Chiral Magnetic Effect and Energy Conservation

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CME in a plasma and neutron stars

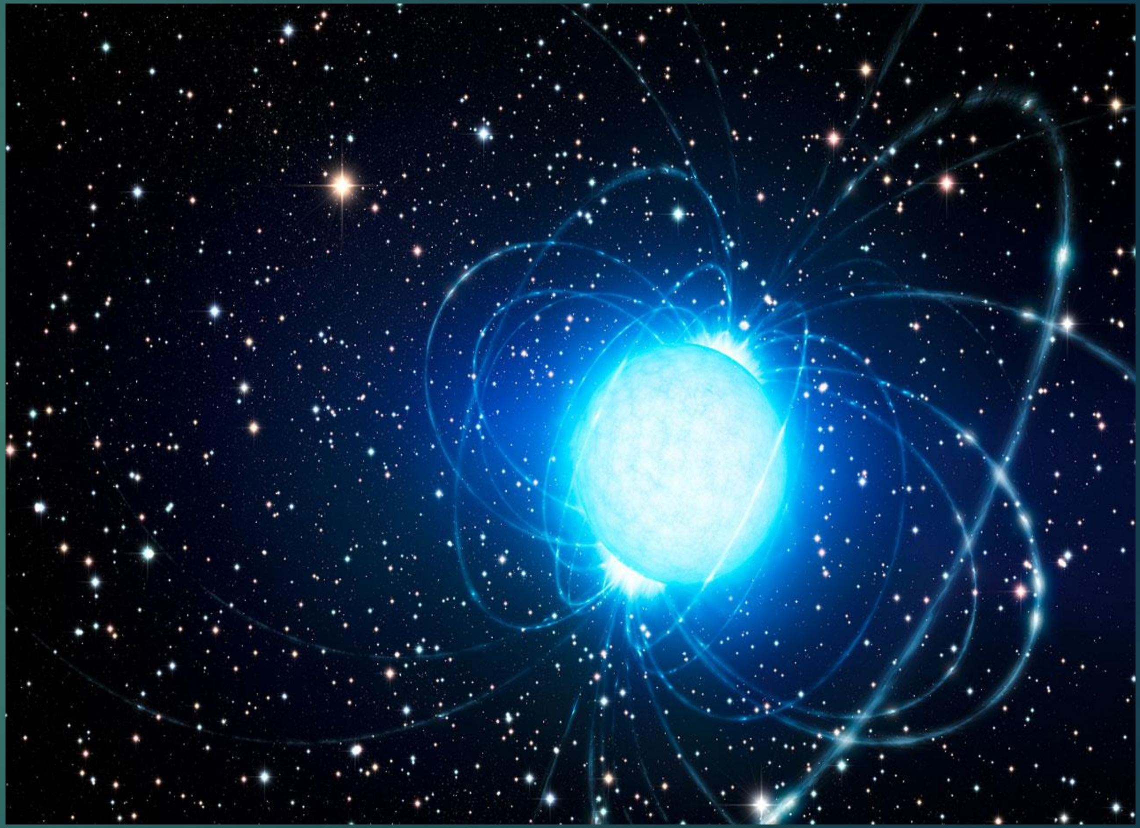
- Can axial asymmetry in electron weak interaction with a background of baryons give rise to the chiral magnetic instability ?

- In other words, is a homogeneous plasma of electrons, protons and neutrons unstable towards generation of strong magnetic fields ?

- The answer turns out to be no in the absence of an initial chiral imbalance in the Fermi energies of the left and right handed electrons as we well see in this talk.

Context : Magnetars

- These are neutron stars with very strong magnetic fields.
- A typical neutron star has about 10^{12}Gauss of magnetic field on its surface.
- But magnetars are even more strongly magnetized with magnetic fields that are as big as 10^{15}gauss.
- Where do these strong magnetic fields come from ?

