Maximal Entanglement in High Energy Physics

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Quantum Entanglement at Collider Energies,
CFNS Stony Brook

ACL, J. I. Latorre, J. Rojo and L. Rottoli, SciPost Phys. **3**, 036 (2017).

Outline

1. Motivation
2. Maximal Entanglement in QED
3. MaxEnt and gauge symmetry
4. MaxEnt in weak interactions
5. Conclusions

Motivation

QI paradigm

Quantum Information is bringing new insights:

$$H|\psi\rangle = E|\psi\rangle$$

- Traditional emphasis on operators $\rightarrow H$
- QI emphasis on states $\rightarrow |\psi\rangle$

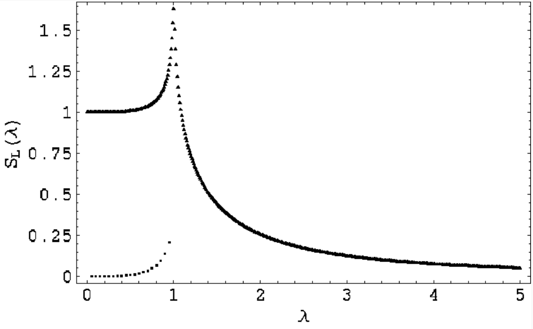
Example: Quantum Phase Transitions

$\mathbf{H}$	$\|\psi\rangle$
Criticality, RG flows on coupling constants, Conformal Symmetry,...	Scaling of entropy, RG flows on states, Distribution of entanglement: MERA,...

Example: QPT

Entanglement is maximal at Quantum Phase Transition

$$H_{QI} = \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda \sigma_i^z$$



$\lambda \to 1 \Rightarrow$ Max Entropy $S \Rightarrow$ Max Entanglement $\Rightarrow$ Conformal Symmetry

Entropy Scaling

- *Von Neumann Entropy* $\rho_A = \mathrm{Tr}_B |\psi\rangle_{AB}\langle\psi|,\ S(\rho_A) = -\mathrm{Tr}_A \rho_A \log \rho_A$
- *Fixed points*: scaling with block size L $\quad S(\rho_A) = \frac{c}{3} \log L \ll L$ [1]

$S \sim n$	Random states, QMA problems, Local translational invariant higher d
$S \sim .8858n$	Prime state
$S \sim n^{\frac{d-1}{d}}$	Area law in d-dimensions
$S \sim \frac{c}{3} \log n$	Critical scaling in $d = 1$
$S \sim \log(\xi) = ct$	Finitely correlated states away from criticality

[1] Callan-Wilczek 94, Vidal-Latorre-Rico-Kitaev 02

What is the most entangled spin chain?

homogeneous and nearest neighbour

Spin $\frac{1}{2}$ numerical evidence sets max entanglement along the conformal line [2]

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z$$

$$\Delta \to -1^+ \Rightarrow S \sim \frac{1}{2} \log(n+1)$$

Not CFT!

Frustration as a limit of conformal symmetry

[2] G. Blázquez and J. I. Latorre

Maximal Entanglement in QED

Quantifying entanglement

Focus

Two-particle scattering processes at tree level

Entanglement of helicity degrees of freedom

$$|\psi\rangle_{final} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

assuming $|0\rangle$, $|1\rangle$ helicity or polarization states.

$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$

Figure of merit to quantify entanglement: **concurrence**

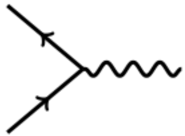
$$\Delta = |\alpha\delta - \beta\gamma|,$$

by construction, $0 \leq \Delta \leq 1$.

Question

Can a product state become entangled?

