CME & CMV neutron star kicks, thermodynamic chiral transport, and polarization

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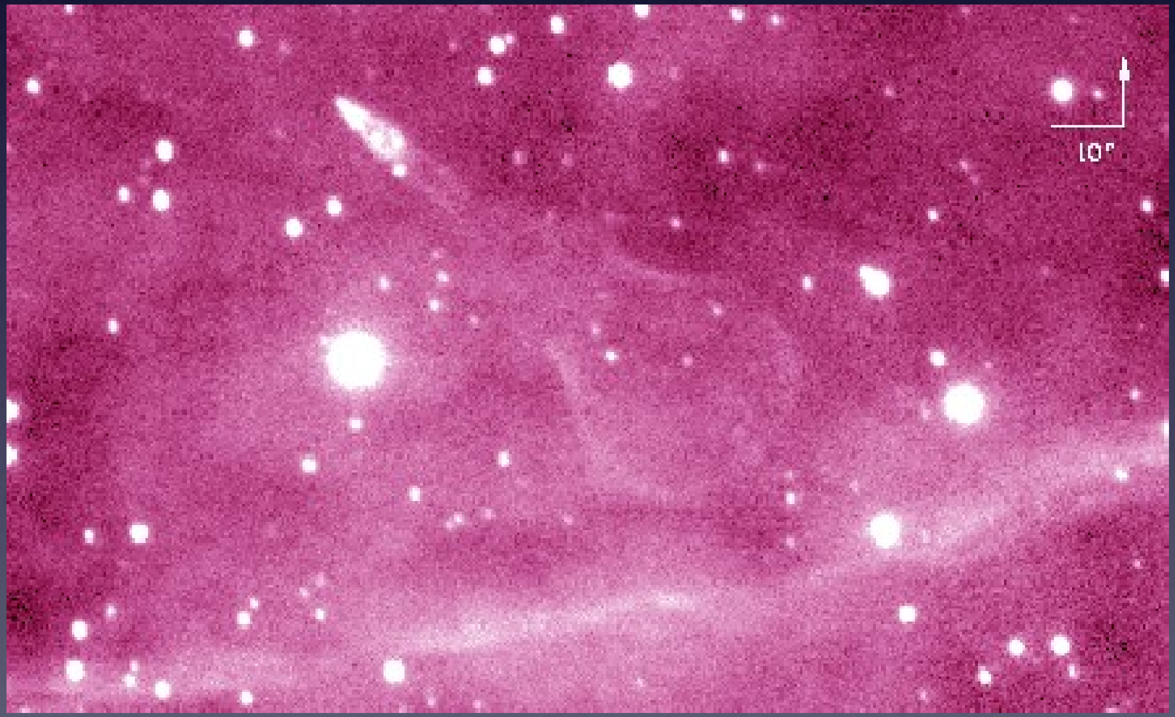
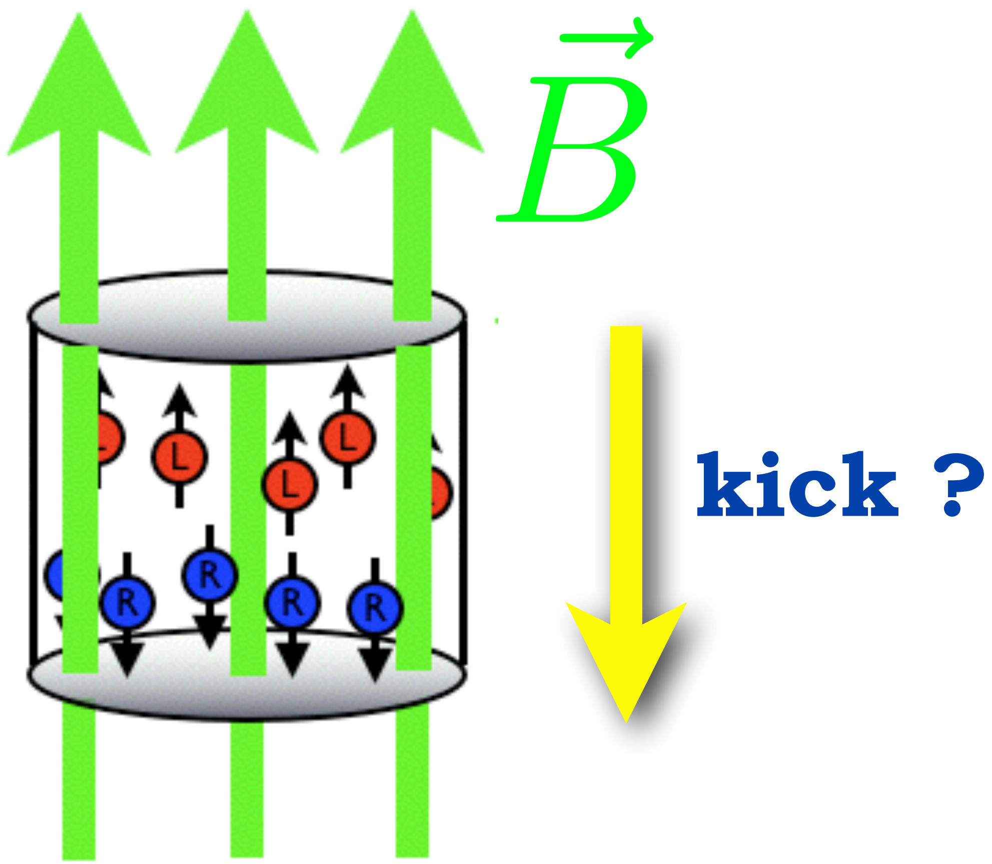
Open Problems and Opportunities in Chiral Fluids, Santa Fe, NM

July 19th, 2018



Matthias Kaminski
University of Alabama

Chiral hydrodynamics & neutron star kicks

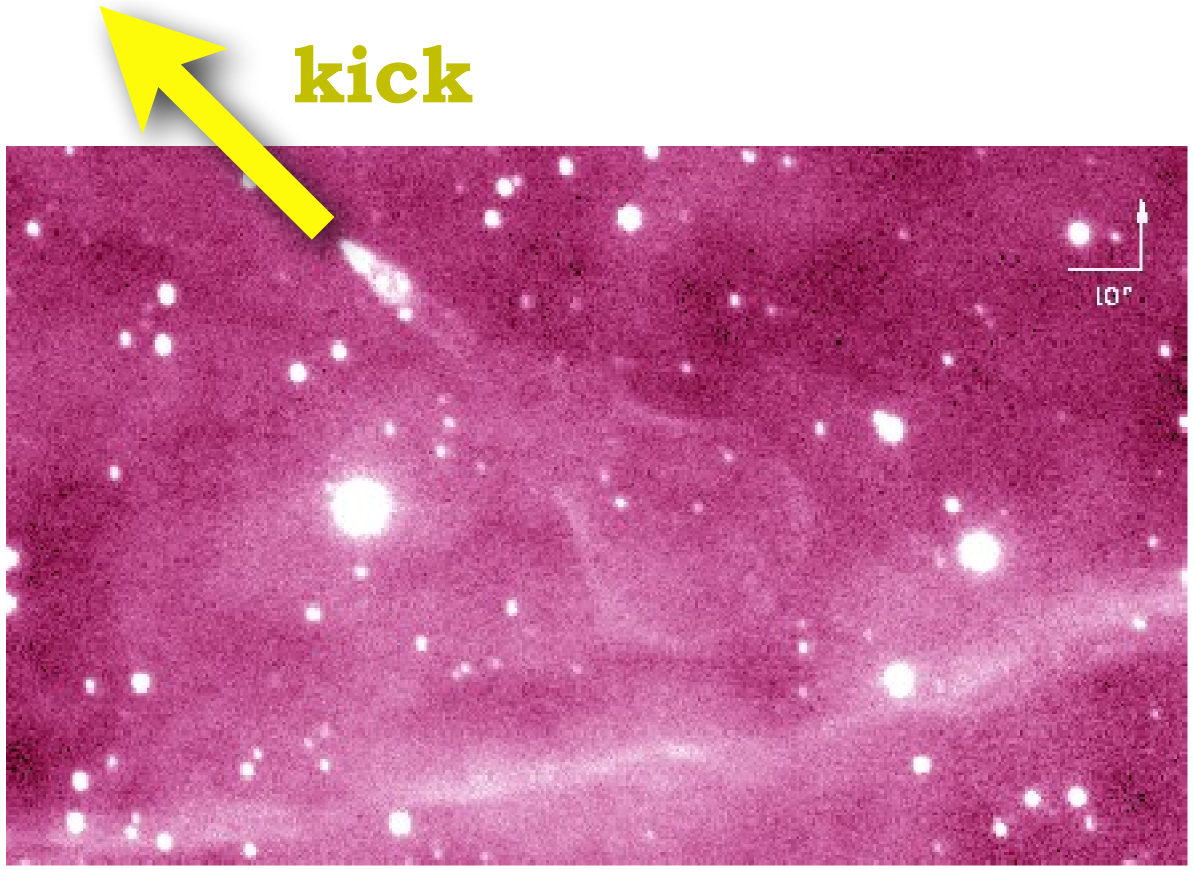


hydrodynamics: fluids with left-handed and right-handed particles produce a **current** along magnetic field

e.g. right/ left-handed electrons, neutrinos, ...

theory: [Son,Surowka; PRL (2009)] [Landsteiner]

experiment: [Huang et al; PRX (2015)]



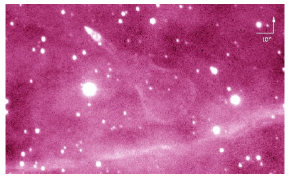
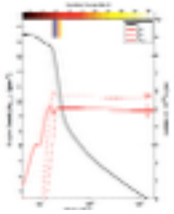
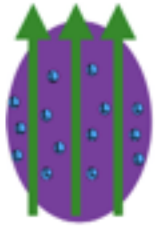
observation: neutron stars undergo a large momentum change (a kick)

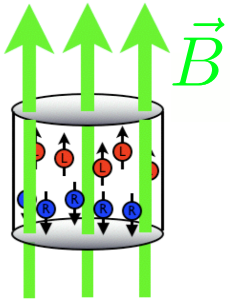
[Chatterjee et al.; Astrophys. J (2005)]

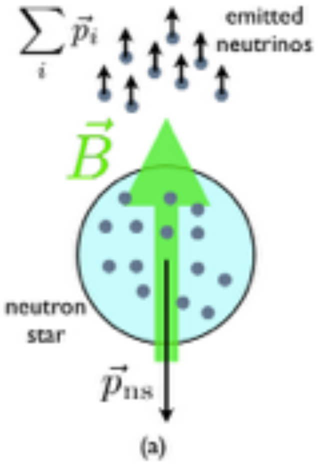
Can chiral hydrodynamics be relevant for neutron star kicks?

Outline

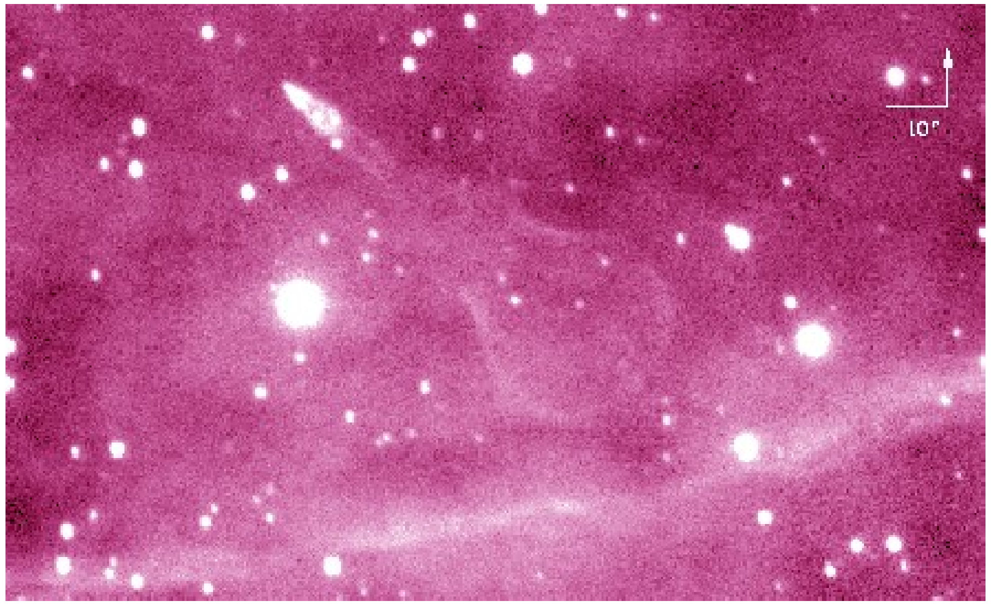
✓ Invitation: chiral hydrodynamics & neutron star kicks

