Large Gauge Symmetry and Memory Effects in Electron-Ion Collisions

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Key Points

1. Generic gauge theories enjoy infinite-dimensional physical symmetries, known as asymptotic or large gauge symmetries.

2. The memory effect is the observable consequence of large gauge symmetry.
   - ⋆ The memory effect is the effect of a vacuum transition on a pair of probes charged under the gauge group.

3. The color memory effect appears in scattering at collider energies in the Regge limit of QCD.

4. In scattering events, large gauge symmetry implies soft radiation (or vacuum transitions) is highly correlated with hard particles.

Key Points

1. Generic gauge theories enjoy infinite-dimensional physical symmetries, known as asymptotic or large gauge symmetries.
   - ⋆ In systems with asymptotic regions or boundaries, part of the gauge symmetry may be physical (as opposed to redundant).
   - ⋆ The physical symmetries have physical consequences.
   - ⋆ Canonical example is the conservation law derived by Noether's theorem.
   - ⋆ Noether has a second theorem for local symmetries.

2. The memory effect is the observable consequence of large gauge symmetry.
   - ⋆ The memory effect is the effect of a vacuum transition on a pair of probes charged under the gauge group.
3. The color memory effect appears in scattering at collider energies in the Regge limit of QCD.
4. In scattering events, large gauge symmetry implies soft radiation (or vacuum transitions) is highly correlated with hard particles.

Noether's First Theorem

- The variation of the action under a symmetry $\phi \to \phi + \hat{\delta}\phi$ is

$$\hat{\delta}S[\phi] \equiv S[\phi + \hat{\delta}\phi] - S[\phi] = \int d^d x\, \partial_\mu K^\mu. \tag{1}$$

- An arbitrary variation of the action takes the form

$$\delta S = \int d^d x[-E(\phi)\delta\phi + \partial_\mu \theta^\mu(\phi;\delta\phi)], \tag{2}$$

  where $E(\phi)$ are the equations of motion and θ^μ is the symplectic current density.
- Consider a deformation of the symmetry by a local function ρ

$$\delta_\rho \phi = \rho\hat{\delta}\phi.$$

  Assuming S depends only on ϕ and $\partial_\mu\phi$, (1) is modified to

$$\delta_\rho S = \int d^d x\Big(\rho\partial_\mu K^\mu + (\partial_\mu\rho)\theta^\mu(\phi;\hat{\delta}\phi)\Big),$$

  and evaluating (2) on $\delta_\rho\phi$ gives

$$\delta S\Big|_{\delta_\rho\phi} = \int d^d x\Big[-E(\phi)\rho\hat{\delta}\phi + (\partial_\mu\rho)\theta^\mu(\phi;\hat{\delta}\phi) + \rho\partial_\mu\theta^\mu(\phi;\hat{\delta}\phi)\Big].$$

[Noether (1918); S. Avery & B. Schwab, hep-th/1510.07038]

Noether's First Theorem (continued)

- Equating the following

$$\delta_\rho S = \int d^d x \Big((\partial_\mu \rho) \theta^\mu(\phi; \hat{\delta}\phi) + \rho \partial_\mu K^\mu \Big),$$

$$\delta S \Big|_{\delta_\rho \phi} = \int d^d x \Big[-E(\phi) \rho \hat{\delta}\phi + (\partial_\mu \rho) \theta^\mu(\phi; \hat{\delta}\phi) + \rho \partial_\mu \theta^\mu(\phi; \hat{\delta}\phi) \Big],$$

one finds

$$0 = \int d^d x \rho(x) \Big[-E(\phi) \hat{\delta}\phi + \partial_\mu \Big(\theta^\mu(\phi; \hat{\delta}\phi) - K^\mu \Big) \Big].$$

Since ρ is arbitrary, the integrand must vanish.

Noether's First Theorem:

The current j^μ of a continuous symmetry is conserved on the equations of motion

$$\partial_\mu j^\mu = E(\phi) \hat{\delta}\phi \approx 0, \qquad \text{where} \qquad j^\mu = \theta^\mu(\phi; \hat{\delta}\phi) - K^\mu.$$

- ⋆ Note, the derivation did not depend on form of $\hat{\delta}\phi$ (other than being infinitesimal), so it applies just as well to local symmetries.

