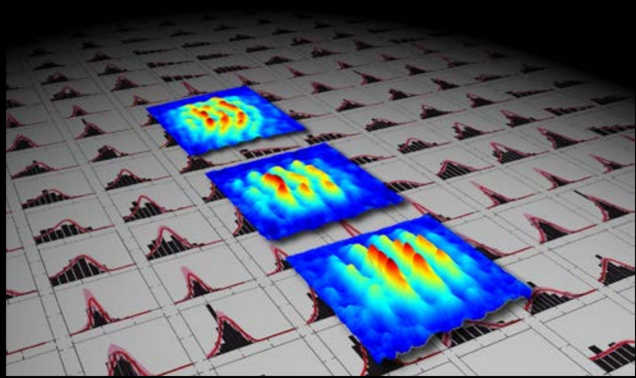
Higher Order Correlations

and what we can learn about the solutions for many body problems from experiment

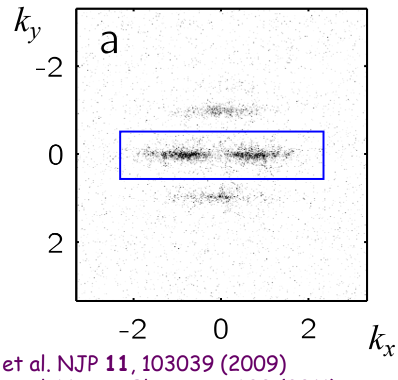


Jörg Schmiedmayer

Vienna Center for Quantum Science and Technology, TU-Wien

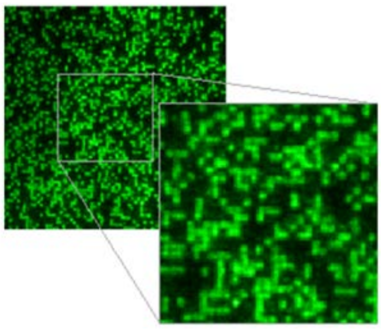
Probing cold atom systems

time of flight -> momentum



R. Bücker et al. NJP **11**, 103039 (2009)
R. Bücker et al. Nature Physics **7**, 608 (2011)

in situ -> position



W. S. Bakr, et al.,Nature **462**, 74 (2009).
J. F. Sherson, et al. Nature **467**, 68 (2010).

single shot measurements of the many body wavefunction
n-point correlation function

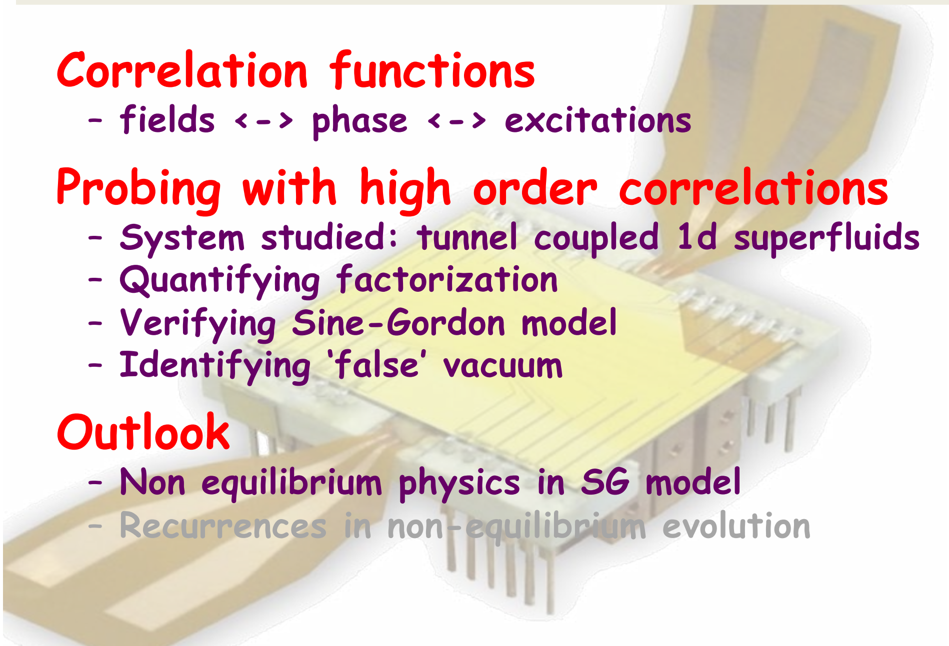
Theory: Single-shot simulations of dynamic quantum many-body systems
K. Sakmann, M. Kasevich, Nature Physics (2016)

Many Body Quantum Systems <-> Correlation Functions

On the Green's functions of quantized fields
J. Schwinger PNAS (1951)

- Solving a quantum many-body problem is equivalent to knowing **all** its correlation functions.
- Real world: Observer can only measure a **finite** number of correlations
  -> describing the propagation and scattering of excitations.
- To (approximately) 'solve' the problem one need to **find degrees of freedom** where only few (low order) correlation functions are relevant.
- If one finds the degrees of freedom (basis) where the **correlation functions factorize**, this is equivalent to **diagonalization of the many body Hamiltonian.**
- Experiment / its 'read-out' always has a **finite accuracy and errors**.
  need clear signatures **what can we learn** from an experiment, and **what is a fair comparison** to the models
  -> example: probing for quantum supremacy

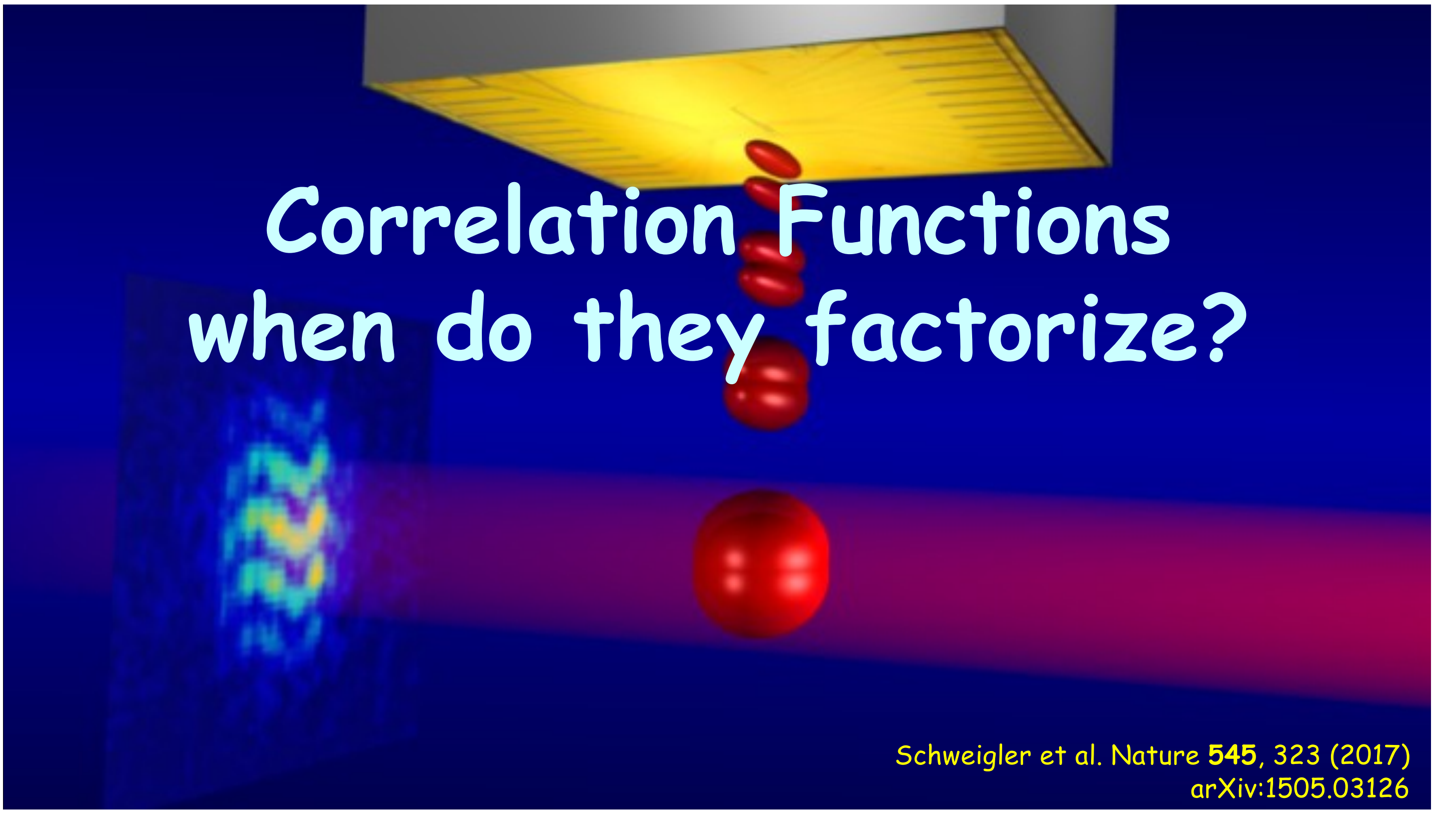
Outline



Correlation functions
- **fields <-> phase <-> excitations**

Probing with high order correlations
- **System studied: tunnel coupled 1d superfluids**
- **Quantifying factorization**
- **Verifying Sine-Gordon model**
- **Identifying 'false' vacuum**

Outlook
- **Non equilibrium physics in SG model**
- **Recurrences in non-equilibrium evolution**



