# The Substructure of Jets: a theorist's cartoon.

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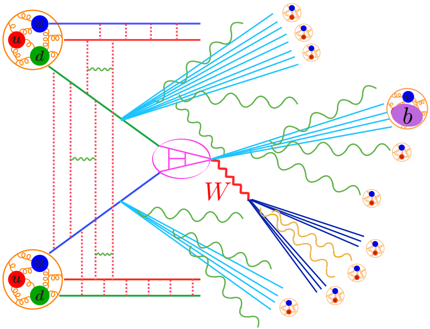
Probing Quark-Gluon Matter With Jets

July 25, 2018

Quantum Chromodynamics:

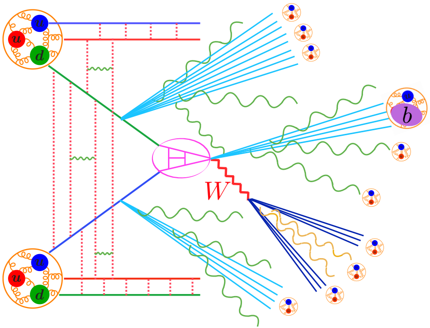
- High energy: quasi-conformal, weakly coupled, massless partons (asymptotic freedom).
- Low energies: strongly confines into bound states.




- Modern particle accelerators probe all these regimes.

(Image courtesy of Ian Moult.)

# The Basic Questions:




- How does an event at high energy transition from interacting partons to strongly bound hadrons?
- How does the reverse happen?
- How does QCD quantum mechanically transfer momentum, spin, charge, flavor, etc., between the ultra-violet and the infra-red?

Jet substructure was motivated (officially) by *none* of these questions.

- arXiv:0802.2470:
  **"Jet substructure as a new Higgs search channel at the LHC"**

But it needed them answered.

- Their Goal: Distinguish the progenitors of the jet based on the jet's shape.
- Our Goal: exploit jet substructure to understand the patterns of QCD radiation,

Luckily these are not mutually exclusive activities.

# Jet substructure, a (personal) path to understanding.

- Use energy-flow variables to classify the QCD states.
- Use power counting to organize shape variable phase-space.
- Use grooming to give well-defined jet structures.
- Calculate.

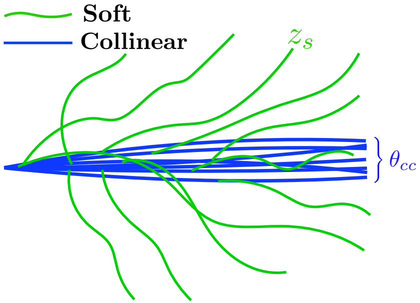
# Power Counting The "States" of Perturbation Theory

- Quarks and gluons are massless when on-shell: $p_g^2 = p_q^2 = 0$.
- High energies, no intrinsic scale, except hard interaction.
- Continuous spectrum and long range-interactions.

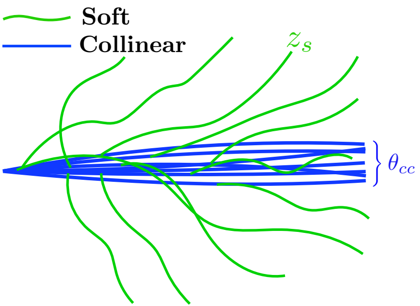
$$m_{ab}^2 = (p_a + p_b)^2 = 4E_aE_b\sin^2\frac{\theta_{ab}}{2} \sim Q^2 z_a z_b \theta_{ab}^2$$

**Close to on-shell:**

- Soft:
  $$\left(\sum p_s\right)^2 \sim Q^2 z_s^2 \ll Q^2$$
- Collinear:
  $$\left(\sum p_c\right)^2 \sim Q^2 \theta_{cc}^2 \ll Q^2$$



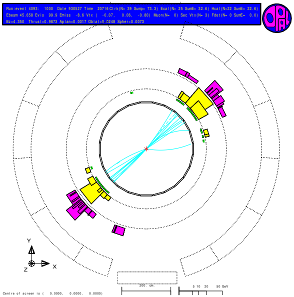
# Power Counting The “States” of Perturbation Theory

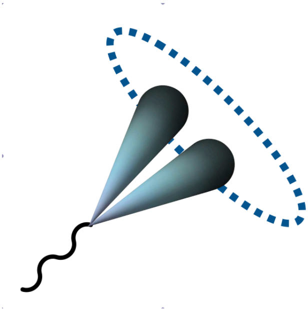



- $z_\mathrm{s}$ and $\theta_{cc}$ will be set by the phase-space region we are in.
- Phase space will be organized by infra-red and collinear safe observables.

