb-Pb HIC (hydrodynamical model)

*Denicol, Gale, Jeon, Paquet, Schenke arXiv:1406.7792*



# How to include SRC

## Can use contact formalism to encode SRCs — simple ansatz

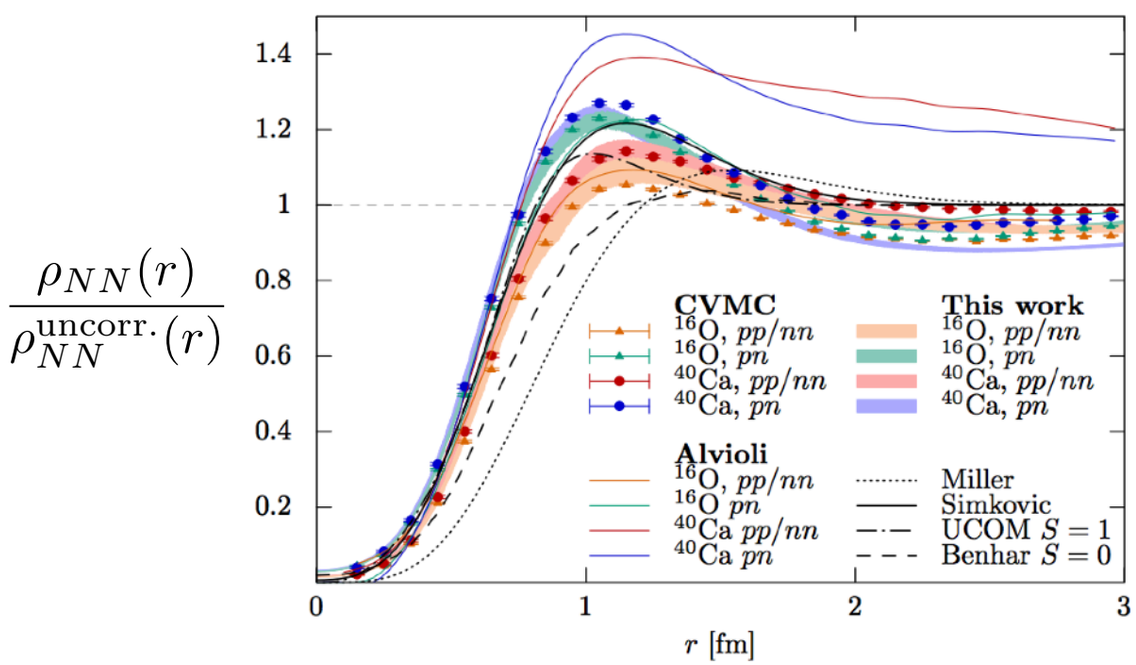
*Weiss, Cruz-Torres, Barnea, Piasetzky, Hen, PLB780 (2018)*
*Weiss, Bazak, Barnea, PRC 92 (2015)*
*Cruz-Torres, Schmidt, Miller, Weinstein, Barnea, Weiss, Piasetzky, Hen arXiv:1710.07966*

$$\rho_{NN}(r) = \rho_{NN}^{\text{uncorr.}}(r) + \mathcal{F}(r)\rho_{NN}^{\text{uncorr.}}(r)$$

N.B. $\rho_{NN}^{\textbf{uncorr.}}$ includes Pauli exchange term

## Good agreement with Cluster Variational Monte Carlo

*Pieper, Wiringa, Pandharipande, PRC46 (1992)*




*Cruz-Torres, Schmidt, Miller, Weinstein, Barnea, Weiss, Piasetzky, Hen arXiv:1710.07966*

These two-body correlations can be included in standard MC-Glauber by resampling nucleon positions to reproduce known distributions — *Work in progress*

*Hen, MM, Schmidt, Venugopalan, in progress.*

# Conclusions

Multiparticle collectivity demonstrated through purely initial state correlations with simple proof of principle parton model

*Dusling, MM, Venugopalan PRL 120, 042002 (2018), PRD 97, 016014 (2018)*

Full dilute-dense CGC framework able to describe system size hierarchy of $v_2$ and $v_3$ at RHIC — systematic uncertainties need to be quantified further

*MM, V. Skokov, P. Tribedy, R. Venugopalan PRL121 (2018); arXiv:1807.00825*

Modeling of HIC reaching point where disentangling initial vs. final state dominance scenarios requires greater input of nuclear wavefunctions

SRCs may have impact for both initial and final state modeling of HICs — potentially noticeable effect in *rare* high-multiplicity events in small systems

*Hen, MM, Schmidt, Venugopalan, in progress.*

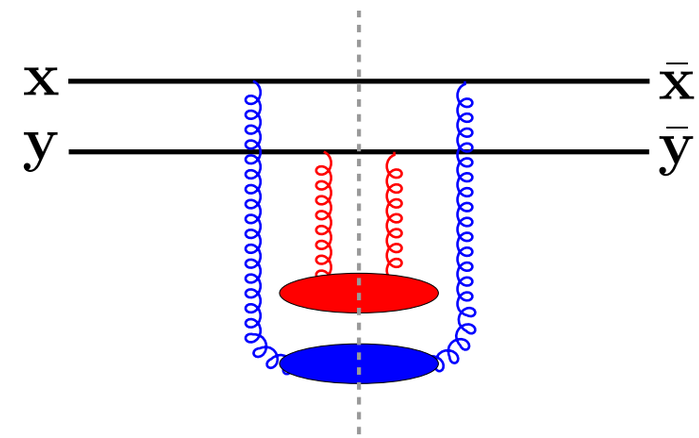
# Thanks!

# Back-up

# Comparison to glasma graphs

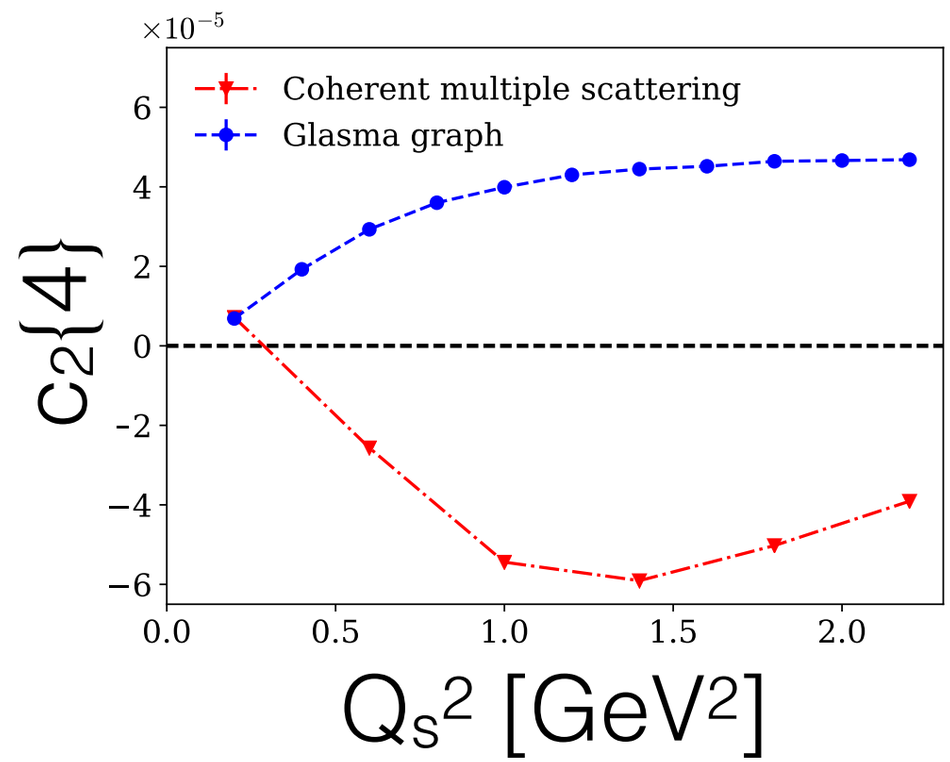
Glasma graph approximation, valid only for $p_\perp > Q_s$, only considers single gluon exchange

*Dumitru, Gelis, McLerran, Venugopalan, NPA 810 (2008), Dusling, Venugopalan PRL 108 (2012), PRD 87 (2013)*




Glasma graphs have very strong correlations, close to a Bose distribution (as in a laser)

*Gelis, Lappi, McLerran NPA 828 (2009)*




Multiple scattering suppresses higher cumulants → $c_2\{2\}<0$

*Dusling, MM, Venugopalan PRD 97 (2018)*

# Dilute-dense CGC scaling

In dilute-dense CGC, consider all orders of color charge density ρ in target, first order for projectile

Odd harmonics come about via additional gluon interaction:
first saturation correction

McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)

$$\frac{dN^{\mathrm{even}}(\mathbf{k}_\perp)}{d^2k dy} \sim \int \Omega^2 \sim \#\rho^2$$

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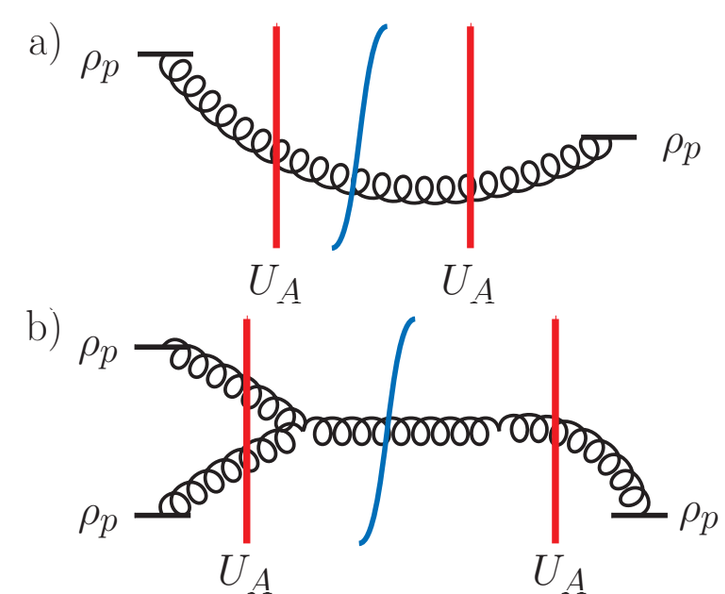


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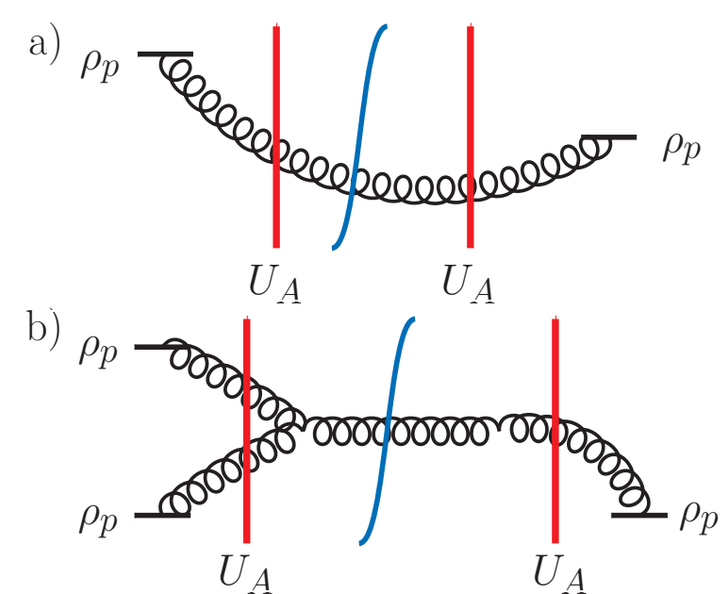


Even/odd harmonics depend on different factors of $\rho_p$

MM, Skokov, Tribedy, Venugopalan, arXiv:1807.00825

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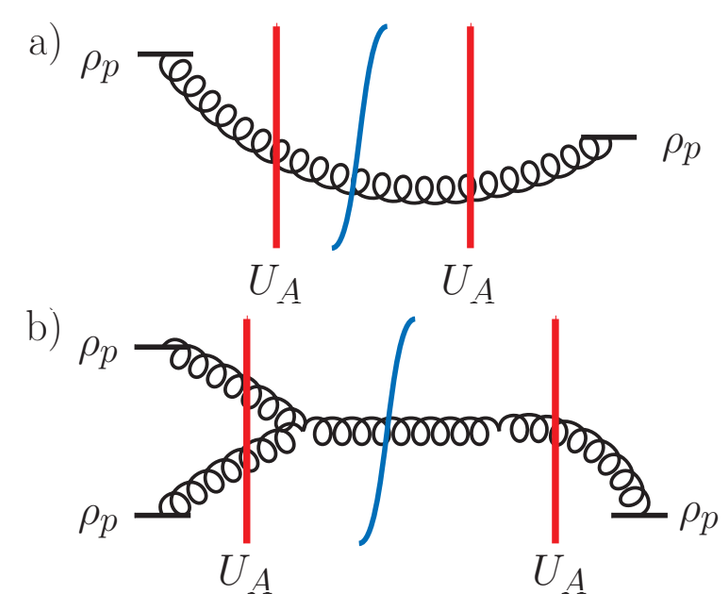



Even/odd harmonics depend on different factors of $\rho_p$

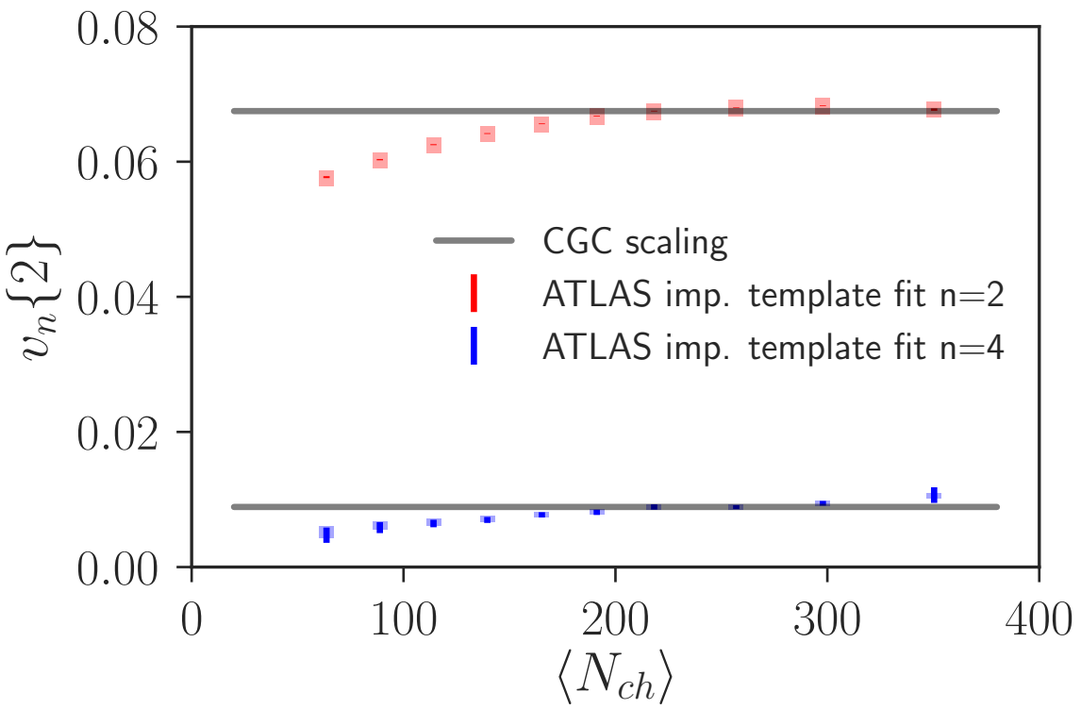
Multiplicity driven by $\rho_p$, so dilute-dense CGC scaling is then

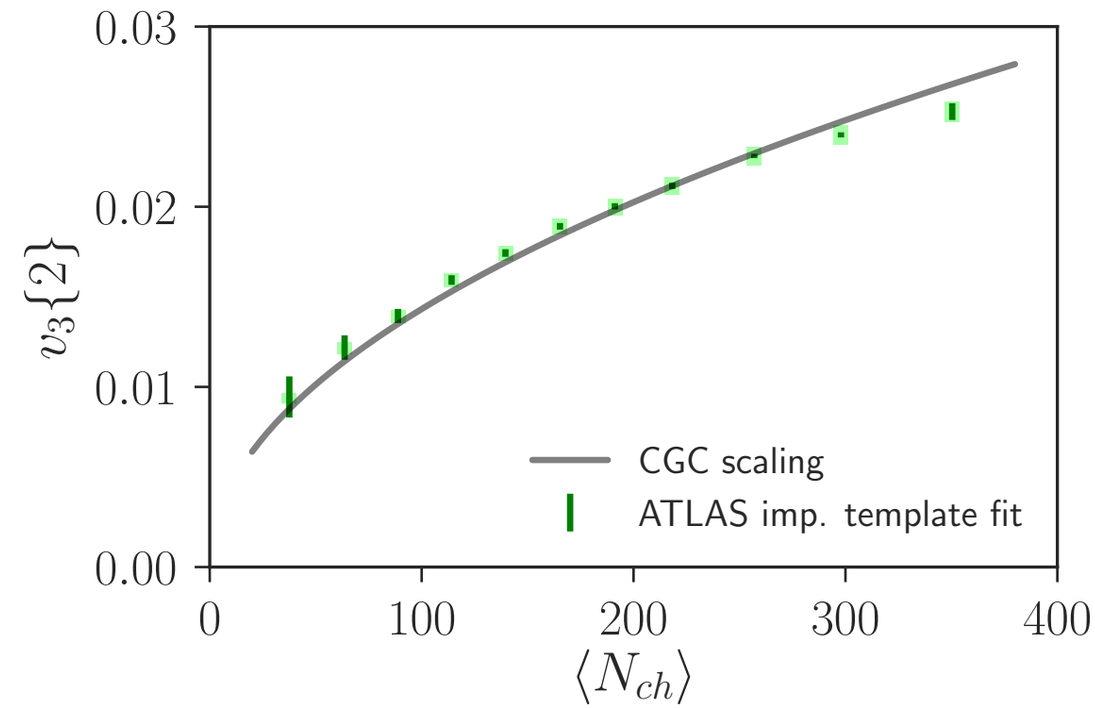
$$\boxed{v_{2n}\{2\} \sim N_{ch}^0 \ , \ v_{2n+1}\{2\} \sim N_{ch}^{1/2}}$$

MM, Skokov, Tribedy, Venugopalan, arXiv:1807.00825

# Dilute-dense CGC scaling

Fixing proportionality coefficient at a single multiplicity for each $v_n$




MM, Skokov, Tribedy, Venugopalan, arXiv:1807.00825

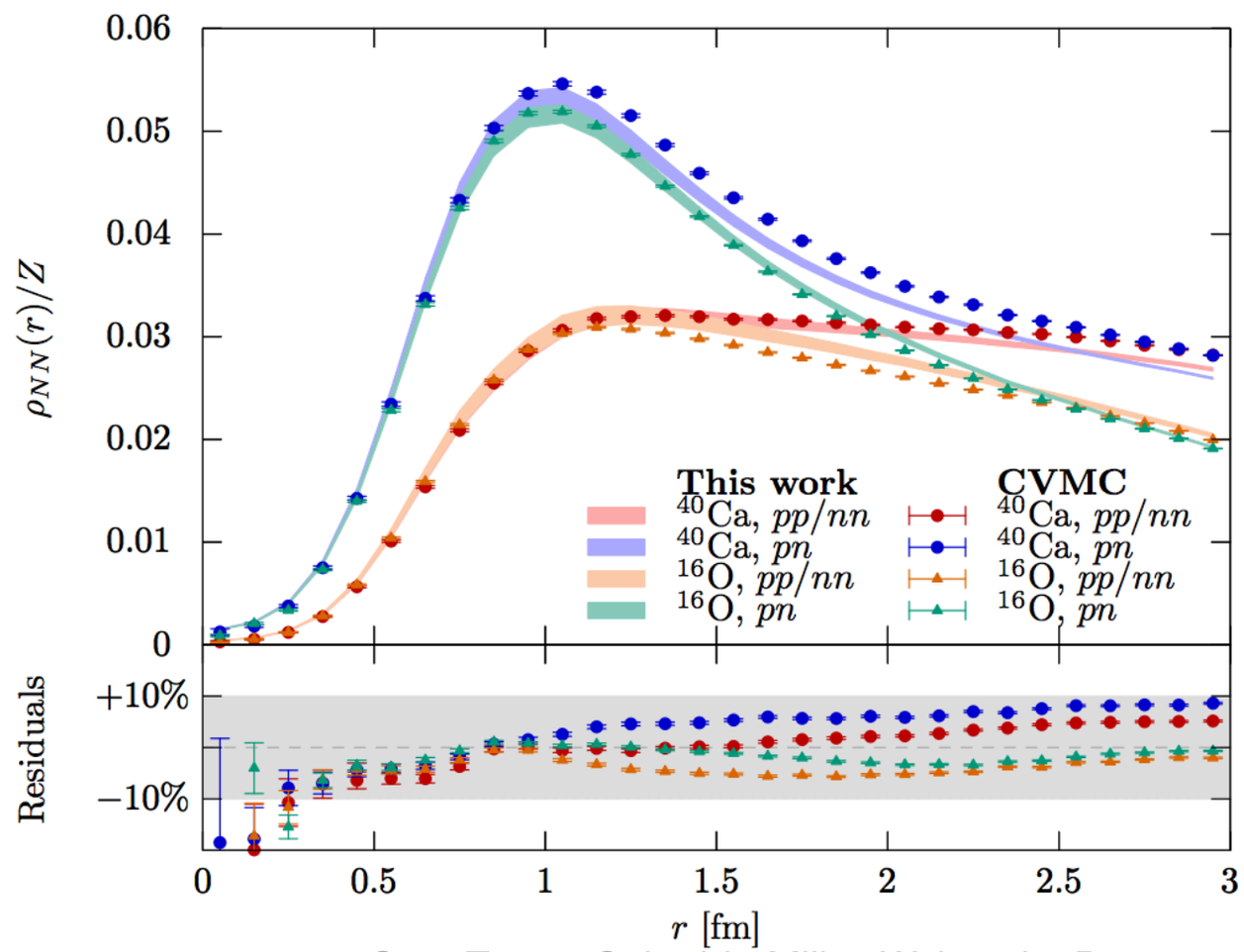
High projectile density effects probably responsible for large $N_{ch}$ deviation

Scaling from fluctuations, may then explain some of peripheral A+A signal

Basar, Teaney PRC 90 (2014)

# Different nuclei

Scaling with nuclei — use same formalism for Au, Pb relevant for HIC




*Cruz-Torres, Schmidt, Miller, Weinstein, Barnea, Weiss, Piasetzky, Hen arXiv:1710.07966*

# Dilute-dense CGC scaling

$$\rho_p \to c\rho_p \longrightarrow \Omega \to c\Omega$$

Multiplicity is then rescaled as $\frac{dN}{dy}[\rho_p, \rho_t] \to c^2 \frac{dN}{dy}[\rho_p, \rho_t] + \mathcal{O}(c^3)$

MM, Skokov, Tribedy, Venugopalan, in preparation

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Rescaling Fourier moments

$$V_{2n} = \frac{\int_{k,\phi,p} e^{i2n\phi} \frac{dN^{\rm even}(\mathbf{k}_\perp)}{d^2kdy}\Big[\rho_p, \rho_t\Big]}{\int_{k,\phi,p} \frac{dN^{\rm even}(\mathbf{k}_\perp)}{d^2kdy}\Big[\rho_p, \rho_t\Big]} \to c^0\ V_{2n}\ \ , V_{2n+1} = \frac{\int_{k,\phi,p} e^{i(2n+1)\phi} \frac{dN^{\rm odd}(\mathbf{k}_\perp)}{d^2kdy}\Big[\rho_p, \rho_t\Big]}{\int_{k,\phi,p} \frac{dN^{\rm even}(\mathbf{k}_\perp)}{d^2kdy}\Big[\rho_p, \rho_t\Big]} \to c\ V_{2n+1}$$

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In terms of multiplicity

$$V_{2n}(p_1, p_2) \sim \left(\frac{dN}{dy}[\rho_p, \rho_t]\right)\ , \quad V_{2n+1}(p_1, p_2) \sim \left(\frac{dN}{dy}[\rho_p, \rho_t]\right)^0$$

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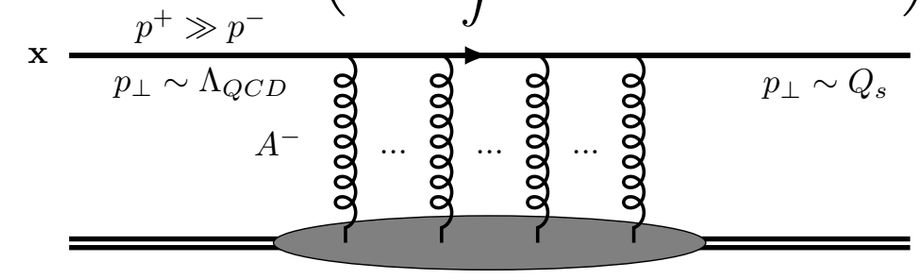
# Dipole correlators

First, need to be able to compute correlation functions expectation values of dipoles

Consider dipole scattering matrix

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \left\langle \frac{1}{N_c}\mathrm{tr}(U(\mathbf{x})U^\dagger(\mathbf{y}))\right\rangle$$

$$U(\mathbf{x}) = \mathcal{P}\exp\Big(-ig\int dz^+ A^{a-}(\mathbf{x},z^+)\,t^a\Big)$$



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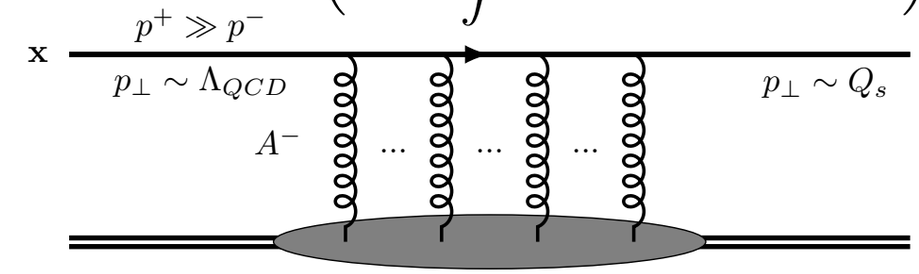


Expand out Wilson line in slices in rapidity

$$U(\mathbf{x}) = \mathcal{P}\exp\left(-ig\int dx^+ A^{a-}(\mathbf{x},x^+)t^a\right) \simeq V(\mathbf{x})[1 - igA^{a-}(\zeta,\mathbf{x})t^a + ...]$$

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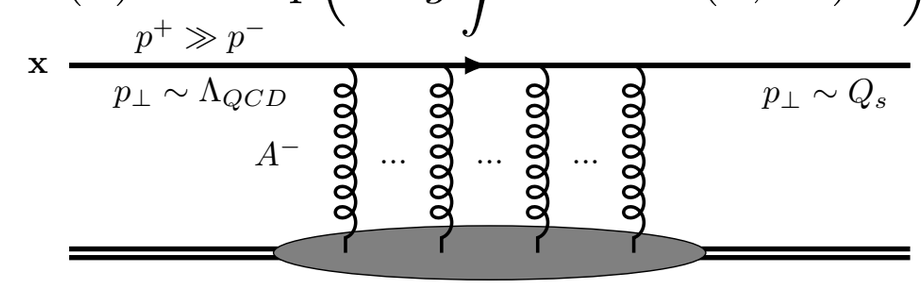


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where $L_{\mathbf{x}_\perp,\mathbf{y}_\perp} = -\frac{(g^2\mu)^2}{16\pi^2}|\mathbf{x}-\mathbf{y}|^2\log\left(\frac{1}{|\mathbf{x}_\perp-\mathbf{y}_\perp|\Lambda}+e\right)$

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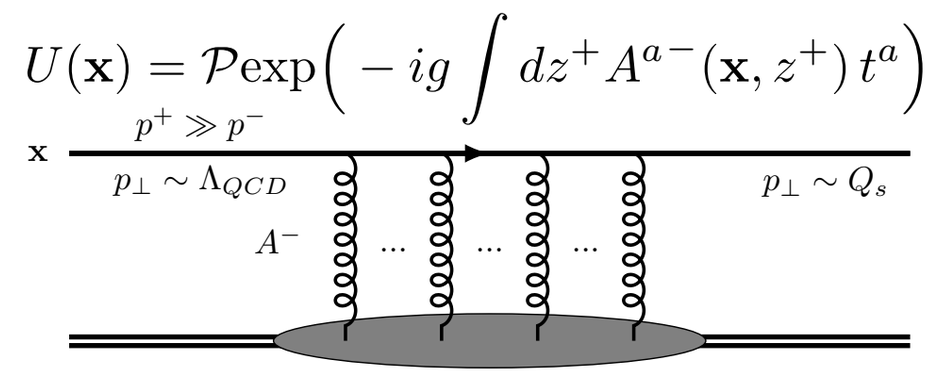


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We can re-exponentiate

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \exp(C_F L(\mathbf{x},\mathbf{y}))$$



See arXiv:1706.06260 for details

# Dipole correlators

Multiple dipole correlation functions encode projectile nucleus scattering, depends on scale $Q_s$

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dominguez, Marquet, Wu NPA 823 (2009), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018), Fukushima, Hidaka JHEP (2007,2017),…

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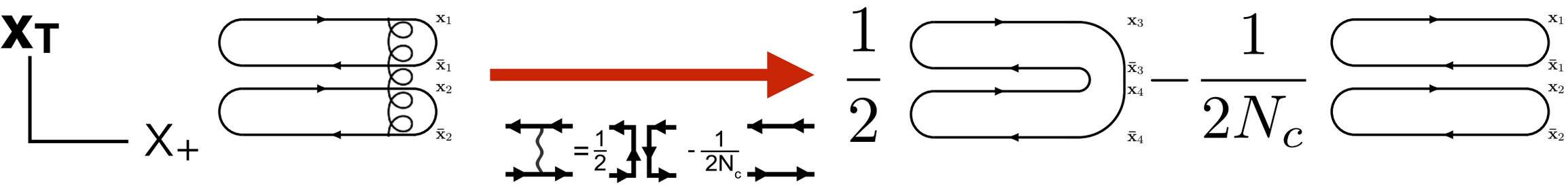


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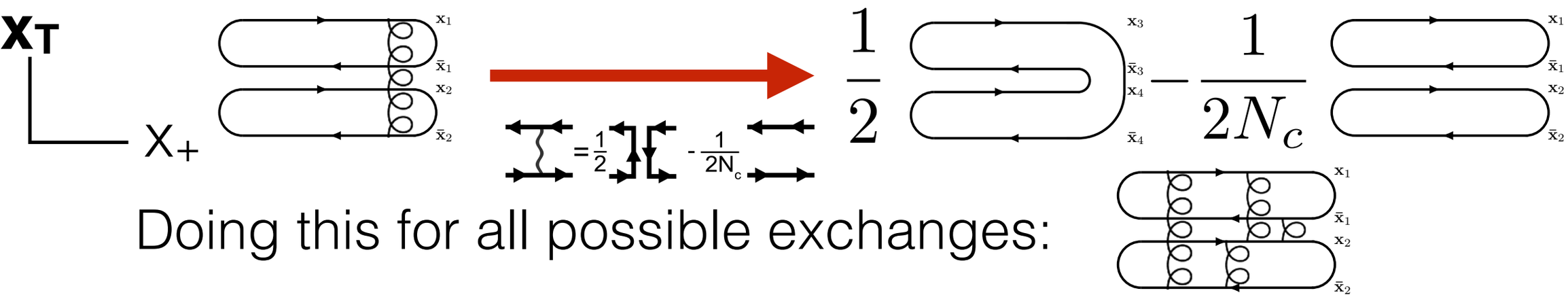


Doing this for all possible exchanges:

$$\begin{pmatrix} \langle D_{x_1\bar{x}_1} D_{x_2\bar{x}_2} \rangle \\ \langle Q_{x_1\bar{x}_2 x_2\bar{x}_1} \rangle \end{pmatrix}_U = \begin{pmatrix} \alpha_{x_1\bar{x}_1 x_2\bar{x}_2} & \beta_{x_1\bar{x}_2 x_2\bar{x}_1} \\ \beta_{x_1\bar{x}_1 x_2\bar{x}_2} & \alpha_{x_1\bar{x}_2 x_2\bar{x}_1} \end{pmatrix} \begin{pmatrix} \langle D_{x_1\bar{x}_1} D_{x_2\bar{x}_2} \rangle \\ \langle Q_{x_1\bar{x}_2 x_2\bar{x}_1} \rangle \end{pmatrix}_V$$