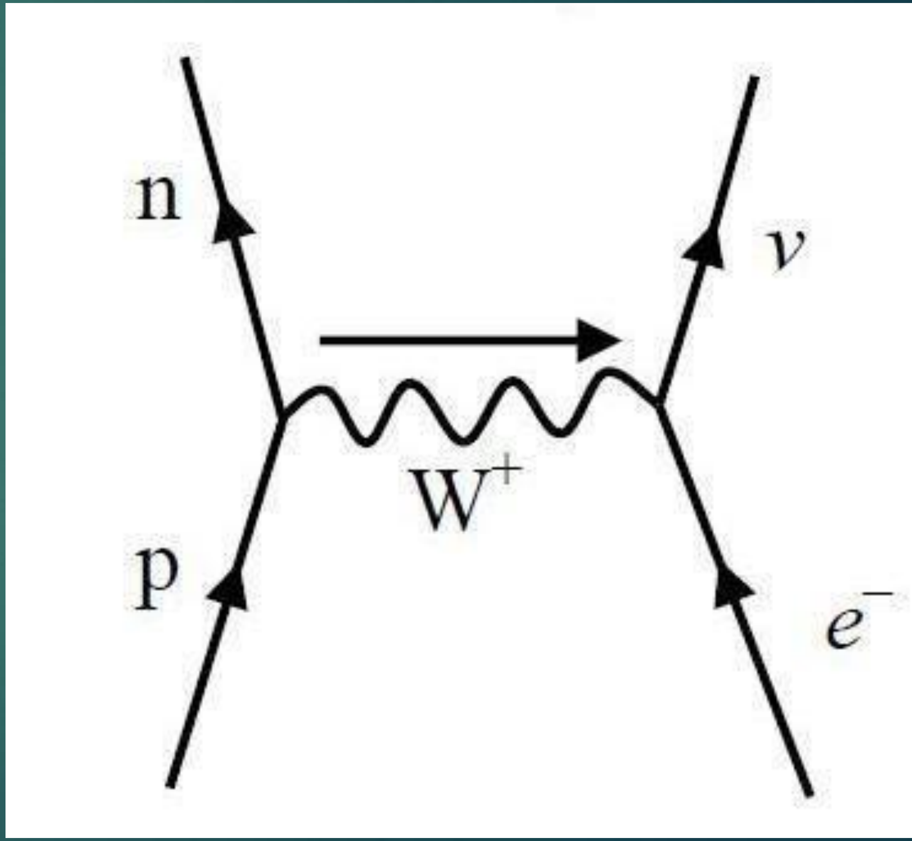


A proposal relying on neutronization

- This proposal in its simplest form requires massless electrons.
- As the core collapses after a supernova explosion the density increases such that protons start capturing electrons via weak interactions producing neutrons.
- At the end one gets a giant ball of neutrons tightly packed together with few protons and electrons hanging around.