1. Observations 
2. Chiral hydrodynamics
3. Simulation data 
4. Kicks from anomalies
5. Opportunities 



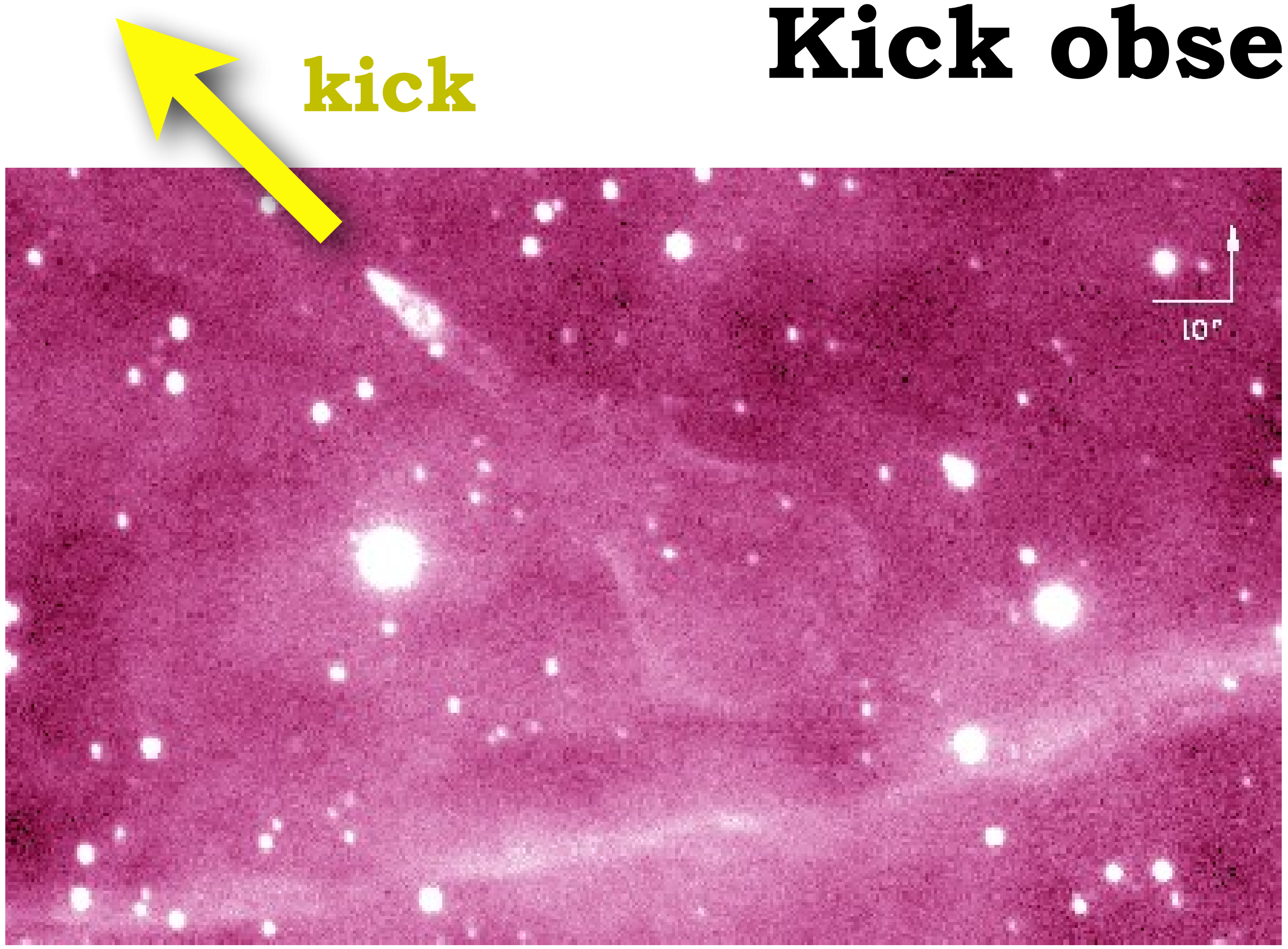


1. Observations



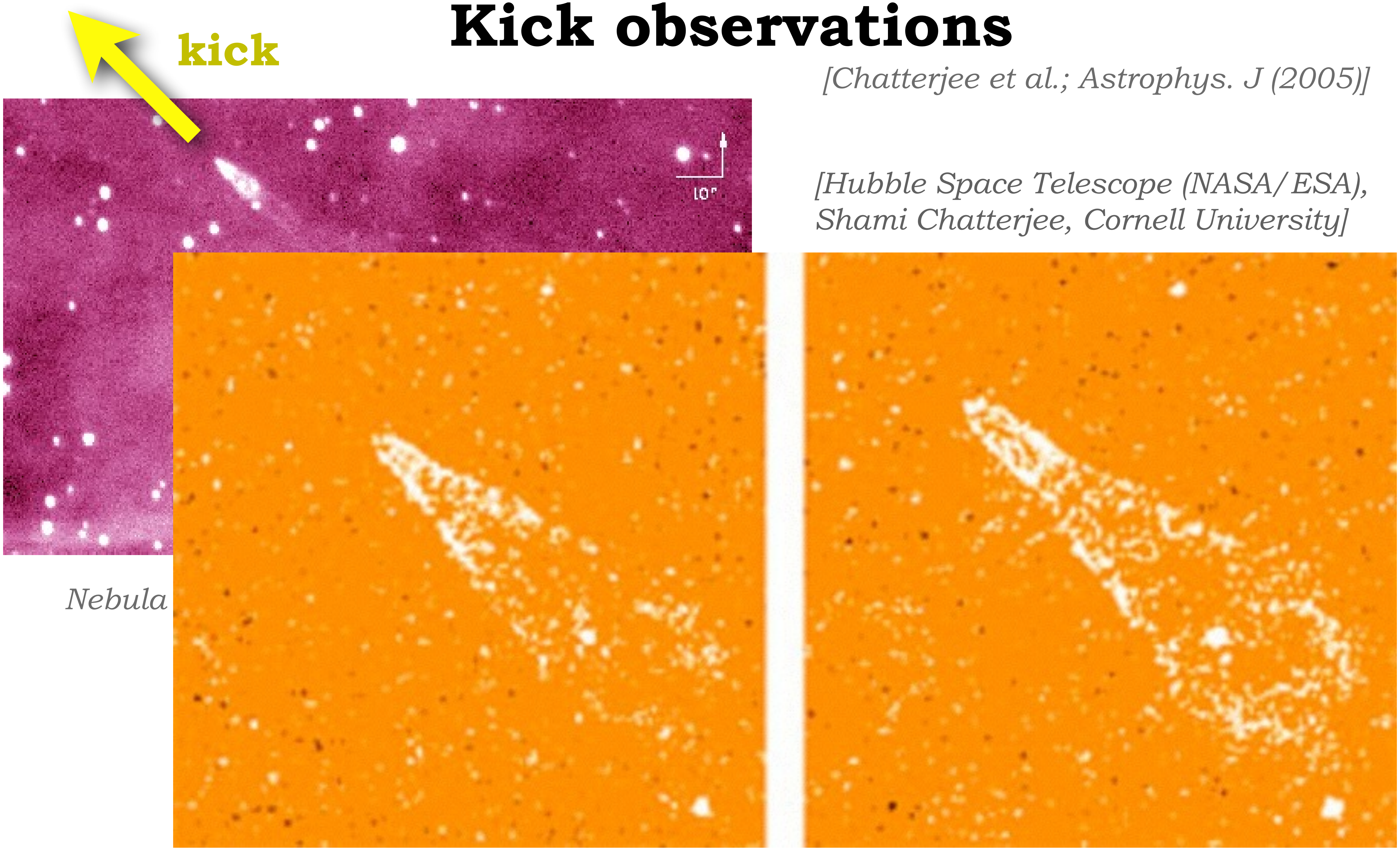
Kick observations

[Chatterjee et al.; Astrophys. J (2005)]



Nebula discovered: [Cordes et al; Nature (1993)]

Kick observations



[Chatterjee et al.; Astrophys. J (2005)]

[Hubble Space Telescope (NASA/ESA), Shami Chatterjee, Cornell University]

December 1994

December 2001

Kick observations

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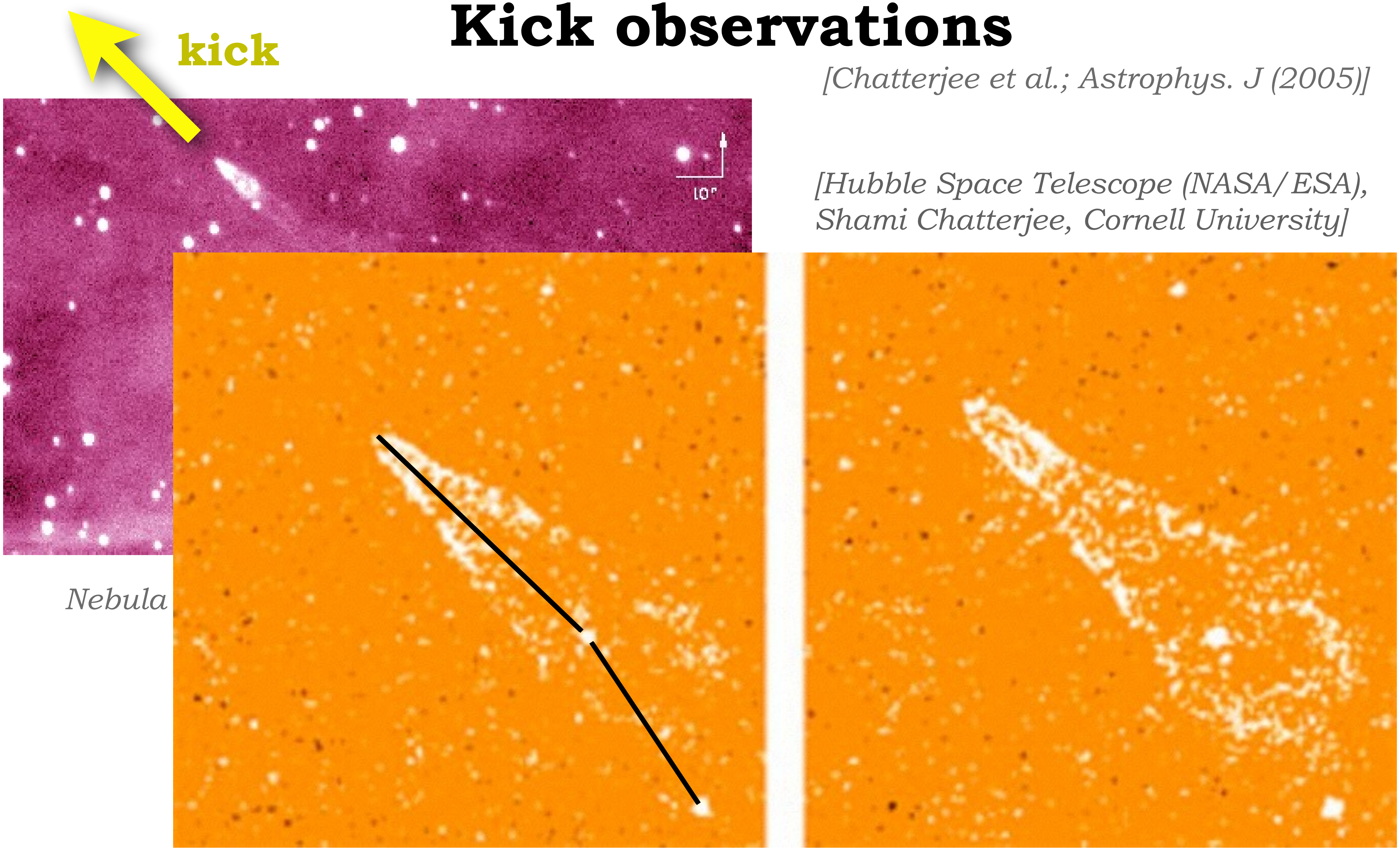

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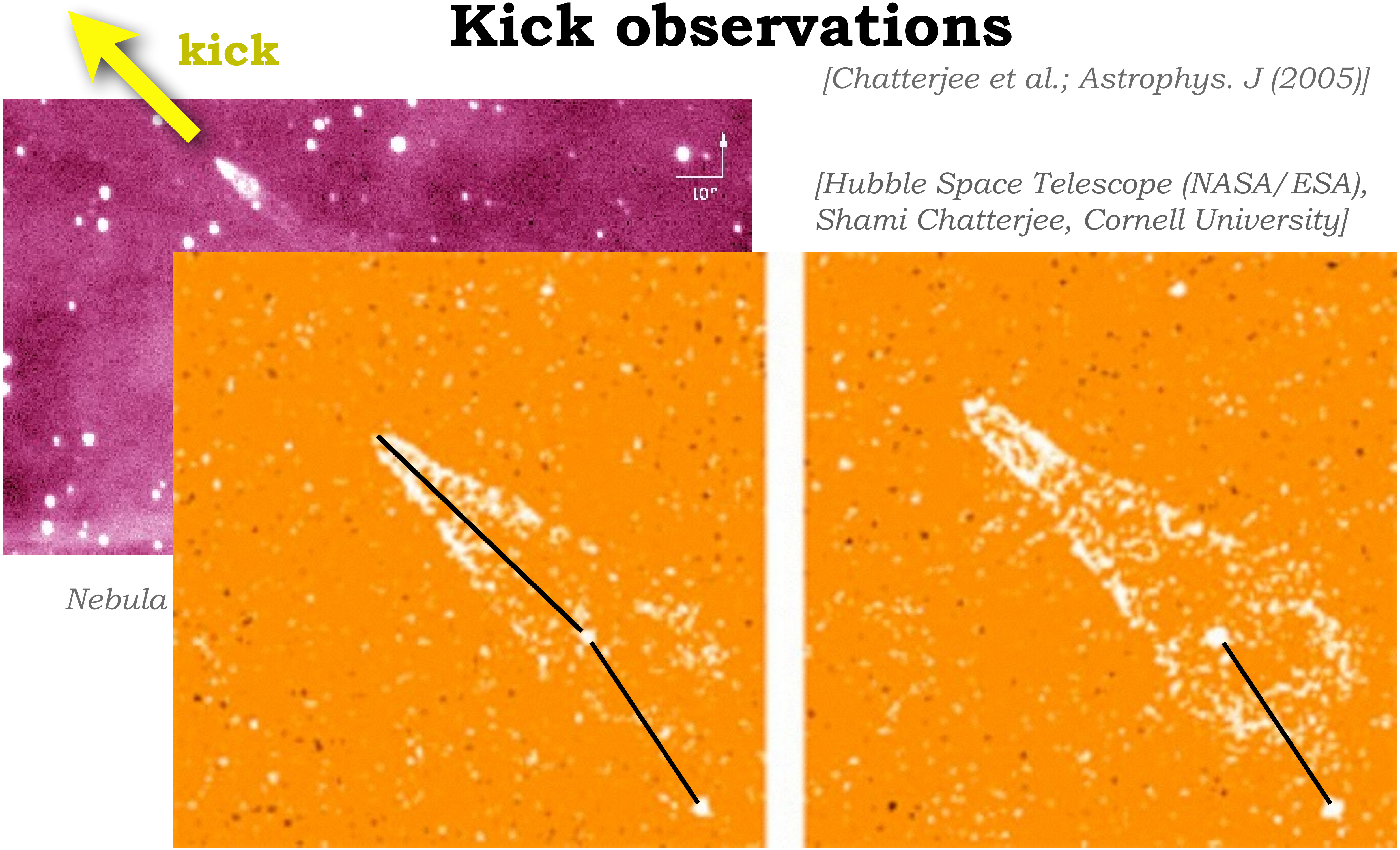

