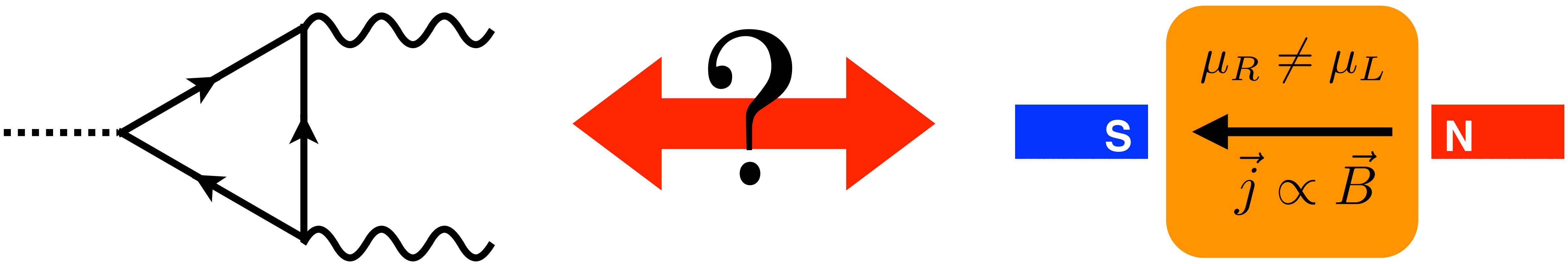
Anomlaous hydrodynamics from projection operator method



Masaru Hongo (RIKEN, iTHEMS)

Open Problems and Opportunities in Chiral Fluids,
2018 July 18th, Santa Fe

Based on arXiv: 1808.***** (in preparation) MH, Noriyuki Sogabe, Naoki Yamamoto

Outline

MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?

APPROACH:

Mori's method as a generalization of current algebra

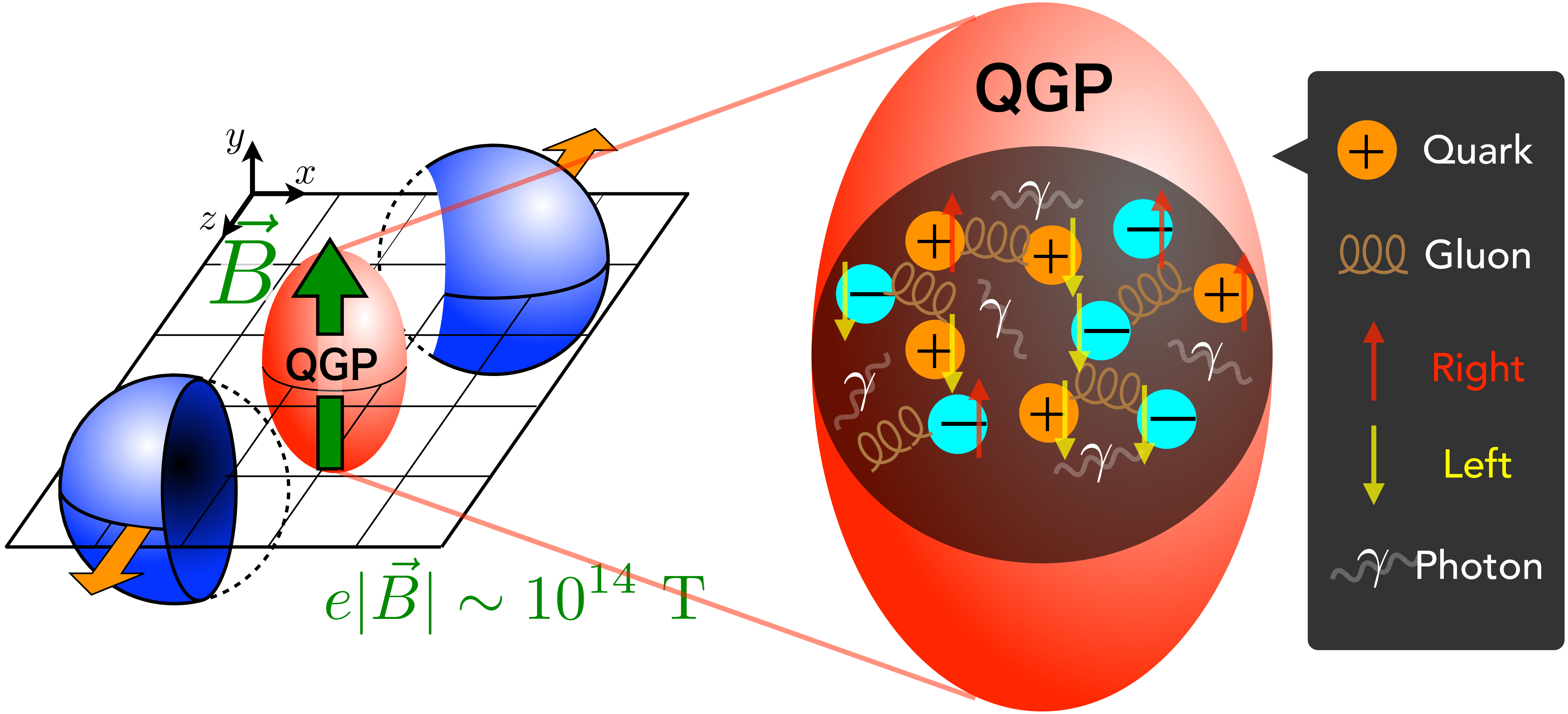
Anomalous commutation:

RESULT:

Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

QGP as Chiral fluid

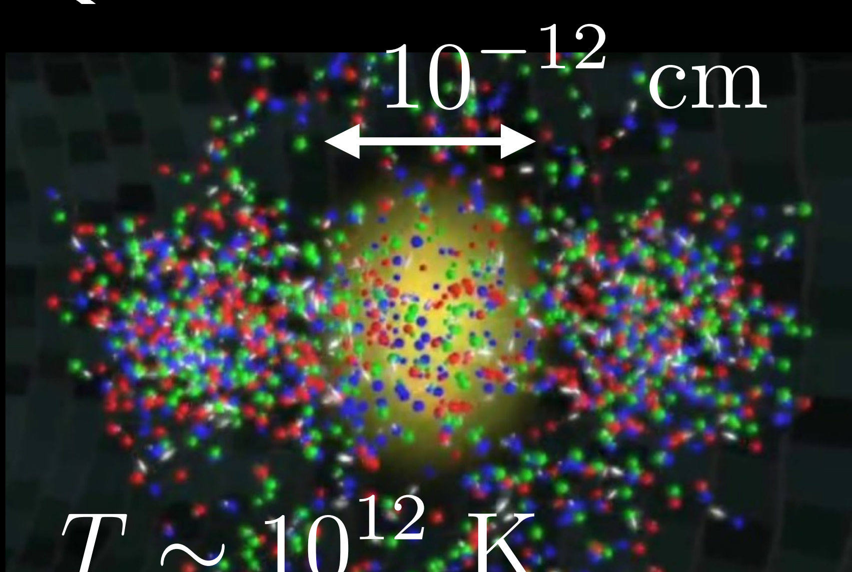


- Existence of extremely strong magnetic field
- Chirality drastically affect hydrodynamic transport

