# Quantum simulation of the universal features of the Polyakov loop

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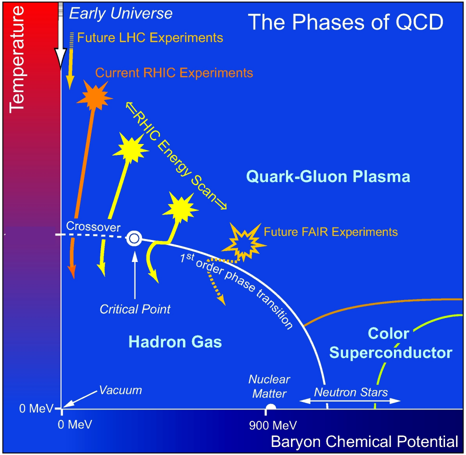
- J. Unmuth-Yockey (Syracuse University)
- J. Zhang (University of California, Riverside)
- J. Zieher (Max Planck Insitute for Quantum Optics, Germany)

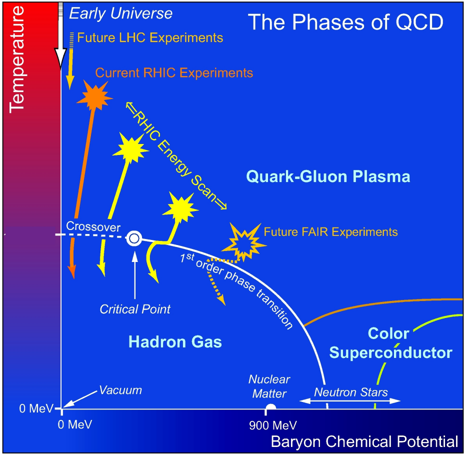
Some results: `1403.5238, 1503.08354, 1703.10577, 1803.11166, 1807.09186`

# Thermodynamics of strong interactions




- Phases of the strongly interacting matter

# Thermodynamics of strong interactions

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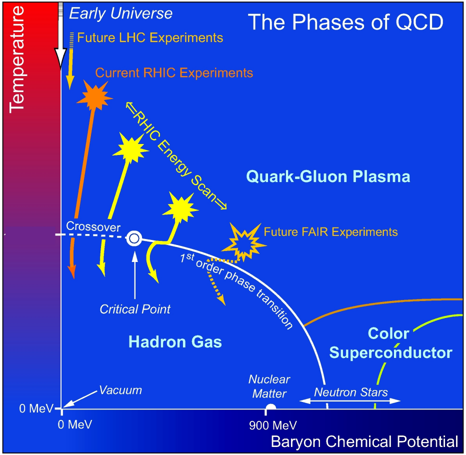


- Phases of the strongly interacting matter
- Properties of quark-gluon plasma
- Experiments: RHIC, LHC, FAIR, NICA

# Quantum Chromodynamics

- The QCD Lagrangian:

$$\begin{aligned}\mathcal{L}^E_{QCD} &= \mathcal{L}^E_{gluon} + \mathcal{L}^E_{fermion} \\ &= -\frac{1}{4} F^{\mu\nu}_a(x) F^a_{\mu\nu} - \sum_{f=u,d,s\dots} \bar{\psi}^\alpha_f(x) \left( \not{D}^E_{\alpha\beta} + m_f \delta_{\alpha\beta} \right) \psi^\beta_f(x)\end{aligned}$$

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- The grand canonical partition function:

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\mathcal{Z}(T, V, \vec{\mu}) = \int \prod_\mu \mathcal{D}A_\mu \prod_{f=u,d,s\ldots} \mathcal{D}\psi_f \mathcal{D}\bar{\psi}_f \, \mathrm{e}^{-S_E(T,V,\vec{\mu})}
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- The expectation value of a physical observable $\mathcal{O}$:

$$\langle\mathcal{O}\rangle = \frac{1}{Z(T,V,\vec{\mu})}\int \prod_\mu \mathcal{D}A_\mu \prod_f \mathcal{D}\psi_f\mathcal{D}\bar{\psi}_f \, \mathcal{O} \, \mathrm{e}^{-S_E(T,V,\vec{\mu})}$$

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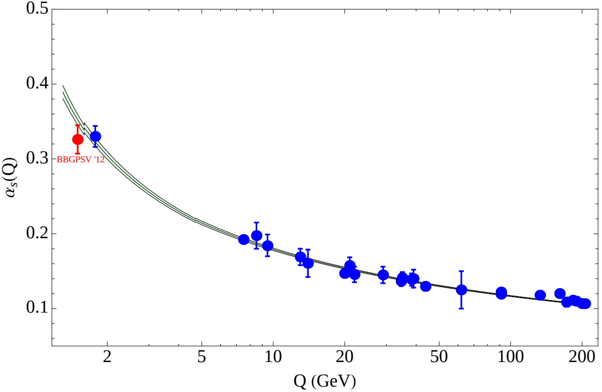
- In QCD the coupling constant is large in the region of interest (i.e. on the energy scales of few hundred MeV)