[Noether (1918); S. Avery & B. Schwab, hep-th/1510.07038]

Noether's Second Theorem

Noether's Second Theorem:

The Noether current j^μ of a local symmetry can always be written as

$$j^\mu = S^\mu + \partial_\nu k^{\nu\mu},$$

where $S^\mu \approx 0$ and $k^{\mu\nu} = -k^{\nu\mu}$.

- Consider local symmetries, parametrized by an arbitrary function $\varepsilon(x)$. For simplicity, consider symmetries of the form

$$\delta_\varepsilon \phi = f(\phi)\varepsilon + f^\mu(\phi)\partial_\mu \varepsilon.$$

- To derive an identity involving the equations of motion (which will be independent of ε), focus on ε of compact support.
- Again, compare (1) with (2) evaluated on $\delta_\varepsilon \phi$, (total derivatives will not contribute)

$$\int d^d x \, E(\phi)\delta_\varepsilon \phi = 0.$$

[Noether (1918); S. Avery & B. Schwab, hep-th/1510.07038]

Noether's Second Theorem (continued)

- Since ε is compactly supported, integrate-by-parts to find

$$\int d^d x \, \varepsilon \, \Delta(E) = 0, \qquad \Delta(\cdot) \equiv f(\phi)(\cdot) - \partial_\mu (f^\mu(\phi) \cdot).$$

- Then, since ε is arbitrary, this implies

$$\Delta(E) = 0.$$

- **Comments:**
  - Identity involving only the equations of motion.
  - Implies equations of motion are not all independent
    $\Rightarrow$ some degrees of freedom are gauge.
- **Takeaway:**
  - Theories with local symmetries of compact support are gauge theories.

[Noether (1918); S. Avery & B. Schwab, hep-th/1510.07038]

Noether's Second Theorem (continued)

- Now use identity to derive decomposition of the Noether current. Recall,

$$\int d^d x\, E(\phi)\delta_\varepsilon \phi \quad \overset{\int \text{ by parts}}{\Longrightarrow} \quad \int d^d x\, \varepsilon\, \Delta(E).$$

- This implies the integrands differ by a total derivative

$$E\delta_\varepsilon \phi = \varepsilon \Delta(E) + \partial_\mu S^\mu(E;\varepsilon).$$

- However, since $\Delta(E) = 0$, this implies

$$\partial_\mu S^\mu(E;\varepsilon) = E\delta_\varepsilon \phi.$$

Comments:

  - ⋆ S^μ obeys the same conservation equation as the Noether current j^μ.
  - ⋆ S^μ vanishes on the equations of motion.

- Then, taking the difference

$$\partial_\mu (j^\mu(\varepsilon) - S^\mu(\varepsilon)) = 0,$$

implies (assuming trivial de Rham cohomology)

$$j^\mu(\varepsilon) = S^\mu(\varepsilon) + \partial_\nu k^{\nu\mu}(\varepsilon), \qquad k^{\mu\nu} = -k^{\nu\mu}.$$

[Noether (1918); S. Avery & B. Schwab, hep-th/1510.07038]

Conserved Charges in Gauge Theory

- Decomposition reveals that we can construct conserved charges from $k^{\mu\nu}$

$$Q_\varepsilon = \int_{\sigma=\partial\Sigma} d\sigma_{\mu\nu} k^{\mu\nu}(\varepsilon) \approx \int_\Sigma d\Sigma_\mu j^\mu(\varepsilon).$$

- Conservation of charge is only non-trivial if ε has support at the boundary $\partial\Sigma$ of the Cauchy surface. Otherwise, charge is zero and trivially conserved.
- **Example**: *QED*

$$e^2 j^\mu = \varepsilon e^2 j^\mu_M - (\partial_\nu \varepsilon) F^{\mu\nu}, \qquad e^2 S^\mu = \varepsilon\left(e^2 j^\mu_M + \partial_\nu F^{\mu\nu}\right), \qquad e^2 k^{\mu\nu} = \varepsilon F^{\mu\nu}.$$

$$\boxed{Q_\varepsilon = \frac{1}{e^2}\int_\sigma d\sigma_{\mu\nu}\ \varepsilon F^{\mu\nu}}$$

- **Example**: *Non-abelian gauge theory*

$$g^2 j^\mu = \mathrm{Tr}\left[\varepsilon g^2 j^\mu_M - (\partial_\nu \varepsilon - i[A_\nu, \varepsilon]) F^{\mu\nu}\right],$$