Hydrodynamics is

- Effective theory for macroscopic dynamics
- Universal description, not depending on details
- Only conserved quantity ~ ~~symmetry~~ of system

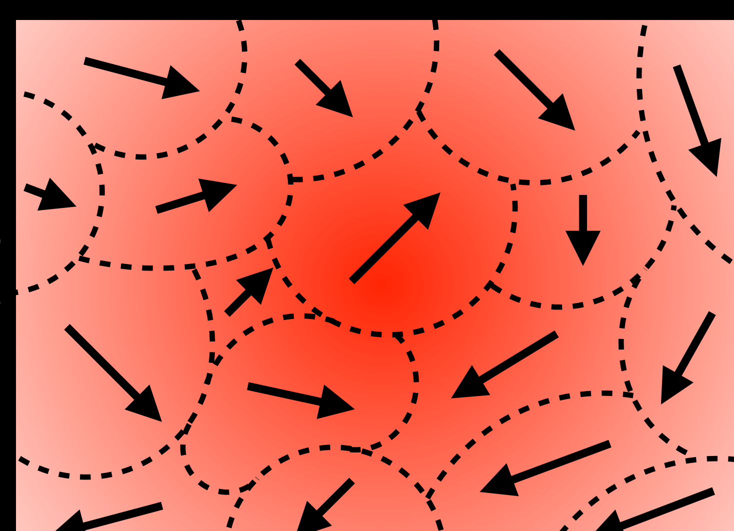
Quark-Gluon Plasma



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

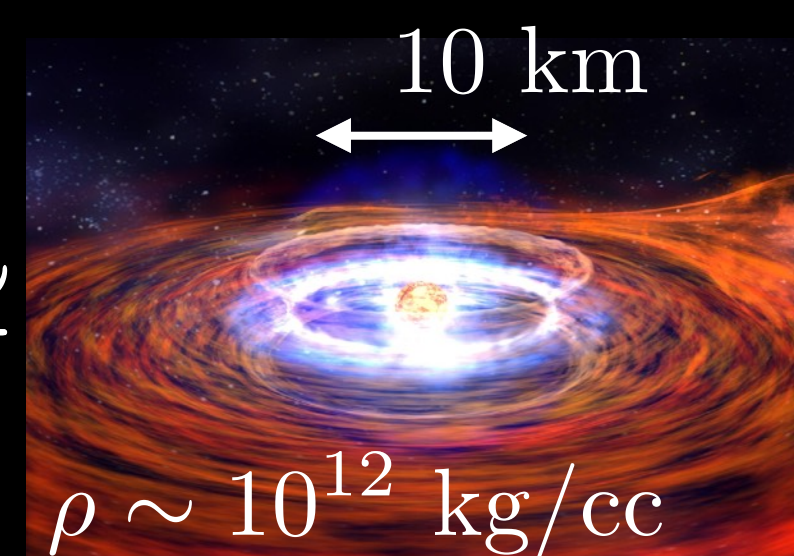
$\simeq$

Hydro: $\{\beta(x),\ \vec{v}(x)\}$



$\simeq$

Neutron Star

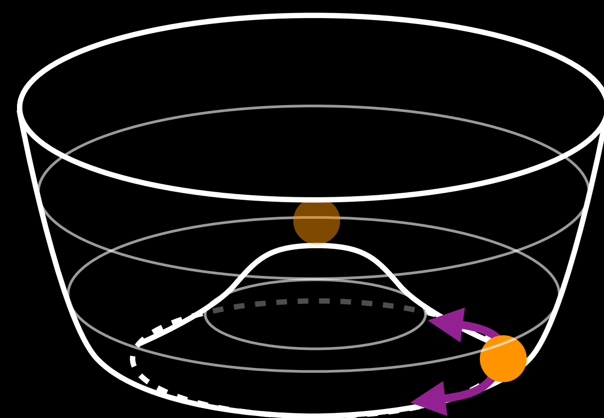


http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

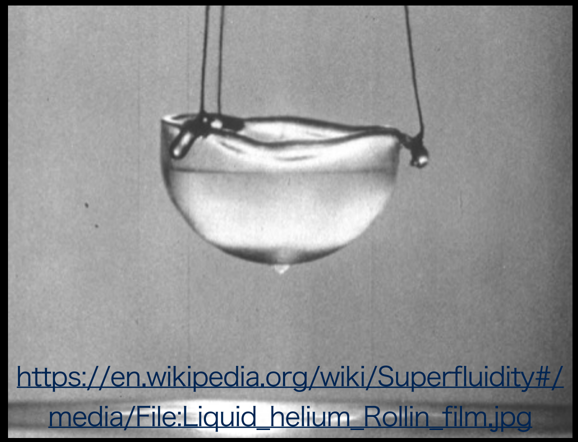
Hydro with symmetry breaking

- Spontaneous symmetry breaking

Micro：Selecting vacuum

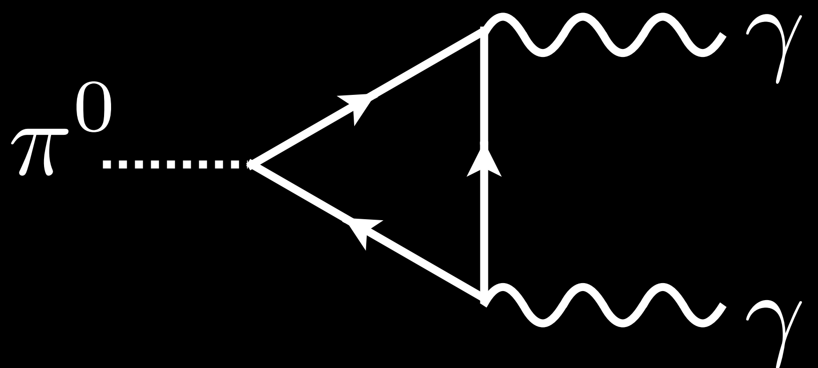


Macro：Superfluid



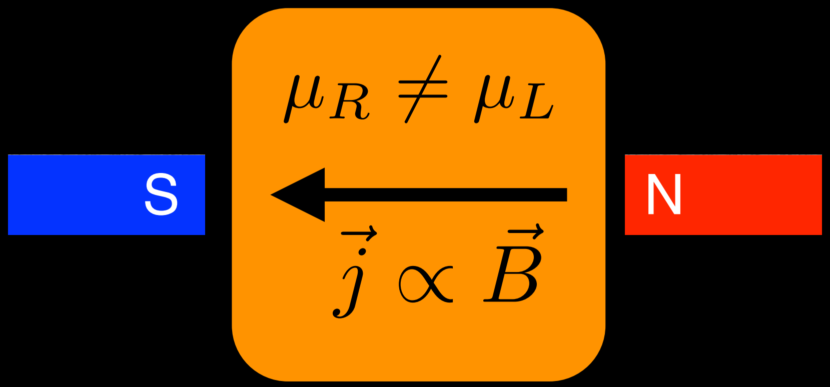
- Symmetry breaking by quantum anomaly

Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport



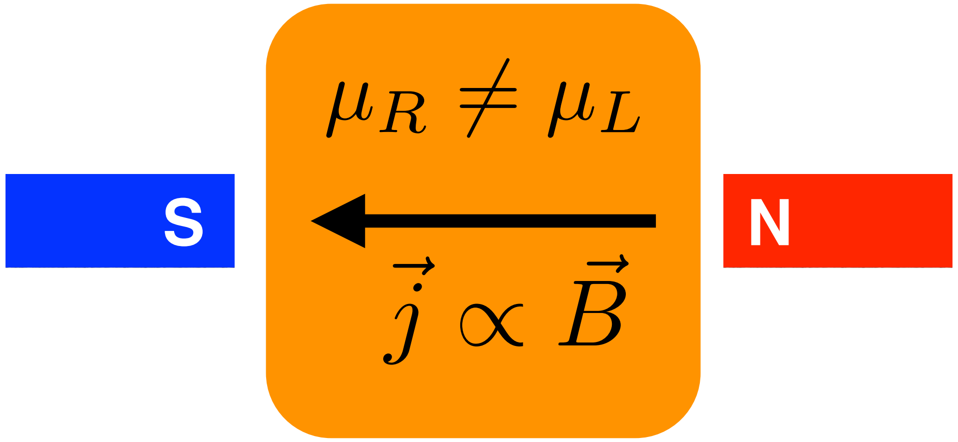
[Erdmenger et al. (2008), Son-Surowka (2009)]

Parity-violating chiral transport

[Fukushima et al. (2008), Vilenkin (1980)]

◆Chiral Magnetic Effect (CME)

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



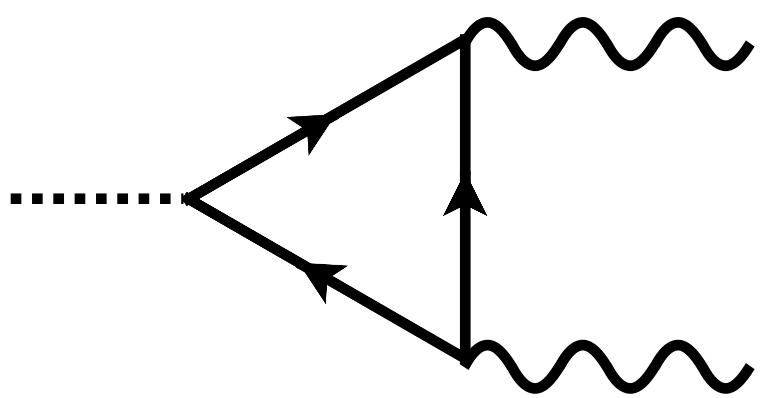
[Erdmenger et al. (2008), Son-Surowka (2009)]

◆ Chiral Vortical Effect (CVE)

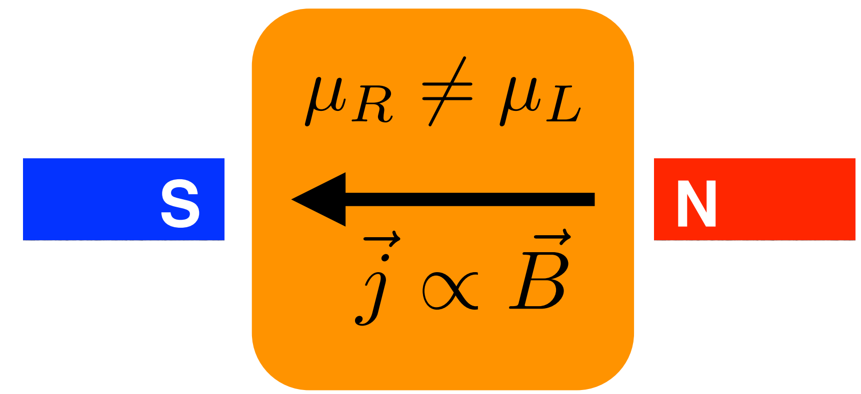
$$\vec{J} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$

Anomaly and chiral transport

Quantum anomaly



CME



Can we understand this based on current algebra (CA)?