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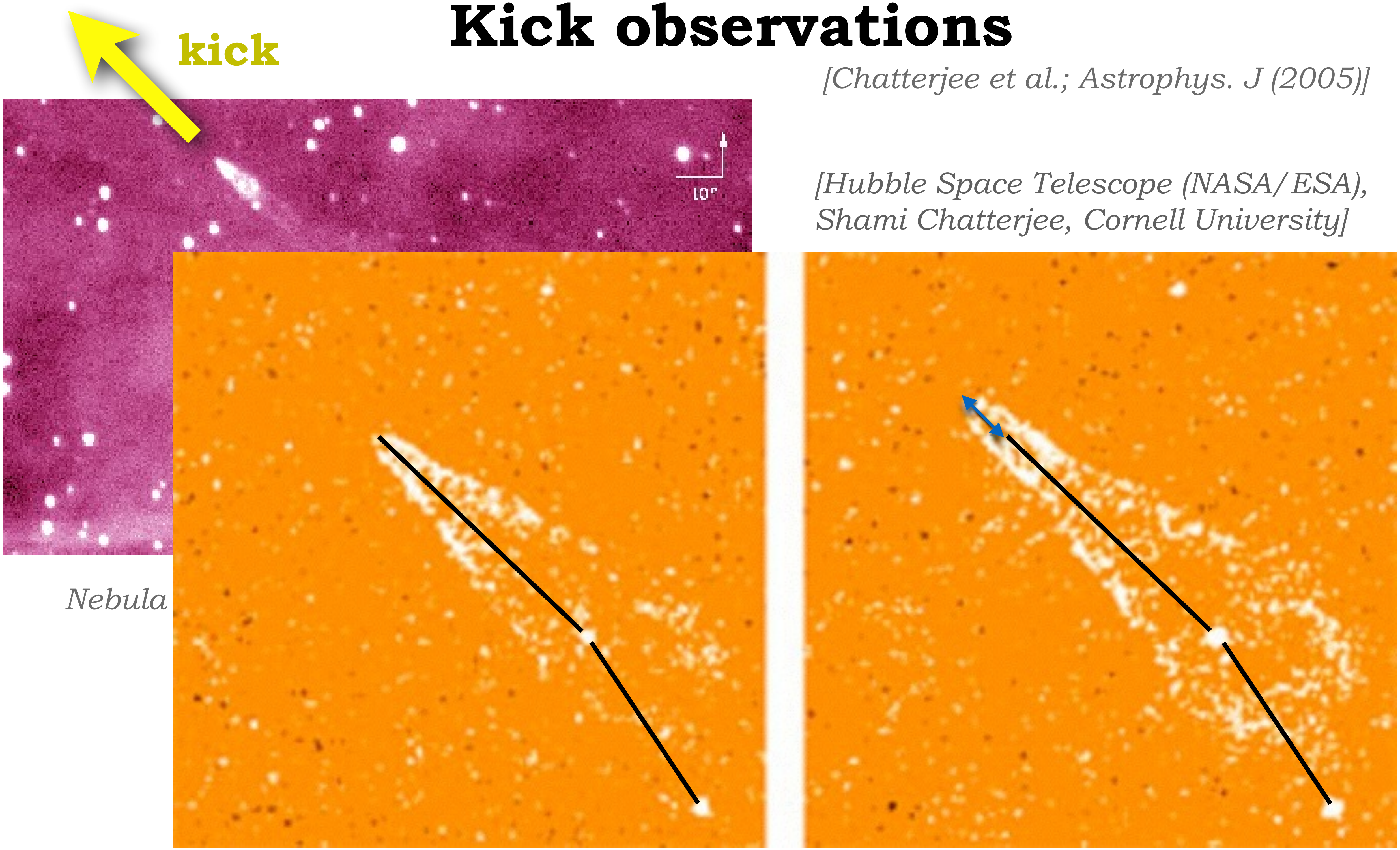


[Chatterjee et al.; Astrophys. J (2005)]

[Hubble Space Telescope (NASA/ESA), Shami Chatterjee, Cornell University]

Nebula

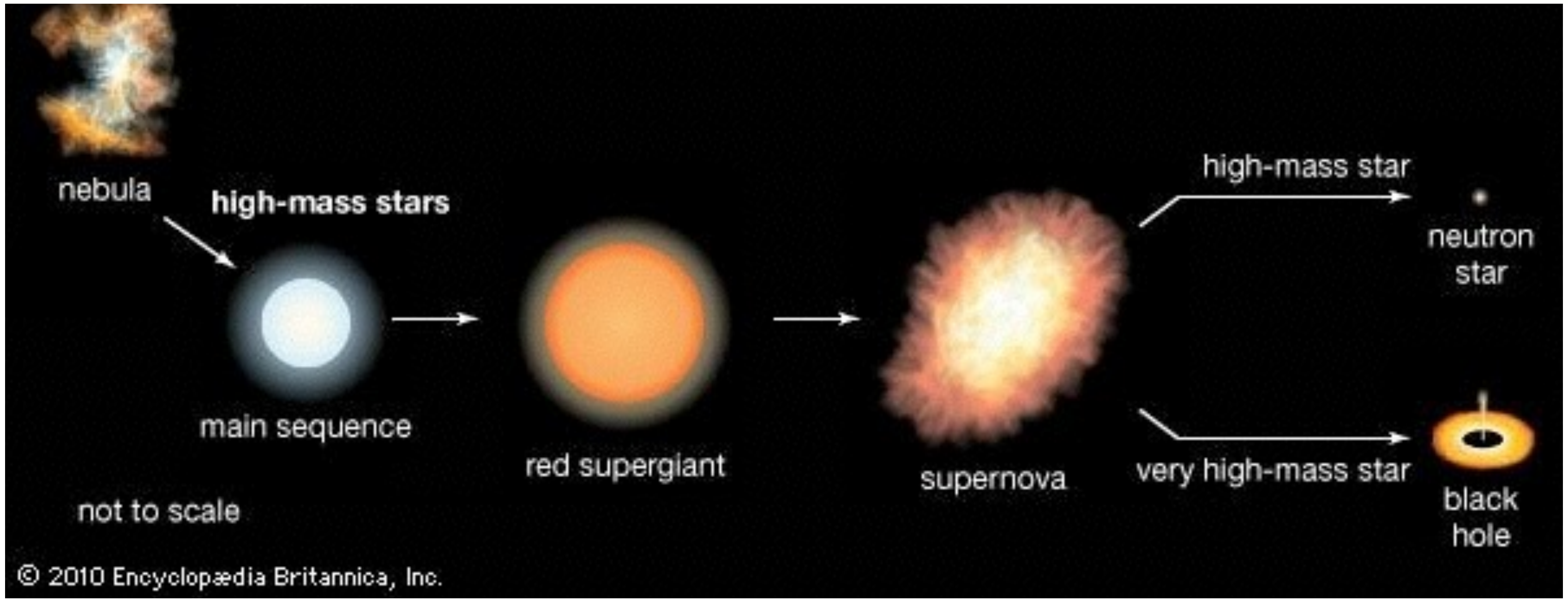
December 1994

December 2001

Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

Neutron star genesis

- compact star
  - * small radius
  - * large mass
  - * high density
- quick rotation
- large B field



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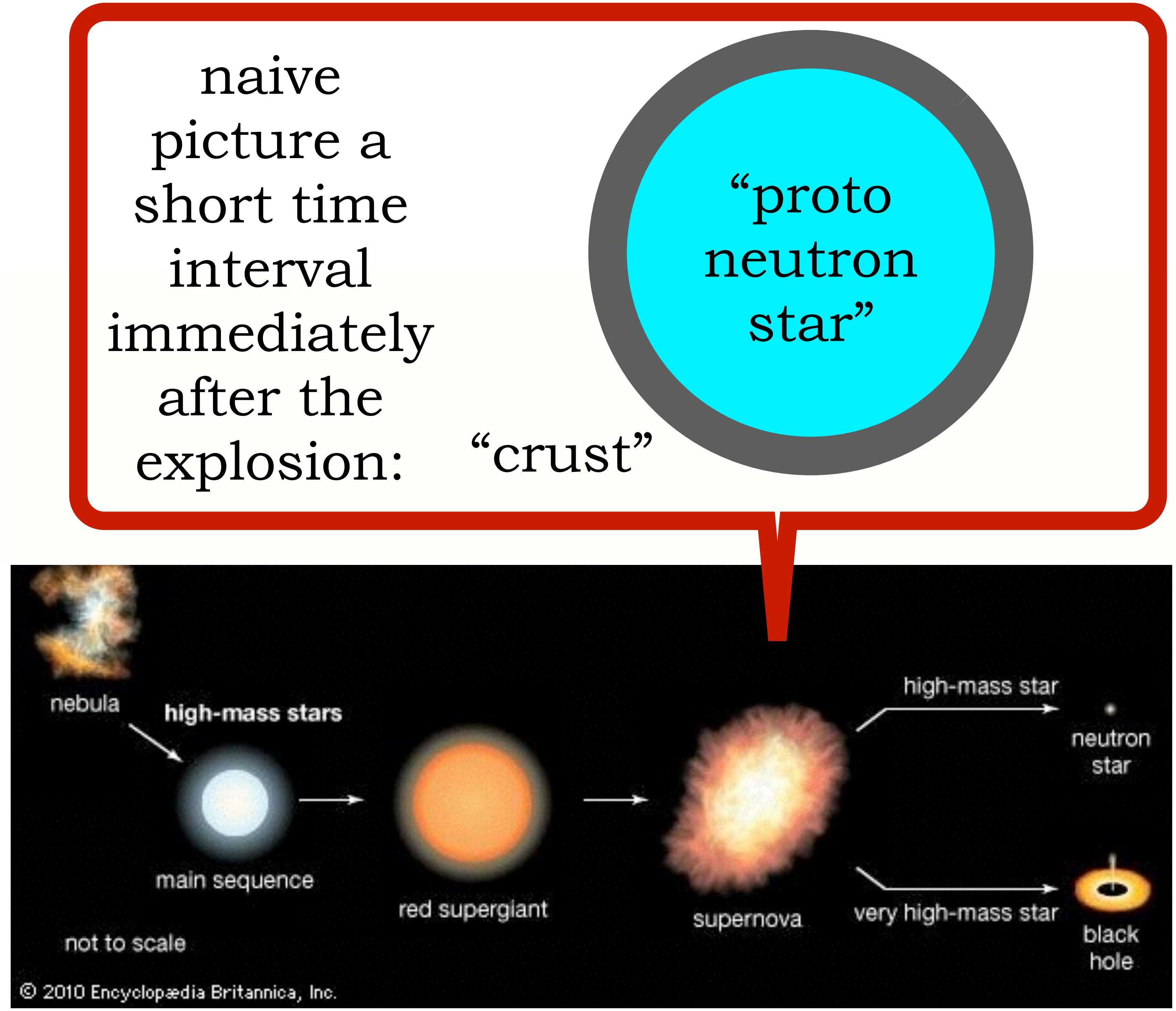

naive picture a short time interval immediately after the explosion:

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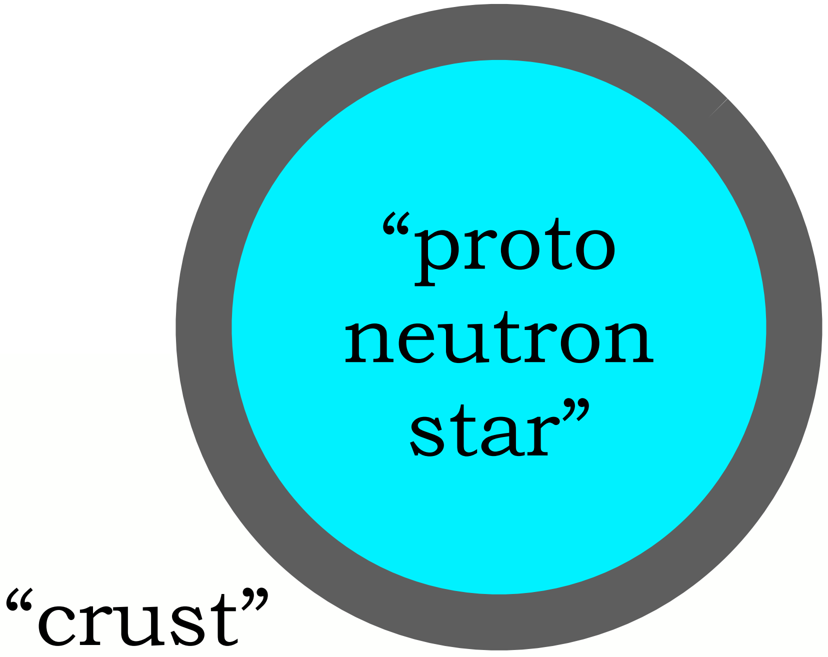

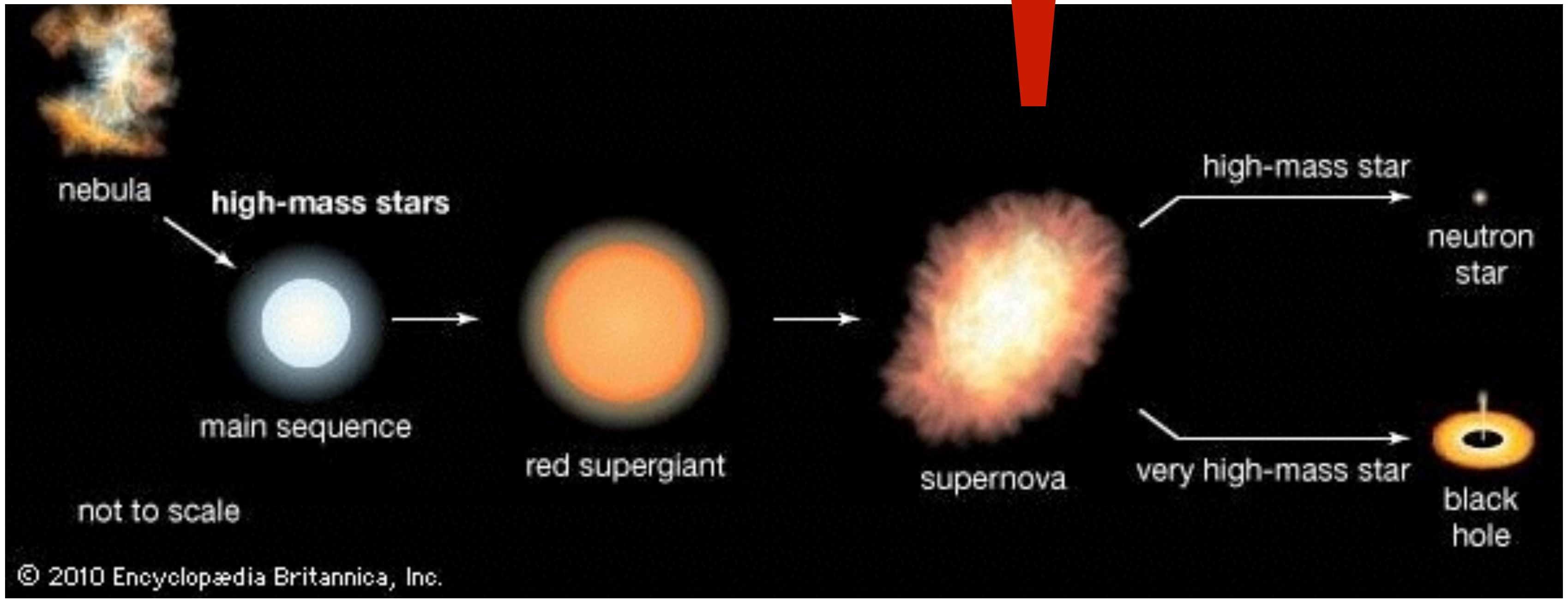

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- compact star
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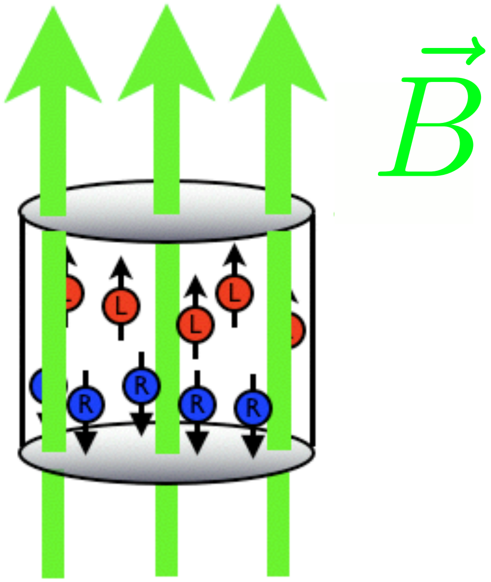
proto-neutron stars are dense objects with “crust” and preferred directions

