Thermal behaviour and entanglement in Pb-Pb and pp collisions

C.Pajares

Dept Física de Partículas and IGFAE University of Santiago de Compostela

Quantum entanglement at hadron collider

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---Introduction

---Two scales, Tth and Th related each other (description of pp and Pb-Pb pt distrbutions at dfferent multiplicities)

---Dependence of n with multiplicity

--Gamma distribution for the temperature as solution of Fokker-Plank associated to Langevin equation for a stochastic white noise

--Multiplicity distribution associated to hard events (gamma distribution)

--Entanglement entropy and its evolution with energy or centrality

-- Clustering of color sources

-- Conclusions

X.Feal, C.P and R.Vazquez arXiv:1805.12444 and to appear

O.K.Baker,D.E.Kharzeev arXiv:1712.04558

D.E.Kharzeev,E.Levin Phys Rev D95 (2017) 114008

$$|\Psi_{HS}\rangle = \sum_n \alpha_n |\Psi_n^H\rangle |\Psi_n^S\rangle$$

$$\rho_H = \mathrm{Tr}_S\, \rho_{SH} = \sum_n \langle\Psi_n^S|\Psi_{HS}\rangle\langle\Psi_{HS}|\Psi_n^S\rangle = \sum_n |\alpha_n|^2 |\Psi_n^H\rangle\langle\Psi_n^H|.$$

$$S \equiv -\sum_n p_n \log p_n.$$

$$\frac{1}{N_{\rm ev}}\frac{1}{2\pi p_t}\frac{d^2N_{\rm ev}}{d\eta dp_t} = A_{\rm th}\exp\left(-m_t/T_{\rm th}\right)$$

$$\frac{1}{N_{\rm ev}}\frac{1}{2\pi p_t}\frac{d^2N_{\rm ev}}{d\eta dp_t} = A_{\rm h}\frac{1}{\left(1+\frac{m_t^2}{nT_{\rm h}^2}\right)^n}$$