Generation of entanglement: s channel



$$j^{\mu}_{ss'} = e\bar{v}^{s'}(p')\gamma^{\mu}u^{s}(p)$$

Process: $e^+e^- \to \mu^+\mu^-$ at high energy

Incoming:

$$j^{\mu}_{RL} = 2ep_0\,(0,1,i,0)$$
$$j^{\mu}_{LR} = 2ep_0\,(0,1,-i,0)$$

Outgoing:

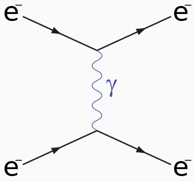
$$j^{\mu}_{RL} = 2ep_0\,(0,\cos\theta,i,-\sin\theta)$$
$$j^{\mu}_{LR} = 2ep_0\,(0,\cos\theta,-i,\sin\theta)$$

$$|RL\rangle \to (1+\cos\theta)|RL\rangle + (-1+\cos\theta)|LR\rangle$$

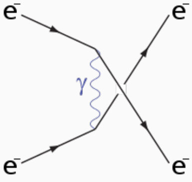
$$\theta = \pi/2 \to \Delta = 1$$

Generation of entanglement: indistinguishability

Process: $e^- e^- \to e^- e^-$ at high energy



t channel



u channel

$$\mathcal{M}\left(|RL\rangle \to |RL\rangle\right) = -2e^2\frac{u}{t}$$
$$\mathcal{M}\left(|RL\rangle \to |LR\rangle\right) = 0$$

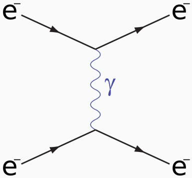
$$\mathcal{M}\left(|RL\rangle \to |RL\rangle\right) = 0$$
$$\mathcal{M}\left(|RL\rangle \to |LR\rangle\right) = -2e^2\frac{t}{u}$$

$$|RL\rangle \to \frac{u}{t}|RL\rangle - \frac{t}{u}|LR\rangle$$

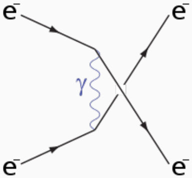
$$t = u \ (\theta = \pi/2) \to \Delta = 1$$

Generation of entanglement: indistinguishability

Process: $e^- e^- \to e^- e^-$ at any energy



t channel



u channel

$$\Delta_{|RL\rangle} = \frac{2tu\left(tu + m^2 \frac{(t-u)^2}{t+u}\right)}{2m^2(t-u)^2\left(2m^2 - 2(t+u) + \frac{tu}{t+u}\right) + (t^4 + u^4)} \xrightarrow{t=u} 1$$

$$\Delta_{|RR\rangle} \xrightarrow{E \ll m,\ t=u} 1 + \mathcal{O}(p^2/m^2)$$

QED interaction can generate maximal entanglement in almost all processes and at different energy regimes.

Is this a property of nature interactions?

MaxEnt and gauge symmetry

It from bit

Could a symmetry emerge from a
Maximum Entanglement Principle
?

It from bit philosophy by J. A. Wheeler:

"All things physical are information-theoretic in origin"

MaxEnt principle

MaxEnt principle

"Nature is such that maximally entangled states exist"

Max Entanglement = Max Entropy = Max Surprise = NO Local Realism

MaxEnt Principle = Nature cannot be described by classical physics

Bell Inequalities will be violated

Test: QED coupling

QED Lagrangian at tree level (high-energy limit, $m = 0$):

	free fermions		free photons		interaction term
$\mathcal{L} =$	$i\bar{\psi}\gamma^\mu\partial_\mu\psi$	$+$	$\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$+$	$-eA_\mu\bar{\psi}G^\mu\psi$
	Dirac eq.		Maxwell eq.		

G^μ: 4×4 arbitrary matrices

Gauge invariance: $G^\mu = \gamma^\mu$

Which are the couplings, G^μ, that generate MaxEnt?

Unconstrained QED

1. In general, G^μ may not be Lorentz invariant. Expand in a basis of 16 matrices:

$$G^\mu = a^\mu \mathbb{I} + a^{\mu\nu}\gamma_\nu + i a^{\mu 5}\gamma^5 + a^{\mu\nu 5}\gamma^5\gamma_\nu + a^{\mu\nu\rho}[\gamma_\nu, \gamma_\rho]$$

2. Assuming conservation of $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ symmetries:

$$G^\mu = a^{\mu\nu}\gamma^\nu \qquad a_{\mu\nu} \in \mathbb{R} \qquad a_{0i} = a_{i0} = 0$$

