Is There Quantum Entanglement in Parton Distribution Functions?

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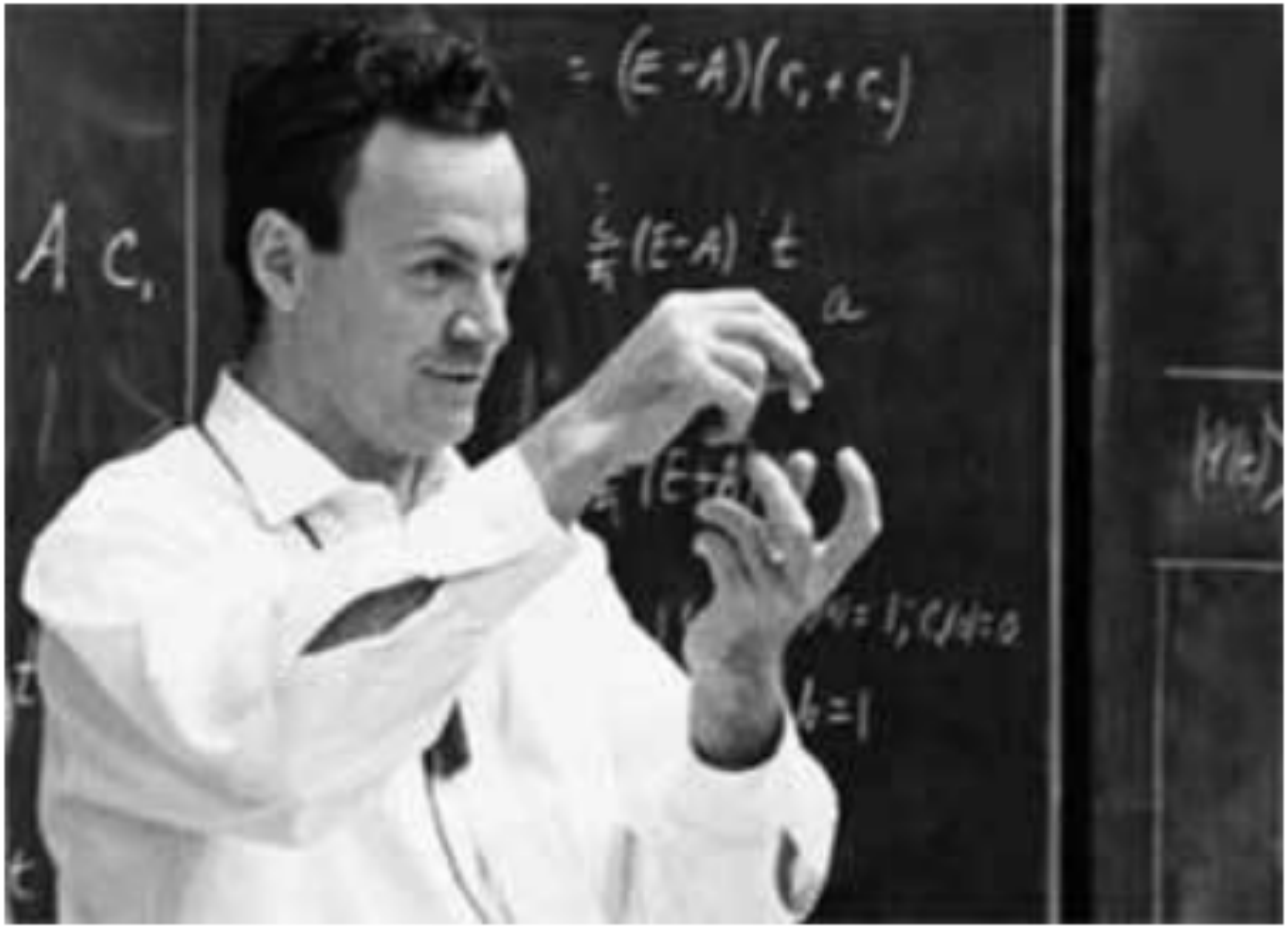