# Lattice gauge theory

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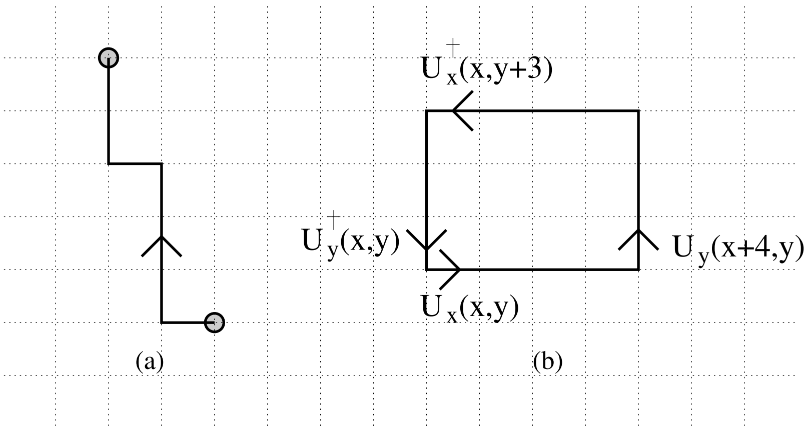
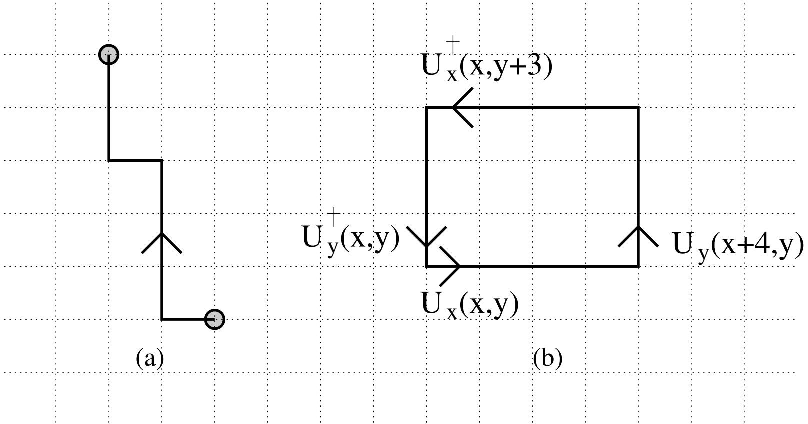



[1]Wilson (1974)

# Lattice gauge theory

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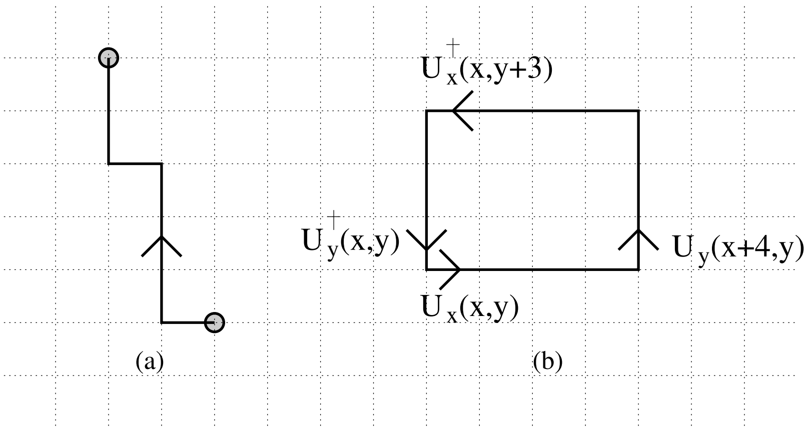


- Lattice gauge theory[1] – a non-perturbative regularization scheme
- Discrete space-time, gauge invariant action

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  - Parton distribution functions
  - ...and many more!

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$$\langle P \rangle (T) = \exp(-F_\infty(T)/(2T))$$

- Not an order parameter in full QCD

# Quantum simulation

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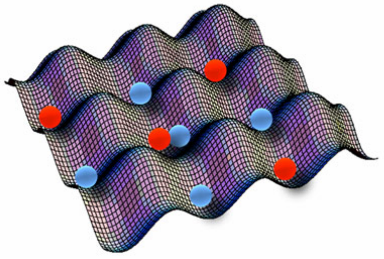

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[2] Picture courtesy of JILA

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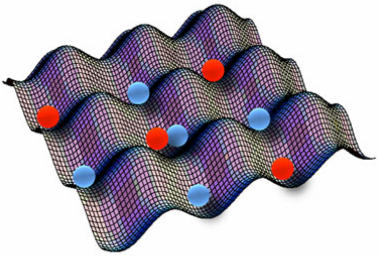