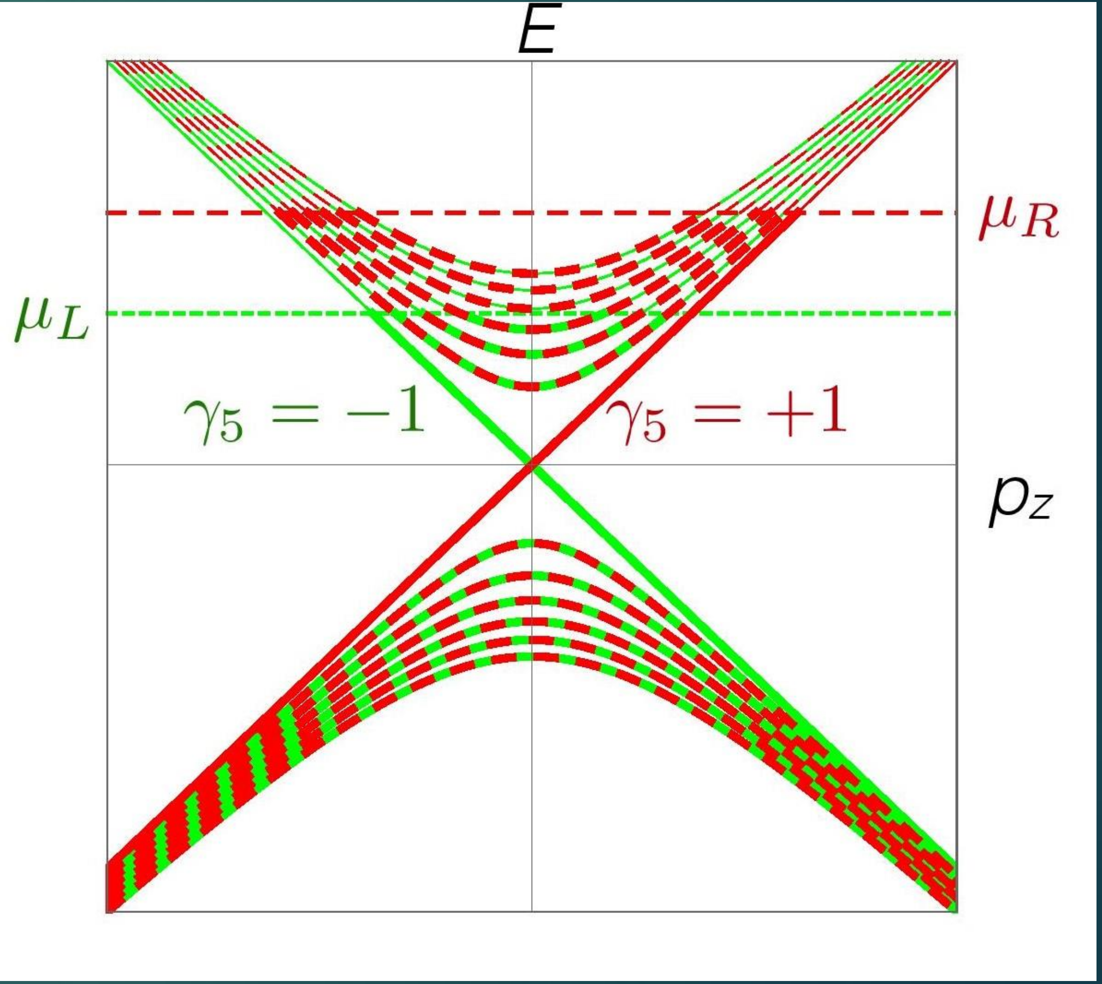
A proposal relying on neutronization

- Weak interactions involve only left handed fermions.
- Only left handed electrons get captured by protons to form neutrons and left handed neutrinos.
- The result is an imbalance in the total population of right and left handed electrons.
- Since neutrinos are neutral, this almost amounts to a net chiral imbalance in the charged fermion sector.



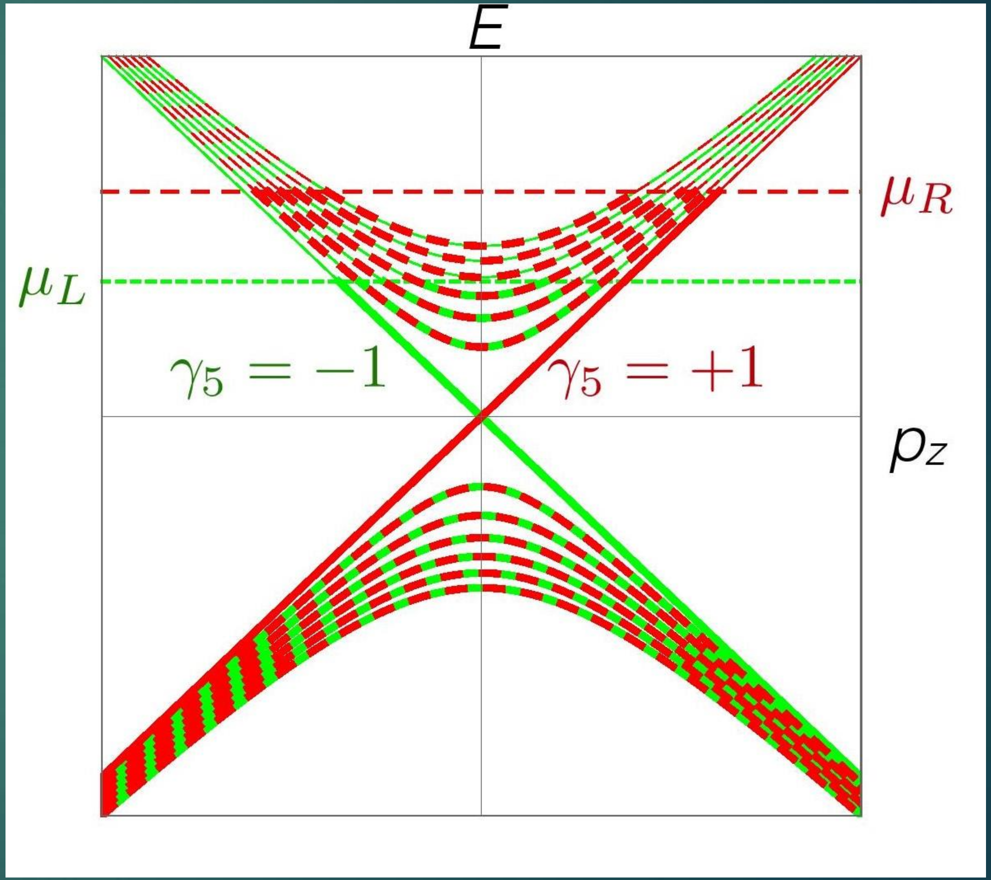
Chiral electrons in a magnetic field :