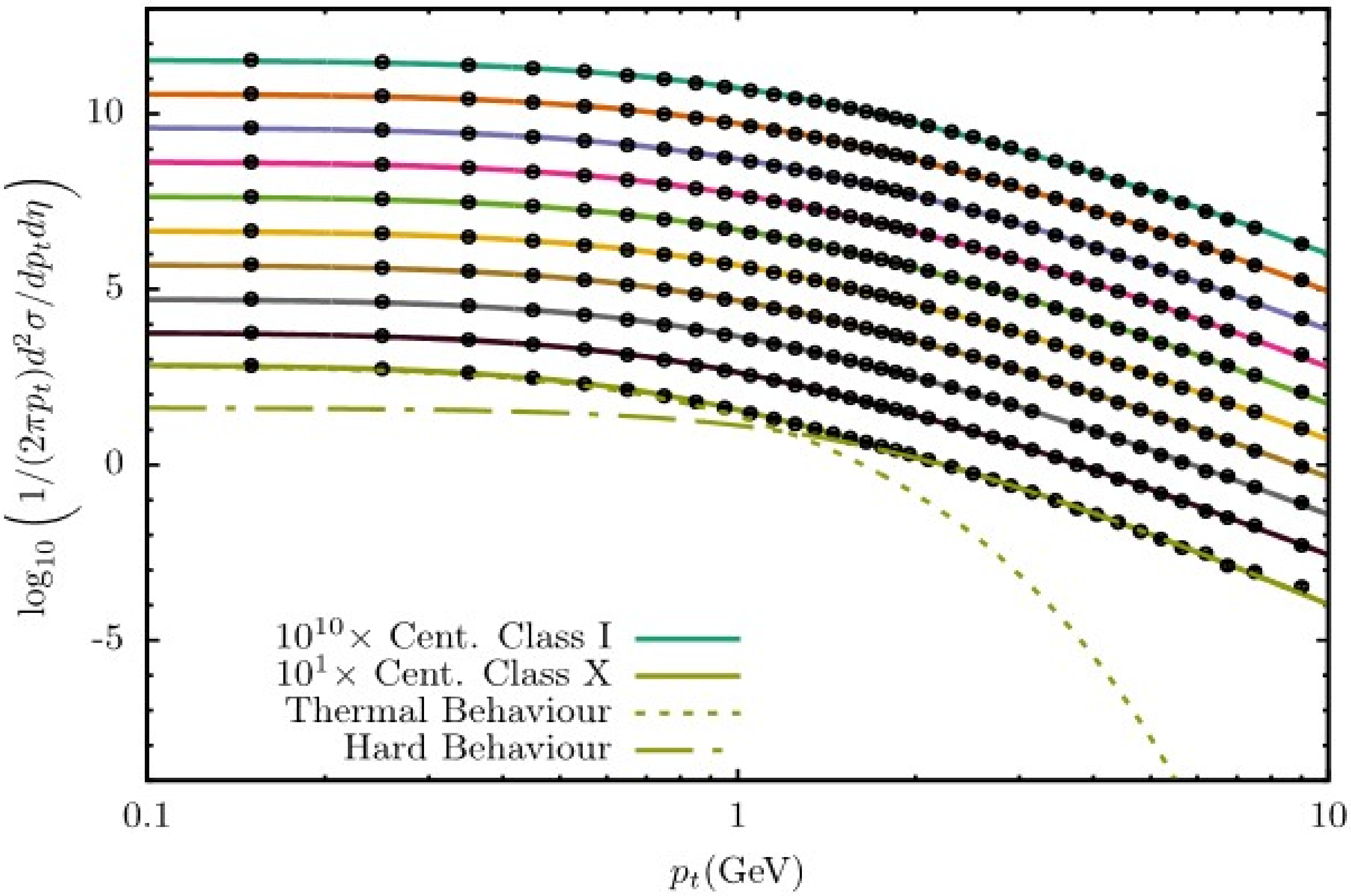
from the extrapolation of the relation

$$T_{\rm th} = 0.098\left(\sqrt{\frac{s}{s_0}}\right)^{0.06}\ ({\rm GeV}),$$

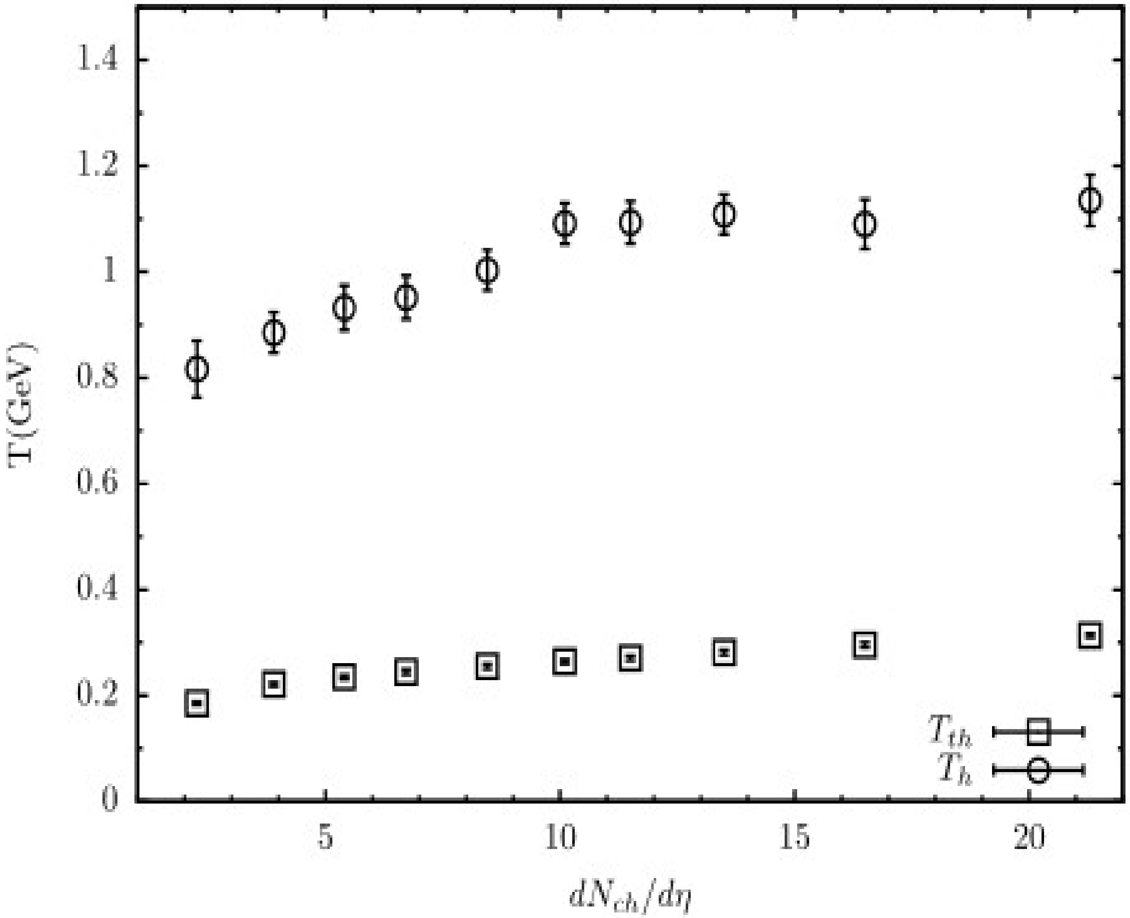
obtained at lower energies. Similarly the hard scale T_h is given by the relation

$$T_{\rm h} = 0.409\left(\sqrt{\frac{s}{s_0}}\right)^{0.06}\ ({\rm GeV}).$$

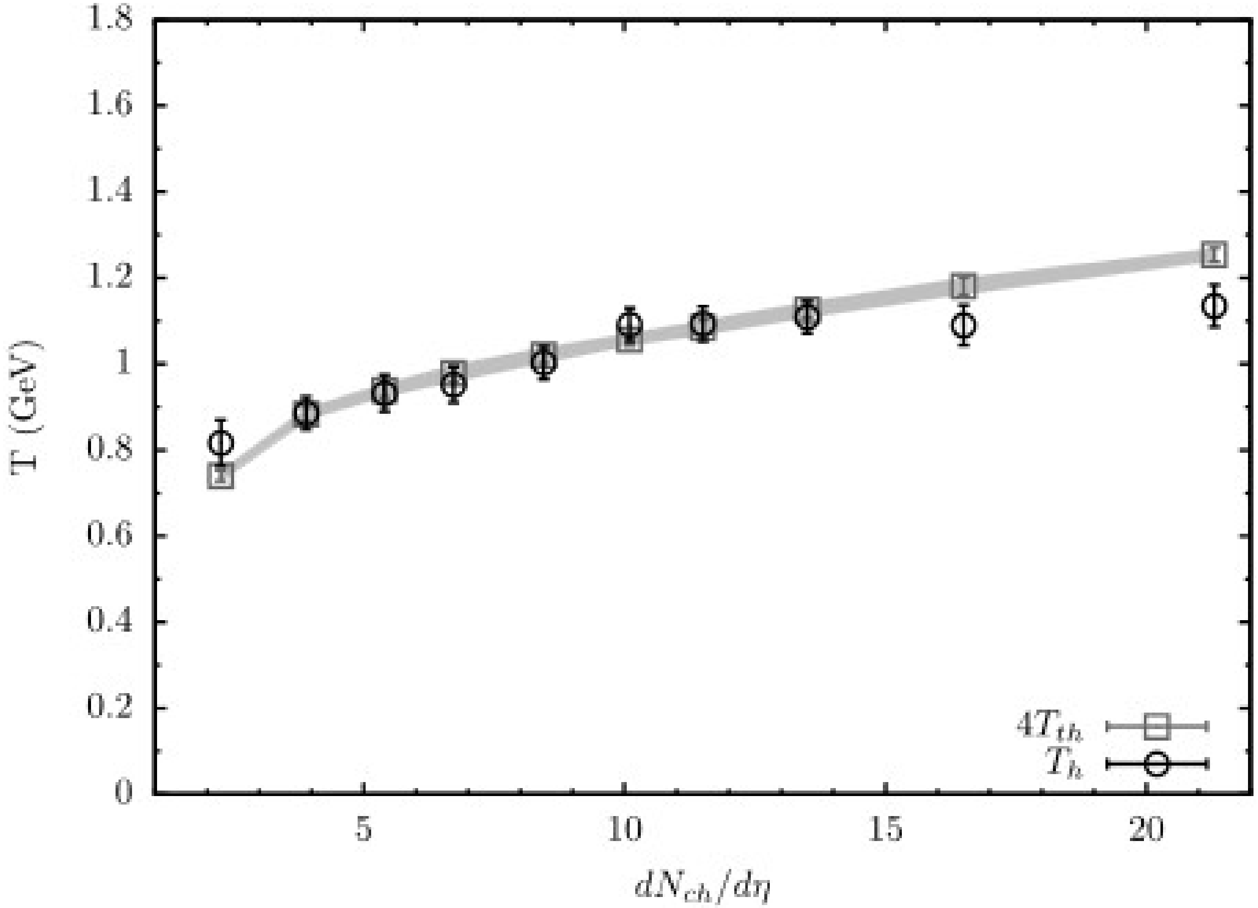
At $\sqrt{s} = 13$ TeV, the values found for the hard scale are $T_{\rm h} = 0.72$GeV and $n = 3.1$.