◆ Advantages

- Only based on universal symmetry structure (cf. Kinetic theory)
- Easy to generalize the situation including other massless modes

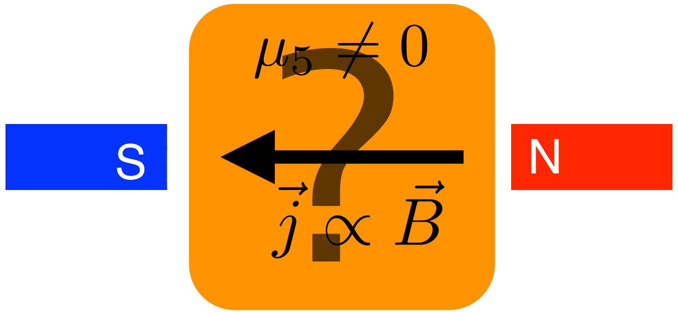
◆ Problem

Not vacuum! → Need generalization of CA for $T \neq 0, \mu \neq 0$

Outline

MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra

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Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

Review: Current algebra for QCD

◆ Current algebra for $SU(N)_R \times SU(N)_L$

$$\left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{L,b}(x)\right] = i f^{c}{}_{ab} \hat{J}^{\mu}_{L,c}(x), \qquad \left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{R,b}(x)\right] = 0$$

$$\left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{R,b}(x)\right] = i f^{c}{}_{ab} \hat{J}^{\mu}_{R,c}(x), \qquad \left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{L,b}(x)\right] = 0$$

Low-energy theorem

- Goldberger-Treiman relation
- Soft Pion theorem, $\cdots$

Universal results for process with low-energy pion scattering!

If current algebra satisfies the above relations,

it does not matter whether UV theory is QCD, NJL model, or anything!

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}_5^0(t,\boldsymbol{x}), \hat{J}_5^0(t,\boldsymbol{y})\right] = 0$$

$$\cancel{\left[\hat{J}_5^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = 0}$$

Proof.