- The magnetic field will align the electron spin along itself.
- This implies that chiral electrons in a magnetic field can have momentum only in the direction of the magnetic field or opposite to it.
- The spectrum hence is practically one dimensional.



The current density in a magnetic field and some chiral imbalance

- Count the number of right moving states and left moving states.
- Take into account the density of states $\sim eB$.
- This produces a net current along the magnetic field when a nonzero chiral imbalance is present : $j \sim e^2 \mu_5 B$



Maxwell's Equations :

- If the chiral imbalance is constant in time, Maxwell's equations exhibit instability. $\overline{\nabla} \times \overline{B} - \frac{\partial \overline{E}}{\partial t} = \sigma \overline{E} + \xi \overline{B}$

- The gauge field solutions look like

$A_0 = 0, \overline{A}(t) = A_k(0)(\hat{x}\cos(kz) - \hat{y}\sin(kz))e^{t/\tau}$ with

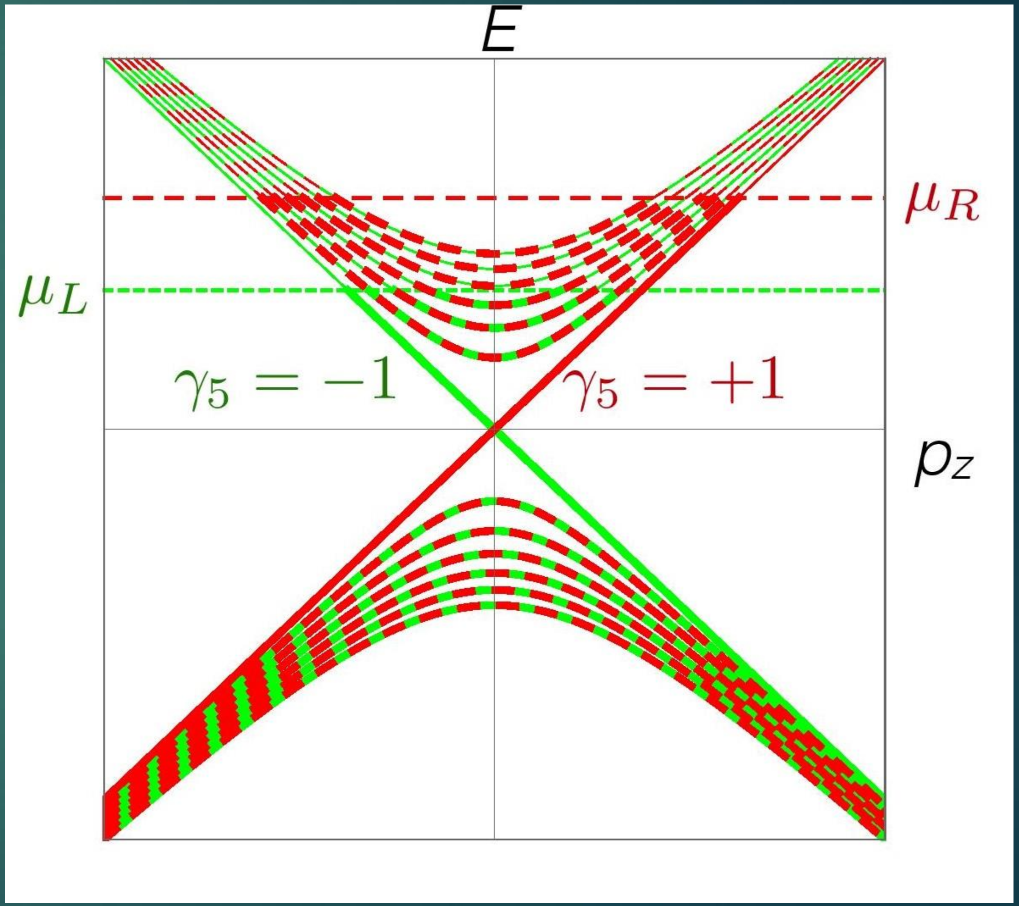
$\tau = k(2k^* - k)/\sigma$ and $k^* = \frac{e^2}{4\pi^2}\mu_5$