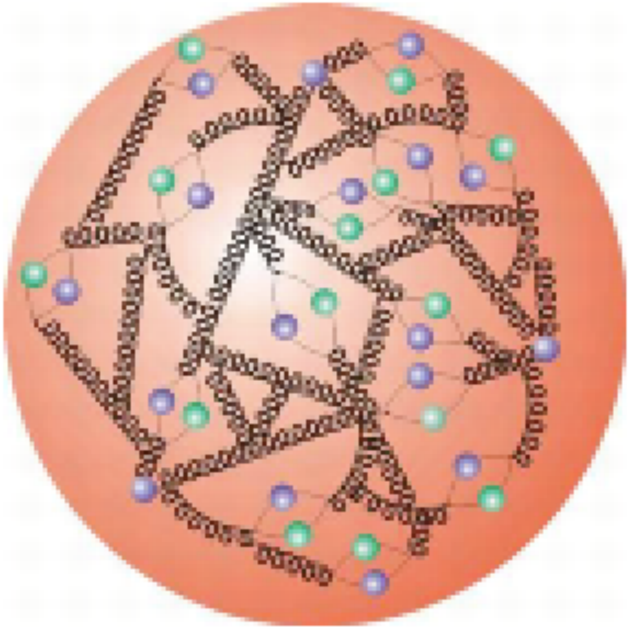
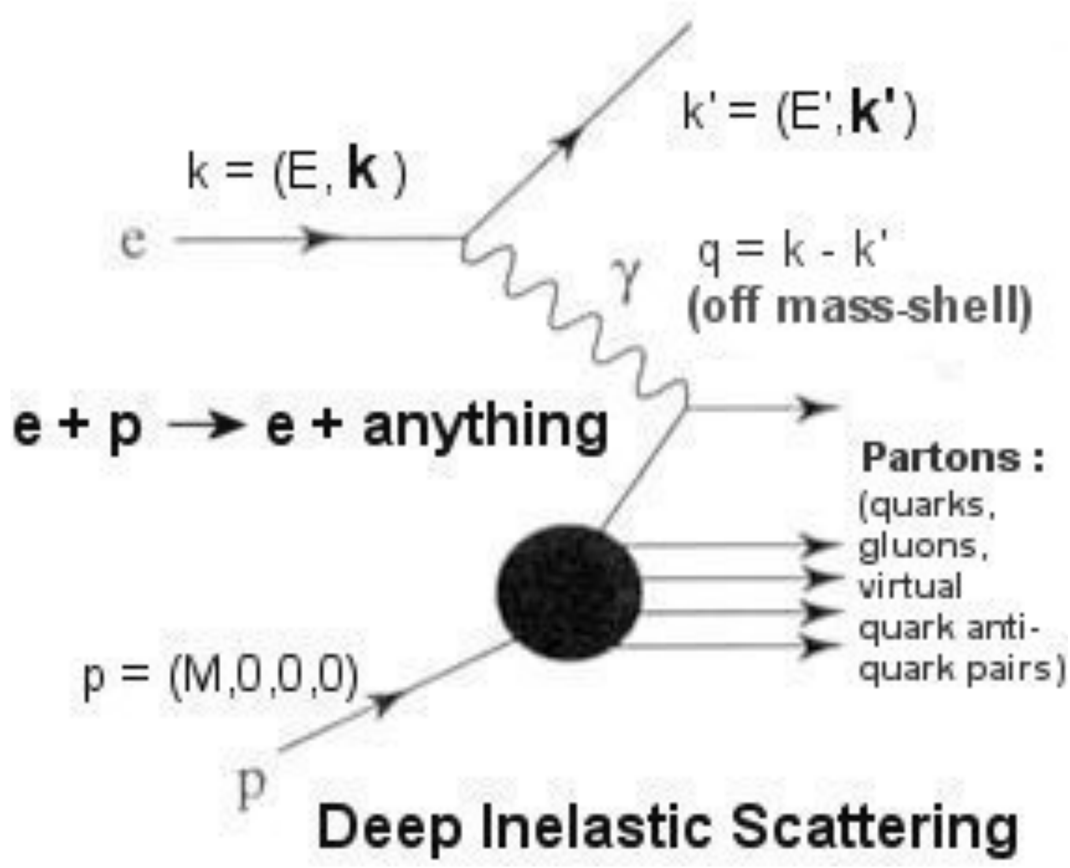
JWC and Jin-Yi Pang, arXiv:1709.03205

