Higher Order Correlations
and what we can learn about the solutions for many body problems from experiment

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Probing cold atom systems

Theory: Single-shot simulations of dynamic quantum many-body systems
Many Body Quantum Systems $\leftrightarrow$ Correlation Functions

- Solving a quantum many-body problem is equivalent to knowing all its correlation functions.
- Real world: Observer can only measure a finite number of correlations → describing the propagation and scattering of excitations.
- To (approximately) 'solve' the problem one need to find degrees of freedom where only few (low order) correlation functions are relevant.
- If one finds the degrees of freedom (basis) where the correlation functions factorize, this is equivalent to diagonalization of the many body Hamiltonian.
- Experiment / its 'read-out' always has a finite accuracy and errors. need clear signatures what can we learn from an experiment, and what is a fair comparison to the models → example: probing for quantum supremacy

Outline

Correlation functions
- fields $\leftrightarrow$ phase $\leftrightarrow$ excitations

Probing with high order correlations
- System studied: tunnel coupled 1d superfluids
- Quantifying factorization
- Verifying Sine-Gordon model
- Identifying 'false' vacuum

Outlook
- Non equilibrium physics in SG model
- Recurrences in non-equilibrium evolution
Correlation Functions

when do they factorize?

arXiv:1505.03126

The $N$th order Correlation function

$$G^{(N)}(z) = \langle O(z_1) O(z_2) \ldots O(z_N) \rangle$$

Characterizes the propagation and the interactions of the degrees of freedom connected to the operators $O(z_i)$

It can be decomposed:

$$G^{(N)}(z) = G^{(N)}_{\text{dis}}(z) + G^{(N)}_{\text{con}}(z)$$

- The disconnected part $G^{(N)}_{\text{dis}}$ is fully determined through lower order correlations
- The connected part $G^{(N)}_{\text{con}}$ contains genuine new information about the system at order $N$
4-point correlation function

\[ G^{(4)}(x_1, x_2, x_3, x_4) = \]

\[ = G^{(2)}(x_1, x_3)G^{(2)}(x_2, x_4) + G^{(2)}(x_1, x_2)G^{(2)}(x_3, x_4) + G^{(2)}(x_1, x_4)G^{(2)}(x_2, x_3) \]

+ \int d^D y_1 \ldots d^D y_4 \; G^{(2)}(x_1, y_1)G^{(2)}(x_2, y_2)G^{(2)}(x_3, y_3)G^{(2)}(x_4, y_4) \; \Gamma^{(4)}(y_1, y_2, y_3, y_4)

What can we learn from connected correlations

Connected diagram can not be decomposed in lower order diagrams

4th order \( \sum \) 2 body interaction

6th order \( \sum \) 3 body interaction

8th order \( \sum \) 4 body interaction

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Correlation function of the phase

\[ G^{(N)}(z, z') = \langle [\varphi(z_1) - \varphi(z'_1)] \ldots [\varphi(z_N) - \varphi(z'_N)] \rangle \]

Calculate the connected part of the correlation:

\[ G^{(N)}_{\text{con}}(z, z') = \sum_\pi \left[ (|\pi| - 1)! \prod_{B \in \pi} \left( \prod_{i \in B} \langle \varphi(z_i) - \varphi(z'_i) \rangle \right) \right] \]

Sum runs over all possible partitions \( \pi \), the first product over all blocks \( B \) of the partition, the second over all elements of the block.

The number of partitions to consider grows rapidly with \( N \).

For \( N=10 \) there are already \( >10^5 \) terms to consider !!!!

Decomposition into 2nd order correlations (Wick decomposition)

\[ G^{(N)}_{\text{wick}}(z, z') = \sum_{\pi_2} \left[ \prod_{B \in \pi_2} \left( \langle [\varphi(z_{B_1}) - \varphi(z'_{B_1})][\varphi(z_{B_2}) - \varphi(z'_{B_2})] \rangle \right) \right] \]

- Density phase representation
- Expanding the Hamiltonian in density fluctuations $\delta \rho_j$ and phase gradients $\partial \theta_j$ up to second order and neglecting mixed terms separates $H$ in symmetric and antisymmetric degrees of freedom
- Neglecting terms $|\delta \rho_j/n_0| \ll 1$

One arrives at **Quantum Sine-Gordon model**:

$$\hat{H}_{SG} = \int dz \left[ \frac{\hbar^2}{4m} (\partial_z \phi)^2 + g \delta \rho^2 \right] - \int dz 2J n_{1D} [1 - \cos \phi]$$

- Phase coherence length
  $$\lambda_T = \frac{\hbar^2 n_{1D}}{m k_B T}$$
- Phase (spin) healing length
  $$\lambda_J = \sqrt{\frac{\hbar}{4mJ}}$$
- Characteristic parameters
  $$q = \lambda_T/\lambda_J$$

**Sine Gordon Model**

**Sine Gordon <=> Massive Thirring Model**

**Sine Gordon <=> Coulomb Gas**

**Sine Gordon <=> XY**
Combine the robustness of nano-fabrication and the quantum tools of atomic physics and quantum optics to build a toolbox for quantum experiments