The corresponding electric and magnetic fields look like

$$\bar{E} = -\frac{\bar{A}}{\tau} \qquad \bar{B} = k\,\bar{A}$$

Chiral imbalance however is not constant in time in this problem.

- First note that the electric and magnetic field solutions obtained are parallel to each other and are growing in time.
- Imagine the spectrum for nonzero magnetic field but no electric field.
- Now apply a parallel electric field : $\partial_t \overline{p}.\hat{B} = -e\,\overline{E}.\hat{B}$



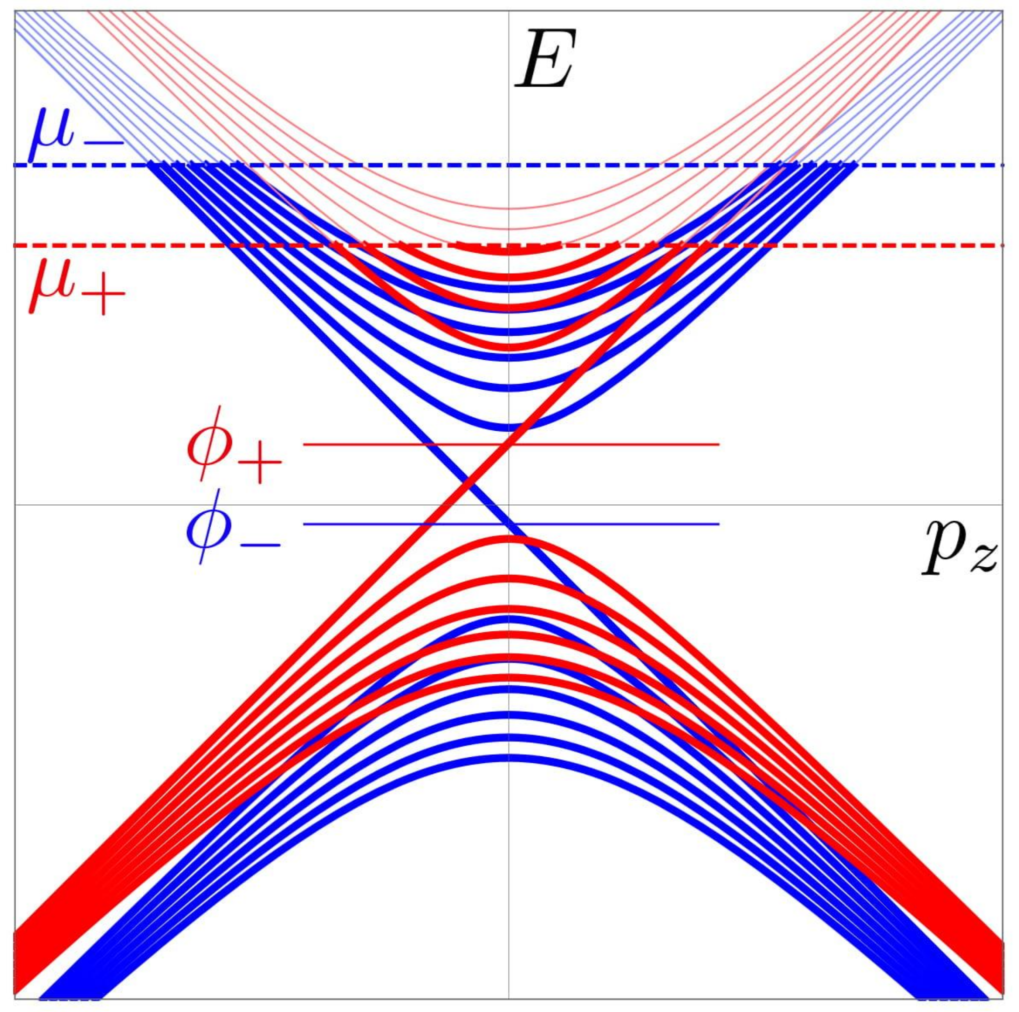
A version of anomaly equations :

