Introduction to Jets at Acceleraors

July 23, 2018

Probing Quark-Gluon Matter with Jets

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1. Seeing the Unseen at Accelerators
2. From Short to Long Distances in Quantum Field Theory:
   Why We Can Calculate Anything
   Infrared Safety: Finding Jets and Jet Substructure
3. Theory of Jets at Colliders: Factorization and Final-state Interactions
4. Outlook for Jets (at an EIC)

Primarily background material for the workshop.

1. Seeing the Unseen at Colliders

(First, a few comments on the Triumph of the Standard Model at Accelerators)

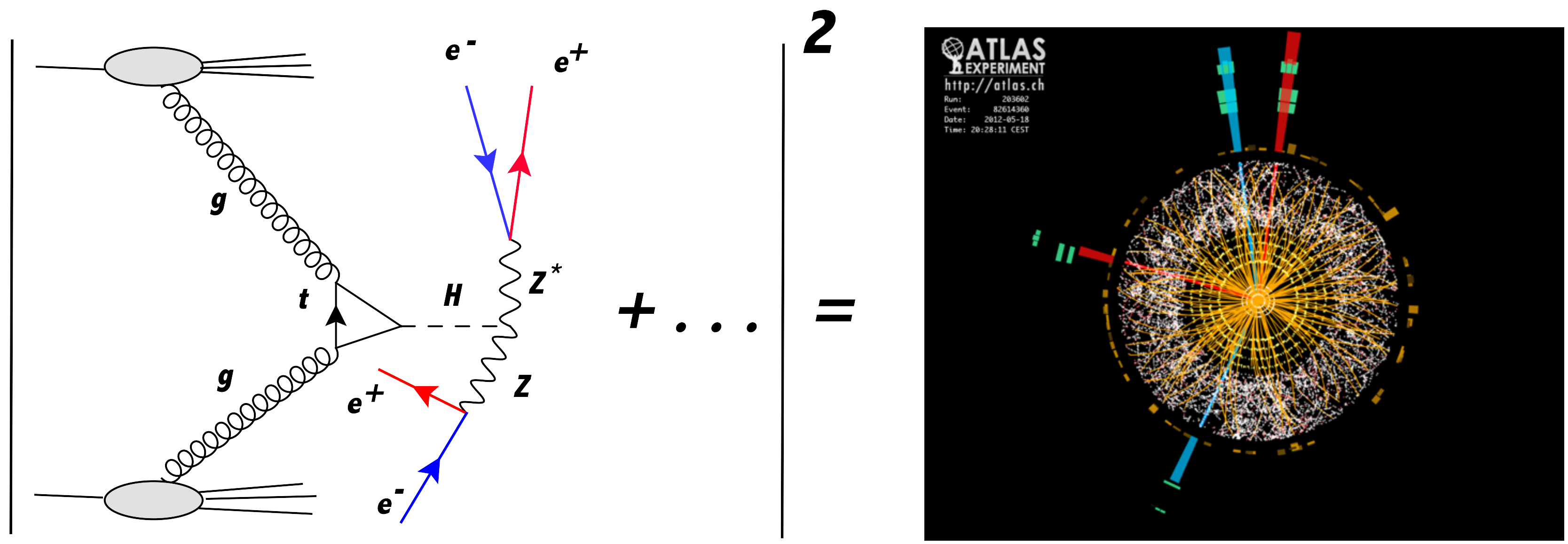
- **High energy accelerators offer the most direct window to short-lived quantum processes, potentially including transient exotic states of matter.**

- **The strategy of probing matter at short distances has resulted in the identification/discovery of the gauge and matter fields of the Standard Model and to a new phase of strongly-interacting matter.**

- **Accelerator programs, however complex and costly, remain experiments following scientific canon. They are capable of design, replication and variation in response to the demands of nature and the imagination.**

- **I will review a little of how quantum field theory is applied in accelerator experiments, how jets emerge in final states, and what they tell us.**

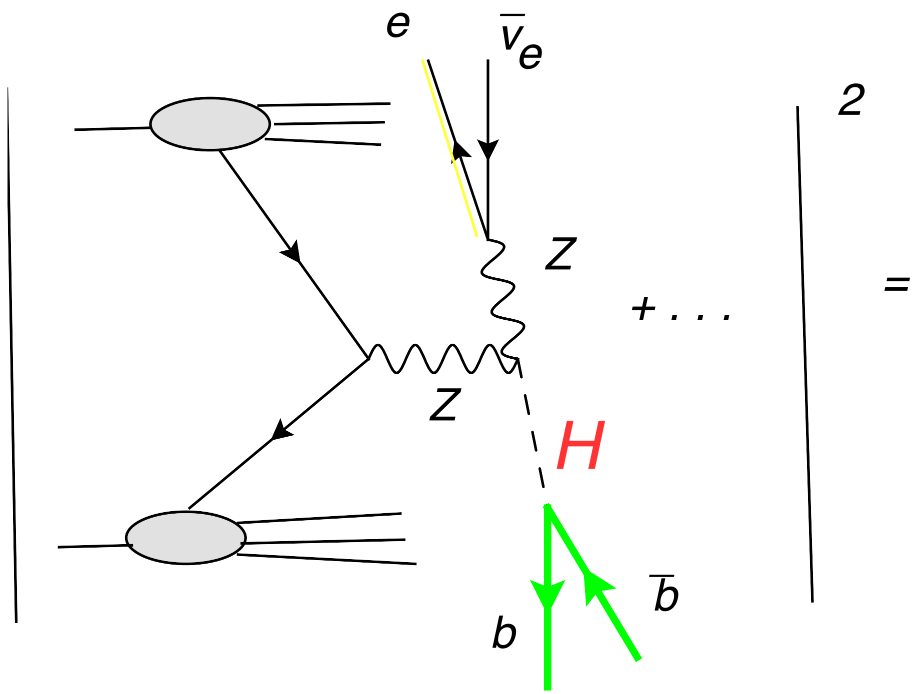
We can sum it up with a picture worth a thousand words:

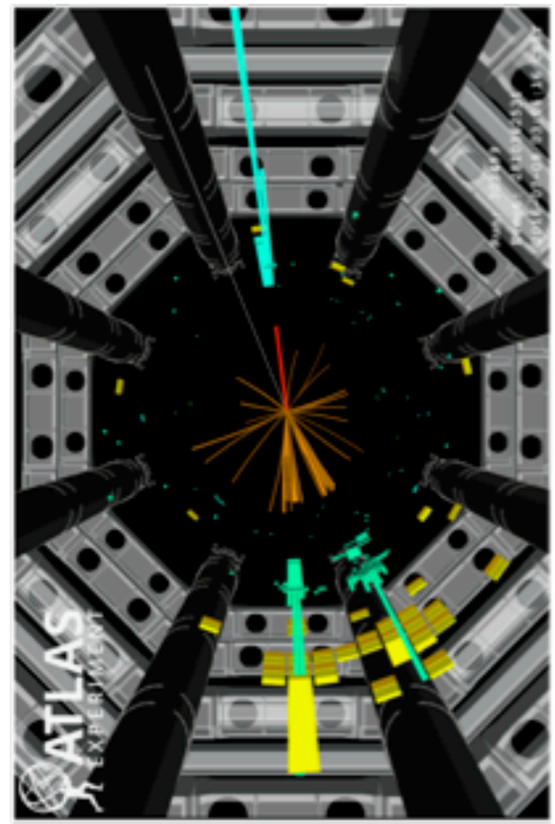


From $SU(3)$ color through the Higgs into $SU(2)_L \times U(1)$.

Every observed final state is the result of a quantum-mechanical set of stories, and so far the stories supplied by the Standard Model, built on an unbroken $SU(3)$ color gauge theory (very much like the original Yang-Mills Lagrangian) and a spontaneously-broken $SU(2)_L \times U(1)$, account for all observations at accelerators.