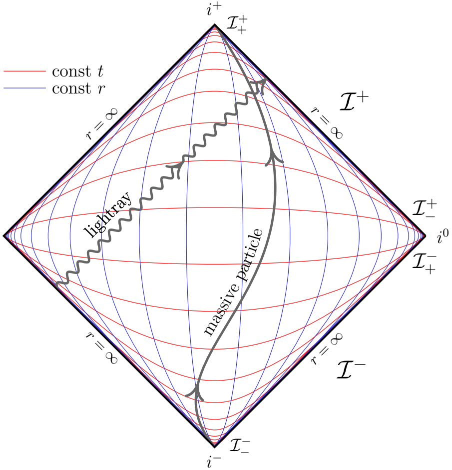
$$g^2 S^\mu = \mathrm{Tr}\left[\varepsilon\left(g^2 j^\mu_M + \partial_\nu F^{\mu\nu} - i[A_\nu, F^{\mu\nu}]\right)\right], \qquad g^2 k^{\mu\nu} = \mathrm{Tr}[\varepsilon F^{\mu\nu}].$$

$$\boxed{Q_\varepsilon = \frac{1}{g^2}\int_\sigma d\sigma_{\mu\nu}\ \mathrm{Tr}[\varepsilon F^{\mu\nu}]}$$

  - ⋆ Recover standard global charge when ε is constant.

Conservation in Minkowski Space

- Use conformal diagram to study asymptotic boundary of flat space.
- Diagram preserves causal structure.



- $\mathcal{I}^\pm$ are natural Cauchy surfaces for massless scattering.
- Charges of local symmetries are

$$Q^\pm_\varepsilon = \int_{\mathcal{I}^\pm_\mp} d\sigma_{\mu\nu} k^{\mu\nu}(\varepsilon).$$

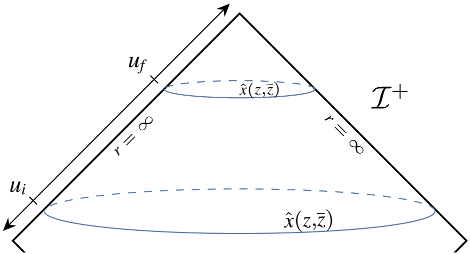
$\mathcal{I}^\pm_\mp$ are boundaries of $\mathcal{I}^\pm$ near i^0.

- One version of conservation is

$$Q^+_\varepsilon = Q^-_\varepsilon .$$

- Instead we'll focus on conservation measured by observers on $\mathcal{I}^+$. (These are natural observers of scattering in the bulk.)

Conservation for Observers at $\mathcal{I}^+$



- Coordinates for observers at $\mathcal{I}^+$:

$$u = t - r, \quad r^2 = \vec{x}^2, \quad \vec{x} = r\hat{x}(z,\bar{z})$$

  - ⋆ $\mathcal{I}^+$: $r \to \infty$ at fixed u
  - ⋆ u is observer time on $\mathcal{I}^+$
  - ⋆ $(z,\bar{z})$ are coordinates on S^2 with covariant derivative D

- Charge:

$$Q^+_\varepsilon = \frac{1}{g^2}\int_{\mathcal{I}^+_-} d^2\hat{x}\,\mathrm{Tr}[\varepsilon F_{ru}]$$

- Study change in charge as a function of observer time u

$$\int_{S^2} d^2\hat{x}\,\mathrm{Tr}[\varepsilon F_{ru}]\Big|_{u_f} - \int_{S^2} d^2\hat{x}\,\mathrm{Tr}[\varepsilon F_{ru}]\Big|_{u_i} = \int_{S^2} d^2\hat{x}\int_{u_i}^{u_f} du\,\mathrm{Tr}\left[D^z\varepsilon F_{zu} + D^{\bar{z}}\varepsilon F_{\bar{z}u} - \varepsilon J_u\right],$$

  where

$$J_u \equiv i[A^z, F_{zu}] + i[A^{\bar{z}}, F_{\bar{z}u}] + g^2 j^M_u\,.$$

- Since ε is arbitrary, obtain local (on S^2) conservation law

$$\Delta F_{ru} = -\int_{u_i}^{u_f} du\,\left[D^z F_{zu} + D^{\bar{z}} F_{\bar{z}u} + J_u\right].$$

- These are the asymptotic symmetries of interest.
- They are physical because they imply a non-trivial conservation law.