Correlation Functions

The N^th^ order Correlation function

$$G^{(N)}(\mathbf{z}) = \langle \mathcal{O}(z_1)\mathcal{O}(z_2)\ldots\mathcal{O}(z_N)\rangle$$

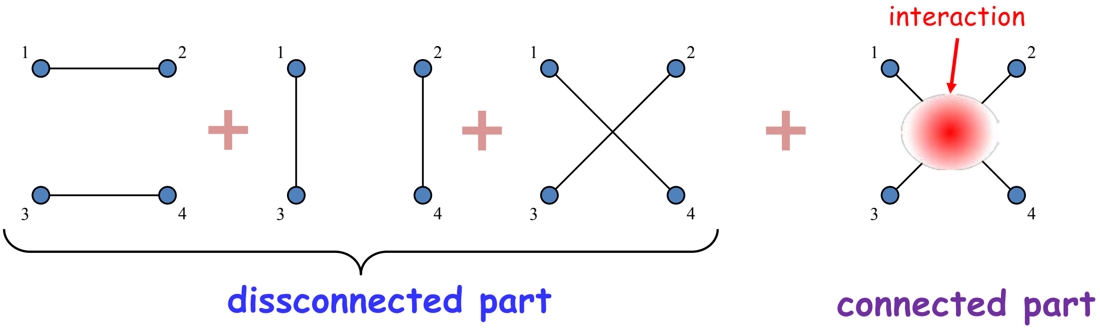
Characterizes the propagation and the interactions of the degrees of freedom connected to the operators $\mathcal{O}(z_i)$

It can be decomposed: $G^{(N)}(\mathbf{z}) = G^{(N)}_{\text{dis}}(\mathbf{z}) + G^{(N)}_{\text{con}}(\mathbf{z})$

- The **disconnected** part $G^{(N)}_{\text{dis}}$ is fully determined through lower order correlations
- The **connected** part $G^{(N)}_{\text{con}}$ contains genuine new information about the system at order N

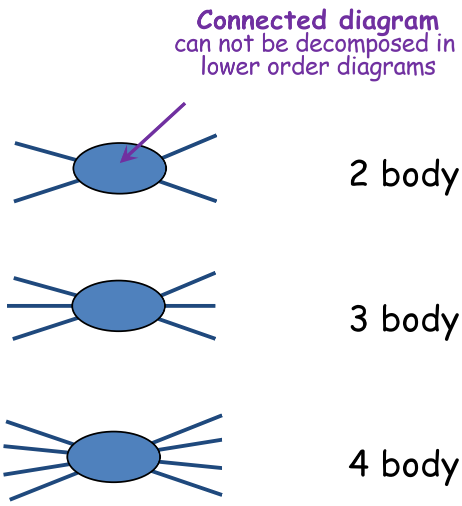
4-point correlation function

$G^{(4)}(x_1, x_2, x_3, x_4) =$



$$
\begin{aligned}
G^{(4)}(x_1, x_2, x_3, x_4) = \\
& G^{(2)}(x_1, x_3) G^{(2)}(x_2, x_4) + G^{(2)}(x_1, x_2) G^{(2)}(x_3, x_4) + G^{(2)}(x_1, x_4) G^{(2)}(x_2, x_3) \\
& + \int d^D y_1 \ldots d^D y_4 \; G^{(2)}(x_1, y_1) G^{(2)}(x_2, y_2) G^{(2)}(x_3, y_3) G^{(2)}(x_4, y_4) \; \Gamma^{(4)}(y_1, y_2, y_3, y_4)
\end{aligned}
$$

What can we learn from connected correlations



4^{th} order $\sum$ 2 body interaction

6^{th} order $\sum$ 3 body interaction

8^{th} order $\sum$ 4 body interaction

...

Probing factorization
extracting connected correlations

Schweigler et al. Nature **545**, 323 (2017)

Correlation function of the phase

$$G^{(N)}(\mathbf{z},\mathbf{z}') = \langle [\varphi(z_1) - \varphi(z_1')] \dots [\varphi(z_N) - \varphi(z_N')] \rangle$$

Calculate the connected part of the correlation:

$$G^{(N)}_{\text{con}}(\mathbf{z},\mathbf{z}') = \sum_{\pi} \left[(|\pi|-1)!\,(-1)^{|\pi|-1} \prod_{B\in\pi} \left\langle \prod_{i\in B} [\varphi(z_i) - \varphi(z_i')] \right\rangle \right]$$

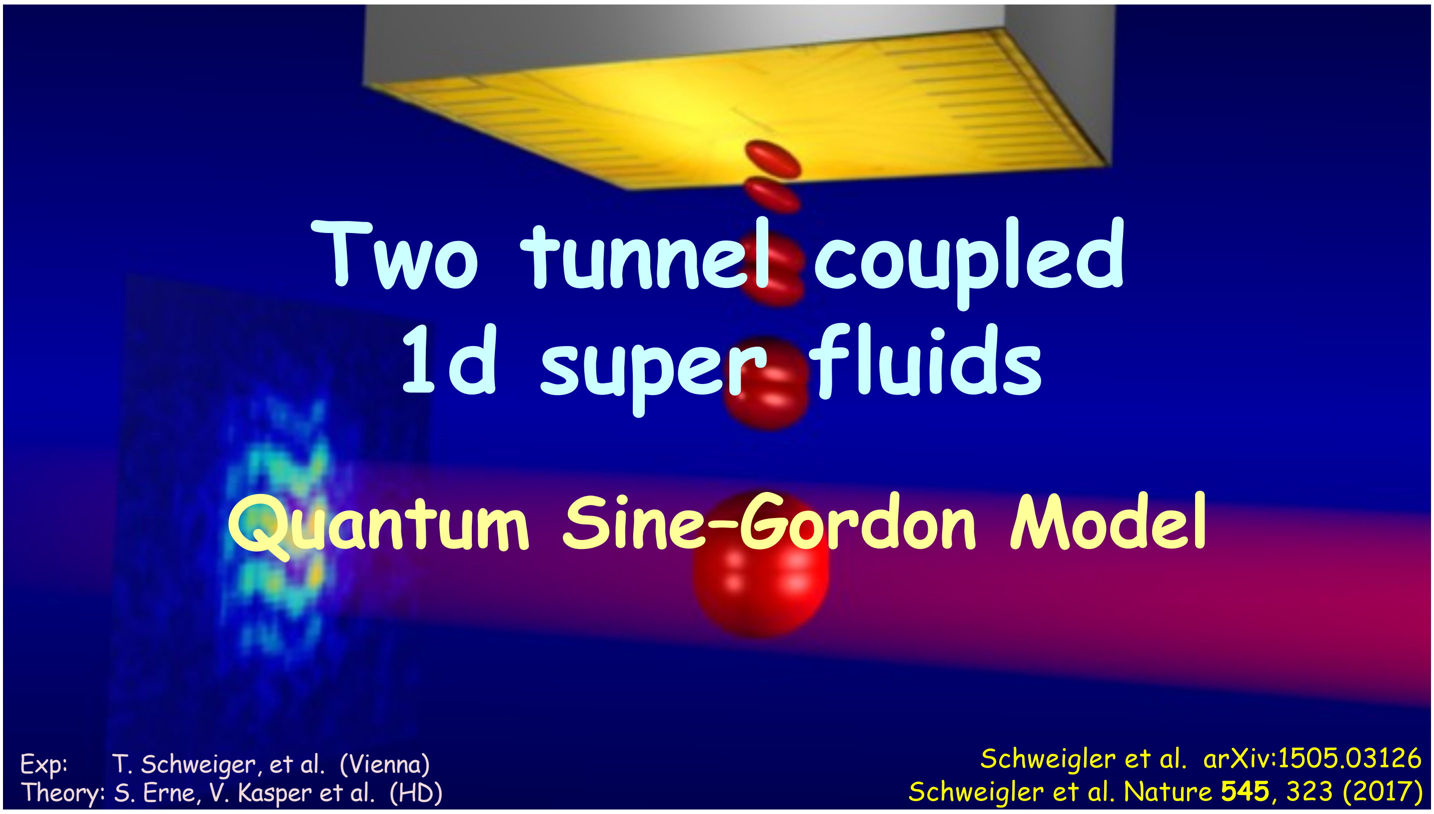
Sum runs over all possible partitions π, the first product over all blocks B of the partition, the second over all elements of the block.

The number of partitions to consider grows rapidly with N.
For N=10 there are already >10^5 terms to consider !!!!

