Transversity GPDs in the Large-N_c limit

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joint work with Peter Schweitzer and Christian Weiss

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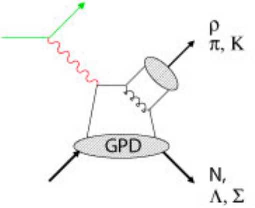
Chiral-odd GPDs

- Accessible through exclusive meson production processes



- There are four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\,\frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\,\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\,\frac{\gamma^+P^i - P^+\gamma^i}{m}\right]u(p,\lambda).$$

where $i = 1, 2$ is the transversity index [Diehl '03]

Properties of chiral-odd GPDs

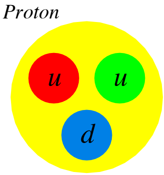
- In the forward limit $\Delta \to 0$, H_T reduces to transversity PDF; $H_T(x, 0, 0) \to h_1(x)$
- It follows from the time reversal invariance that the GPDs $H_T, \tilde{H}_T, E_T$ are invariant under the transformation $\xi \to -\xi$. Whereas $\tilde{E}_T$ is subject to sign change, i.e.

$$\begin{aligned} GPD(x,\xi,t) =& GPD(x,-\xi,t) \qquad \text{for } GPD = H_T, \tilde{H}_T, E_T \\ GPD(x,\xi,t) =& -\ GPD(x,-\xi,t) \quad \text{for } GPD = \tilde{E}_T \end{aligned}$$

- First moments of the chiral-odd GPDs are the nucleon's tensor form factors, i.e.

$$\begin{aligned} \int_{-1}^{1} \left\{ H_T, \tilde{H}_T, E_T \right\}(x,\xi,t) dx =& H_T(t), \tilde{H}_T(t), E_T(t) \\ \int_{-1}^{1} \left\{ \tilde{E}_T \right\}(x,\xi,t) dx =& 0 \end{aligned}$$

Bag Model



- Quarks are constrained inside of a finite size "bag"
- Quarks are free inside the bag (Asymptotic freedom), however are subject to sharp boundary conditions on the surface to implement the confinement.
- Evaluate the correlators by using this model in order to calculate chiral-odd GPDs.
- Bag model has been used to obtain the first estimations for chiral-even GPDs [Ji, Melnitchouk, Song '97]

Chiral-odd GPDs in Bag Model

- The momentum space wave function in the bag is given by

$$\varphi(\vec{k}) = \sqrt{4\pi} N R^3 \begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k} \quad t_1(k)\chi_m \end{pmatrix}$$

where

$$t_0(k) = \frac{j_0(w_0)\cos(kR) - j_0(kR)\cos(w_0)}{w_0^2 - \vec{k}^2 R^2}$$

$$t_1(k) = \frac{j_0(kR) j_1(w_0) kR - j_0(w_0) j_1(kR) w_0}{w_0^2 - \vec{k}^2 R^2}$$

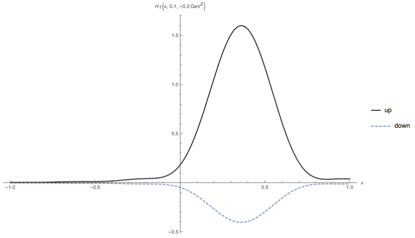
- Use this wave function to evaluate the correlators on the left hand side;

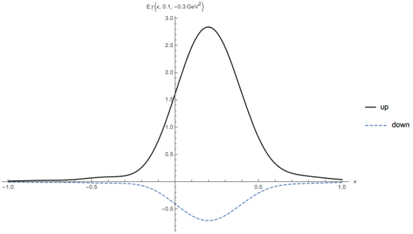
$$\varphi^\dagger(k') S(\Lambda_{-\vec{v}}) \gamma^0 \Gamma S(\Lambda_{\vec{v}}) \varphi(k)$$

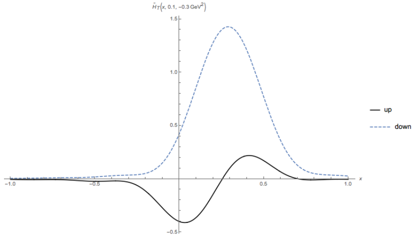
where $\Gamma = i\sigma^{+i}$ and $S(\Lambda_{\vec{v}})$ is the Lorentz boost transformation
- We have 2 equation (for $i = 1, 2$) and 4 unknowns; project on different spin components to obtain 4 equations with 4 unknowns

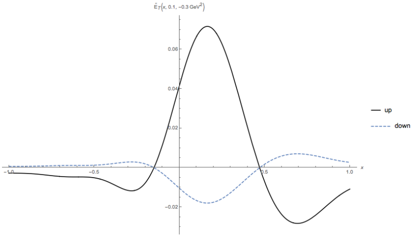
Chiral-odd GPDs in Bag Model

- Chiral-odd GPDs in Bag Model at $\xi = 0.1, t = -0.3 GeV^2$









Large-N_c expansion

- Usually once we can not solve a problem analytically, we tend to use perturbation theory; anharmonic oscillator in QM, ϕ^4 theory in QFT, ect.
- In QCD, however, the coupling constant g is high at low energies. Hence is not a good expansion parameter.
- The only known expansion parameter valid in all regions in QCD is $1/N_c$ obtained by generalizing the color gauge group $SU(3) \to SU(N_c)$ [t'Hooft '74]
- As $N_c \to \infty$, QCD simplifies significantly and can be approached nonperturbatively; with an expansion parameter $1/N_c$