- $\Delta n_5 = -\int_{-\infty}^{t} \bar{E}.\bar{B}dt'$
- This implies that as electric and magnetic fields change in time they sap out the chiral imbalance from the system.
- As the chiral imbalance is reduced the rate of the growth of electric and magnetic fields decrease finally saturating the instability.

Imagine a different scenario.

- Neutron stars contain very dense matter that interacts differently with right and left handed electrons.

- This affects the spectrum of the right and left handed electrons significantly. One of them get pushed up in energy, the other down.



Is there a current ?

- The standard argument leads people to think, count the number of particle states with positive and negative momentum and subtract one from the other to get the current.
- The estimate for the current that one obtains this way is $j = e^2 B\,(\mu_5 - \varphi_5)$.
- But this is WRONG. Then what is the right estimate of the current ?

Correct estimate of the current.

- The right way to estimate the current is to use energy conservation.
- Write out Maxwell's equations $\vec{\nabla} \times \vec{B} - \frac{\overrightarrow{\partial E}}{\partial t} = \vec{J}$ and $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- Manipulate to get $\frac{1}{2}\partial_t(\vec{E}.\vec{E} + \vec{B}.\vec{B}) - \nabla.\underbrace{(\vec{E} \times \vec{B})}_{0} = -\vec{E}.\vec{J}$

So we have $\frac{1}{2}\partial_t(\epsilon^2) = -\vec{E}.\vec{J}$

Use energy conservation

- As electric field parallel to the magnetic field changes, the population of right and left handed electrons change. What is the change in energy as this happens ?

- The change in energy in time dt is $d\epsilon = \frac{2\,\alpha}{\pi}\mu_5 EB\; dt$

- This implies that the current is given by $\vec{J} = e^2\,\vec{B}\,\mu_5$

This is different from the naïve estimate of counting states.

Stability analysis :

- Let us now analyze the anomaly equation and well as Maxwell's equation describing the dynamics.
- The ansatz : $A_0 = 0, \vec{A}(t) = A_k(t)(\hat{x}\cos(kz) - \hat{y}\sin(kz))$
- Plugging this in Maxwell's equations we get:

$$\ddot{A_k} = -A_k(t)\left(k - \frac{2\alpha}{\pi}\mu_5(t)\right)k$$

Number of left and right handed particles :

$$n_\pm = \frac{\left(\mu_\pm(t) - \phi_\pm\right)^3}{6\pi^2}$$

Stability analysis :

- The anomaly equation $\dot{n}_{\pm} = \pm \frac{\alpha}{\pi}\,\vec{E}.\vec{B} = -\frac{\alpha k}{2\pi}\frac{d}{dt}A_k^2$
- Constants of motion : $N_{\pm} = n_{\pm}(t) \pm \frac{\alpha k}{2\pi}A_k(t)^2$
- Fermi momenta $k_{F\pm}(t) = \left(6\pi^2(N_{\pm} \mp \frac{\alpha k}{2\pi}A_k(t)^2)\right)^{1/3}$
- Hence we get an equation of motion for the ansatz :

$$\ddot{A}_k(t) = -A_k(t)\left[k - \frac{2\alpha}{\pi}\left(\phi_{\pm} + \frac{k_{F+}(t)-k_{F-}(t)}{2}\right)\right]k$$