And recently, $Z + H \to b\bar{b}$ as revealed in boosted dijet decays:





- **The Standard Model developed through the latter half of the Twentieth Century in parallel with modern field-theoretic ideas of flow: couplings within theories (renormalization group) and between theories (Wilsonian).**

- **A primary theme of Twenty-first Century physics is strongly coupled theories with emergent degrees of freedom. This is part and parcel of the contemporary understanding of the strong interactions.**

- **The historic picture of strong interactions: nucleons, nuclei bound by meson exchange, with multiple excitations evolved into:**

- **THE QUARK MODEL, with (mostly) qqq' baryons and $q\bar{q}'$ mesons.**

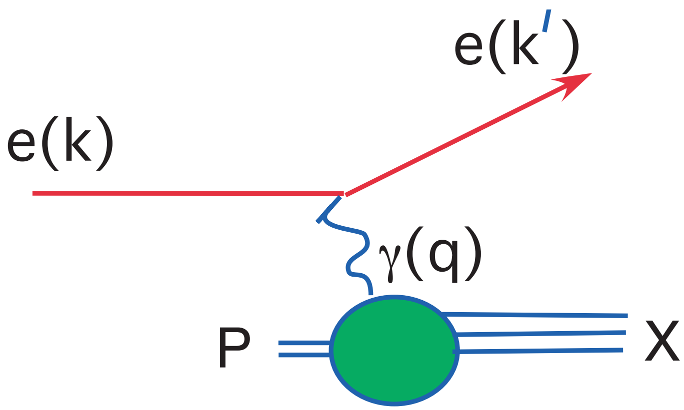
- **QUANTUM CHROMODYNAMICS a part of the Standard Model, is in some ways the exemplary QFT, still not fully understood, but illustrating the fundamental realization that quantum field theories are protean: manifesting themselves differently on different length scales, yet experimentally accessible at all scales.**

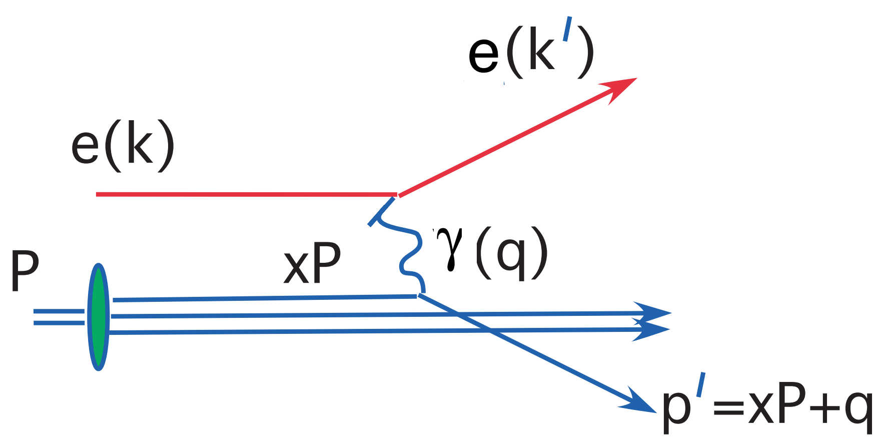
- **To make a long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable. Here was the problem.**

1. Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.

2. In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":

$$\frac{d\sigma_{e+p}(Q, p\cdot q)}{dQ^2}\Big|_{\text{inclusive}} \quad \propto \quad F\left(x = \frac{Q^2}{2p\cdot q}\right) \frac{d\sigma^{\text{free}}_{e+\text{spin}\,\frac{1}{2}}}{dQ^2}\Big|_{\text{elastic}}$$





- **If the "spin-$\frac{1}{2}$ is a quark, how can a confined quark scatter freely?**

- **This paradoxical combination of confined bound states at long distances and nearly free behavior at short distances was explained by asymptotic freedom: In QCD, the force between quarks behaves at short distances like**

$$F(r) \sim \frac{\alpha_s(r)}{r^2}, \qquad \alpha_s(r^2) = \frac{4\pi}{\ln\left(\frac{1}{r^2\lambda^2}\right)}$$

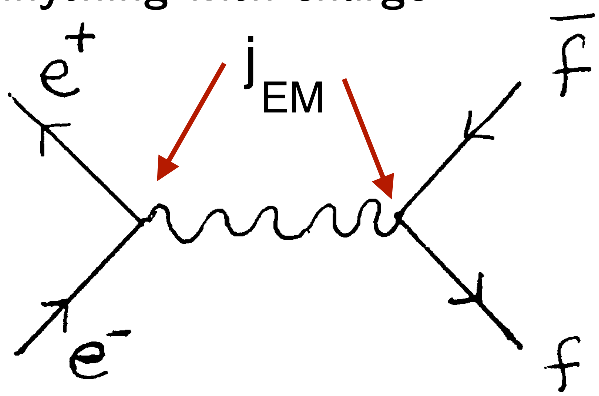
where $\Lambda \sim 0.2$ GeV. For distances much less than $1/(0.2GeV) \sim 10^{-8}$cm the force weakens. These are distances that began to be probed in deep inelastic scattering experiments at SLAC in the 1970s.

- **The short explanation of DIS: Over the times $ct \leq \hbar/GeV$ it takes the electron to scatter from a quark-parton, the quark really does seem free. Later, the quark is eventually confined, but by then it's too late to change the probability for an event that has already happened.**

- **The function $F(x)$ is interpreted as the probability to find quark of momentum xP in a target of total momentum P – a parton distribution.**

- **To explore further, SLAC used the quantum mechanical credo: anything that can happen, will.**

- **Quarks have electric charge, so if they are there to be produced, they will be. This can happen when colliding electron-positron pairs annihilate to a virtual photon, which ungratefully decays to just anything with charge**



- **But of course because of confinement it's not that simple. But more generally, we believe that a virtual photon decays through a local operator:** $j_{\rm em}(x)$ **.**

- **This enables translating measurements into correlation functions . . . In fact, the cross section for electron-positron annihilation probes the vacuum with an electromagnetic current.**

- **On the one hand, all final states are familiar hadrons, with nothing special about them to tell the tale of QCD, $|N\rangle = |\text{pions, protons, neutrons}\ldots\rangle$,**

$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \propto \sum_N |\langle 0|j^\mu_{\mathrm{em}}(0)|N\rangle|^2\, \delta^4(Q - p_N)$$

- **On the other hand, $\sum_N |N\rangle\langle N| = 1$, and using translation invariance this gives**

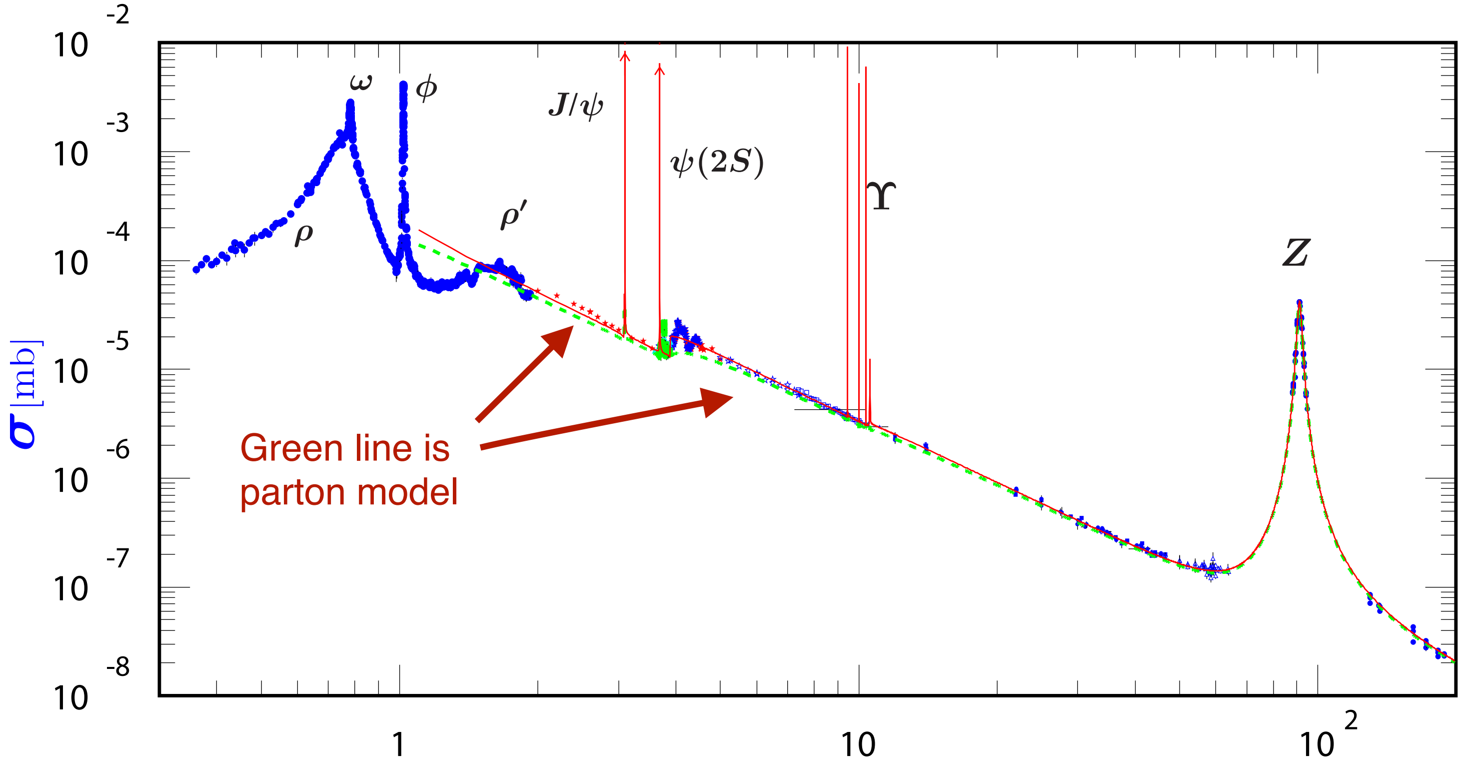
$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \propto \int d^4x\, e^{-iQ\cdot x}\ \langle 0|j^\mu_{\mathrm{em}}(0)\ j^\mu_{\mathrm{em}}(x)|0\rangle$$