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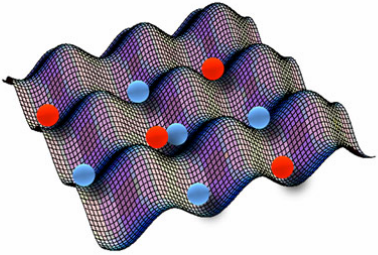

- Technology: Ultra-cold atoms trapped in optical lattices (counter propagating laser beams)[2]
- Possibility of tunable interactions
- Goal: Quantum simulator for lattice gauge theory

[2] Picture courtesy of JILA

## Analog quantum simulation of (1+1)D Abelian-Higgs model

# (1+1)D Abelian-Higgs model

The partition function:

$$Z = \int D\phi^\dagger D\phi D U e^{-S},$$

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$$S_g = -\beta_{pl} \sum_x \mathrm{Re}\left[U_{pl,x}\right],$$

$$\begin{aligned} S_h = & - \kappa_\tau \sum_x \left[\mathrm{e}^{\mu} \phi_x^\dagger U_{x,\hat{\tau}} \phi_{x+\hat{\tau}} + \mathrm{e}^{-\mu} \phi_{x+\hat{\tau}}^\dagger U_{x,\hat{\tau}}^\dagger \phi_x\right] \\ & - \kappa_s \sum_x \left[\phi_x^\dagger U_{x,\hat{s}} \phi_{x+\hat{s}} + \phi_{x+\hat{s}}^\dagger U_{x,\hat{s}}^\dagger \phi_x\right], \end{aligned}$$

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$$S_\lambda = \lambda \sum_x \left( \phi_x^\dagger \phi_x - 1 \right)^2 + \sum_x \phi_x^\dagger \phi_x.$$

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- $\kappa = 0$: $U(1)$ pure gauge theory
- $\lambda < \infty, \beta = \infty$: $\phi^4$ theory
- $\lambda = \infty, \beta = \infty$: $O(2)$ model, Kosterlitz-Thouless transition

# (1+1)D Abelian-Higgs model

- In the hopping part of the action $S_h$, we can separate the compact and non-compact variables

$$
\begin{aligned}
S_h = & - 2\kappa_\tau |\phi_x||\phi_{x+\hat{\tau}}| \sum_x \cos(\theta_{x+\hat{\tau}} - \theta_x + A_{x,\hat{\tau}} - i\mu) \\
& - 2\kappa_s |\phi_x||\phi_{x+\hat{s}}| \sum_x \cos(\theta_{x+\hat{s}} - \theta_x + A_{x,\hat{s}})
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\end{aligned}
$$

- and then Fourier transform the Boltzmann weight, *i.e.*

$$
\begin{aligned}
& \exp[2\kappa_\tau |\phi_x||\phi_{x+\hat{\tau}}| \cos(\theta_{x+\hat{\tau}} - \theta_x + A_{x,\hat{\tau}} - i\mu)] \\
= & \sum_{n=-\infty}^{\infty} I_n(2\kappa_\tau |\phi_x||\phi_{x+\hat{\tau}}|) \exp[in(\theta_{x+\hat{\tau}} - \theta_x + A_{x,\hat{\tau}} - i\mu)]
\end{aligned}
$$

# (1+1)D Abelian-Higgs model

- The effective action for the gauge and hopping part

$$e^{-S_{eff}} = \sum_{\{m_\Box\}} \left[ \prod_{\Box} I_{m_\Box}(\beta_{pl}) \prod_{x} \left( I_{n_{x,\hat{s}}}(2\kappa_s |\phi_x||\phi_{x+\hat{s}}|) \right.\right.$$
$$\left.\left. \times I_{n_{x,\hat{\tau}}}(2\kappa_\tau |\phi_x||\phi_{x+\hat{\tau}}|) \exp(\mu n_{x,\hat{\tau}}) \right) \right]$$

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$$\left.\left. \times I_{n_{x,\hat{\tau}}}(2\kappa_\tau|\phi_x||\phi_{x+\hat{\tau}}|) \exp(\mu n_{x,\hat{\tau}}) \right) \right]$$

- Using the hopping parameter expansion for $\kappa = \kappa_s = \kappa_\tau$ and with $M_x \equiv \phi_x^\dagger \phi_x$:

$$S_{eff} = \sum_{\langle xy \rangle} \left( -\kappa^2 M_x M_y + \frac{1}{4}\kappa^4 (M_x M_y)^2 \right)$$
$$-2\kappa^4 \frac{I_1(\beta_{pl})}{I_0(\beta_{pl})} \sum_{\Box(xyzw)} M_x M_y M_z M_w + O(\kappa^6)$$

$$Z = \int D\phi^\dagger D\phi D U e^{-S} \simeq \int DM e^{-S_{eff}(M) - S_\lambda(M)}$$