2. Chiral hydrodynamics

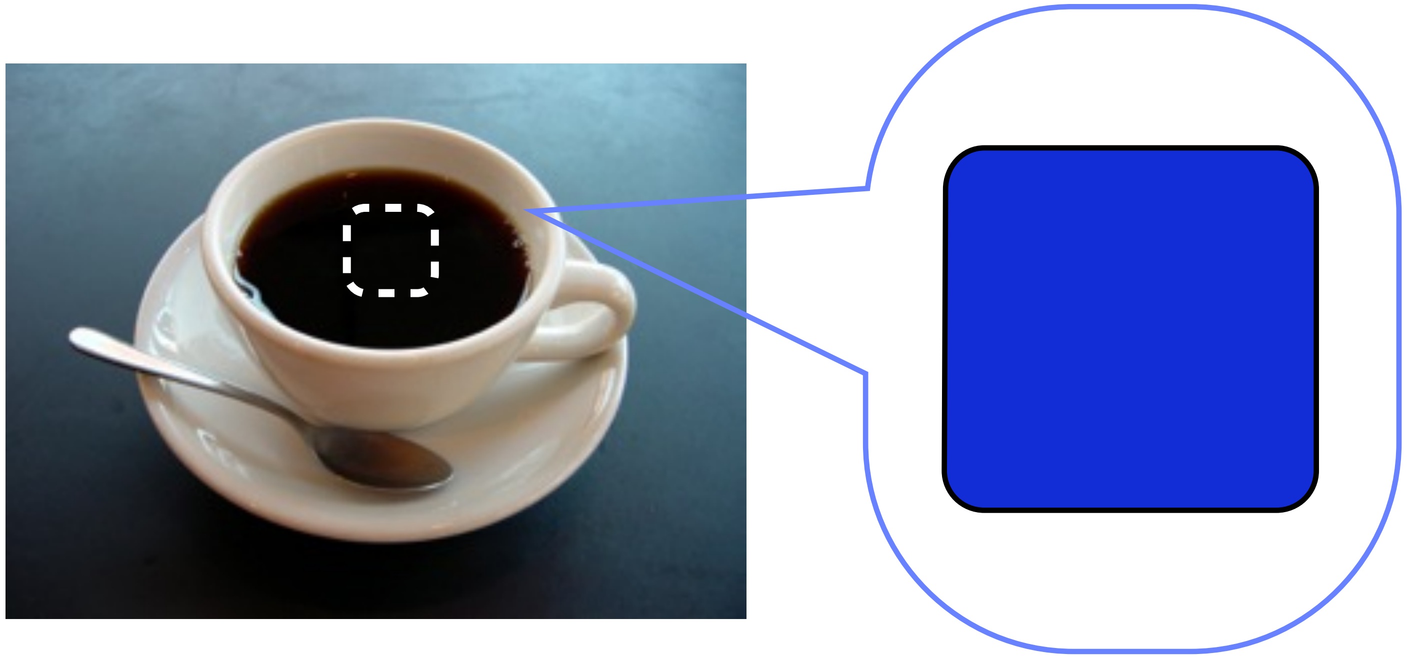


Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$

thermodynamic variables:
temperature, chemical potential,
fluid velocity



Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

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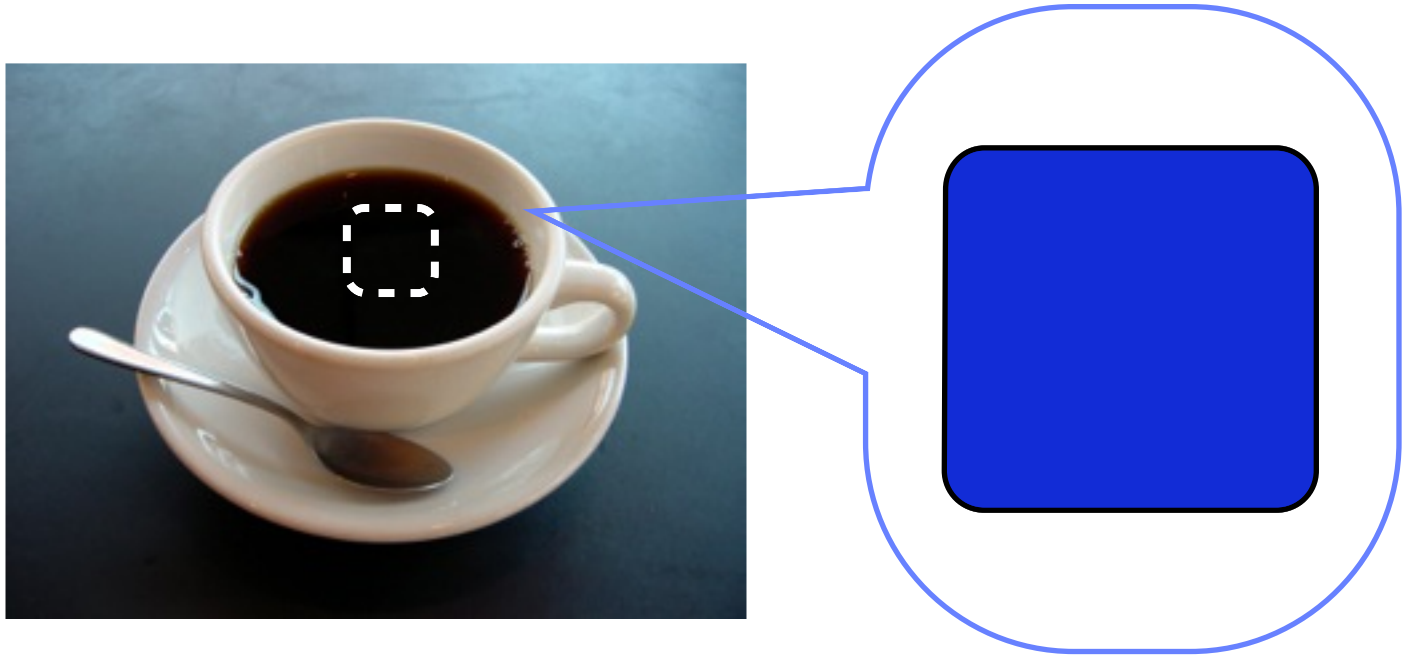
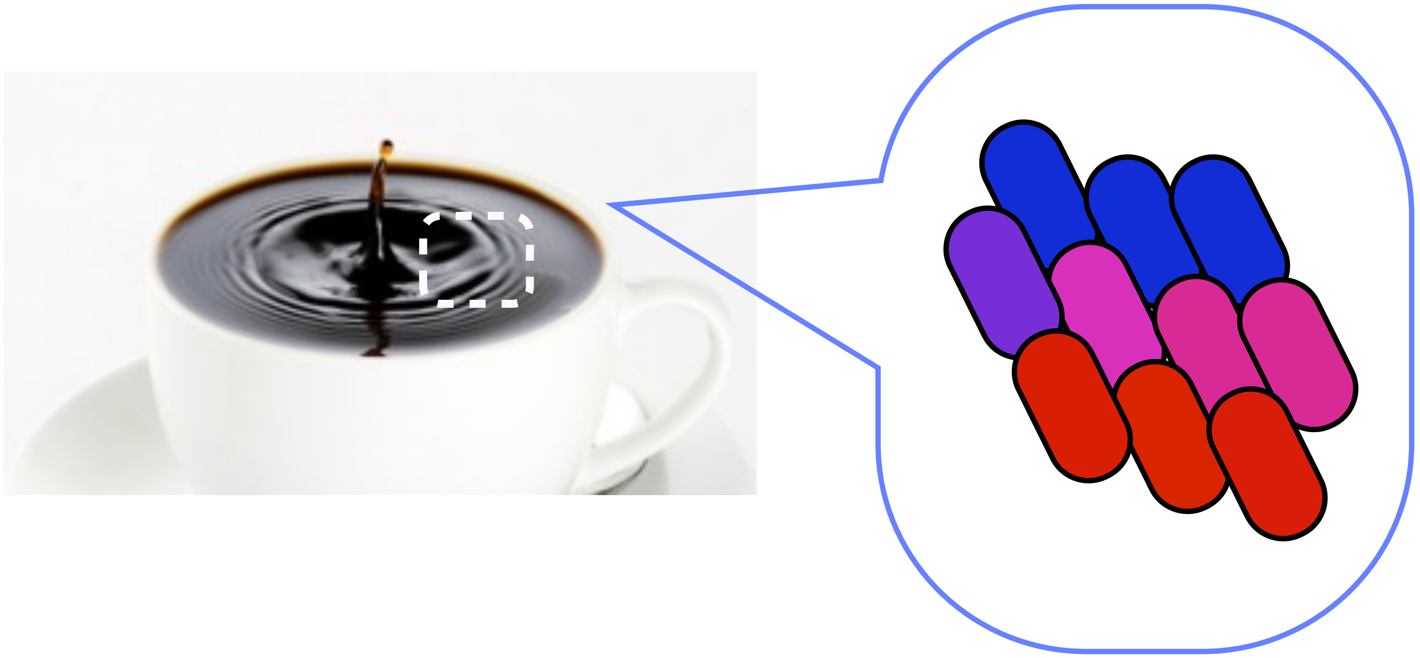


Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu\overbrace{(t,\vec{x})}^{x}$$

hydrodynamic fields
-protected by symmetry



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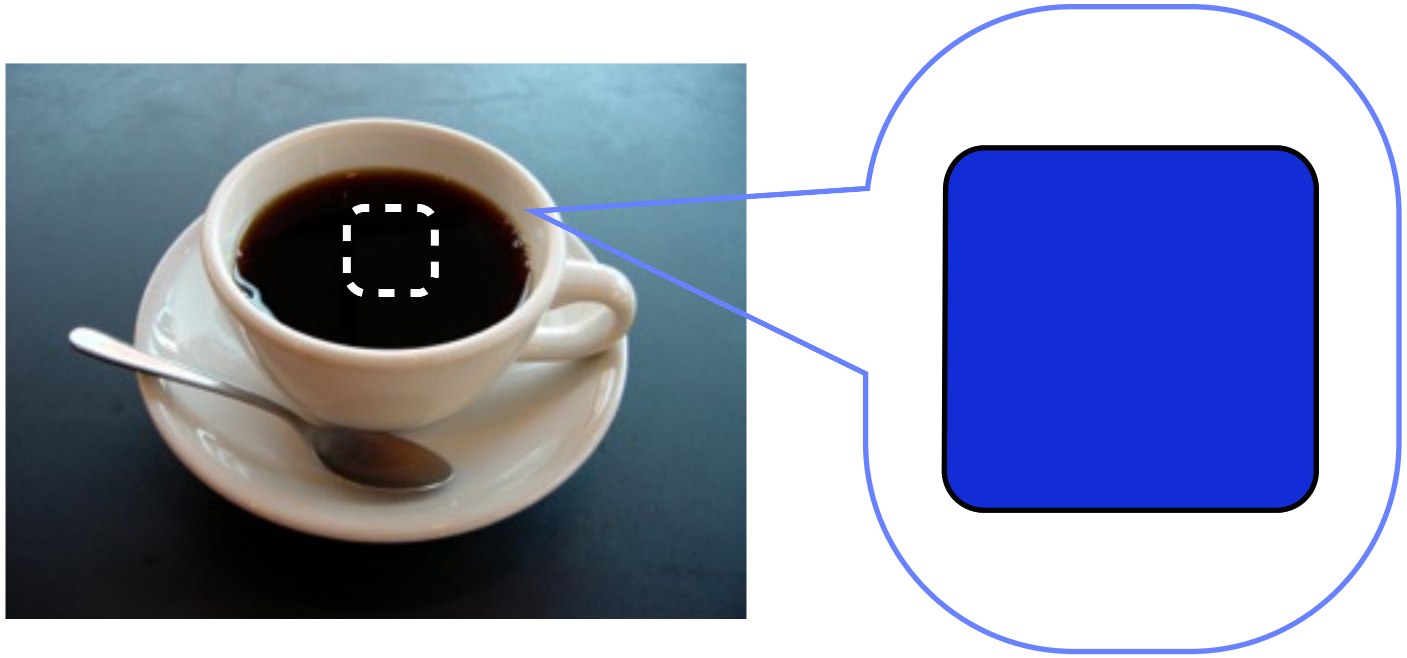

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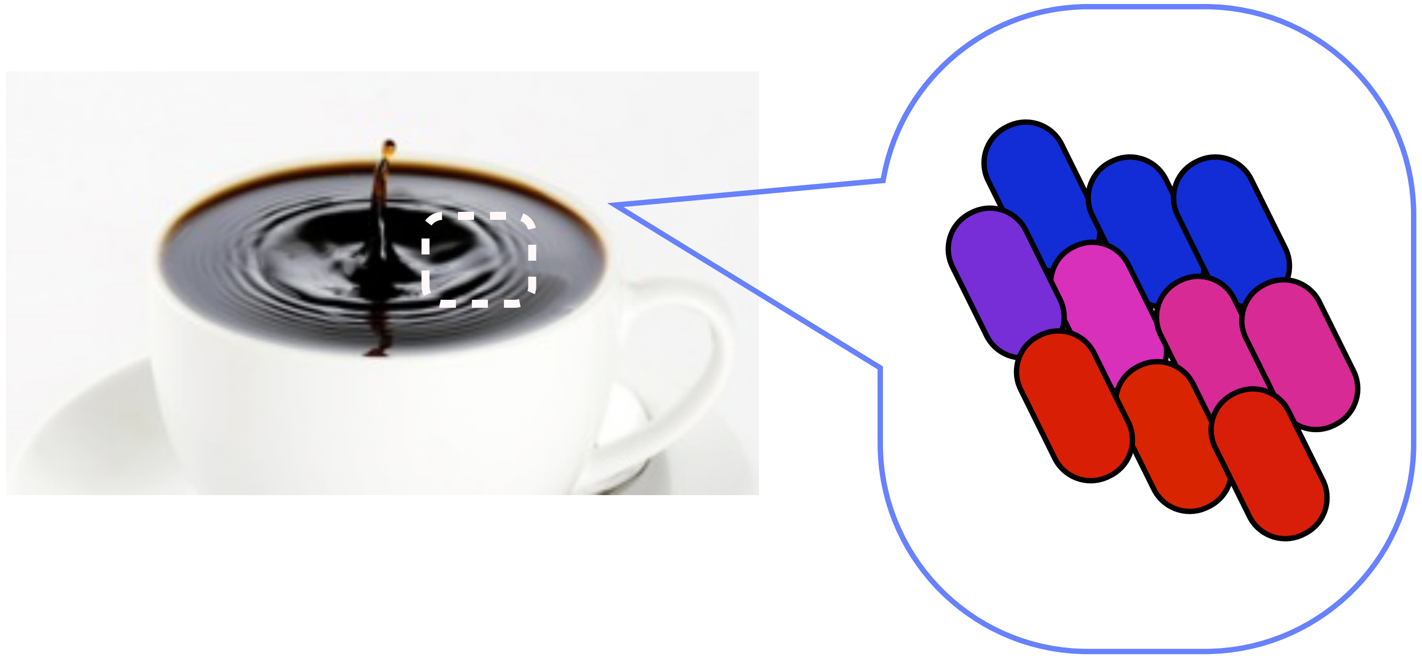

Hydrodynamics

see Piotr Surowka's talk

effective field theory, expansion in gradients of fields

- fields $T(x),\ \mu(x),\ u^\nu(x)$

  temperature *chemical potential* *fluid velocity*

- sources $A_\mu(x),\ \ldots$

  gauge field

- conservation equation $\nabla_\nu j^\nu = 0$

- constitutive equation (Landau frame)

Hydrodynamics

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Conserved current

$$j^\mu = n u^\mu + \nu^\mu$$

gradient terms

Form can be derived and restricted from first principles.