3. Computation of amplitudes of all tree-level processes:

$$\mathcal{M}_{|initial\rangle \to |final\rangle} = f(\theta, a_{\mu\nu})$$

Unconstrained QED

Constrain G^{μ} imposing MaxEnt in **ALL** tree level processes

$$\max_{a^{\mu\nu}} \{\Delta_{Bhabha}, \Delta_{Compton}, \Delta_{pair\ \ annhilation}, \Delta_{Moller}, ...\}$$

Each process will deliver different kind of MaxEnt at different angles

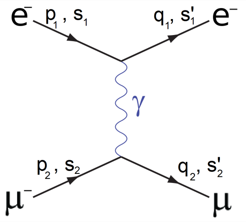
$\rightarrow$ Choose optimal settings

(Logic: Bell Ineq. seek to discard classical physics using optimal settings)

Unconstrained QED

$$e^- \mu^- \to e^- \mu^-$$

$$\begin{aligned}
\mathcal{M}_{|RL\rangle \to |RR\rangle} &= 0 \\
\mathcal{M}_{|RL\rangle \to |RL\rangle} &= f(a) \\
\mathcal{M}_{|RL\rangle \to |LR\rangle} &= 0 \\
\mathcal{M}_{|RL\rangle \to |LL\rangle} &= 0
\end{aligned}$$



No entanglement can be generated!

No constraints emerge from this process

Unconstrained QED

$$e^- e^+ \to \mu^- \mu^+$$

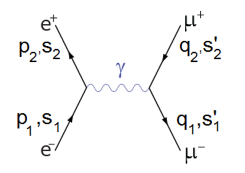
Amplitudes quadratic in a's:

$$\begin{aligned}
\mathcal{M}_{|RL\rangle \to |RL\rangle} &= \left(-a_{i2}^2 - a_{i1}^2 \cos\theta + a_{i1} a_{i3} \sin\theta\right) + i \left(a_{i1} a_{i2} (1 - \cos\theta) + a_{i2} a_{i3} \sin\theta\right) \\
\mathcal{M}_{|RL\rangle \to |LR\rangle} &= \left(-a_{i2}^2 + a_{i1}^2 \cos\theta - a_{i1} a_{i3} \sin\theta\right) + i \left(a_{i1} a_{i2} (1 + \cos\theta) - a_{i2} a_{i3} \sin\theta\right) \\
\mathcal{M}_{|RL\rangle \to |RR\rangle} &= \mathcal{M}_{|RL\rangle \to |LL\rangle} = 0
\end{aligned}$$

Arbitrary angle dependent solutions are discarded by other processes

MaxEnt $\quad \begin{matrix} \theta = \pi/2 \\ \Delta = 1 \end{matrix} \implies \begin{matrix} A = aa^T \geq 0 \\ A_{22}A_{13} - A_{12}A_{23} = 0 \end{matrix}$

QED $\quad a_{ij} = \begin{cases} 0 \ \forall i \neq j \\ 1 \ \forall i = j \end{cases} \implies A_{ij} = \begin{cases} 0 \ \forall i \neq j \\ 1 \ \forall i = j \end{cases}$



MaxEnt consistency

MaxEnt is well defined for a given process since it is a quadratic maximization, but

Is this consistent?

Yes

Does MaxEnt pull in different directions depending on the process?

No

Is there a unique maximum?

$G^\mu = \left(\pm\gamma^0, \pm\gamma^1, \pm\gamma^2, \pm\gamma^3\right)$

Final solution

Considering all tree level 2-particles processes (Bhabha, Moller, Compton, pair annihilation, ...)

$$\left(G^0, G^1, G^2, G^3\right) = \begin{cases} \left.\begin{matrix} \left(\pm\gamma^0, \gamma^1, \gamma^2, \gamma^3\right) \\ \left(\pm\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3\right) \end{matrix}\right\}\text{QED} \\ \\ \left.\begin{matrix} \left(\pm\gamma^0, -\gamma^1, \gamma^2, \gamma^3\right) \\ \left(\pm\gamma^0, \gamma^1, -\gamma^2, -\gamma^3\right) \end{matrix}\right\}\ ? \end{cases}$$

All two-body processes are blind to the signs $\begin{matrix} \gamma^i \to -\gamma^i \\ e \to -e \end{matrix}$

$-\gamma^1$ solution:

- No rotational invariance!
- Fermion scattering processes are identical to QED
- Leads to non-conservation of current
- Could be discarded at higher orders or appealing to rotational symmetry?