# Hopping parameter expansion

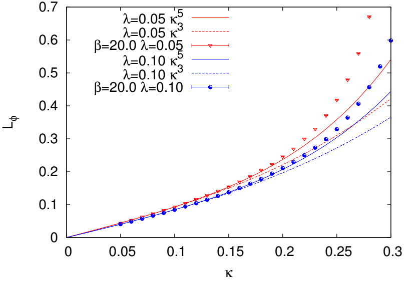
- The hopping parameter expansion[3]

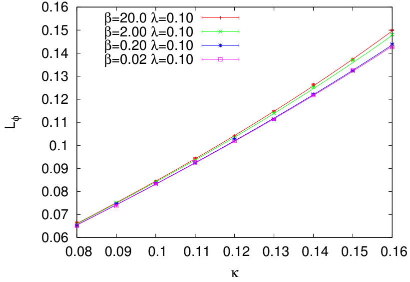
$$
\begin{aligned}
\frac{Z_{\kappa,\lambda}}{Z_\lambda} &= 1 + Vd\gamma_2^2\kappa^2 \\
&+ Vd\left\{\left[\frac{1}{2}(Vd-4d+1)+(d-1)\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})}\right]\gamma_2^4+(2d-1)\gamma_2^2\gamma_4+\frac{1}{4}\gamma_4^2\right\}\kappa^4 \\
&+ Vd\left\{\left[\frac{1}{6}(Vd-1)(Vd-2)-\frac{2}{3}(d-1)(2d-1)-(2d-1)^2-(2d-1)(Vd-6d+2)\right.\right. \\
&+ 2(d-1)(2d-3)\left(\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})}\right)^2+(d-1)(Vd-8d+4)\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})} \\
&+ \left.\frac{4}{3}(d-1)(d-2)\left(\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})}\right)^3\right]\gamma_2^6+(2(d-1)\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})}+(2d-1)^2+\frac{1}{4}(Vd-4d+1))\gamma_2^2\gamma_4^2 \\
&+ (8(d-1)^2\frac{I_1(\beta_{pl})}{I_0(\beta_{pl})}+(2d-1)(Vd-6d+2))\gamma_2^4\gamma_4+\frac{2}{3}(2d-1)(d-1)\gamma_2^3\gamma_6 \\
&+ \left.\frac{1}{2}(2d-1)\gamma_2\gamma_4\gamma_6+\frac{1}{36}\gamma_6^2\right\}\kappa^6
\end{aligned}
$$

where $\gamma_{2k} \equiv \langle\rho^{2k}\rangle_{Z_\lambda}$.

[3] Heitger (1997)

# Tests of the hopping parameter expansion




- Left: $L_\phi$ at $\lambda = 0.05$ and 0.1 for $\beta = 20$ compared with the $O(\kappa^3)$ and $O(\kappa^5)$ expansions
- Right: $L_\phi$ at $\lambda = 0.1$ for $\beta = 0.02 - 20$ compared with the $O(\kappa^5)$ expansion

# The partition function in the dual representation

- The partition function can be rewritten exactly in a gauge-invariant way in terms of integer fields living on the plaquettes:

$$Z = \sum_{\{m\}} \left( \prod_{x,\nu<\mu} t_m(\beta_{pl}) \right) \left( \prod_{x,\nu} t_{m-m'}(2\kappa) \right),$$

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- The expectation value of the Polyakov loop:

$$\langle P \rangle = \frac{1}{Z} \int \mathcal{D}[\phi^\dagger]\mathcal{D}[\phi]\mathcal{D}[U] \left( \prod_{n=0}^{N_\tau - 1} U_{x^* + n\hat{\tau},\hat{\tau}} \right) e^{-S}$$

where $x^*$ is a single specific spatial site

# The Polyakov loop

- The Polyakov loop insertion modifies the link integrals:

$$\int \frac{\theta_x}{2\pi} e^{i(n-m_r+m_l+1)\theta_x} = \delta_{n,m_r-m_l-1},$$

where the subscripts $l$ and $r$ denote the "left" and "right" plaquette quantum numbers, respectively, to the vertical (temporal) link in question