- **We are probing the vacuum at short distances, imposed by the Fourier transform as $Q \to \infty$. The currents are only a distance $1/Q$ apart.**

- **Asymptotic freedom suggests a "free" result: QCD at lowest order ("quark-parton model") at cm. energy Q,**

$$\sigma^{tot}_{e^+e^-\to\mathrm{hadrons}} = \frac{4\pi\alpha^2_{\mathrm{EM}}}{3Q^2}$$

- **This works for σ_{tot} to quite a good approximation (with calculable corrections)**



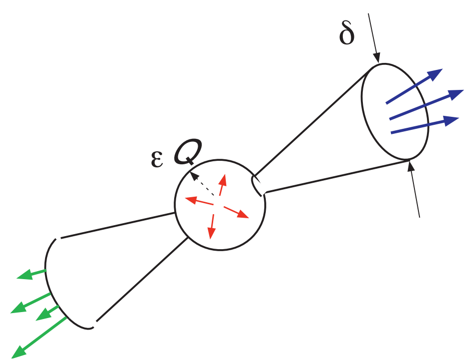
- **So the "free" theory again describes the inclusive sum over confined (nonperturbative) bound states – another "paradox".**

- Is there an imprint on these states of their origin? Yes. What to look for? The spin of the quarks is imprinted in their angular distribution relative to the electron:

$$\frac{d\sigma(Q)}{d\cos\theta} = \frac{\pi\alpha_{\rm EM}^2}{2Q^2}\left(1+\cos^2\theta\right)$$

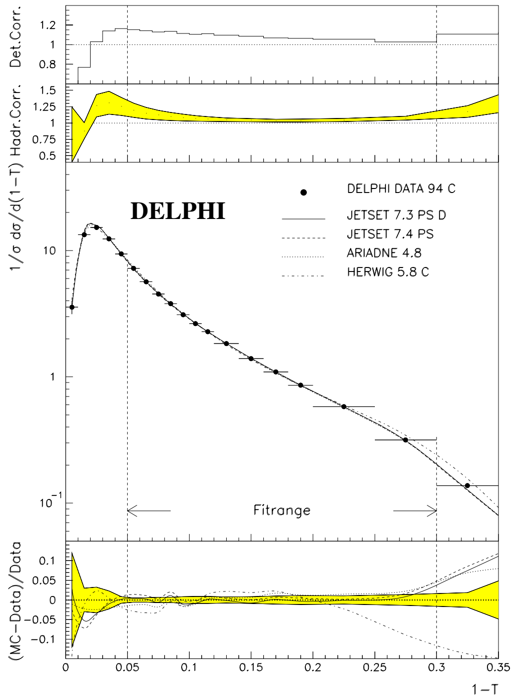
- It's not quarks, but can look for a back to back flow of energy by finding an axis that maximizes the projection of particle momenta ("thrust") measuring a "jet-like" structure

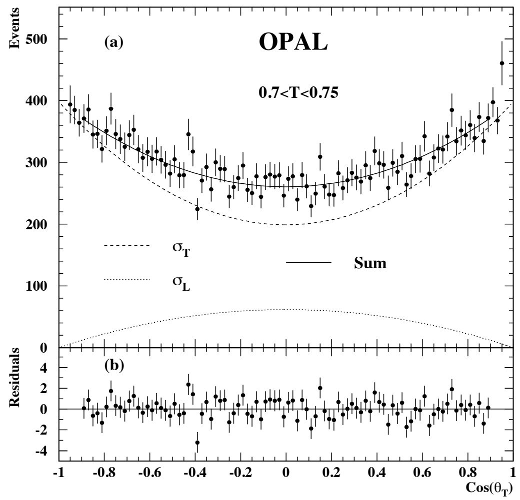
$$\frac{d\sigma_{e^+e^-\to\ \rm hadrons}(Q)}{dT} \propto \sum_N |\langle 0|j^\mu_{\rm em}(0)|N\rangle|^2\,\delta^4(Q-p_N)\;\delta\left(T-\frac{1}{Q}\max_{\hat{\rm n}}\sum_{i\in N}|\vec{p}_i\cdot\hat{\rm n}|\right)$$

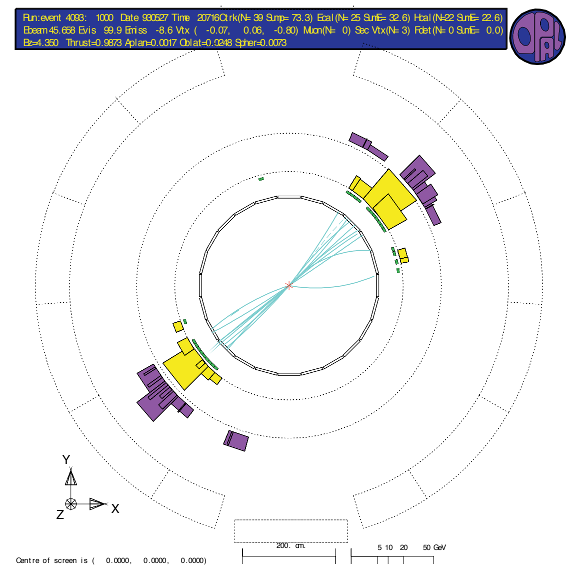


- When the particles all line up $T\to 1$ (neglecting masses). So what happens?

- **Here's what was found (from a little later, at LEP):**

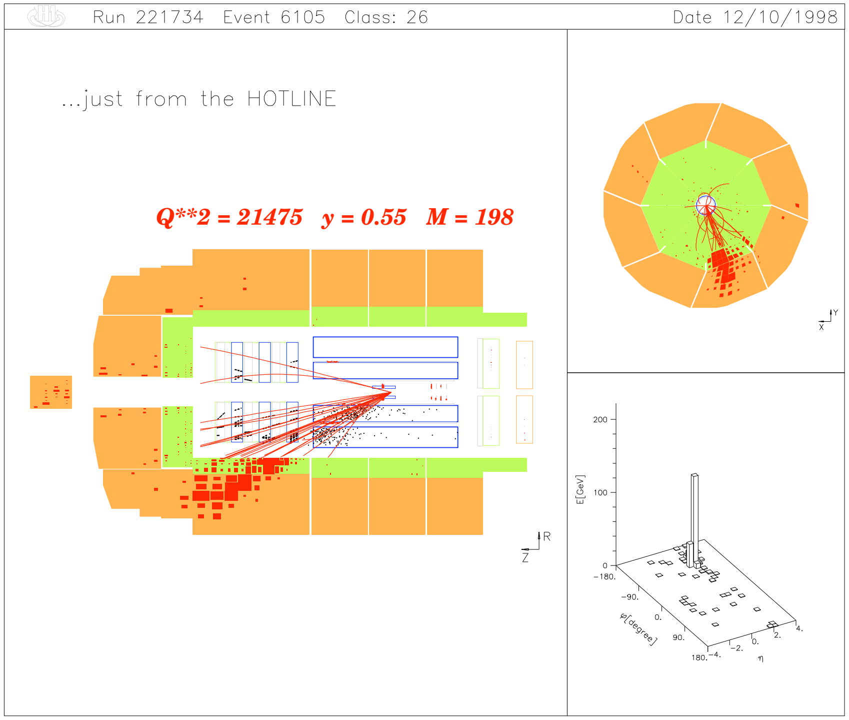






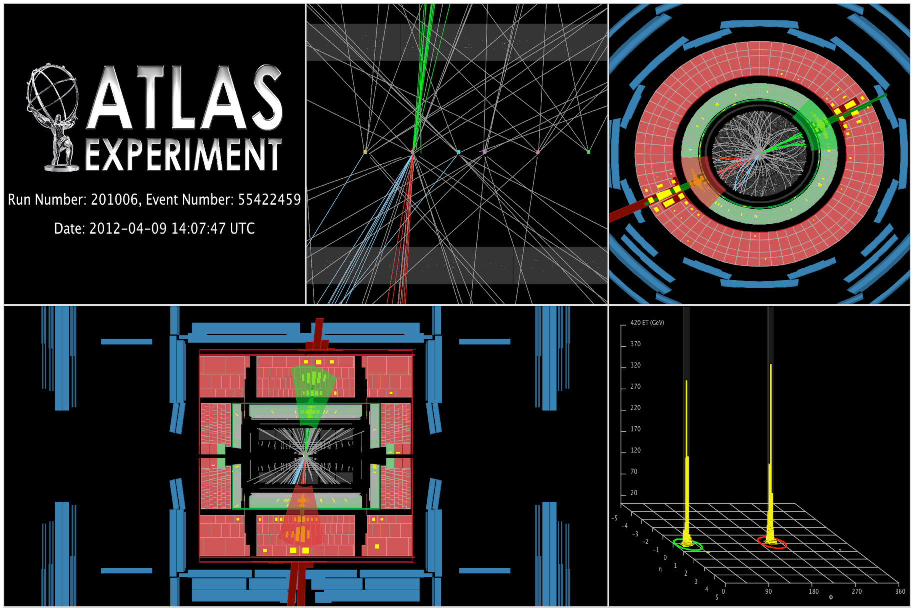
- **Thrust is peaked near unity and follow the $1 + \cos^2\theta$ distribution – reflecting the production of spin $\frac{1}{2}$ particles – back-to-back. All this despite confinement. Quarks have been replaced by "jets" of hadrons. What could be better? But what's going on? How can we understand persistence of short-distance structure into the final state, evolving over many many orders of magnitude in time? Check it with other initial states.**

- **1990's – 2005: The great Standard Model machines: HERA, the Tevatron Run I, and LEP I and II provided jet cross sections over multiple orders of magnitude. The scattered quark appears.**



- **And now . . . a new era of jets at the anticipated limits of the SM, ushered in by Tevatron Run II, and then the LHC: 2 $\rightarrow$ 7 $\rightarrow$ 8 $\rightarrow$ 13 TeV .**

- **Events at the scale** $\delta x \sim \frac{\hbar}{1 \text{ TeV}} \sim 2 \times 10^{-19}$ **meters . . . observed about 10 meters away.**



Let's explore the relationship to the underlying principles of quantum field theory.