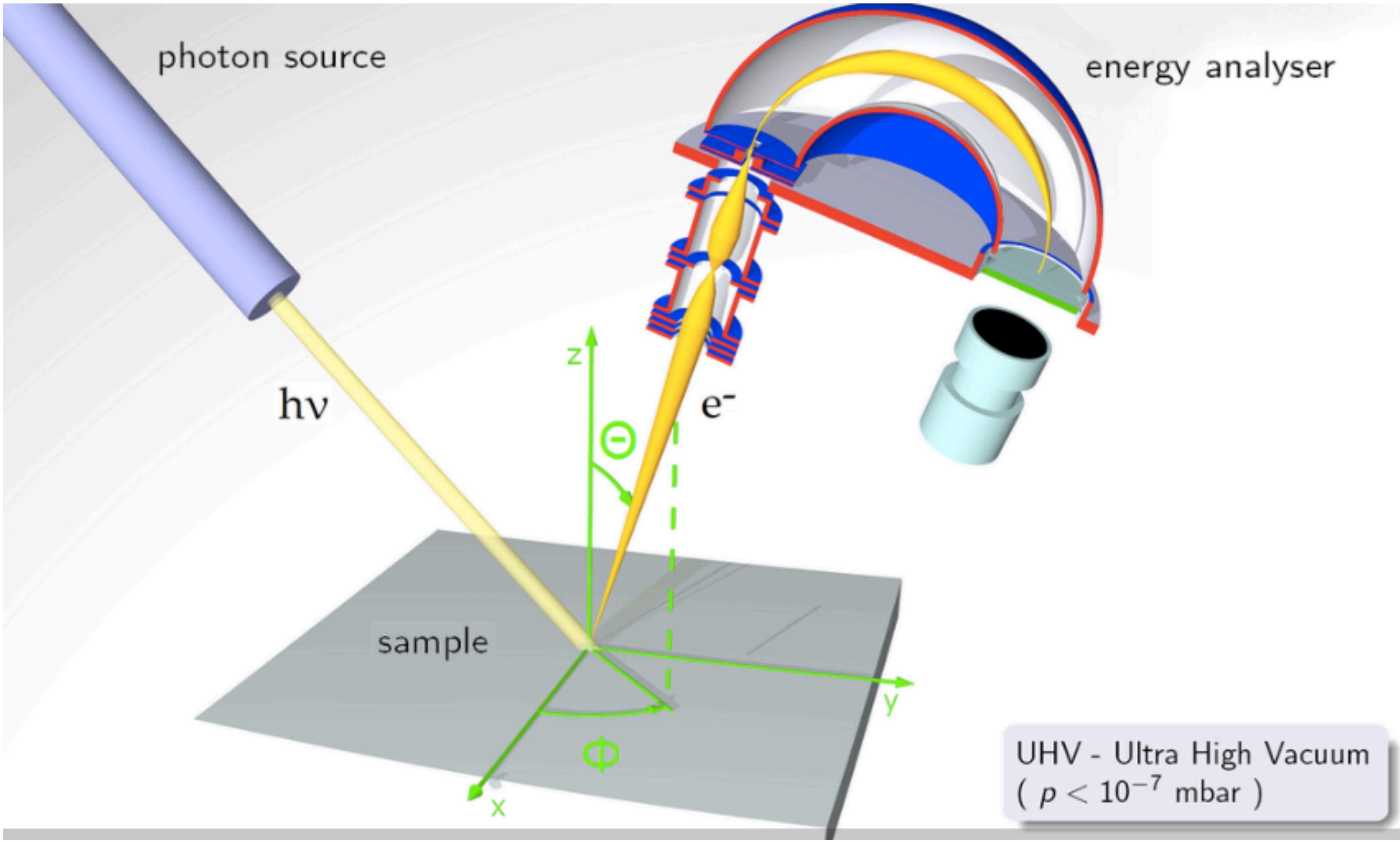




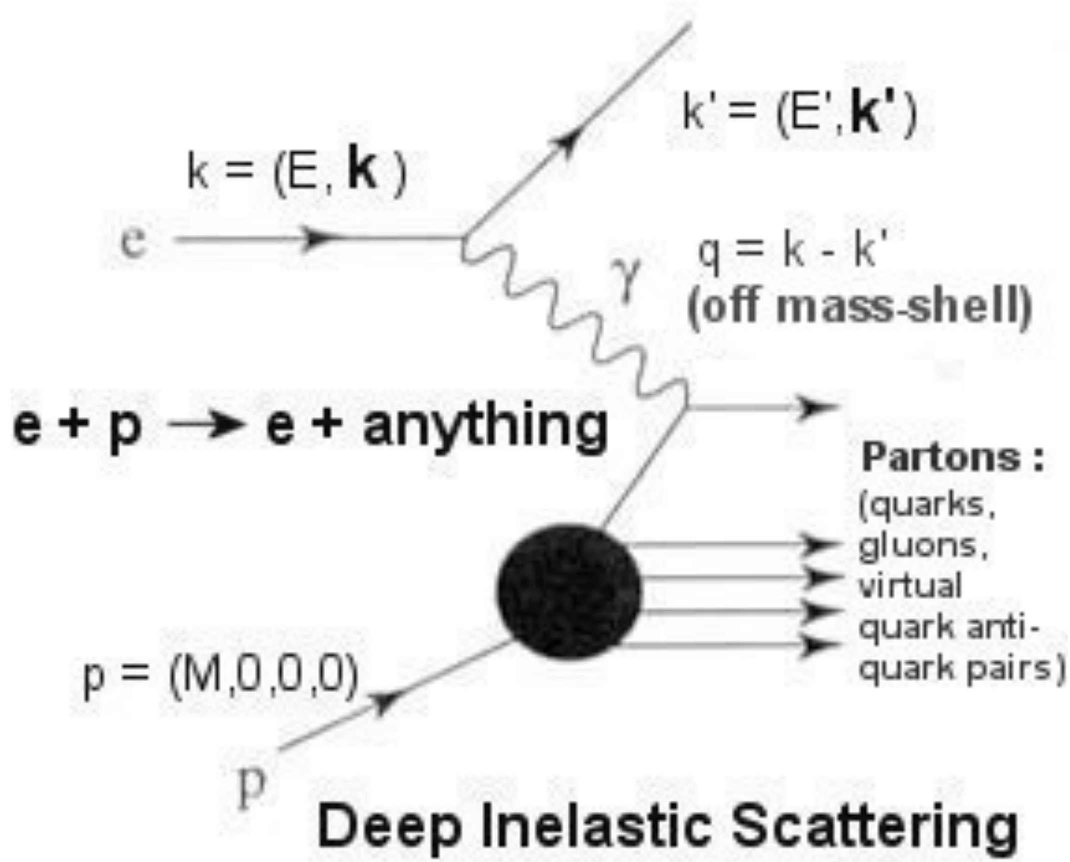
Deep Inelastic Scattering



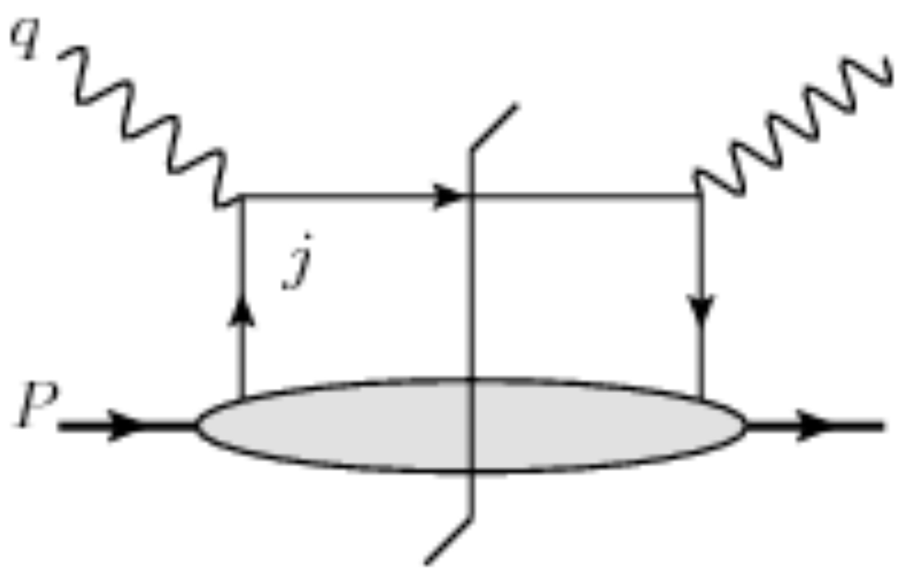
ARPES


photon source
energy analyser
hν
e⁻
z
Θ
sample
y
Φ
x
UHV - Ultra High Vacuum
($p < 10^{-7}$ mbar)

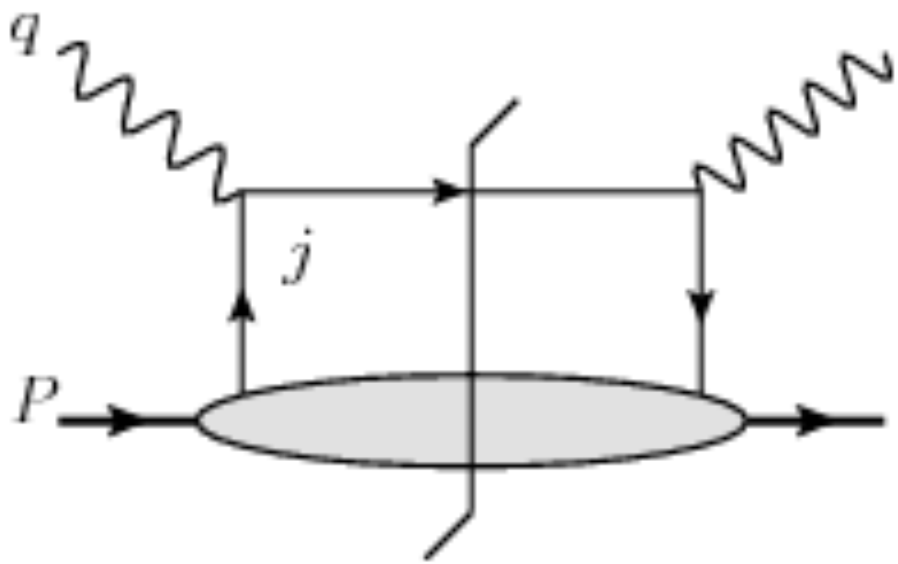
Deep Inelastic Scattering



Parton Distribution Function (PDF) in QCD



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \right\rangle$$

PDF = Partially Traced Density Matrix