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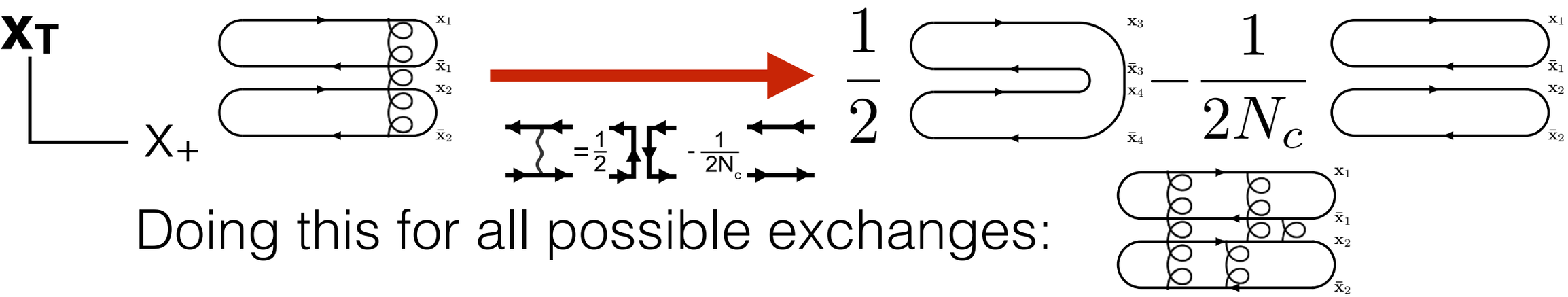


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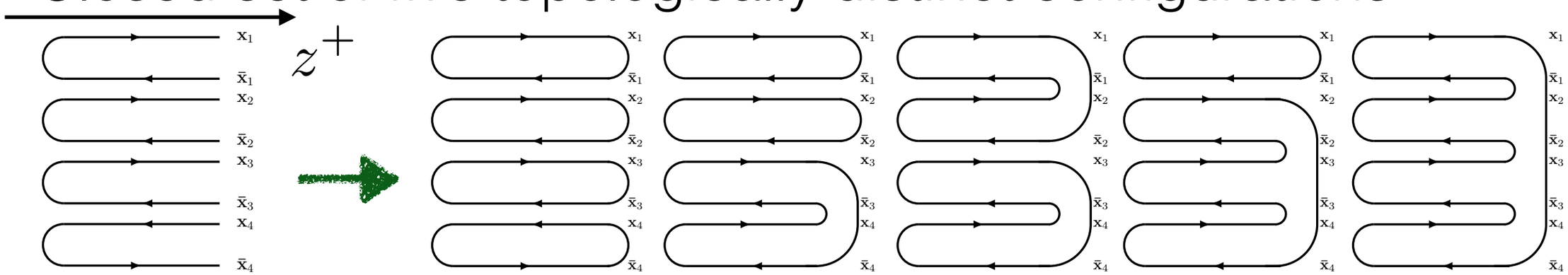
Straightforward to generalize $\frac{d^4N}{d^2\mathbf{p}_1 \cdots d^2\mathbf{p}_4} \simeq \int \langle DDDD \rangle$

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dominguez, Marquet, Wu NPA 823 (2009), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018), Fukushima, Hidaka JHEP (2007,2017),…

# Four dipole correlators

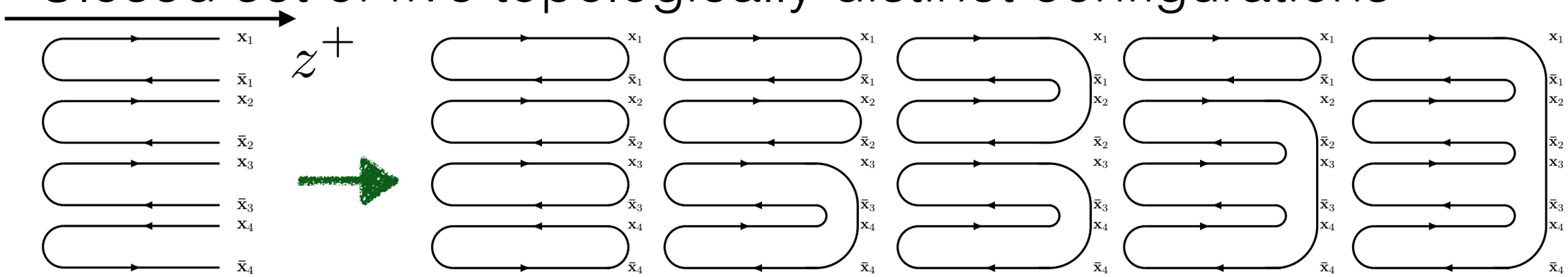
Closed set of five topologically distinct configurations




Permutations for each topology for closing on $z^+ = +\infty$

# Four dipole correlators

Closed set of five topologically distinct configurations




Permutations for each topology for closing on $z^+ = +\infty$

Define single gluon exchange matrix in terms of $L_{\mathbf{x},\mathbf{y}}$

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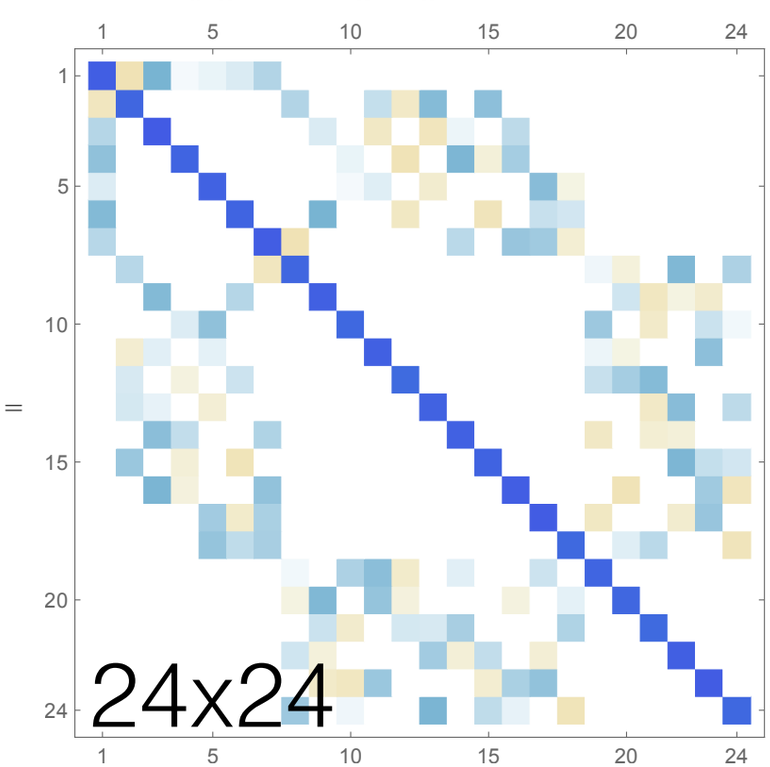
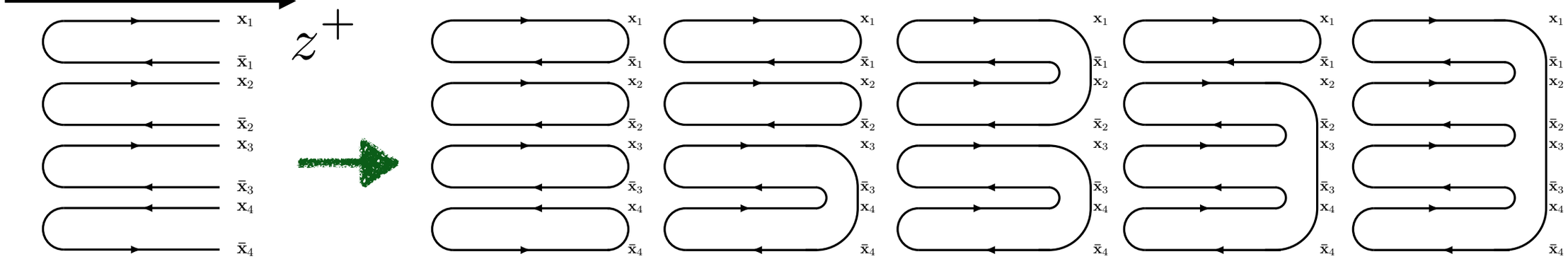



$$\begin{bmatrix} \langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{x}} D_{y\bar{z}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{w}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{z}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{x}} D_{z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}} \rangle \end{bmatrix}_V$$

# Four dipole correlators

Closed set of five topologically distinct configurations

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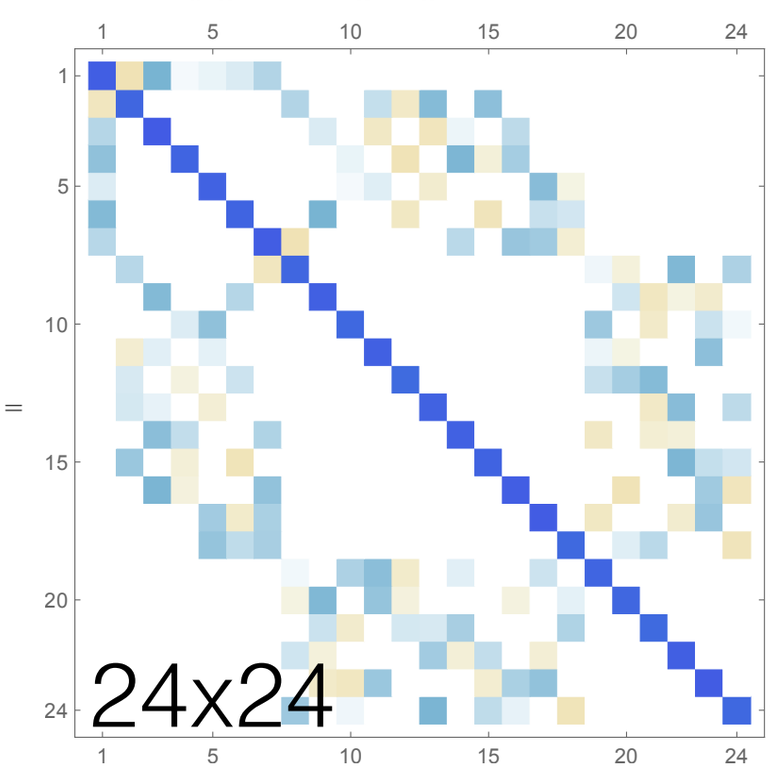


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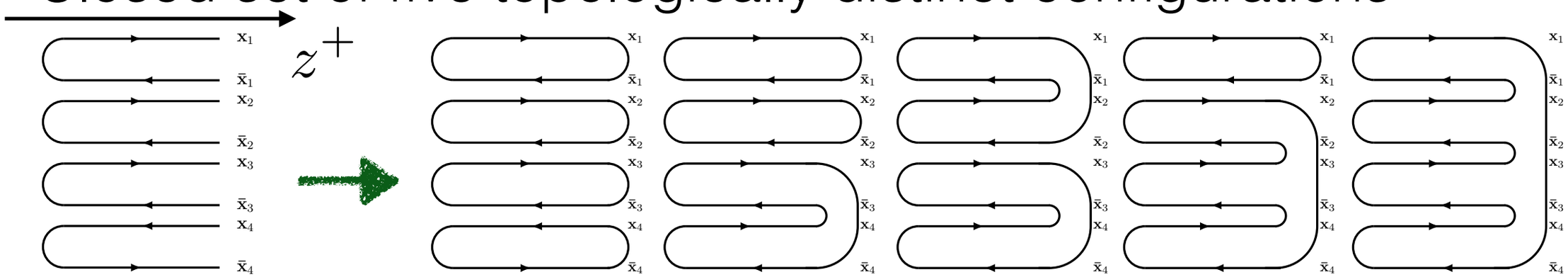
# Four dipole correlators

Closed set of five topologically distinct configurations




Permutations for each topology for closing on $z^+ = +\infty$

Define single gluon exchange matrix in terms of $L_{\mathbf{x},\mathbf{y}}$

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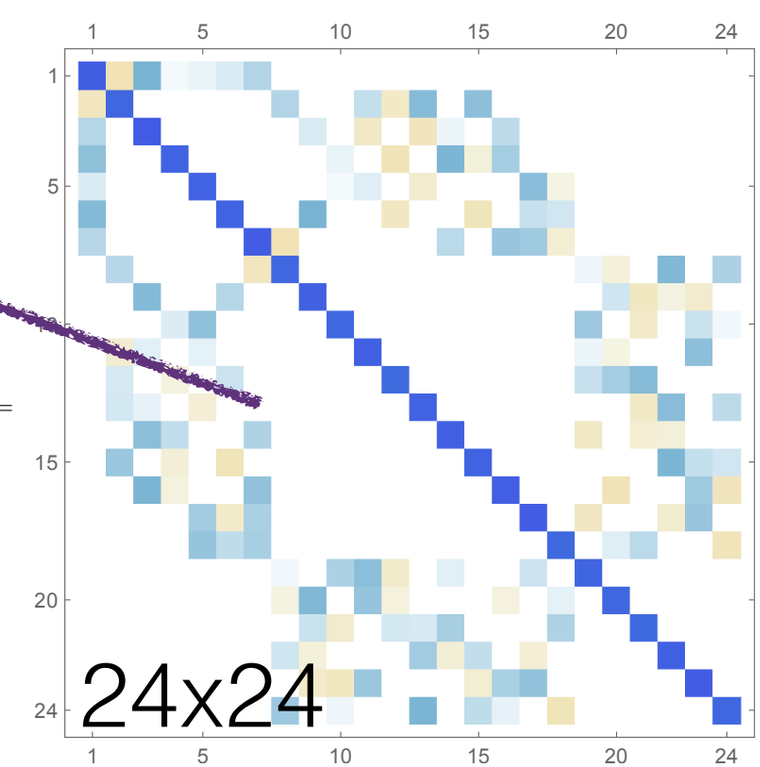


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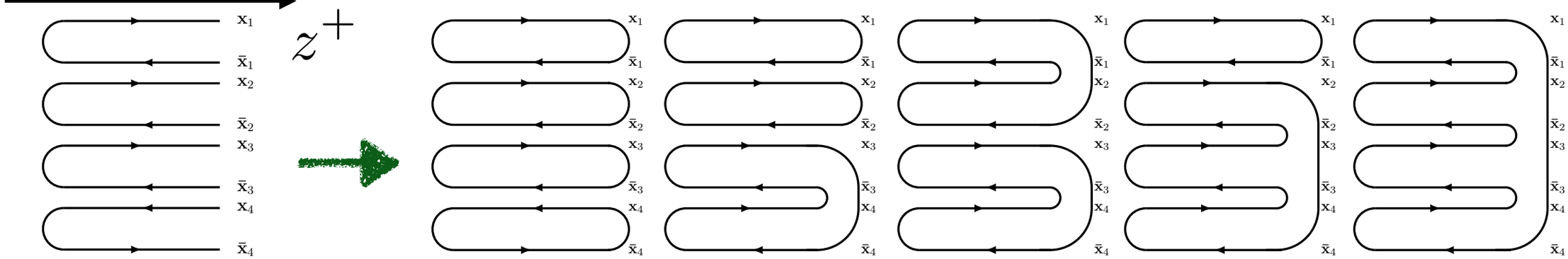


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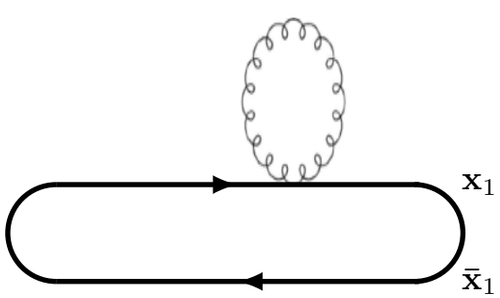
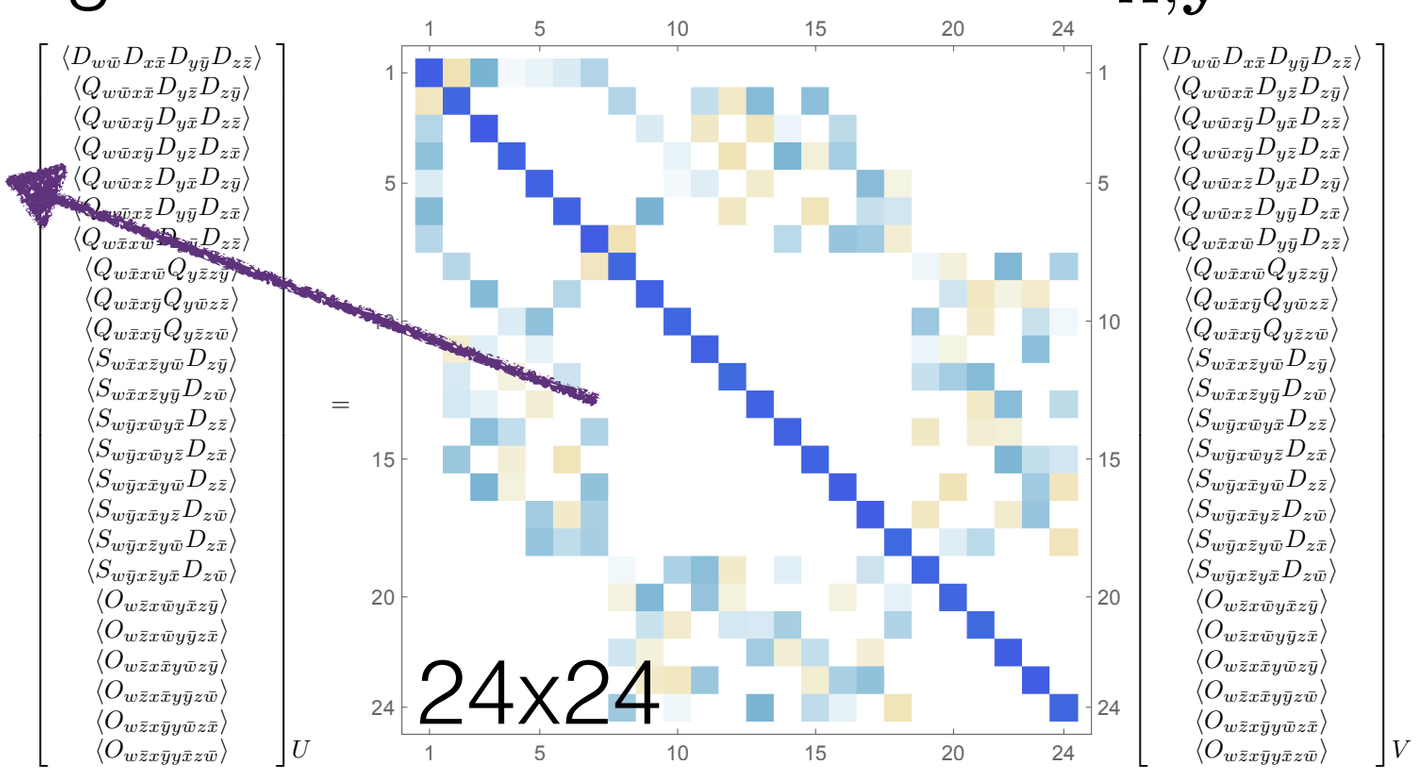
Also includes tadpole contribution



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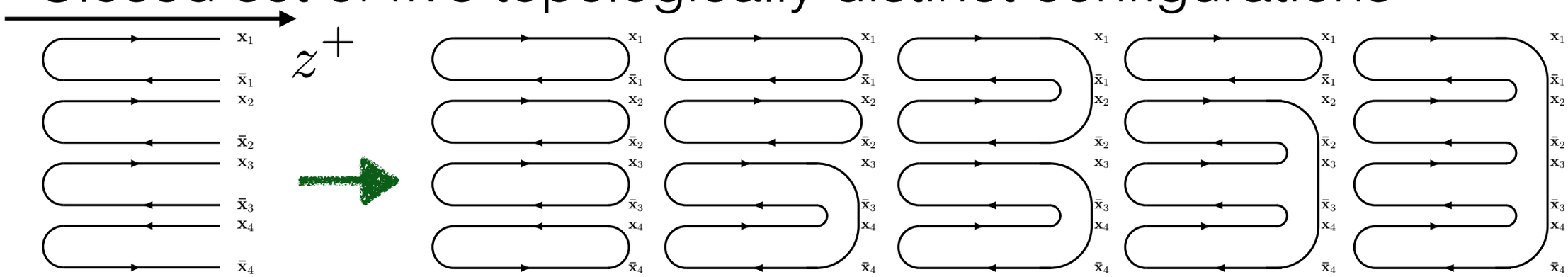


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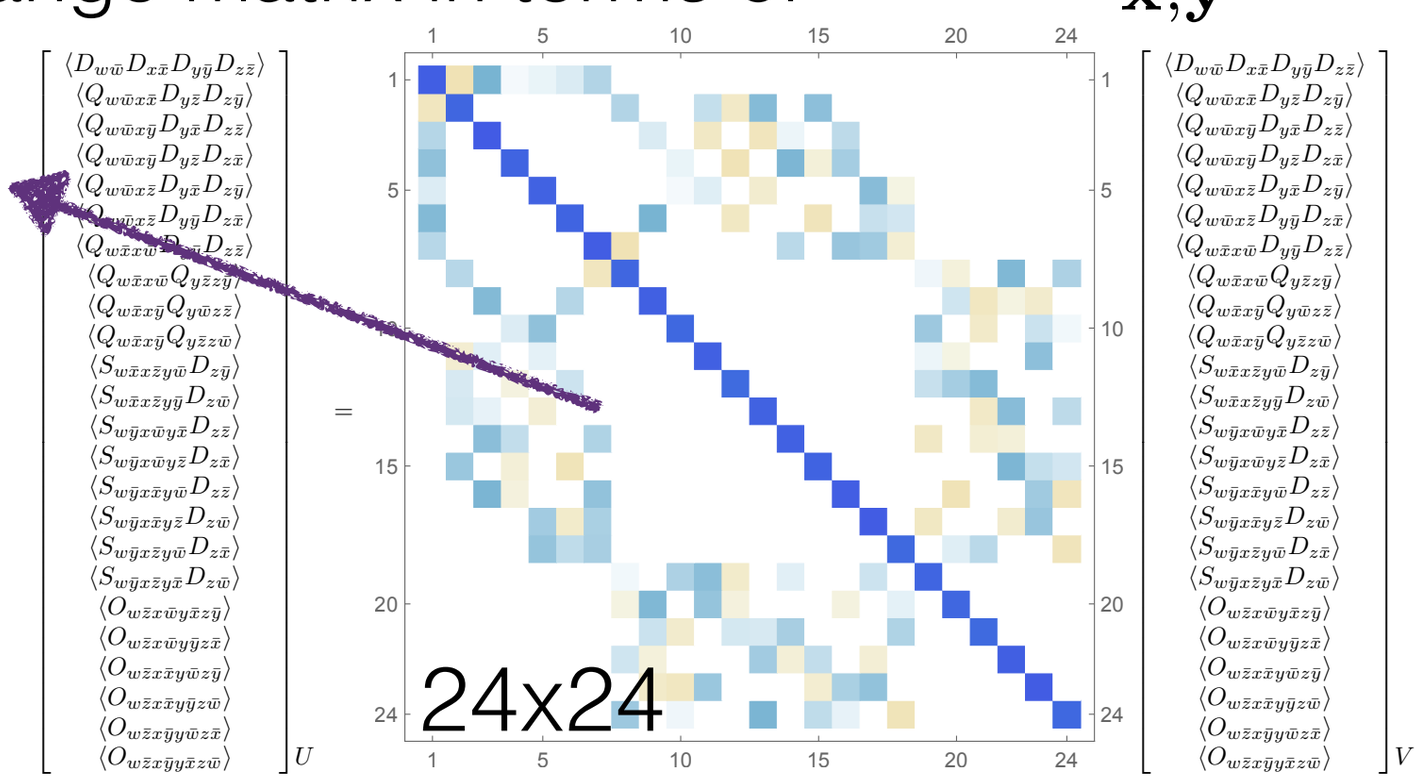


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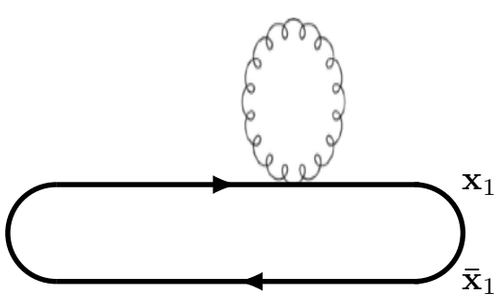



Algorithm can be used to compute other configurations, arbitrary number of Wilson lines

# Dilute dense for gluons

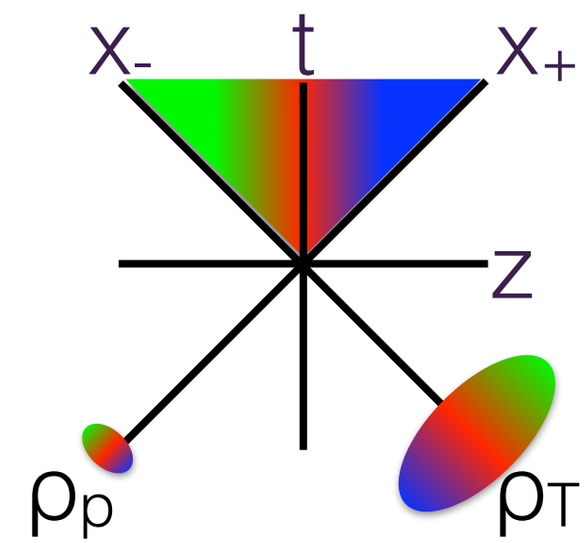
CGC EFT: solve CYM with static color sources

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

All orders in $\rho_T$, $\rho_p$ only known numerically

Dilute-dense limit: $\rho_T \gg \rho_p$




Kovchegov,. Mueller NPB 529 (1998), Kovner, Wiedemann PRD 64 (2001), Dumitru, McLerran NPA 700 (2002),, Blaizot, Gelis, Venugopalan NPA 743 (2004), McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018),…

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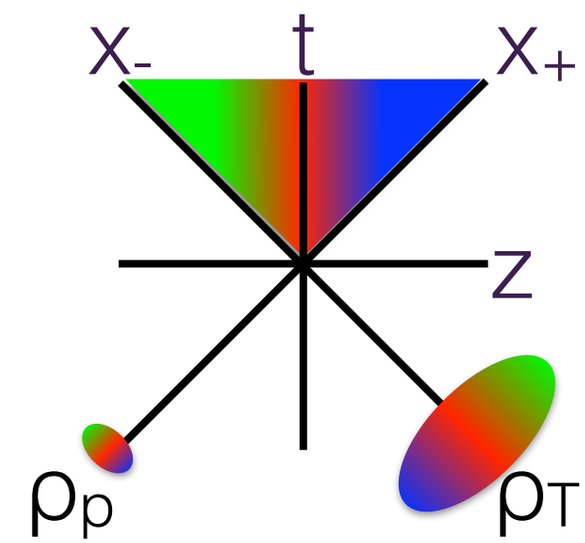


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Calculate for $A^\mu$ to all order in $\rho_T$, first order in $\rho_p$ a.k.a. the dilute-dense, analytically accessible

e.g. Dumitru, McLerran NPA 700 (2002), McLerran, Skokov NPA 959 (2017)

$$\frac{dN}{d^2k} \sim g^2\rho_p^2 f_{(1)}(\rho_T) + g^4\rho_p^4 f_{(2)}(\rho_T) + \cdots$$

$f_{(1)}$ well known, no complete results for $f_{(2)}$ yet

Kovchegov,. Mueller NPB 529 (1998),Dumitru, McLerran NPA 700 (2002), Blaizot, Gelis, Venugopalan NPA 743 (2004)
Balitsky, PRD 70 (2004), Chirilli, Kovchegov, Wertepny, JHEP 03 (2015)

# The $v_3$ Problem

Leading order dilute-dense limit highly amenable to numerics

Lappi EPJC 55 (2008)

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For double inclusive, $\frac{d^2N}{d^3k_1 d^3k_2}$, leading order is also known

Kovner. Lublinsky, IJMPE 22 (2013), Kovchegov, Wertepny, NPA 906, (2013),

$$\frac{d^2N}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{d^2N}{k_1 dk_1 dy_1 k_2 dk_2 dy_2}$$
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For a non-zero $v_3$

McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)

$$\int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\Big(\delta\phi\Big) = \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\Big(\delta\phi\Big) - \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\Big(\delta\phi + \pi\Big)$$

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Must be non-vanishing

However, at leading order ($\rho_p^4$) it is exactly zero, but not in dense-dense

Kovner, Lublinsky, PRD 83 (2011), Kovchegov, Wertepny, NPA 906 (2013), Kovchegov, Skokov PRD 97 (2018)

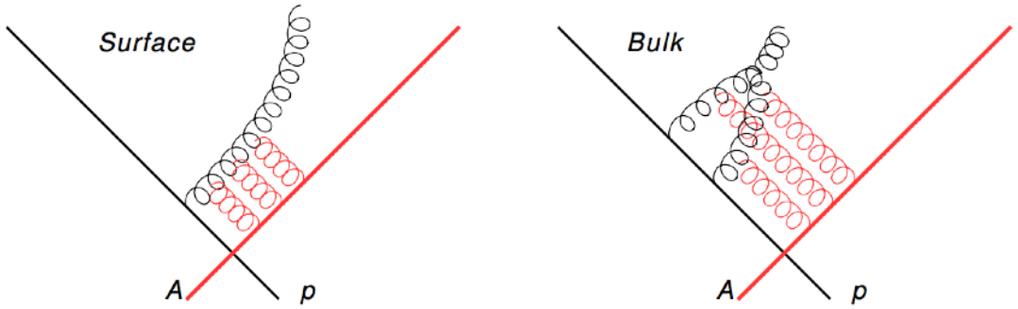
Lappi, Srednyak, Venugopalan JHEP 1001 (2010), Schenke, Schlichting, Venugopalan PLB 747 (2015)

# Dilute dense for gluons

Issue resolved at next order in $\rho_p$

Symmetry broken in $\frac{d^2N}{d^3k_1 d^3k_2}$ by first saturation correction $O(\rho_p^6)$

McLerran, Skokov NPA 959 (2017)

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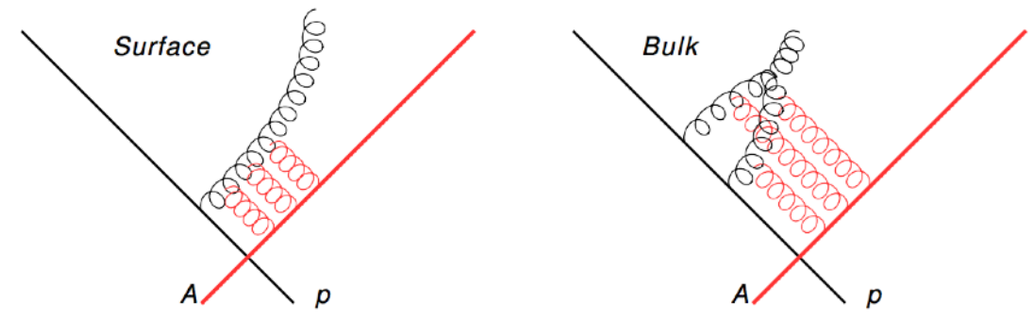


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Projectile Target

In terms of: $$\Omega^a_{ij}(\mathbf{x}) = g\left[\frac{\partial_i}{\partial^2}\rho^b_p(\mathbf{x})\right]\partial_j U^{ab}(\mathbf{x})$$

Valence sources rotated by target

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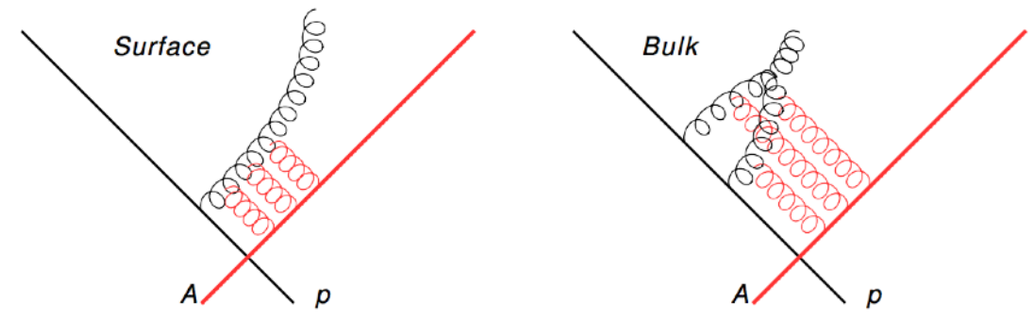


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Valence sources rotated by target

Same results in LC gauge ($A^+$=0), resolution similar to STSA

Kovchegov, Skokov PRD 97 (2018), Brodsky, Hwang, Kovchegov, Schmidt, Sievert, PRD 88 (2013)

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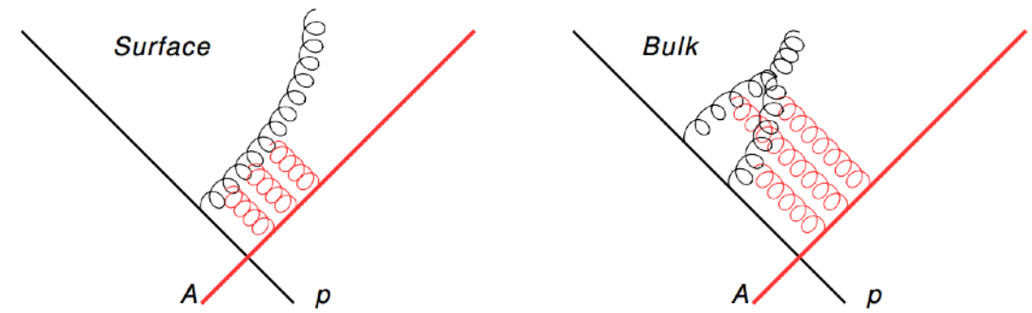


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Valence sources rotated by target

Also non-zero contribution to $v_3$ from proj. JIMWLK evolution

Kovner, Lublinsky, Skokov PRD 96 (2017)