2. From Short to Long Distances in Quantum Field Theory: Why We Can Compute Anything

- At the short distances accessible to accelerators, we assume that we can expand around the free field theory. Virtual states are "stories" that provide predictions.

- Perturbation theory really just follows from Schrödinger equation for mixing of free particle states,

$$i\hbar \frac{\partial}{\partial t} |\psi(t) >= \left(H^{(0)} + V \right) |\psi(t) >$$

  Usually with free-state "IN" boundary condition :

$$|\psi(t = -\infty) >= |m_0 >= |p_1^{\rm IN}, p_2^{\rm IN}\rangle$$

- Notation : $V_{ji} = \langle m_j | V | m_i \rangle$ (vertices)

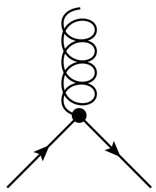
- Theories differ in their list of particles and their (hermitian) Vs.

For QCD, the Lagrange density

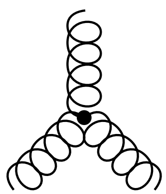
$$\mathcal{L}_{\mathrm{QCD}} = \overline{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - \tfrac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu} - g_{\mathrm{s}}\,\overline{\psi}_i\lambda^a_{ij}\psi_j\,\gamma^\mu A^a_\mu$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - 2g_{\mathrm{s}}\,f_{abc}A_b^\mu A_c^\nu$$

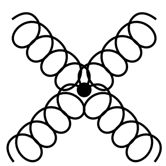
And vertices



$g_{\mathrm{s}}\,\bar{\psi}_i\lambda^a_{ij}\psi_j\gamma^\mu A^a_\mu$ quark-gluon vertex



$g_{\mathrm{s}}\,(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu)f_{abc}A^b_\mu A^c_\nu$ 3-gluon vertex



$g_{\mathrm{s}}^2\,f_{abc}A_b^\mu A_c^\nu\,f_{ade}A^d_\mu A^e_\nu$ 4-gluon vertex

- Solutions to the Schrödinger equation are sums of ordered time integrals.

"Old-fashioned perturbation theory."

$$\langle m_n | m_0 \rangle = \sum_{\tau \text{ orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$

$$\times \prod_{\text{loops } i} \int \frac{d^3 \ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j} \times \prod_{\text{vertices } a} iV_{a \to a+1}$$

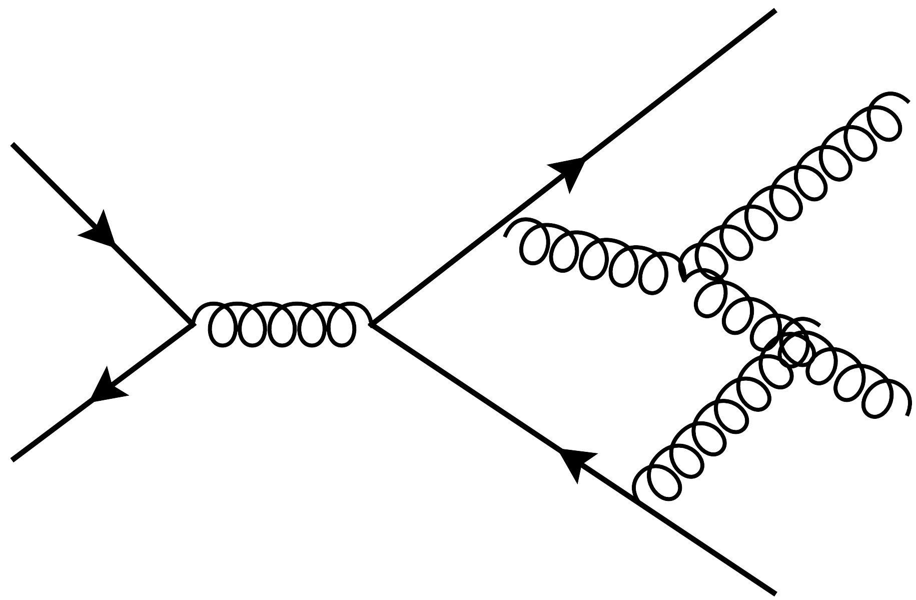
$$\times \exp\left[i \sum_{\text{states } m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right]$$

- Time integrals of elementary transition amplitudes summed over intermediate states with energy-dependent phases

- Perturbative QFT in a nutshell: integrals are divergent in QFT from:

$\tau_i \to \tau_j$ (UV) and $\tau_i \to \infty$ (IR).

Each term in the solution to the Schrödinger equation corresponds to diagram



The vertices are time-ordered. Sums of orderings give (topologically equivalent) "Feynman diagrams", which exhibit Lorentz invariance manifestly. But the basic problem and its solution can be seen at this deeper level.

For QCD, asymptotic freedom enters through renormalization: cut off the limit $\tau_i \to \tau_{i+1} \to 0$ at a "renormalization: scale $1/\mu$. In QCD, the "bigger the better" for μ.

- **So what are we after? – something like an "ideal cross section" made from our time-ordered amplitudes:**

- one with only a single kinematic scale, to which we can set μ:

$$Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) \ = \ \sum_n c_n(Q^2/\mu^2)\ {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$
$$= \ \sum_n c_n(1)\ {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- **The key is to find quantities that are observable, and for which the coefficients are well-behaved, and do not depend on momentum scales at which the coupling is too large.**

- **Such quantities are commonly called "infrared safe"**

- **This is far from automatic.**

- **What is the specific problem?**
- **Degenerate states in perturbation theory**
- **Now go back to solutions to the Schrödinger equation:**

$$\langle m_n | m_0 \rangle \;=\; \sum_{\tau\ \text{orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$
$$\times \prod_{\text{loops}\ i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines}\ j} \frac{1}{2E_j} \times \prod_{\text{vertices}\ a} iV_{a\to a+1}$$
$$\times \exp\left[i \sum_{\text{states}\ m} \left(\sum_{j\ \text{in}\ m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right]$$

- **Time integrals extend to infinity, but Long-time, "infrared" divergences (logs) come about when phases vanish and the t integrals diverge. This doesn't always happen – oscillations can damp them to give finite answers.**

- **When do we get divergences?** **Here's the phase:**

$$\exp\left[\, i \sum_{\text{states}\, m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right] =$$

$$\exp\left[\, i \sum_{\text{vertices}\, m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \;-\; \sum_{j \text{ in } m-1} E(\vec{p}_j) \right)\, \tau_m \right]$$

- **Divergences for $\tau_i \to \infty$ require two things:**

i) (RHS) the phase must vanish $\leftrightarrow$ "degenerate states"

$$\sum_{j\, \in m} E(\vec{p}_j) = \sum_{j\, \in m+1} E(\vec{p}_j)\,, \quad \text{and}$$

ii) (LHS) the phase must be stationary in the sum over states:

$$\frac{\partial}{\partial \ell_{i\mu}}\, [\,\text{phase}\,] = \sum_{\text{states}\, m}\; \sum_{j \text{ in } m} (\pm\beta_j^{\mu})(\tau_{m+1} - \tau_m) = 0$$

where the β_js are normal 4-velocities:

$$\beta_j = \pm\partial E_j / \partial \ell_i\,.$$

- **Condition of stationary phase:**

$$\sum_{\text{states } m} \sum_{j \text{ in } m} (\pm \beta_j^\mu)(\tau_{m+1} - \tau_m) = 0$$

- $\beta^\mu \Delta\tau = x^\mu$ **is a classical translation. For IR divergences, there must be free, classical propagation as** $t \to \infty$**. Easy to satisfy if all the** β_j**'s are equal.**

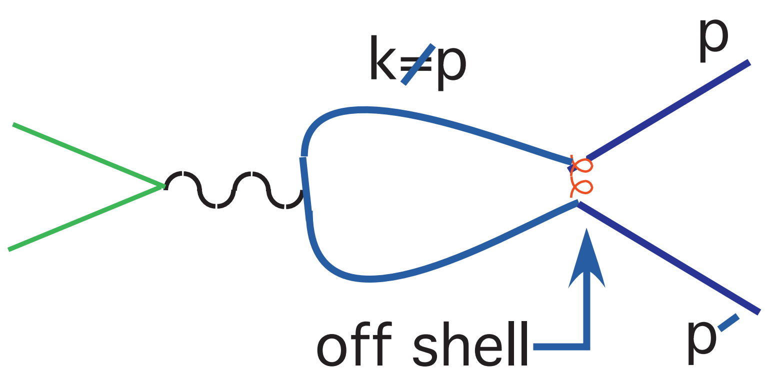
- **Whenever fast partons (quarks or gluons) emerge from the same point in space-time,**

  they will rescatter strongly with collinear partons.