Key Points

1. Generic gauge theories enjoy infinite-dimensional physical symmetries, known as asymptotic or large gauge symmetries.
   - ⋆ Noether's 2nd theorem ⇒ physical symmetries are non-vanishing at the boundary.
   - ⋆ They are physical because they imply a non-trivial conservation law.
   - ⋆ We will focus on non-trivial conservation law that arises at null infinity ($\mathcal{I}$).
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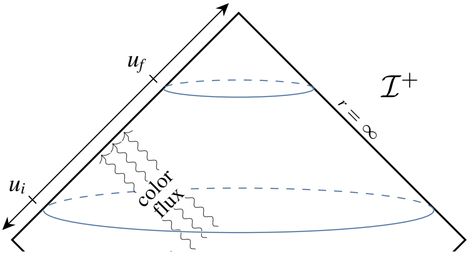
The Color Memory Effect

- Consider scenario w/ vacuum transition
  - Radiative vacuum before u_i and after u_f

$$F_{uz} = F_{z\bar{z}} = 0.$$

$$\Rightarrow A_z = iU\partial_z U^{-1}$$

  - Color flux at intermediate times



- In temporal gauge ($A_u = 0$), rearrange conservation law

$$(D^z \Delta A_z + D^{\bar{z}} \Delta A_{\bar{z}}) = \Delta F_{ru} - \int_{u_i}^{u_f} du\, J_u.$$

- Find vacuum transition as a function of color flux. ("memory")
- **Claim:** Vacuum transition induces permanent rotation in relative colors of 'test' quarks.

[MP, A. Raclariu, & A. Strominger, hep-th/1707.08016]

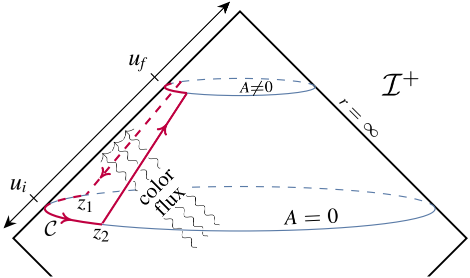
The Color Memory Effect (continued)

- **Claim:** Vacuum transition induces permanent rotation in relative colors of 'test' quarks.
- Quarks begin in singlet at u_i
- At u_f, quarks have acquired relative color rotation whose trace is

$$\mathcal{W}_{\mathcal{C}} \equiv \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P}\exp\left(i \oint_{\mathcal{C}} A \right)$$

where $\mathcal{C}$ is closed contour on $\mathcal{I}^+$.

$\Rightarrow$ measures vacuum transition



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Color Memory in the Regge Limit

- The Regge limit is a limit of fixed momentum transfer, with center-of-mass energy taken to ∞

$$t \ \text{fixed}, \quad s \to \infty.$$

- In deeply inelastic scattering, we find

$$x_{\mathrm{Bj}} \equiv -\frac{q^2}{2P \cdot q} \sim \frac{t}{s} \to 0$$

where q is momentum of exchanged photon and P is the hadron momentum.
- Since x_{Bj} is fixed by kinematics to be the longitudinal momentum fraction carried by the struck parton, the Regge limit probes partons (gluons) carrying a small fraction x of the hadron momentum.
- Introduce lightcone coordinates

$$x^{\pm} = \frac{t \pm x^3}{\sqrt{2}}, \qquad \vec{x} = (x^1, x^2).$$

- To resolve dynamics at small-x, work in infinite momentum frame (IMF). ($P^+ \to \infty$ for hadron moving in x^+ direction).

Color Memory in the Regge Limit (continued)

- To see why IMF resolves dynamics at small-x, notice typical lifetime of a parton with lightcone momentum $k^+ = xP^+$

$$\Delta x^+ \sim \frac{1}{k^-} = \frac{2k^+}{m_\perp^2} = x\frac{2P^+}{m_\perp^2}$$

- Hence, we can use a Born-Oppenheimer type approximation and treat large-x d.o.f. as static sources for gluons at small-x.
- Next from longitudinal spread

$$\Delta x^- \sim \frac{1}{k^+} = \frac{1}{xP^+}$$

  find large-x d.o.f. are highly localized in x^-.
- For purposes of small-x dynamics, fixing $A_+ = 0$ gauge, we can approximate large-x d.o.f. by a color shockwave traveling in the x^+ direction

$$g^2 J_M^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{x}).$$

  - ⋆ Resembles localized color flux through $\mathcal{I}^+$ that induces vacuum transition.