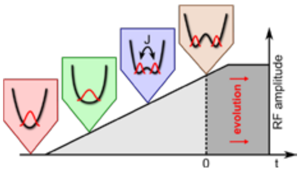
Complexity of a many body problem

Decomposition into 2$^{\text{nd}}$ order correlations (**Wick decomposition**)

$$G^{(N)}_{\text{wick}}(\mathbf{z},\mathbf{z}') = \sum_{\pi_2} \left[\prod_{B\in\pi_2} \left\langle [\varphi(z_{B_1}) - \varphi(z_{B_1}')][\varphi(z_{B_2}) - \varphi(z_{B_2}')] \right\rangle \right]$$



1d double-well with tunable coupling J



$$H=\sum_{j=1}^{2}\int dz\left[\frac{\hbar^2}{2m}\frac{\partial\psi_j^\dagger}{\partial z}\frac{\partial\psi_j}{\partial z}+\frac{g_{1D}}{2}\psi_j^\dagger\psi_j^\dagger\psi_j\psi_j+U(z)\psi_j^\dagger\psi_j-\mu\psi_j^\dagger\psi_j\right]-\hbar J\int dz\left[\psi_1^\dagger\psi_2+\psi_2\psi_1^\dagger\right]$$

Following: Gritsev, Polkovnikov, Demler Phys. Rev. B **75**, 174511 (2007)

- Density phase representation
- Expanding the Hamiltonian in density fluctuations $\delta\rho_j$ and phase gradients $\partial_z\varphi_j$ up to second order and neglecting mixed terms separates H in symmetric and antisymmetric degrees of freedom
- Neglecting terms $|\delta\rho/n_0| \ll 1$

One arrives at **Quantum Sine-Gordon model**:

$$\hat{H}_{SG}=\int dz\left[\frac{\hbar^2 n_{1D}}{4m}(\partial_z\hat{\varphi})^2+g\delta\hat{\rho}^2\right]-\int dz\ 2Jn_{1D}\left[1-\cos\hat{\varphi}\right]$$

"uncoupled harmonic oscillators"

anharmonic, non-gaussian, gapped,

phase coherence length

$$\lambda_T = 2\hbar^2 n_{1D}/(mk_BT)$$

phase (spin) healing length

$$l_J = \sqrt{\hbar/(4mJ)}$$

Characteristic parameters

$$q = \lambda_T/l_J$$

Sine Gordon Model

Sine Gordon <=> Massive Thirring Model

S. Colman Phys. Rev. D **217** 11, 2088 (1975).

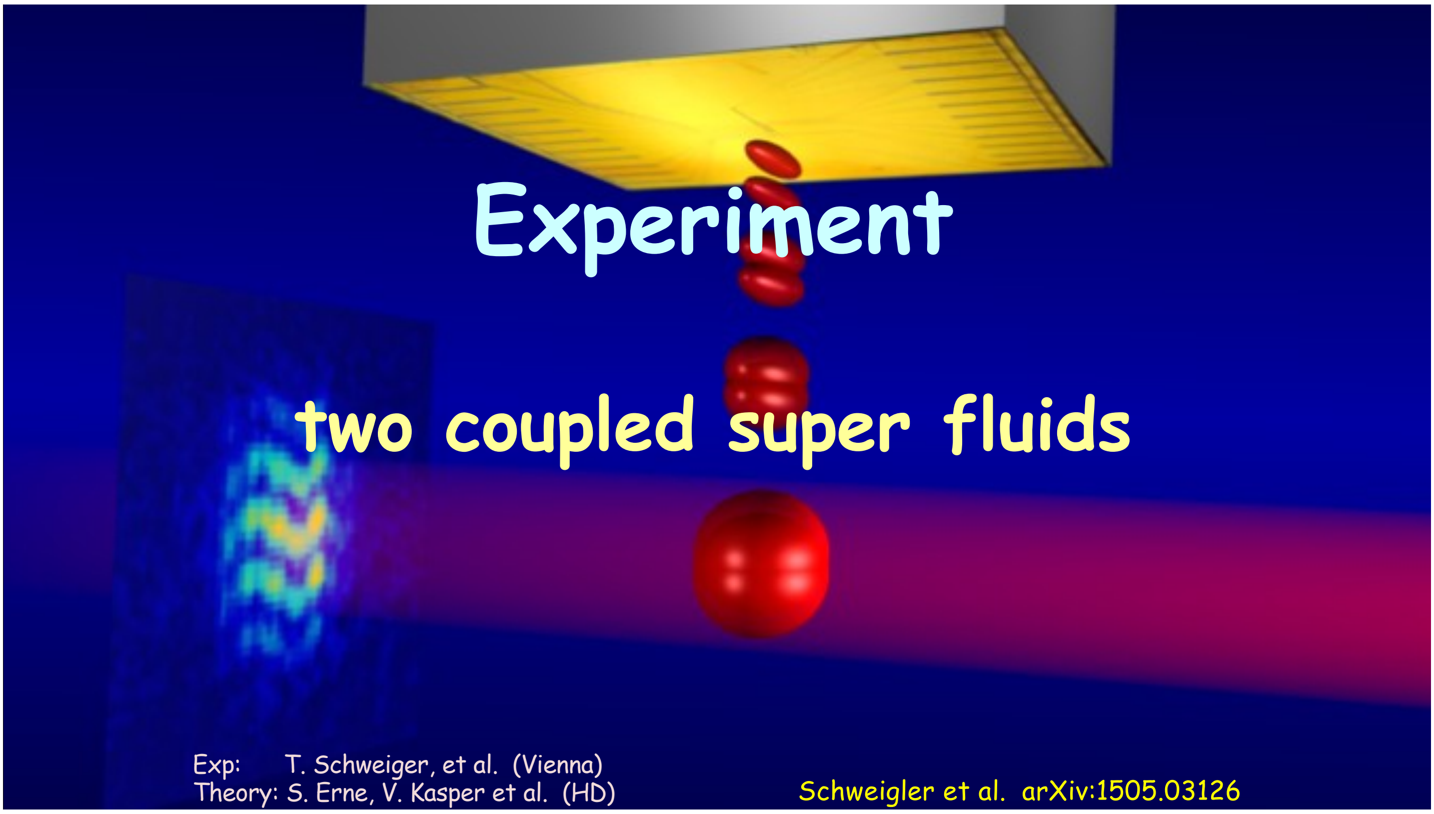
Sine Gordon <=> Coulomb Gas

Polyakov, A. M. *Nuclear Physics B*, ***120***, 429-458 (1977).

Samuel, S. *Physical Review D*, ***18***, 1916 (1978).

Sine Gordon <=> XY

José, J. V. et al., *Physical Review B*, ***16***, 1217 (1977).



AtomChip

Integrated Circuits for ultra-cold Quantum Matter

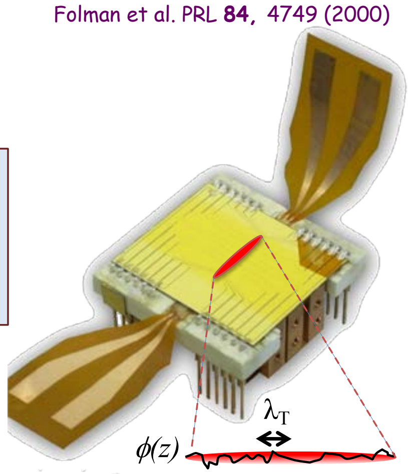
Folman et al. PRL **84**, 4749 (2000)

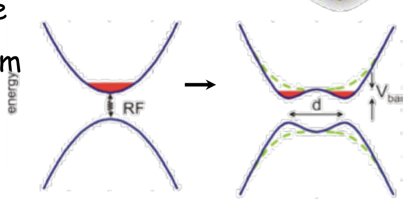
Combine the robustness of nano-fabrication an the quantum tools of atomic physics and quantum optics to build a toolbox for quantum experiments

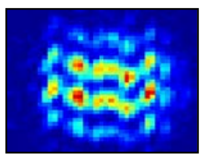
- 1d elongated traps
- Easy to create a BEC
- Very stable and reproducible laboratory for quantum experiments
- Fast operation
- Single atom detection with unit efficiency
- Well controlled splitting and interference
- experiment optimized by genetic algorithm

Rohringer et al. APL **93**, 264101 (2008)

3000-10000 atoms
T = 20-100 nK
$\omega_R \sim 2\pi \times 2$ - 3 kHz
$\omega_L \sim 2\pi \times 5$ - 10 Hz
$k_B T \sim 0.1 - 0.7\ \hbar\omega_R$
$k_B T \sim 0.1 - 1\ \mu$







Experimental procedure

Schweigler et al. Nature **545**, 323 (2017)

Experiment

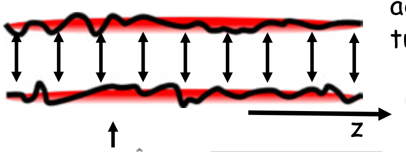
U

`Imaging´

evaluation

extracted values of measurements

1D gas of ^{87}Rb atoms

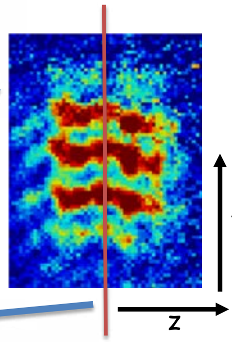


$$\psi(z) = e^{i\hat{\Theta}(z)}\sqrt{\rho_0(z) + \delta\hat{\rho}(z)}$$

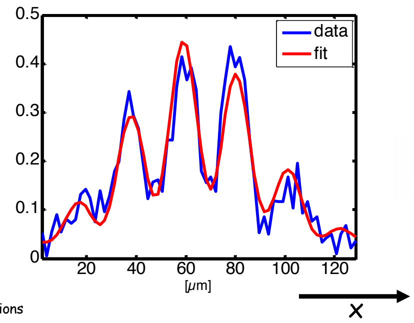
adjustable tunnelling J

Tunnel coupling lead to **phase locking** characterized by $\langle \cos(\varphi) \rangle$

time of flight



fit



=> phase difference between condensates

$$\varphi(z) = \theta_1(z) - \theta_2(z)$$

Correlation functions
fields <-> phase

experiments in a trap
-> non translation invariant correlation functions

$$C(z_1, z_2) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_2(z_2)|^2\rangle}$$

with
$$\Psi(z) = e^{i\theta(z)}\sqrt{\rho_0(z) + \delta\hat{n}(z)}$$
$$\varphi(z) = \theta_1(z) - \theta_2(z)$$

neglecting $\delta\hat{n}(z)$ $\Rightarrow$ $\boxed{C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)]\rangle}$