  But note, all these states describe the same energy flow.

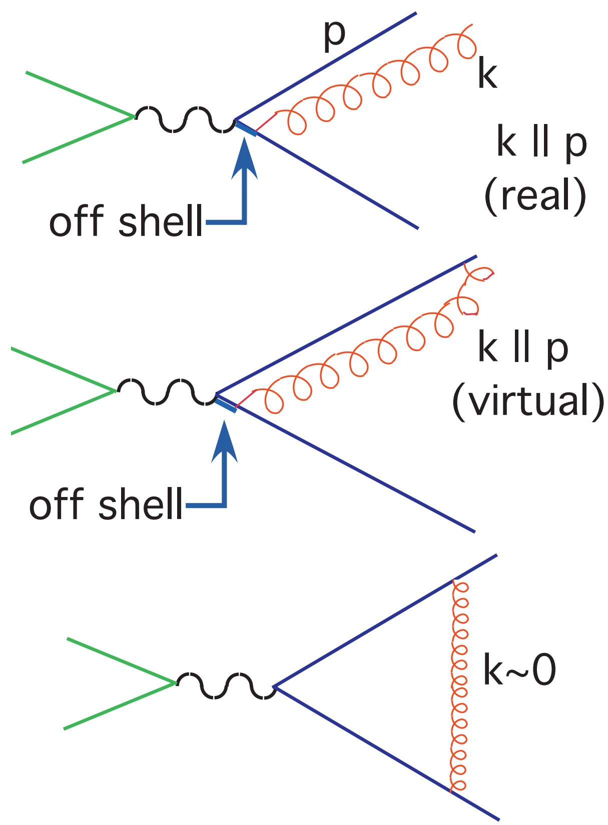
- **Let's illustrate the role of classical propagation.**

- **Example: degenerate states that cannot give long-time divergences:**



- **This makes identifying enhancements a lot simpler! Jets don't change direction.**

- **RESULT: For particles emerging from a local scattering, (only) collinear or soft lines can give long-time behavior and enhancement.** **Example:**



- **This generalizes to any order, and any field theory, but gauge theories alone have soft ($k \to 0$) divergences.**

- **These are what we can't compute (as physical processes).**

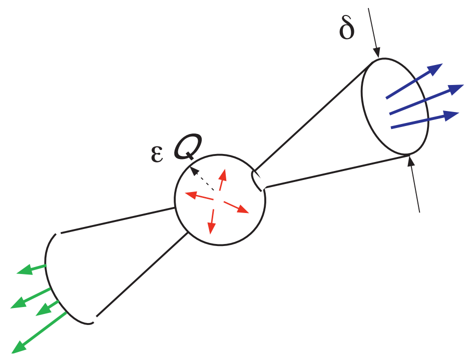
- **But, if we include all the states that can result from these collinear rescatterings, the $\tau \to \infty$ divergences are guaranteed to cancel, because the total probability for something to happen has to be one (unitarity).**

- **If we calculate detailed final states (exactly how many quarks, exactly how many gluons) we get totally unphysical answers, but if we sum over all possibilities so as to preserve energy flow, perturbation theory can give good answers.**

- **Just as in the original parton model, an inclusive process that is nonperturbative at long distances can be described by the lowest order in the perturbative coupling, with calculable corrections.**

- **The same applies jet cross sections, if they are designed to respect the flow of energy**
- **These are what we can compute.**

  (technically, all these singularities can be derived from rotationally non-invariant – but still hermitian – truncations of the QFT hamiltonian. See also Soft-Collinear Effective Theory.)



- **The smaller (larger) the "resolutions" ε and δ, the more (less) sensitivity to long times. We follow the stories only to times like $1/Q\delta$ or $1/Q\varepsilon$.**

ENERGY FLOW IS THE ORGANIZING PRINCIPLE OF THE CLASSICAL STORIES

- General condition for IR safety: treat states with the same flow of energy the same way.

  Introduce IR safe weight $e_n(\{p_i\})$:

$$\frac{d\sigma}{de} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(e_n(\{p_1 \ldots p_n\}) - w\right)$$

  with

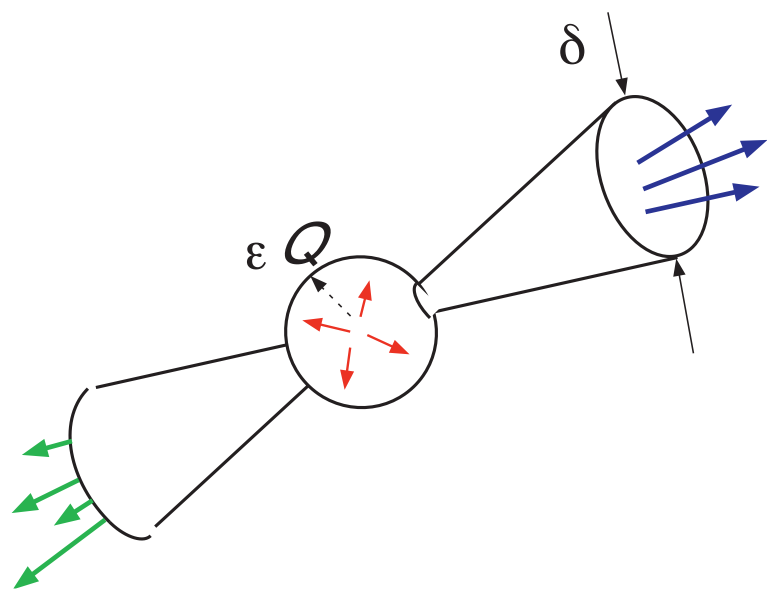
$$e_n(\ldots p_i \ldots p_{j-1}, \alpha p_i + \delta p, p_{j+1} \ldots) = e_{n-1}(\ldots (1+\alpha) p_i \ldots p_{j-1}, p_{j+1} \ldots) + \mathcal{O}\left(\left[\frac{\delta p}{E_{\rm tot}}\right]^p\right)$$

  for some $p > 0$.

- Neglect long times in the initial state for the moment and see how this works in $\mathrm{e}^+\mathrm{e}^-$ annihilation: event shapes and jet cross sections.

- Weight function e_n can pick out jets and/or fix their properties.

- **“Seeing” Quarks and Gluons With Jet Cross Sections**
- **Simplest example: cone jets in $\mathrm{e}^+\mathrm{e}^-$ annihilation. All but fraction ϵ of energy flows into cones of size δ.**



- **Intuition: eliminating long-time behavior ⇔ recognize the impossibility of resolving collinear splitting/recombination of massless particles**

Check at order α_s:

Virtual gluon contribution looks like:

$$\text{virtual}: \quad = \; -\int_0^Q \frac{dk}{2k} \int_{-1}^{1} \frac{2\pi\, d\cos\theta}{1 \;-\; \cos^2\theta_{pk}}$$

k = gluon energy, θ = angle to the quark direction.

Real gluon emission has smaller phase space, but still includes all regions where

$$k \;=\; 0 \quad \text{and} \quad |\cos\theta| \;=\; 1$$

corresponding to soft and collinear configurations (and divergences).

$$\text{real}: \quad \sim \; + \int_0^{\varepsilon Q} \frac{dk}{2k} \int_{-1+\delta^2/2}^{1-\delta^2/2} \frac{2\pi\, d\cos\theta}{1 \;-\; \cos^2\theta_{pk}}$$

$$+ \; \int_0^Q \frac{dk}{2k} \left(\int_{1-\delta^2/2}^{1} + \int_{-1}^{-1+\delta^2/2} \right) \frac{2\pi\, d\cos\theta}{1 \;-\; \cos^2\theta_{pk}}$$

Singularities cancel (even without IR regularization).

- No factors Q/m or $\ln(Q/m)$ Infrared Safety.