[Landau, Lifshitz]

[Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]

$\nabla_\nu j^\nu = 0$ classical theory

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]

Derived for any theory with chiral anomaly

(e.g. the standard model of particle physics)

$$\nabla_\mu j^\mu = \boxed{C\,\epsilon^{\nu\rho\sigma\lambda}F_{\nu\rho}F_{\sigma\lambda}}$$

quantum theory

Chiral hydrodynamics

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Derived for any theory with chiral anomaly

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$$\nabla_\mu j^\mu = C\,\epsilon^{\nu\rho\sigma\lambda}F_{\nu\rho}F_{\sigma\lambda}$$

quantum theory

Generalized constitutive equation with external fields:

$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \dots$$

(non) conserved current

(ideal) charge flow

conductivity term

chiral magnetic conductivity

Agrees with gauge/gravity prediction

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

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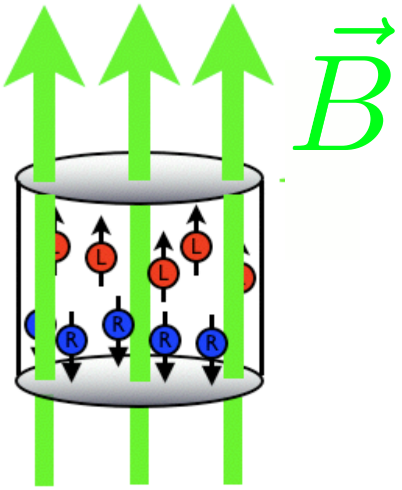
Chiral magnetic conductivity:

$$\sigma^B = C\,\mu$$

anomaly-coefficient C



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chiral magnetic conductivity

chiral vortical conductivity

Agrees with gauge/gravity prediction

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Chiral magnetic conductivity:

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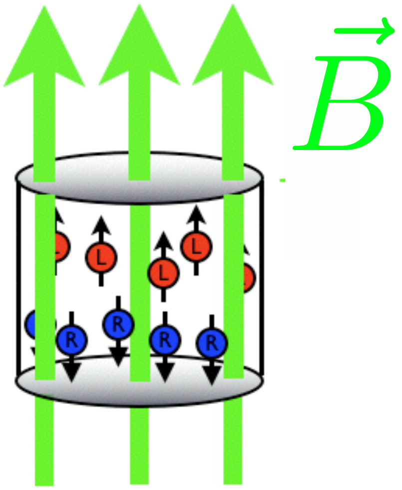

Measured in Weyl semi metals !

e.g. [Huang et al; PRX (2015)] [Landsteiner] various others …

energy extraction ?

neutron stars ?

Dirty details: chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]
[Neiman, Oz; JHEP (2010)]

Vector current (e.g. QCD *U(1)*)

$$J^\mu_V = \cdots + \xi_V \omega^\mu + \underbrace{\xi_{VV} B^\mu}_{\text{chiral magnetic effect}} + \xi_{VA} B^\mu_A$$

Axial current (e.g. QCD axial *U(1)*)

$$J^\mu_A = \cdots + \underbrace{\xi \omega^\mu}_{\text{chiral vortical effect}} + \underbrace{\xi_B B^\mu}_{\text{chiral separation effect}} + \xi_{AA} B^\mu_A$$

Note:

* hydrodynamic frame choice
[Ammon, Kaminski et al.; JHEP (2017)]

* consistent vs covariant

[Landsteiner; APhysPolC (2016)]
[Landsteiner et al; JHEP (2011)]

see e.g. Juan Torres Rincon's talk

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chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J^\mu_A = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B^\mu_A$$

chiral vortical effect

chiral separation effect

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More details: Chiral effects in various currents

[Neiman, Oz; JHEP (2010)]

More than one current $$\langle \partial_\mu J^\mu_a \rangle = \tfrac{1}{8} C_{abc} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma}$$

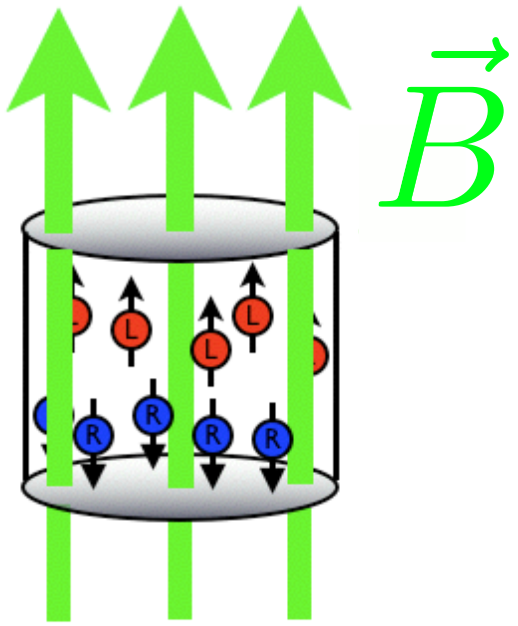
Constitutive relation:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V^\mu_b + \boxed{\sigma^V_a \omega^\mu + \sigma^B_{ab} B^{b\,\mu}} + \mathcal{O}(\partial^2)$$

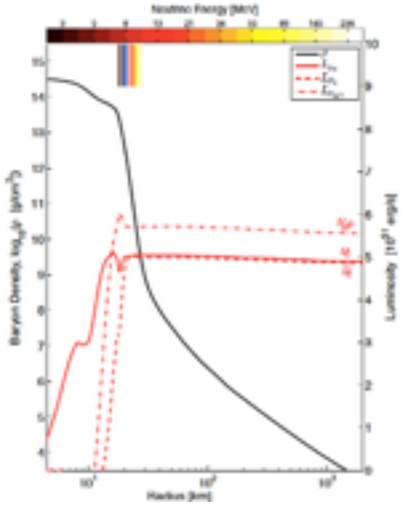
Chiral magnetic conductivity:

$$\sigma^B_{\boxed{ab}} = C_{abc}\, \mu^c$$

various charges
(e.g. lepton number,
electromagnetic charge, ...)



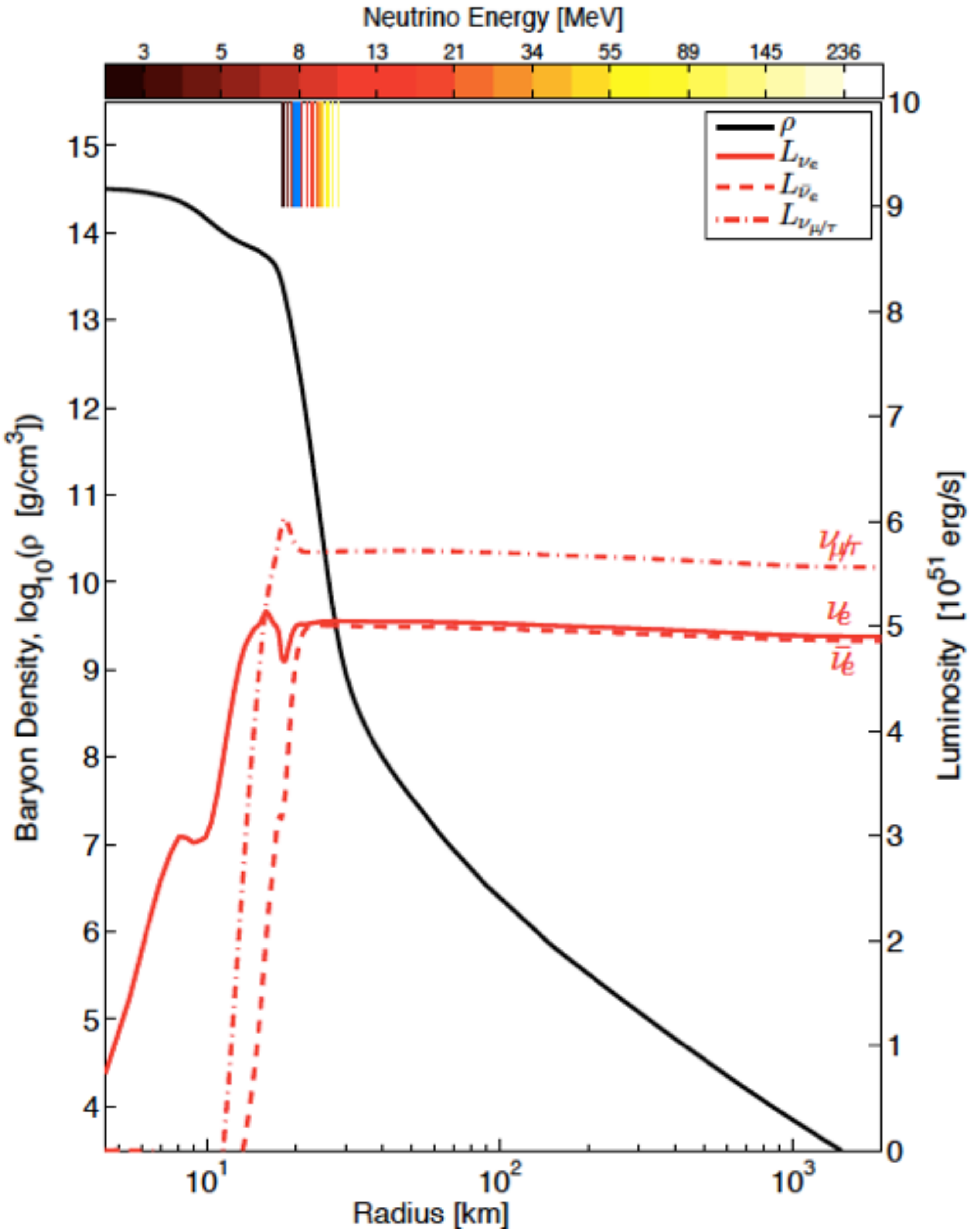
3. Simulation data



First 10 seconds inside proto-neutron stars

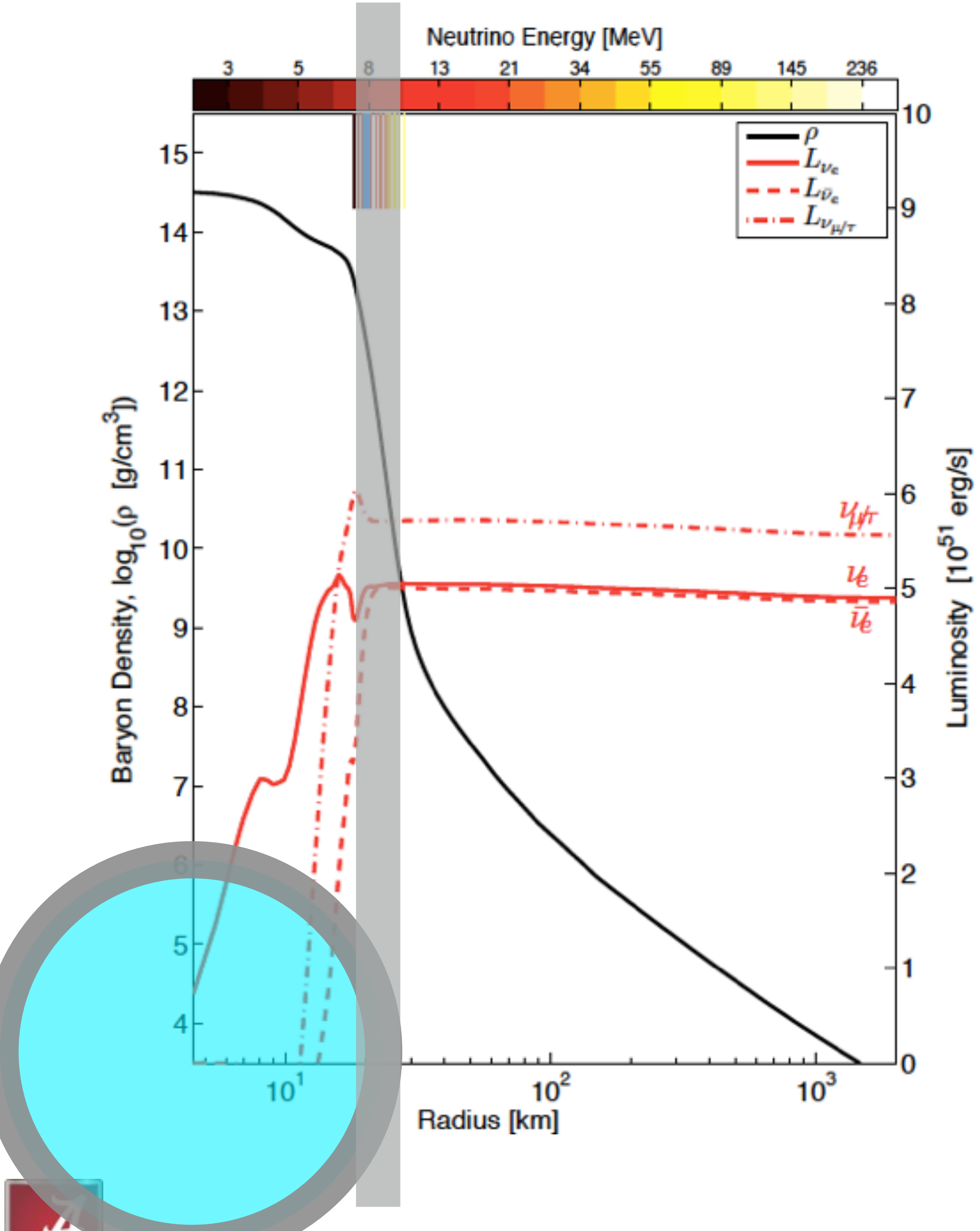
[Fischer et al.; PRD (2011)]

cf. [Wongwathanarat, Janka, Muller; (2012)]



- baryonic matter: 10 km radius
- neutrinos: last scattering surfaces around 10 km
- no anti-neutrinos
- only electron flavor inside 10 km
- high densities
- neutrinos trapped! (everything trapped)

First 10 seconds inside proto-neutron stars



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- baryonic matter: 10 km radius
- neutrinos: last scattering surfaces around 10 km
- no anti-neutrinos
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- neutrinos trapped! (everything trapped)

confirms our naive picture;
apply hydrodynamics!