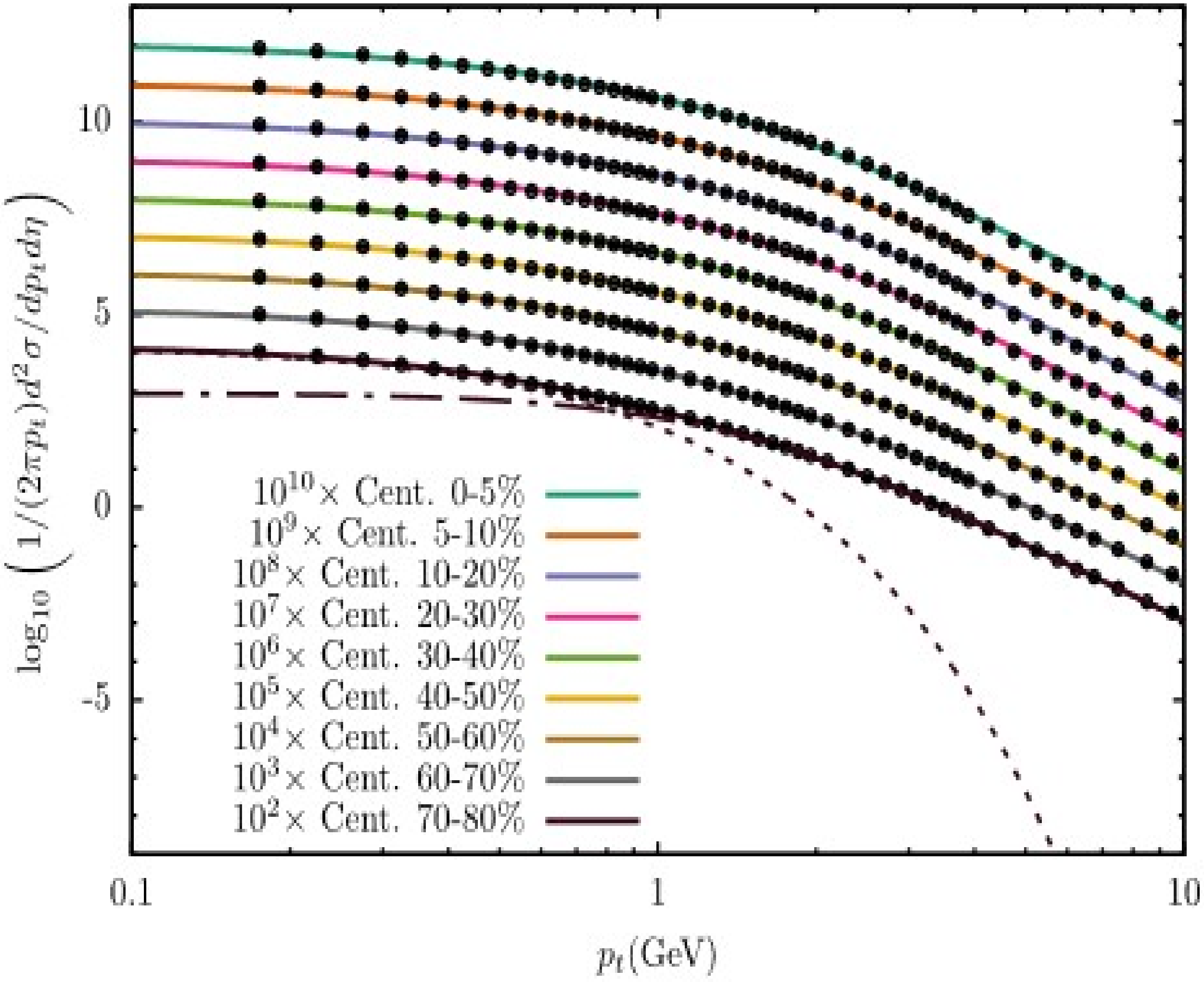
$$\frac{T_{\rm h}}{T_{\rm th}} \approx 4.2$$