# Example: Finding Color Singlet Objects

In rest frame, color singlet objects decay back-to-back:





Boosted, will still exhibit two pronged decays within a fatter jet.

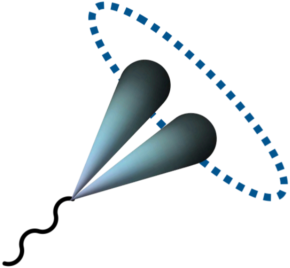
- So how do we seperate QCD jets from the singlet resonances?

# Color Decaying Resonances Versus QCD

**Z-jets:** Mass and energy gives natural opening angle:

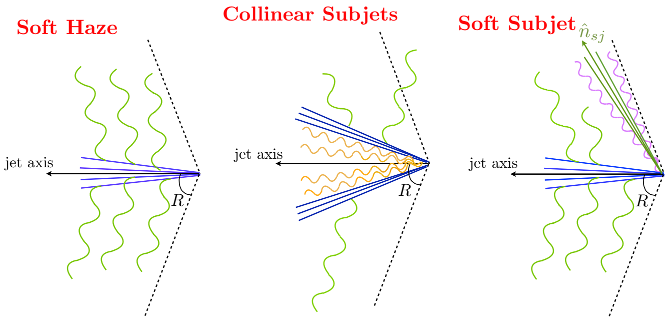
$$\frac{M_Z^2}{Q^2} = \frac{(p_q + p_{\bar{q}})^2}{Q^2} \sim R^2$$

**QCD-jets:** No characteristic mass or opening angle, no preference for structure, populates all regions.



# QCD will populate both 1 and 2 prong jets.

When are subsequent emissions at the same scale or strongly ordered?



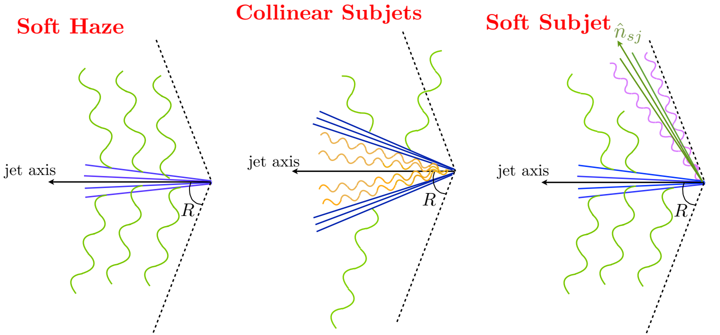

- There exists a general organization of the phase-space, applicable to many observables.
- Parametrically separate distinct regions.
- Bulk energy flow can be identical. (Mass, Broadening, etc.)

**[Larkoski, Moult, DN 1409.6298]**

# Energy Correlation Functions

$$e_2^{(\alpha)} = \sum_{i<j\in J} z_i z_j \theta_{ij}^{\alpha}$$

$$e_3^{(\alpha)} = \sum_{i<j<k\in J} z_i z_j z_k \Big(\theta_{ij}\theta_{jk}\theta_{ki}\Big)^{\alpha}$$



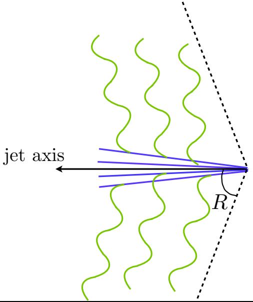
# Energy Correlation Functions: One Prong

$$e_2^{(\alpha)} = \sum_{i<j\in J} z_i z_j \theta_{ij}^{\alpha} \ll 1$$

Power Counting:

$$z_s \sim e_2^{(\alpha)} \qquad\qquad \theta_{cc}^{\alpha} \sim e_2^{(\alpha)}$$