Definition of Noether current gives

$$\hat{J}^0(x) = -\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} i\hat{\phi} = -i\hat{\pi}(x)\hat{\phi}(x), \quad \hat{J}_5^0(x) = -i\hat{\pi}(x)\gamma_5\hat{\phi}(x)$$

Using canonical commutation relation $\left[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})\right] = i\delta(\boldsymbol{x}-\boldsymbol{y})$

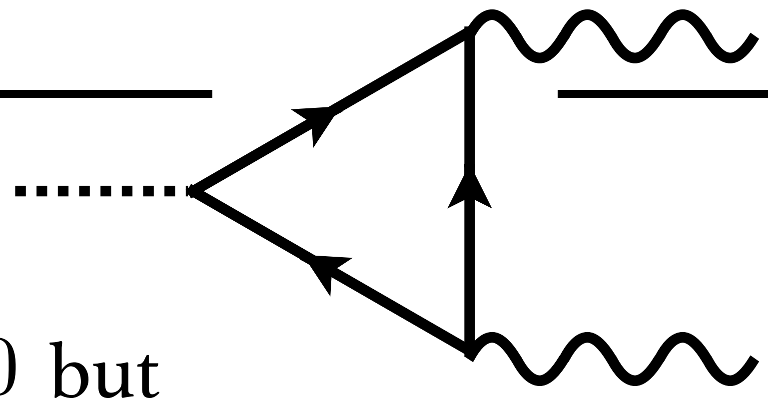
we can directly show the above current algebraic structure! $\square$

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}_5^0(t,\boldsymbol{x}), \hat{J}_5^0(t,\boldsymbol{y})\right] = 0$$

$$\left[\hat{J}_5^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y}) \partial_i^x \delta(\boldsymbol{x}-\boldsymbol{y})$$



Sketch of Proof.

Ward-Takahashi identity is not $\langle \partial_\mu J_5^\mu(x) \rangle_A = 0$ but

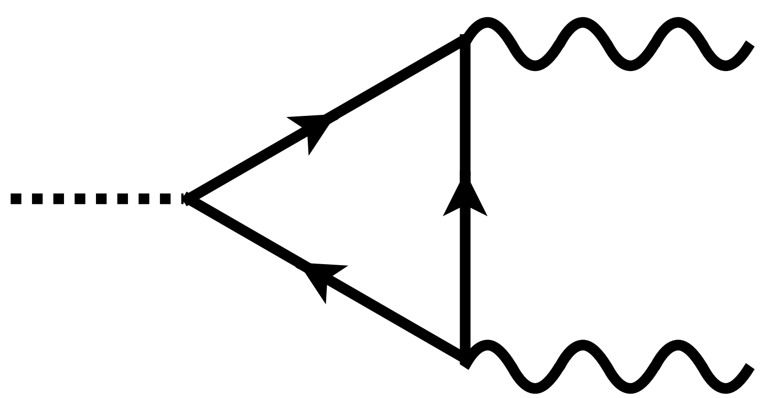
$$\langle \partial_\mu J_5^\mu(x) \rangle_A = C\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x) \sim C dA dA$$

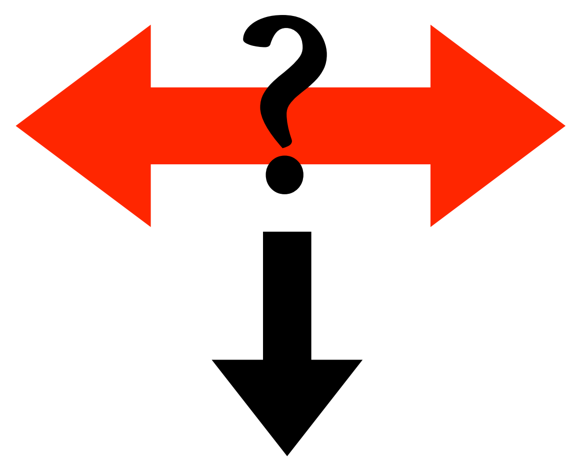
Variation w.r.t A_0 **gives** $\partial_\mu \langle J_5^\mu(x) J^0(y) \rangle_A \sim C ddA \sim C dB$

"Corr. function = T-product in operator formalism" gives the above $\square$

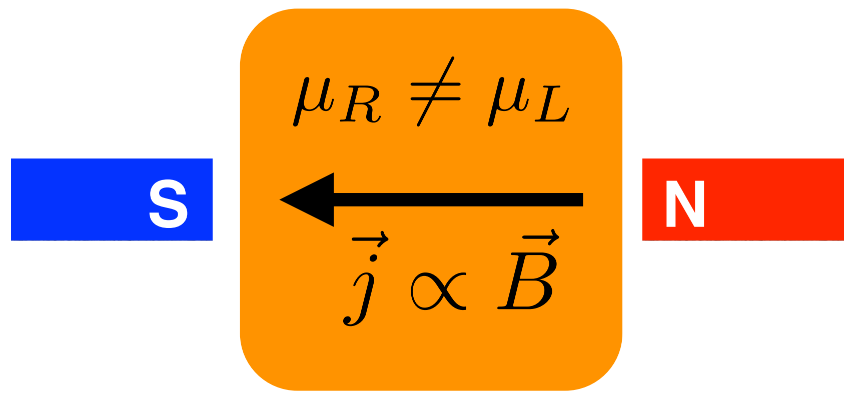
Anomaly and chiral transport

Quantum anomaly





CME



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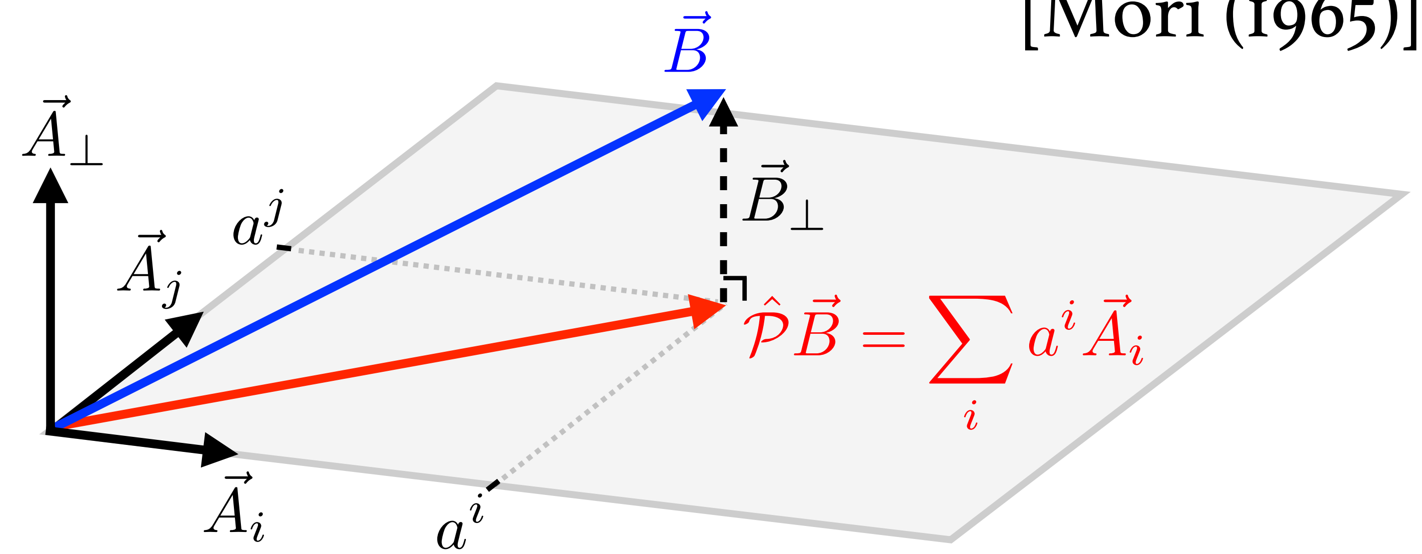
Mori's projection operator method