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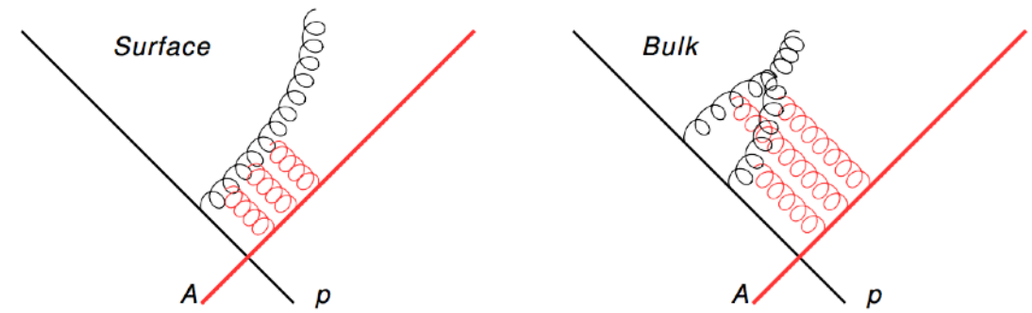


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Multi-particle distributions then defined as

$$\frac{d^2N}{d^2k_1dy_1...d^2k_ndy_n} = \left\langle\left\langle\frac{dN}{d^2k_1dy_1}\Big|_{\rho_p,\rho_T}...\frac{dN}{d^2k_ndy_n}\Big|_{\rho_p,\rho_T}\right\rangle_p\right\rangle_T$$

Only well defined for ensemble over $W[\rho_T,\rho_p]$

# Glauber IP-Sat model

For data-guided initial conditions, consider initial conditions based on very successful IP-Glasma model

Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)

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Sample nucleons through Monte-Carlo Glauber

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Sample nucleons through Monte-Carlo Glauber

IP-Sat model provides $Q_s^2(x,\mathbf{b})$ for each nucleon

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

Based on dipole model fits to HERA DIS data

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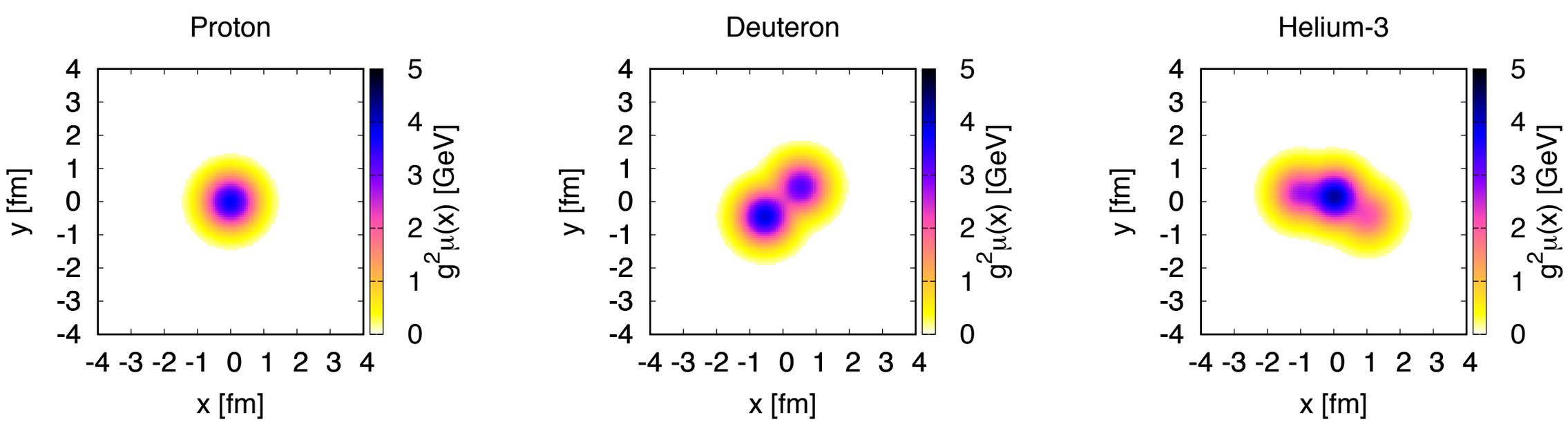
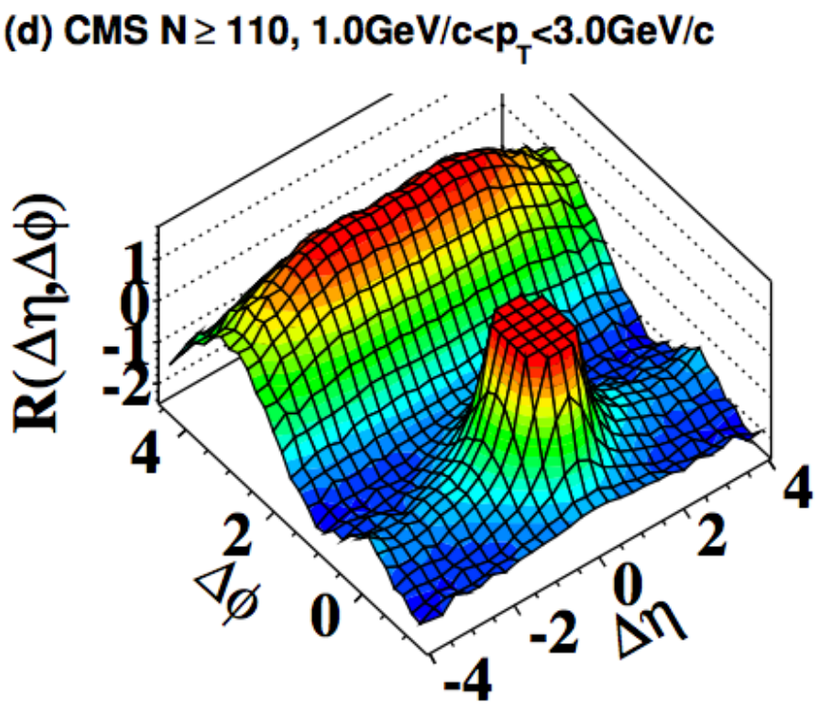
Based on dipole model fits to HERA DIS data

Example of three high multiplicity (0-5%) configurations

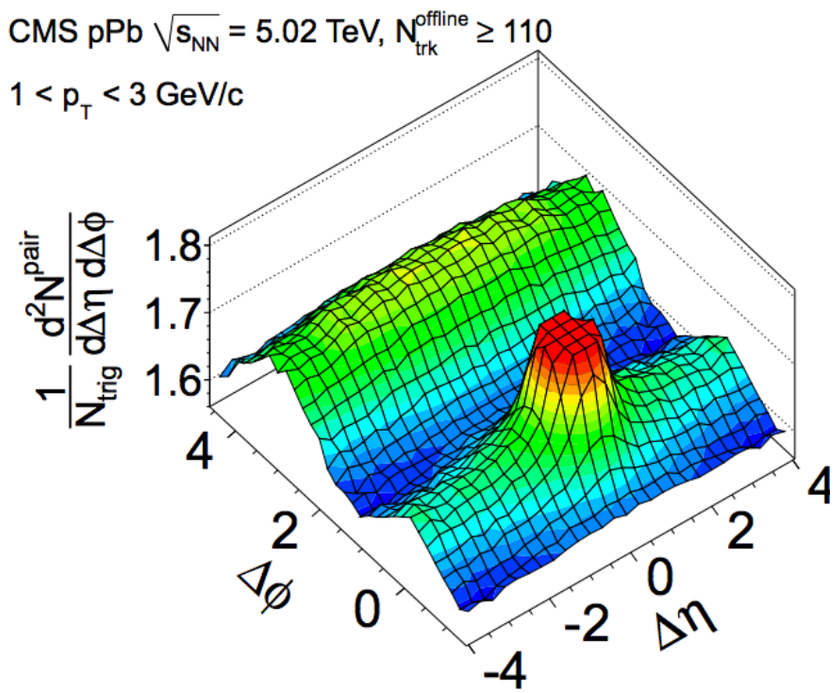


# Similarity in all systems

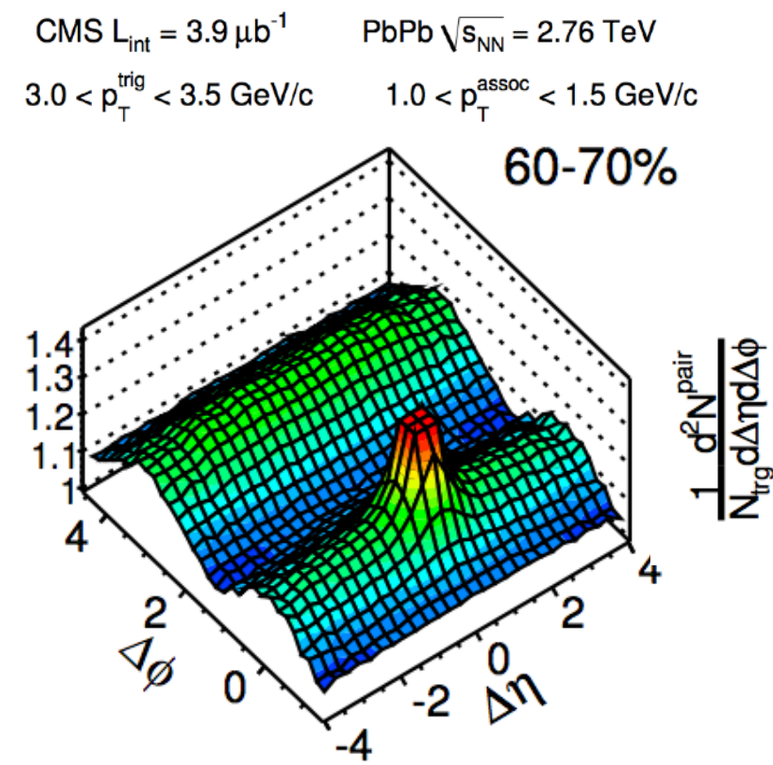
Two particle correlations across all systems look very similar




CMS JHEP 1009:091 (2010)




CMS PLB 718 (2013) 795




CMS EPC 72 (2012) 10052

Are we seeing smallest droplets of QGP? Rare QCD configurations? Both?

# Long range in rapidity?

Experiment is, and thus theory should be, long range in rapidity

Model is based on hybrid framework, valid at forward rapidity

Dumitru, Jalilian-Marian PRL 89 (2002), Kovchegov, Wertepny NPA 906 (2013), Kovner, Lublinsky IJMPE 22 (2013)

Valence partons in projectile long lived and have a boost invariant wave function, coherence length $\Delta y \sim 1/\alpha_s \sim \infty$

Quantum corrections can change this picture, however beyond scope of hybrid model

Dusling, Gelis Lappi, Venugopalan NPA 836 (2010)

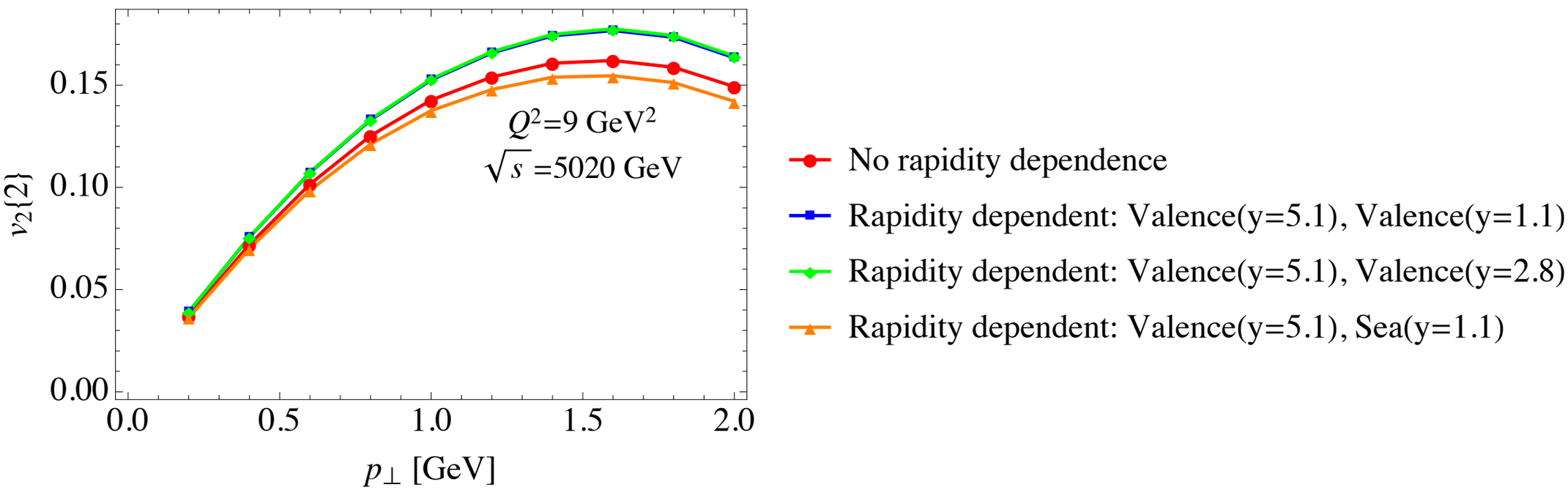
Suppose $x_q \geq 0.01$, $x_q = \frac{p_\perp}{\sqrt{s}} e^y$, taking $p_\perp = 3 \text{ GeV}$, $\sqrt{s} = 5.02 \text{ TeV}$

Framework valid for $y \geq 2.8$

# Rapidity dependence

Convolute spectrum with PDF: $$\frac{dN^{pA\to q+X}}{dyd^2\mathbf{p}_\perp} = x'_q f(x'_q)\frac{dN^{qA\to q+X}}{dyd^2\mathbf{p}_\perp}$$

Compare $v_2\{2\}(p_T)$ for with rapidity dependent distributions




Only quantitive, not qualitative, differences when considering both small and large x quarks

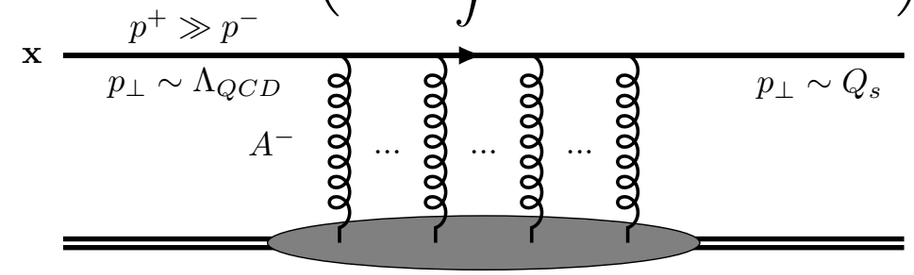
# Dipole correlators

First, need to be able to compute correlation functions expectation values of dipoles

Consider dipole scattering matrix

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \left\langle \frac{1}{N_c}\mathrm{tr}(U(\mathbf{x})U^\dagger(\mathbf{y}))\right\rangle$$

$$U(\mathbf{x}) = \mathcal{P}\exp\Big(-ig\int dz^+ A^{a-}(\mathbf{x},z^+)\,t^a\Big)$$



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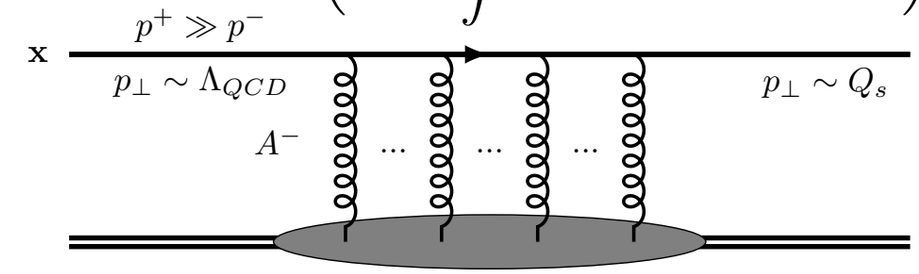


Expand out Wilson line in slices in rapidity

$$U(\mathbf{x}) = \mathcal{P}\exp\left(-ig\int dx^+ A^{a-}(\mathbf{x},x^+)t^a\right) \simeq V(\mathbf{x})[1 - igA^{a-}(\zeta,\mathbf{x})t^a + ...]$$

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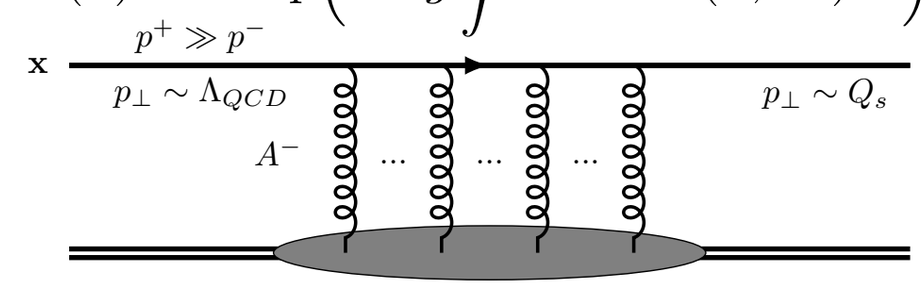


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Then gluons emissions with MV model

$$g^2\langle A_a^-(x^+,\mathbf{x}_\perp)A_b^-(y^+,\mathbf{y}_\perp)\rangle = \delta_{ab}\delta(x^+-y^+)L_{\mathbf{xy}}$$

where $L_{\mathbf{x}_\perp,\mathbf{y}_\perp} = -\frac{(g^2\mu)^2}{16\pi^2}|\mathbf{x}-\mathbf{y}|^2\log\left(\frac{1}{|\mathbf{x}_\perp-\mathbf{y}_\perp|\Lambda}+e\right)$

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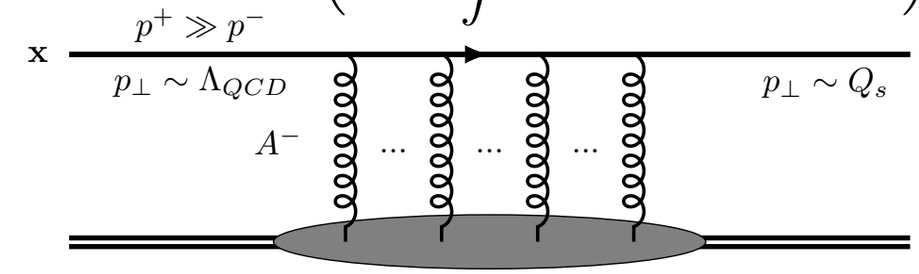


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We can re-exponentiate

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \exp(C_F L(\mathbf{x},\mathbf{y}))$$



See arXiv:1706.06260 for details

# Dipole correlators

Multiple dipole correlation functions encode projectile nucleus scattering, depends on scale $Q_s$

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018)

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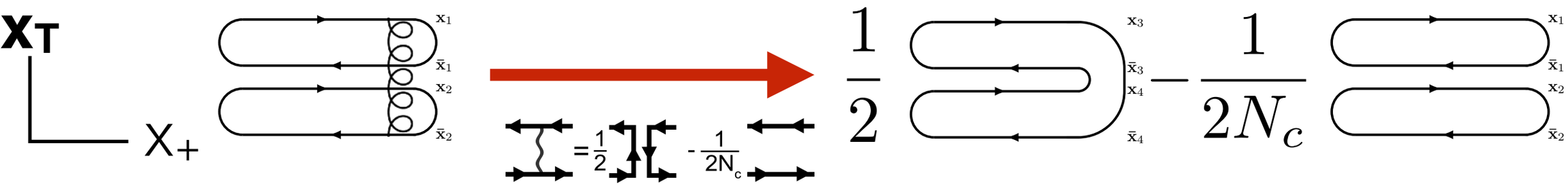
Then we can obtain $\langle DD \rangle$ similarly, first considering single gluon exchange, given by Fierz identity




Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018)

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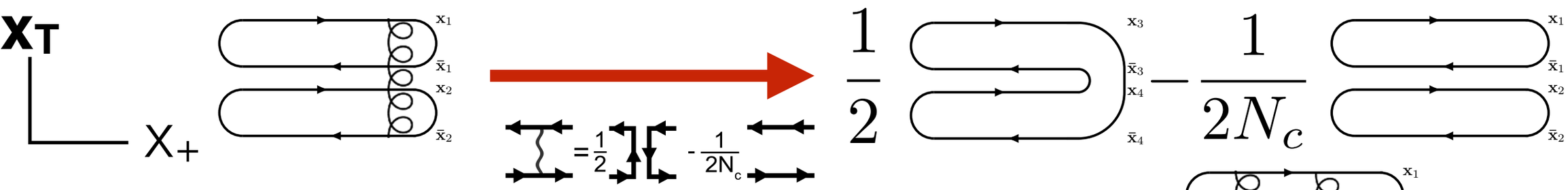
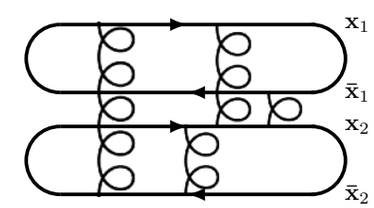



Doing this for all possible exchanges:

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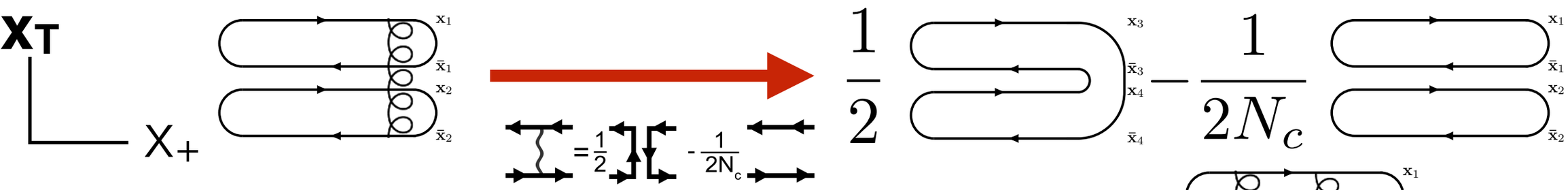


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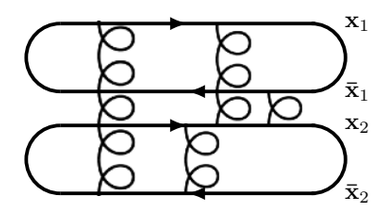


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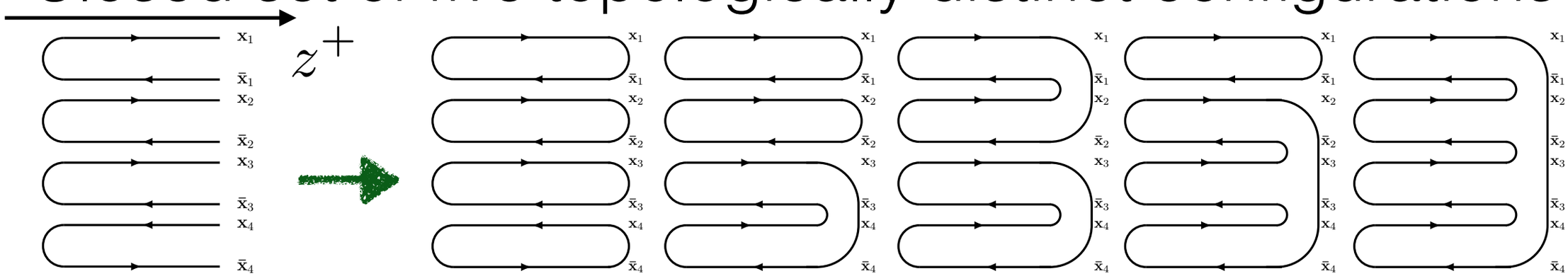
# Four dipole correlators

Closed set of five topologically distinct configurations

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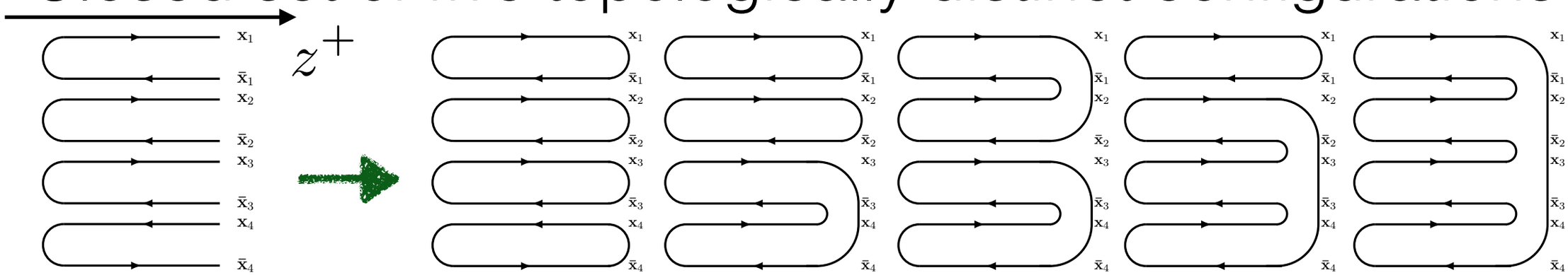


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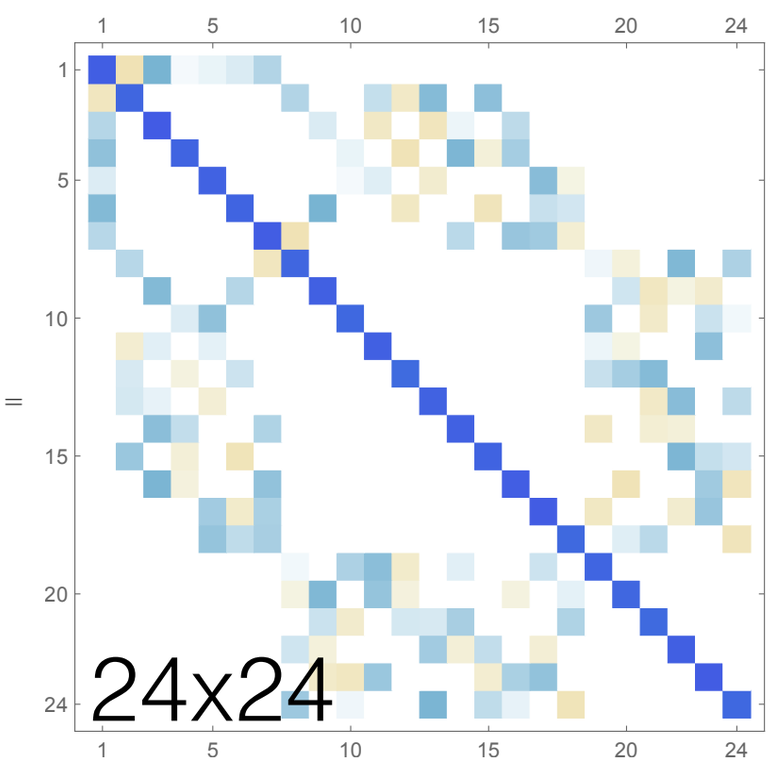


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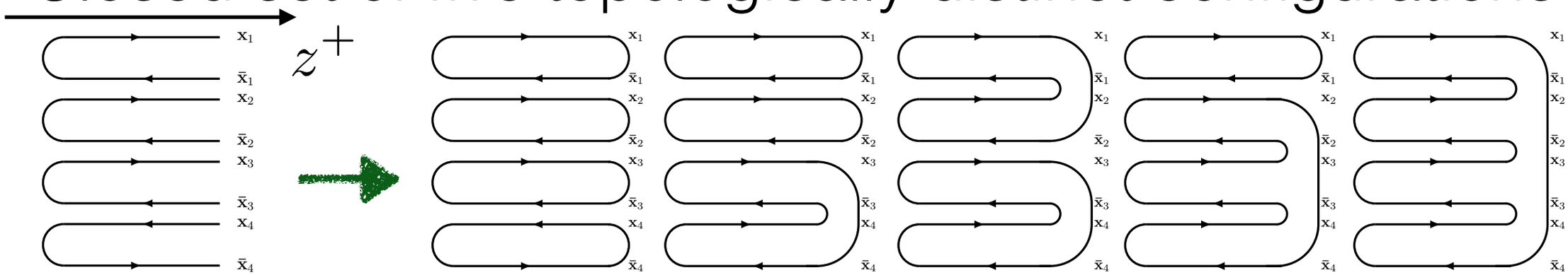
# Four dipole correlators

Closed set of five topologically distinct configurations

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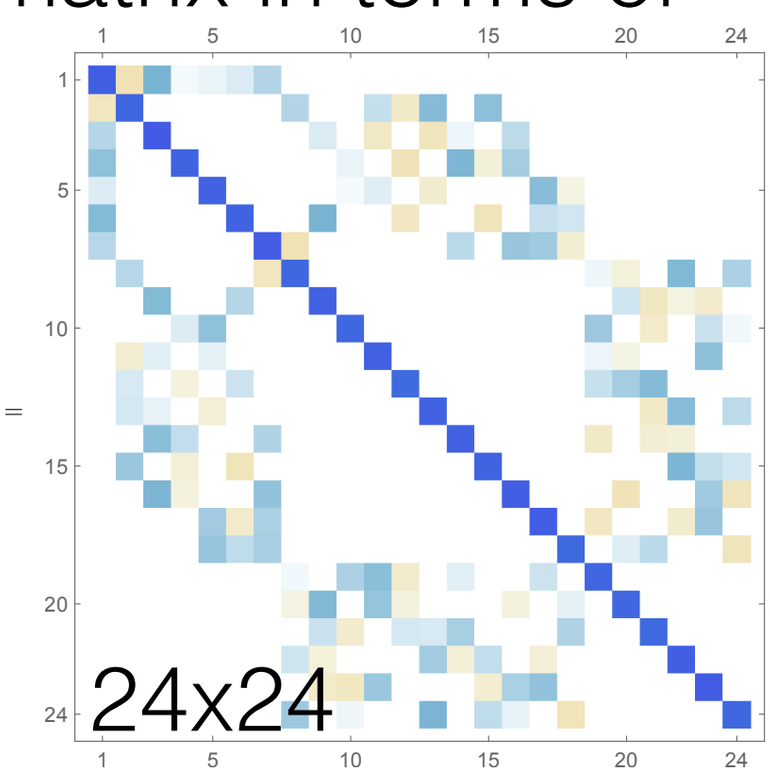


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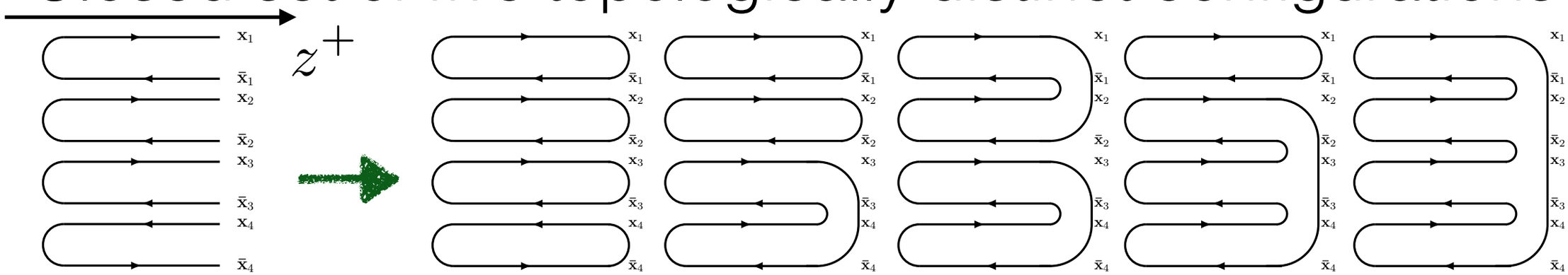


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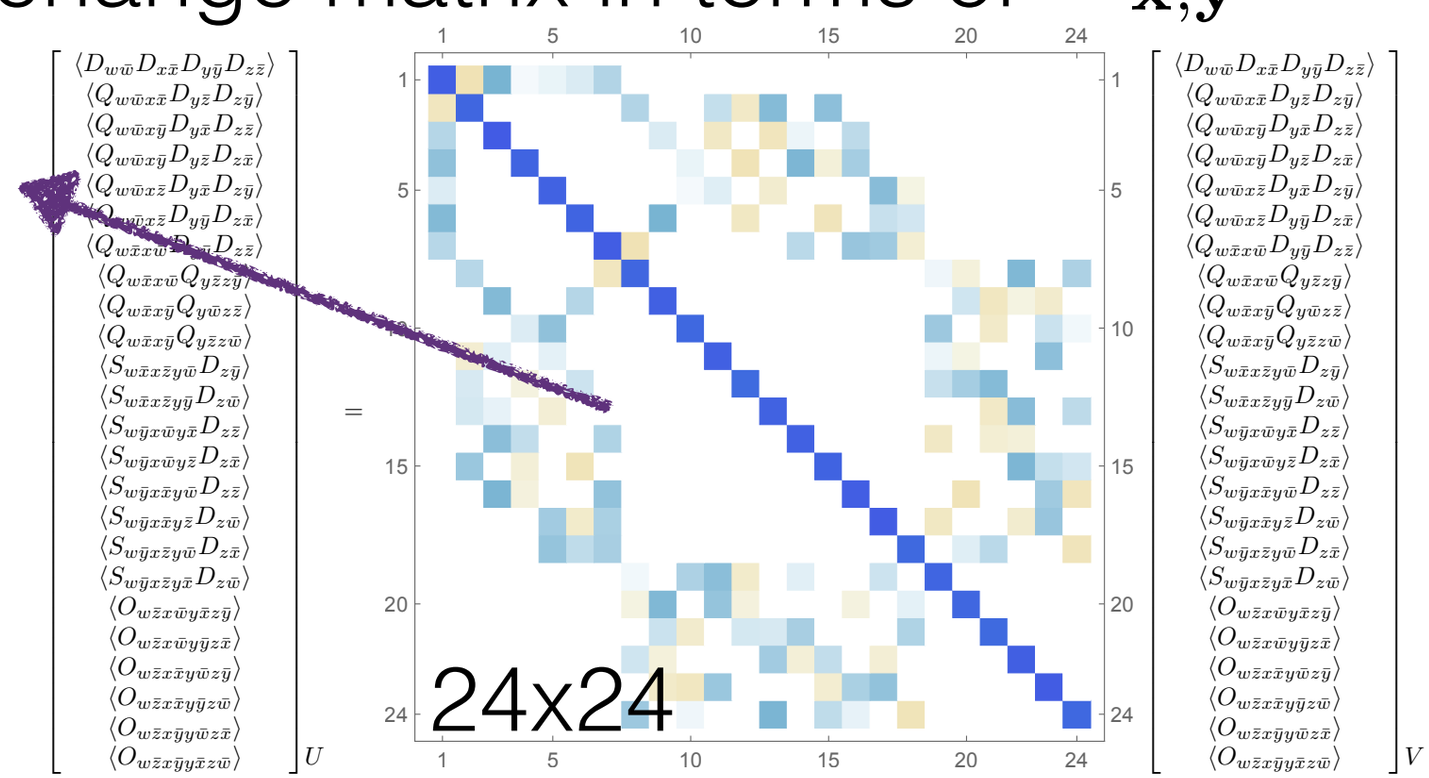