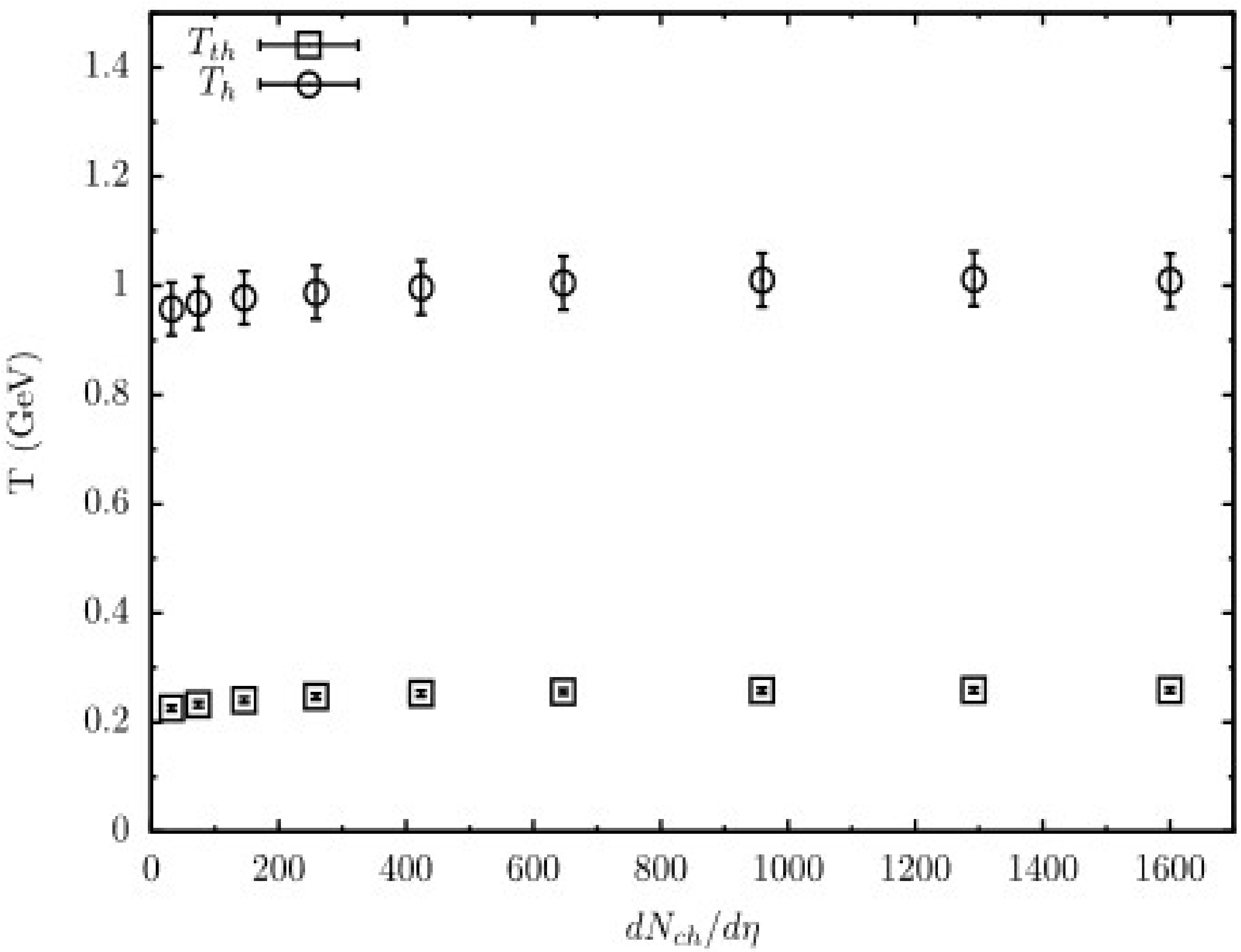
: T_{th} and T_{h} as a function of centrality for K_S^0 production in p-p collisions at $\sqrt{s_{NN}}$ =7