4th order:

$$C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\Psi_1(z_3)\Psi_2^\dagger(z_3)\Psi_1^\dagger(z_4)\Psi_2(z_4)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_1(z_2)|^2\rangle\langle|\Psi_2(z_3)|^2\rangle\langle|\Psi_2(z_4)|^2\rangle}$$

$$C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)]\rangle$$

Correlation functions
excitations <-> phase

in experiment we measure the phase $\varphi(z)$ directly
-> look at phase correlators

$$C^{(2)}(z_1, z_2) = \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2\rangle$$

with $\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$ Note: $\Delta\varphi$ is NOT restricted to 2π

using
$$\varphi(z) = \frac{1}{\sqrt{L}} \sum_{k\neq 0} \left[(-i)\sqrt{\frac{\pi}{|k|K}}(b_k^\dagger - b_{-k})e^{ikz}\right]$$

$$\Longrightarrow \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \sum_{k_1,k_2} \frac{\pi}{K\sqrt{|k_1 k_2|}} b_{k_1}^\dagger b_{-k_2} e^{ik_1 z_1 + ik_2 z_2} + \ldots$$

-> phase correlators are related to the **quasi particles**

4th order

$$C^{(4)}(z_1, z_2, z_3, z_4) = \langle[\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2\rangle$$
$$\propto b_{k_1}^\dagger b_{k_2}^\dagger b_{-k_3} b_{-k_4} + \ldots$$

-> quasi particle scattering

Correlation functions
fields <-> phase

correlation functions for the fields:

$$C(z_1, z_2) = \frac{\langle\Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_2(z_2)|^2\rangle}$$

$$\boxed{C(z_1, z_2) \approx \langle\exp[i\varphi(z_1) - i\varphi(z_2)]\rangle}$$

$C(z_1,z_2)$ contains **all orders of connected** parts

$$C(z_1, z_2) = \exp\left[\sum_{k=1}^{\infty}(-1)^k \frac{\langle(\Delta\varphi)^{2k}\rangle_c}{(2k)!}\right]$$

for Gaussian fluctuations

$$C(z_1, z_2) = \exp\left[-\frac{1}{2}\langle(\Delta\varphi)\rangle^2\right]$$

4^{th} order correlations
Connected and disconnected part

Schweigler et al. Nature **545**, 323 (2017)

to study factorization of correlation functions we look at:

$G^{(2)}(z_1, z_2) = \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle$

$G^{(4)}(z_1, z_2, z_3, z_4) = \langle[\varphi(z_1) - \varphi(z_2)]^2\,[\varphi(z_3) - \varphi(z_4)]^2\rangle$

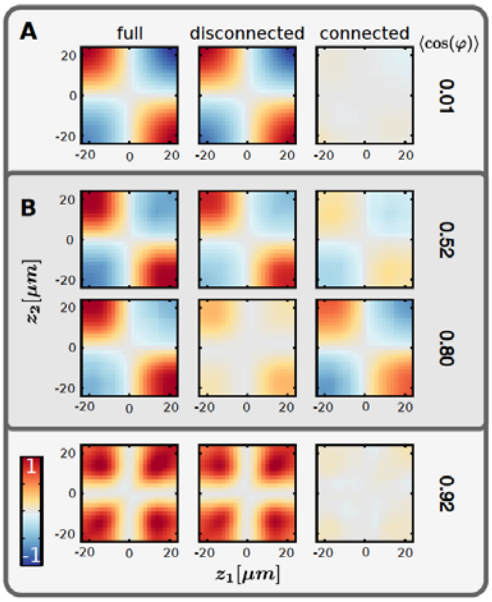
$\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$

$\Delta\varphi$ *is* NOT restricted to $[-\pi, \pi)$

Connected/Disconnected part

$$G^{(N)}(\mathbf{z}) = G^{(N)}_{\rm con}(\mathbf{z}) + G^{(N)}_{\rm dis}(\mathbf{z})$$

$C^{(4)}(z_1, z_2, -15, 15)$



Characterizing Connected Correlations

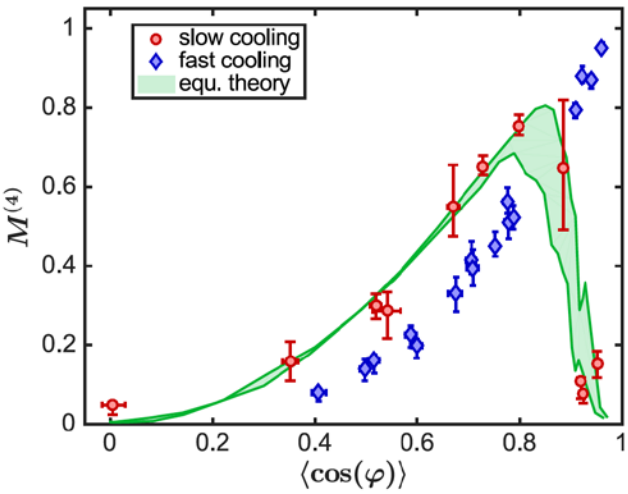
Schweigler et al. Nature **545**, 323 (2017)

4^{th} order correlations

Integrated measure

$$M^{(N)} = \frac{\sum_{\mathbf{z}} \left|G^{(N)}_{\rm con}(\mathbf{z}, 0)\right|}{\sum_{\mathbf{z}} \left|G^{(N)}(\mathbf{z}, 0)\right|}$$

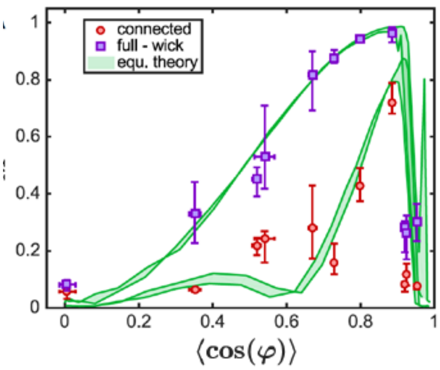
Compared to predictions fo
a thermal equilibrium state
of the sine-Gordon model



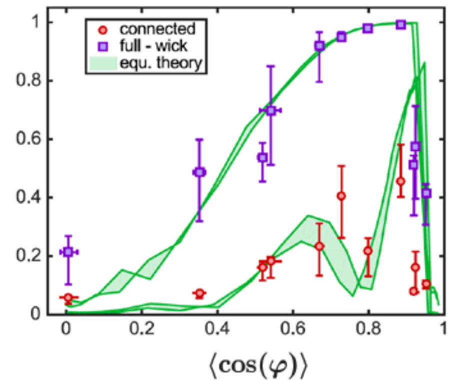
Higher order connected correlations

Schweigler et al. Nature **545**, 323 (2017)

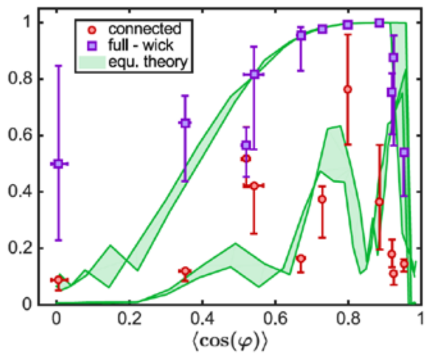
6th order



8th order

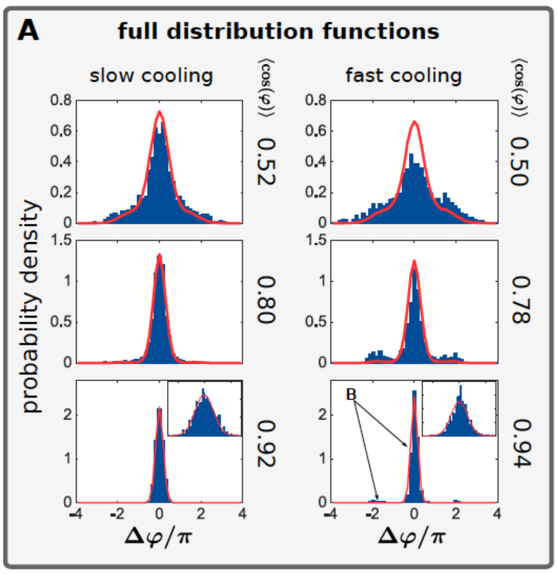


10th order

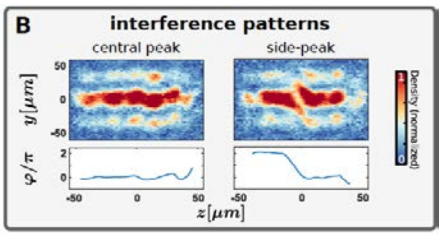


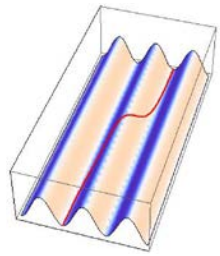
Quantifying factorization of correlation functions

Schweigler et al. Nature **545**, 323 (2017)



- the breakdown of factorization is evident in the **full distribution functions** of the phase by new peaks at multiples of 2π
- caused by the 2π ***periodic*** SG Hamiltonian -> 2π phase jumps, 'kinks' = SG solitons