[Mori (1965)]

A method to write down

Equation of Motion (EoM)

only focusing on $\hat{A}_n(t)$



◆ **EoM given by Mori's projection operator method**

$$\partial_0 \hat{A}_n(t) = i\Omega_n{}^m \hat{A}_m(t) - \int_0^t ds \Phi_n{}^m(t-s)\hat{A}_m(s, \boldsymbol{y}) + \hat{R}_n(t)$$

Reversible Dissipative Noise

$$\begin{cases} i\Omega_n{}^m = -\dfrac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu\big([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0)\big) \\ \textbf{Fluctuation Dissipation relation: } \Phi_n^m(t-s) = \big(\hat{R}_n(t-s), \hat{R}^m(0)\big) \end{cases}$$

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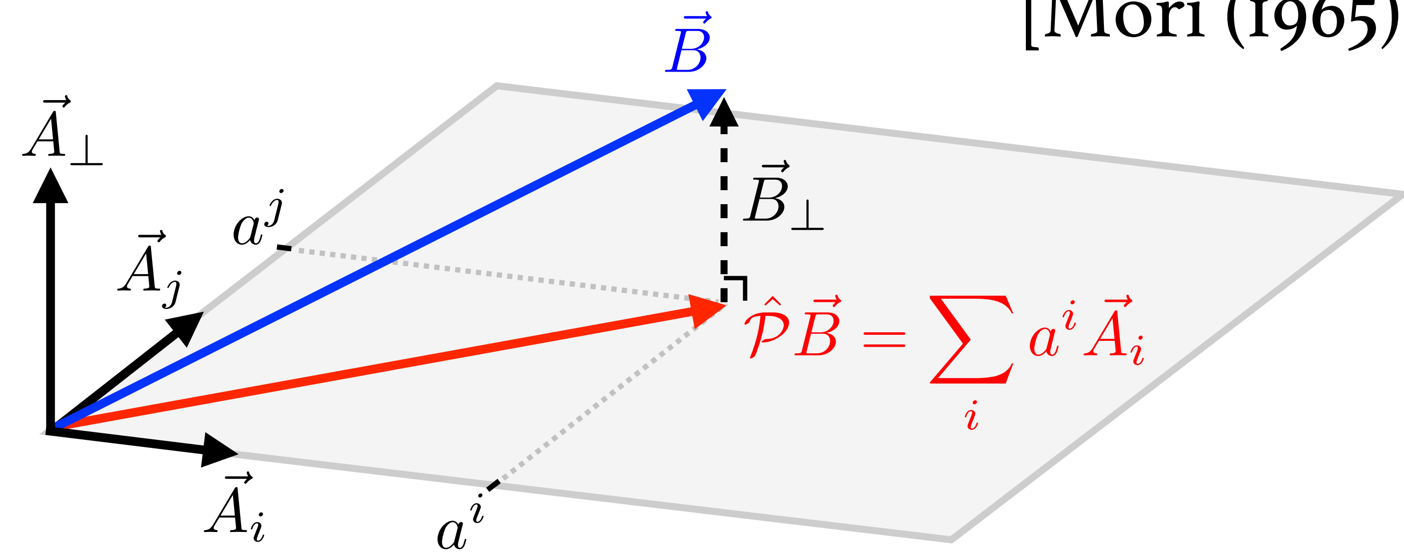

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Reversible **Dissipative** **Noise**

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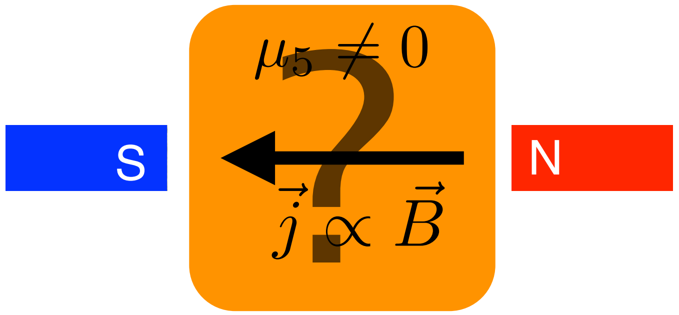
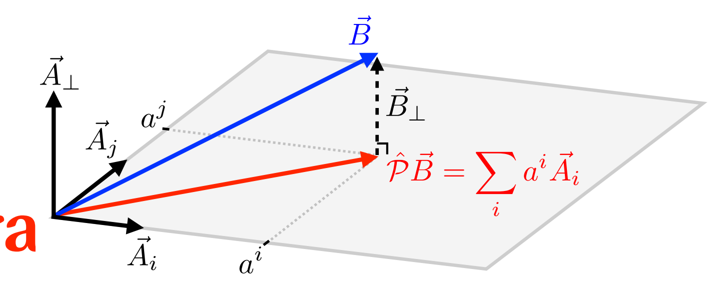


APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $\left[\hat{J}_5^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\dfrac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial_i^x \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

- Derivation of anomalous hydrodynamics
- New(?) perspectives on CMW
- Anomalous mixing of neutral pion & CMW

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Mori's method and current algebra

◆ Leading Order term in EOM

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$$ (χ^{lm} : inv. suscep.)