Two categories of kick mechanisms

Something has to cause an asymmetry in the momentum distribution.

1.) asymmetric supernova explosion

2.) asymmetric emission of matter

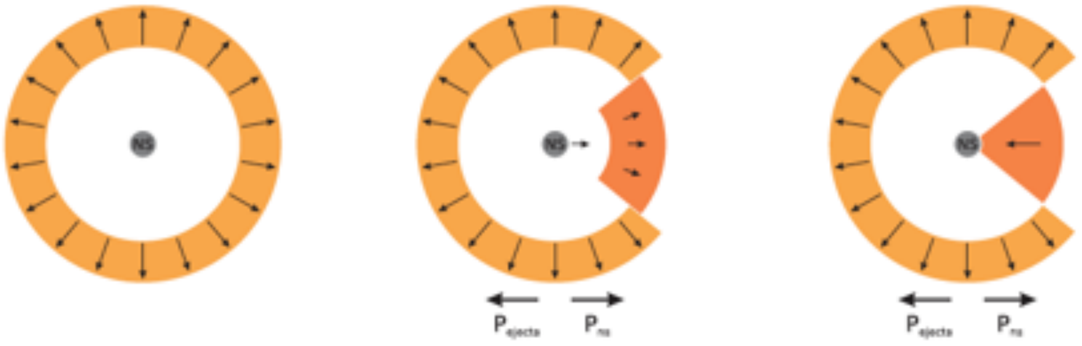
Two categories of kick mechanisms

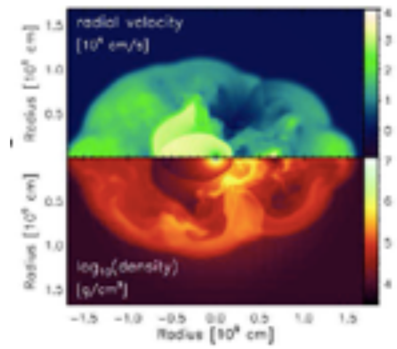
Something has to cause an asymmetry in the momentum distribution.

1.) asymmetric supernova explosion

[Scheck, Kifonidis, Janka, Muller; (2003)]
[Wongwathanarat, Janka, Muller; (2010)]
[Wongwathanarat, Janka, Muller; (2012)]

- kicks of about 1000 km/s
- random seed perturbations plus hydro instabilities (SASI)
- hydro model, neutrino transport, boundary cond's *[Blondin et al; ('02)]*
- timescale: ~5 seconds





2.) asymmetric emission of matter

- neutrino emission *[Vilenkin (1978)]*
- beyond the standard model *[Fuller, Kusenko, Mocioiu, Pascoli (2003)]*
- neutrino kicks, nucl-th *[Sagert, Schaffner-Bielich (2007)]*

Problems with previous kick mechanisms

ad 1.) asymmetric supernova explosion

- numerical analysis, no analytic understanding
- no magnetic dipole fields, no chiral hydro
- instabilities disturbed by other hydro effects?

ad 2.) asymmetric emission of matter

- neutrino kick too small, neutrinos "too cold"
- microscopic asymmetry washed out
- need physics beyond the standard model

[Kusenko, Segre, Vilenkin (1998)]

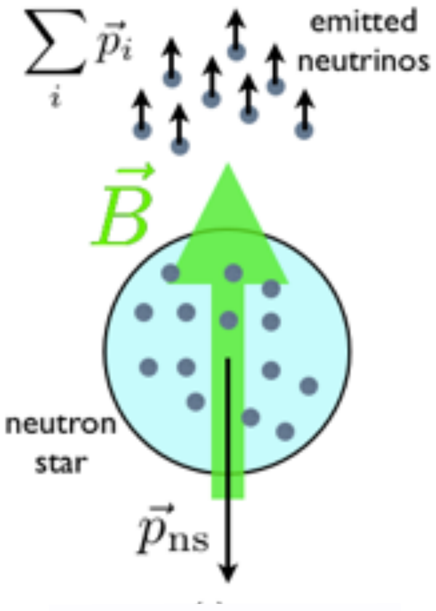
[Fuller, Kusenko, Mocioiu, Pascoli (2003)]

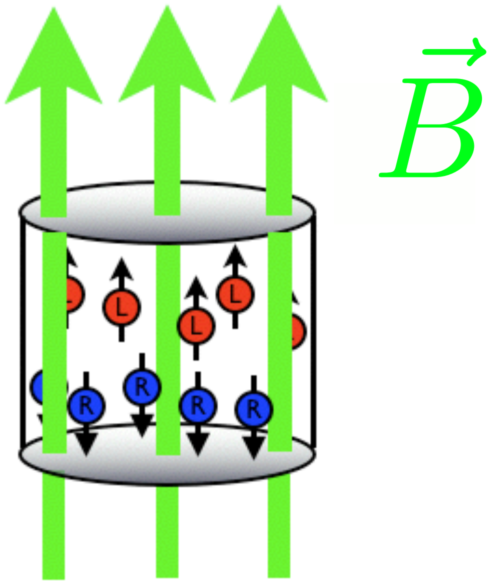
Our anomalous hydrodynamic formalism addresses and overcomes these problems.

Our assumptions

- simulation data from previous section valid
- hydrodynamic approximation applicable
- strong magnetic field exists inside proto-neutron star for about 10 seconds

4. Kicks from anomalies

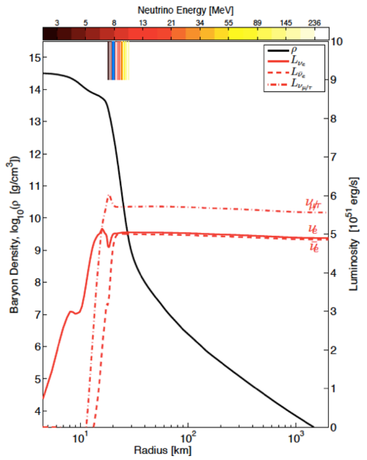




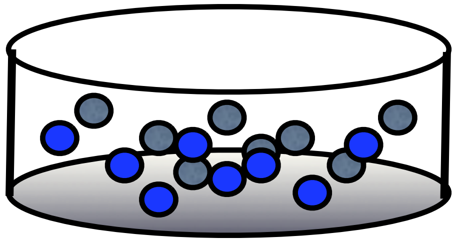
Relevant currents in neutron stars

$\boxed{B \sim \mathcal{O}(\partial)}$

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]



A bucket full of electrons and electron neutrinos with short mean free path



$B = 0.1\,\mathrm{MeV}^2$

$\mu^\ell \approx 300\,\mathrm{MeV}$

Microscopic (standard model) currents: lepton/axial/EM:

$$J^\mu_\ell = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R + \bar{\nu}\gamma^\mu \nu$$
$$J^\mu_{\ell 5} = \bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R + \bar{\nu}\gamma^\mu \nu$$
$$J^\mu_{EM} = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R$$

neutrinos and electrons "inseparable"

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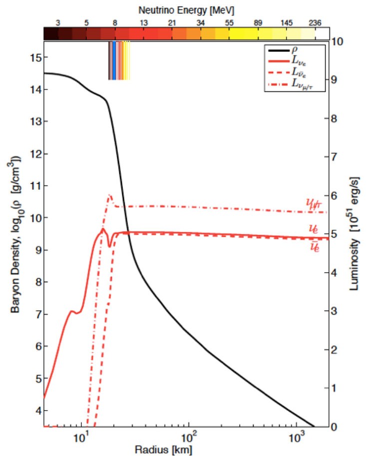
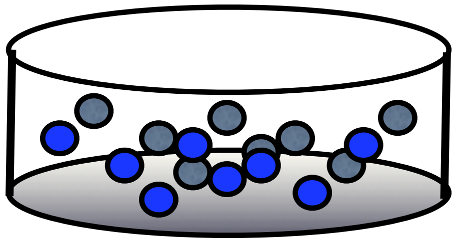


A bucket full of electrons and electron neutrinos with short mean free path



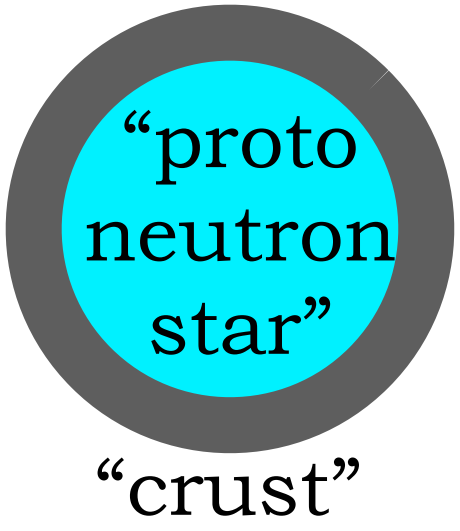
$B = 0.1\,\mathrm{MeV}^2$

$\mu^\ell \approx 300\,\mathrm{MeV}$

Microscopic (standard model) currents: lepton/axial/EM:

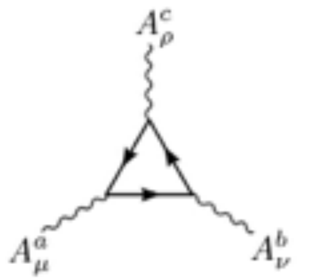
$$J^\mu_\ell = \bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R + \bar{\nu}\gamma^\mu\nu$$
$$J^\mu_{\ell 5} = \bar{e}_L\gamma^\mu e_L - \bar{e}_R\gamma^\mu e_R + \bar{\nu}\gamma^\mu\nu$$
$$J^\mu_{EM} = \bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R$$

neutrinos and electrons “inseparable”



Macroscopic (hydrodynamic) description: $\sigma^B_{ab} = C_{abc}\,\mu^c$

$$J_\ell \sim \sigma^B_{\ell,EM} B \approx C\mu^{\ell 5} B$$
$$J_{\ell 5} \sim \sigma^B_{\ell 5,EM} B \approx C\mu^{\ell} B$$



$$C_{\ell,\ell 5,EM} = \frac{1}{2\pi^2}$$

Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

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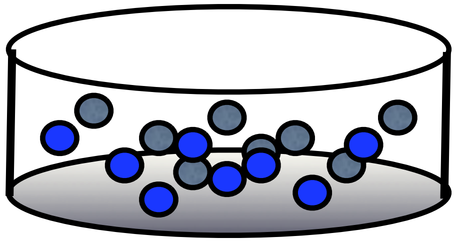

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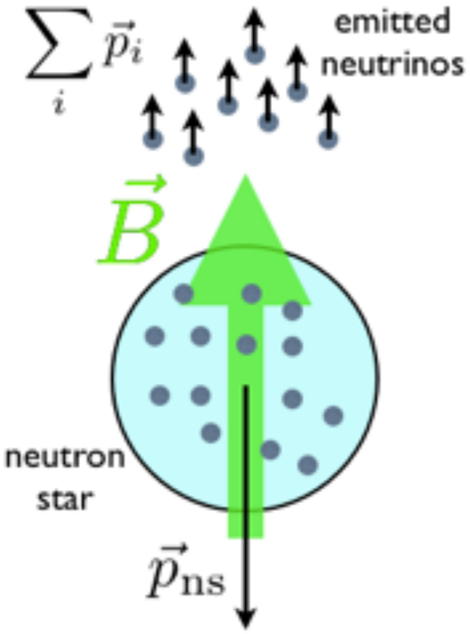

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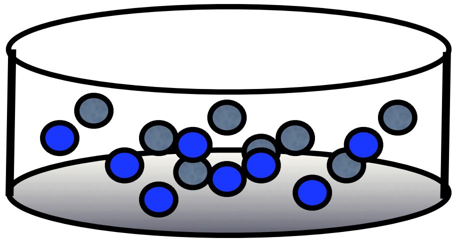

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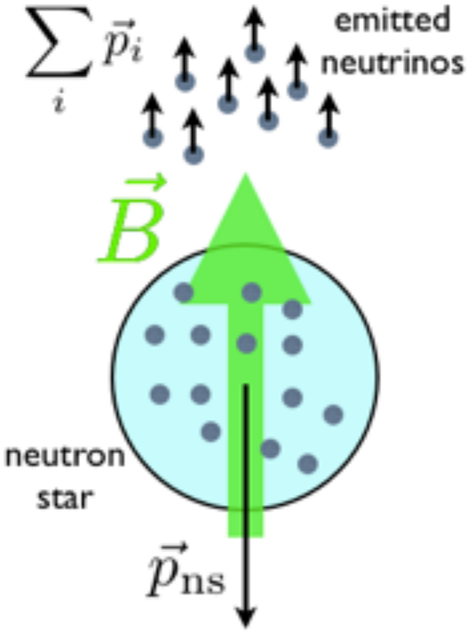

Chiral conductivity:

$$\sigma^B_{\ell 5, EM} = C_{\ell,\ell 5, EM}\,\mu^\ell = \frac{1}{2\pi^2}\mu^\ell$$

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[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

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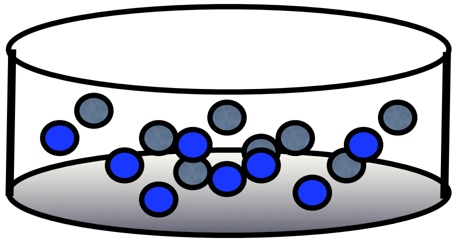

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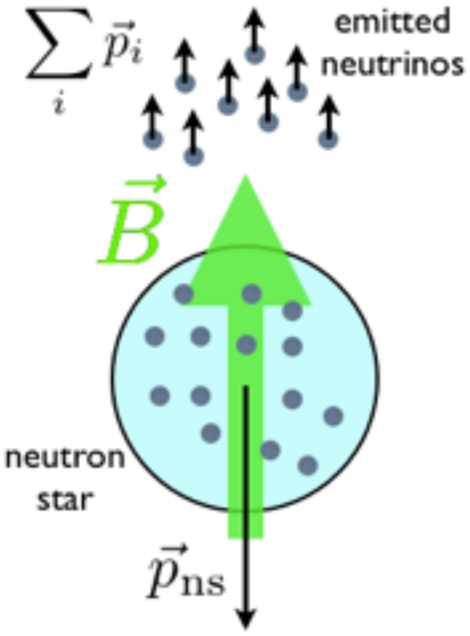

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Axial lepton current:

$$\vec{J}_{\ell 5} = C \mu^\ell \vec{B} \approx \vec{e}_B \cdot 1\,\mathrm{MeV}^3$$

Estimate of the neutron star kick

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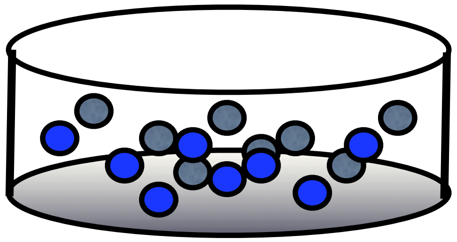