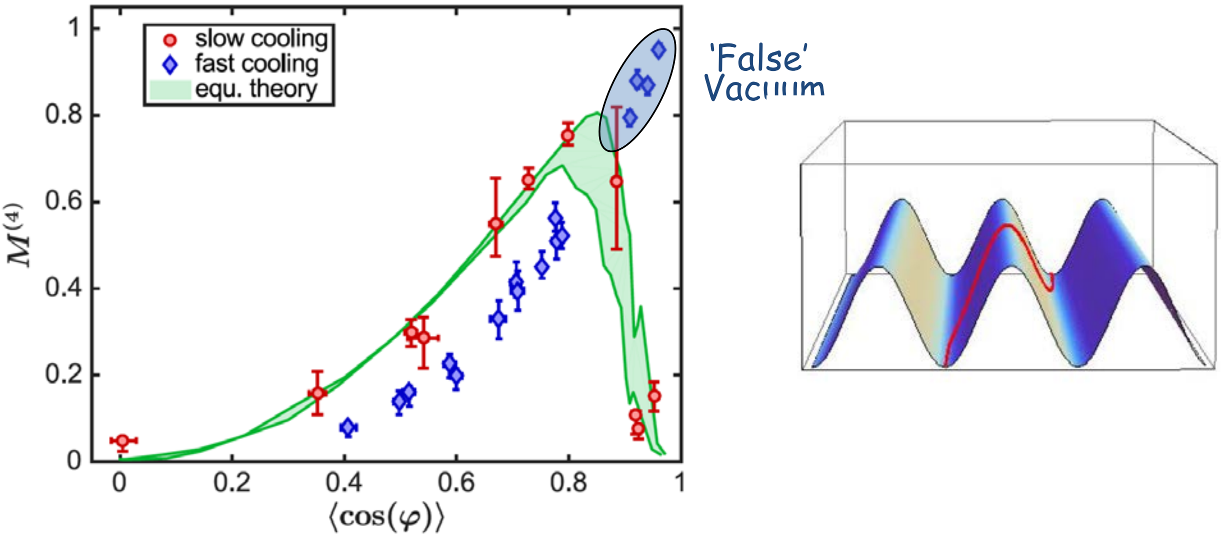


- ***SG Solitons*** are topological excitations
- Phase fluctuations around *topologically different Vaccua*

'False' Vacuum

Schweigler et al. Nature **545**, 323 (2017)

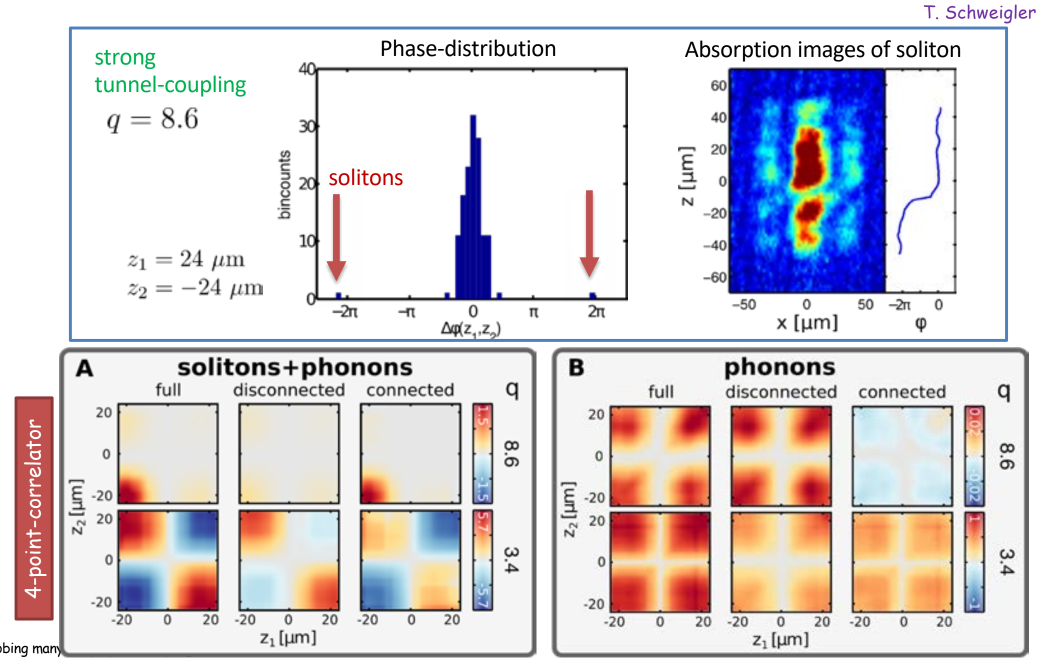
4th order correlations



Solitons <-> quasi particles

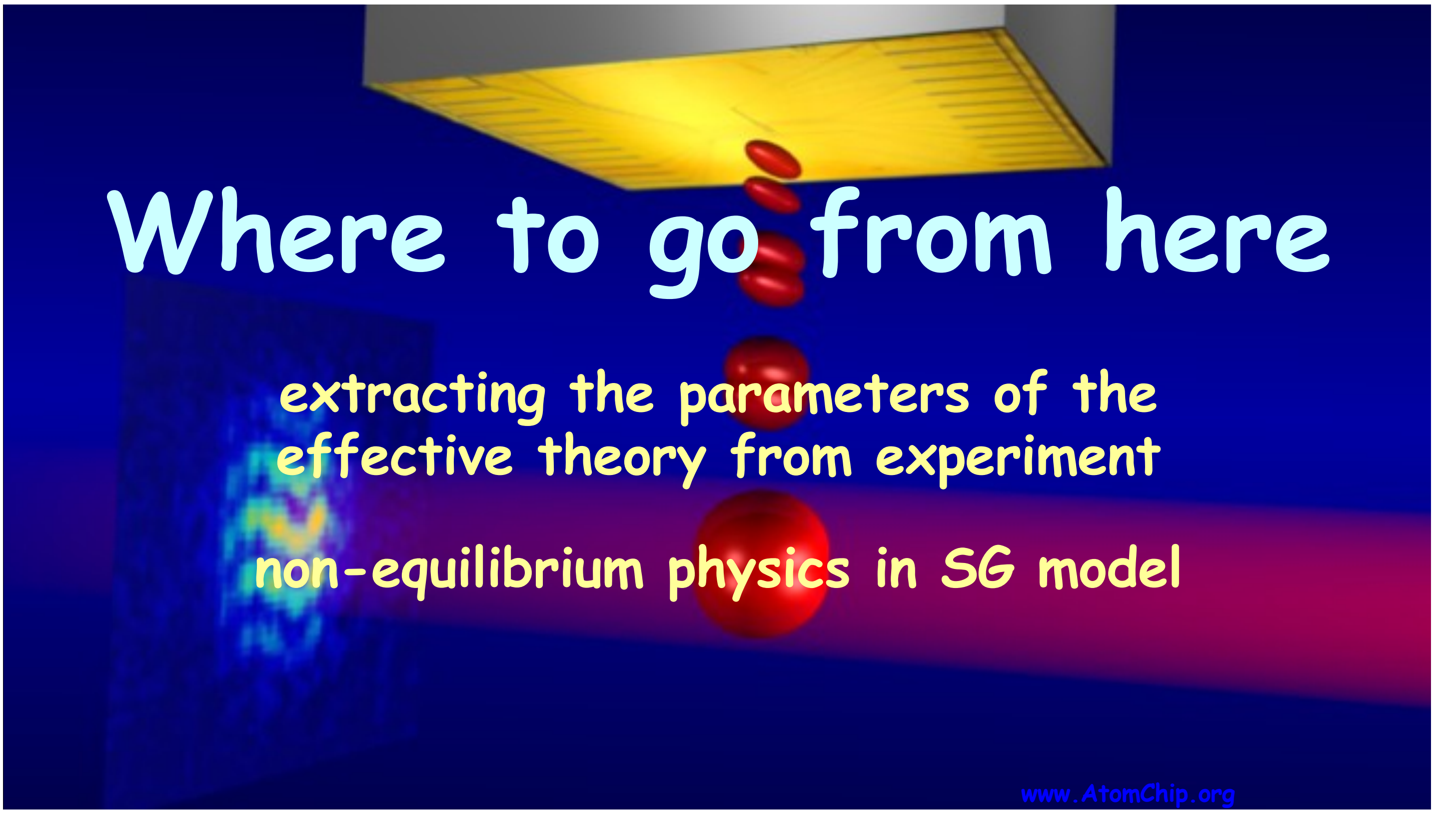
'false' vacuum states <-> excitations

T. Schweigler et al. arXiv:1505.03126



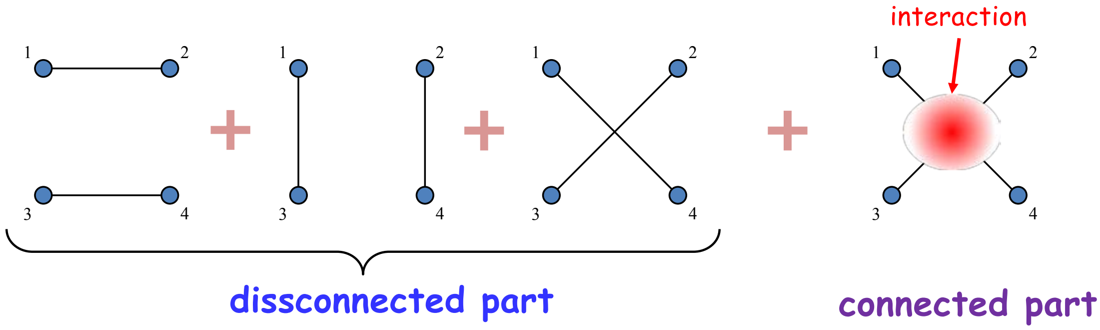
What have we learned

- **high order** (>10) **correlation** functions are accessible in experiment
- **full distribution functions** and the **connected part** of the higher order correlation functions contain genuine information about the quantum field theory
  - quasi particles
  - interaction of quasi particles
  - vacuum states
- gives insight in the **effective theories** describing the many body system
  - for our data 10^{th} order connected correlation is still significant
    -> necessary to take terms up to 5^{th} order into account (5-5 scattering)
  - -> what is needed on the theory side to describe data