- 1d elongated traps
- Easy to create a BEC
- Very stable and reproducible laboratory for quantum experiments
- Fast operation
- Single atom detection with unit efficiency
- Well controlled splitting and interference
- Experiment optimized by genetic algorithm

3000-10000 atoms
\[ T = 20-100 \text{ nK} \]
\[ \omega_R \sim 2\pi \times 2 - 3 \text{ kHz} \]
\[ \omega_L \sim 2\pi \times 5 - 10 \text{ Hz} \]
\[ k_b T \sim 0.1 - 0.7 \hbar \omega_L \]
\[ k_b T \sim 0.1 - 1 \mu \]

**Experiment**

*two coupled super fluids*

Exp: T. Schweiger, et al. (Vienna)
Theory: S. Erne, V. Kasper et al. (HD)
Schweigler et al. arXiv:1505.03126

**AtomChip**

Integrated Circuits for ultra-cold Quantum Matter

Folman et al. PRL 84, 4749 (2000)
Experimental procedure

1D gas of $^{87}\text{Rb}$ atoms

$\psi(z) = e^{i\phi(z)} \sqrt{\rho_0(z) + \delta n(z)}$

adjutable tunnelling $J$

Tunnel coupling lead to phase locking characterized by $\langle \cos(\phi) \rangle$

time of flight

$\Rightarrow$ phase difference between condensates $\varphi(z) = \theta_1(z) - \theta_2(z)$

Tunnel coupling lead to phase locking characterized by $\langle \cos(\phi) \rangle$

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Experimental procedure

U

`Imaging`

evaluation

extracted values of measurements

Correlation functions

fields $\leftrightarrow$ phase

experiments in a trap

$\Rightarrow$ non translation invariant correlation functions

$C(z_1, z_2) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle}$

with

$\Psi(z) = e^{i\theta(z)} \sqrt{\rho_0(z) + \delta n(z)}$

$\varphi(z) = \theta_1(z) - \theta_2(z)$

neglecting $\delta n(z)$

$C(z_1, z_2) \approx \langle \exp[i \varphi(z_1) - i \varphi(z_2)] \rangle$

4th order:

$C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_2) \Psi_3^\dagger(z_3) \Psi_4^\dagger(z_4) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle \langle |\Psi_3(z_3)|^2 \rangle \langle |\Psi_4(z_4)|^2 \rangle}$

$C(z_1, z_2, z_3, z_4) \approx \langle \exp[i \varphi(z_1) - i \varphi(z_2) + i \varphi(z_3) - i \varphi(z_4)] \rangle$
Correlation functions

in experiment we measure the phase $\varphi(z)$ directly

$\rightarrow$ look at phase correlators

$C^{(2)}(z_1, z_2) = \langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle = \langle [\Delta \varphi(z_1, z_2)]^2 \rangle$

with $\Delta \varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$

Note: $\Delta \varphi$ is NOT restricted to $2\pi$

using

$\varphi(z) = \frac{1}{\sqrt{E}} \sum_{k \neq 0} (-i) \sqrt{\frac{\pi}{|k|}} \langle b^+_k - b_k \rangle e^{ikz}$

$\langle |\varphi(z_1) - \varphi(z_2)|^2 \rangle = \sum_{k_1, k_2} \frac{\pi}{K} \sqrt{|k_1| |k_2|} \langle b^+_k - b_k \rangle e^{ik_1z_1 + ik_2z_2} + \ldots$

$\rightarrow$ phase correlators are related to the quasi particles

4th order

$C^{(4)}(z_1, z_2, z_3, z_4) = \langle [\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2 \rangle$

$\propto b^+_k b^+_l b^-_{-k_3} b^-_{-k_4} + \ldots$

$\rightarrow$ quasi particle scattering

Correlation functions for the fields:

$C(z_1, z_2) = \frac{\langle \Psi_1^+(z_1) \Psi_1^+(z_2) \Psi_2(z_2) \Psi_2(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle}$

$C(z_1, z_2) \approx \langle \exp[i \varphi(z_1) - i \varphi(z_2)] \rangle$

$C(z_1, z_2)$ contains all orders of connected parts

$C(z_1, z_2) = \exp \left[ \sum_{k=1}^{\infty} (-1)^k \frac{((\Delta \varphi)^{2k})_{\text{c}}}{(2k)!} \right]$

for Gaussian fluctuations

$C(z_1, z_2) = \exp \left[ -\frac{1}{2} ((\Delta \varphi))^2 \right]$
4th order correlations
Connected and disconnected part

to study factorization of correlation functions we look at:

\[ G^{(2)}(z_1, z_2) = \langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle \]

\[ G^{(4)}(z_1, z_2, z_3, z_4) = \langle [\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2 \rangle \]

\[ \Delta \varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2) \]

\[ \Delta \varphi \text{ is NOT restricted to } [-\pi, \pi) \]

Connected/Disconnected part

\[ G^{(N)}(z) = G_{\text{con}}^{(N)}(z) + G_{\text{dis}}^{(N)}(z) \]