$$q(x,\mu^2) = \frac{1}{4\pi N}\langle P\left|\bar{\tilde{\psi}}(xP^+)\gamma^+\tilde{\psi}(xP^+)\right|P\rangle$$

$$\rho_q(x,\mu^2) = q(x,\mu^2).$$

Diagonalized already.

Is It Quantum Entanglement?

- Momentum conservation also imposes correlation among classical momentum distributions
- Bell Inequality (BI) test? Requires measurements that are not commute.
- However, while violation of BI implies entanglement, entanglement not necessarily violates BI.
- If we assume the proton is a pure state, then reduced DM not in a pure state does imply entanglement.

Entanglement Entropy for PDF?

- Antiparticles are tricky…

$$\rho_q(x<0)\le 0 \qquad q(-x) = -\bar{q}(x)$$

$$\mathrm{tr}\hat{\rho}_d = \int_{-1}^{1} dx\; d(x,\mu^2) = \int_0^1 dx\,\left[d(x,\mu^2) - \bar{d}(x,\mu^2)\right] = 1$$

$$S_d^{EE} \underset{?}{=} -\int_0^1 dx\,\left[d(x,\mu^2)\log d(x,\mu^2) + \bar{d}(x,\mu^2)\log \bar{d}(x,\mu^2)\right]$$

Still divergent!