Observations

- **NO** incompatible pulls!! MaxEnt can be achieved consistently in different channels.
- Entanglement generated either in s channel or in superposition of t and u channels.
- A process may display MaxEnt at some angle with a contrived solution for a's. This solution will fail in other processes.
- Using COM or LAB reference frames do not change the analysis.
- Need of three-body processes to discard wrong signs.

Furthermore,

QED is an isolated maximum

All deformations around QED produce lower entanglement

Apparently, MaxEnt can fix the structure of an interaction like QED

Could we use it to obtain an estimation of free parameters in other interactions?

MaxEnt in weak interactions

Weak interaction

Weak neutral current

$$J_\mu^{NC} = \bar{u}_f \gamma_\mu \left(g_V^f - \gamma^5 g_A^f \right) u_f$$

$$g_A^f = T_3^f/2 \qquad g_V^f = T_3^f/2 - Q_f \sin^2\theta_w$$

For electrons: $T_3^\ell = -1/2$, $Q_\ell = -1$.
Experimentally, $\sin^2\theta_w \simeq 0.23$

Guessing

MaxEnt might be achievable on a line in the plane $\theta - \theta_w$
Non-trivial tests: Bhabha (Z/γ interference)
Special case, no kinematics: Z decay

Z decay to leptons

$m \ll M_Z$, $g_R = (g_V - g_A)/2$ and $g_L = (g_V + g_A)/2$

Longitudinal polarization:

$$\left.\begin{aligned} \mathcal{M}_{|0\rangle \to |RL\rangle} &= g_R M_Z \sin\theta \\ \mathcal{M}_{|0\rangle \to |LR\rangle} &= g_L M_Z \sin\theta \end{aligned}\right\} \Delta_0 = \frac{2|g_L g_R|}{g_L^2 + g_R^2}$$

$\Delta_0 = 1$ if $|g_L| = |g_R| \Rightarrow g_A = 0$ or $g_V = 0$.

$$g_A = T_3/2 \neq 0 \Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = \frac{T_3}{2Q} \xrightarrow{\text{for charged leptons}} \sin^2\theta_w = 1/4.$$

Z decay to leptons

$m \ll M_Z$, $g_R = (g_V - g_A)/2$ and $g_L = (g_V + g_A)/2$

Circular polarizations:

$$\mathcal{M}_{|R\rangle\to|RL\rangle} = g_R M_Z \sqrt{2}\sin^2(\theta/2) \qquad \mathcal{M}_{|L\rangle\to|RL\rangle} = g_R M_Z \sqrt{2}\cos^2(\theta/2)$$
$$\mathcal{M}_{|R\rangle\to|LR\rangle} = -g_L M_Z \sqrt{2}\cos^2(\theta/2) \quad \mathcal{M}_{|L\rangle\to|LR\rangle} = -g_L M_Z \sqrt{2}\sin^2(\theta/2)$$

$$\Delta_{\substack{R\\L}} = \frac{2|g_L g_R|\sin^2\theta}{|2(g_L^2 - g_R^2)\cos\theta \pm (g_L^2 + g_R^2)(1+\cos^2\theta)|}$$

$$\Delta_{\substack{R\\L}} = 1 \text{ if } \begin{cases} \dfrac{g_R}{g_L} = \pm\cot^2(\theta/2) \\[2ex] \dfrac{g_R}{g_L} = \pm\tan^2(\theta/2) \end{cases}$$

Assuming g_R and g_L are independent of the initial polarization:

$$\frac{g_R}{g_L} = \pm 1 \Rightarrow |g_L| = |g_R| \Rightarrow g_V = 0 \Rightarrow \sin^2\theta_w = 1/4$$