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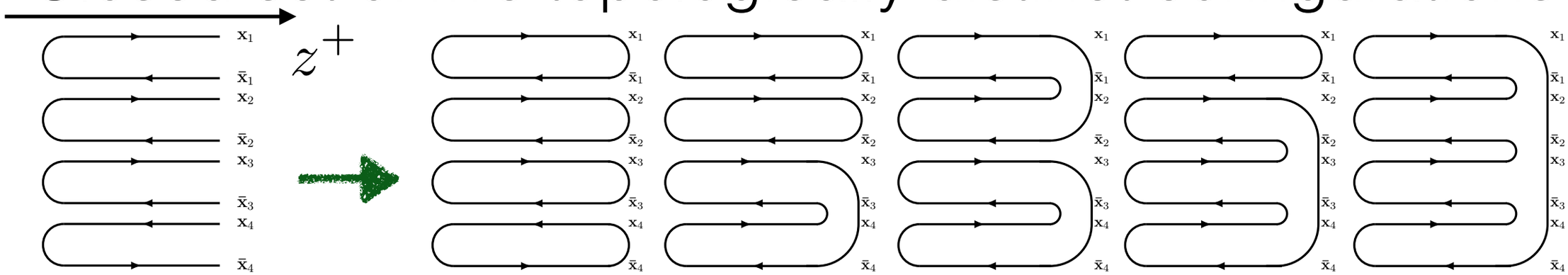


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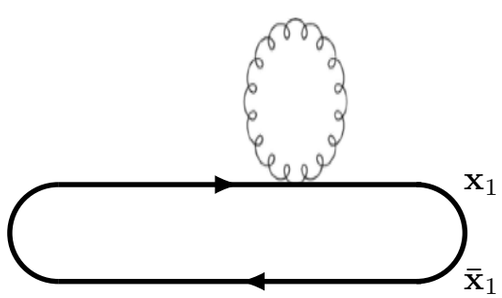


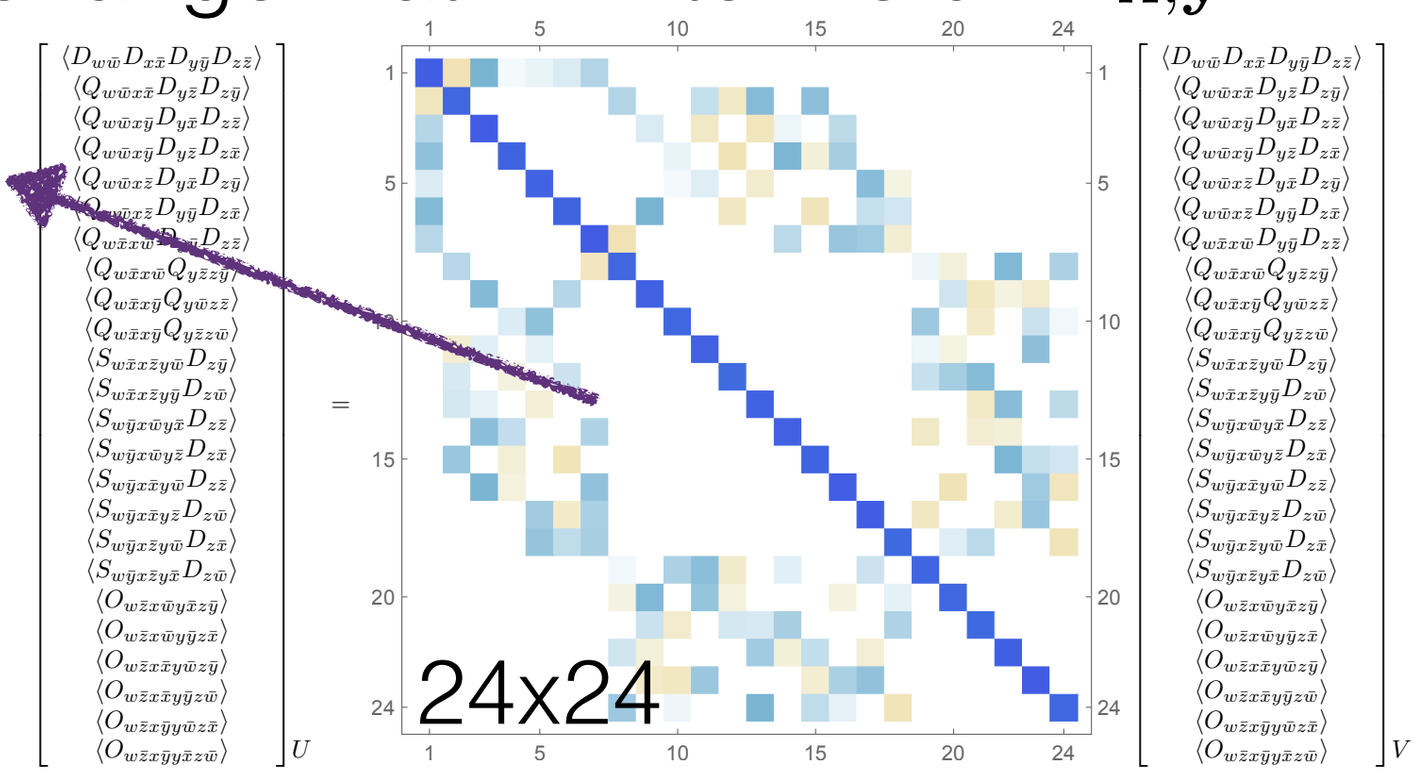

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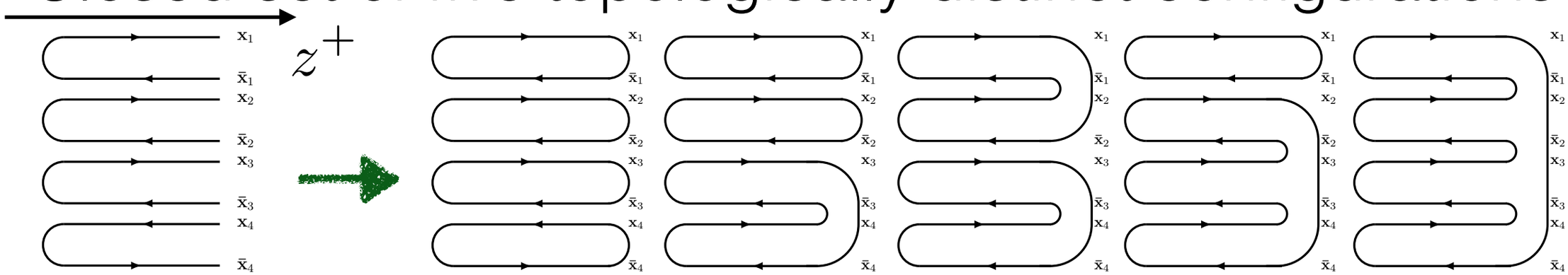


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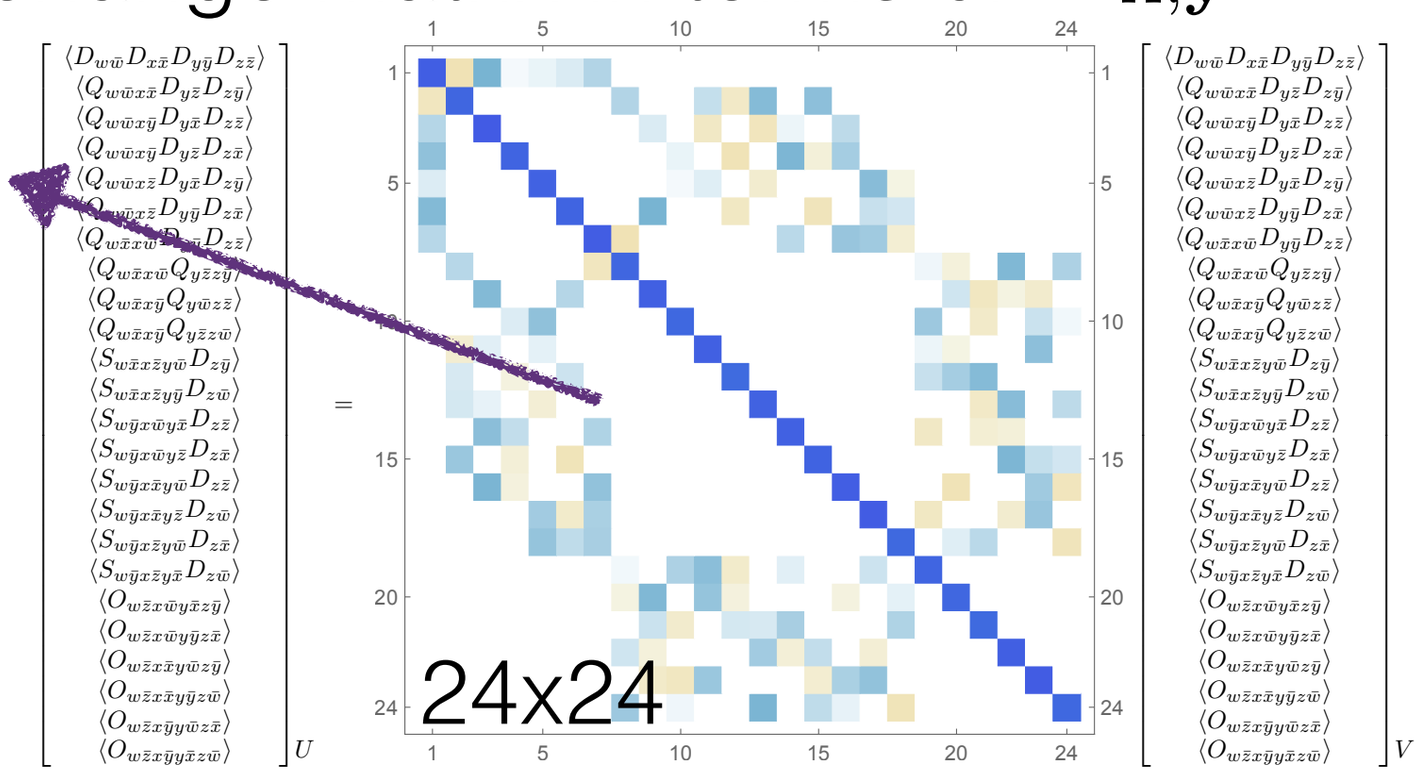


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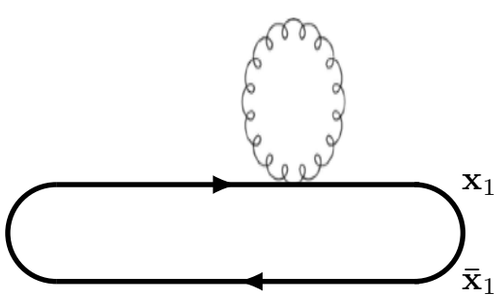
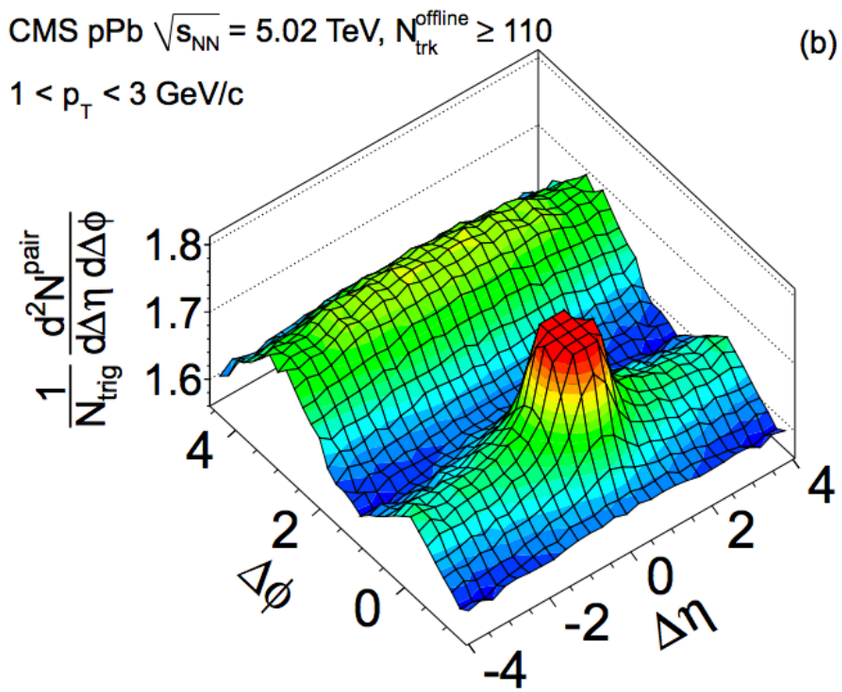


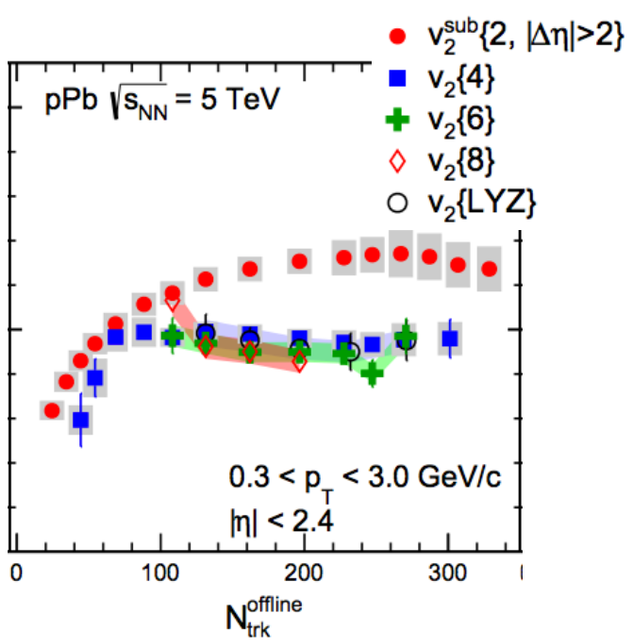

Algorithm can be used to compute other configurations, arbitrary number of Wilson lines

# the Ridge and Collectivity

pPb




CMS PLB 718 (2013) 795




CMS PRL 115 (2015) 012301

Flow paradigm: Event-by-event fluctuations of initial transverse geometry transported via hydrodynamics, resulting in a final state momentum correlations

Alver, Roland, PRC 81 (2010), Alver, Gombeaud, Luzum, Ollitrault, PRC 82 (2010)

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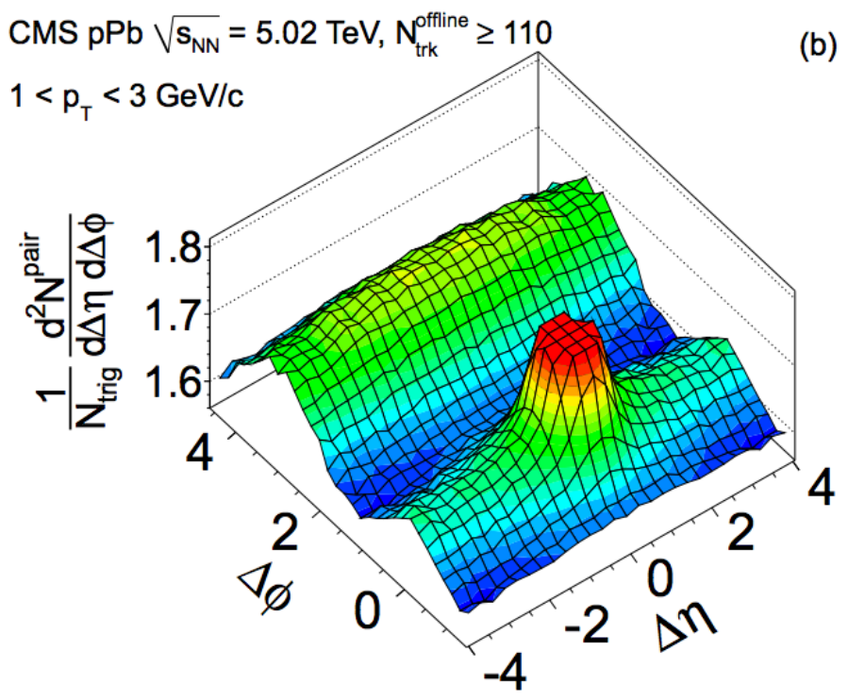


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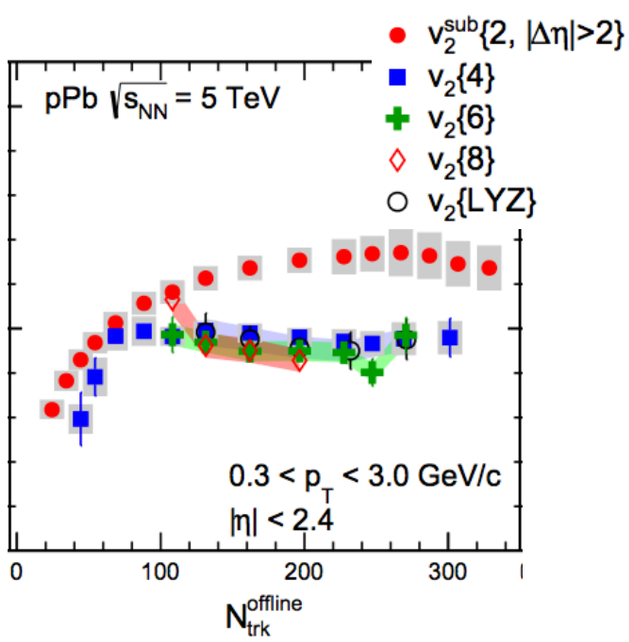


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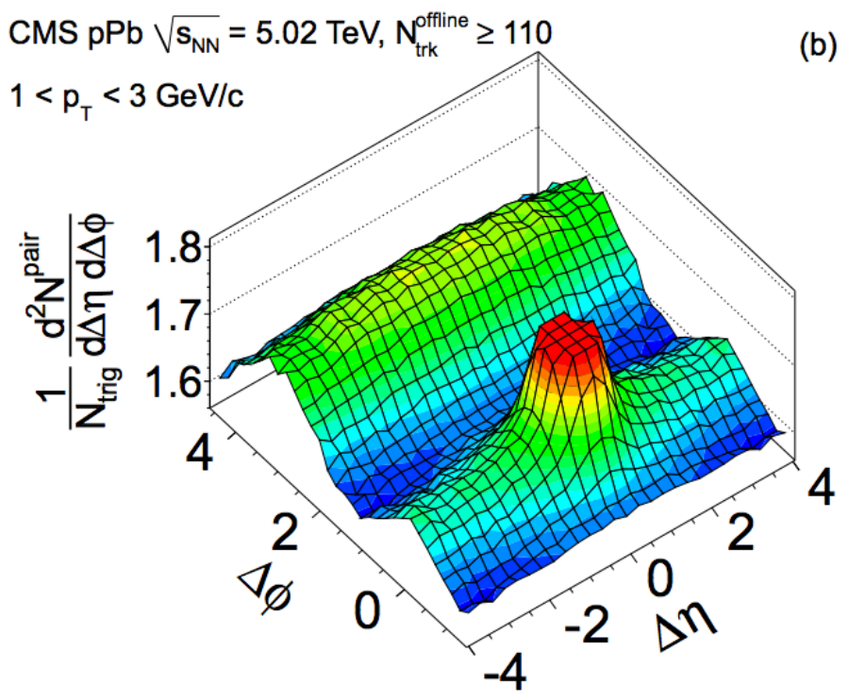


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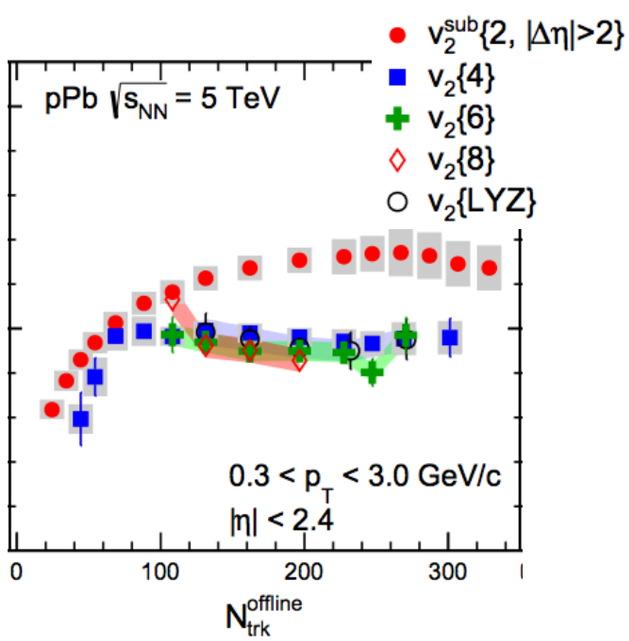


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In small systems, take a working definition for collectivity

c.f. Yan, Ollitrault PRL 112 (2014) 082301, Bzdak, Skokov NPA 943

$$v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$$

# Scale dependence

$p_\perp^{\max}$

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For a fixed $p_\perp^{\max}$ , single scale in problem, $Q_s^2 B_p$

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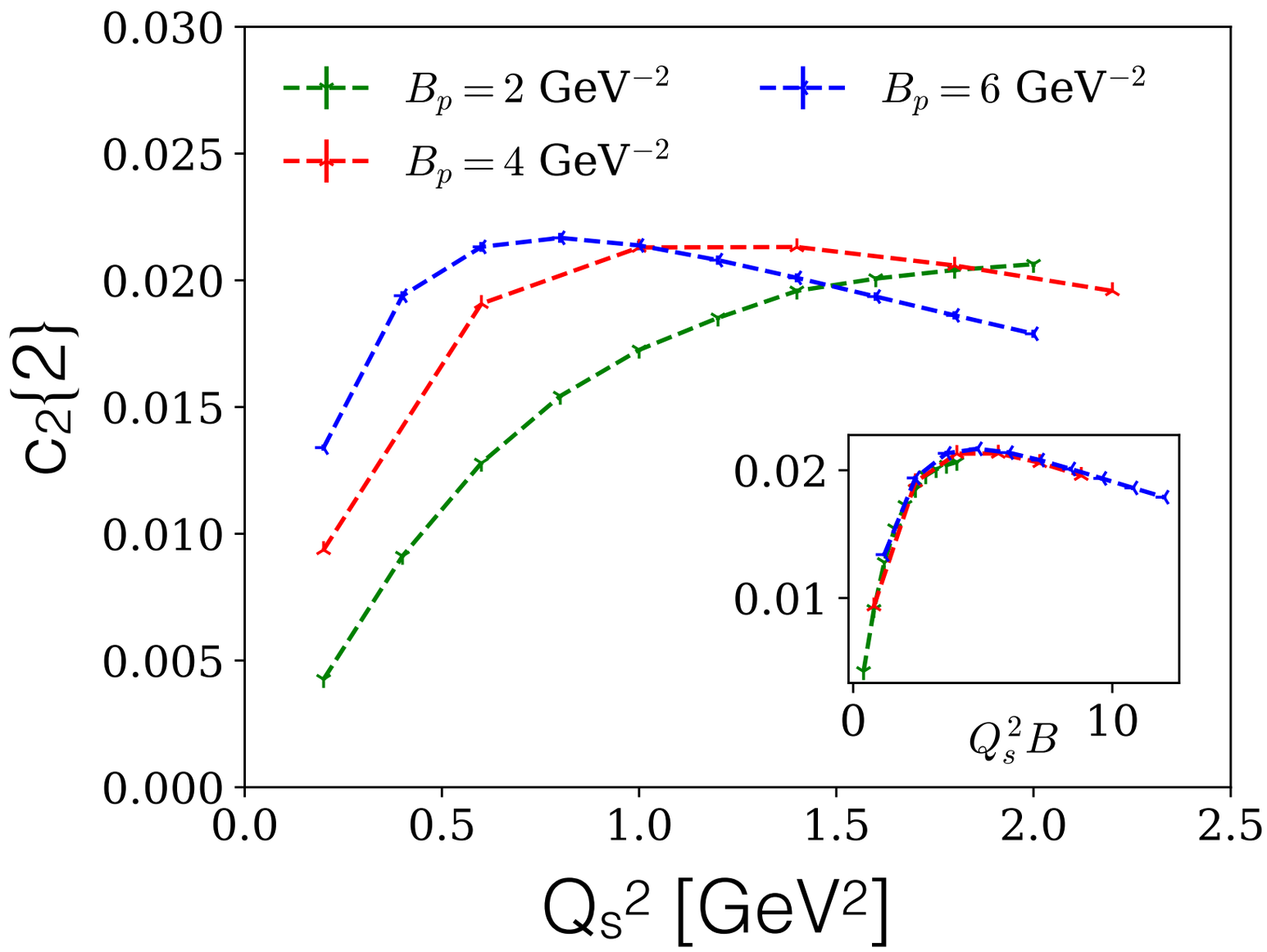
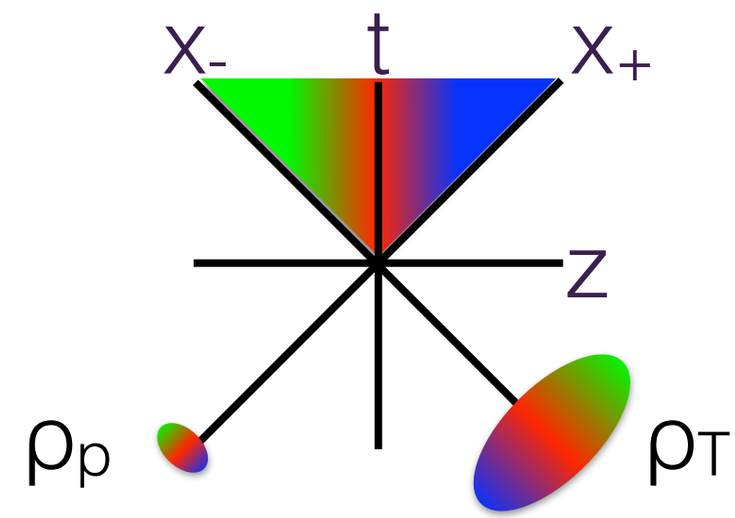


# Dilute dense for gluons

Solve CYM with static color sources

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$



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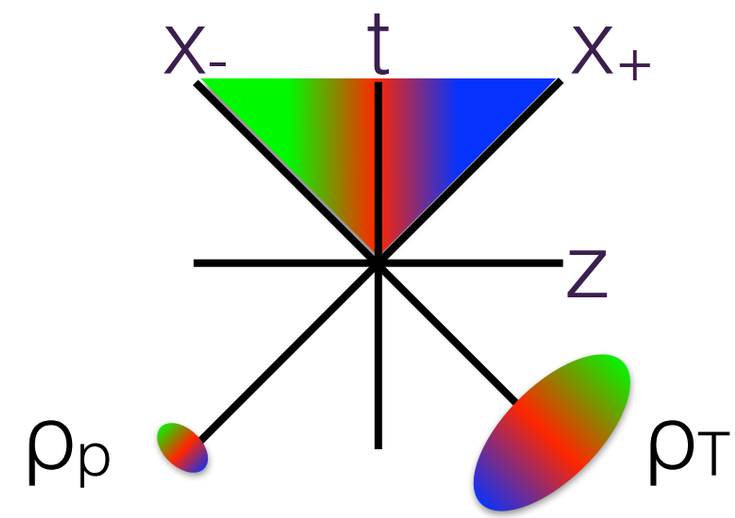
Dumitru, McLerran NPA 700 (2002), Blaziot, Gelis, Venugopalan NPA 743 (2004



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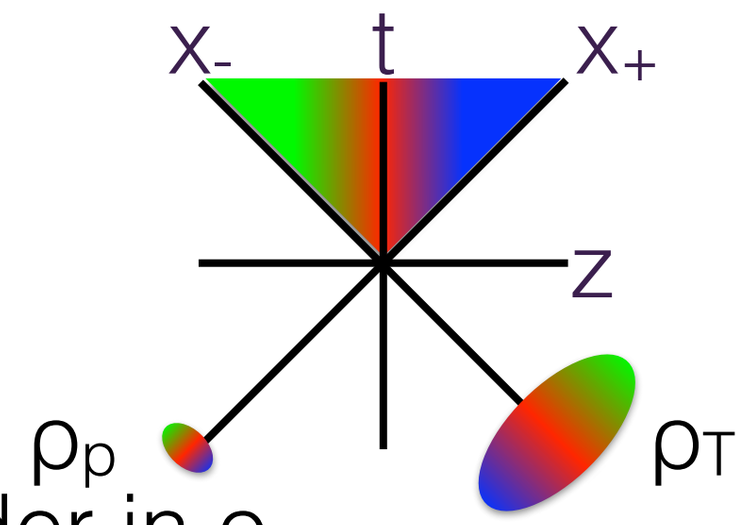


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Calculate for $A^\mu$ to all order in $\rho_T$, first order in $\rho_p$

-> analytically accessible

Need NLO in $\rho_p$ to generate $v_3$

McLerran, Skokov NPA 959 (2017)

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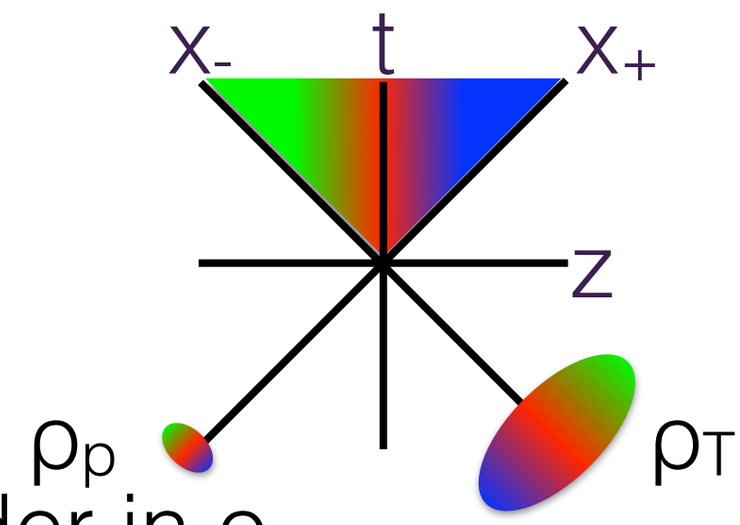


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McLerran, Skokov NPA 959 (2017)

Single inclusive spectrum readily calculable

$$\left.\frac{dN}{d^2k dy}\right|_{\rho_p,\rho_T} = \frac{1}{2(2\pi)^3}\frac{1}{|\mathbf{k}|^2}(\delta_{ij}\delta lm + \epsilon_{ij}\epsilon_{lm})\Omega^a_{ij}(\mathbf{k})[\Omega^a_{lm}(\mathbf{k})]^*$$

$$\Omega^a_{ij}(\mathbf{x}) = g\left[\frac{\partial_i}{\partial^2}\rho^b_p(\mathbf{x})\right]\partial_j U^{ab}(\mathbf{x})$$

Projectile Target

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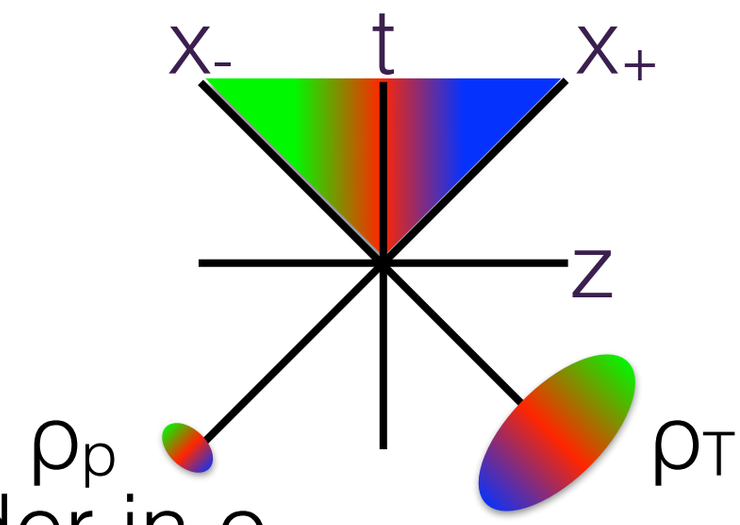