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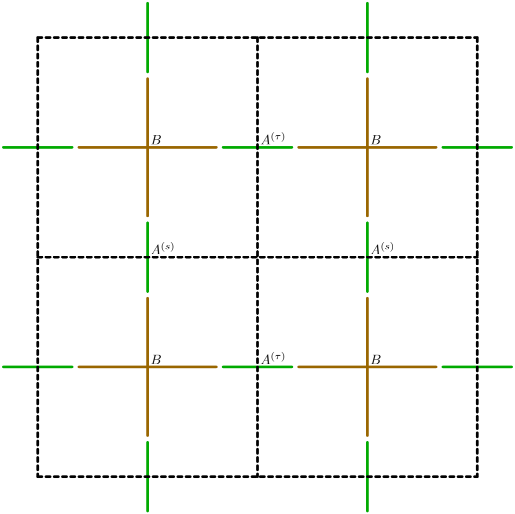
- The Polyakov loop in terms of the new variables:

$$P = \prod_{n=0}^{N_\tau-1} \frac{t_{m-m'-1}(2\kappa)}{t_{m-m'}(2\kappa)}$$

# Tensor Renormalization Group (TRG) method

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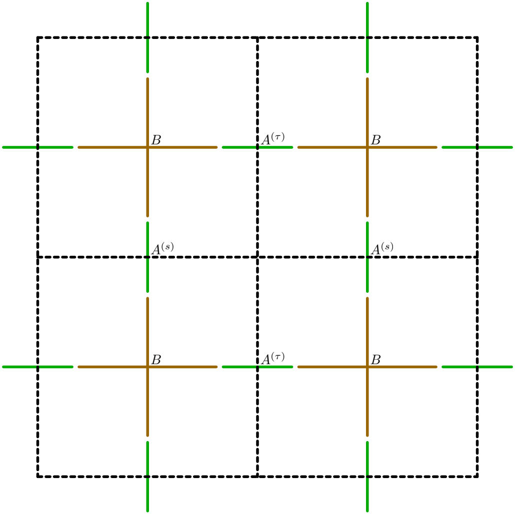


- Rewrite the partition function in a tensor form
- Solve by blocking and truncation in the number of states

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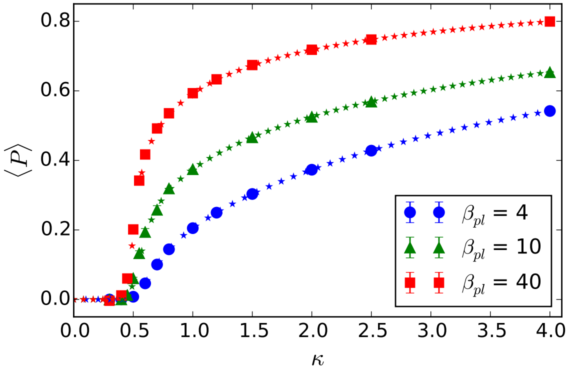


- Comparison TRG and MC for a range of $\kappa$ and $\beta_{pl}$ values for $N_s = N_\tau = 16$.

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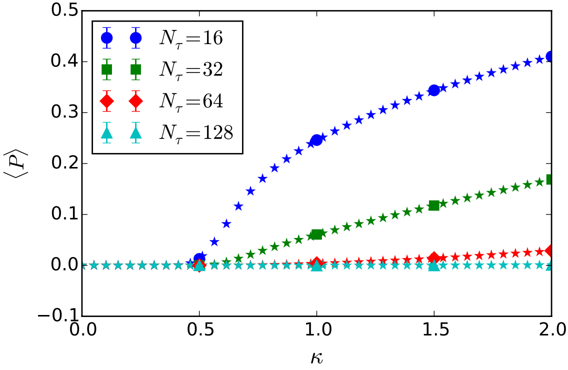


- Comparison of TRG and MC data with fixed spatial length and various temporal lengths, $N_s = 16$, $\beta_{pl} = 5$ and $D_{\text{bond}} = 41$

# The Polyakov loop

- The Polyakov loop can be represented as the ratio of two partition functions: one with the inclusion of the static charge, and the other without:

$$\langle P \rangle = \frac{\tilde{Z}}{Z} = \frac{\mathrm{Tr}[\tilde{\mathbb{T}}^{N_\tau}]}{\mathrm{Tr}[\mathbb{T}^{N_\tau}]} = \frac{\sum_{i=0}^{N} \tilde{\lambda}_i^{N_\tau}}{\sum_{i=0}^{N} \lambda_i^{N_\tau}}$$

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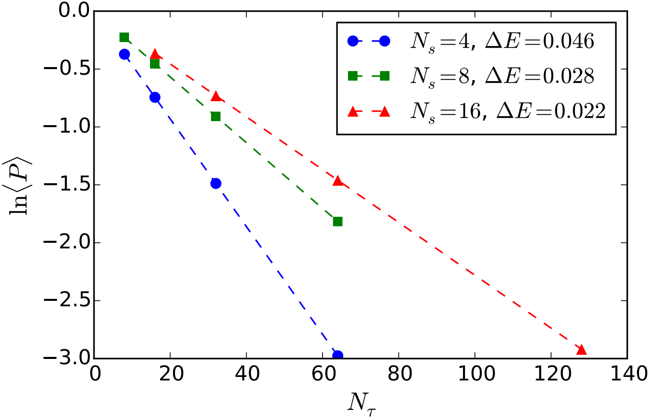
- In the large $N_\tau$ limit the Polyakov loop expectation value is dominated by the largest eigenvalues:

$$\log\langle P \rangle \simeq N_\tau \log(\tilde{\lambda}_0/\lambda_0) = -N_\tau \Delta E$$

where $\Delta E$ is the energy gap between the ground state of the system with the static charge, and that without:

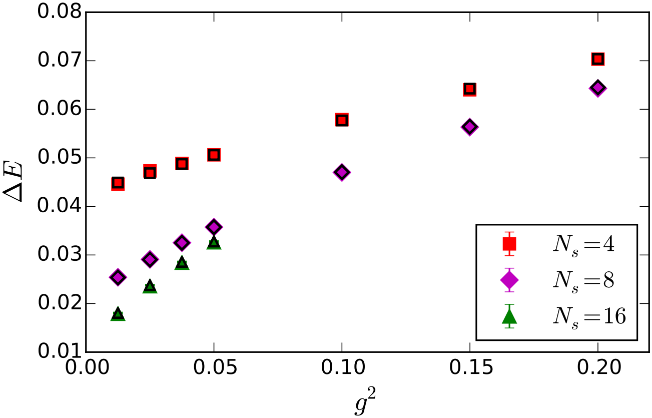
$$\langle P \rangle \simeq e^{-N_\tau \Delta E}$$

# The energy gap




- The energy gap $\Delta E$ for various spatial lattice sizes $\kappa = 1.6$, $\beta_{pl} = 44$

# The energy gap




- Comparison of TRG and MC data for $\Delta E$ at $\kappa = 1.6$

# Universal scaling

- For $\kappa$ large enough (greater than the Kosterlitz-Thouless (KT) transition value) and $g^2 N_s$ small enough, we expect the following scaling:

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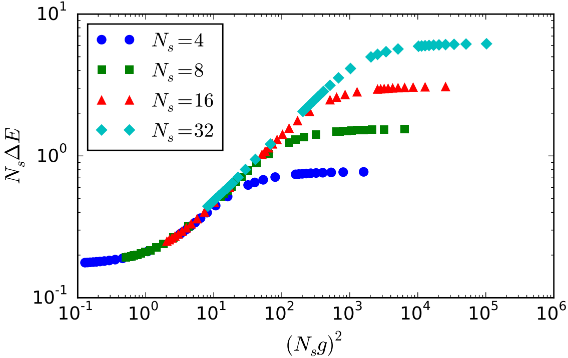
- If we multiply this equation by $N_s$, then the right hand side depends only on $g^2 N_s^2$
- We conjecture that this scaling persists beyond the lowest order:

$$\Delta E N_s = f(g^2 N_s^2)$$

# Universal scaling




- Data collapse for the energy gap $\Delta E$ for different $N_s$

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- For $g \gg 1$ the lowest energy state corresponds to having all plaquette quantum numbers set to zero
- This is possible when the matter loop follows exactly the Polyakov loop in the opposite direction
- This state contributes $\left(t_1(2\kappa)\right)^{N_\tau}$ to the partition function, thus for large $g$ we expect

$$\Delta E \to -\ln(t_1(2\kappa)),$$

  independent of $N_s$

# The continuous-time limit

- The continuous-time limit:

$$\kappa_\tau, \beta_{pl} \to \infty, \quad \kappa_s, a_\tau \to 0$$

keeping fixed:

$$U \equiv \frac{1}{\beta_{pl} a} = \frac{g^2}{a}, \quad Y \equiv \frac{1}{2\kappa_\tau a}, \quad X \equiv \frac{2\kappa_s}{a}$$

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$$[\hat{L}, \mathrm{e}^{\pm i\hat{\theta}}] = \pm\mathrm{e}^{\pm i\hat{\theta}}$$

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- In `1403.5238` we considered a spin-1 truncation and represented this algebra with the angular momentum algebra