Color Memory in the Regge Limit (continued)

- Taking static field configurations, $A_- = 0$ and no long. mag. fields ($F_{ij} = 0$), and integrating the only non-trivial component of the YM equations, one finds

$$-\partial_i \Delta A_i = \int_{x_i^-}^{x_f^-} J_-, \qquad J_- = -\delta(x^-)\rho(\vec{x}) - i[A_i, \partial_- A_i].$$

- Resembles vacuum transition memory formula, but to make precise, must place at $\mathcal{I}^+$.
- First must identify analogue of IMF for $\mathcal{I}^+$ observer
  - ⋆ LC coordinates are nice coordinates for the IMF because they transform simply under boosts in x^3 direction

$$(x^+, x^-) \to (\lambda x^+, \lambda^{-1} x^-).$$

  - ⋆ The IMF is reached by taking $\lambda \to \infty$. In LC coordinates, can readily obtain IMF configurations from configurations in other inertial frames.

[A. Ball, MP, A. Raclariu, A. Strominger & R. Venugopalan, hep-th/1805.12224]

The Regge Limit and Null Infinity

- $(u,r,z,\bar{z})$ do not transform nicely (*i.e.* by scaling) under boosts in x^3 direction.
- *Trick:* obtain nice coordinates by singular coordinate transformation

$$(r,u,z,\bar{z}) \to (\lambda r, \lambda^{-1}u, \lambda^{-1}z, \lambda^{-1}\bar{z}), \qquad \lambda \to \infty.$$

- In new coordinates, further rescalings are boosts in x^3.
- S^2 is flattened to transverse plane.
- Related to LC coordinates by

$$x^+ = \sqrt{2}r, \qquad x^- = \frac{1}{\sqrt{2}}(u + rz\bar{z}), \qquad x^1 + ix^2 = 2rz.$$

- Can identify $r \to \infty$ limit with $x^+ \to \infty$ limit!
- However, vacuum transition formula was x^+-independent
  $\Rightarrow$ can place at $\mathcal{I}^+$ for free!

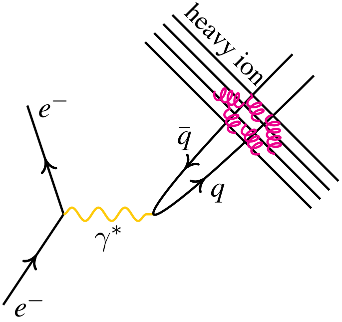
Small-x gluon field configurations are the vacuum-to-vacuum field configurations governed by the conservation law for large gauge symmetry.

- Can we measure these?
- In other words, what are the analogues of the "test" quarks?

[A. Ball, MP, A. Raclariu, A. Strominger & R. Venugopalan, hep-th/1805.12224]

Color Memory in Collider Observables

- *Goal:* seek observables sensitive to vacuum-to-vacuum configurations.
- Consider electron-ion DIS.



- Focus on process where virtual photon fluctuates into singlet quark-antiquark pair.
- In eikonal approximation, shockwave induces color rotation on each quark.
- Forward scattering amplitude:

$$\mathcal{S}(\vec{x}_1,\vec{x}_2) = \frac{1}{N_c}\mathrm{Tr}\Big[U(\vec{x}_1)U^\dagger(\vec{x}_2)\Big] = \mathcal{W}_\mathcal{C}.$$

Identify amplitude with quark dipole color rotation!

[A. Ball, MP, A. Raclariu, A. Strominger & R. Venugopalan, hep-th/1805.12224]

Color Memory in Collider Observables (continued)

- To obtain observable, must average over color sources (CGC).
  - Large random rotations average to zero, small rotations approx. identity
  - Emergent scale: size of dipole where transition occurs.
- *Dipole cross-section*: (via optical theorem)

$$\sigma_{\text{dipole}}(x,\vec{r}) = 2\int d^2\vec{b}\,[1 - \langle \text{Re}\,\mathcal{S}(\vec{x}_1,\vec{x}_2)\rangle],$$

  where $\vec{r} = \vec{x}_1 - \vec{x}_2$ and $\vec{b} = (\vec{x}_1 + \vec{x}_2)/2$.
- *Inclusive DIS virtual photon-heavy ion cross-section*:

$$\sigma_{\gamma^*\text{ion}}(x,Q^2) = \int_0^1 dz \int d^2\vec{r}\,|\Psi(z,\vec{r},Q^2)|^2_{\gamma^*\to q\bar{q}}\sigma_{\text{dipole}}(x,\vec{r}),$$

  where $|\Psi(z,\vec{r},Q^2)|^2_{\gamma^*\to q\bar{q}}$ is probability for $\gamma^* \to q\bar{q}$ with dipole of size $\vec{r}$ and quark carrying momentum fraction z.