$e^+e^- \to \mu^+\mu-$ mediated by Z

$m \ll M_Z$,

$$\begin{aligned}\mathcal{M}_{RL} &\sim (1+\cos\theta)g_R^2 \quad |RL\rangle + (-1+\cos\theta)g_R g_L \quad |LR\rangle \\ \mathcal{M}_{LR} &\sim (-1+\cos\theta)g_R g_L \quad |RL\rangle + (1+\cos\theta)g_L^2 \quad |LR\rangle\end{aligned}$$

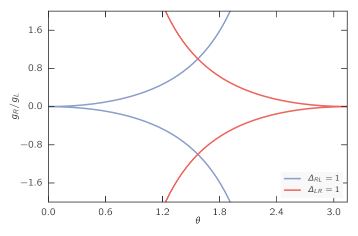
$$\Delta_{RL} \sim \frac{\sin^2\theta |g_L g_R|}{2\left(s^4 g_L^2 + c^4 g_R^2\right)} \quad \Delta_{LR} \sim \frac{\sin^2\theta |g_L g_R|}{2\left(c^4 g_L^2 + s^4 g_R^2\right)}$$

$c = \cos(\theta/2),\ s = \sin(\theta/2)$

$$\begin{aligned} s^2 g_L \pm c^2 g_R = 0 &\to \Delta_{RL} = 1 \\ c^2 g_L \pm s^2 g_R = 0 &\to \Delta_{LR} = 1 \end{aligned}$$

Imposing MaxEnt at the same COM angle:

$$\theta = \frac{\pi}{2},\ \sin^2\theta_w = \frac{1}{4}$$



$e^+e^- \to \mu^+\mu-$ with Z/γ interference

Photon contribution add terms to both RL and LR, which are independent of $\sin^2\theta_w$

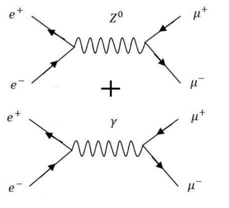
$$\mathcal{M} \sim \left(\mathcal{M}_Z^{RL}(\theta,\theta_w) + \mathcal{M}_\gamma^{RL}(\theta)\right)|RL\rangle + \left(\mathcal{M}_Z^{Lr}(\theta,\theta_w) + \mathcal{M}_\gamma^{LR}(\theta)\right)|LR\rangle$$

$$\Delta_{RL} = \frac{4\sin^2\theta}{6\cos\theta + 5(1+\cos^2\theta)} \quad \Delta_{RL} = 1 \to \quad \theta = \arccos\left(-\frac{1}{3}\right)$$

$$\Delta_{LR} = \frac{\sin^2\theta\sin^2\theta_w}{c^4 + 4s^4\sin^4\theta_w} \quad \Delta_{LR} = 1 \to \quad \theta_w = \arcsin\left(\frac{1}{\sqrt{2}}\cot(\theta/2)\right)$$

Imposing MaxEnt at the same COM angle:

$$\theta = \arccos\left(-\frac{1}{3}\right), \; \sin^2\theta_w = \frac{1}{4}$$



Conclusions

Summary

Maximal entanglement:

- Discards classical physics by principle redictive
- Consistent with QED, which is an isolated solution
- MaxEnt is found in every channel where it was possible
- Open Questions:
  - Relax C, P and T to CPT symmetry?
  - Other interaction theories: chiral, gravity, effective,...
  - RG? IR divergences?
  - Formulate on probabilities and Bell inequalities?

Weak mixing angle

- MaxEnt in weak interactions predict $\sin^2 \theta_w = 0.25$.
- Discarding MaxEnt for Z decay with longitudinal polarization, it is always possible to achieve MaxEnt $\forall \theta_w$.
- How to get closer to experimental value $\sin^2 \theta_w^{exp} \simeq 0.23$?
  - Going to next order
  - Compute more processes: maximization of entanglement over θ_w

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Ideas

Can we apply MaxEnt to

- Quarks?
  No asymptotic states, confinement, no Bell inequalities.
- Flavors?
- WW annihilation, neutrinos,...?
- QED Form Factors: F_1 vs F_2?
- CKM relation to mass ratios?

Thanks!