$$[\hat{L}^z, \hat{L}^\pm] = \pm \hat{L}^\pm$$

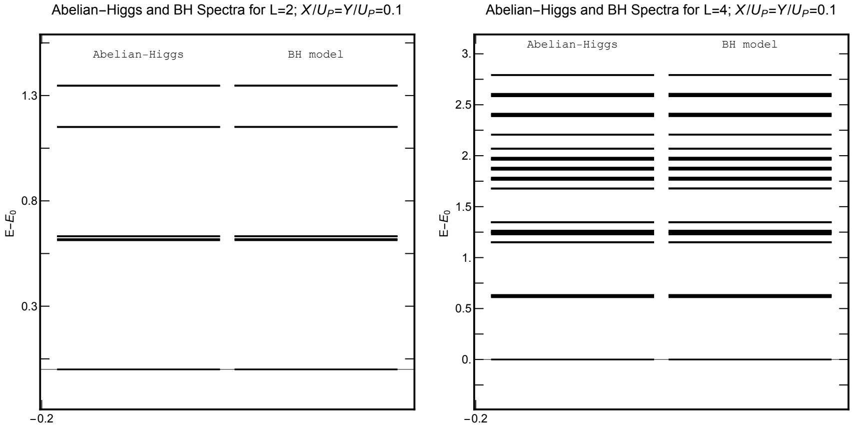
# The continuous-time limit

- The three-state spin-1 Hamiltonian is then:

$$H = \frac{U}{2} \sum_{i=1}^{N_s} (L_i^z)^2 + \frac{Y}{2} {\sum_i}' (L_{i+1}^z - L_i^z)^2 - \frac{X}{\sqrt{2}} \sum_{i=1}^{N_s} L_i^x$$

- This Hamiltonian is mapped onto the two-species Bose-Hubbard model that can be potentially quantum simulated with a "ladder" structure

# Bose-Hubbard realization for the $U(1)$-Higgs model




- Comparison of the energy spectra for two-site (left) and four-site (right) system calculated in the Abelian-Higgs model and in the spin-1 approximation

# Improvements

- In order to go beyond the spin-1 approximation we need the following modification:

$$L^x \quad \to \quad U^x = \frac{1}{2}(U^+ + U^-),$$

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# The Polyakov loop insertion

- We take the continuous-time limit for the $P$ operator:

$$P \to 1 + \frac{1}{2(2\kappa_\tau)}(2(m - m') - 1) + \mathcal{O}((2\kappa_\tau)^{-2})$$

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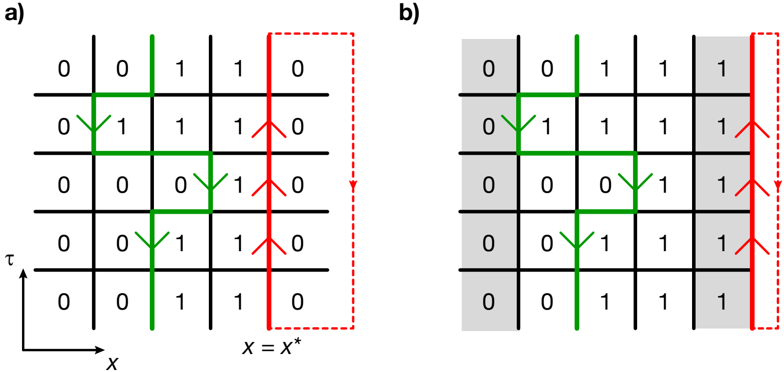
- To avoid boundary effects the Polyakov loop is inserted in the middle of the spatial lattice:

$$\tilde{H} = \frac{U}{2}\sum_{i=1}^{N_s}(L^z_i)^2 + \frac{Y}{2}\sum_{i\neq\frac{N_s}{2}}{}'(L^z_{i+1} - L^z_i)^2$$

$$+ \frac{Y}{2}(L^z_{\frac{N_s}{2}+1} - L^z_{\frac{N_s}{2}} - 1)^2 - X\sum_{i=1}^{N_s} U^x_i$$

# Polyakov loop vs boundary conditions

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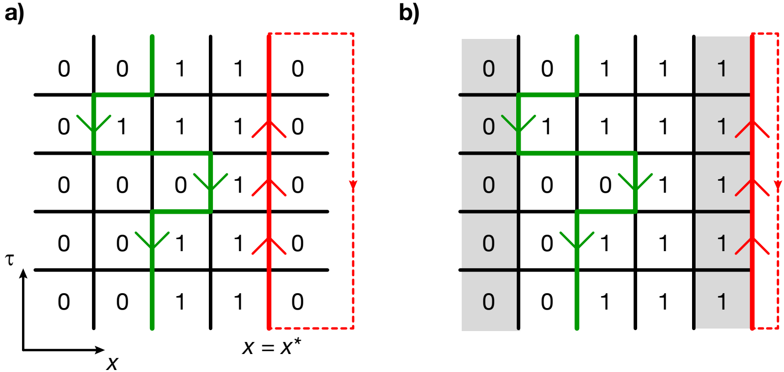
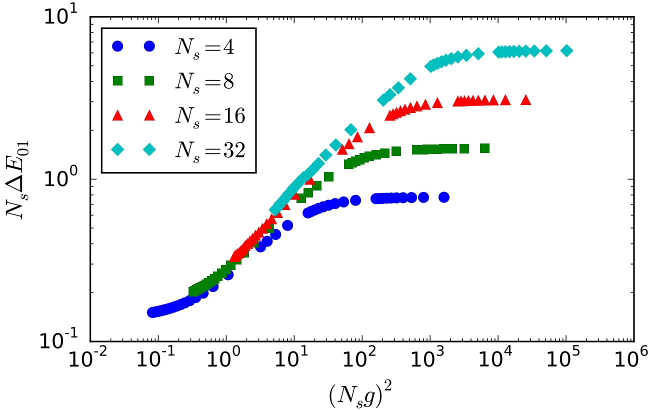