◆ Current algebra related to relativistic hydrodynamics

Choose $\hat{A}_n(t)$ as conserve charges: $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x})\}$

➡ **EoM(LO) is controlled by energy-momentum density algebra!**

<u>**Current algebra**</u>

$$\begin{cases} [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_0(t,\boldsymbol{y})] = -i\big(\hat{T}^k{}_0(t,\boldsymbol{x}) + \hat{T}^k{}_0(t,\boldsymbol{y})\big)\partial_k\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{y})] = -i\big(\hat{T}^j{}_i(t,\boldsymbol{x})\partial_j - \hat{T}^0{}_0(t,\boldsymbol{y})\partial_i\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{T}^0{}_j(t,\boldsymbol{y})] = -i\big(\hat{T}^0{}_j(t,\boldsymbol{x})\partial_i + \hat{T}^0{}_i(t,\boldsymbol{y})\partial_j\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \end{cases}$$

➡ **EoM for perfect fluid (Sound wave) is derived!!**

Perfect fluid from Mori's method

◆ Relativistic hydrodynamic from Mori's method

[Minami-Hidaka (2013)]

$$\partial_0 \delta \hat{T}^0{}_0 = -ik^i \delta \hat{T}^0{}_i$$

$$\partial_0 \delta \hat{T}^0{}_i = -ik_i h_{\mathrm{eq}} \chi^{ee} \delta \hat{T}^0{}_0 - \left[k_i k^k \left(\frac{\zeta}{h_{\mathrm{eq}}} + \frac{d-3}{d-1} \frac{\eta}{h_{\mathrm{eq}}} \right) + \boldsymbol{k}^2 \delta_i^k \frac{\eta}{h_{\mathrm{eq}}} \right] \delta \hat{T}^0{}_k + \hat{R}_{\pi_i}$$

◆ Green-Kubo formula for transport coefficients (viscosity)

$$\zeta = \beta_{\mathrm{eq}} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{0}))$$

$$\eta = \frac{\beta_{\mathrm{eq}}}{(d+1)(d-2)} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{\pi}_{ik}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{\pi}_{jl}(0, \boldsymbol{0})) \Delta^{ij} \Delta^{kl}$$

➡ **Reversible → Sound wave / Dissipative → Diffusion mode**

CME from anomalous commutation

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x}), J^0(t,\boldsymbol{x}), J^0_5(t,\boldsymbol{x})\}$

Current algebra with anomalous commutation rel.

$$\begin{cases} [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -i\hat{J}^0(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})] = -i\hat{J}^0_5(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\dfrac{i}{2\pi^2}B^i(t,\boldsymbol{y})\partial_i^x\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = [\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})] = 0 \end{cases}$$

◆ **EoM for** $\hat{J}^0(t,\boldsymbol{x})$

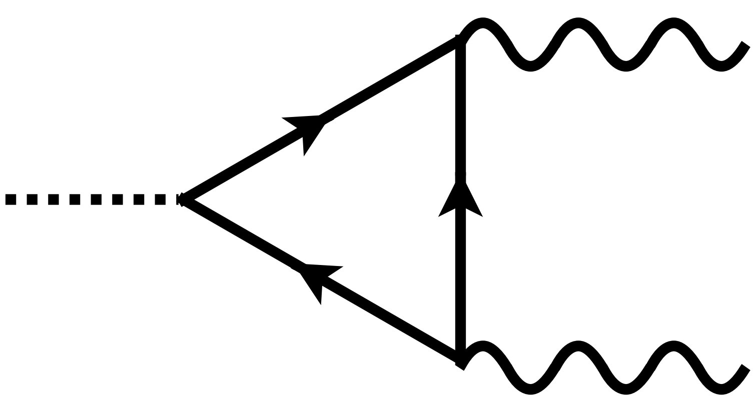
$$\partial_0 \hat{J}^0(x) + \partial_i^x\left[\frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2}B^i(x)\right] + \cdots = 0$$

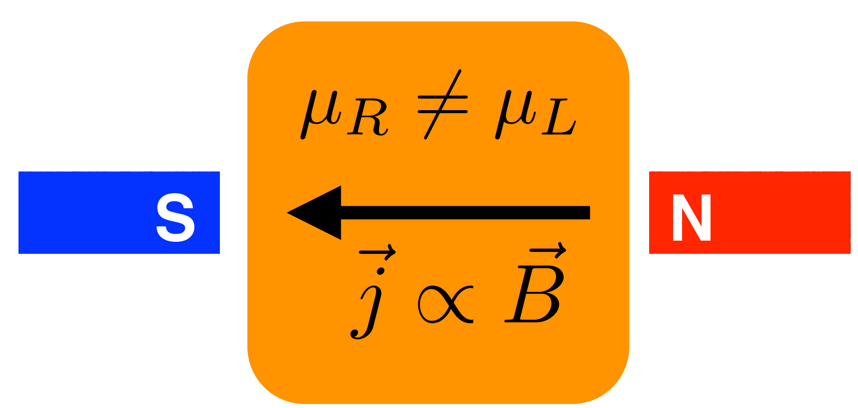
CME from anomalous commutation

$$\partial_0 \hat{J}^0(x) + \partial_i^x \Bigg[\underbrace{\frac{\chi^{nn_5} \hat{J}^0_5(x)}{2\pi^2} B^i(x)}_{= \hat{J}^i(x)} \Bigg] + \cdots = 0$$

◆ **Summary of result**

- **Conservation law:** $\partial_\mu \hat{J}^\mu(x) = 0$
- **Const. relation:** $\hat{J}^i(x) = \dfrac{\chi^{nn_5} \hat{J}^0_5(x)}{2\pi^2} B^i(x)$ **Chiral Magnetic Effect (CME)**





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Chiral Magnetic Wave (CMW)