4-point correlation function

$G^{(4)}(x_1, x_2, x_3, x_4) =$

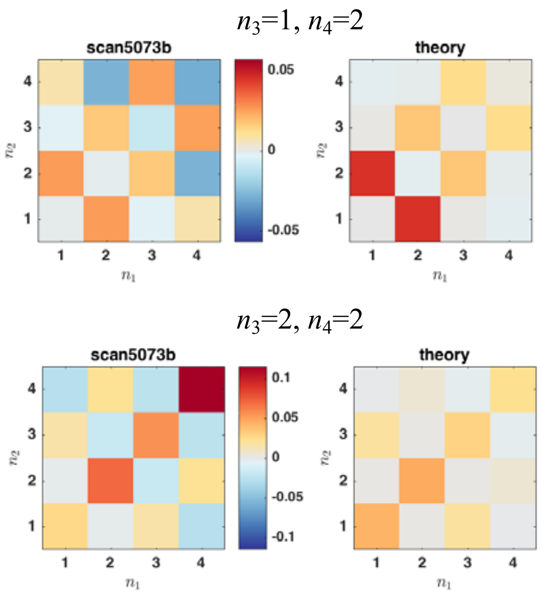


$$
\begin{aligned}
G^{(4)}(x_1, x_2, x_3, x_4) = \\
& G^{(2)}(x_1, x_3) G^{(2)}(x_2, x_4) + G^{(2)}(x_1, x_2) G^{(2)}(x_3, x_4) + G^{(2)}(x_1, x_4) G^{(2)}(x_2, x_3) \\
& + \int d^D y_1 \ldots d^D y_4 \, G^{(2)}(x_1, y_1) G^{(2)}(x_2, y_2) G^{(2)}(x_3, y_3) G^{(2)}(x_4, y_4) \Gamma^{(4)}(y_1, y_2, y_3, y_4)
\end{aligned}
$$

Extracting the Coupling Constants

T. Schweigler, S. Erne preliminary

- The measured connected correlators contain contributions from the propagators (the 'legs')
- To extract the information about the coupling constants in the scattering vertices one has to ‚amputate' the correlators
- Best done in momentum representation.
- In our finite system we have a discrete momentum spectrum (the modes of the system)
- Transform the correlators to the space of the modes

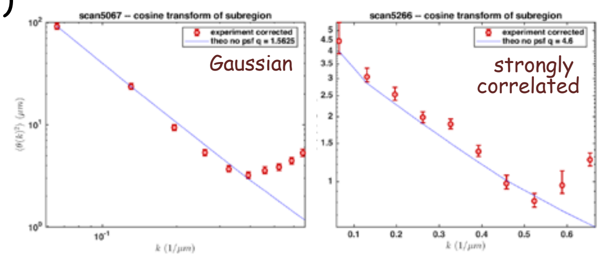


Extracting the Coupling Constants

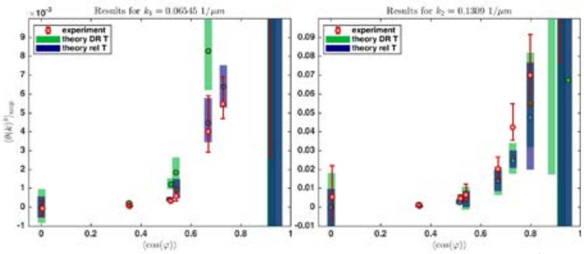
T. Schweigler, S. Erne preliminary

- The measured connected correlators contain contributions from the propagators (the 'legs')
- To extract the information about the coupling constants in the scattering vertices one has to ‚amputate' the correlators
- Best done in momentum representation.
- In our finite system we have a discrete momentum spectrum (the modes of the system)
- Transform the correlators to the space of the modes

Transformed 2-point correlator (propagator)



‚amputated' connected 4-point correlator



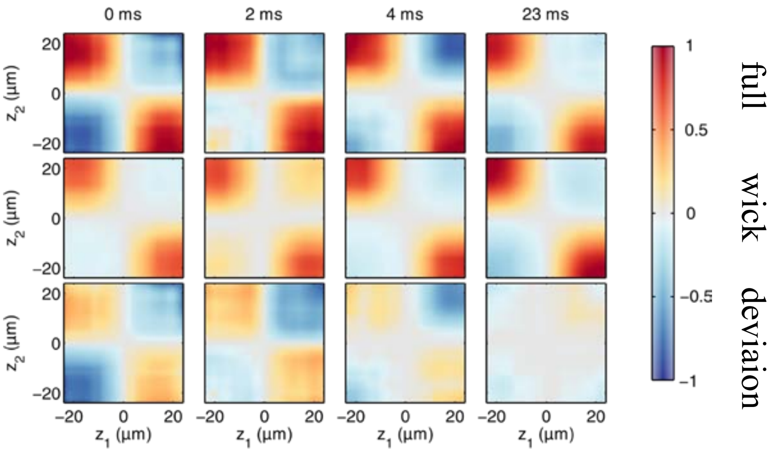
Quench from J>0 to J=0

preliminary

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

Initial state non-Gaussian, dynamics Gaussian

4^{th} order correlation function of phase

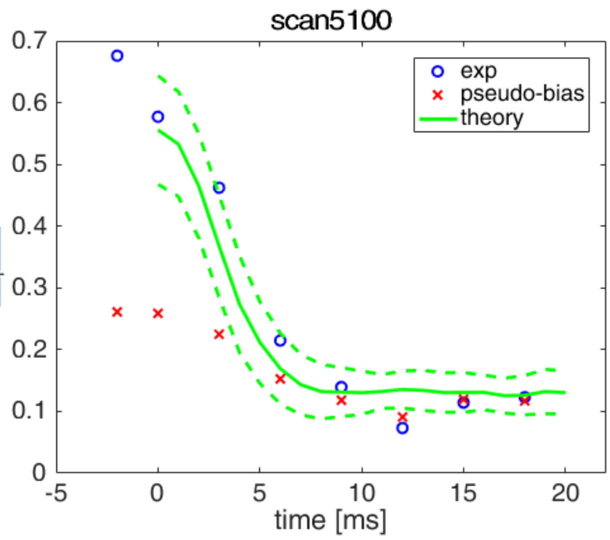


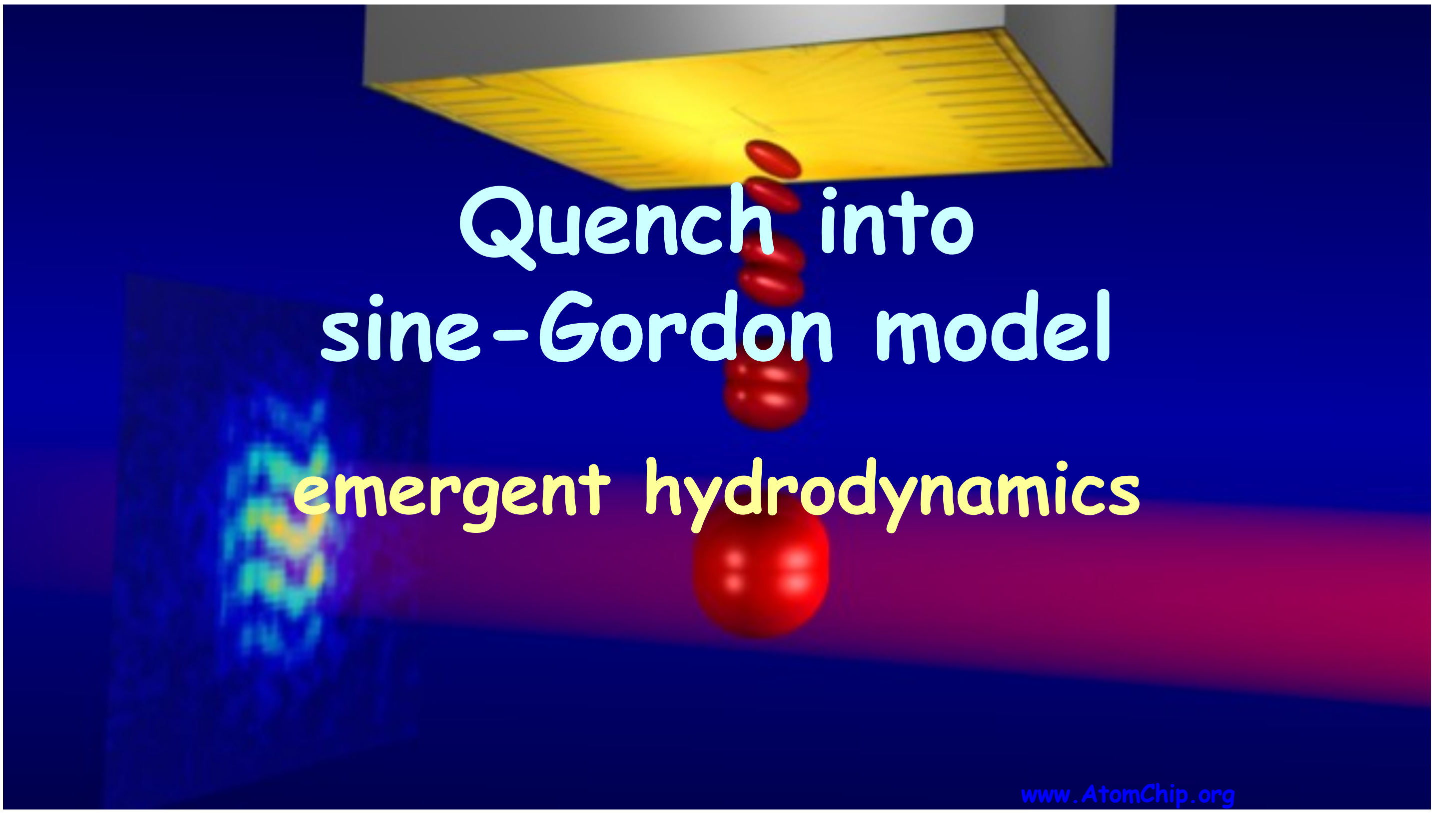
collaboration with Berges & Gasenzer groups, Heidelberg
Eisert group, Berlin