- Insertion of the Polyakov loop probes the response of the system to the addition of a single static charge
- Alternatively, one can probe $Q \neq 0$ sectors by changing the boundary conditions (similar to subjecting the system to an external electric field)

# Polyakov loop vs boundary conditions

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- The Hamiltonian is modified to

$$H_{10} = \frac{U}{2}\sum_{i=1}^{N_s}(L_i^z)^2 + \frac{Y}{2}\sum_{i=1}^{N_s-1}(L_{i+1}^z - L_i^z)^2 + \frac{Y}{2}(L_{N_s}^z)^2 + \frac{Y}{2}(L_1^z - 1)^2 - X\sum_{i=1}^{N_s} U_i^x$$

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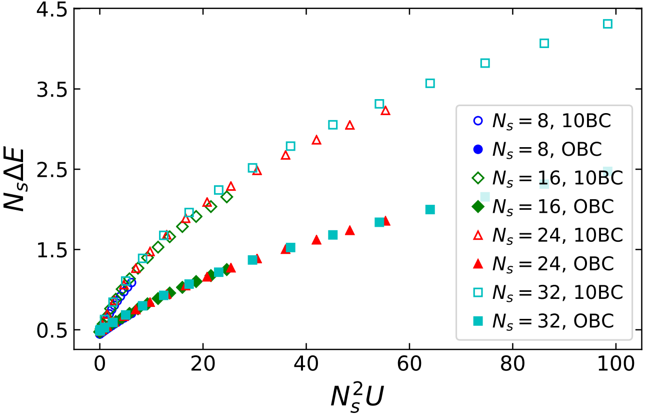
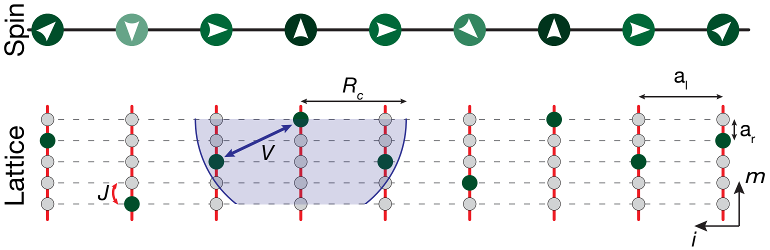



- Data collapse for the energy gap between 01BC and 0BC systems in the continuous-time limit

# Quantum simulation




- Multi-leg ladder implementation for spin-2

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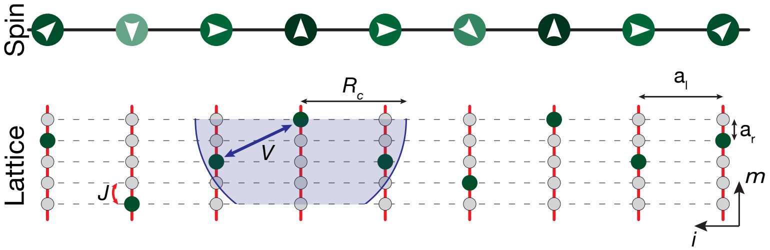


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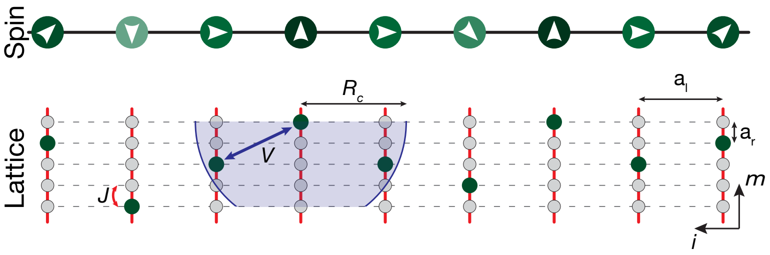


- Multi-leg ladder implementation for spin-2
- The atoms hop along the rungs but not the legs of the ladder
- Coupling between the atoms in different rungs is implemented via an interaction $V$

# Conclusion

- Analog quantum simulations have potential to become useful for studying models relevant for particle and nuclear physics

[4] 1803.11166, 1807.09186

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- Both of these features are important for control and measurement in optical lattice simulators

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