**Soft Haze**




Correlate Two Soft and One Collinear:

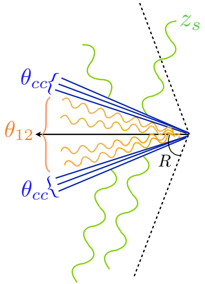
$$e_3^{(\alpha)} \sim \left(z_c\, z_s^2\, \theta_{sc}^{2\alpha}\theta_{ss}^{\alpha}\right) \sim (e_2^{(\alpha)})^2$$

# Energy Correlation Functions: Two Prongs

$$e_2^{(\alpha)} = \sum_{i<j\in J} z_i z_j \theta_{ij}^{\alpha} \quad \sim z_1 z_2 \theta_{12}^{\alpha} + \ldots$$

$$e_3^{(\alpha)} = \sum_{i<j<k\in J} z_i z_j z_k \Big(\theta_{ij}\theta_{jk}\theta_{ki}\Big)^{\alpha} \quad \sim z_1 z_2 \theta_{12}^{\alpha}\Big(\theta_{12}^{\alpha}\theta_{cc}^{\alpha} + z_s\Big)\ldots$$

**Collinear Subjets**

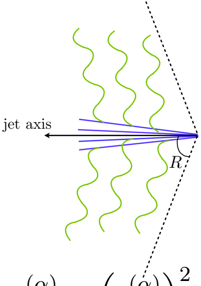



$$z_s \ll e_2^{(\alpha)} \text{ or } \theta_{cc} \ll e_2^{(\alpha)}$$

Soft haze and subjet regions parametrically separated if

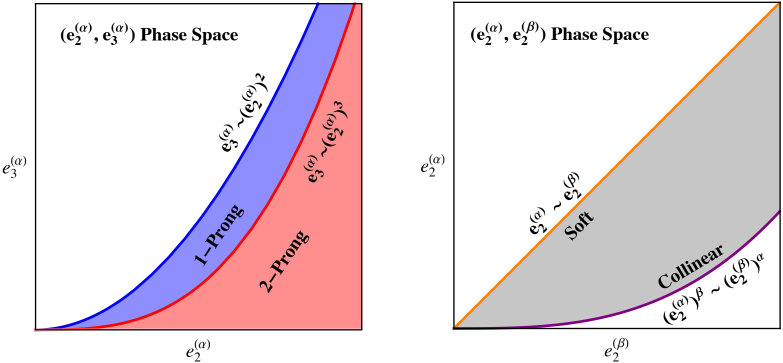
$$e_3^{(\alpha)} \sim \Big(e_2^{(\alpha)}\Big)^3$$

**Soft Haze**



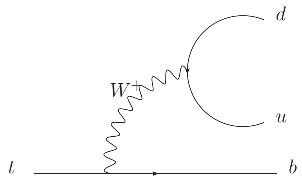

$$e_3^{(\alpha)} \sim \Big(e_2^{(\alpha)}\Big)^2$$

# Phase Space for $1 \to 2$ Splitting



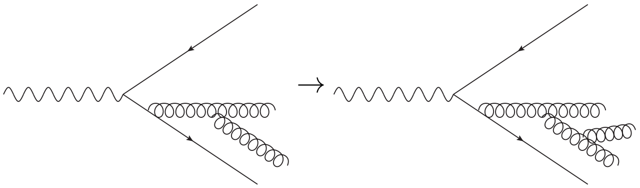

Single observable distinguishes 1-prong (**background**) from the 2-prong (**signal**) regions:

$$D_2 = \frac{e_3^{(\alpha)}}{\left(e_2^{(\alpha)}\right)^3}$$




Need resolution scale one splitting lower:

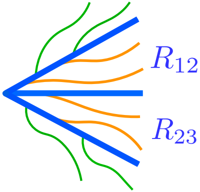
$$e_4^{(\alpha)} = \sum_{i<j<k<\ell\in J} z_i z_j z_k z_\ell \Big(\theta_{ij}\theta_{jk}\theta_{k\ell}\theta_{\ell i}\theta_{\ell j}\theta_{ik}\Big)^\alpha$$