- The equation of motion is integrable and we find

$$\frac{\dot{A}_k^2}{2} + V(A_k) = \epsilon_{total}$$

- For small A_k, $V(A_k) = V(0) + \frac{1}{2}A_k^2[(k-k_*)^2 - k_*^2]$+..

$$k_* = -\frac{\alpha\mu_5(0)}{\pi}$$

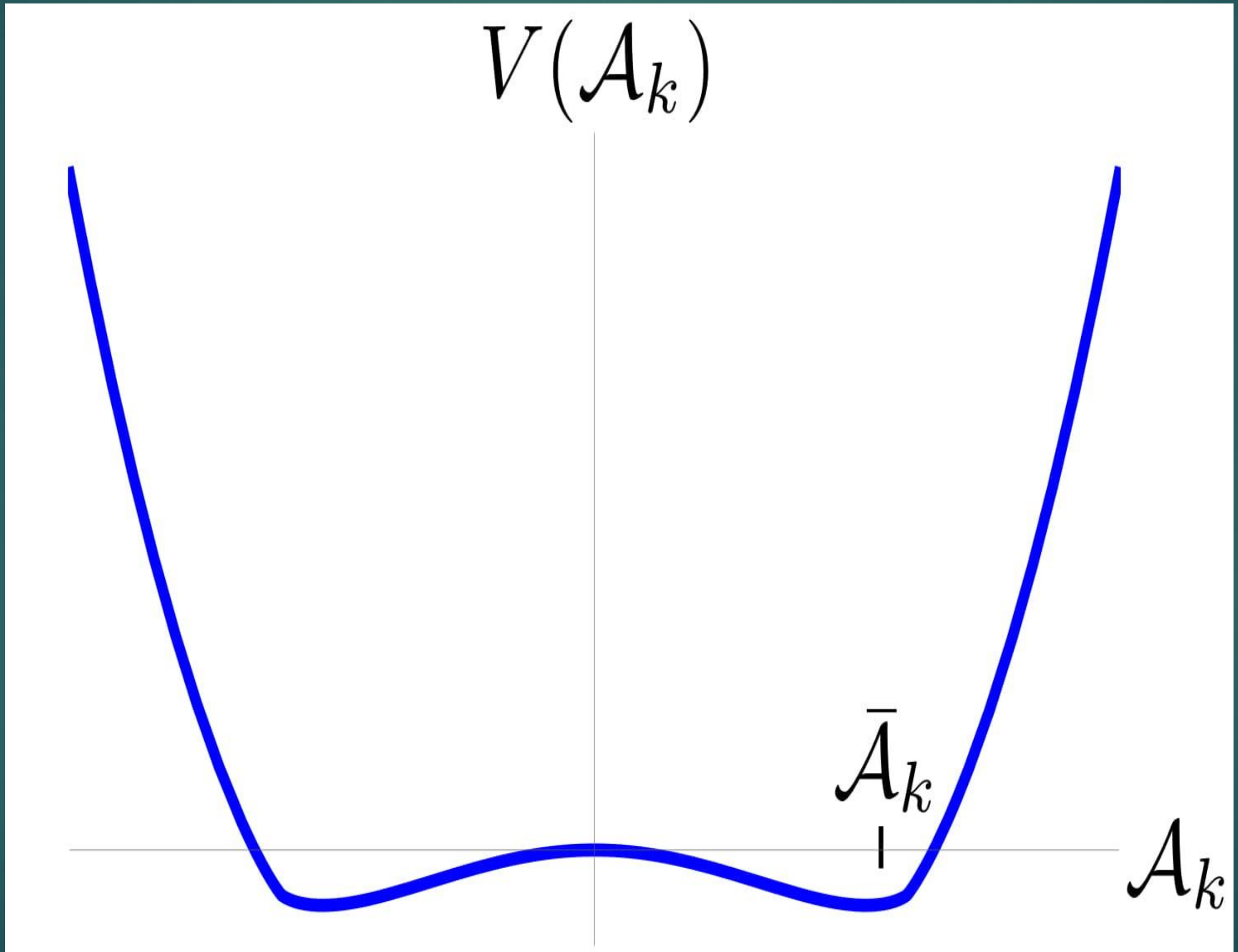
Since chiral imbalance in Fermi energies or background potential is typically small, a small μ_5, ϕ_5, k expansion can be used to obtain a simplified potential for the field.