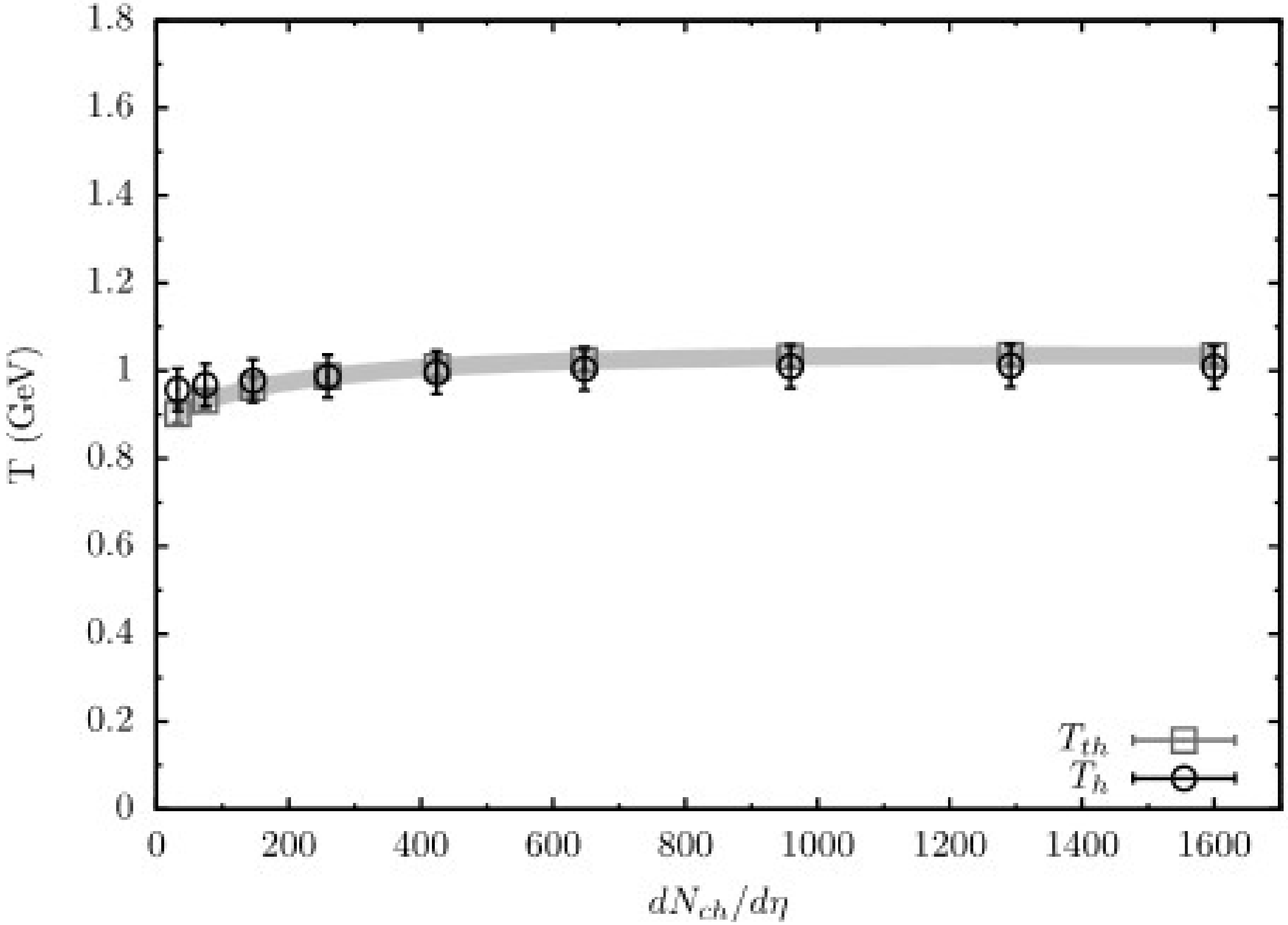
T_{h} and $4T_{\mathrm{th}}$ as a function of centrality for K_S^0 production in p-p collisions at $\sqrt{s_{NN}}$ =7



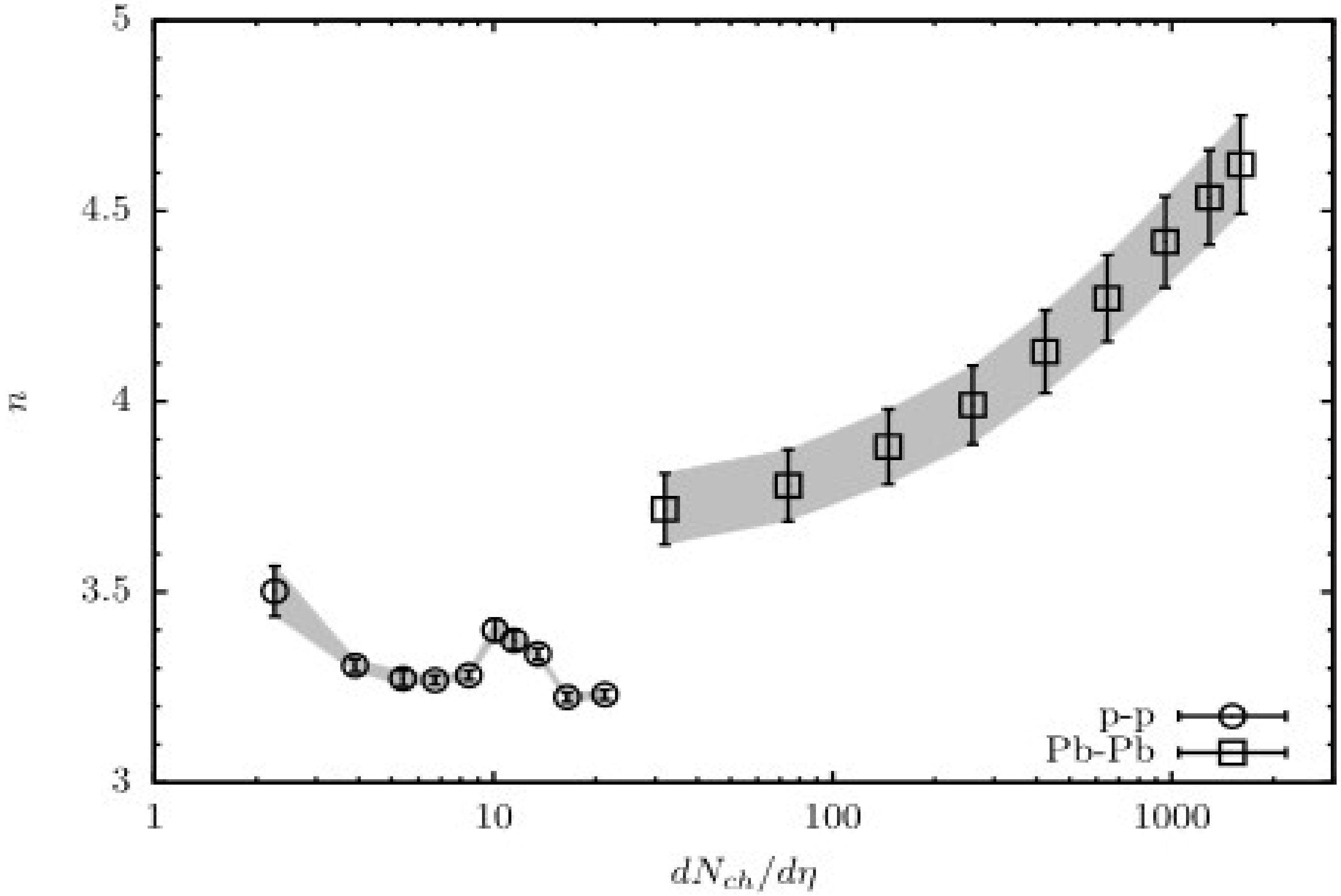
Normalized differential charged particle production in Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76