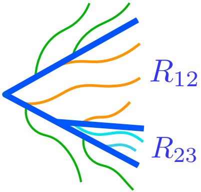
**[Larkoski, Moult, DN 1411.0665]**

# Varieties of $1 \to 3$ Splittings

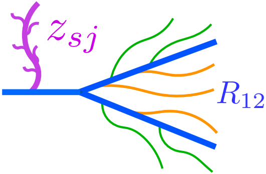
Unordered Subjets:

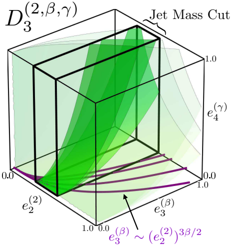



Strongly Ordered:



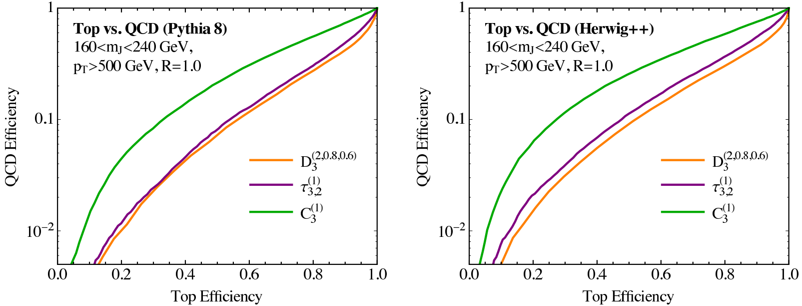

Soft & Collinear Subjets:




$R$ is opening angle.

Comparison of Discriminators At 8 TeV LHC:




$$D_3^{\alpha,\beta,\gamma} = e_4^{(\gamma)} \left( \frac{\left(e_2^{(\alpha)}\right)^{\frac{3\gamma}{\alpha}}}{\left(e_3^{(\beta)}\right)^{\frac{3\gamma}{\beta}}} + x \frac{\left(e_2^{(\alpha)}\right)^{\frac{2\gamma}{\alpha}-1}}{\left(e_3^{(\beta)}\right)^{\frac{2\gamma}{\beta}}} + y \frac{\left(e_2^{(\alpha)}\right)^{\frac{2\beta}{\alpha}-\frac{\gamma}{\alpha}}}{\left(e_3^{(\beta)}\right)^{2}} \right)$$

$$x \sim \left( \frac{(p_T^{cut})^2}{m_{top}^2} \right)^{\frac{\alpha\gamma}{\beta}-\frac{\alpha}{2}}, \quad y \sim \left( \frac{(p_T^{cut})^2}{m_{top}^2} \right)^{\frac{5\gamma}{2}-2\beta}$$

# Jets versus Color Singlet Resonances

- Power Counting gives quick organization to phase-space.

What about **calculating**?

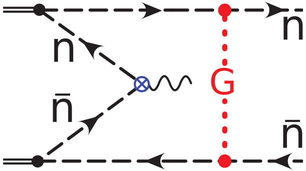
- Resonance: invoke Narrow Width.
- Calculate decay distribution isolated from production.

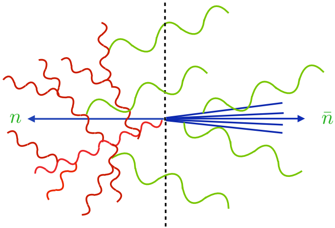
QCD??

# Calculating: Jets versus Color Singlet Resonances

Sensitivity to soft radiation destroys universality:

- Contamination from underlying event → distinct spectra for $e^+e^-, pp, ep, \ldots$.
- Non-global logarithms: soft radiation entangles distinct jet regions. [**Dasgupta, Salam hep-ph/0104277**]



# Calculating for Jets in $pp$

Two options:

- (A) Figure out underlying event and NGLs, then calculate.
- (B) Groom the Jet.

Option (A) has seen lots of *theoretical* progress:
(Incomplete list....)