- In this case,

$$\sigma_{2J}(Q,\delta,\epsilon) = \frac{3}{8}\sigma_0(1+\cos^2\theta) \times \left(1 - \frac{4\alpha_s}{\pi}\left[4\ln\delta\ln\varepsilon + 3\ln\delta + \frac{\pi^2}{3} + \frac{5}{2}\right]\right)$$

- Perfect for QCD: asymptotic freedom $\rightarrow d\alpha_s(Q)/dQ < 0$.
- Some Lessons:
  - No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.
  - The relation of a jet to quarks and gluons is always approximate but corrections to the approximation computable.
  - A single jet may have an enormous amount of information.
  - Judicious choices of IR safe event shapes can reveal that information.

- Simplest Case: The general form of an $\mathrm{e^+e^-}$ annihilation jet cross section:

$$\sigma_{\rm jet} = \sigma_0 \sum_{n=0}^{\infty} c_n(y_i, N, C_F)\alpha_s^n(Q)$$

- Dimensionless variables y_i include direction and information about the 'size' and 'shape' of the jet:
- δ, cone size as above
- To specify the jet direction, may use a Shape variable, e.g. thrust

$$T = \frac{1}{s}\max_{\hat{n}} \sum_i |\hat{n}\cdot\vec{p}_i| = \frac{1}{s}\max_{\hat{n}} \sum_i E_i\,|\cos\theta_i|$$

with θ_i the angle of particle i to the "thust" axis, which we can define as a jet axis.

- $T = 1$ for "back-to-back" jets.
- Once jet direction is fixed, we can generalize thrust to any smooth weight function:

$$\tau[f] = \sum_{\text{particles } i \text{ in jets}} E_i\, f(\theta_i)$$

(For example "Angularities" – see recent work by Z.-B. Kang, K. Lee, R. Ringer)

- **The distribution as seen at high energies, compared to experiment (Davison & Webber, 0809):**

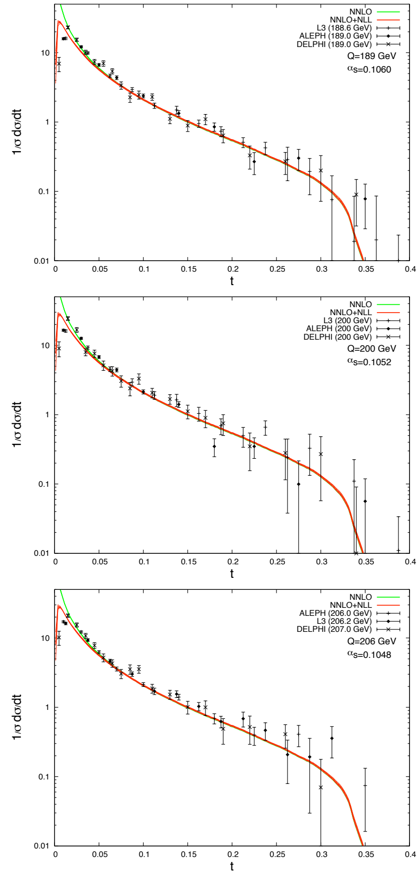


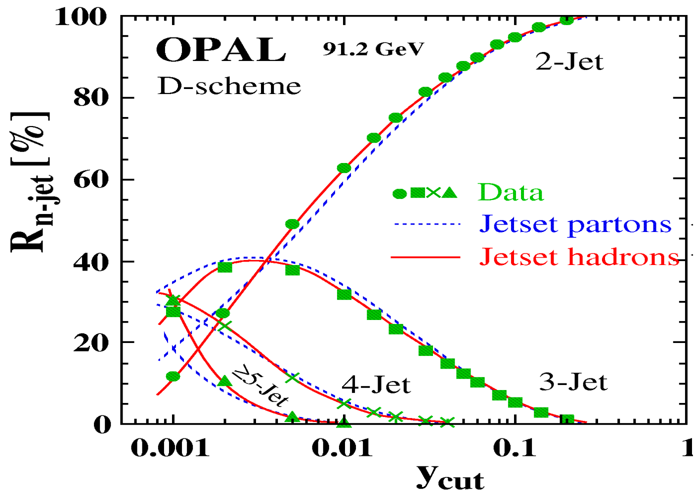
Fig. 3. Fixed-order (NNLO), resummed (NNLO+NLL) and experimental thrust distributions: $Q = 189 - 207$ GeV.

- **Strongly peaked near, but not at, $T = 1$, due to radiation.**

- For possibly multi-jet events, "cluster algorithms".
- $y_{\rm cut}$ Cluster Algorithm: Combine particles i and j into jets until all $y_{ij} > y_{\rm cut}$, where (e.g., "Durham alogrithm" for $\rm e^+e^-$):

$$y_{ij} = 2\min\left(E_i^2, E_j^2\right)(1 - \cos\theta_{ij})$$

- The number of jets depends on the variable $y_{\rm cut}$, and the dependence on the number of jets was an early application of jet physics. (Reproduced from Ali & Kramer, 1012)



- More generally, conventional cluster variables for (symmetric) collisions involving hadron(s) in the initial state:

$$d_{ij} = \min\left(k_{ti}^{2p}\,,\, k_{tj}^{2p}\right) \frac{\Delta_{ij}^2}{R^2}$$

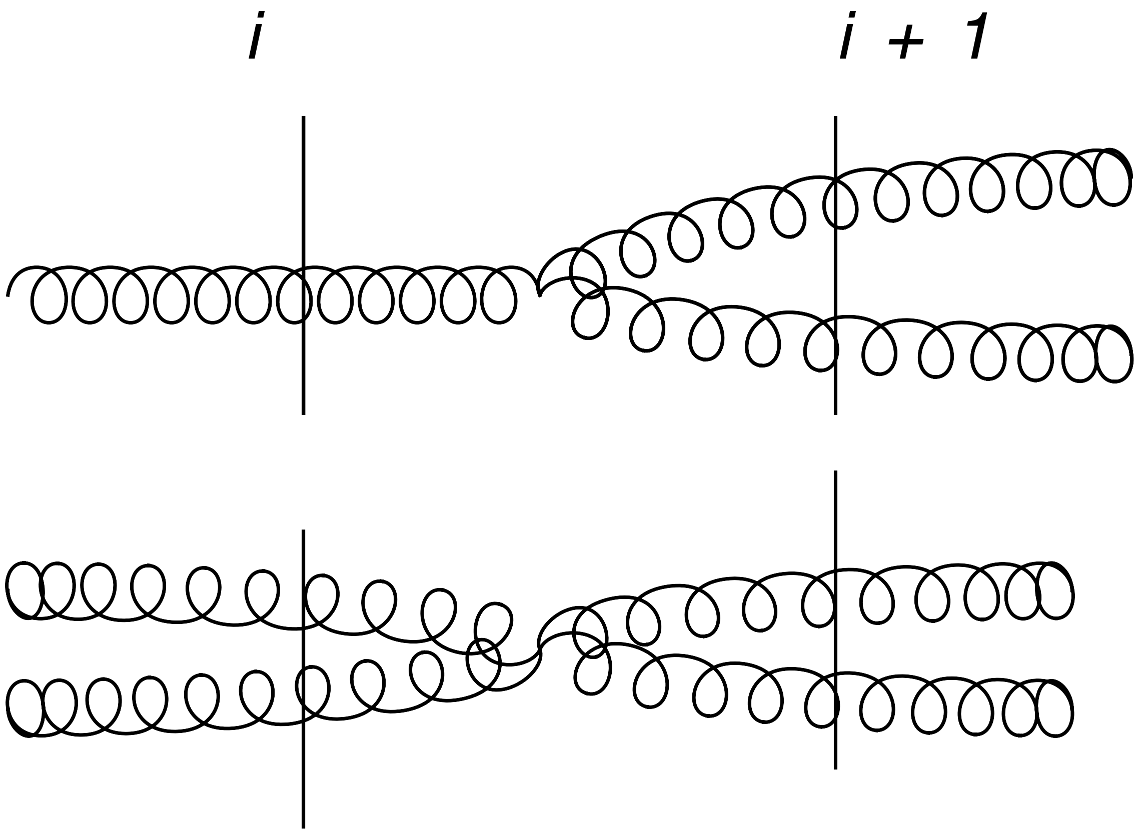
$\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$. R is an adjustable parameter.

- These avoid clustering jets with fragments of initial hadrons. The "classic" choices:
  - $p = 1$ "k_t algorithm:
  - $p = 0$ "Cambridge/Aachen"
  - $p = -1$ "anti-k_t"

- Each step in a clustering process is IR safe, so can "groom" jets by calculating jet properties in terms of only energetic clusters. Such constructions are actually more inclusive in soft radiation. "Mass drop" is one such technique, where "dropping" a cluster of particles means keeping them in the cross section.

Summarize: what makes a cross section infrared safe?

- **Independence of long-time interactions:**



More specifically: should depend on only the flow of energy into the final state. This implies independence of collinear re-arrangements and soft parton emisssion.

But if we prepare one or two particles in the initial state (as in DIS or proton-proton scattering), we will always be sensitive to long time behavior inside these particles. The parton model suggests what to do: factorize.