- This looks like a classic double well potential

$$V(A_k) = V(0) + \frac{1}{2}A_k^2\left((k-k_*)^2 - k_*^2\right) + A_k^4\frac{\alpha^2 k^2}{2\mu\text{^}2}$$

$$\bar{A}_k = \frac{\mu}{\sqrt{2}\,\alpha}$$

Double well potential for CME



Bottomline

It is the initial chiral imbalance in the Fermi energies and not the neutral current mean field potential that sources an chiral magnetic instability.

There is no reason for a homogeneous “standard model plasma” to be unstable unless there is an initial chiral imbalance in the Fermi energies.

Add in a finite conductivity:

An initial chiral imbalance in the absence of conductivity leads to an instability and produces oscillating electric and magnetic helical fields that oscillate in the double well while conserving energy.

The equation of motion changes in the presence of a finite conductivity to which allows dissipation

$$\ddot{A}_k + \sigma\,\dot{A}_k = -A_k\left(k - 2\frac{\alpha}{\pi}\mu_5(t)\right)k$$

The field settles at the minimum of the potential after a time of

$$\tau_k^* = \frac{\sigma}{2{k^*}^2} \approx 10^{-12} \text{ seconds}$$

Finite conductivity:

- At late times after the field has settled to its minimum, the chiral imbalance settles to half its original value :

$$\mu_5(t) = \frac{\mu_5(0)}{2}$$

- The various modes of the magnetic field :

$$\bar{B}_k^2 = \frac{\mu^2}{2\alpha^2}\left({k^*}^2 - (k-k^*)^2\right) \quad \text{for } {k^*}^2 = (k-k^*)^2$$

and $\bar{B}_k^2 = 0$ otherwise

- For optimal wavenumber of $k = k^*$, $\bar{B}_k^2 = \frac{\mu_5(0)^2\mu^2}{2\pi^2}$

Typical wavelengths that grow :

- $\lambda^* = (6.2 \times 10^{-7})\left(\frac{\mu}{100}\right)(\frac{10^{12}}{B_k^*})$ meter Gauss/ MeV
- This indicates that assuming maximally helical field, the above mechanism cannot generate strong fields of astrophysical scales.
- A second mechanism is needed to convert small wavelengths to large wavelengths. However any such mechanism has to be very rapid as electrons have mass.

Electron mass :

- Electron mass can destroy chiral imbalance pretty quickly.
- The way this happens is via collisions.
- Electrons hit protons in what is known as Rutherford scattering and reverse their direction of momentum. If the spin is not flipped, chirality of the electron changes in such a collision. The probability for this process to occur is zero for massless electrons, but is not zero for massive electrons.

Electron mass :

- Electrons have a mass of about 0.5 MeV.
- The typical relaxation time : $\tau_m = 10^{-12}\left(\frac{\mu}{100}\right)^3 \frac{T}{10^8}$ K^-1 (MeV)^-3 seconds.
- The anomaly equation turns into

$$\dot{\mu}_5(t) = -\frac{\mu_5(t)}{\tau_m} - \frac{2\pi\alpha}{\mu^2} k\, \dot{A}_k A_k \text{ for } \mu_5 \ll \mu$$

Electron mass :

- Chiral imbalance wants to produce a helical magnetic field where as the mass of electrons wants to flip the chirality driving the chiral imbalance to zero.
- If the time scale of chirality flipping due to mass is smaller than time scale of generation of magnetic field, no field will be generated at all.
- In typical situations for an isolated neutron star in equilibrium, this is indeed the case.

Way out ?

- To get around the problem posed by a finite electron mass we need to consider more violent events.
- One needs to look at turbulent phenomena.
- Binary mergers can also possibly produce a sustained chiral imbalance which can be a source of magnetic field.
- Requires further study !!

Thank You !