$$S_d^{EE} \underset{?}{=} -\Sigma_i\left[d(x_i)\epsilon\log(d(x_i)\epsilon) + \bar{d}(x_i)\epsilon\log(\bar{d}(x_i)\epsilon)\right]$$

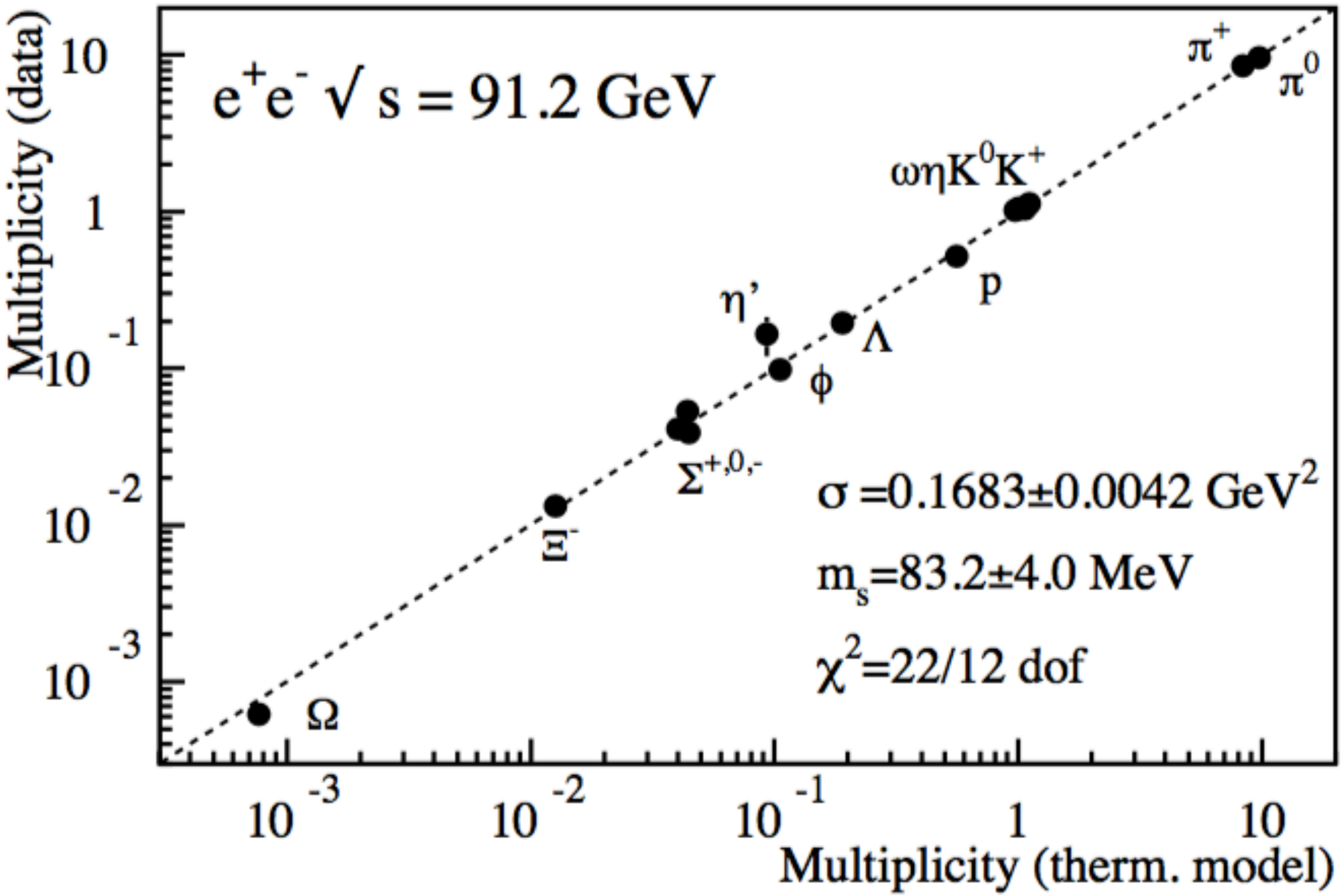
Entanglement Entropy for PDF?