**[Banfi, Marchesini, Smye hep-ph/0206076 ]**, **[Weigert hep-ph/0312050 ]**
**[Hatta 0810.0889]**, **[Hagiwara, Hatta, Ueda 1507.07641]**,
**[Avsar, Hatta, Masuo 0903.4285]**,
**[Forshaw, Kyrieleis, Seymour 0808.1269]**, **[Forshaw, Seymour, Siodmok 1206.6363]**,
**[Angeles-Martinez, Forshaw, Seymour 1510.07998]**
**[Hatta, Iancu, Mueller, Triantafyllopoulos 1710.06722]**,
**[Larkosk, Moult, Neill 1501.04596, 1609.04011]**,
**[Neill 1508.07568,1610.02031]**,
**[Caron-Huot 1501.03754]**,
**[Becher, Neubert, Rothen, Shao 1508.06645, 1605.02737]**,
**[Rothstein, Stewart 1601.04695 ]**,

**[Schwartz, Yan, Zhu 1703.08572]**

Two options:

- (A) Figure out underlying event and NGLs, then calculate.
- (B) Groom the Jet.

Option (B) has seen lots of *practical* progress:

(Incomplete list....)

**[Dasgupta, Fregoso, Marzani, Salam 1307.0007]**, **[Dasgupta, Fregoso, Marzani, Powling 1307.0013]**,
**[Larkoski, Marzani, Soyez, Thaler 1402.2657]**,
**[Frye, Larkoski, Schwartz, Yan 1603.06375, 1603.09338]**,
**[Dasgupta, Powling, Schunk, Soyez 1609.07149]**,
**[Salam, Schunk, Soyez 1612.03917]**,
**[Baron, Marzani, Theeuwes 1803.04719]**,
**[Marzani, Schunk, Soyez 1704.02210, 1712.05105]**,
**[Larkoski, Marzani, Thaler, Tripathee, Xue 1704.05066]**,
**[Larkosk, Moult, Neill 1708.06760, 1710.00014]**,
**[Moult, Neill, Nachman 1710.06859]**,
**[Makris, Neill, Vaidya 1712.07653]**,
**[Hoang, Mantry, Pathak, Stewart 1708.02586]**,
**[Kang, Lee, Liu, Ringer, 1803.03645]**

# Calculating for Jets in $pp$

- Option (A), buy me a beer.
- Rest of talk: option (B)

Need:

- List of Particles $P$, List of Jets $J$.
- Single particle metric on momentum space $d(p_i) \equiv d_i$, and two particle metric $d(p_i, p_j) \equiv d_{ij}$.
- Merging prescription $M(p_i, p_j) \to p_{ij}$.
- Radius $R$.

$$d(p_i) = (p_i^0)^{2\alpha}, \qquad d(p_i, p_j) = \min\Big((p_i^0)^{2\alpha}, (p_j^0)^{2\alpha}\Big)\frac{\theta_{ij}}{R}$$

Common $\alpha = -1, 0, 1$.
**Most important choice is 0.**

Don't be mislead by -1.

# Aside: Clustering Jet Algorithms

$$d(p_i) = (p_i^0)^{2\alpha}, \qquad d(p_i, p_j) = \min\Big((p_i^0)^{2\alpha}, (p_j^0)^{2\alpha}\Big)\frac{\theta_{ij}}{R}$$

Start: $P = \{p_1, p_2, ...., p_n\}$ and $J = \{\}$

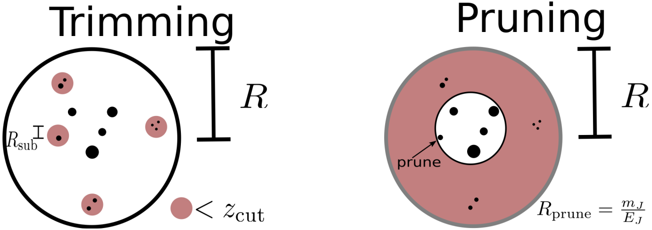
1. Compute $d_i$ for all $p_i \in P$, compute $d_{ij}$ for all $p_i, p_j \in P$.
2. If the smallest is some $d_i$, move $i$ from $P$ to $J$
3. If smallest is some pair $p_i, p_j$, merge them within $P$
4. Repeat till $P$ is empty.