3. The Theory of Jets at Colliders

- **Machines with hadrons involve the scattering of "pre-existing" quarks and gluons from hadrons, whose interactions extend back to nucleosythesis, requiring:**

  Factorization: Following the New Stories into the Final State

  The essence of predictions for event weights e (selecting jets, say),

$$Q^2\sigma_{\rm phys}(Q,m,e) \ = \ \hat{\sigma}(Q/\mu,\alpha_s(\mu),e) \otimes f_{\rm LD}(\mu,m) \ + \ \mathcal{O}\left(\frac{1}{Q^p}\right)$$

  $\mu =$ factorization scale; $m=$ IR scale **(m *may* be perturbative)**

  This is a "first this and then that" multiplication of probabilities – the essence of factorization. It requires a "sufficiently" inclusive cross section, much as in the calculation of jets in e^+e^- annihilation.

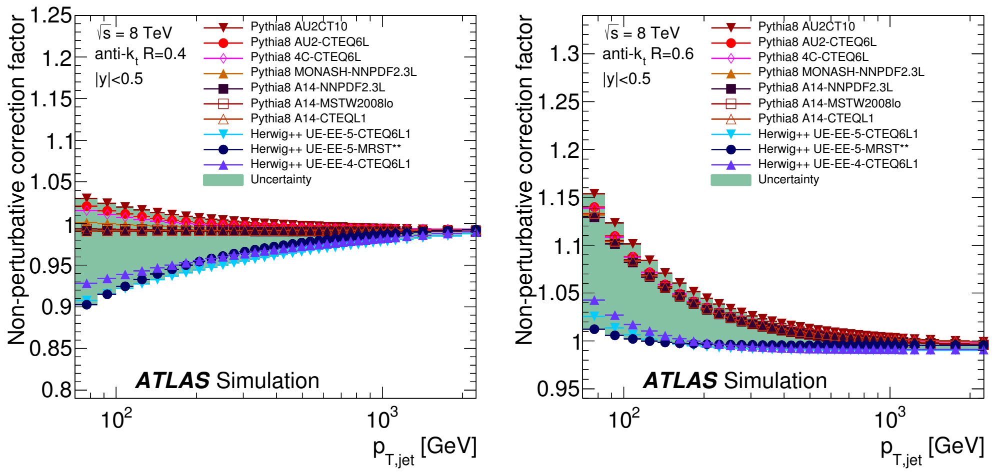
- **Newly-minted jets are in $\hat{\sigma}$; $f_{\rm LD}$ are "universal"**

- Again, a factorized cross section, including the possibility of fragmentation fuctions:

$$Q^2\sigma_{\rm phys}(Q,m,e) \;=\; D_{\rm LD}(\mu,m) \otimes \hat\sigma(Q/\mu,\alpha_s(\mu),e) \otimes f_{\rm LD}(\mu,m) \;+\; \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- What we do:
  - Compute σ and $f_{\rm LD}$ $(D_{\rm LD})$ in an IR-regulated variant of QCD, where we can prove the factorization explicitly, then extract $\hat\sigma$, assuming it is the same in true QCD as in its IR-regulated version.
  - We compare the formula with unknown physical parton distributions to a suite of data and do a "global fit" for the $f(x,\mu)$ for different quarks and the gluon.
- What we get: absolute predictions for the creation of jets and heavy particles from QCD (and for new degrees of freedom in BSM hypotheses.)
  - The process is a "bootstrap", resulting in feedback between parton distributions, predictions and measurements. This must be done for each hadron, a task to which the EIC will contribute greatly.
- Final-state interactions involving more than one parton are in the correction, and are power formally suppressed. As in AA collisions, these can pile up (jet quenching). There is no expectation of quenching in eA, for generic jet kinematics. (Viz. Fermlab E665)

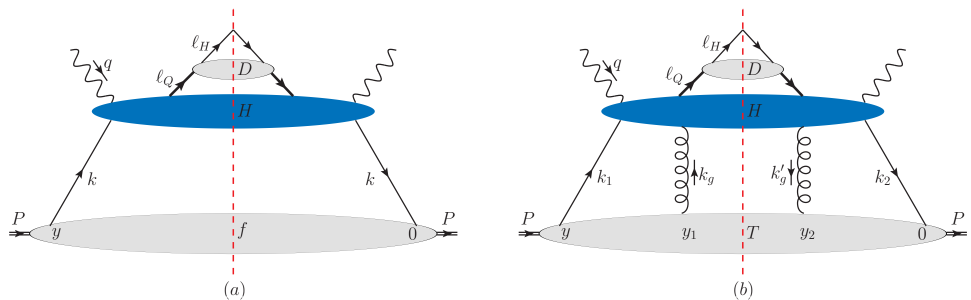
- **Although power-suppressed, nonperturbative corrections due to hadronization (and for proton-proton, underlying event) remain important, but not overwhelming. Their systematic study should be an EIC targert.**



(a) $R = 0.4$, $|y| < 0.5$ (b) $R = 0.6$, $|y| < 0.5$

- **Estimated by Atlas (1706.03192) by comparing output of event generators. No unified theory framework is available (yet).**

- **An illustration of other power-suppressed, but significant corrections – a story in which a parton scatters in one nucleon, and exchanges another gluon on the way out:**
  (Y.-L. Du et al 1807.06917)



- **How does such a process interfere with radiation, hadronization? Nuclei should provide tests in a "controlled" environment. At high energy, we can ask many questions of the final state.**

- **Some precedent in Fermilab E665, 1994 (Phys. Rev.) With $z = p_{\text{had}} \cdot p_{\text{target}}/q \cdot p_{\text{target}}$. "KIn" refers to event selection – Kin1 is low-x, Kin2 is high Q^2.**

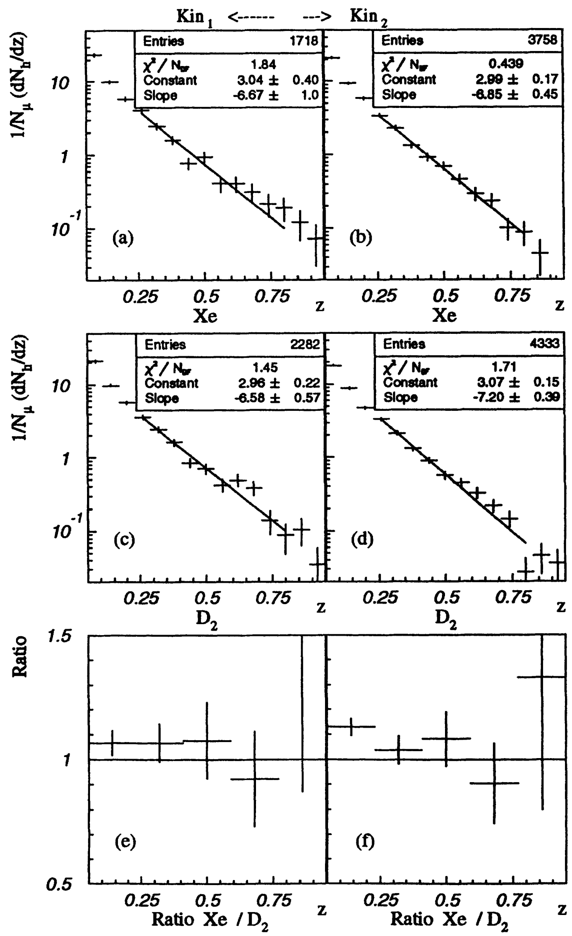


FIG. 7. z distributions: Xe and D_2. These plots show the z distributions from xenon and deuterium and the ratios of the distributions. The distributions on the left are from events in the low kinematic region: Kin_1, while those on the right are from events in the high kinematic region: Kin_2. The data have been corrected for acceptance but not for target length effects; they are tabulated in Tables XIX and XXX.

The range of these predictions is greatly extended by Evolution & Resummation: If we have factorization, we can automatically extrapolate from one energy scale to another.