Large-N_c expansion

- In this picture, baryons appear as solitons in the background of weakly interacting mesons [Witten '79]
- Large-N_c results can be checked in various ways: Diagrammatic techniques, chiral soliton model, Large-N_c quark model
- Large-N_c expansion connects QCD with chiral soliton model and quark model
- This is due to spin-flavor structure of the nucleon at Large-N_c of QCD has the same group structure with chiral soliton model and quark model at Large-N_c
- We use Bag Model as a tool to investigate model independent (N_c-scaling) results of GPDs in the Large-N_c framework

Large-N_c expansion in Bag Model

- In Large-N_c limit, we have the following scaling of parameters

$$\begin{aligned} M_N &\sim N_c \\ x &\sim N_c^{-1} \\ \xi &\sim N_c^{-1} \\ t &\sim N_c^{0} \end{aligned}$$

- In the Large-N_c framework, a GPD G is asymptotically equivalent to a product

$$G(x,\xi,t) \sim N_c^k \times F(N_c x, N_c \xi, t)$$

  where $k \in \mathbb{Z}_+$ and F is the limiting function which arise in the limit of $N_c \to \infty$.

- Here k depends on the GPD in question and the function F depends on the dynamical model used

Large-N_c expansion in Bag Model

- Then, by using the results obtained in Bag Model, we have the following scaling properties of chiral-odd GPDs

$$\begin{aligned} H_T^q &\sim N_c^2 \\ E_T^q &\sim N_c^4 \\ \tilde{H}_T^q &\sim N_c^4 \\ \tilde{E}_T^q &\sim N_c^3. \end{aligned} \tag{1}$$

- Here we note that among chiral-odd GPDs there is a special linear combination $\bar{E}_T^q = E_T^q + 2\tilde{H}_T^q$ which shows a cancellation of leading order scalings in the Large-N_c expansion

$$\bar{E}^q \sim N_c^3.$$

Large-N_c expansion in Bag Model

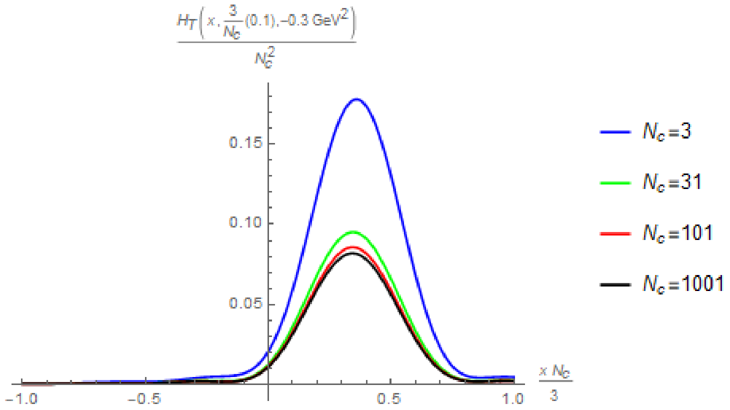


Figure: N_c-scaling of the chiral-odd GPD H_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

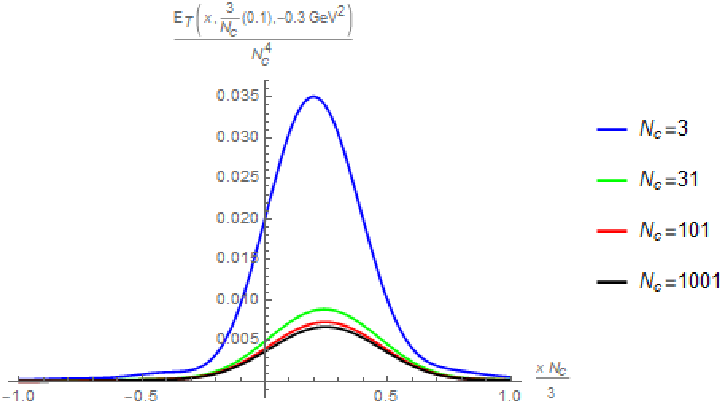


Figure: N_c-scaling of the chiral-odd GPD E_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3 GeV^2$.