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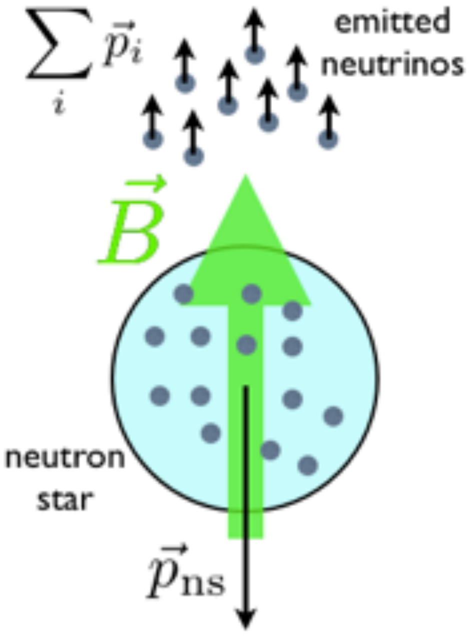

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Particle flux and momentum transfer:

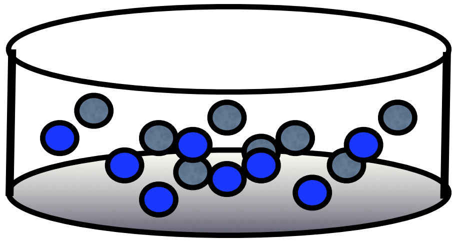
$$\dot{N}_\nu = |\vec{J}|\,A_{\mathrm{surface}}$$
$$\approx 10^{54}/\mathrm{s}$$

$$\Delta P_{\mathrm{NS}} = \Delta t \dot{N}_\nu \langle p_\nu \rangle,$$
$$\Delta t \approx 10\,\mathrm{s}$$

Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

A bucket full of electrons and electron neutrinos with short mean free path



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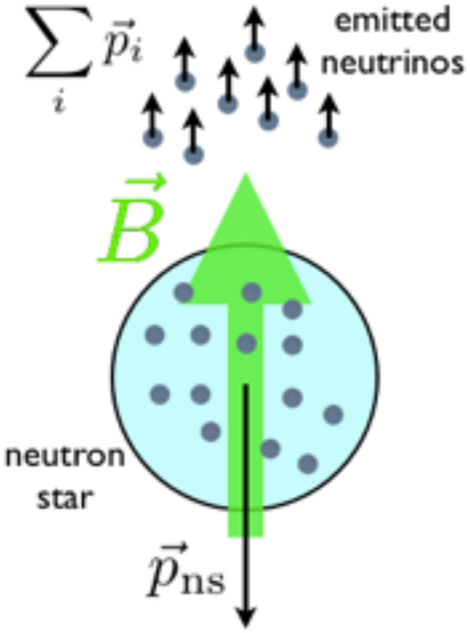

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Neutron star mass: $m_{\rm NS} = 3 \cdot 10^{30}\,\mathrm{kg}$

Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

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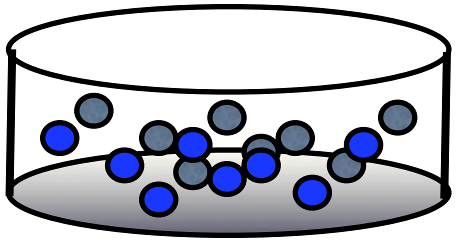

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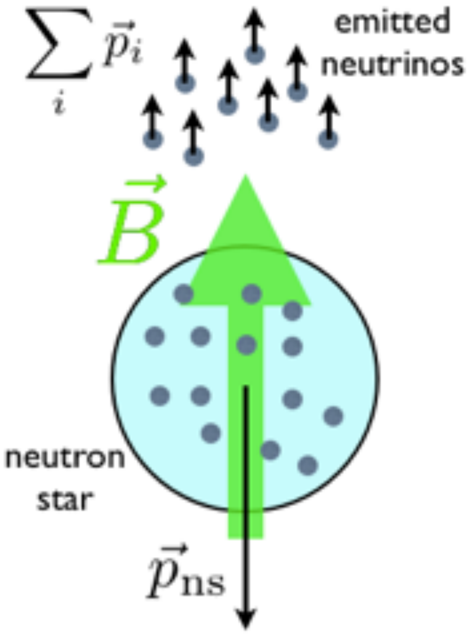

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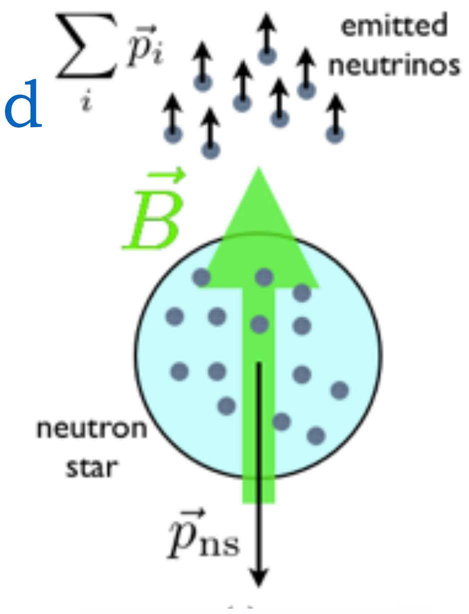
$$\Delta t \approx 10\,\mathrm{s}$$

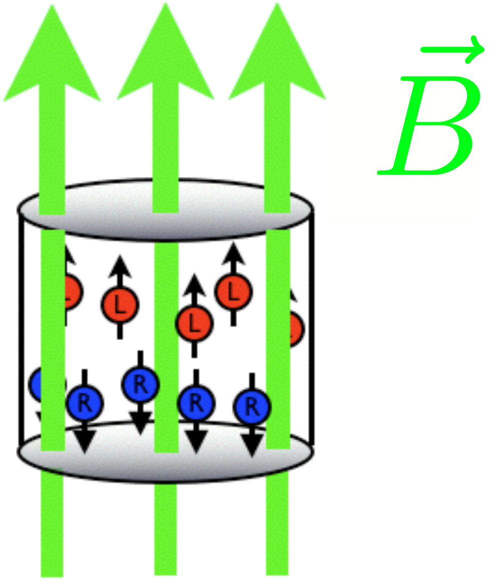
Neutron star mass: $m_{\rm NS} = 3 \cdot 10^{30}\,\mathrm{kg}$

Kick velocity agrees with observations: $\Rightarrow$ $v_{\rm kick} \approx 1000 \frac{\rm km}{\rm s}$

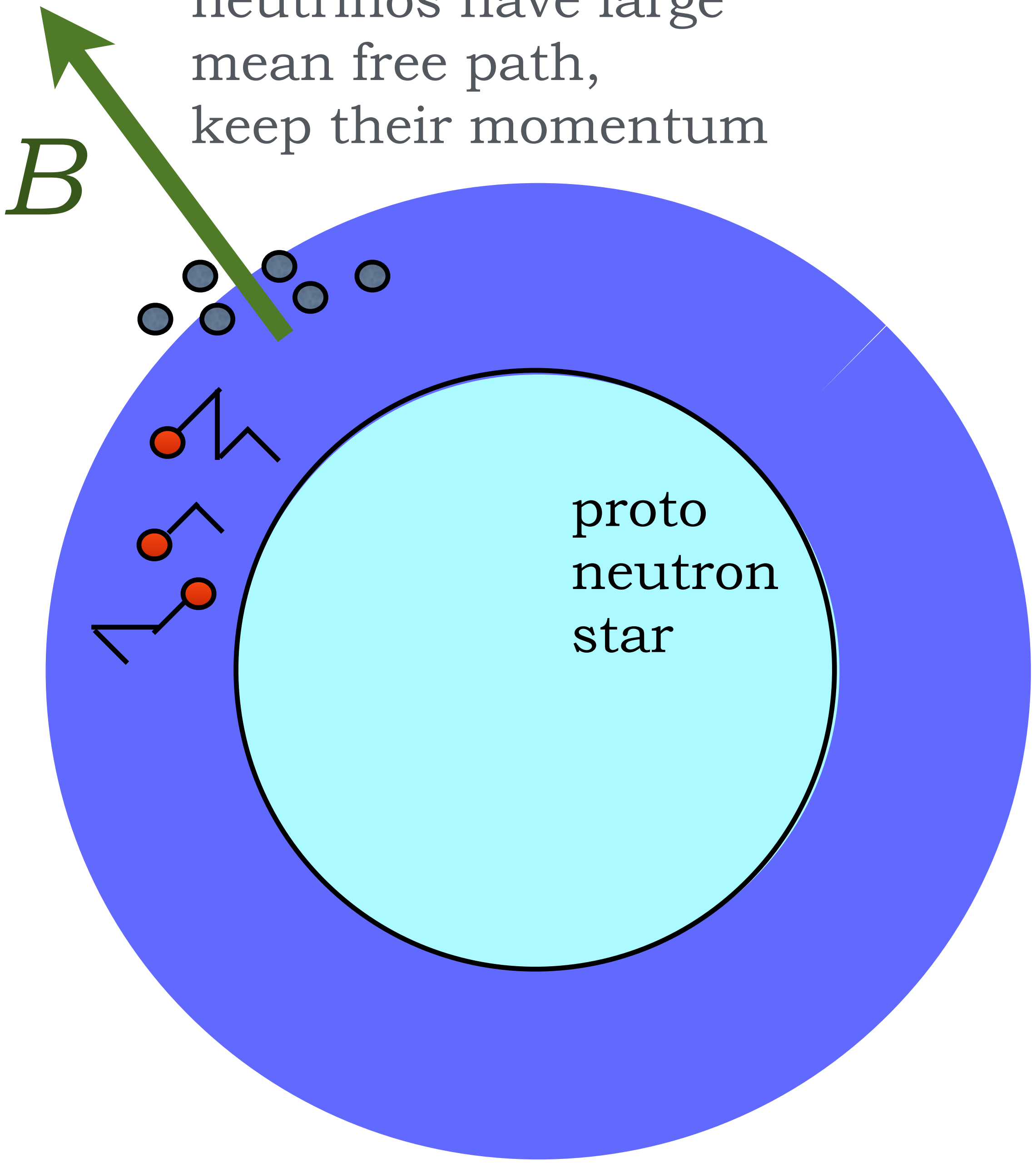
Physics behind the kick

thermal energy of neutrinos is transformed
to kinetic energy of the neutron star
by chiral hydrodynamic process





Filtering at the "crust"

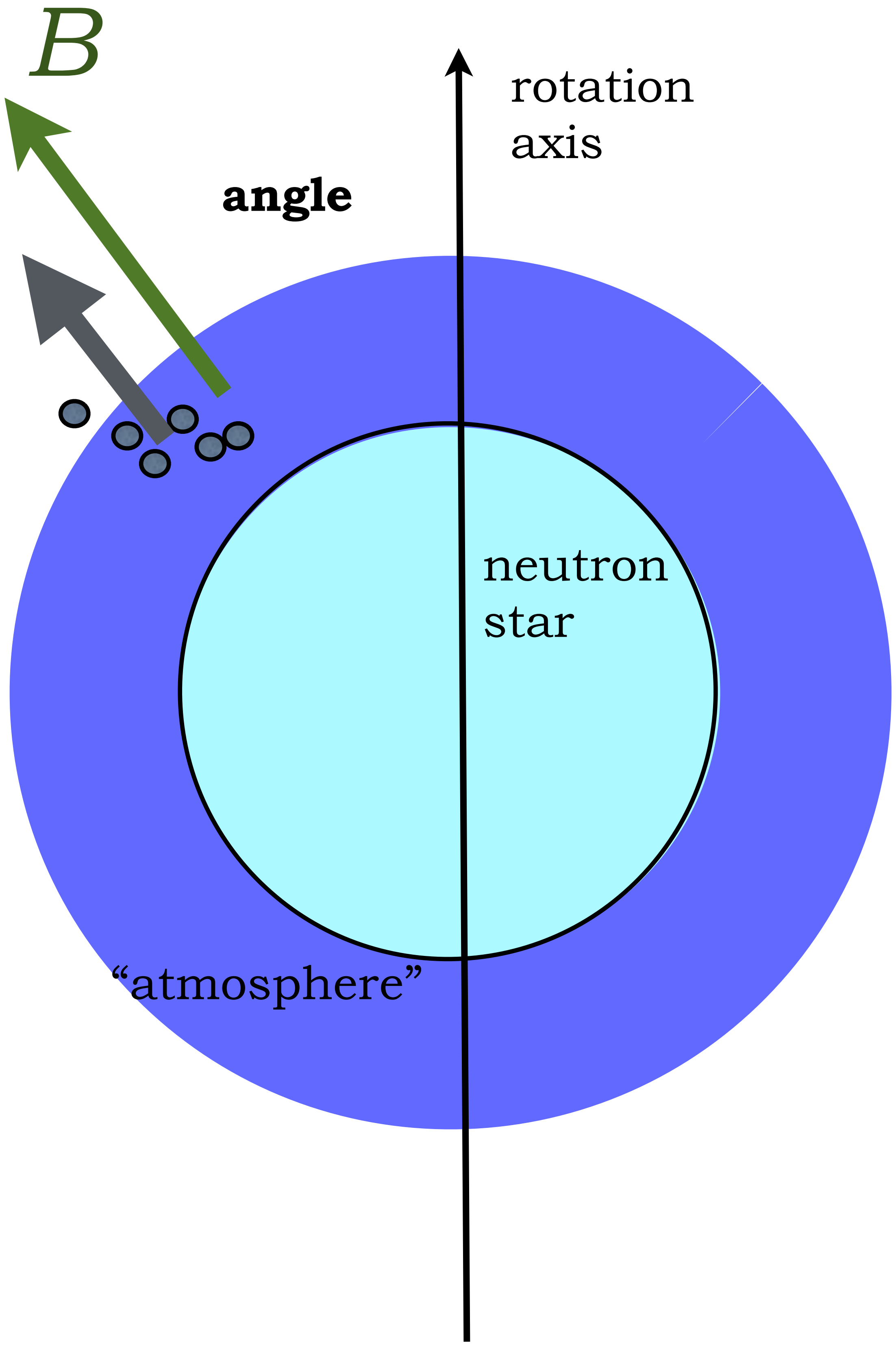


neutrinos have large
mean free path,
keep their momentum

electrons
scatter,
dissipate
their
momentum

"crust":
hydrodynamic
approximation
breaks down;
neutrinos decouple

Observable signal?



Prediction: Kick magnitude depends on angle between rotation axis and internal magnetic field axis.

For fast spinning neutron stars, kick directed along rotation axis.