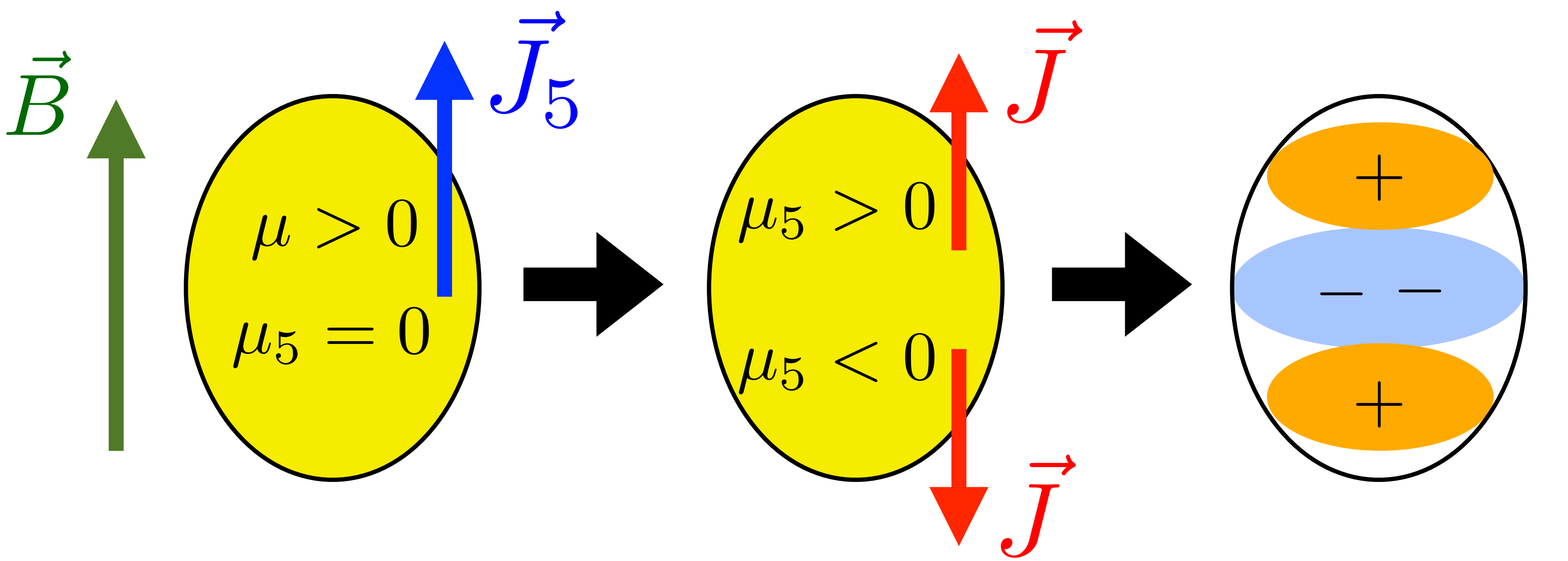
Chiral Magnetic Effect

$$\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}_5^0(x)}{2\pi^2}B^i(x)$$

Chiral Separation Effect

$$\hat{J}_5^i(x) = \frac{\chi^{n_5 n}\hat{J}^0(x)}{2\pi^2}B^i(x)$$

Ex.



Charge propagation along B

||

Collective excitation

➡ **Chiral Magnetic Wave** = **Analogue of Charge density wave in Tomonaga-Luttinger liquid**

[Kharzeev-Yee, (2011)]

$$[\hat{J}_R^0(t,x), \hat{J}_R^0(t,y)] = -iC\partial_x\delta(x-y)$$

Interpretation as Type-B NG mode?

◆ Spontaneous symmetry breaking & Nambu-Goldstone mode

For some conserved charge $\hat{Q}_a$

$$\exists\, \hat{\Phi}(x) \ \text{ such that } \ \langle [i\hat{Q}_a, \hat{\Phi}_i(x)] \rangle \equiv \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a, \hat{\Phi}_i(x)]\big) \neq 0$$

Spontaneous Symmetry Breaking (SSB)

➡ **Massless mode = Nambu-Goldstone (NG) mode appears!**

◆ Classification of NG mode

[Hidaka (2012), Watanabe-Murayama(2012)]

$$\begin{cases} \text{- Type-A NG mode : } \ \forall\, \hat{Q}_b \ \text{ satisfy } \ \langle [i\hat{Q}_a, \hat{Q}_b] \rangle = 0 \\ \text{- Type-B NG mode : } \ \exists\, \hat{Q}_b \ \text{ such that } \ \langle [i\hat{Q}_a, \hat{Q}_b] \rangle \neq 0 \end{cases}$$

➡ **Generalization of Nambu-Goldstone's theorem for type-B NG mode**

Sound & CMW ≒ Type-B NG mode?

- Type-B NG mode： $\exists\, \hat{Q}_b$ such that $\langle [i\hat{Q}_a, \hat{Q}_b] \rangle \neq 0$

Hydrodynamics of chiral plasma contains

massless collective excitation known as
- Sound wave
- Chiral Magnetic Wave (CMW)

◆ Origin of Sound wave and CMW

$$\langle \big[\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{y})\big] \rangle = -i\big(\langle \hat{T}^j{}_i(t,\boldsymbol{x})\rangle \partial_j - \langle \hat{T}^0{}_0(t,\boldsymbol{y})\rangle\big)\partial_k \delta(\boldsymbol{x}-\boldsymbol{y}) \neq 0$$

$$\langle \big[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\big] \rangle = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial_i^x \delta(\boldsymbol{x}-\boldsymbol{y}) \neq 0$$

➡ The above definition states they are a friend of Type-B NG mode!

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm} \langle [\hat{A}_n(0), \hat{A}_m(0)] \rangle \hat{A}_l(t) + \cdots$$

- Derivation of anomalous hydrodynamics
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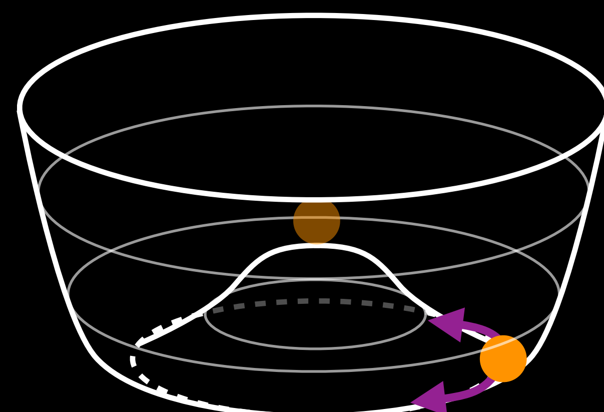
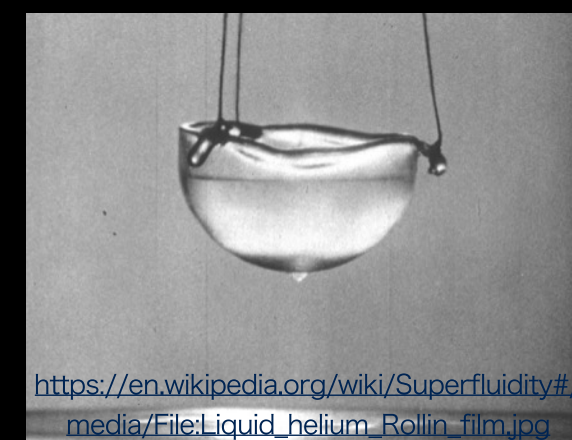
Anomalous superfluid?