- Antiparticles are tricky…

$$S_d^{EE} = -\Sigma_i d_v(x_i)\epsilon \log(d_v(x_i)\epsilon) = -\int_0^1 dx d_v(x) \log d_v(x) + \log \epsilon.$$

- Then valence u quarks are more entangled than the valence d quarks in a proton.
- Trace of area law from coordinate space EE not clear

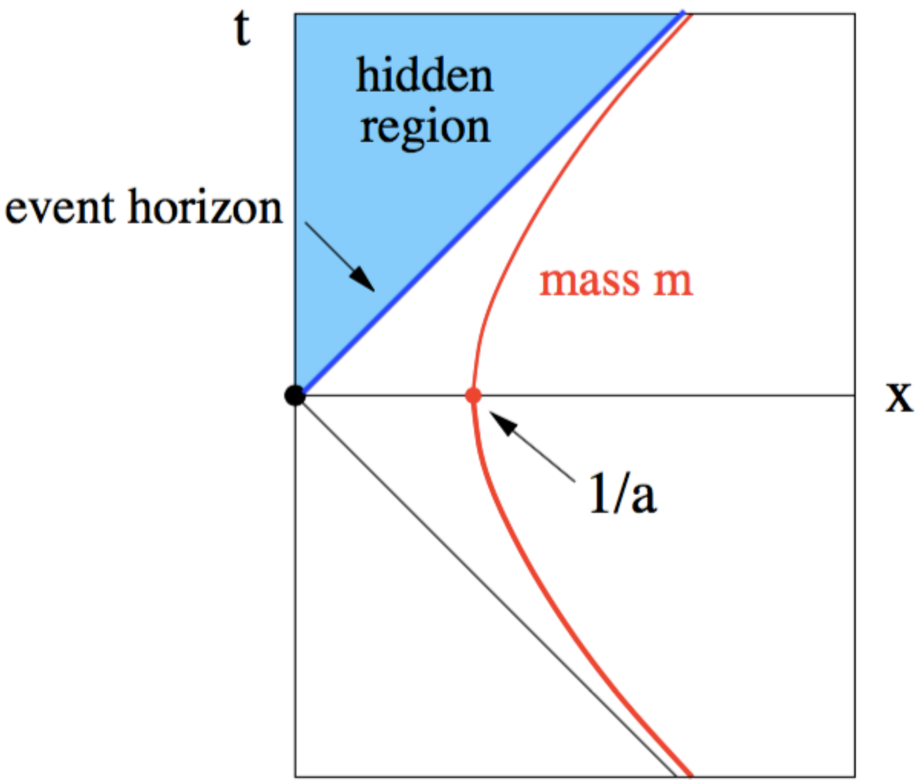
A puzzle in e^+e^- collisions



Becattini, Castorina, Manninen, Satz, 0805.0964

Hawking-Unruh Radiation Explanation

- Castorina, Kharzeev, Satz, 0704.1426



$$T_U = \frac{a}{2\pi}$$

Connection to Entanglement?

- See e.g. Berges, Floerchinger, Venugopalan, 1707.05338
- Lesson from black hole, entropy = entanglement entropy

't Hooft; Sussking

$$S_{BH} = S_E = A/4G_R$$

Black Hole Thermal Entropy

Outside the black hole horizon:

$$R^2 \times S^2$$

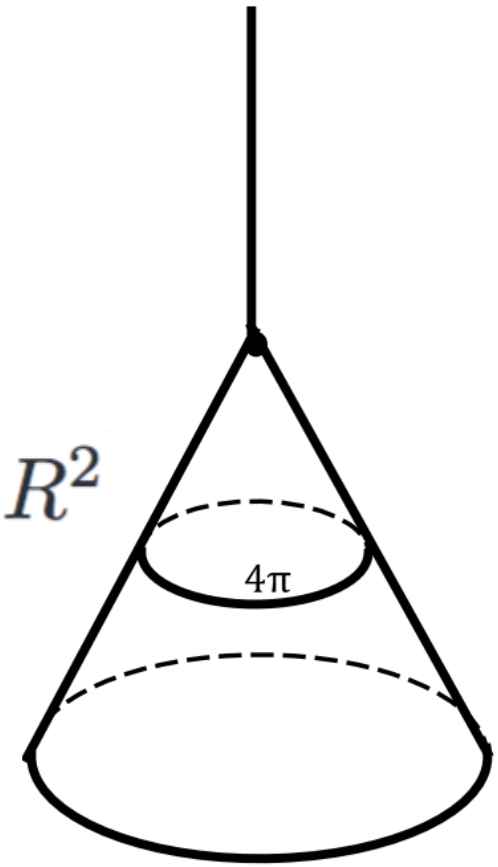
$$S = \left(-\beta \frac{\partial}{\partial \beta} + 1\right) \ln Z$$