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   - ⋆ The color memory effect is a *measure* of the vacuum transition.
3. The color memory effect appears in scattering at collider energies in the Regge limit of QCD.
   - ⋆ Vacuum-to-vacuum transitions governed by the large gauge conservation law appear in the Regge limit.
   - ⋆ Observables in the Regge limit are sensitive to these transitions.
4. In scattering events, large gauge symmetry implies soft radiation (or vacuum transitions) is highly correlated with hard particles.

Large Gauge Symmetry and Quantum Information

- Color rotation $\rightarrow$ learn something about flux of color radiation through $\mathcal{I}^+$
- Imagine dense array of "test" quarks $\rightarrow$ can fully determine ΔA_z

- *Ask:* To what extent can we distinguish scattering events in the bulk?
- Answered by Carney, Chaurette, Neuenfeld, & Semenoff (hep-th/1706.03782, hep-th/1710.02531), (see also Strominger, hep-th/1706.07143)
  - Study scattering in QED and gravity
  - Determine reduced density matrix for scattered hard particles by tracing over soft radiation
  - Find soft radiation decoheres nearly all momentum superpositions of outgoing hard particles
  - Intuition: radiation is essentially classical and distinguishes different scattering events

Large Gauge Symmetry and Quantum Information

- For simplicity, focus on QED.
- In scattering, analogue of conservation law is the Ward identity

$$\langle\beta|\big(Q_\varepsilon^+\mathcal{S} - \mathcal{S}Q_\varepsilon^-\big)|\alpha\rangle = 0.$$

- As before, use equations of motion (Gauss constraint) to write as

$$Q_\varepsilon^+ = \int_{\mathcal{I}^+} dud^2\hat{x}\;\varepsilon j_u^M - \frac{1}{e^2}\int_{\mathcal{I}^+} dud^2\hat{x}\big(D^z\varepsilon F_{uz} + D^{\bar{z}}\varepsilon F_{u\bar{z}}\big) \equiv Q_H^+ + Q_S^+.$$

- Need similar expansion of charge Q_ε^- near $\mathcal{I}^-$ to determine action on $|\alpha\rangle$.
- Rearrange and simplify

$$\begin{aligned}\langle\beta|\big(Q_S^+\mathcal{S} - \mathcal{S}Q_S^-\big)|\alpha\rangle &= -\langle\beta|\big(Q_H^+\mathcal{S} - \mathcal{S}Q_H^-\big)|\alpha\rangle \\ &= \Omega_\varepsilon(\beta,\alpha)\langle\beta|\mathcal{S}|\alpha\rangle,\end{aligned}$$

where

$$\Omega_\varepsilon(\beta,\alpha) = \frac{1}{4\pi}\int d^2\hat{x}\;\varepsilon\,D^2\tilde{\Omega}_{\beta\alpha} \sim \int d^2\hat{x}\;\varepsilon r^2 E_r,$$

$$\tilde{\Omega}_{\beta\alpha}(\hat{x}) = -\sum_{k\in\beta} Q_k \log(p_k\cdot\hat{q}) + \sum_{k\in\alpha} Q_k \log(p_k\cdot\hat{q}), \qquad \hat{q}^\mu = (1,\hat{x}).$$

[CCNS - hep-th/1706.03782, hep-th/1710.02531]

Large Gauge Symmetry and Quantum Information

- Why soft? $\to$ Express in terms of standard creation and annihilation operators.

$$Q_S^+ = \frac{1}{8\pi e}\int d^2\hat{x}\left[D^z\varepsilon\partial_z\hat{x}^i + D^{\bar{z}}\varepsilon\partial_{\bar{z}}\hat{x}^i\right]\lim_{\omega\to 0}\sum_{\alpha=\pm}\left[\omega\varepsilon_i^{\alpha *}a_\alpha(\omega\hat{x}) + \omega\varepsilon_i^{\alpha}a_\alpha^\dagger(\omega\hat{x})\right]$$