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McLerran, Skokov NPA 959 (2017)

Multi-particle distribution then easily accessible

$$\frac{d^2N}{d^2k_1dy_1...d^2k_ndy_n} = \left\langle \left\langle \frac{dN}{d^2k_1dy_1}\Big|_{\rho_p,\rho_T} ... \frac{dN}{d^2k_ndy_n}\Big|_{\rho_p,\rho_T} \right\rangle_p \right\rangle_T$$

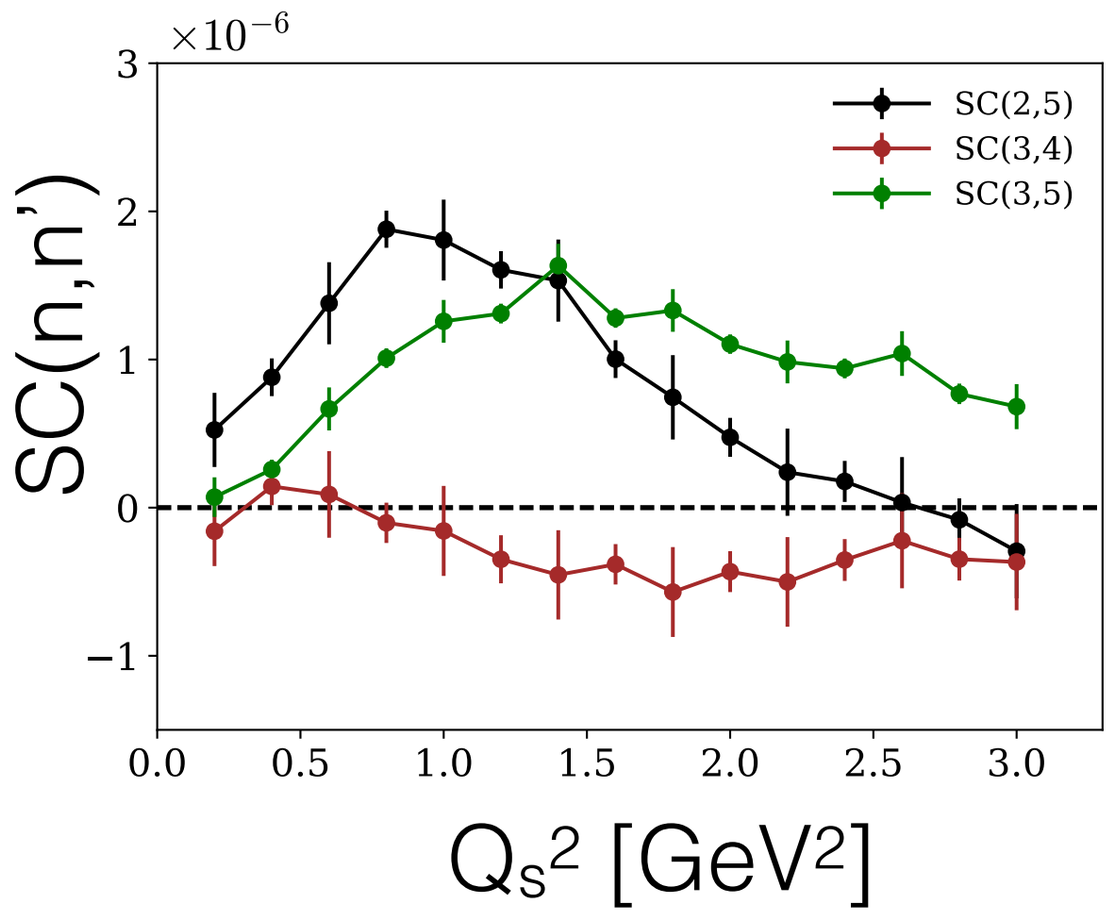
Only well defined for ensemble over $W[\rho_T,\rho_p]$

# Symmetric Cumulants

$$\mathrm{SC}(n,n') = \langle e^{i(n(\phi_1-\phi_3)-n'(\phi_2-\phi_4))}\rangle - \langle e^{in(\phi_1-\phi_3)}\rangle\langle e^{in'(\phi_2-\phi_4)}\rangle$$

Bilandzic et al, PRC 89, no. 6, 064904 (2014)

Prediction for higher moments in small systems




n=2,n'=5

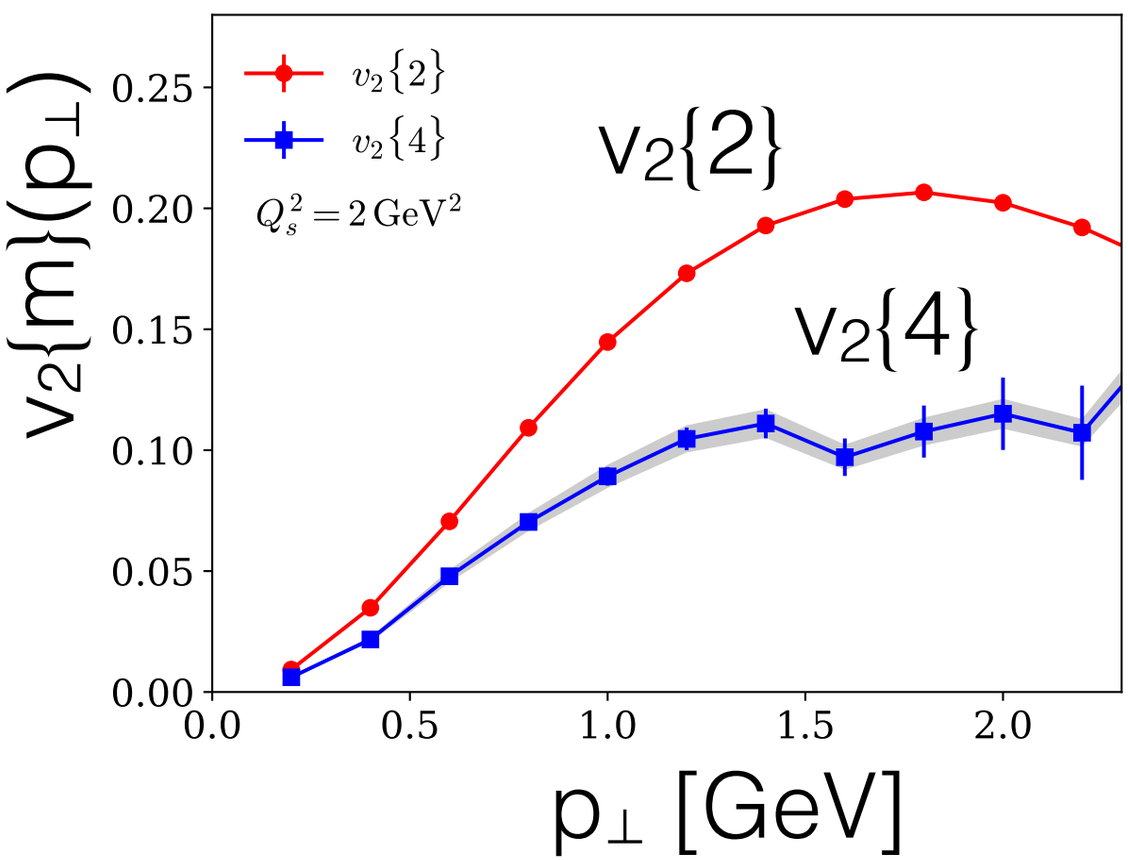
n=3,n'=4

n=3,n'=5

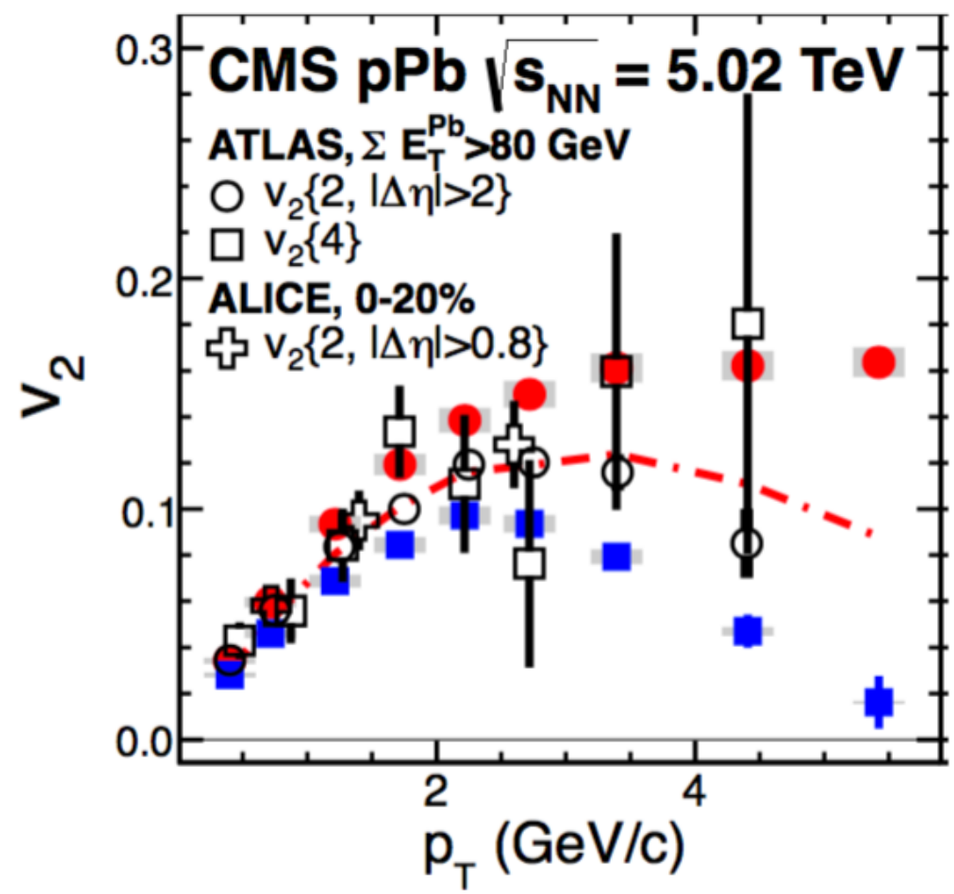
Dusling, MM, Venugopalan PRD 97 (2018) arXiv:1706.06260

# Multiparticle correlations

Integrating momentum of m-1 particles



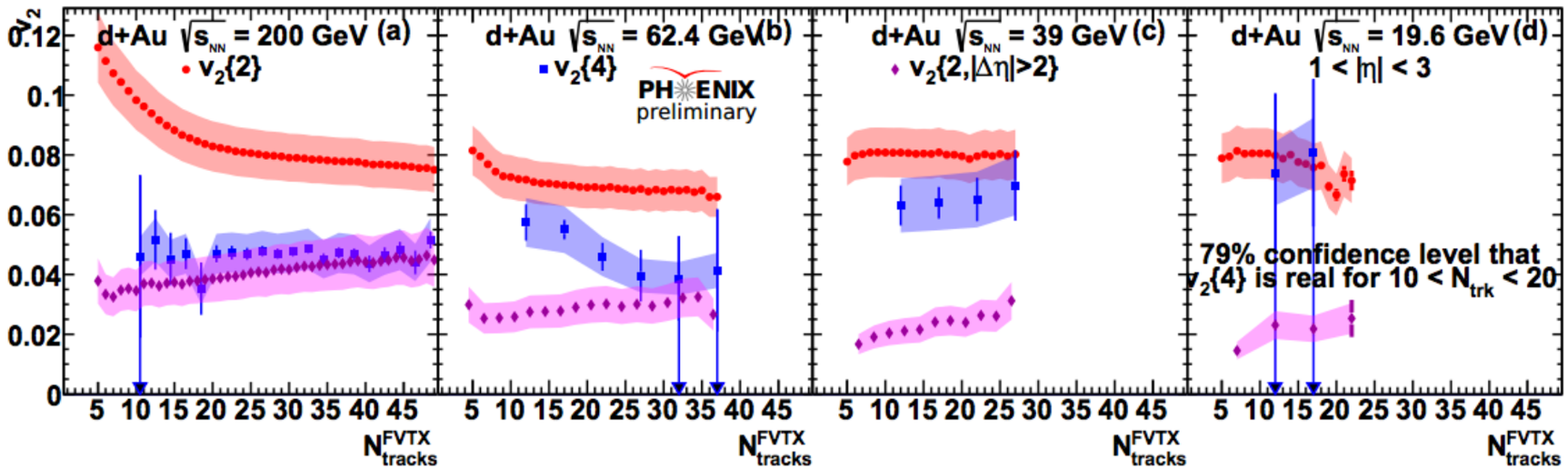

Dusling, MM, Venugopalan PRD 97 (2018)




CMS PLB 724 (2013) 213

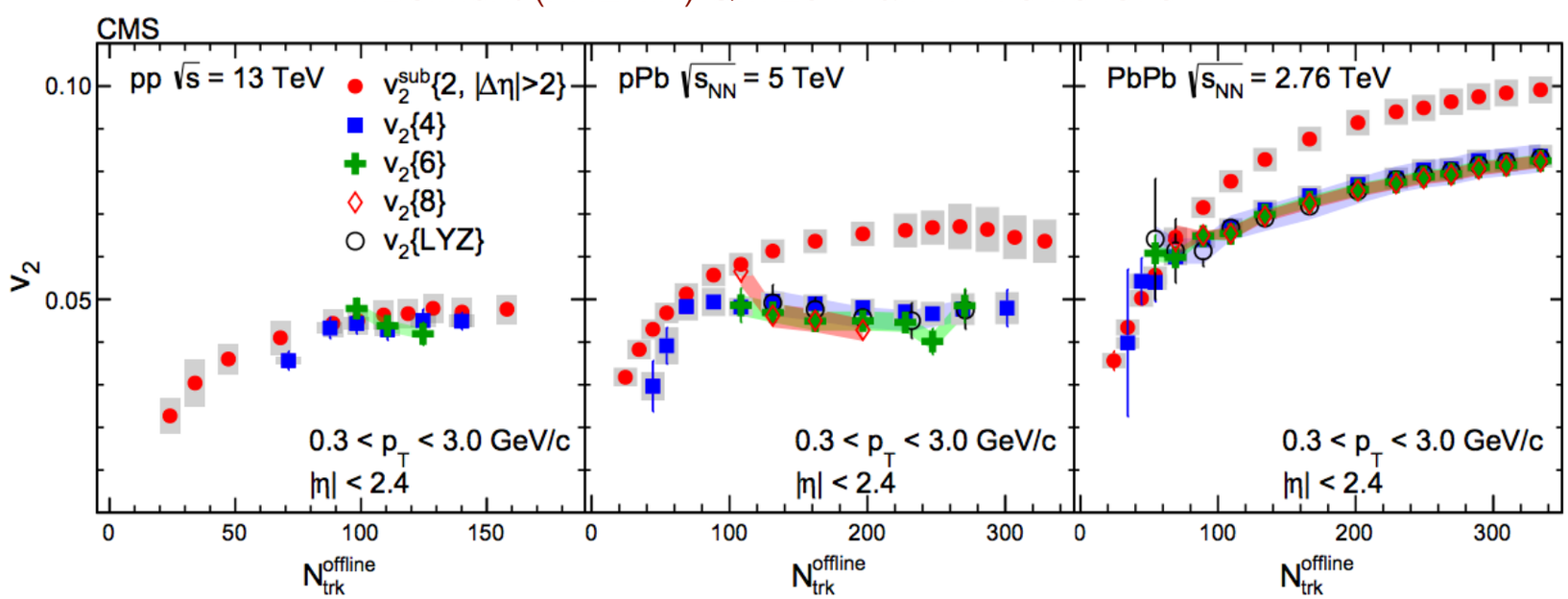
Similar characteristic shape

# “Collectivity” is everywhere




RHIC

R. Belmont (PHENIX) QM 2017 arXiv:1704.04570




LHC

CMS PRL 115 (2015) 012301

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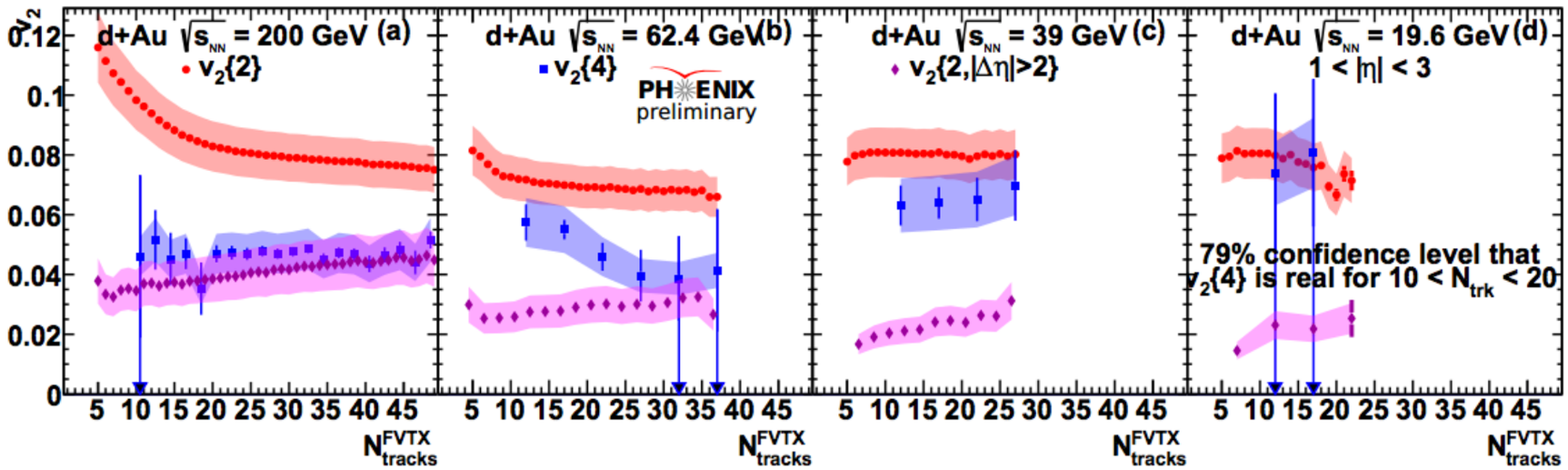


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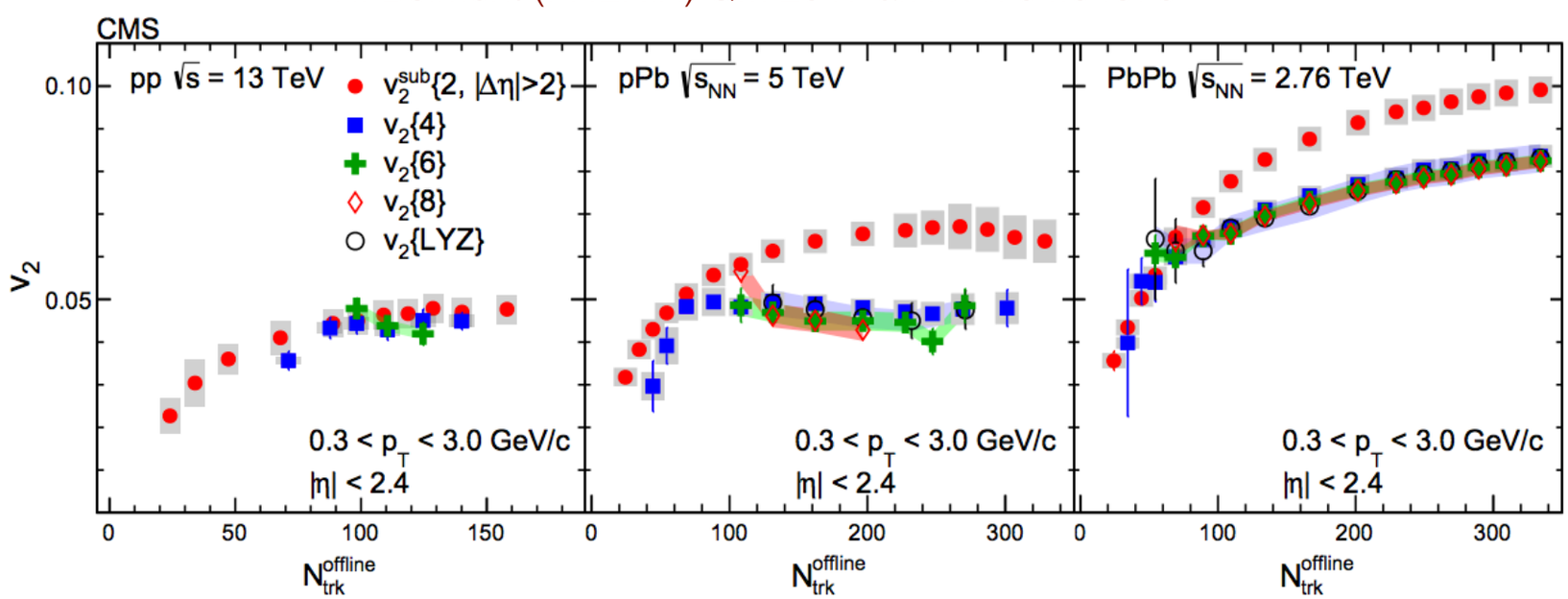


LHC

CMS PRL 115 (2015) 012301

Smallest droplets of QGP?
Pre-existing correlations from rare QCD configurations?
Both?

# Glauber IP-Sat model

For data-guided initial conditions, consider initial conditions based on very successful IP-Glasma model

Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)

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Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

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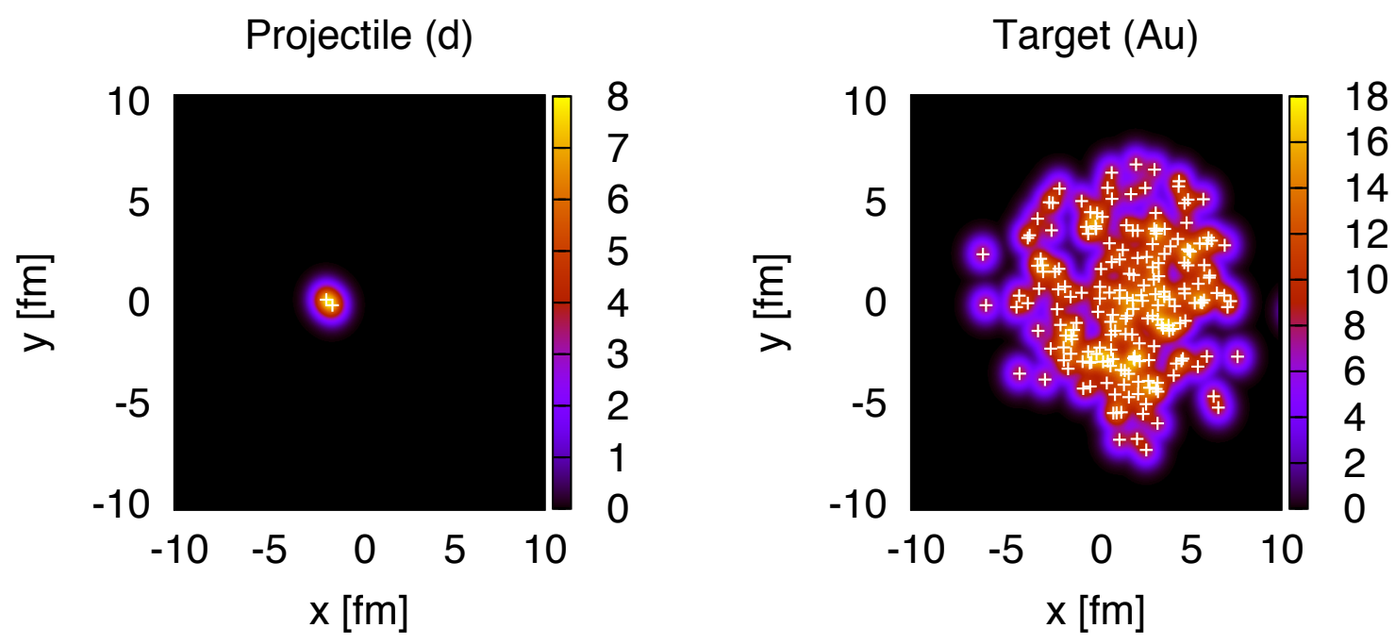
Color charge density



# Color Glass Condensate

McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204

CGC is an effective field theory in the non-linear regime of QCD ($Q_s^2(x) \gg \Lambda_{QCD}^2$) describing dynamical gluon *fields* (small-x partons) effected by static color *sources* (large-x partons)

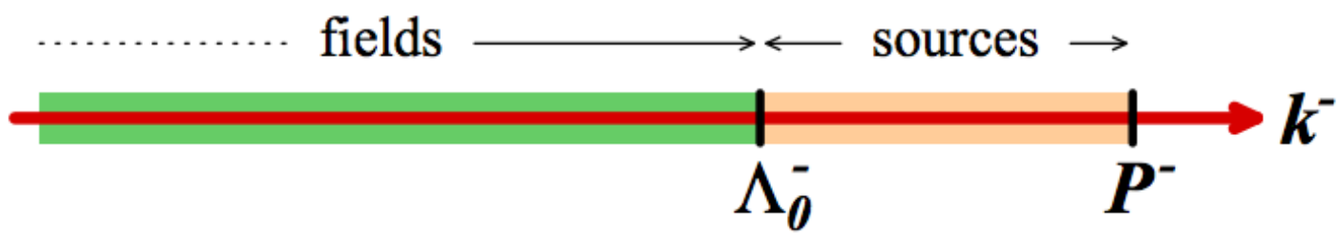
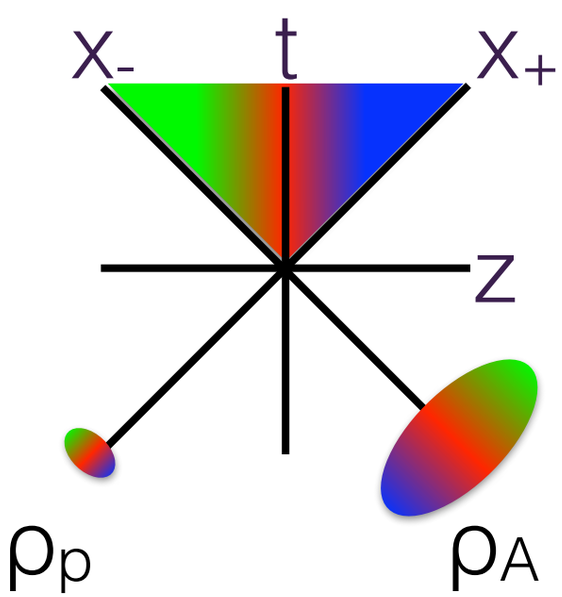



Fig: Gelis, Iancu, Jalilian-Marian, Venugopalan ARNPS. 60 (201



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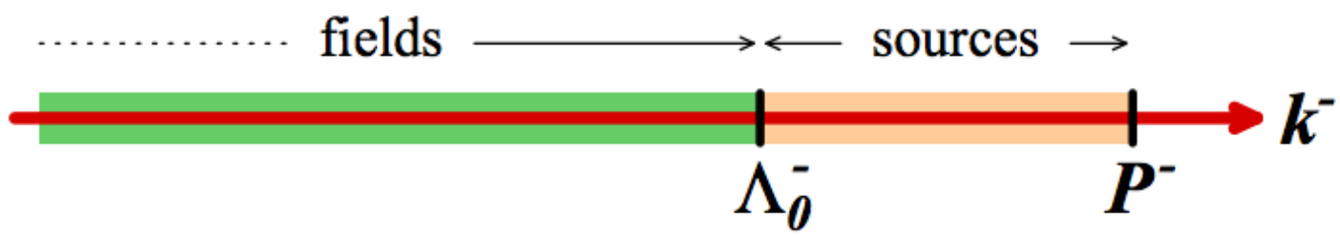


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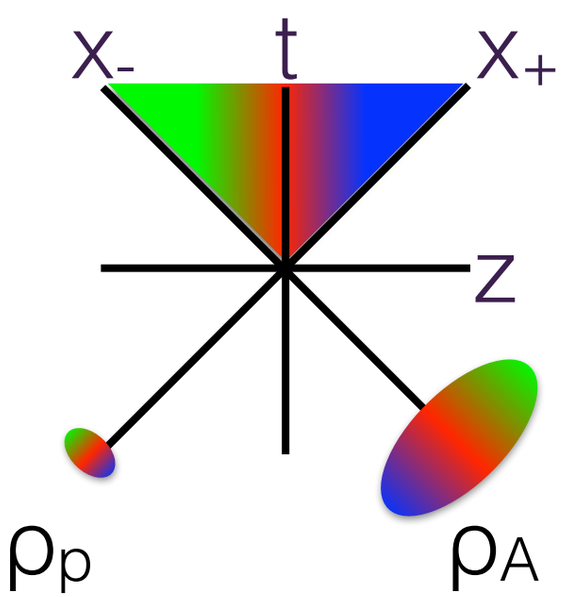


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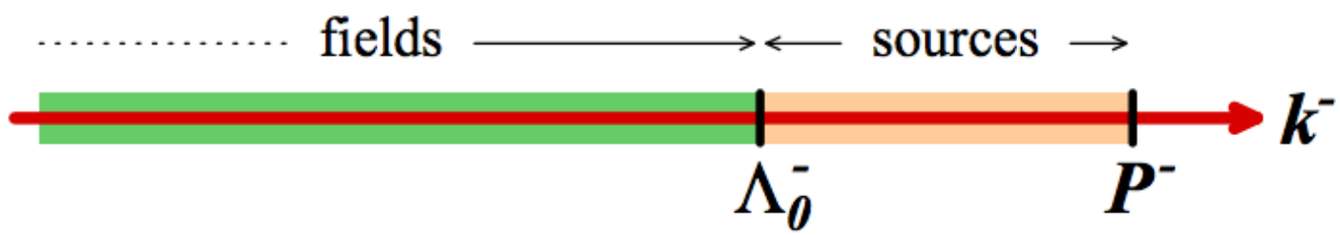


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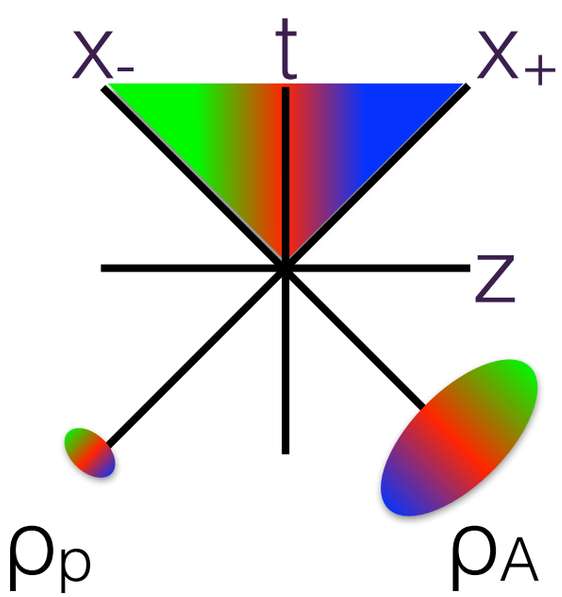


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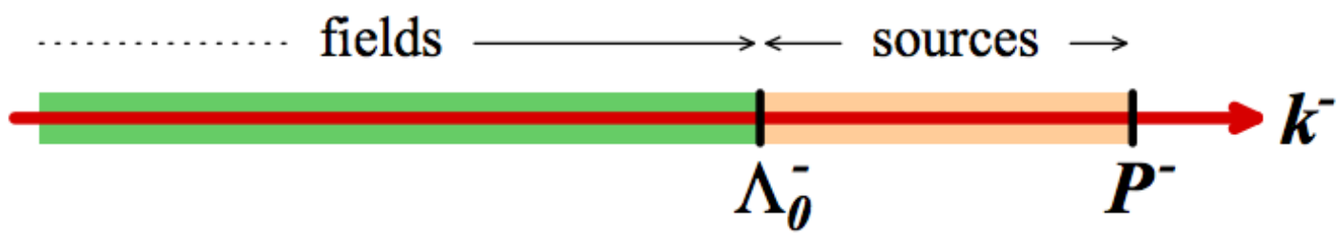


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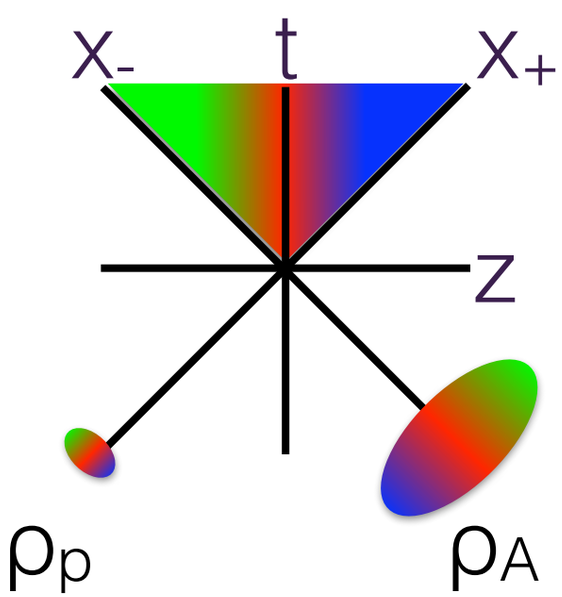