◆ Spontaneous symmetry breaking

Micro : Selecting vacuum

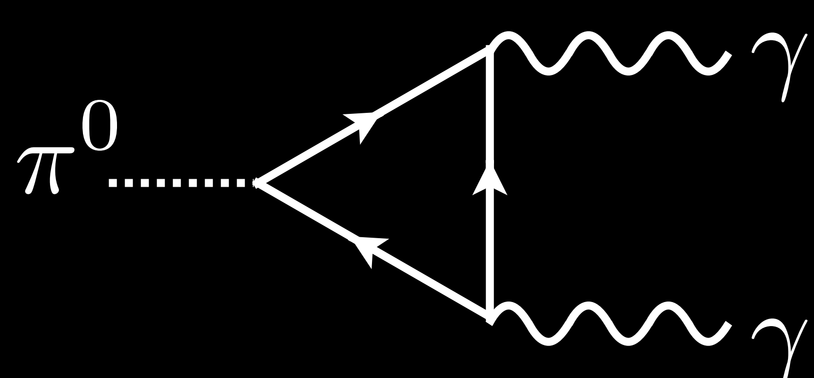


Macro : Superfluid



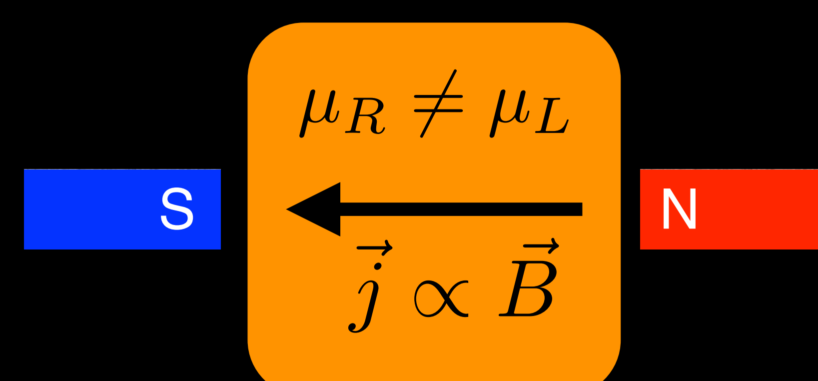
◆ Symmetry breaking by quantum anomaly

Micro : π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro : Anomalous transport



[Erdmenger et al. (2008), Son-Surowka (2009)]

Symmetry of QCD under B

- Symmetry structure of chiral limit QCD under magnetic field

Explicitly broken by magnetic field | Spontaneously broken

$$SU(2)_R \times SU(2)_L \to U(1)_V^3 \times U(1)_A^3 \to U(1)_V^3$$

Original symmetry of QCD

→ 1 SSB = 1 NG mode = π_0 (=Neutral pion) appears!

cf. Spontaneous symmetry breaking & Nambu-Goldstone mode

For some conserved charge $\hat{Q}_a$

$\exists\, \hat{\pi}_i(x)$ such that $\langle [i\hat{Q}_a, \hat{\pi}_i(x)] \rangle = \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a, \hat{\pi}_i(x)]\big) \neq 0$

Spontaneous Symmetry Breaking (SSB)

→ This can be also captured by the projection operator method!

Towards anomalous superfluid

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(x), \hat{T}^0{}_i(x), \hat{J}^0(x), \hat{J}^0_5(x), \hat{\pi}(x)\}$

<u>**Current algebra**</u>

<u>**with**</u>

<u>**Anomaly & SSB**</u>

$$\begin{cases} \chi^{\pi\pi}(\boldsymbol{k}) = \boldsymbol{k}^2 + O(\boldsymbol{k}^4) \\ \langle[\hat{J}^0_5(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})]\rangle = \sigma_0 \delta(\boldsymbol{x}-\boldsymbol{y}) \\ \text{Others: the same as before} \end{cases}$$

◆ **EoM for** $\{\hat{J}^0(x), \hat{J}^0_5(x), \hat{\Phi}(x)\}$

$$\partial_0 \hat{J}^0_V(x) + \partial_i\left(C\chi^{AA}B^i\hat{J}_A\right) + \cdots = 0$$

$$\partial_0 \hat{J}^0_A(x) + \partial_i\left(C\chi^{VV}\hat{J}_V(x)B^i(x) - \sigma_0\partial_i\hat{\pi}(x)\right) + \cdots = 0$$

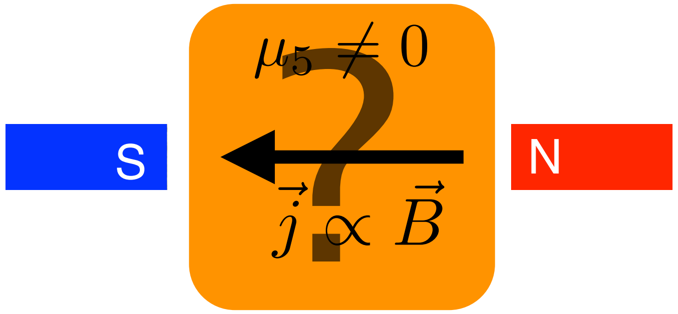
$$\partial_0 \hat{\pi}(x) + \chi^{AA}\sigma_0\hat{J}_A + \cdots = 0$$

$\omega^2 = \sigma_0^2\chi^{AA}\boldsymbol{k}^2 + C^2\chi^{VV}\chi^{AA}(\boldsymbol{k}\cdot\boldsymbol{B})^2$ **: Mixing CMW and NG mode!**

Summary

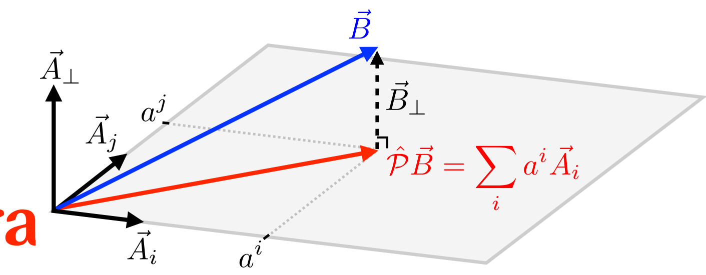
MOTIVATION:

Origin of chiral transport (**Chiral Magnetic Effect**)?



APPROACH:

Mori's method as a generalization of **current algebra**



Anomalous commutation: $[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

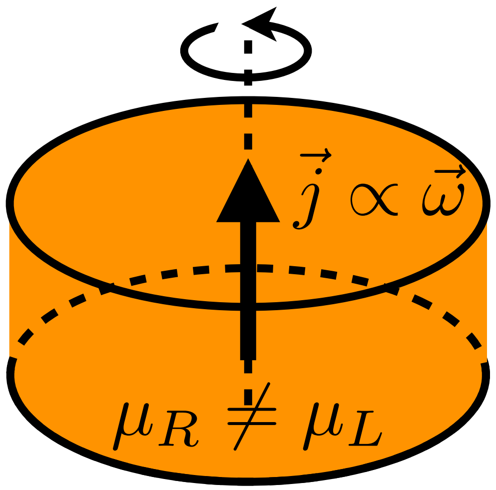
Chiral Magnetic Effect in operator formalism: $\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)$

Anomalous superfluid: mixing between CMW and NG mode

Outlook

◆ Chiral vortical effect?

$$\vec{J}_5 = \underbrace{\frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2+\mu_5^2}{4\pi^2}}_{\text{From chiral anomaly}} + \underbrace{\frac{T^2}{12}}_{\text{From where?}}\right)\vec{\omega}$$



◆ Path-integral treatment?

	Operator formalism	Path-integral formalism
QCD (T=0)	CA w/ Anomalous CR	Chiral pert. w/ Wess-Zumino term
Hydro	Mori's projection w/ Anomalous CR	MSRJD effective action w/ ? ? [Crossley et al. (2015)] Paolo's talk

◆ Effect on critical dynamics?

[MH-Sogabe-Yamamoto arXiv: 1803.07267]

QCD (2-flavor chiral limit) without EM tensor: Model E → Model A

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