Variation of T_{th} and T_{h} with centrality for charged particle production in Pb-Pb collisions



T_{h} and $4T_{\mathrm{th}}$ as a function of centrality, for charged particle production in Pb-Pb collisions



5
4.5
4
3.5
3
n
1
10
100
1000
$dN_{ch}/d\eta$
p-p
Pb-Pb

Experimental facts

--Two scales Tth an Th related each other (pp and PbPb data at different centralities)

-- n parameter decreases with multiplicity in pp and increases in PbPb

Thermal behaviour and Langevin equation

Thermal behaviour--gaussian distribution in momenta--stationary solution of Fokker-Planck associated to Langevin equation

$$\frac{d\sigma}{dt} + \left(\frac{1}{\tau} + \xi(t)\right)\sigma = \phi$$

$$\langle \sigma(t) \rangle = \sigma(0)\exp(-t/\tau), \qquad \langle \sigma^2(\infty) \rangle = \frac{\tau D}{2}$$

$$\langle \zeta(t)\zeta(t+\Delta t) \rangle = 2D\delta(\Delta t), \qquad \langle \zeta(t) \rangle = 0$$

$$\frac{\partial f(\sigma)}{\partial t} = -\frac{\partial}{\partial \sigma} K_1(\sigma) f(\sigma) + \frac{1}{2} \frac{\partial^2}{\partial \sigma^2} K_2(\sigma) f(\sigma)$$

$$K_1(\sigma) = \phi - 2\frac{\sigma}{\tau} + D\sigma, \qquad K_2(\sigma) = 2D\sigma^2$$

$$f(\sigma) = \frac{1}{\Gamma(n)} \mu \left(\frac{\mu}{\sigma}\right)^{n-1} \exp\left(-\frac{\mu}{\sigma}\right)$$

$$\mu = \frac{\phi}{D} \qquad n = \frac{1}{\tau D}$$

$$\sigma = T_h^2 = 1/x \qquad T_h^2 = \frac{1}{\tau \phi}$$

Conditioned probability for hard collisions

$$N(n) \equiv \sum_{i=0}^{n} \binom{n}{i} \alpha_c^i (1-\alpha_c)^{n-i} N(n)$$

$$N(n) = \alpha_c n N(n) + (1-\alpha_c n) N(n)$$

$$N_c(n) = \alpha_c n N(n)$$

$$p_c(n) = \frac{\alpha_c n N(n)}{\alpha_c \langle n \rangle N} = \frac{n}{\langle n \rangle} p(n)$$

$$p(n) \to \frac{n}{\langle n \rangle} p(n) \to \frac{n^2}{\langle n^2 \rangle} p(n) \to \cdots \frac{n^k}{\langle n^k \rangle} p(n)$$

$p_{t,2}$, $p_{t,2} > p_{t,1}$, and so on

$$\langle n \rangle p_n = \psi\left(\frac{n}{\langle n \rangle}\right) = \psi(z)$$

$$\frac{1}{k} = \frac{\langle z^2 \rangle - \langle z \rangle^2}{\langle z \rangle^2}$$

$$\psi(z) = \frac{\beta^k}{\Gamma(k)} z^{k-1} e^{-\beta z}, \;\; k > 1 \qquad \beta = k.$$

Entanglement entropy

$$S^c = -\sum_n p_n^c \log p_n^c = -\sum_n \frac{np_n}{\langle n \rangle} \log\left(\frac{np_n}{\langle n \rangle}\right) = -\sum_n \frac{n}{\langle n \rangle^2} \psi(z) \log\left(\frac{n\psi(z)}{\langle n \rangle^2}\right)$$

$$= -\int_0^\infty dz z\psi(z) \log\left(\frac{z\psi(z)}{\langle n \rangle}\right) = \log\langle n \rangle - \int_0^\infty dz z\psi(z) \log\left(z\psi(z)\right)$$

$$S^c = \log\langle n \rangle + k + \log\Gamma(k) - \frac{k}{\Gamma(k)}\partial_k \Gamma(k) \simeq \log\langle n \rangle + \frac{1}{2}\left[1 + \log\left(\frac{2\pi}{k}\right)\right]$$

$$\to \log\frac{\langle n \rangle}{\sqrt{k}} = \log\langle n \rangle^{1/2}$$

--Leading term log(n) (the n partons are the n microstates and are equal probably and the entropy is maximal)

--Additional term which depends only on k, the (Inverse of)fluctuations on the number of partons

--At very high k(no fluctuations) instead of n microstates we have n/2(saturation or clustering of color sources)