# Grooming the Jet

The Jet Algorithm builds a Clustering Tree and List of Jets. **Modern Grooming:** Examine the Tree or List of Jets for **evil**, remove it.

- Trimming
- Pruning
- Modified Mass Drop Tagger/Soft Drop

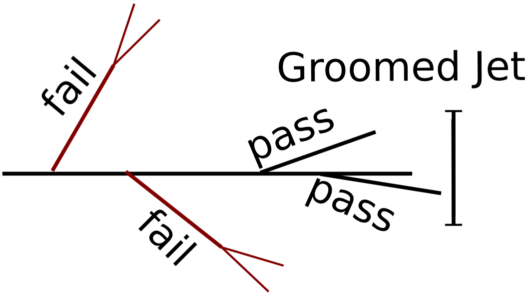
- **Trimming: (Krohn et al. 0912.1342)**
  Recluster jet at smaller $R_{\text{sub}}$, drop subjets that are too soft.
- **Pruning: (Ellis et al. 0912.0033)**
  Recluster jet at smaller $R_{\text{prune}} \propto \frac{m_J}{E_J}$.
  When merging, drop soft branches **before** clustering.



# Modified Mass Drop Tagger/Soft Drop

- Recluster Jet with $\alpha = 0$ (only angles!)
- *Decluster* jet, dropping branches that are too soft.
- When both branches are hard enough, that is the groomed jet.

$$\frac{\min(p_L^0, p_R^0)}{p_L^0 + p_R^0} > z_{\rm cut}$$



# Passing and Failing jet algorithms

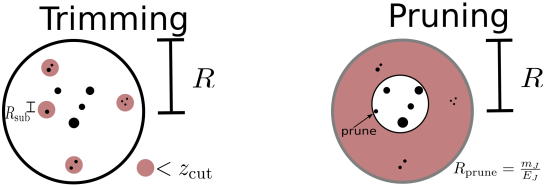
- Modified Mass Drop Tagger: **No angular parameter sets width of groomed jet.**
- Pruning and Trimming do have angular parameters.

*And that is why they fail.*

# Pruning and Trimming **fail**

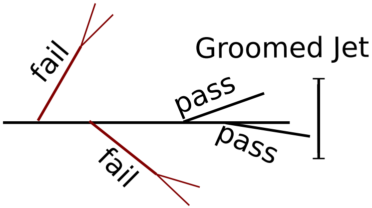
Why? Pruning and Trimming jet has a ***set*** angular area.

- Trimming makes holes of size $R_{\text{sub}}^2$
- Pruning shrinks jet down to $R_{\text{prune}}^2$
- Modified Mass Drop Tagger/Soft Drop: the groomed jet has **no** area.




Area of Jet: **[Cacciari, Salam, Soyez 0802.1188]**

# Examining Modified Mass Drop Tagger/Soft Drop




**No angular parameter sets width of groomed jet.**

- *However*, it forces a collinear limit.
- C/A: decreasing angular scale as we decluster.
- QCD has dynamically imposed Angular Ordering: Soft radiation top of tree.
- First declustered branches most likely to fail.

# "Collinear Factorization the Best Factorization!"

mMDT/Soft Drop forces collinear limit.

- Lessons from fragmentation: Universality
- $z_h$ fraction of energy/transverse momentum carried by hadron $h$:
- $C$ experimental cuts or tagged process.

$$\frac{d\sigma}{dz_h}(C) = \sum_{i=g,q} \int_{z_h}^{1} \frac{dx}{x} \Big[H \otimes f \otimes f\Big]_i \Big(x, C, Q, \mu\Big) D_{i\to h}\Big(\frac{z_h}{x}, \Lambda, \mu\Big)$$

- $D_{i\to h}\Big(z_h, \Lambda, \mu\Big)$ is universal quantity.
- $\Big[H \otimes f \otimes f\Big]_i$ process dependent number of quarks and gluons.