$$\epsilon = 2\pi(\beta_H - \beta)/\beta_H$$

$$S_{BH} = \left(2\pi \frac{\partial}{\partial \epsilon} + 1\right) \ln Z(\epsilon)\Big|_{\epsilon \to 0}$$

Ground State Entanglement Entropy
Replica Trick (Callan &Wilczek)

$C_\epsilon \times R^2$



$$S_E = \left(2\pi \frac{\partial}{\partial \epsilon} + 1\right) \ln Z(\epsilon)\Big|_{\epsilon \to 0}$$

Renormalization of EE: Scalar theory in a curve space

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 \\ &+ \frac{1}{2}Z_\phi(\partial\phi)^2 + \frac{1}{2}Z_m\phi^2 + \frac{1}{4}Z_\lambda\phi^4 + Z_4 \\ &+ \epsilon\delta^{(2)}(x_\parallel)\left(Z_2 + Z_0\phi^2\right) + \mathcal{O}(\epsilon^2).\end{aligned}$$

The Correspondence in the Black Hole Case

$$\mathcal{L} = \mathcal{L}_\phi - \frac{R}{16\pi G} + \frac{\alpha}{4\pi}\phi^2 R + \mathcal{O}(R^2).$$

$$R = 4\pi\epsilon\delta^{(2)}(x_\parallel)$$

$$\frac{S_{BH}}{A_\perp} = \frac{1}{4G} - C - \frac{m^2}{48\pi} + \mathcal{O}(\lambda^2) = \frac{1}{4G_R} + \mathcal{O}(\lambda^2),$$

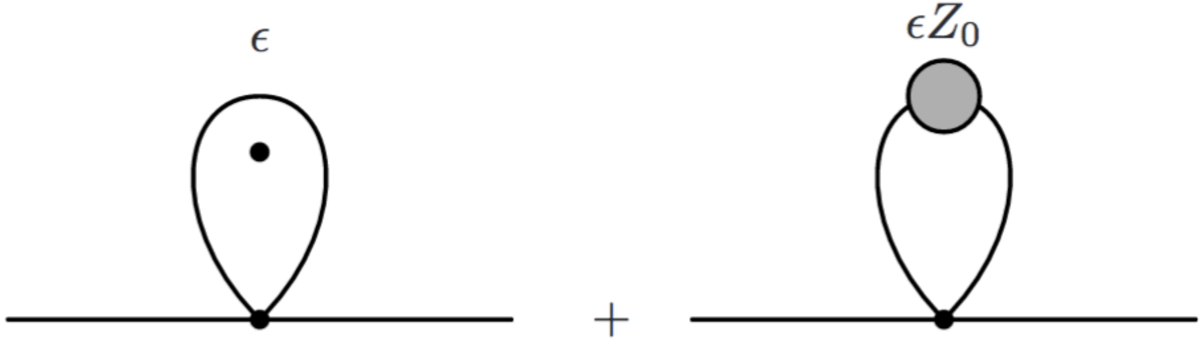
$$S_{BH} = S_E = \frac{A_\perp}{4G_R}.$$

Summary

- Interpreting PDF as partially traced density matrix elements should be useful to connect entanglement. Need to think more about how to deal with anti-particles.
- e^+e^- collision might be a good place to start.

Backup

2-point function renormalization at $O(\epsilon\lambda)$



$$Z_0 = \pi/3$$

0-point function renormalization at $O(\epsilon\lambda)$