- Whenever there is factorization, there is evolution

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q,m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \hat{\sigma}}{d\mu}$$

- **We can calculate P because we can calculate $\hat{\sigma}$.**
  (Dokshitzer, Gribov, LIpatov, Altarelli, Parisi)

- Wherever there is evolution there is resummation,

$$\sigma_{\rm phys}(Q,m) = \sigma_{\rm phys}(q,m) \ \otimes \ \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- **For example: $\sigma_{\rm phys} \equiv \tilde{F}_2(Q^2,N) = \int_0^1 dx\; x^{N-1} F(Q^2,x)$, a moment in ep deep-inelastic scattering. The success of this formalism is extraordinary.**

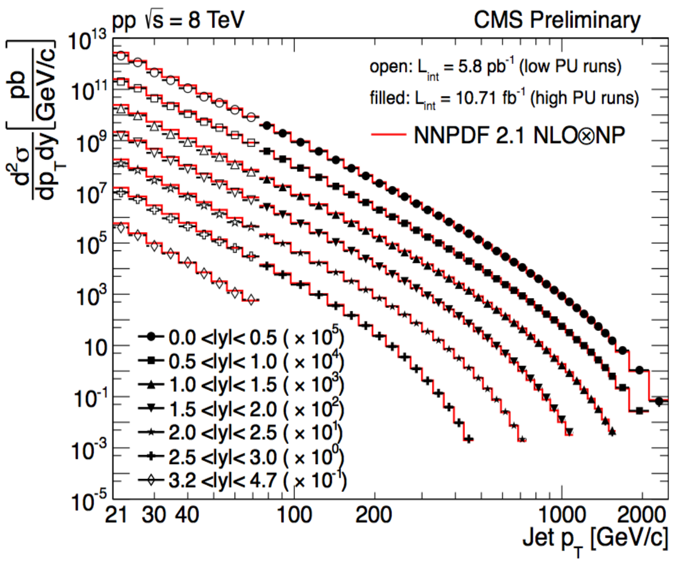
Computing jet cross sections

- **Factorized jet cross sections look like this:**

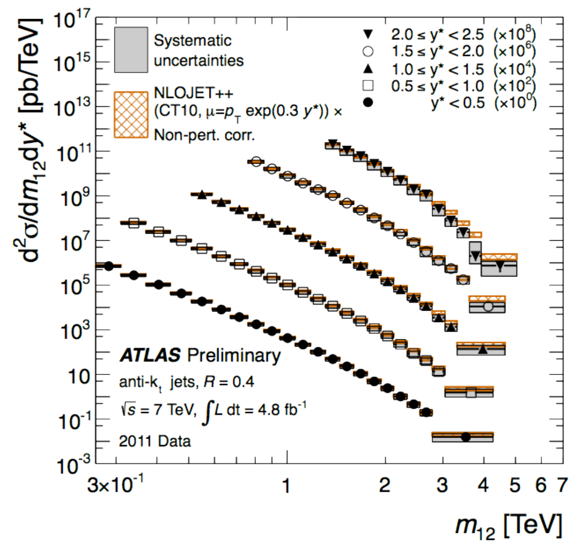
$$
\begin{aligned}
d\sigma(A+B\to\{p_i\}) & \\
&= \int dx_a dx_b\; f_{a/A}(x_a,\mu_F)\, f_{b/B}(x_b,\mu_F) \\
&\quad \times\; C\left(x_a p_A, x_b p_B, \frac{Q}{\mu_F}, \frac{p_i\cdot p_j}{p_k\cdot p_l}\right)_{ab\to c_1\ldots c_{N_{\text{jets}}}+X} \\
&\quad \times d\left[\prod_{i=1}^{N_{\text{jets}}} J_{c_i}(p_i,\mu_F)\right]
\end{aligned}
\tag{1}
$$

- **Parton distributions, short distance "coefficients" and functions of the jet momenta tell a story of autonomous correlated on-shell propagations punctuated by a single short-distance interaction.**

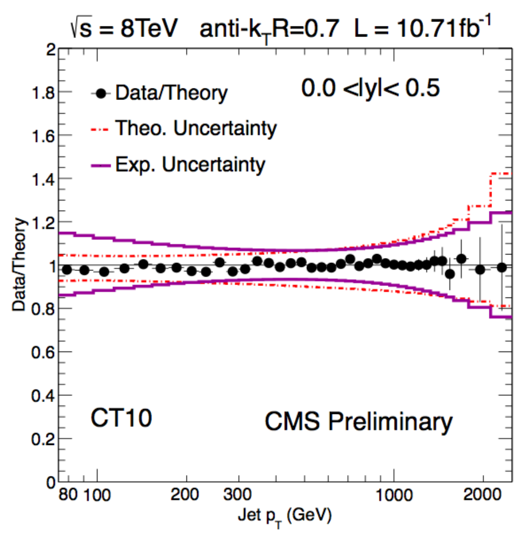
Correlated and "autonomous" dynamics. The data confront calculations . . .

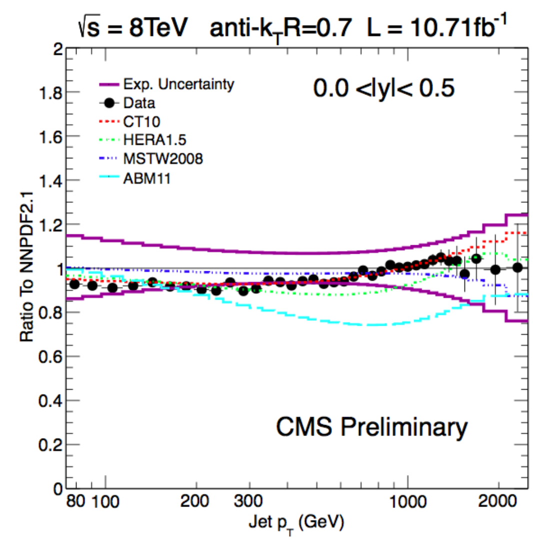


(CMS-PAS-SMP-12-012)



(ATLAS-CONF-2012-021)





- **We have seen that enhancement of particle correlations is built into QFT, and mutual autonomy is a feature of classical pictures. Different jets follow different paths.**

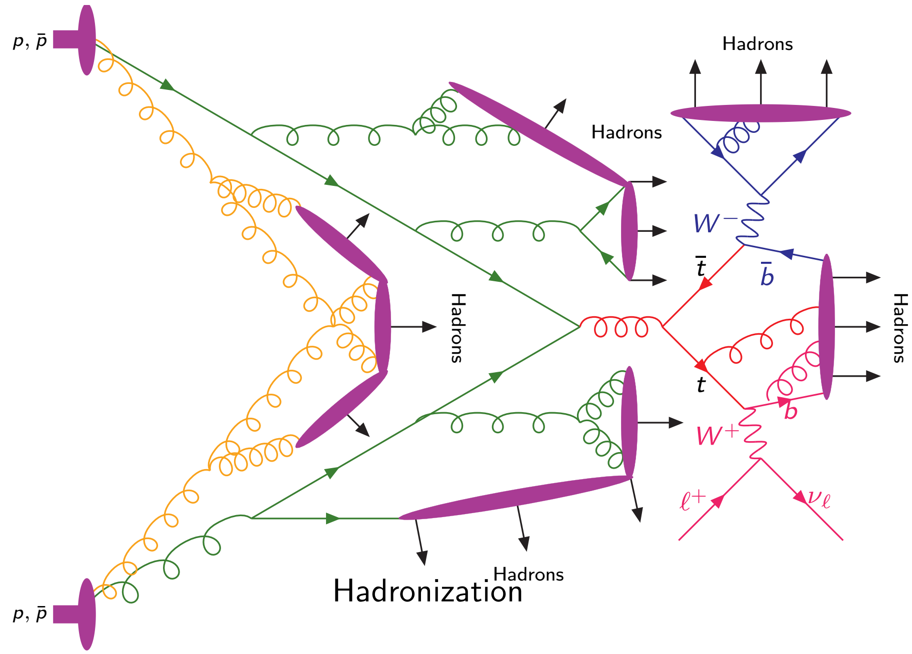
- **The same factorization $\to$ evolution step applies to our jets, and they "evolve"**

$$J(\text{scale } \mu_2) \;\sim\; J(\text{scale } \mu_1) \exp\left[\int_{\mu_1}^{\mu_2} \frac{d\mu'}{\mu'} \int dx\; P(x, \alpha_s(\mu'))\right]$$

- **Each term in the exponent corresponds to the potential emission of a new "subjet", which factors from the remaining jet and evolves nearly autonomously into the final state, branching further subjets along the way.**

- **These double-logarithmic exponentiations give the theory curves for event shapes (as thrust above). Extensions to hadronic environments invite many further analyses.**

- **This is also exploited systematically to build event generators (PYTHIA, Herwig . . .), which simulate the details of events by probabilistic steps specified in detail by the calculable "spitting functions" $P(x, \alpha_s)$.**



Here's a representation of an Event generated by Herwig. Although it looks like an amplitude, each step is probabilistic, and given by splitting functions as above.
(P. Richardson, 2015)

- **Which brings us full circle. To model "real" final states, the step has to be made between perturbative jets given by gluons and quarks, and hadrons. Modern event generators exploit the calculable momentum and quantum number distributions provided by perturbation theory to make the final step: hadronization, shown here between final-state partons that are "close enough" in phase space.**
It is close to here that the tide of our theory reaches its current high water mark.

- **It is here that nuclear targets can provide filters on the mechanisms of hadronization. A systematic theory for these effects will be invaluable.**

4. Outlook for Jets (at an EIC)

Jet cross sections will be an important part of DIS at EIC energies. The theory described above provides a secure starting point.

At expected EIC energies, jet cross sections will be remain sensitive to a range of nonperturbative effects associated with both vacuum and medium dynamics.

The dependence of jet properties on target size will shed light on both the time evolution of hadronization and on target structure.

Of special interest will be the dependence of jet-specific fragmentation functions on kinematic properties of the scattering.

Tests of parton-hadron duality analyses are particularly promising. How is the perturbative probability distribution for parton production (re)distributed among observed hadrons?