Large-N_c expansion in Bag Model

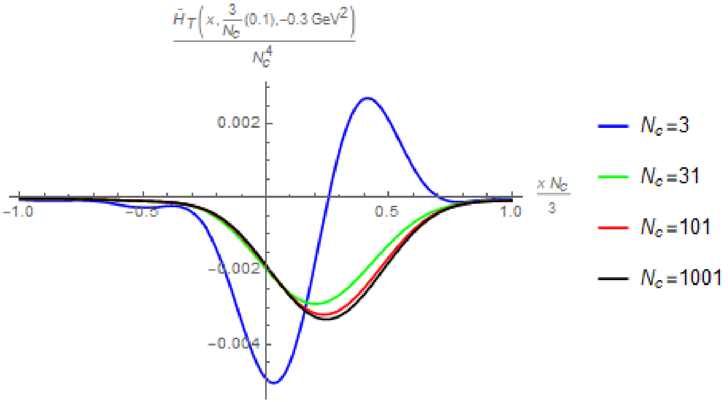


Figure: N_c-scaling of the chiral-odd GPD $\tilde{H}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

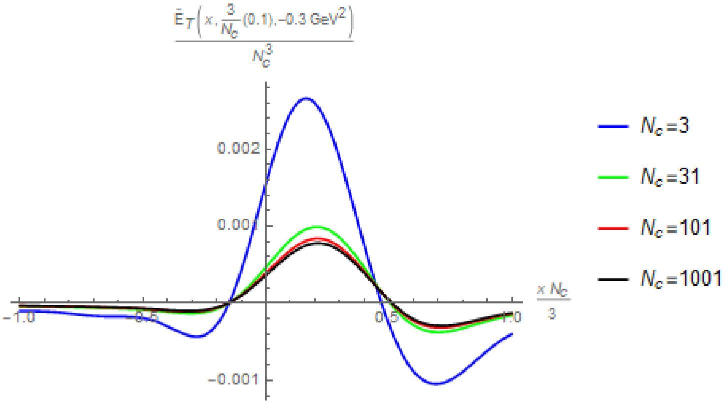


Figure: N_c-scaling of the chiral-odd GPD $\tilde{E}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

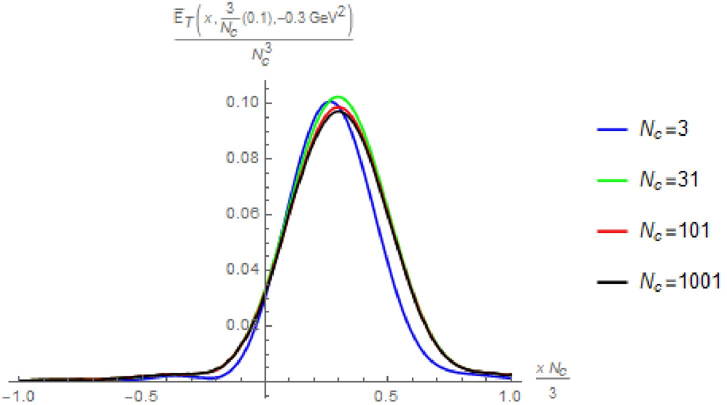


Figure: N_c-scaling of the chiral-odd GPD $\bar{E}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

- On the other hand, dominant isospin combinations of chiral-odd GPDs in the Large-N_c limit appear as

$$\begin{aligned} H_T^{u-d}(x,\xi,t) &\sim N_c^2 \\ E_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{H}_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{E}_T^{u-d}(x,\xi,t) &\sim N_c^3 \\ \bar{E}_T^{u+d}(x,\xi,t) &\sim N_c^3. \end{aligned} \tag{2}$$

- Whereas, opposite isospin combinations are suppressed by order one
- The N_c scaling behaviors of chiral-odd GPDs $\bar{E}_T, H_T$ and $\tilde{E}_T$ were discussed by [Schweitzer, Weiss '16] using a solitonic field with known symmetry properties. The results are confirmed in the bag model

Phenomenological implications

- What are the phenomenological implications of our findings?
- Since we have Large-N_c relations among flavor-singlet and flavor-nonsinglet components of GPDs, this order among them predicts the relative sign of flavor decomposed GPDs
- For instance, dominance of flavor-nonsinglet $(u - d)$ component of the GPD H_T in the Large-N_c limit implies a sign difference in the flavor decomposition of H_T
- Similarly for $\bar{E}_T$, flavor-singlet $(u + d)$ component is dominant in the Large-N_c limit. This implies that the flavor decomposition is expected to have the same sign

Phenomenological implications

- Preliminary π^0, η electroproduction data at JLab confirms our predictions for H_T and $\bar{E}_T$

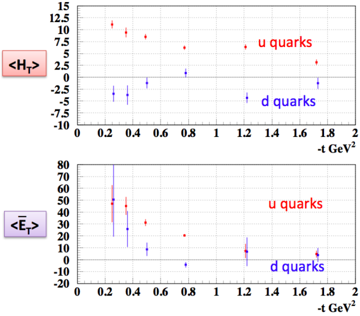


Figure: Preliminary [Kubarovsky '15], talk given at EMP and Short Range Hadron Structure. $Q^2 = 1.8 GeV^2$

where $< H_T >, < \bar{E}_T >$ denotes the weighted integral of the GPD over x.

Conclusions

- Chiral-odd GPDs at leading twist are evaluated in the MIT Bag Model
- The model satisfies important properties: like polynomiality, the sum rule $\int dx \tilde{E}_T^q(x, \xi, t) = 0$ (not all models satisfy it)
- In the Large-N_c limit, scaling properties of GPDs analyzed
- Flavor structure dictated by quark model SU(4)-spin flavor symmetry is in qualitative agreement with JLab data