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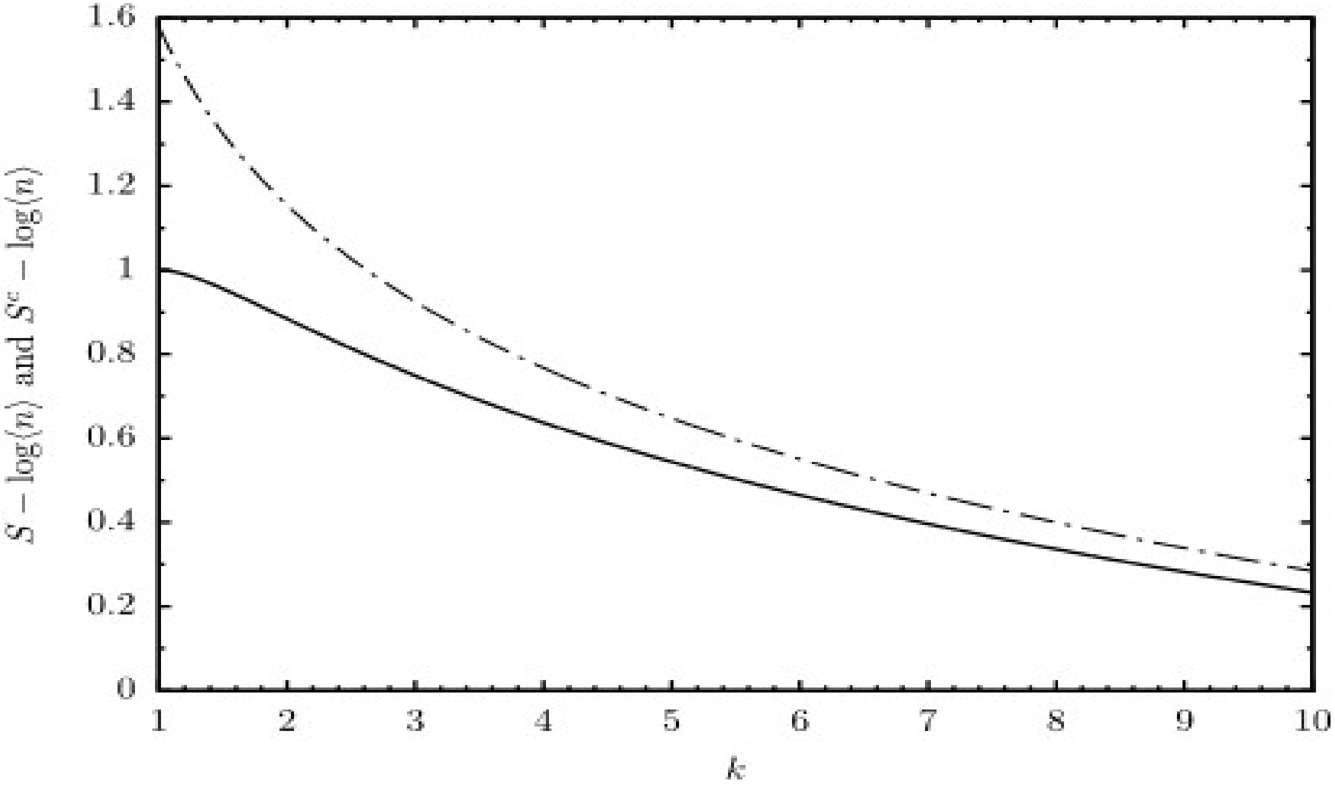
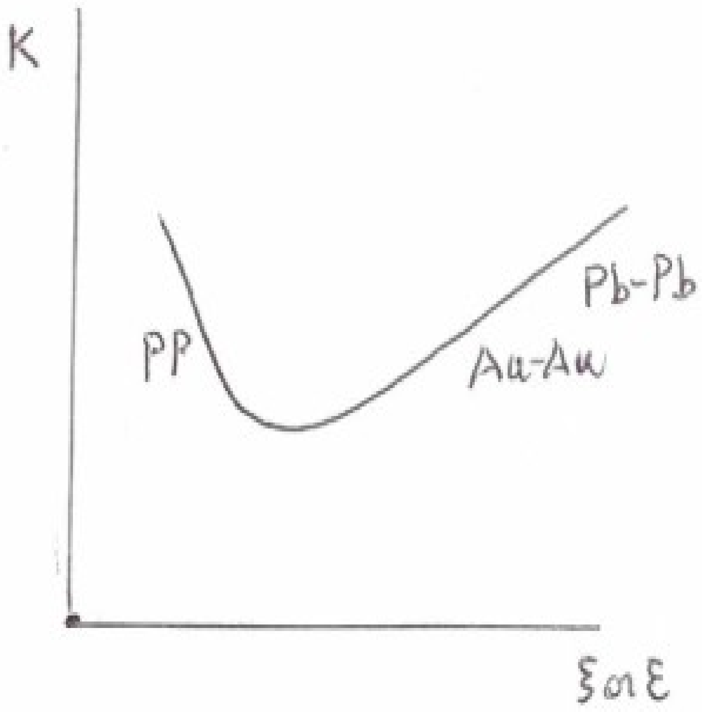
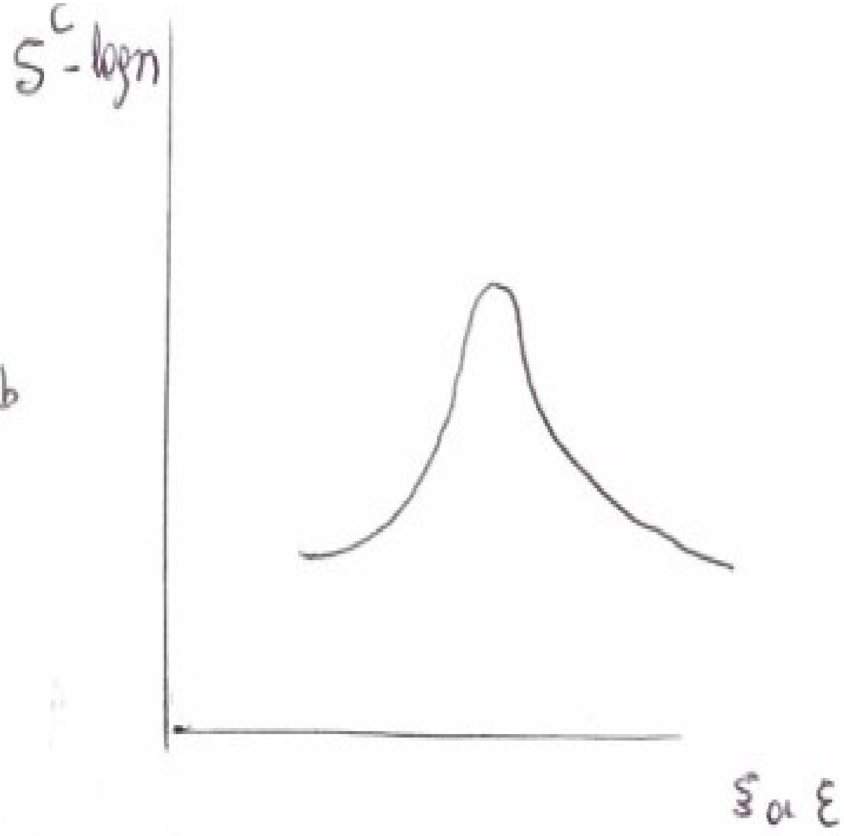
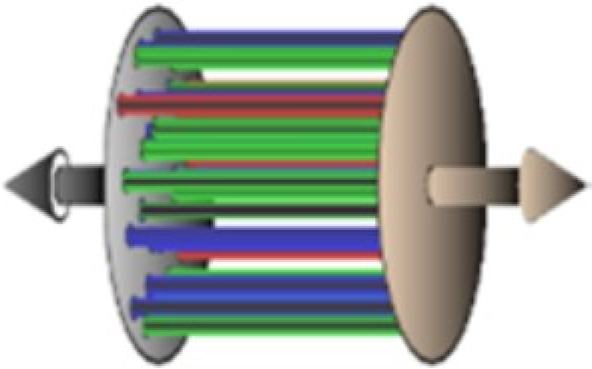