# Collinear Factorization and mMDT/Soft Drop

mMDT/Soft Drop enjoys a collinear factorization:

$$\frac{d\sigma}{d\mathcal{M}}(C) = \sum_{i=g,q} \Big[H \otimes f \otimes f\Big]_i \Big(C, Q, R, z_{\text{cut}}, \mu\Big) \otimes \mathcal{J}_i\Big(\mathcal{M}, R, P_T, z_{\text{cut}}, \mu\Big)$$

- $\mathcal{M}$ an (almost) arbitrary set of measurements on the groomed constituents.
- $R$ jet radius,
- $P_T$ transverse momentum of jet (to beam, or jet energy)
- $z_{\text{cut}}$ grooming parameter.

**[Larkoski, Marzani, Soyez, Thaler 1402.2657]**,

**[Frye, Larkoski, Schwartz, Yan 1603.06375, 1603.09338]**

# Collinear Factorization and mMDT/Soft Drop

mMDT/Soft Drop enjoys a collinear factorization:

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Note that $C$ and $\mathcal{M}$ can have other hierarchies of scales!
Examples:

- $\mathcal{M} = m_J \ll P_T$, jet mass of groomed constituents

  **[Frye, Larkoski, Schwartz, Yan 1603.06375, 1603.09338]**

- $R \ll 1$

  **[Kang, Lee, Liu, Ringer, 1803.03645]**

- $\mathcal{M} = \{m_J, D_2\}$ two versus one prong jets

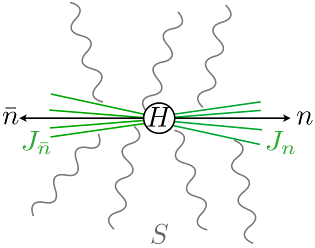
  **[Larkosk, Moult, Neill 1708.06760, 1710.00014]**

$\rightarrow$ **Further factorization/resummation of $\mathcal{J}_i$**

# Soft-Collinear Effective Field Theory

SCET, a grammar for factorization:

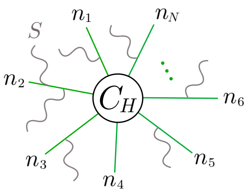
- Operators defined by the IR limits of QCD.
- Constrained by symmetries of QCD.
- Constructs a systematic expansion of QCD observables.
- All momentum transfers between sectors expanded according to the power counting.
- Renormalization Scales and Subtractions to distinguish modes.



# Factorization as a Theorem

**Factorization:**
Separating a cross-section into a finite product of non-trivial distributions, with support in distinct momentum regions.

$$\frac{d\sigma}{d\tau} = \mathrm{tr}[C_H(Q^2,\mu)S(\tau,\mu)C_H^\dagger(Q^2,\mu)] \otimes J_{n_1}(\tau,\mu) \otimes J_{n_2}(\tau,\mu) \otimes ... \\ + O\Big(\frac{\tau^2}{Q^2}\Big)...$$

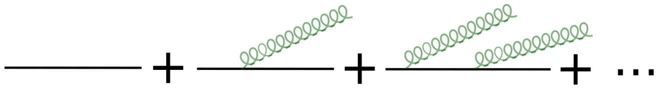



- $J$ energetic jets or beams.
- $S$ isotropic soft radiation.
- Non-perturbative effects contained in $J$ or $S$.
- Universality of Asymptotics of QCD (Reusability of $H, J, S$)
- Resummation with Renormalization Group.

# Coherent Branching

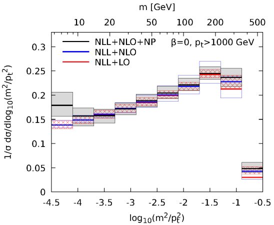
Probabilistic Exploration of Phase-Space

- Splitting Probability $dP \propto dS\, e^{-S}$
- $S$ integrated splitting function
  $S \sim \int_0^1 dz \int \frac{d\theta}{\theta} \mathcal{M}(z,\theta) P(z;\theta)$
- **“Path-Integral for Jets”**: $\rightarrow$
  Sum over all histories consistent with observed jet.
- Angular-ordered integration/sum over multiple emissions.