Characterizing Connected Correlations

Integrated measure

\[ M^{(N)} = \frac{\sum_z |G_{\text{con}}^{(N)}(z, 0)|}{\sum_z |G^{(N)}(z, 0)|} \]

Compared to predictions for a thermal equilibrium state of the sine-Gordon model.
Higher order connected correlations

6th order

8th order

10th order

Quantifying factorization of correlation functions

- the breakdown of factorization is evident in the full distribution functions of the phase by new peaks at multiples of $2\pi$
- caused by the $2\pi$ periodic SG Hamiltonian $\rightarrow 2\pi$ phase jumps, ‘kinks’ = SG solitons

- SG Solitons are topological excitations
- Phase fluctuations around topologically different Vacua
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**‘False’ Vacuum**

4th order correlations


Solitons <-> quasi particles

‘false’ vacuum states <-> excitations

strong tunnel-coupling

$\phi = 8.6$

$z_1 = 24 \mu m$

$z_2 = -24 \mu m$


Absorption images of soliton
What have we learned

- high order (>10) correlation functions are accessible in experiment

- full distribution functions and the connected part of the higher order correlation functions contain genuine information about the quantum field theory
  - quasi particles
  - interaction of quasi particles
  - vacuum states

- gives insight in the effective theories describing the many body system
  - for our data 10th order connected correlation is still significant
    -> necessary to take terms up to 5th order into account (5-5 scattering)
  - -> what is needed on the theory side to describe data

Where to go from here

extracting the parameters of the effective theory from experiment
non-equilibrium physics in SG model
4-point correlation function

\[ G^{(4)}(x_1, x_2, x_3, x_4) = G^{(2)}(x_1, x_3)G^{(2)}(x_2, x_4) + G^{(2)}(x_1, x_2)G^{(2)}(x_3, x_4) + G^{(2)}(x_1, x_4)G^{(2)}(x_2, x_3) + \int d^Dy_1 \ldots d^Dy_4 G^{(2)}(x_1, y_1)G^{(2)}(x_2, y_2)G^{(2)}(x_3, y_3)G^{(2)}(x_4, y_4) \Gamma^{(4)}(y_1, y_2, y_3, y_4) \]

disconnected part

connected part

Extracting the Coupling Constants

- The measured connected correlators contain contributions from the propagators (the 'legs')
- To extract the information about the coupling constants in the scattering vertices one has to 'amputate' the correlators
- Best done in momentum representation.
- In our finite system we have a discrete momentum spectrum (the modes of the system)
- Transform the correlators to the space of the modes
Extracting the Coupling Constants

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Quench from SG to free system

Gaussification of correlations
Quench from $J>0$ to $J=0$

Initial state non-Gaussian, dynamics Gaussian

4th order correlation function of phase

Integral measure

slowly cooled thermal equilibrium initial state
Quench into sine-Gordon model
emergent hydrodynamics
Comparison to *equilibrium* theory:

find effective, time- local parameters $\lambda_T$ and $q$ for every time step by fitting $\langle \cos(\varphi) \rangle$ and the second order correlation function $G^{(2)}$.
Non-equilibrium evolution from a free system into a (very) strongly correlated system proceeds along a path that looks ‘time-local’ in equilibrium

Emergent Hydro Dynamics

Phase locking much faster and much better then predicted by della Torre et al. PRL 2013
Recurrences in a quantum many body system and
Many body tomography

Recurrences in 1d superfluid
Phase correlation function

Phase correlation dynamics in a 50 μm box trap after decoupling:

\[ C(\bar{z}, t) = \langle \cos(\varphi(z) - \varphi(z')) \rangle \]

\[ \bar{z} = z - z' \]

expected revival times calculated from Luttinger-liquid simulations
In interference experiments we measure phase quadrature.

Idea: 'free evolution' rotates the Wigner function of the modes in the low energy effective field theory description.

Repeated measurement is equivalent to a tomographic slicing, allowing reconstruction of the density matrix. This will give excess to entanglement entropy, etc...

What have we learned:

- Higher order correlation functions and the full distribution functions, especially the question if they factorize gives insight into the effective quantum field theories describing the many body system.

- Verified Sine-Gordon model as emergent from the microscopic physics of two tunnel coupled super fluids.

- Identified the topological excitations in the Sine-Gordon model.

Building and probing quantum field theories in the lab:

- Observation of recurrences in coherences in long range order for a many body system of 5000 atoms.

- Time evolution allows 'tomography'.

Example: data from recurrence experiment.

Hofferberth et al., Nature 449, 324 (2007)
Gring et al., Science 337, 1318 (2012)
Kuhnert et al., Nat. Phys. 10, 036405 (2013)
Smith et al., Nature 464, 460 (2013)
Geiger et al., NJP 16, 053034 (2014)
Langen et al., Science 348, 204 (2015)
Langen et al., Science 348, 207 (2015)
Schweigler et al., Nature 545, 323 (2017)
M. Gulza et al. arXiv:1807.04567

M. Gulza et al. arXiv:1807.04567
Atom Chip Experiment

Atom Chip Fabrication
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EU: SIQS, QIBEC, AQuS
AT: FWF, CoQuS, Wittgenstein, Stadt Wien
ERC AdG: QuantumRelax

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