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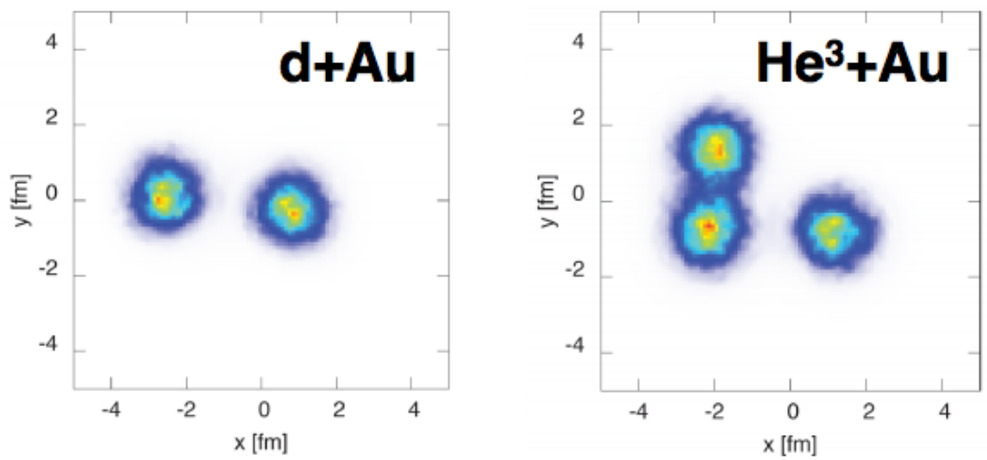
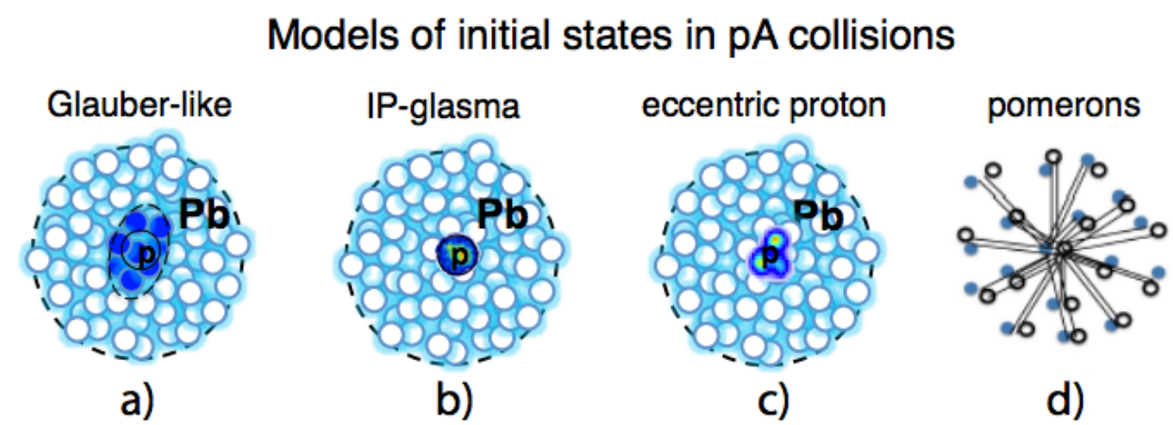
McLerran-Venugopalan (MV) Model: interactions between nucleons is a Gaussian random walk in color space

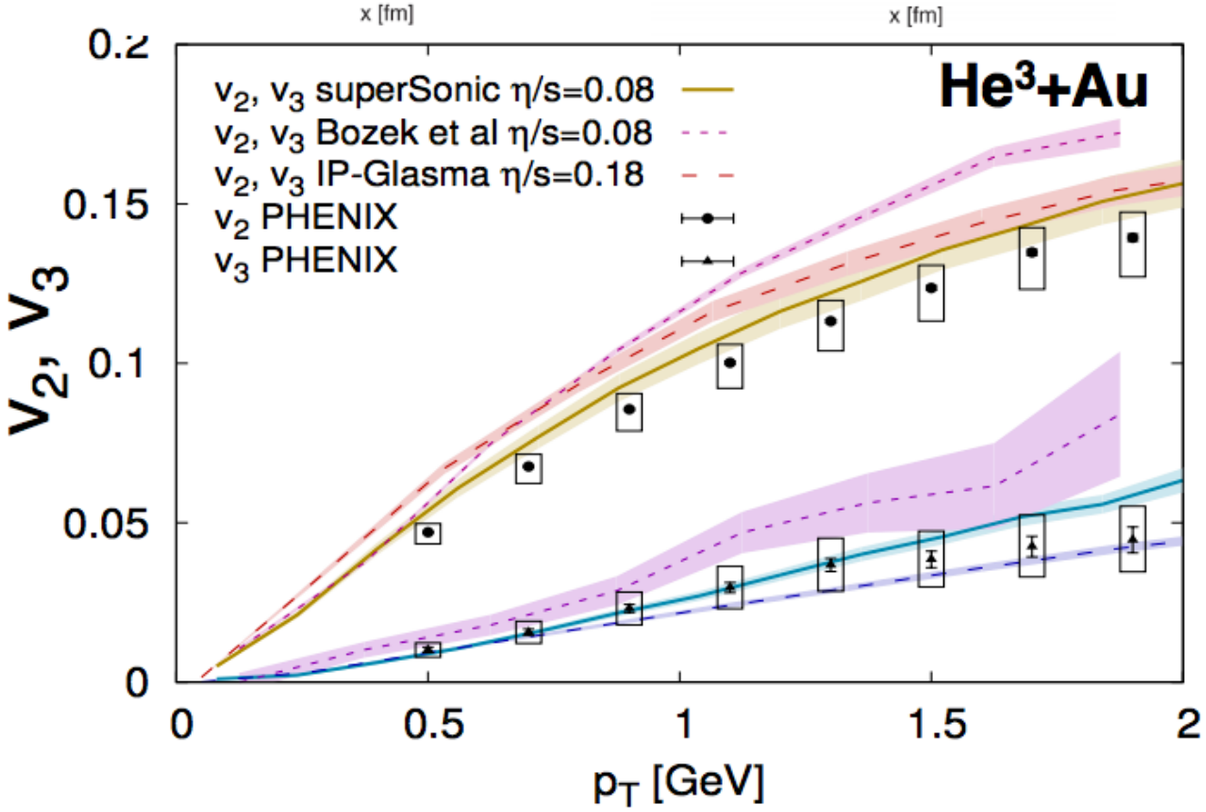
$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = g^2\delta^{ab}\mu^2\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

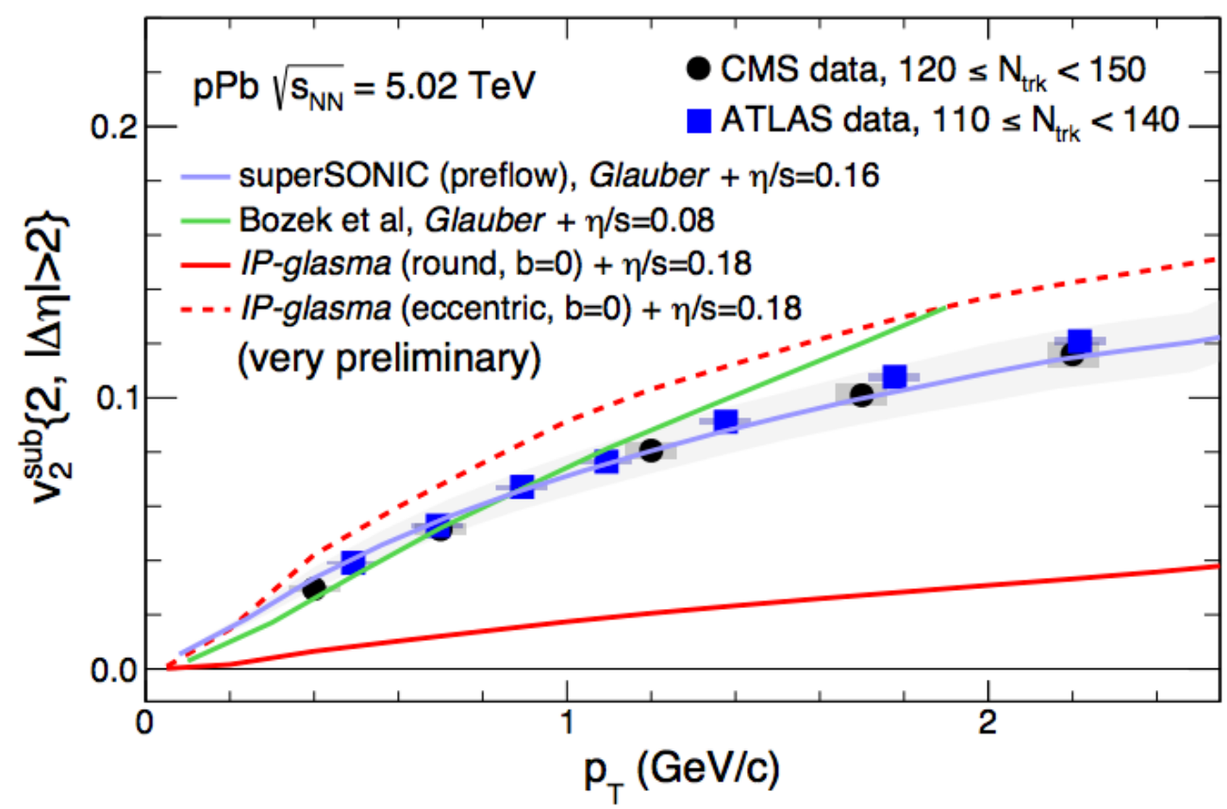
# Hydro in small systems

Applying methods/models used in AA collisions to smaller systems



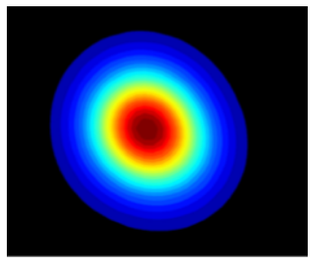



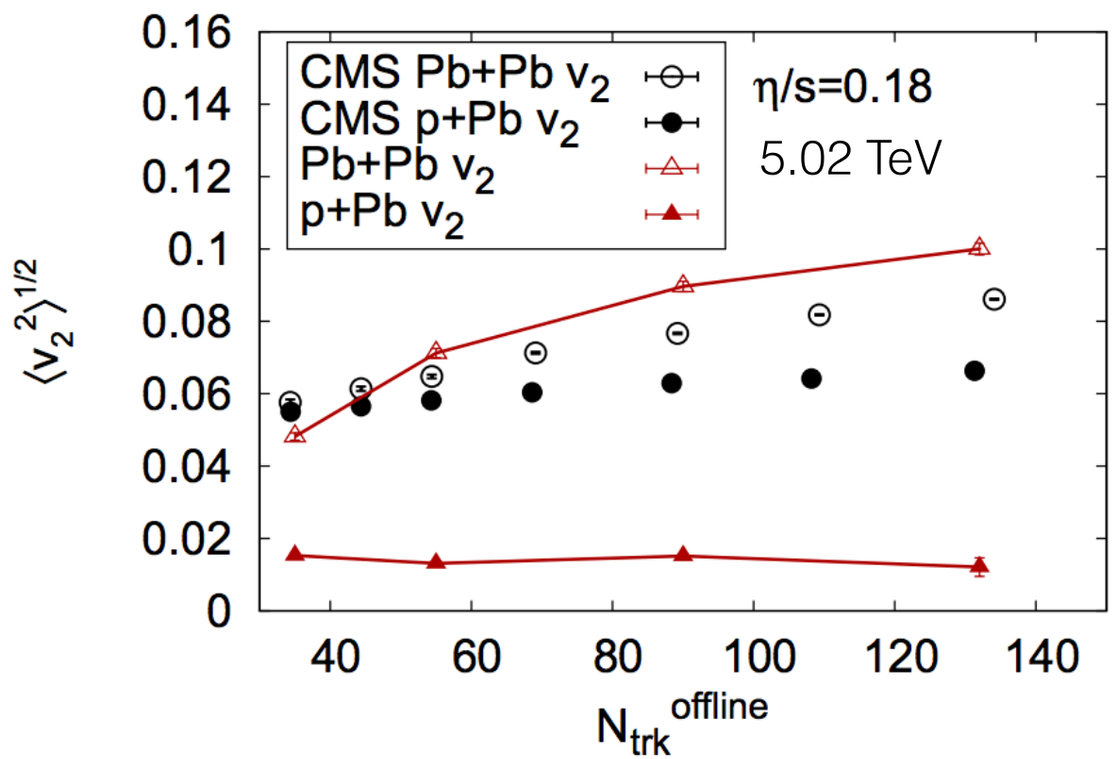




Dusling, Li, Schenke, IJMP E25 (2016) no.01, 1630002

# Hydro in small systems

Round proton






Schenke, Venugopalan PRL 113 (2014) 102301

IP-Glasma+Hydro

# Hydro in small systems

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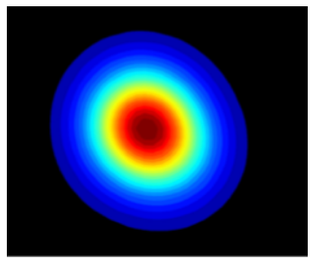
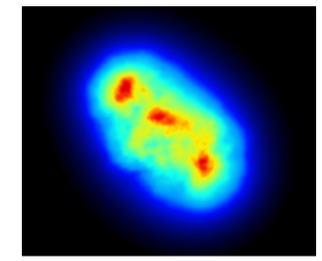
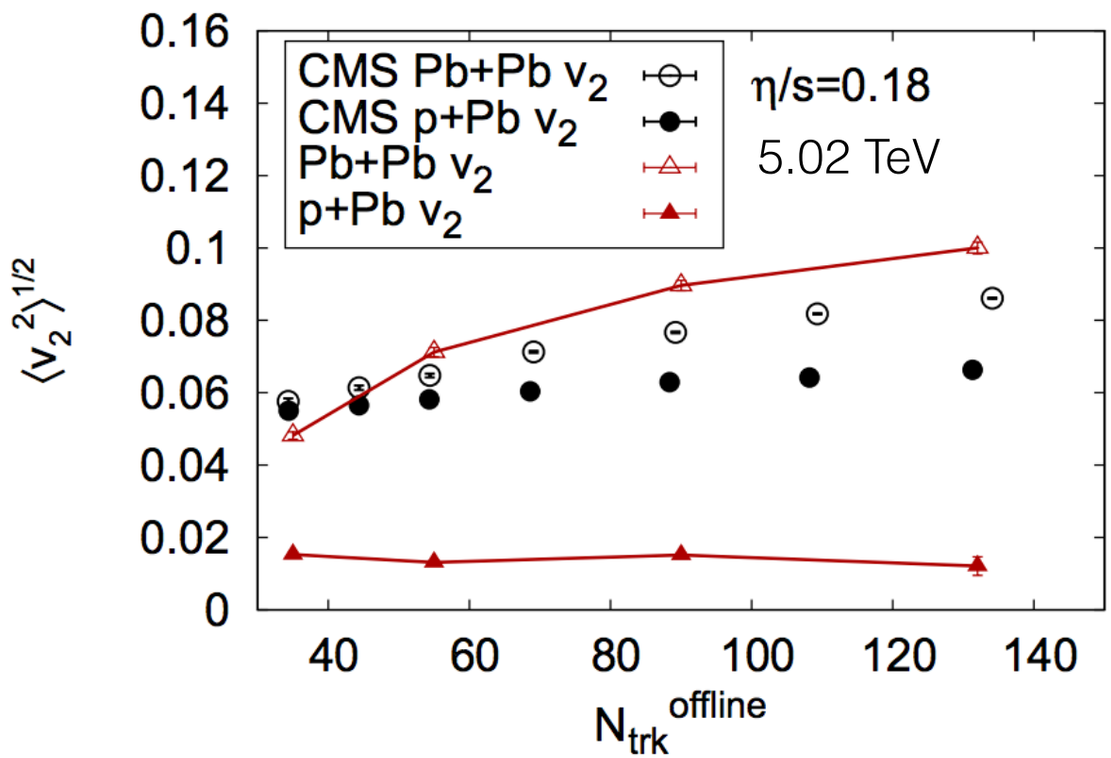


Constrain proton shape fluctuations using exclusive J/Ψ production (HERA)

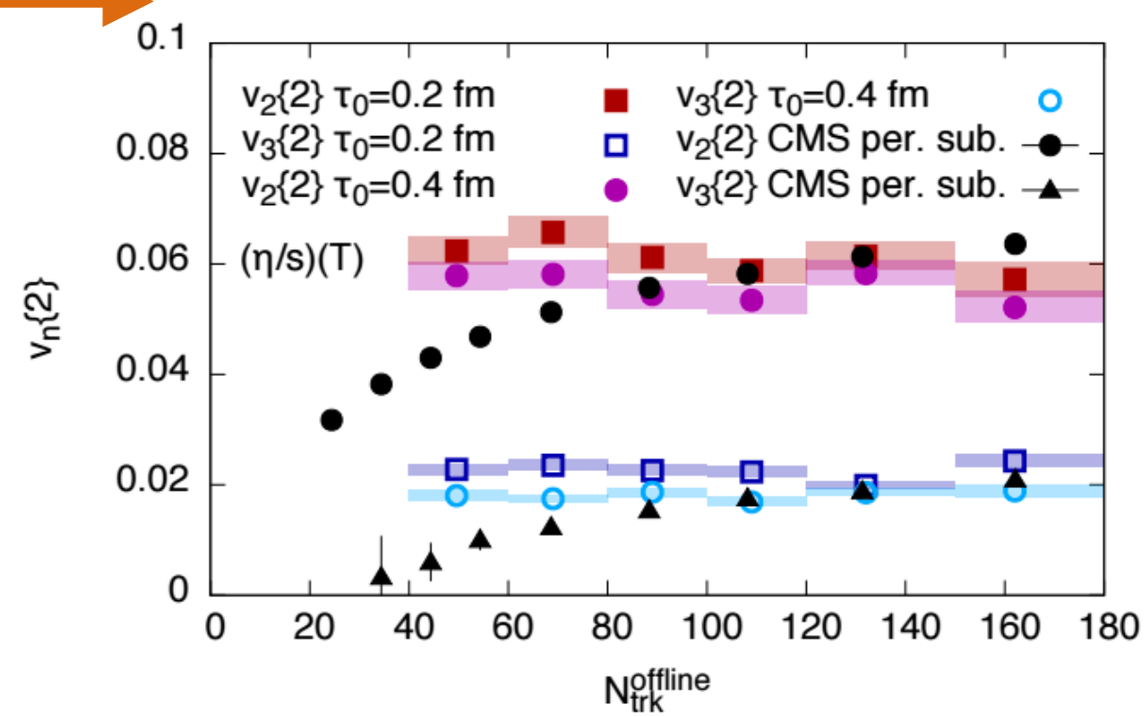
Fluctuating proton






Schenke, Venugopalan PRL 113 (2014) 102301

IP-Glasma+Hydro

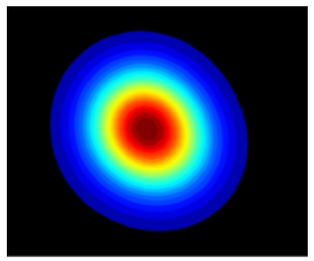



Mäntysaari, Schenke, Shen, Tribedy arXiv: 1705.03177

IP-Glasma**+Fluct. proton**+Hydro+UrQMD

# Hydro in small systems

Round proton



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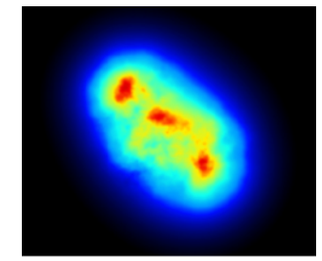

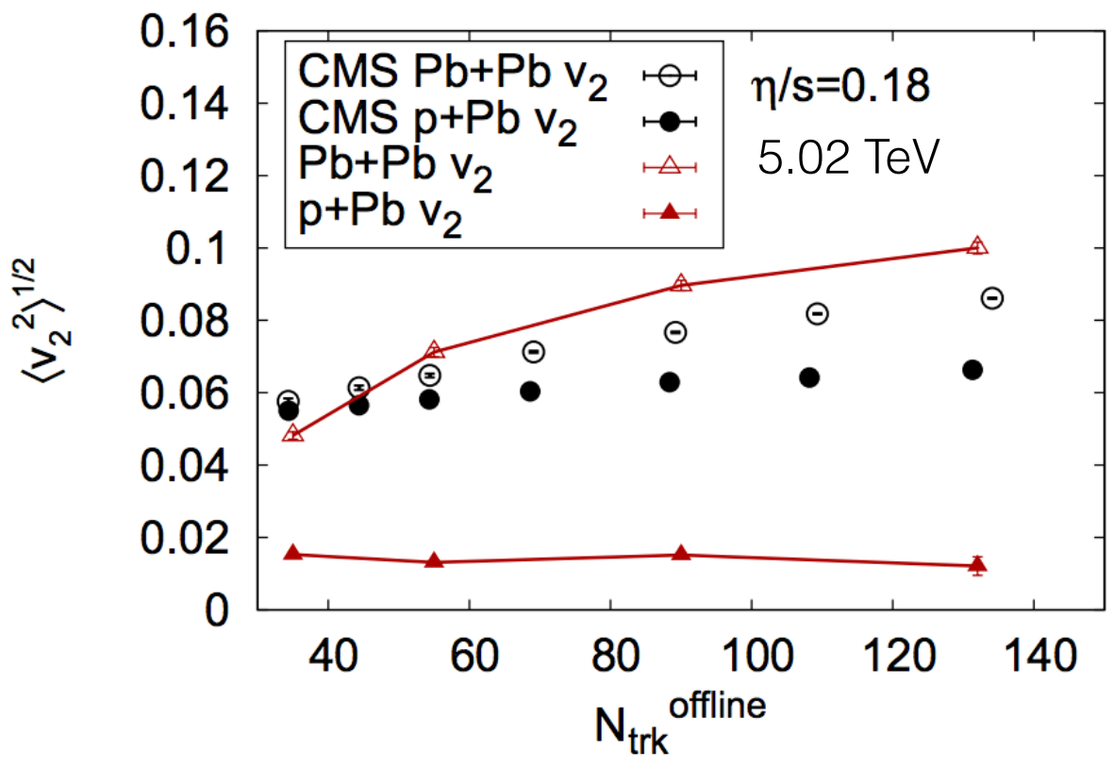


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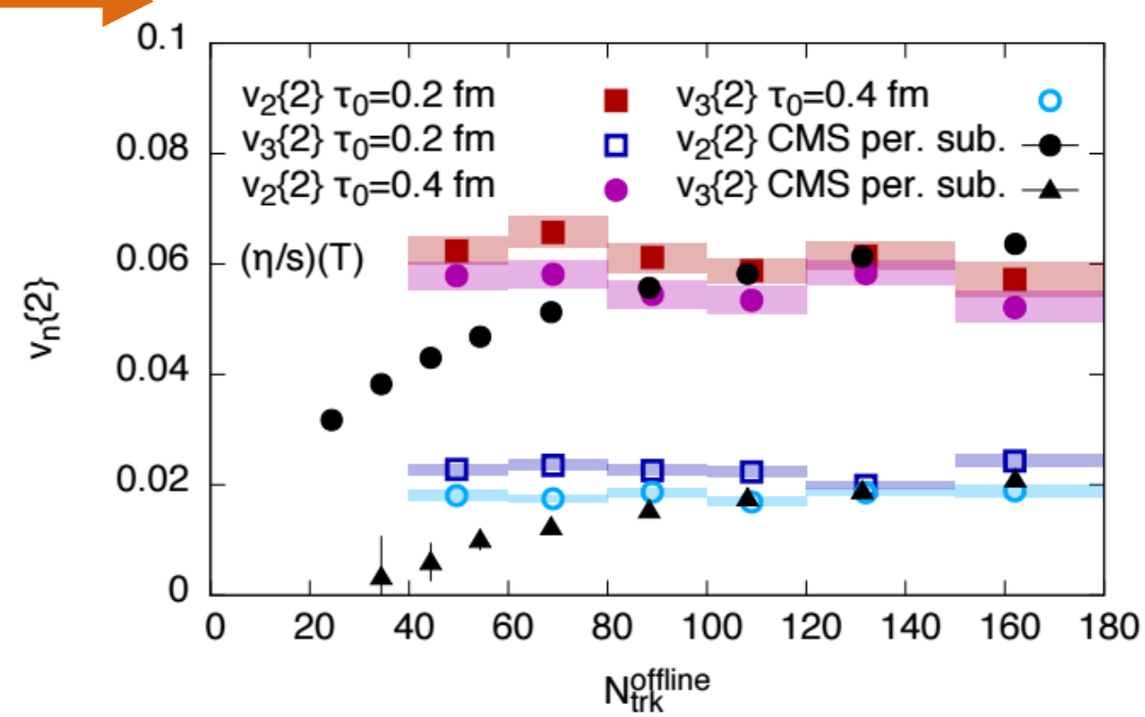


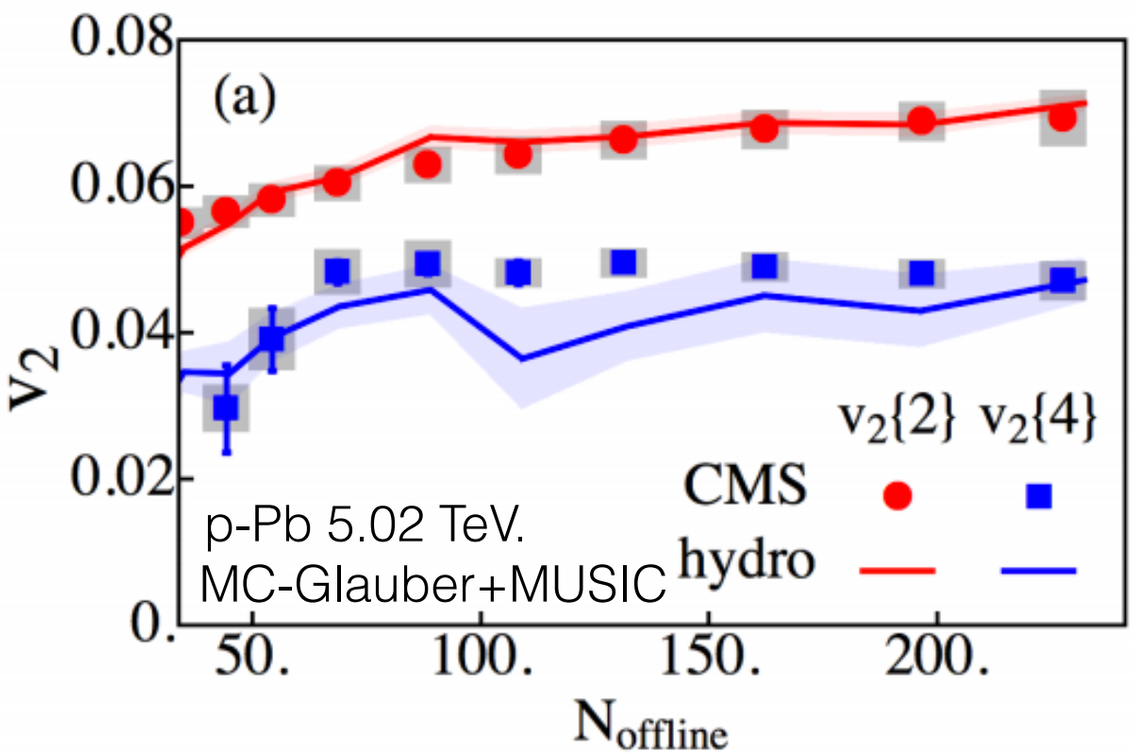

Mäntysaari, Schenke, Shen, Tribedy arXiv: 1705.03177

IP-Glasma**+Fluct. proton**+Hydro+UrQMD

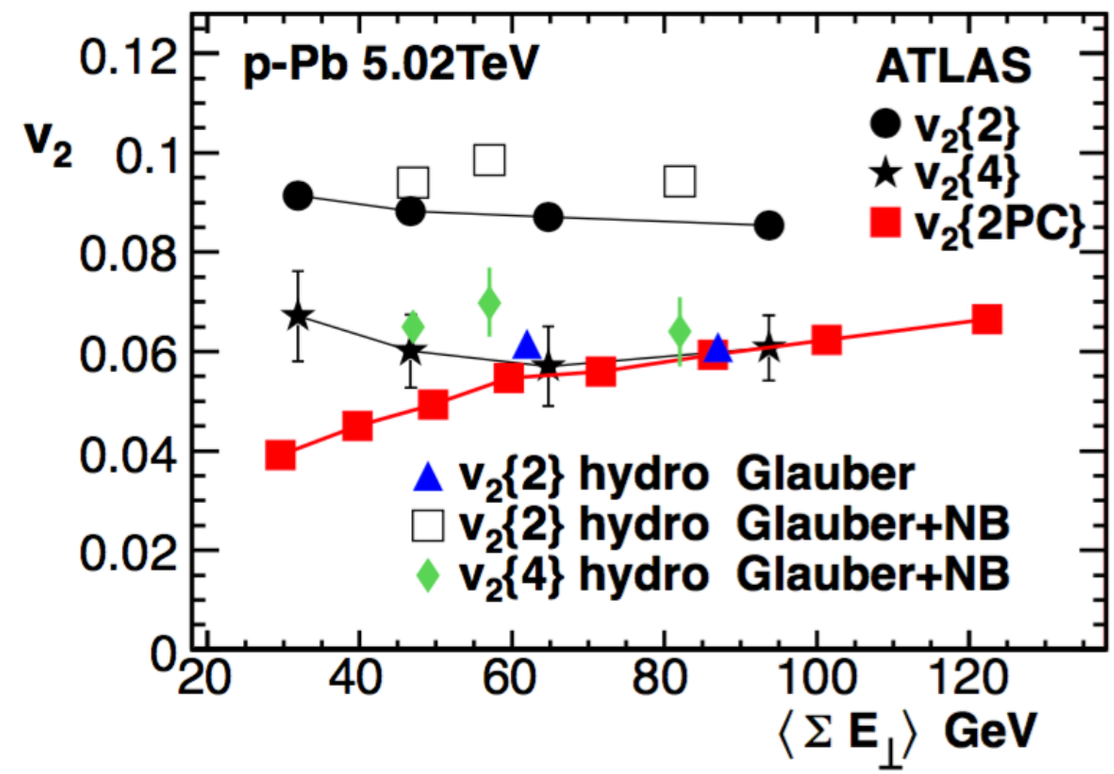
At face value, hydro calculations do okay

# Collectivity from hydro

Two and four particle correlations in p+A in hydro




Kozlov, Denicol, Luzum, Jeon, Gale
NPA 931 (2014) 1045-1050



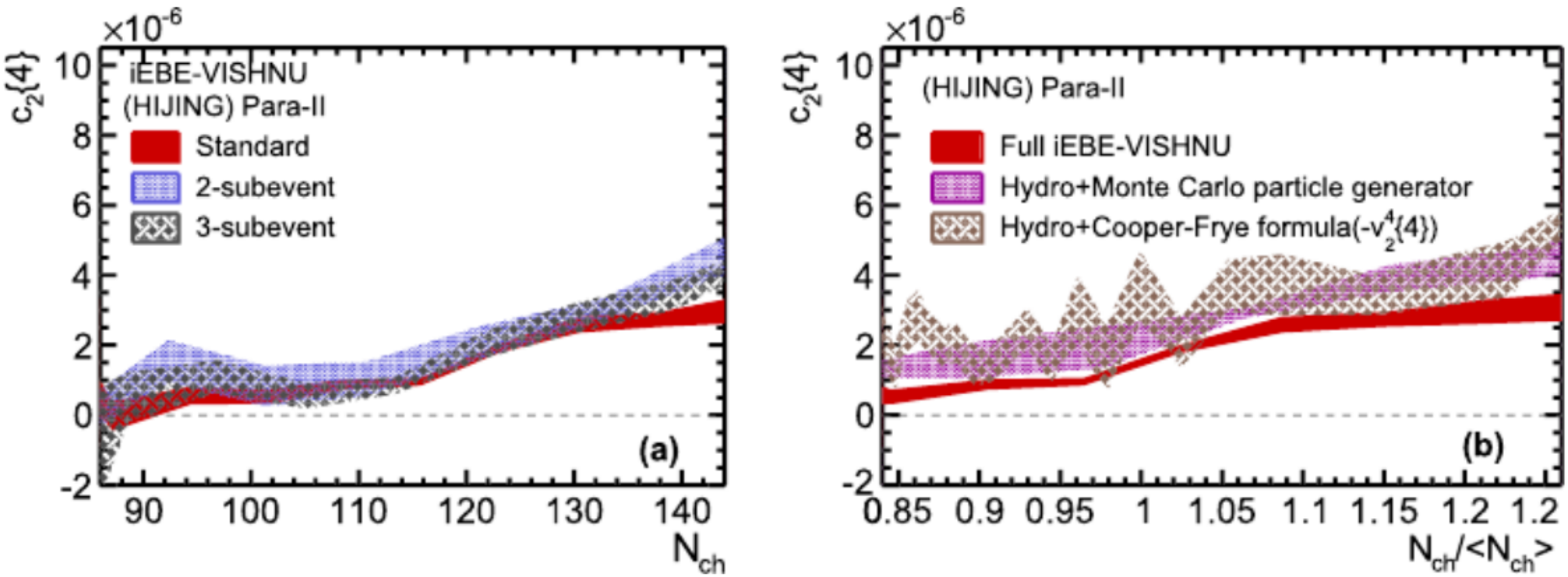

Bozek, Broniowski
Phys. Rev. C 88, 014903 (2013)