Integral measure

slowly cooled thermal equilibrium initial state

$$M^{(N)} = \frac{\sum_{\mathbf{z}} \left| G^{(N)}_{\mathrm{con}}(\mathbf{z},0) \right|}{\sum_{\mathbf{z}} \left| G^{(N)}(\mathbf{z},0) \right|}$$



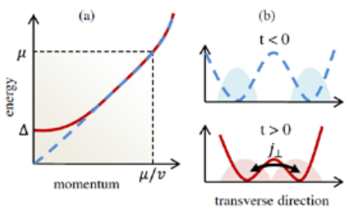


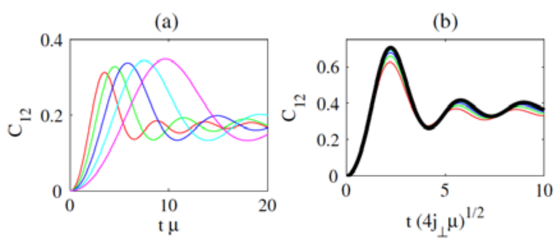
# Free -> Sine Gordon
J=0 -> J finite

PRL 110, 090404 (2013) PHYSICAL REVIEW LETTERS week ending 1 MARCH 2013

Universal Rephasing Dynamics after a Quantum Quench via Sudden Coupling of Two Initially Independent Condensates

Emanuele G. Dalla Torre,[1] Eugene Demler,[1] and Anatoli Polkovnikov[2]
[1]Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
[2]Department of Physics, Boston University, Boston, Massachusetts 02215, USA

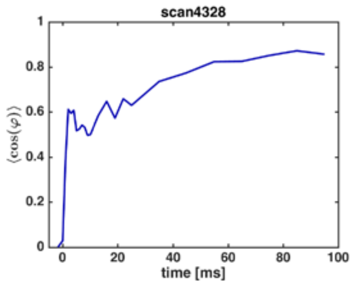




$$C_{12}(t) = \langle \psi_1^\dagger \psi_2 + \mathrm{H.c.}\rangle / 2N = \langle \cos(\sqrt{2}\phi)\rangle$$

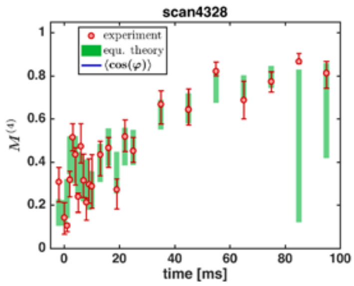
Free -> Sine Gordon
experiment

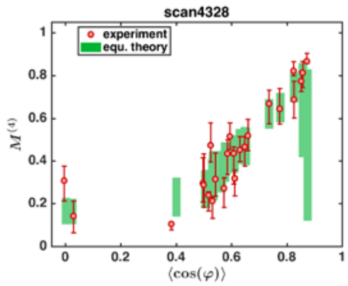
T. Schweigler



Comparison to *equilibrium* theory:

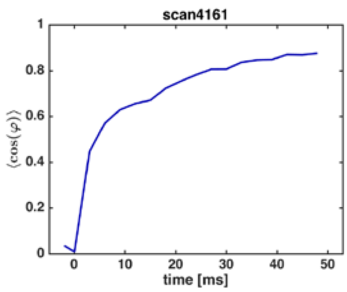
find effective, time − local parameters λ_T and q for every time step by fitting $\langle\cos(\varphi)\rangle$ and the second order correlation function $G^{(2)}$





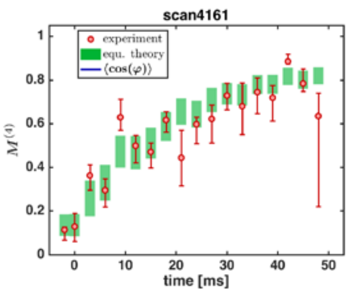
Free -> Sine Gordon
experiment

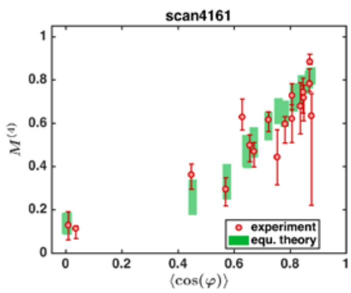
T. Schweigler



Comparison to *equilibrium* theory:

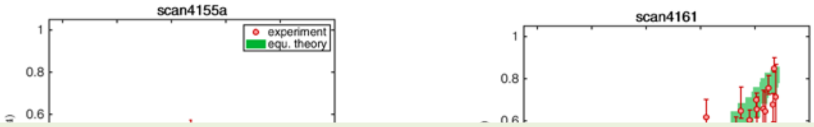
find effective, time − local parameters λ_T and q for every time step by fitting $\langle\cos(\varphi)\rangle$ and the second order correlation function $G^{(2)}$





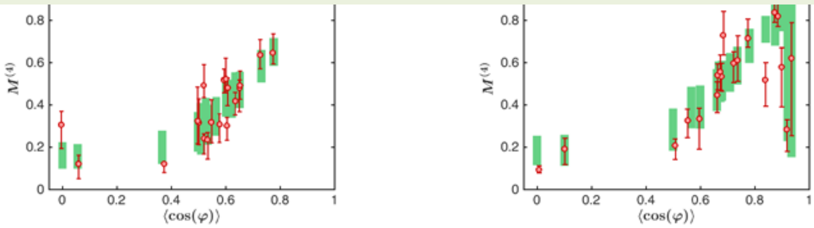
Free -> Sine Gordon
experiment

T. Schweigler



Non-equilibrium evolution from a free system into a (very) **strongly correlated** system proceeds along a path that looks 'time-local' in equilibrium

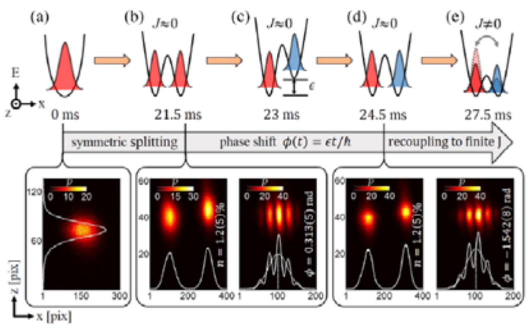
Emergent Hydro Dynamics



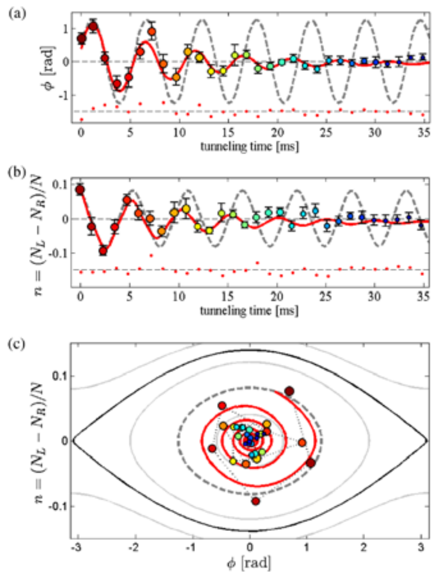
Phase locking in a 1d Josephson junction