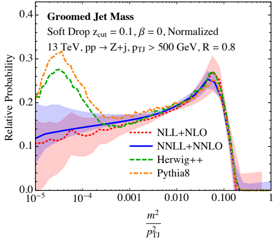


**Sophisticated form: Equations for evolution of x-sec generating functional.**

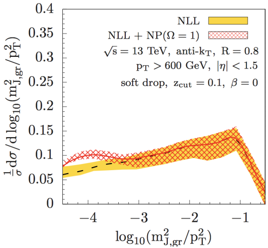
# Selected Results: Theory Jet Mass




[Marzani, Schunk, Soyez 1704.02210, 1712.05105]

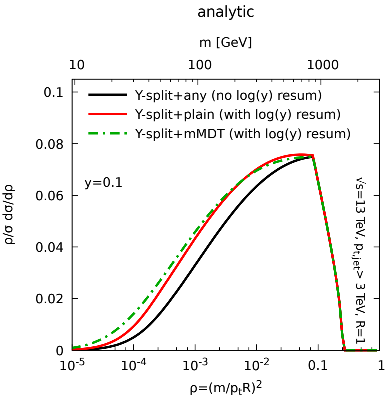



[Frye, Larkoski, Schwartz, Yan 1603.06375, 1603.09338]

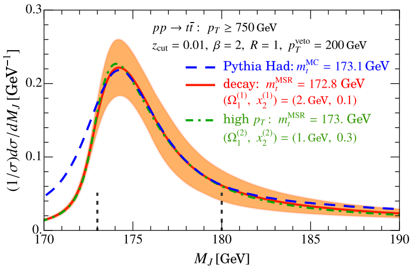



[Kang, Lee, Liu, Ringer, 1803.03645]

# Selected Results: Theory Jet Mass With Tagged 2-prongs

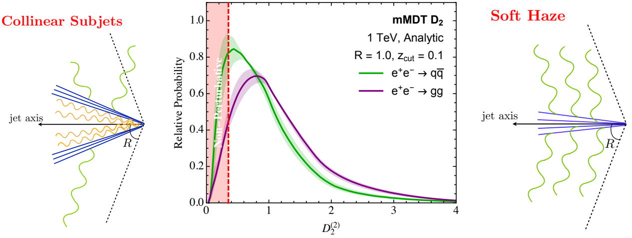



**[Dasgupta, Powling, Schunk, Soyez 1609.07149]**

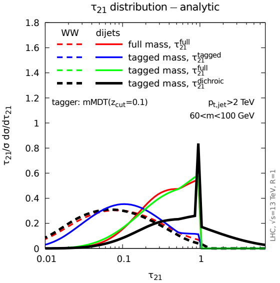



**[Hoang, Mantry, Pathak, Stewart 1708.02586]**

# Selected Results: Theory One versus Two pronged Jets

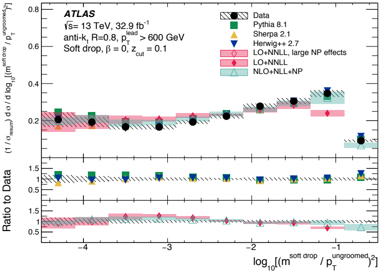



**[Larkoski, Moult, Neill 1708.06760, 1710.00014]**

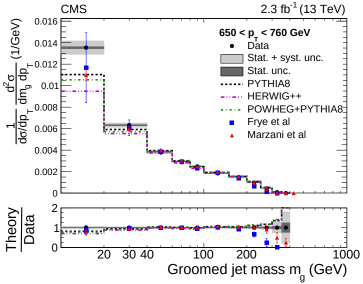



**[Salam, Schunk, Soyez 1612.03917]**

# Selected Results: Experiment Jet Mass




ATLAS




CMS

# Outlook

**A serious question:**

- $\alpha_S$ from jet substructure?

**Things I left out...**

- Fragmentation, fractal observables, track observables.
- Machine Learning.
- Fine/Coarse graining energy distributions: Aggregrate representations of jet.

**Review [Larkoski, Moult, Nachman 1709.04464]**