Theory issues linger however about applicability of hydrodynamics in small systems (size of gradients,…)

No demonstration of higher particle number correlations, consistency with small and large systems in question…

# Collectivity from hydro

Absence of four-particle $v_2$ from hydro in pp



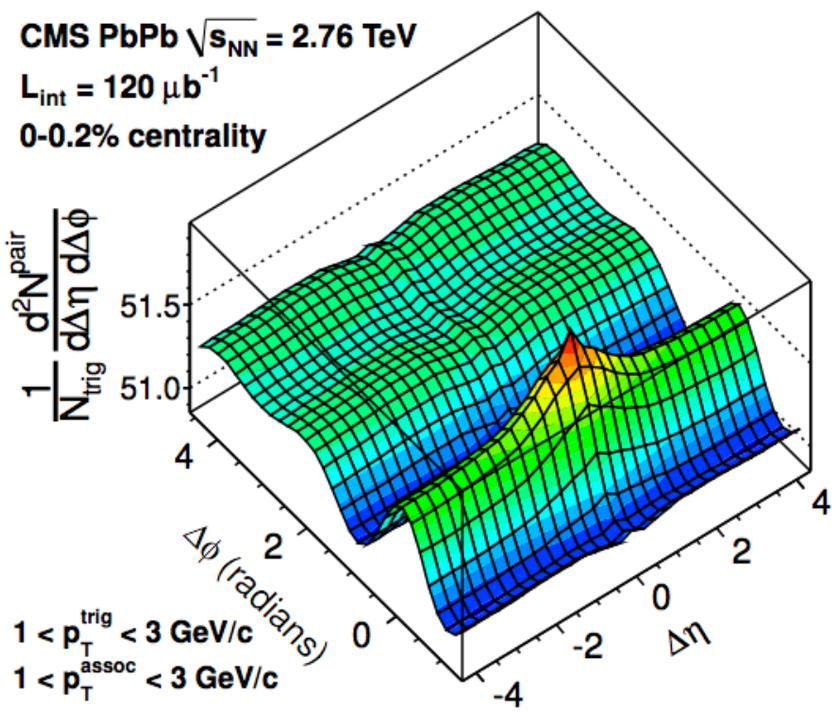

Zhao, Zhou, Xu, Deng, Song PLB 780 (2018)

Theory issues linger however about applicability of hydrodynamics in small systems (size of gradients,…)

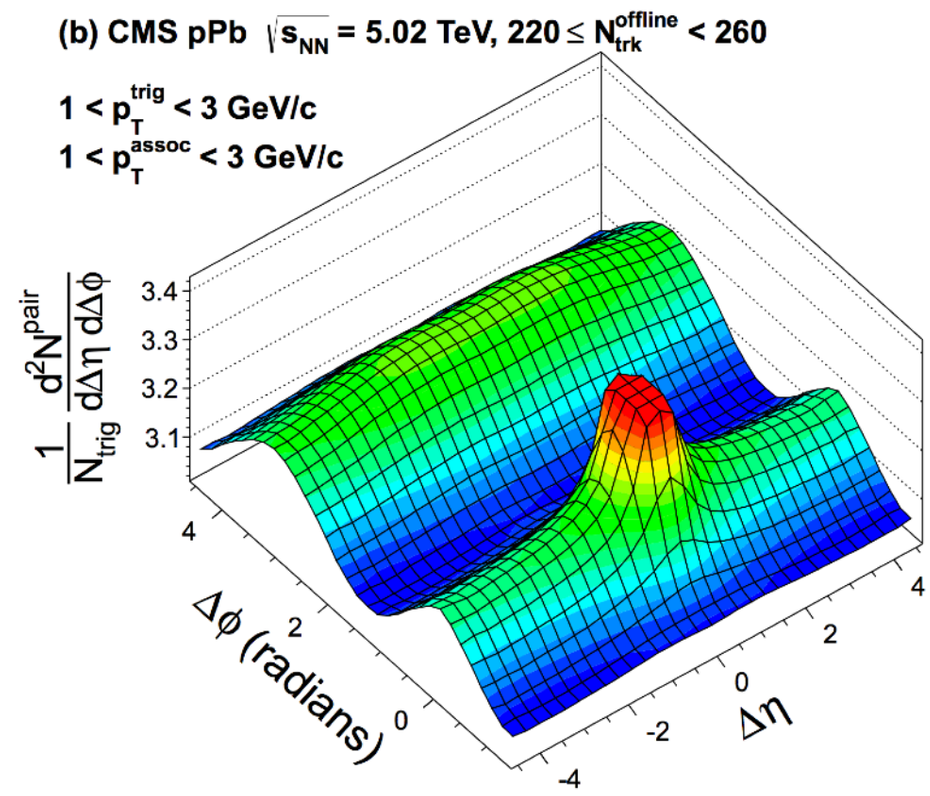
No demonstration of higher particle number correlations, consistency with small and large systems in question…

# Jets in pA

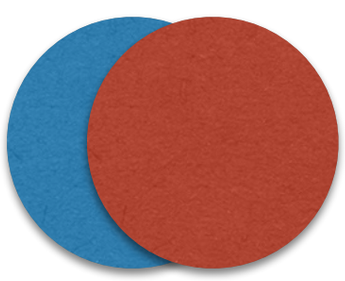
- Lack of away-side ridge

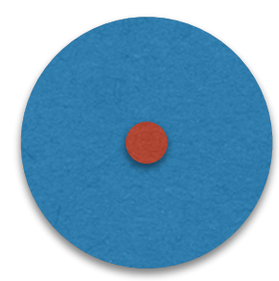



CMS JHEP 02 (2014) 088



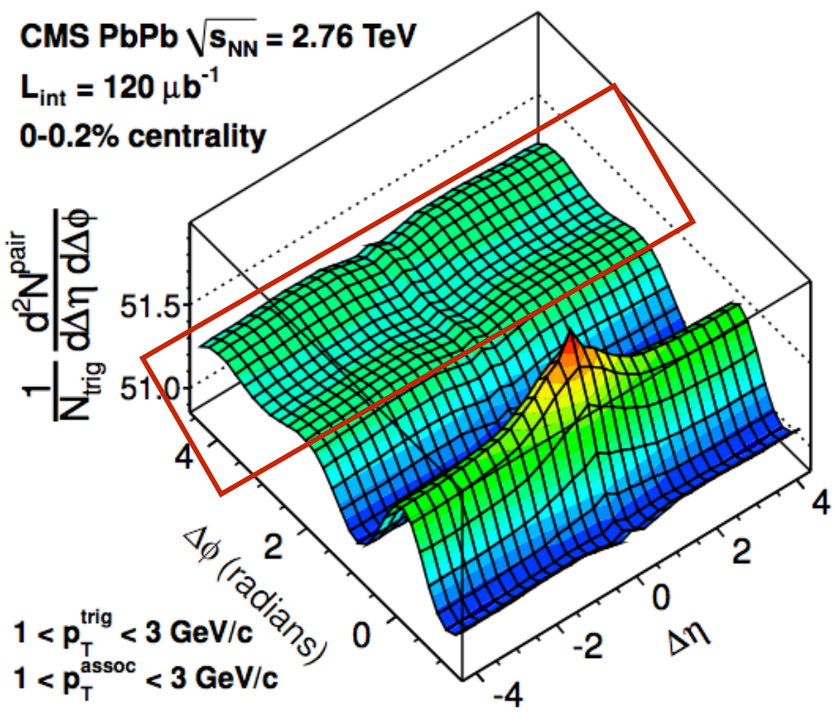

CMS PLB 724 (2013) 213



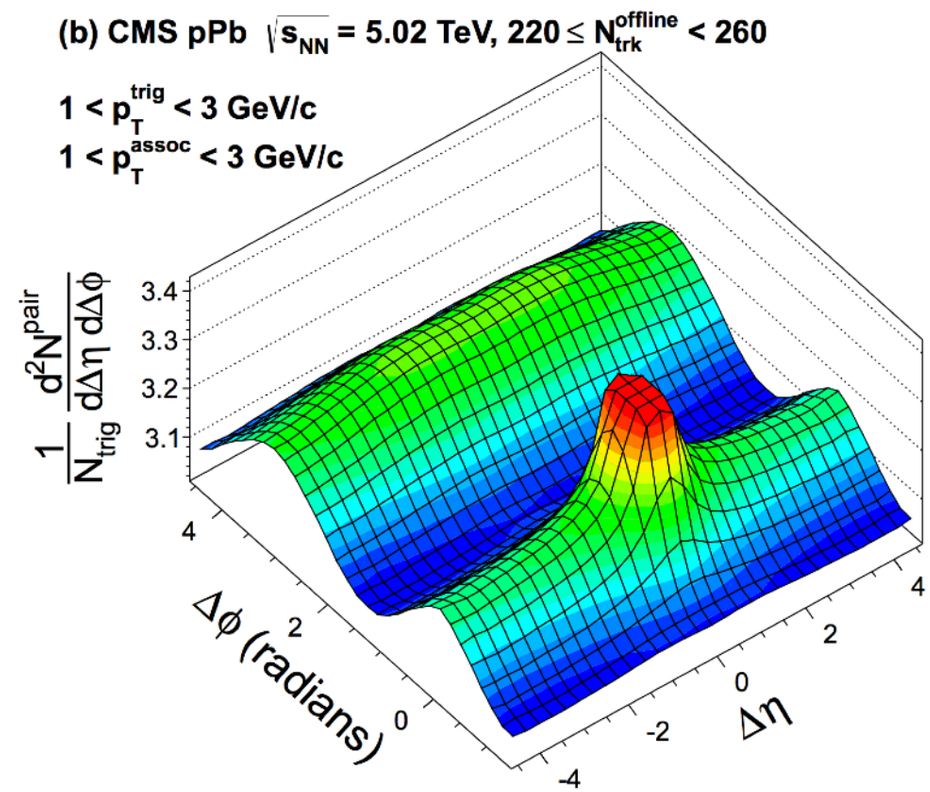


# Jets in pA

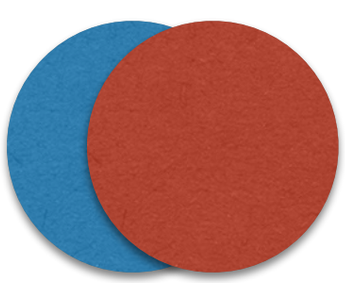
- Lack of away-side ridge

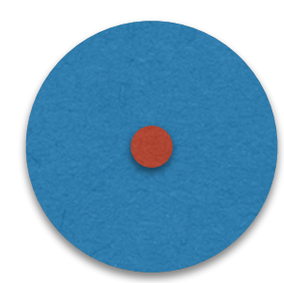



CMS JHEP 02 (2014) 088



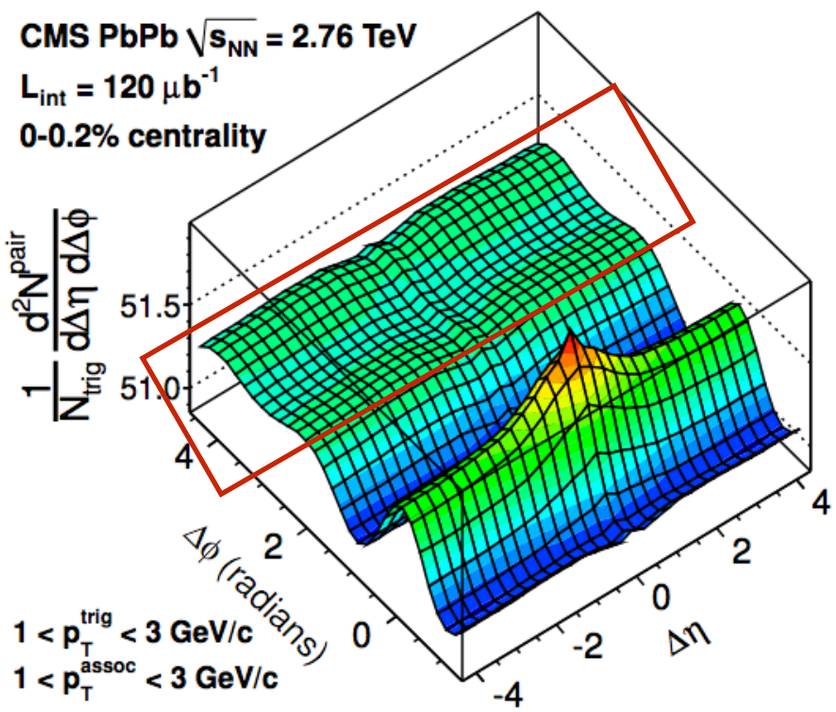

CMS PLB 724 (2013) 213



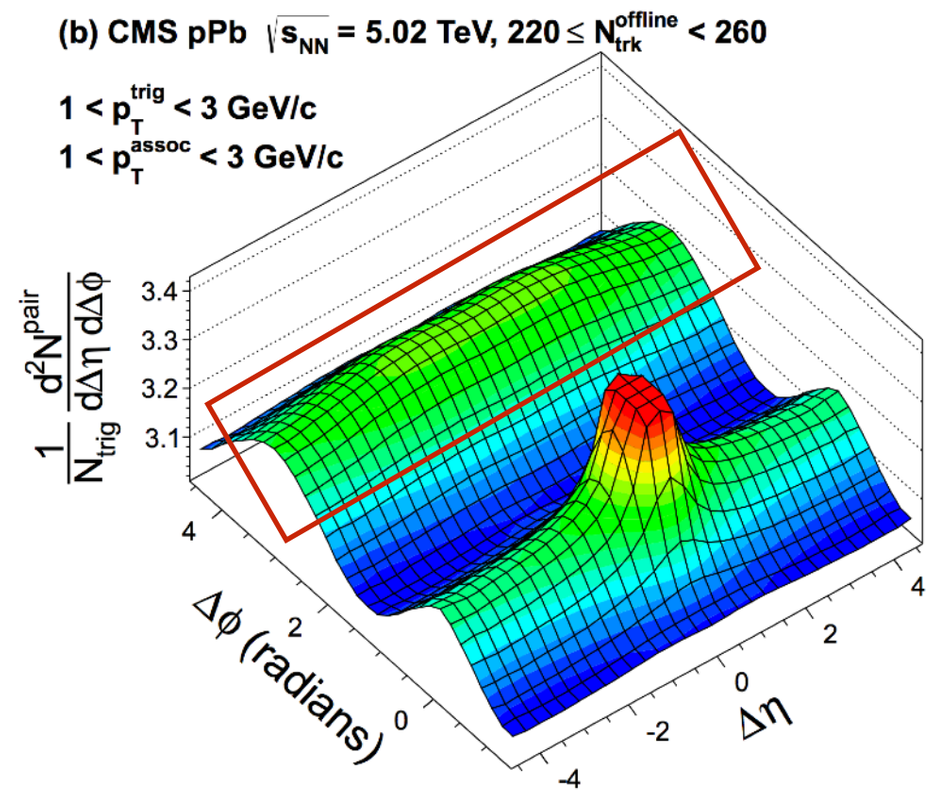


# Jets in pA

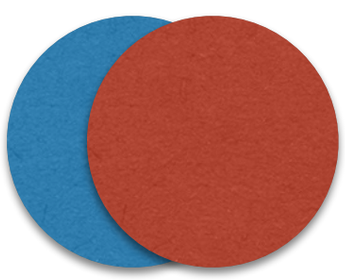
- Lack of away-side ridge

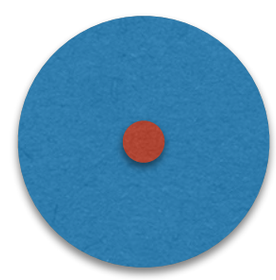



CMS JHEP 02 (2014) 088




CMS PLB 724 (2013) 213

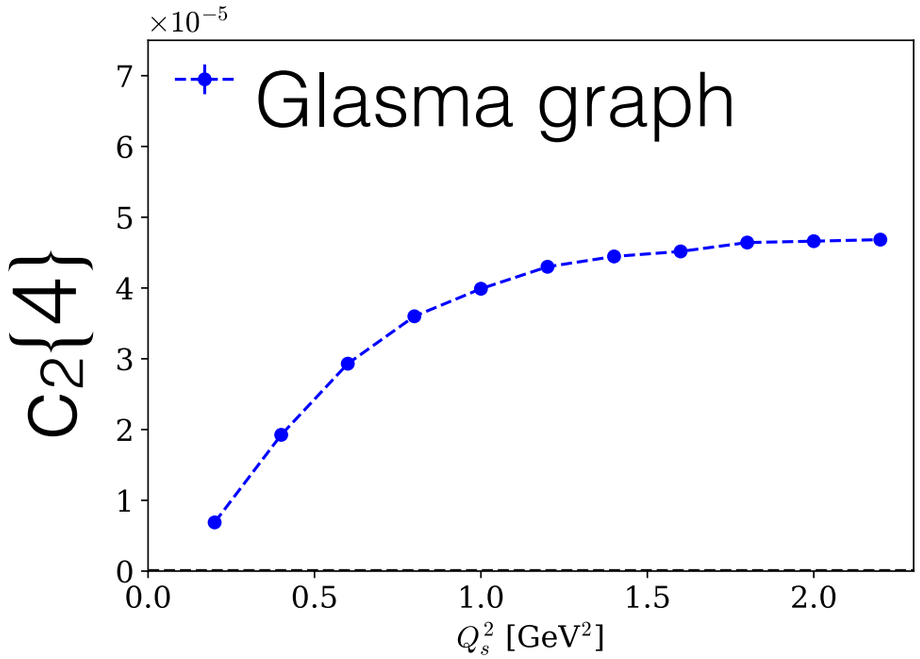




# Collectivity from initial state

For higher particle cumulants and harmonics, Glasma graph gives $c_2\{4\}>0$

Dusling, MM, Venugopalan, arXiv:1706.06260



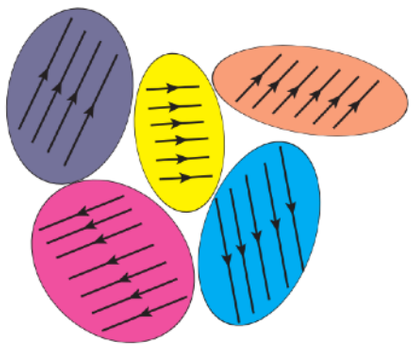

Adding multiple scattering, qualitatively

A. Dumitru, L. McLerran, V. Skokov, Phys.Lett. B743 (2015) 134-137

$$c_2\{4\} \equiv -(v_2\{4\})^4 = -\frac{1}{N_D^3}\left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right)$$

"Glasma graph"
Always positive

Negative contribution: Non-linear, non-Gaussian from color domain



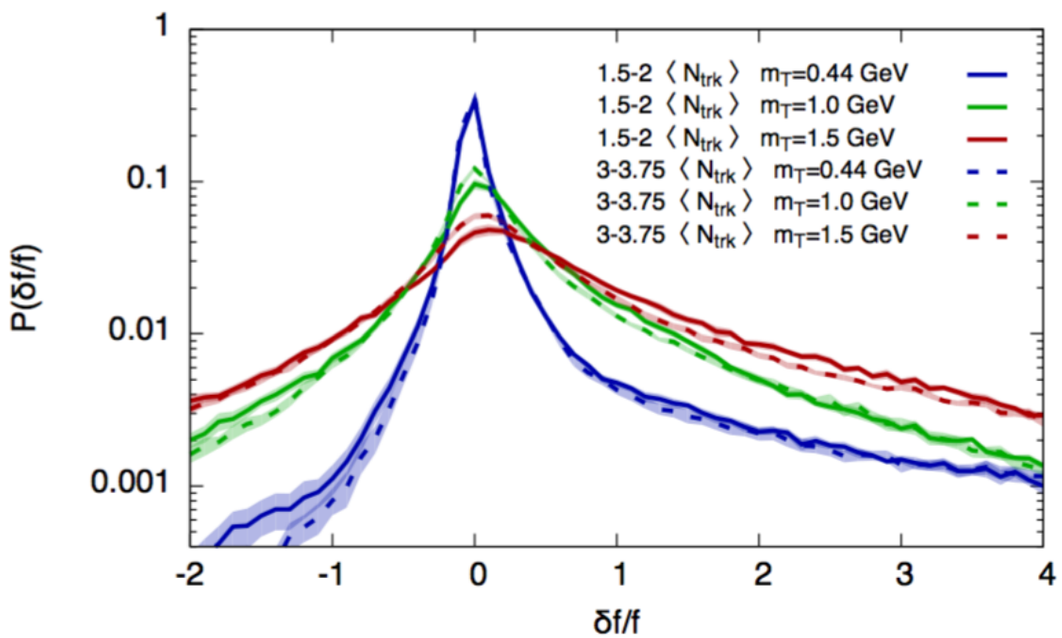
Non-linear Gaussian can also get two particle $v_2, v_3, \ldots$, what about four particles?

# Hydro in small systems

A+A collisions well described by hydrodynamics

But in small systems large gradients, highly anisotropic,…

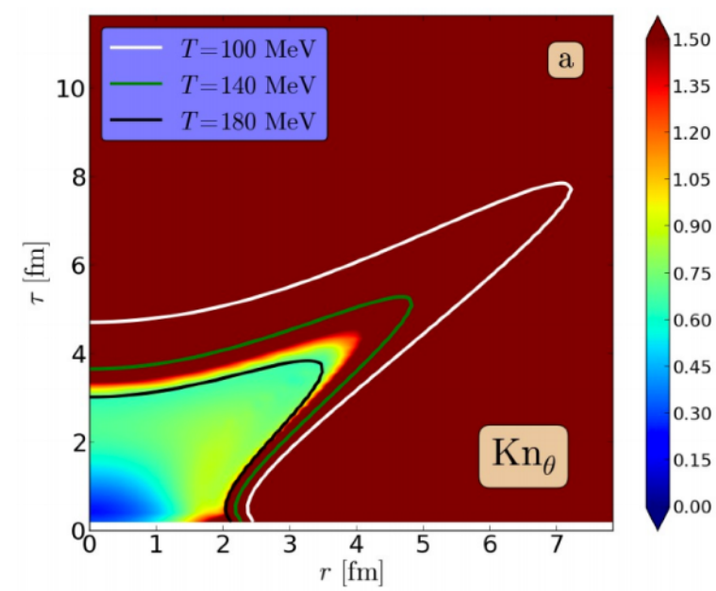
Corrections to freeze-out




Schenke, QM2017 plenary

At freeze-out δf can be » f

Measure of hydro applicability




Large Kn

$$\mathrm{Kn} = \frac{L_{\mathrm{micro}}}{L_{\mathrm{macro}}}$$

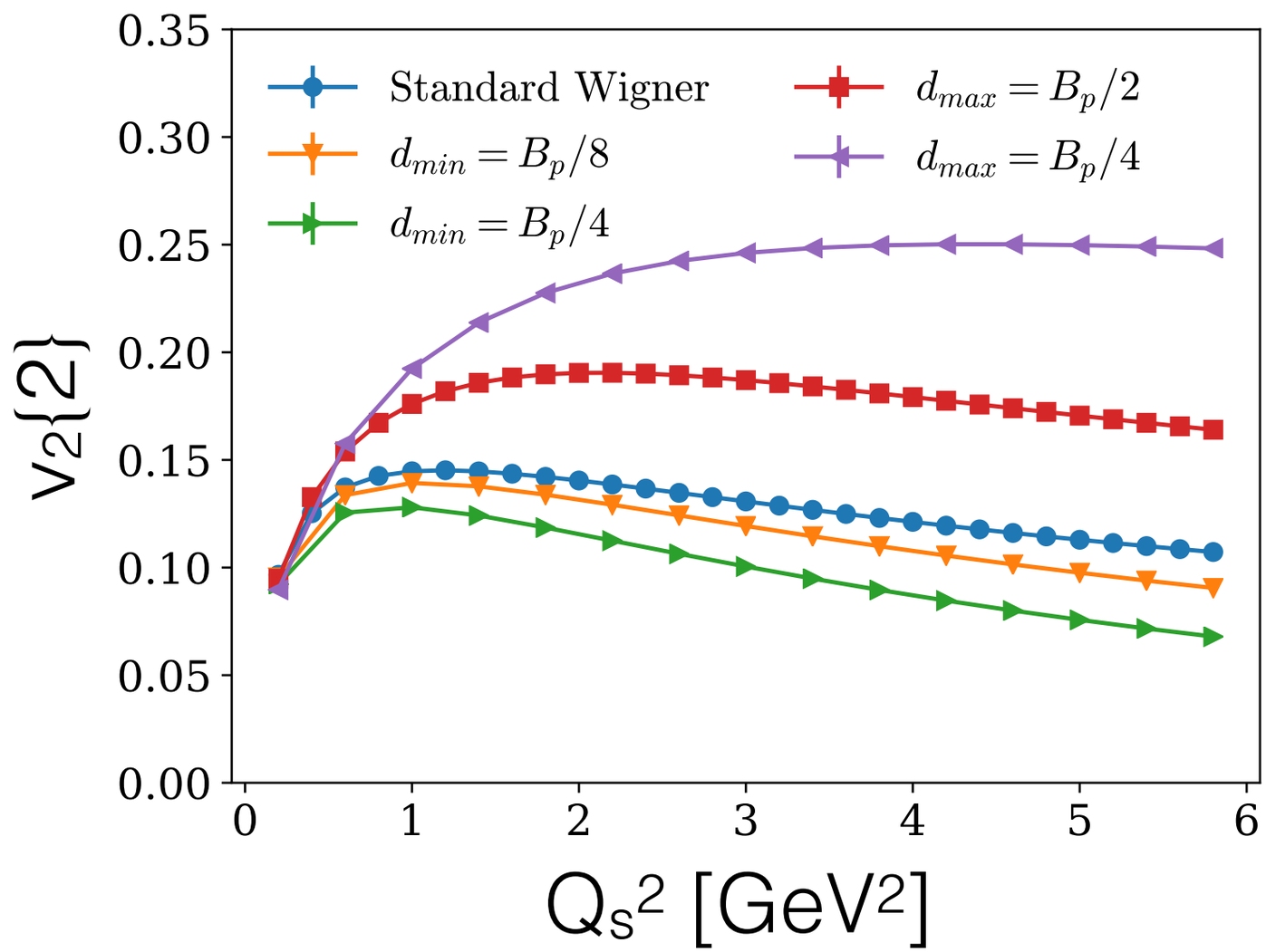
Small Kn

Denicol, Niemi, arXiv:1404.7327

Large Kn in p-Pb systems

# Role of the projectile

In order to study role of choice of the projectile, consider hard minimum cutoff ($d_{min}$) and hard maximum distance cutoff ($d_{max}$) between quarks




By keeping quarks separated by a maximum distance, correlations decrease

Confining quarks to a smaller separation increases correlations

# Multiplicity dependence

In $k_t$ factorization, multiplicity given by

$$N_{mult} \approx Q_{s,p}^2 S_\perp \log\left(\frac{Q_{s,T}^2}{Q_{s,p}^2}\right)$$

$B_p$ can be a proxy for $S_\perp$, but is held fixed

In our model, we have no dependence on projectile $Q_s$: $Q_{s,p}$

$Q_{s,T}$ not good proxies for multiplicity

More realistic model for projectile needed to study multiplicity dependence

$Q_{s,T}$ is a function of Bjorken x and impact parameter, better understood as a was to study energy dependence

# Long range in rapidity?

To make meaningful comparison to experiment, correlations should be long range in rapidity

Model is based on hybrid framework, valid at forward rapidity

Dumitru, Jalilian-Marian PRL 89 (2002), Kovchegov, Wertepny NPA 906 (2013), Kovner, Lublinsky IJMPE 22 (2013)

Valence partons in projectile long lived and have a boost invariant wave function, coherence length $\Delta y \sim 1/\alpha_s \sim \infty$

Quantum corrections can change this picture, however beyond scope of hybrid model

Dusling, Gelis Lappi, Venugopalan NPA 836 (2010)

Valid for large-x quarks, taking $x_q \geq 0.01$

From $x_q = \frac{p_\perp}{\sqrt{s}} e^y$ taking $p_\perp = 3 \text{ GeV}$ $\sqrt{s} = 5.02 \text{ TeV}$

Framework valid for $y \geq 2.8$

# Rapidity dependence

Assume eikonal quarks

Initial $k^\mu = (k^+ = \frac{\sqrt{s}}{\sqrt{2}} x_q, 0, \mathbf{0}_\perp)$

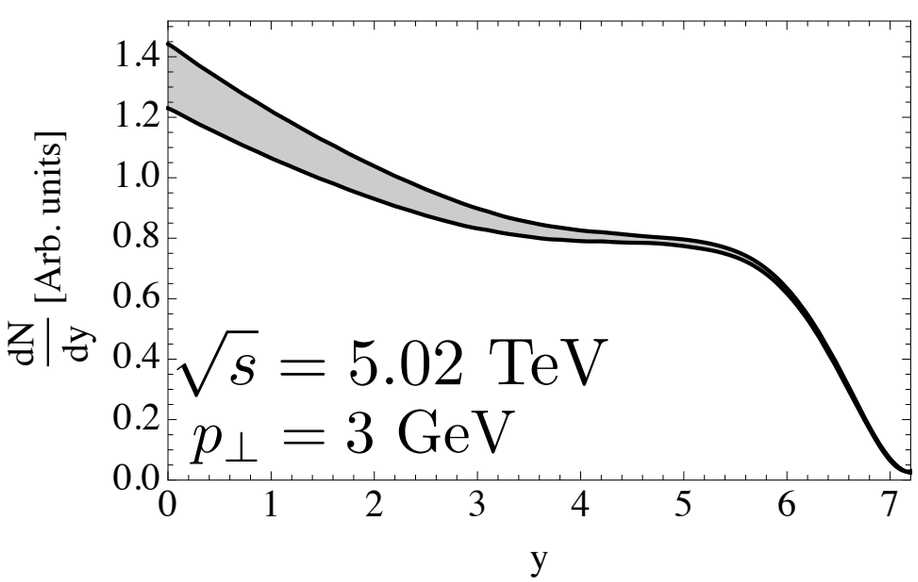
Final $p^\mu = (p^+ = \frac{p_\perp}{\sqrt{2}} e^y, 0, \mathbf{p}_\perp)$

$$\frac{dN}{d^3\mathbf{p}} = \frac{dN}{dp^+ d^2\mathbf{p}_\perp} = \delta(p^+ - k^+) \frac{dN}{d^2\mathbf{p}_\perp} .$$

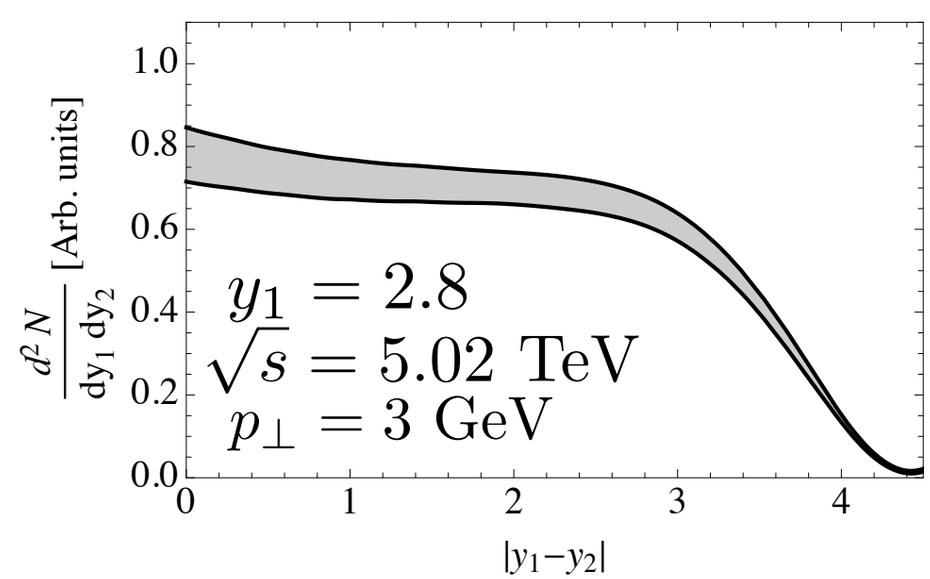
Straightforward to add rapidity dependence

$$\frac{dN^{pA\to q+X}}{dy d^2\mathbf{p}_\perp} = x'_q f(x'_q) \frac{dN^{qA\to q+X}}{dy d^2\mathbf{p}_\perp}$$

$$\frac{d^2N^{pA\to q+X}}{dy_1 d^2\mathbf{p_1}_\perp dy_2 d^2\mathbf{p_2}_\perp} = x'_{q,1} f(x'_{q,1}) x'_{q,2} f(x'_{q,2}) \frac{d^2N^{qA\to q+X}}{dy_1 d^2\mathbf{p_1}_\perp dy_2 d^2\mathbf{p_2}_\perp}$$



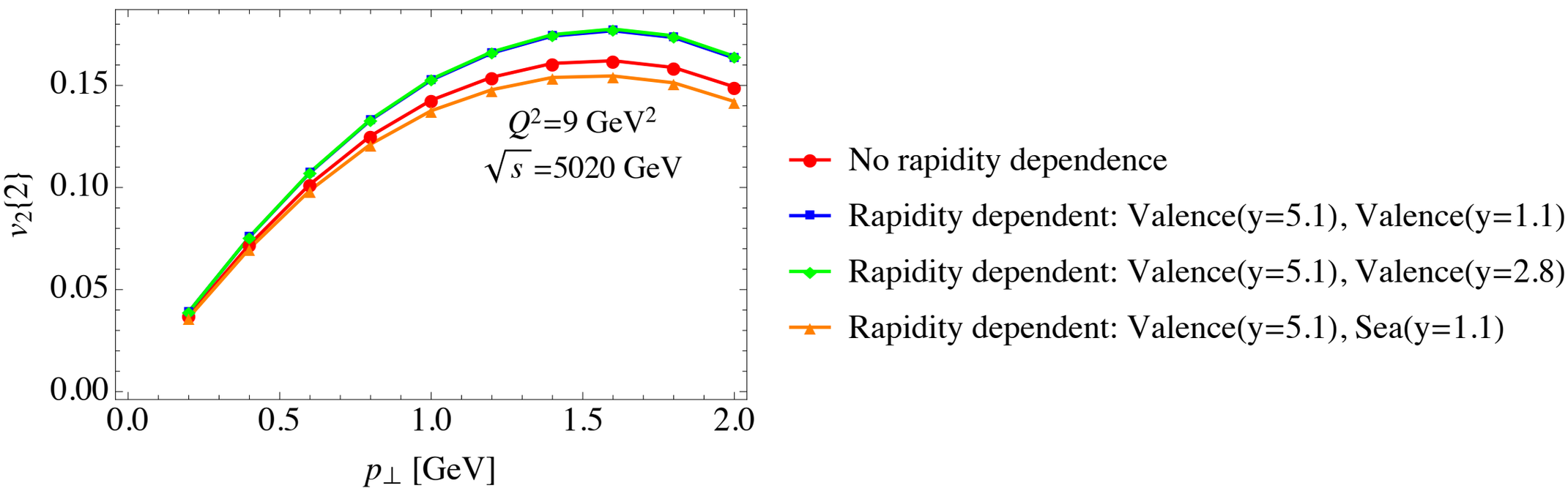

Single quark




Two quarks

# Rapidity dependence

Compare $v_2\{2\}(p_T)$ for with rapidity dependent distributions




Only quantitive, not qualitative, differences when considering both small and large x quarks

# Fluctuating initial shape

Constrain proton shape fluctuations from comparison to exclusive J/Ψ production (HERA)

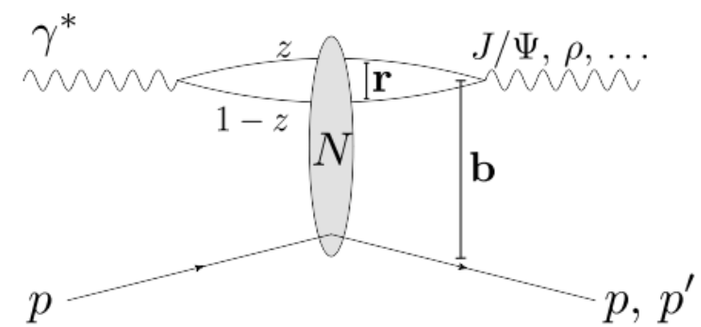



Fig. 1. Diffractive vector meson production in dipole picture.