- In quantum scattering, conservation law determines soft photon content.
- Namely, suppose $Q_S^-|\alpha\rangle = 0$, then, from

$$\langle\beta|\left(Q_S^+\mathcal{S} - \mathcal{S}Q_S^-\right)|\alpha\rangle = \Omega_\varepsilon(\beta,\alpha)\langle\beta|\mathcal{S}|\alpha\rangle,$$

  we find

$$|\beta\rangle = \exp\left[e\int_0^\Lambda \frac{d\omega d^2\hat{x}}{2(2\pi)^3}\left[D^z\tilde{\Omega}_{\beta\alpha}\partial_z\hat{x}^i + D^{\bar{z}}\tilde{\Omega}_{\beta\alpha}\partial_{\bar{z}}\hat{x}^i\right]\sum_{\alpha=\pm}\left[\varepsilon_i^{\alpha *}a_\alpha(\omega\hat{x}) - \varepsilon_i^{\alpha}a_\alpha^\dagger(\omega\hat{x})\right]\right]|\hat{\beta}\rangle$$
$$\equiv W_{\beta\alpha}|\hat{\beta}\rangle.$$

- These are a generalization of the Fadeev-Kulish states.

[CCNS - hep-th/1706.03782, hep-th/1710.02531]
[Fadeev & Kulish (1970), KPRS-hep-th/1705.04311]

Large Gauge Symmetry and Information Theory

- Consider evolution of density matrix associated to an incoming state

$$|\alpha\rangle\langle\alpha| \quad \to \quad \mathcal{S}|\alpha\rangle\langle\alpha|\mathcal{S}^\dagger \equiv \rho.$$

- Trace over soft radiation to compute reduced density matrix

$$\rho_{\rm red} = {\rm Tr}_{\rm soft}(\rho) = \sum_{\beta\beta'}\langle 0|W^\dagger_{\beta'\alpha}W_{\beta\alpha}|0\rangle\, \mathcal{S}_{\beta\alpha}\mathcal{S}^*_{\beta'\alpha}|\hat{\beta}\rangle\langle\hat{\beta}'|, \qquad \mathcal{S}_{\beta\alpha} = \langle\beta|\mathcal{S}|\alpha\rangle,$$

$$\langle 0|W^\dagger_{\beta'\alpha}W_{\beta\alpha}|0\rangle = \left(\frac{\lambda_{IR}}{\Lambda}\right)^{\Gamma_{\beta\beta'}} = \begin{cases} 1, & \Gamma_{\beta\beta'} = 0 \\ 0, & \Gamma_{\beta\beta'} > 0 \end{cases},$$

$$\Gamma_{\beta\beta'} = \frac{e^2}{2}\int \frac{d^2\hat{x}}{(2\pi)^3}\gamma^{z\bar{z}}\partial_z(\tilde{\Omega}_{\beta\beta'})\partial_{\bar{z}}(\tilde{\Omega}_{\beta\beta'}).$$

- α contribution cancels between two dressings and drops out.
- Integrand of $\Gamma_{\beta\beta'}$ is the norm of a vector on $S^2 \Rightarrow \Gamma_{\beta\beta'} \geq 0$.
- $\Gamma_{\beta\beta'} = 0$ when $\tilde{\Omega}_{\beta\beta'}$ is constant. ($\sim$ radial electric fields match)
- In QED, one finds $\rho_{\rm red}$ is diagonal, hence completely decohered.

[CCNS - hep-th/1706.03782, hep-th/1710.02531]

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4. In scattering events, large gauge symmetry implies soft radiation (or vacuum transitions) is highly correlated with hard particles.
   - ⋆ Soft radiation nearly completely decoheres outgoing momentum superpositions.

Conclusions/Outlook

- Large gauge (asymptotic) symmetries are symmetries of generic gauge theories with physical consequences.
- Color memory, the observable consequence of large gauge symmetry in Yang-Mills theory, appears in scattering at collider energies in the Regge limit of QCD.
- Appearance of asymptotic symmetries in Regge limit is intriguing and deserves further investigation.
- Entanglement between degrees of freedom at small-x and degrees of freedom at large-x due to large gauge symmetry?