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
k
$S - \log(n)$ and $S^c - \log(n)$

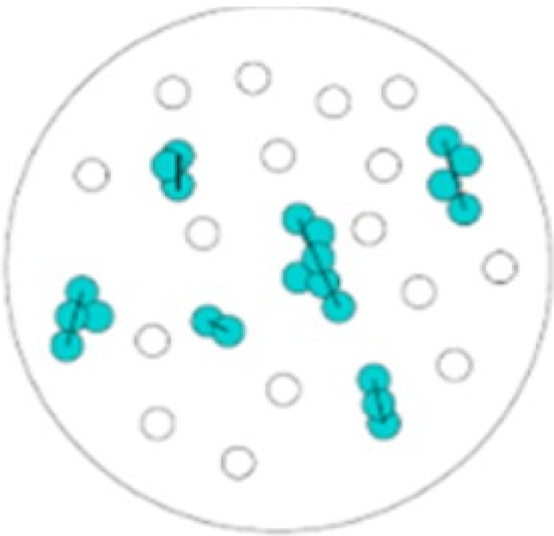

K
PP
Au-Au
Pb-Pb
ξ or ε


S^c - log n
ξ or ε

Clustering of color sources



- Projectile and target interact via color field created by th constituent partons of the nuclei.
- Color field is confined in a region with transverse size $r_0 \sim$ **0.2 fm**.
- We can see them as small areas in transverse plane.



- With growing energy and/or atomic number of colliding particles, the number of **sources** grows → The **number of strings** grows with energy and/or atomic number.
- The **number of strings** also increases with increasing centrality.
- Strings are **randomly distributed** in transverse plane so they can overlap forming clusters.

$\vec{Q}_n^2 = (\sum_1^n \vec{Q}_i)^2$ the average $\vec{Q}_i \cdot \vec{Q}_j$ is zero, so $\vec{Q}_n^2 = n\vec{Q}_1^2$.

$$\mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 \qquad \langle p_t^2 \rangle_n = \sqrt{\frac{nS_1}{S_n}}\langle p_t^2 \rangle_1$$

which in the limit of high density, $\xi = N_s S_1/S$, becomes

$$\mu_n = N_s F(\xi)\mu_1 \qquad \langle p_t^2 \rangle_n = \frac{1}{F(\xi)}\langle p_t^2 \rangle_1,$$

where N_s is the number of color sources and $F(\xi)$ is an universal factor

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}.$$

The factor $1 - e^{-\xi}$ is the fraction of the total collision area covered by color sources

$$f(p_t) = \int dx W(x) \exp(-p_t^2 x)$$

$$W(x') \to \frac{x' W(x')}{\langle x' \rangle} \to \cdots \frac{x'^k W(x')}{\langle x'^k \rangle} \to \cdots$$

$$W(x) = \frac{\gamma}{\Gamma(n)} (\gamma x)^n \exp(-\gamma x)$$

$$\gamma = \frac{n}{\langle x \rangle}, \qquad \frac{1}{n} = \frac{\langle x^2 \rangle - \langle x \rangle^2}{\langle x \rangle^2}.$$

$$f(p_t) = \frac{1}{(1 + p_t^2/\gamma)^n} = \frac{1}{(1 + F(\xi) p_t^2/)^n} \qquad T_h^2 = \frac{\langle p_t^2 \rangle_1}{F(\xi)},$$

$$f(p_t) \approx \exp(-p_t^2 F(\xi)/\langle p_t^2\rangle_1)$$

$$\sqrt{\frac{2}{\pi\langle x_h^2\rangle}} \int_0^\infty \exp(-\frac{x_h^2}{2\langle x_h^2\rangle}) \exp(-\frac{\pi p_t^2}{2x_h^2}) = \exp(-p_t\sqrt{\frac{2\pi}{\langle x_h^2\rangle}})$$

$$T_{\rm th} = \frac{T_{\rm h}}{\pi\sqrt{2}}.$$

-----In the clustering of color sources is naturally explained:

a)the relation between Th and Tth

b)the dependence of n with multiplicity

c)the gamma distribution obtained for the Temperature distribution, coincides with the Fokker-Planck equation solution for a gaussian stochastic white noise and with the distribution for events with hard collisions

Conclusions

--The data on pp and different multiplicities and Pb-Pb show that the two scales Tth and Th are related each other

--The distribution of temperatures is a gamma distribution(Fokker Plank solution, multiplicity associated to hard events,cluster size distribution)

--The entanglement entropy changes from log(n) to log(n)/2