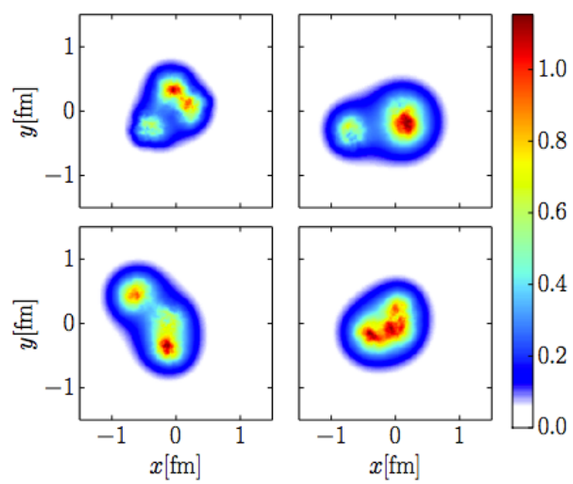
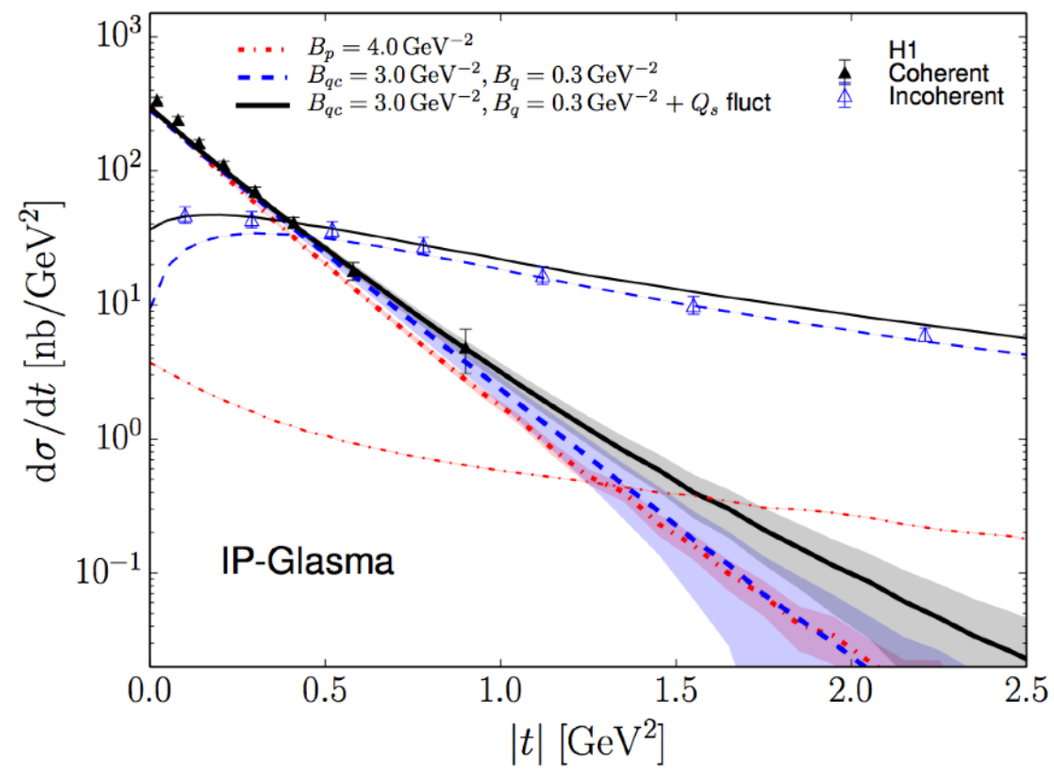



Fig. 3. Example of the proton density profiles at $x \approx 10^{-3}$. The quantity shown is $1 - \mathrm{Re}\,\mathrm{Tr}V(\mathbf{x})/N_c$.

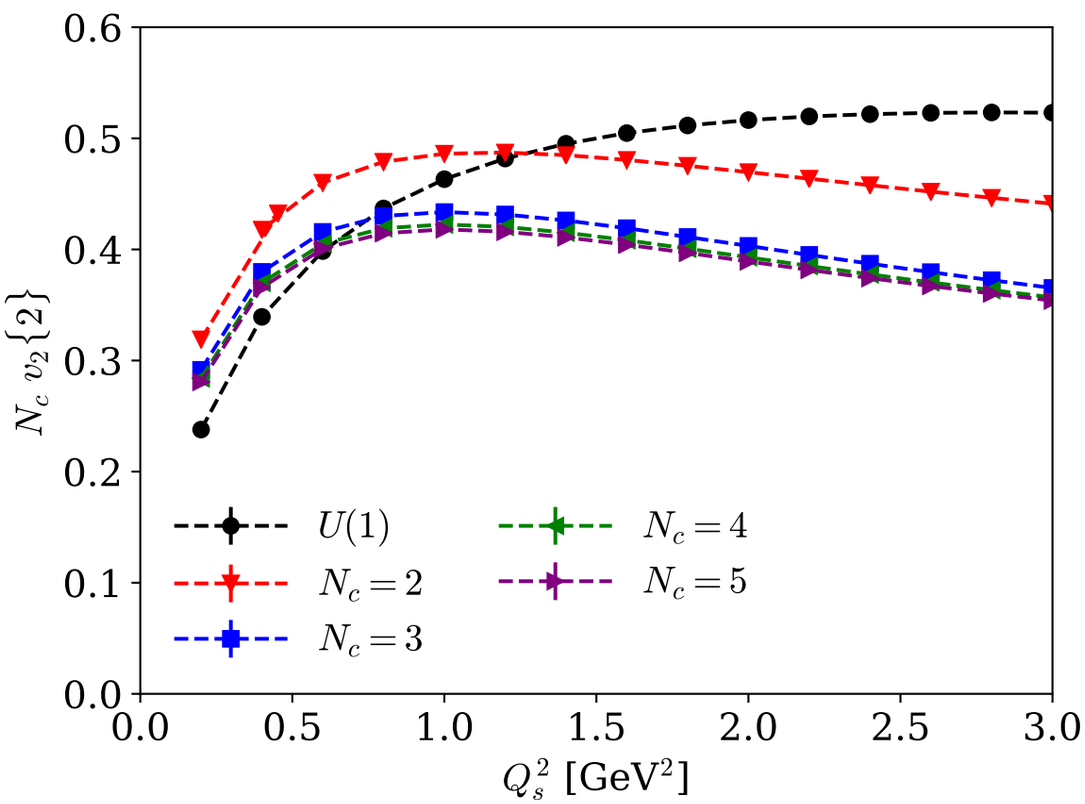



Incoherent cross section sensitive to fluctuations

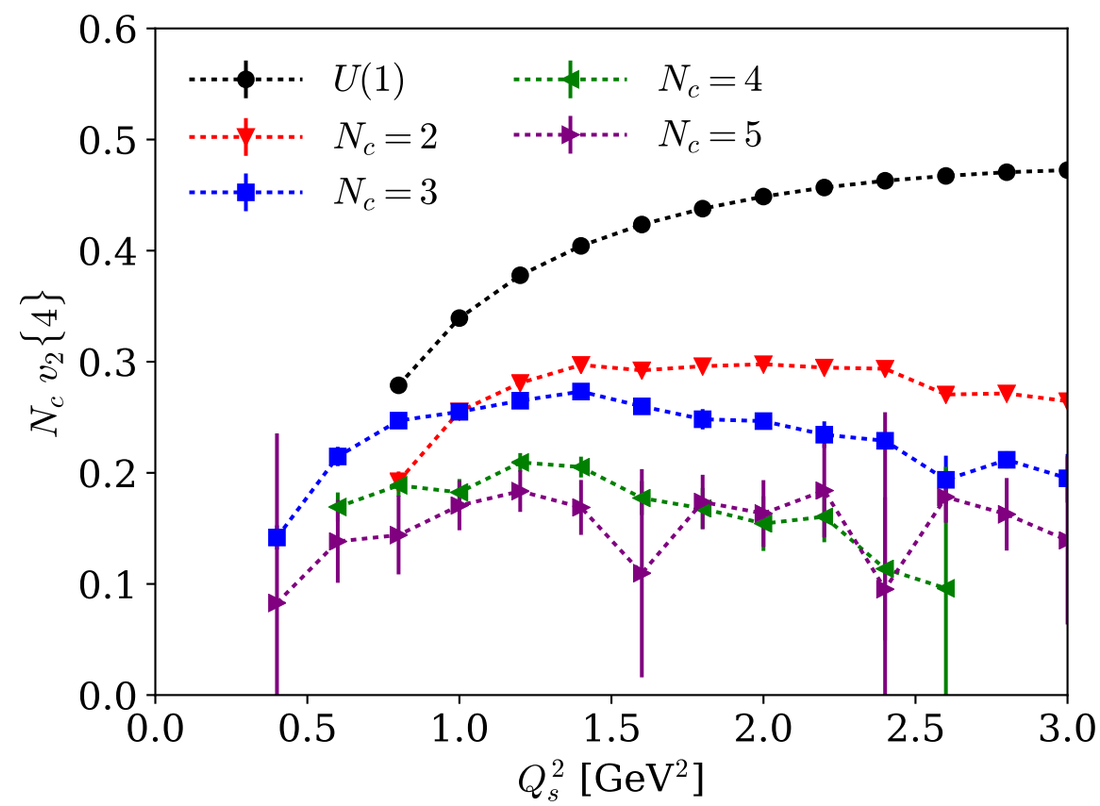
Mäntysaari, Schenke, PRL 117 (2016) 052301; PRD 94 (2016) 034042

# $N_c$ Scaling

$v_2\{2\}$ scales with $1/N_c$

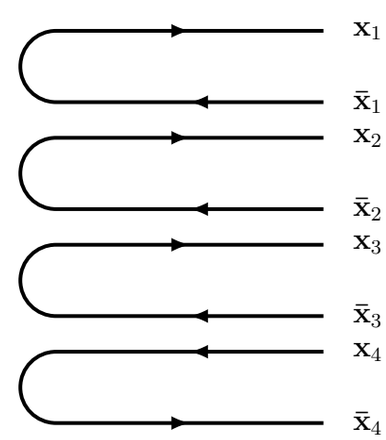



$v_2\{4\}$ appears to also scale with $1/N_c$

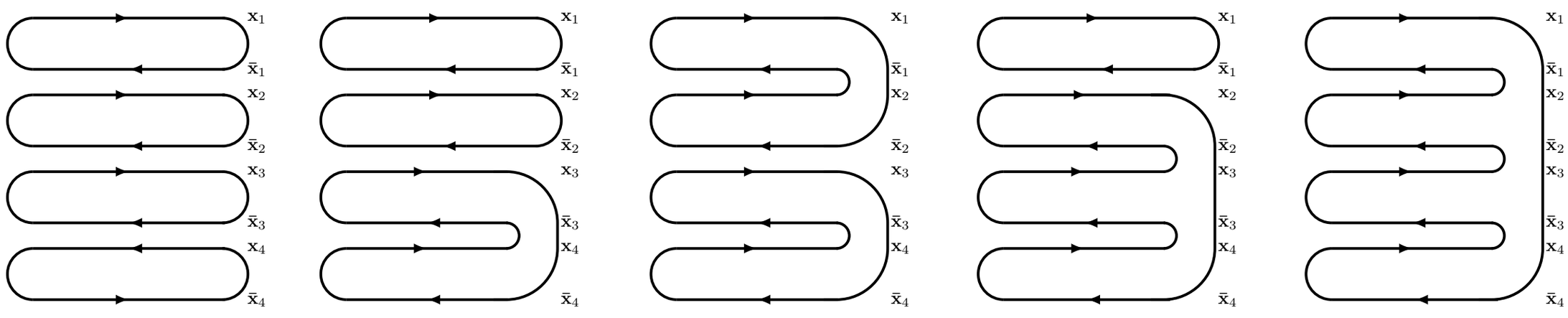


# Four dipoles

Consider initially four dipoles at $z^+=-\infty$

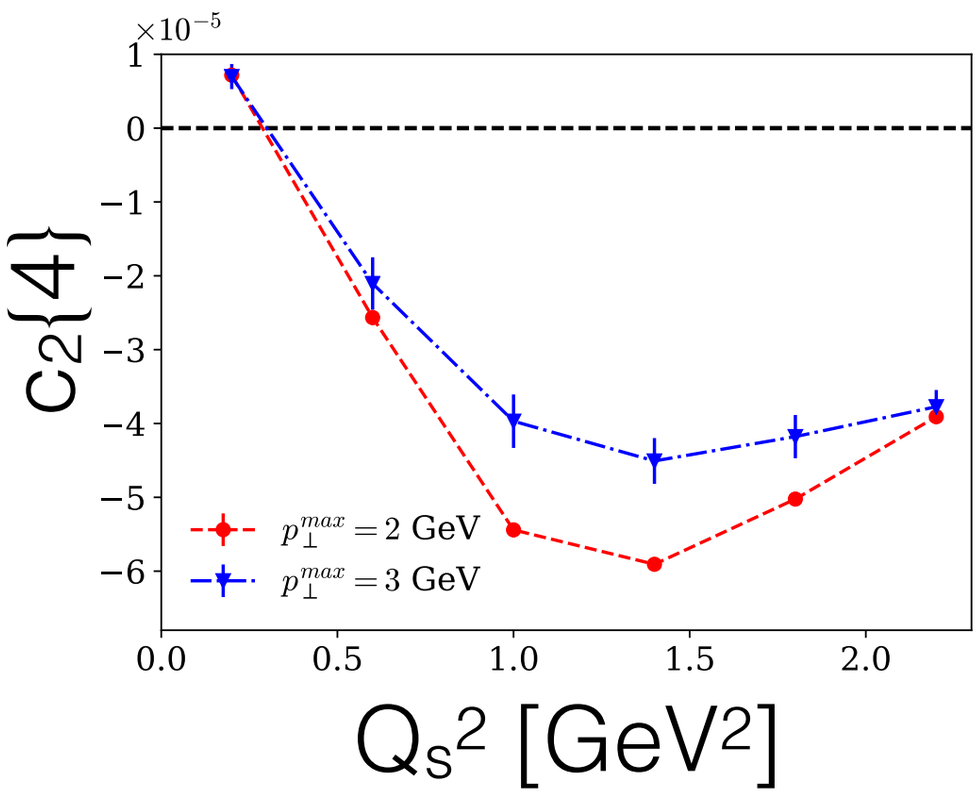



Five topologically distinct configurations

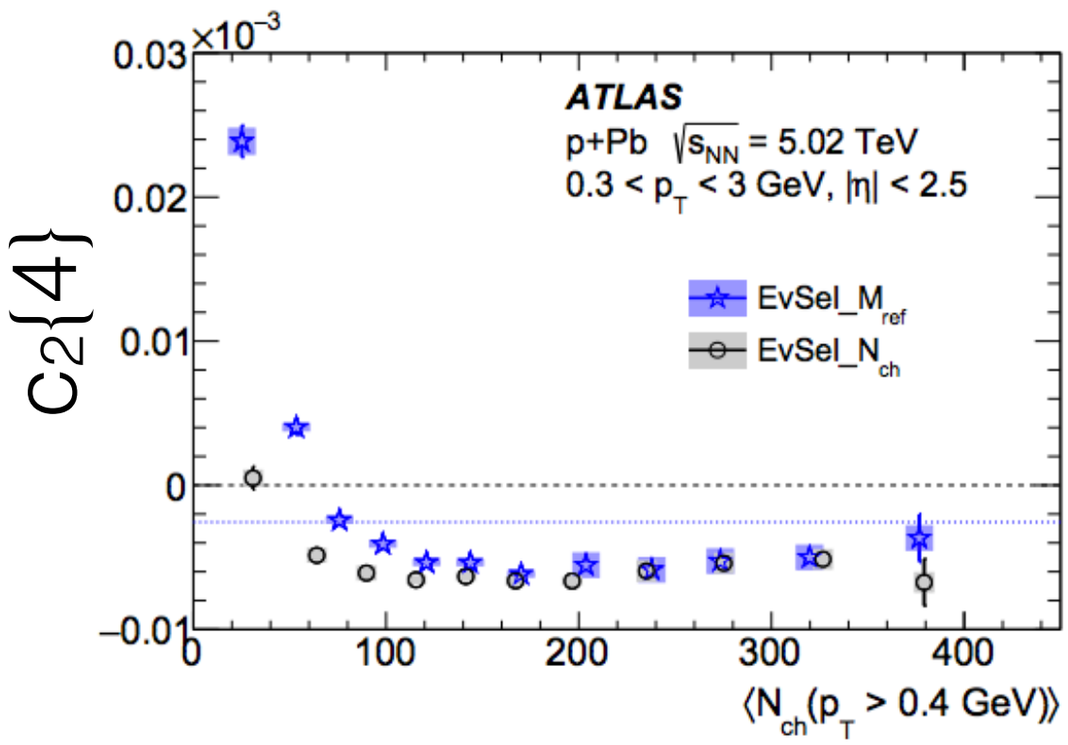


# Multi-particle quark correlations

$c_2\{4\}$ becomes negative for increasing $Q_s$




*Dusling, MM, Venugopalan PRD 97 (2018)*




*ATLAS EPJC 77 (2017)*

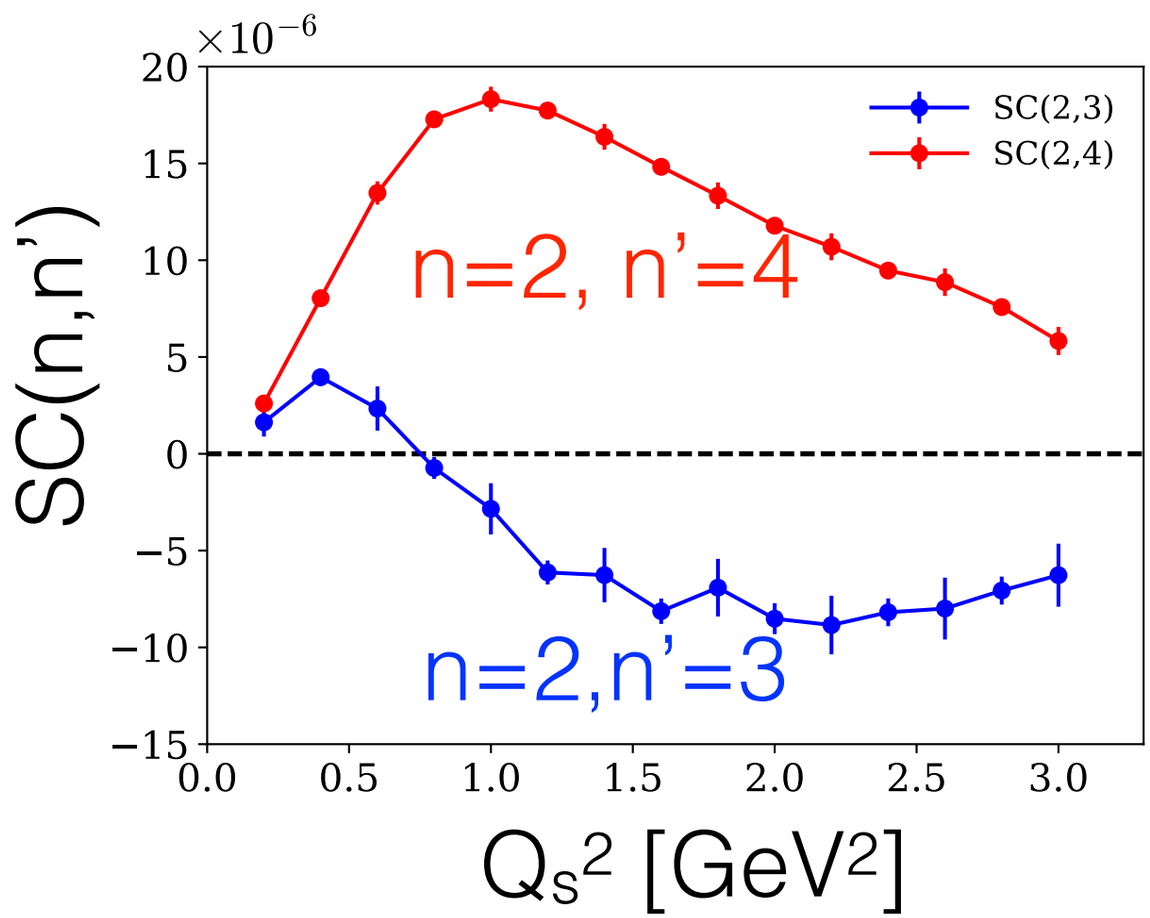
Mild dependence on maximum integrated $p_\perp$

# Symmetric Quark Cumulants

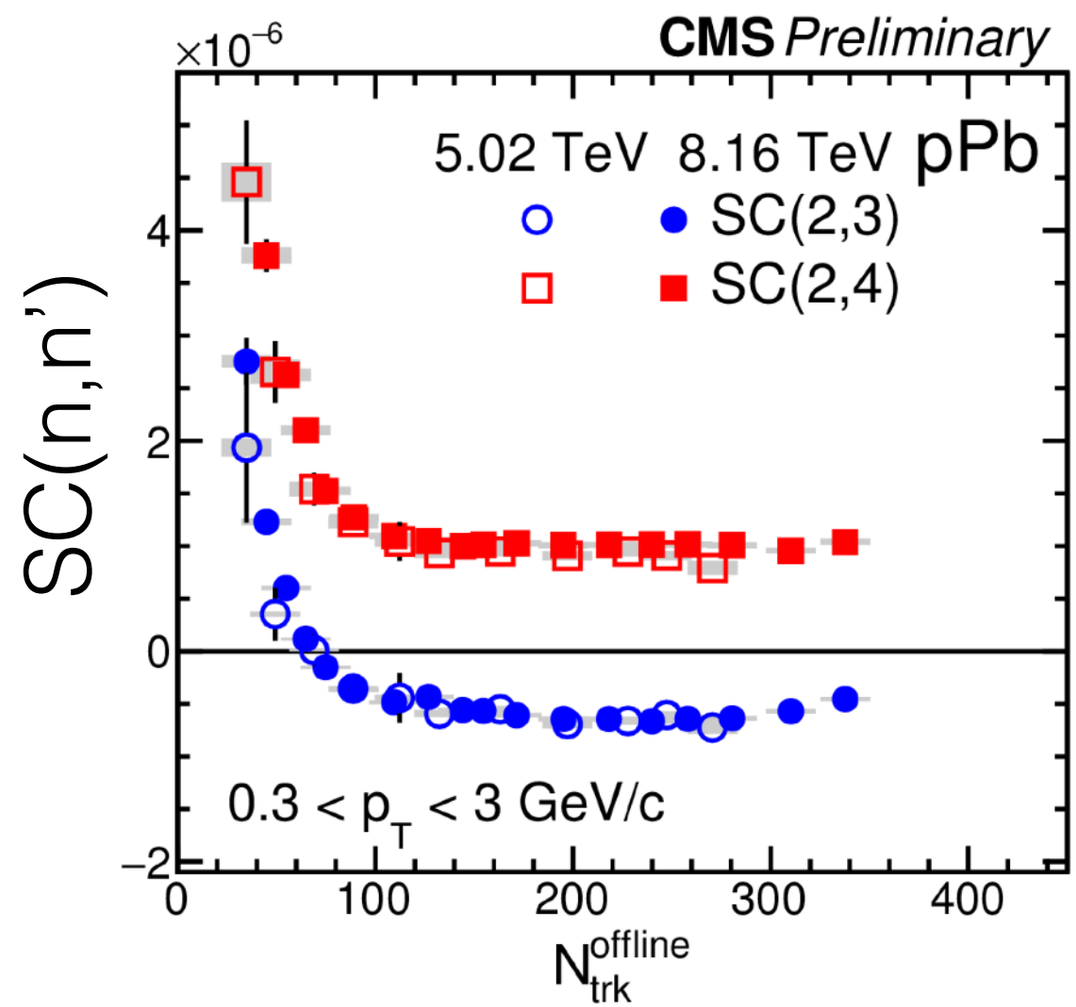
Symmetric cumulants: mixed harmonic cumulants

$$\mathrm{SC}(n,n') = \langle e^{i(n(\phi_1-\phi_3)-n'(\phi_2-\phi_4))}\rangle - \langle e^{in(\phi_1-\phi_3)}\rangle\langle e^{in'(\phi_2-\phi_4)}\rangle$$

*Bilandzic et al, PRC 89, no. 6, 064904 (2014)*

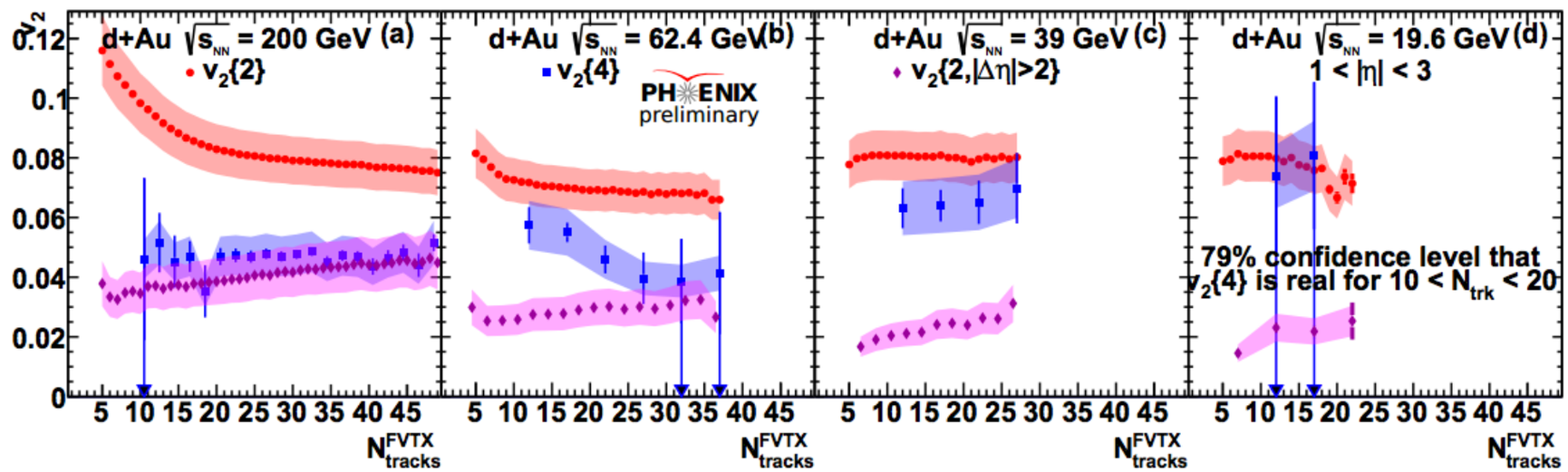



*Dusling, MM, Venugopalan PRD 97 (2018)*



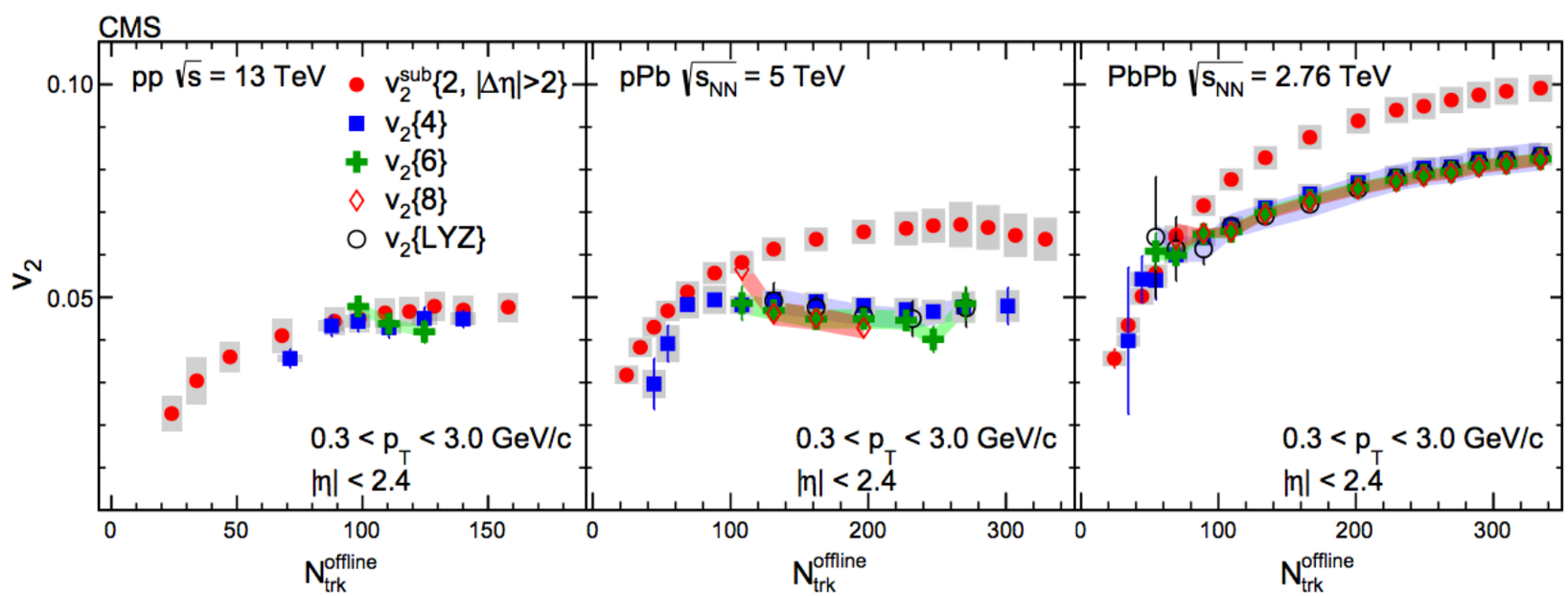

*CMS-PAS-HIN-16-022*

# Collectivity is everywhere




RHIC

*PHENIX arXiv:1704.04570*




LHC

*CMS PRL 115 (2015) 012301*

Smallest droplets of QGP? Pre-existing correlations from rare QCD configurations? Both?