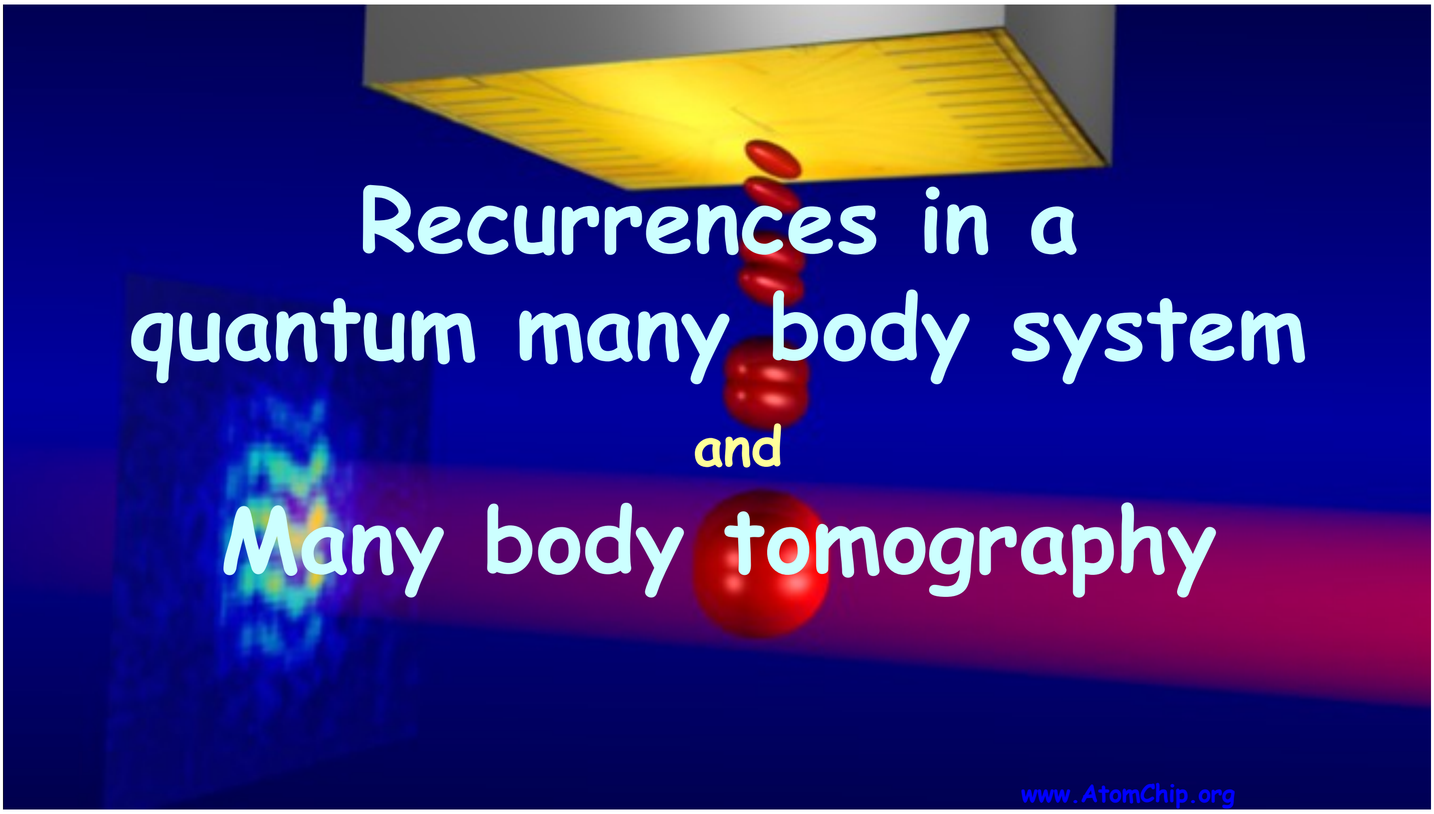
M. Pigneur et al. PRL **120**, 173601 (2018)

Simpler initial state:
Start with an imprinted phase



Phase locking much faster and much better then predicted by della Torre et al. PRL 2013

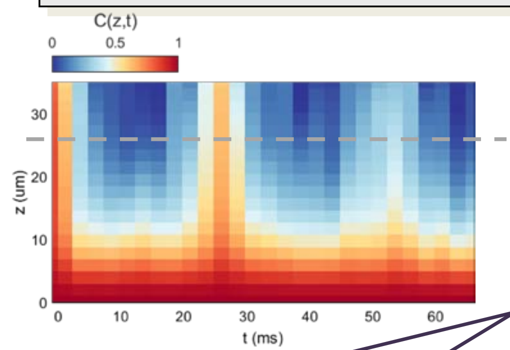




Recurrences in 1d superfluid

Phase correlation function

B. Rauer et al. Science **360**, 307 (2018)

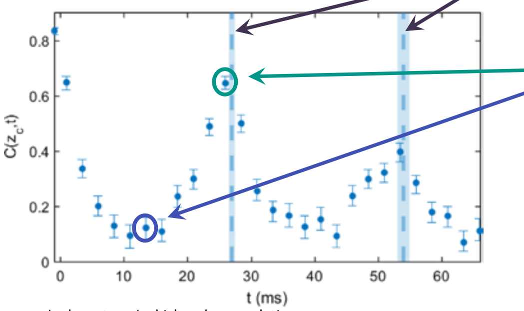


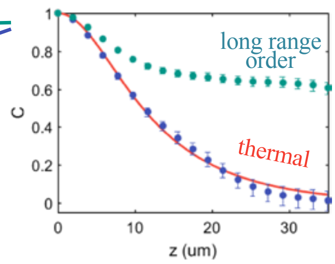
Phase correlation dynamics in a 50 μm box trap after decoupling:

$$C(\bar{z},t) = \langle cos(\varphi(z) - \varphi(z')) \rangle$$

$$\bar{z} = z - z'$$

expected revival times calculated from Luttinger-liquid simulations



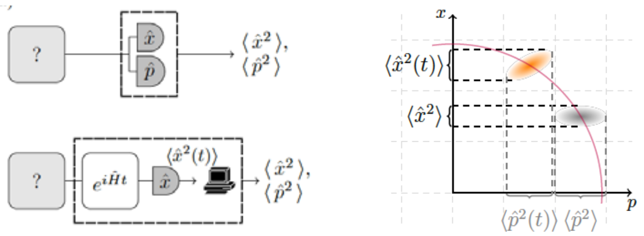


Many Body Tomography

M. Gulza et al. arXiv:1807.04567

In interference experiments we measure **phase** quadrature

Idea:
'free evolution' rotates the Wigner function of the modes in the low energy effective field theory description

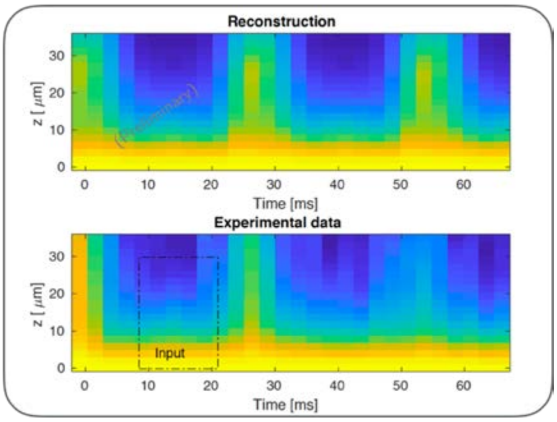


Repeated measurement is equivalent to a tomographic slicing

allows reconstruction of the density matrix

Will give excess to entanglement entropy etc …

Example:
data from recurrence experiment



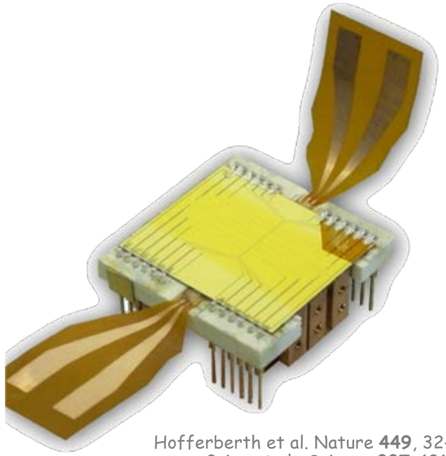
What have we learned

- **Higher order correlation functions** and the **full distribution functions**, especially the question if they factorize gives insight in the effective quantum field theories describing the many body system
- Verified Sine-Gordon model as emergent from the microscopic physics of two tunnel coupled super fluids
- Identified the topological excitations in the Sine-Gordon model.

Building and probing quantum field theories in the lab

- Observation of recurrences in coherences in long range order for a many body system of 5000 atoms
- Time evolution allows 'tomography'

Schweigler et al., Nature **545**, 323 (2017)
Rauer, et al., Science **360**, 307 (2018)
M. Gulza et al. arXiv:1807.04567



Hofferberth et al. Nature **449**, 324 (2007)
Gring et al., Science **337**, 1318 (2012)
Kuhnert et al., PRL **110**, 090405 (2013)
Smith et al., NJP **15**, 075011 (2013)
Langen et al., Nature Physics **9**, 460 (2013)
Berrada, et al., Nat. Comm **4**, 2077 (2013)
Geiger et al., NJP **16** 053034 (2014)
Van Frank, et al., Nat. Comm **5**, 4009 (2014)
Langen et al., Science **348**, 207 (2015)
Steffens, et al., Nature Comm. **6**, 7663 (2015)
Rauer, et al., PRL **116**, 030402(2016)

Atom Chip Experiment

S. Manz, T. Betz, R. Bücker, **T. Berrada**, S. vanFrank,
M. Pigeur, A. Perrin, T Schumm, JF Schaff, R. Wu,
M. Bonneau

M. Kuhnert, M. Gring, **B. Rauer**, **Th. Schweigler**
D. Smith, R. Geiger, **T. Langen**

Atom Chip Fabrication

D. Fischer, M. Trinker, M. Schamböc
S. Groth (HD), Israel Bar Jose

Theory Coll

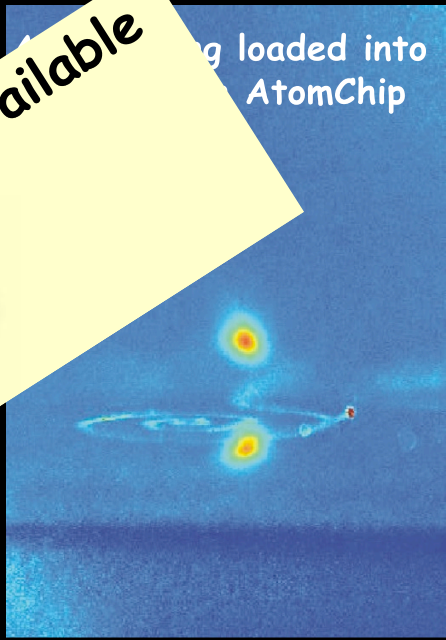
I. Mazets
J. Grond, U
E. Deml
T. Gasenzer, J. R
T.

AT: ... /ien

PhD and PostDoc position available

CoQuS ComplexQuantumSystems

www.atomchip.org
www.CoQuS.at



FWF

CoQuS ComplexQuantumSy

WITTGENSTEIN

erc

ISOQUANT SFB1225

www.AtomChip.org