Data on rotation/B/kick axes:

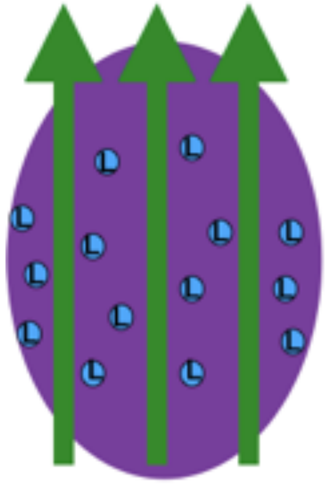
http://adsabs.harvard.edu/abs/2012ApJ...755..141B

http://adsabs.harvard.edu/abs/2007ApJ...670..635W

http://adsabs.harvard.edu/abs/1988srim.conf...65W

http://adsabs.harvard.edu/abs/2007ApJ...656..399W

5. Opportunities



CVE neutron star kicks

$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \ldots$$

“chiral vortical conductivity”

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

CVE kick possible but suppressed by orders of magnitude

$\omega \approx 10^{-17}\,\mathrm{MeV}$

$B \sim 10^{12}\,\mathrm{G} \approx 0.1\,\mathrm{MeV}^2$

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

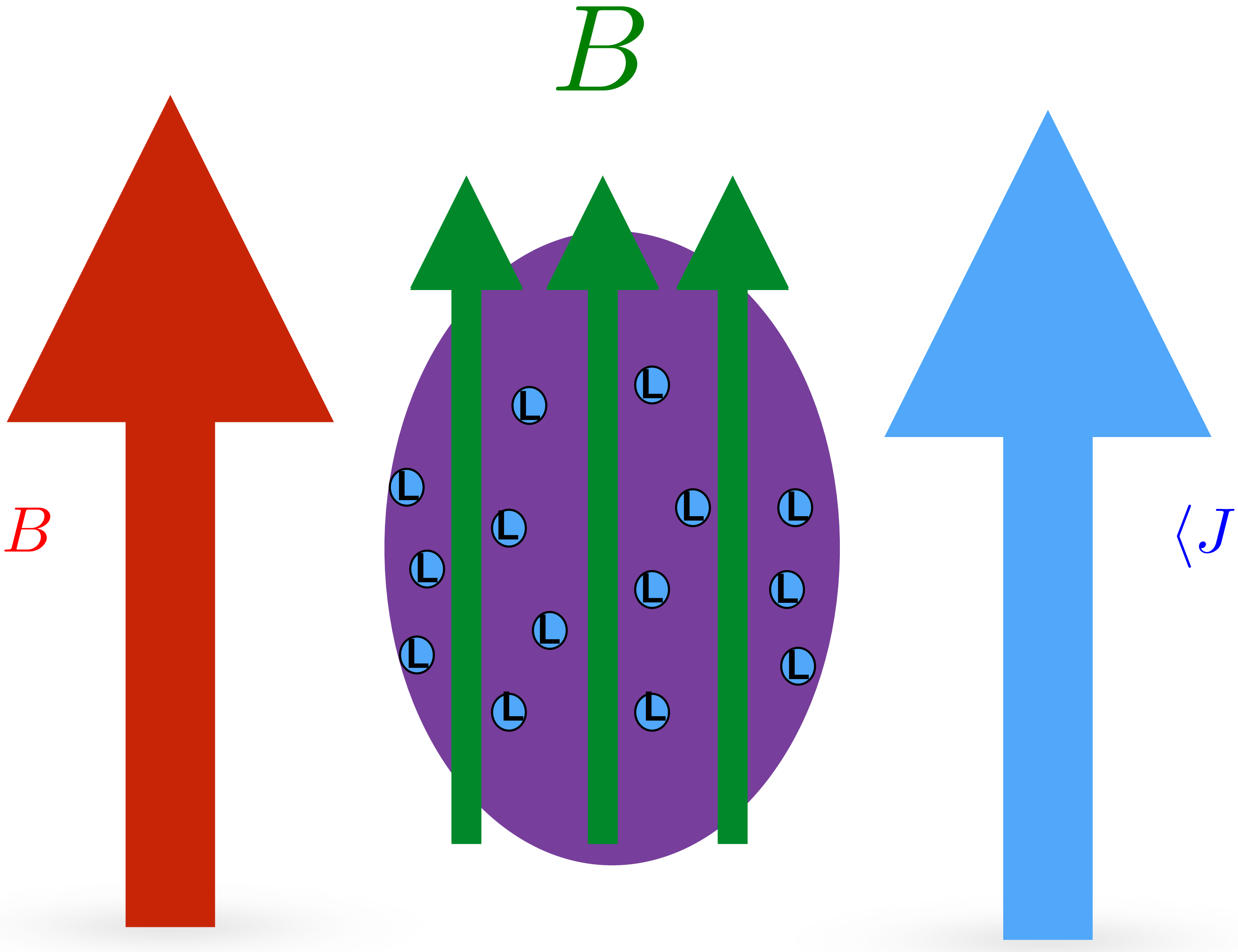
stronger CVE kick possible due to formation of ergo sphere

[Shaverin, Yarom; arXiv (2014)]

see also [Shaverin; arXiv (2018)]

Zeroth order CME $\boxed{B \sim \mathcal{O}(1)}$
-thermodynamic chiral currents

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]



$\langle T^{0z}\rangle = \xi_V^{(0)} B$

heat current

$\langle J^z\rangle = \xi_B^{(0)} B$

axial current

EFT result: strong B thermodynamics

see Ho-Ung Yee's talk

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$$\boxed{B \sim \mathcal{O}(1)}$$

$$\langle T^{\mu\nu}_{\mathrm{EFT}} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

Axial current:

$$\langle J^{\mu}_{\mathrm{EFT}} \rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B} \right) + \mathcal{O}(\partial)$$

EFT result: strong B thermodynamics

see Ho-Ung Yee's talk

[Ammon, Kaminski et al.; JHEP (2017)]
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Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$$\boxed{B \sim \mathcal{O}(1)}$$

equilibrium heat current

$$\langle T^{\mu\nu}_{\text{EFT}} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^{\mu}_{\text{EFT}} \rangle = \left(n_0, 0, 0, \underline{\xi_B^{(0)} B} \right) + \mathcal{O}(\partial)$$

equilibrium charge current

- ➡ **chiral thermodynamic equilibrium observables**
- ➡ **confirmed by holographic model**
- ➡ **opportunity: order one vorticity** $\langle J^{\mu}_{\text{EFT}} \rangle \sim \xi_V^{(0)} \omega^{\mu}$
- ➡ **opportunity: polarization/magnetization**

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]

Previous work: polarized matter at strong B

Generating functionals W~P (pressure) for thermodynamics $\boxed{B \sim \mathcal{O}(1)}$

(i) No anomaly: *[Kovtun; JHEP (2016)]*

$$T^{\mu\nu} = Pg^{\mu\nu} + (Ts + \mu\rho)u^\mu u^\nu + T^{\mu\nu}_{\rm EM}$$

$$J^\alpha = \rho u^\alpha - \underbrace{\nabla_\lambda M^{\lambda\alpha}}_{\textit{bound current}}$$

$$T^{\mu\nu}_{\rm EM} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu} \right)$$

[Israel; Gen.Rel.Grav. (1978)]

Polarization tensor:

$$M_{\mu\nu} = p_\mu u_\nu - p_\nu u_\mu - \epsilon_{\mu\nu\rho\sigma} u^\rho m^\sigma \qquad M^{\mu\nu} = 2\frac{\partial P}{\partial F_{\mu\nu}}$$

Including vorticity:
$W \sim M_\omega \, B \cdot \omega$

[Kovtun, Hernandez; JHEP (2017)]

see Enrico Speranza's talk

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[Kovtun, Hernandez; JHEP (2017)]
see Enrico Speranza's talk

(ii) With anomaly: *[Jensen, Loganayagam, Yarom; JHEP (2014)]*

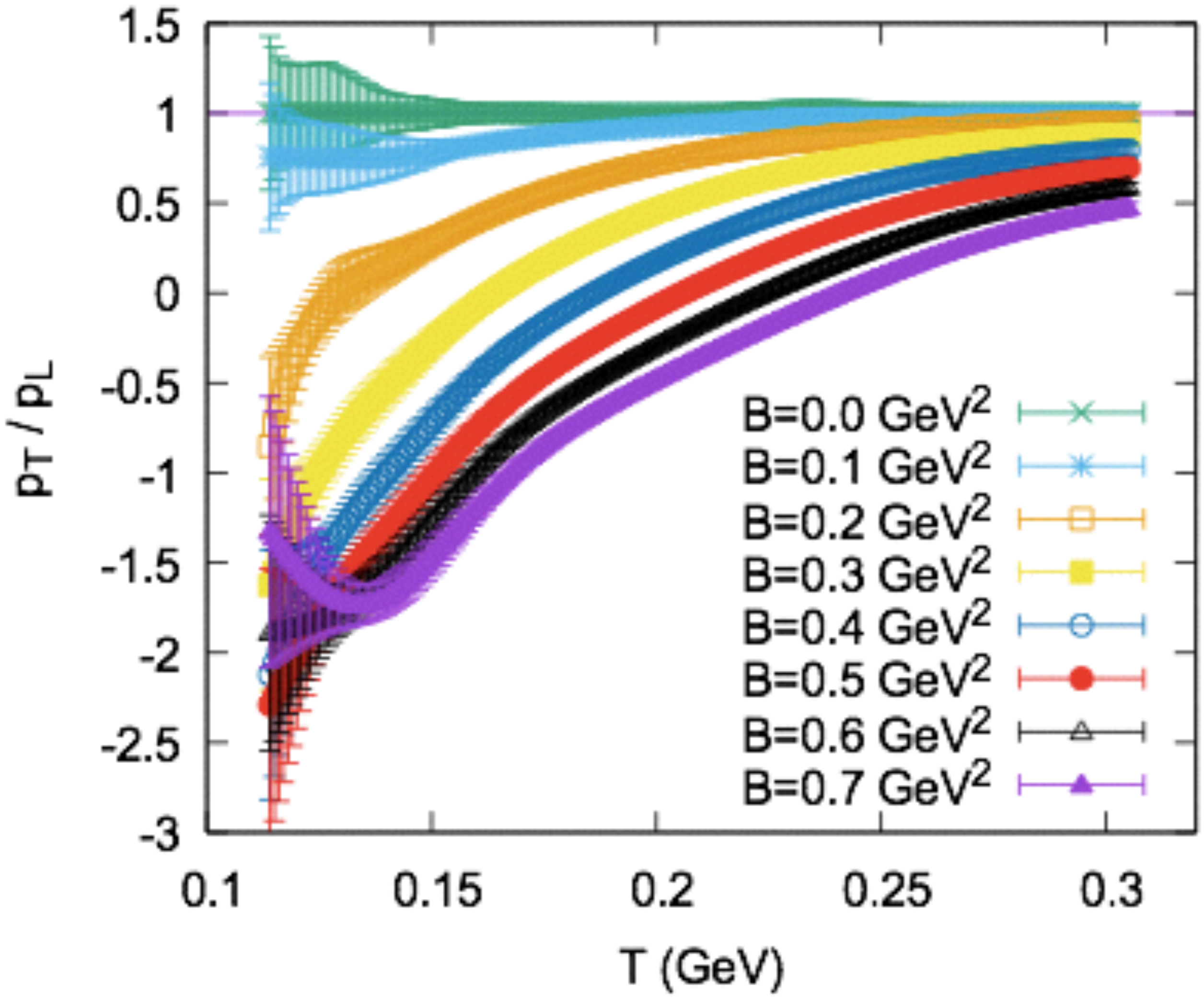
➡ opportunity: single framework allows for polarization, magnetization, external vorticity, *E*, *B*, *and* chiral anomaly

➡ opportunity: dynamical *E* and *B;* magnetohydrodynamics
[Kovtun, Hernandez; JHEP (2017)]

➡ opportunity: study equilibrium and near-equilibrium transport
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]

Universal magnetoresponse in QCD …

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_\mathrm{T} = -\frac{L_\mathrm{T}}{V}\frac{\partial F_\mathrm{QCD}}{\partial L_\mathrm{T}}$

longitudinal pressure: $p_\mathrm{L} = -\frac{L_\mathrm{L}}{V}\frac{\partial F_\mathrm{QCD}}{\partial L_\mathrm{L}}$

F_QCD **…** *free energy*

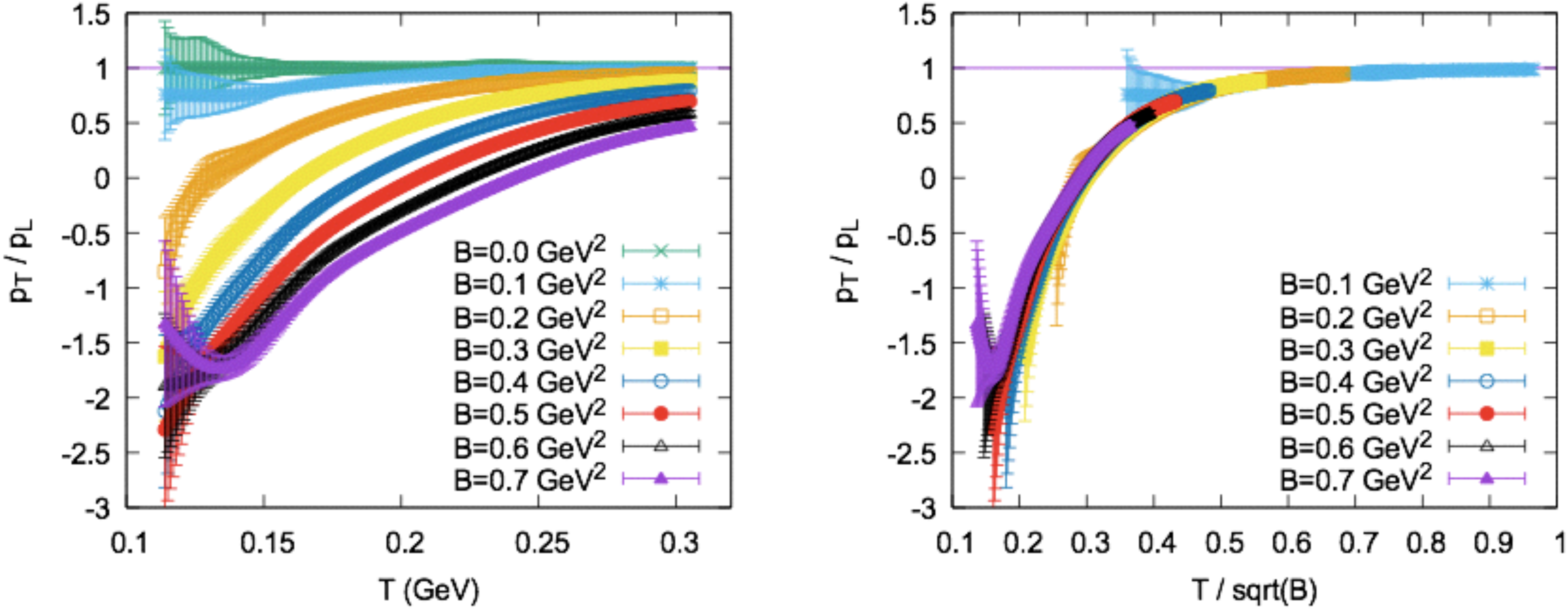
L_T **…** *transverse system size*

L_L **…** *longitudinal system size*

V **…** *system volume*

Universal magnetoresponse in QCD …

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



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transverse pressure: $p_{\rm T} = -\frac{L_{\rm T}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm T}}$

longitudinal pressure: $p_{\rm L} = -\frac{L_{\rm L}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm L}}$

$F_{\rm QCD}$ **…** *free energy*

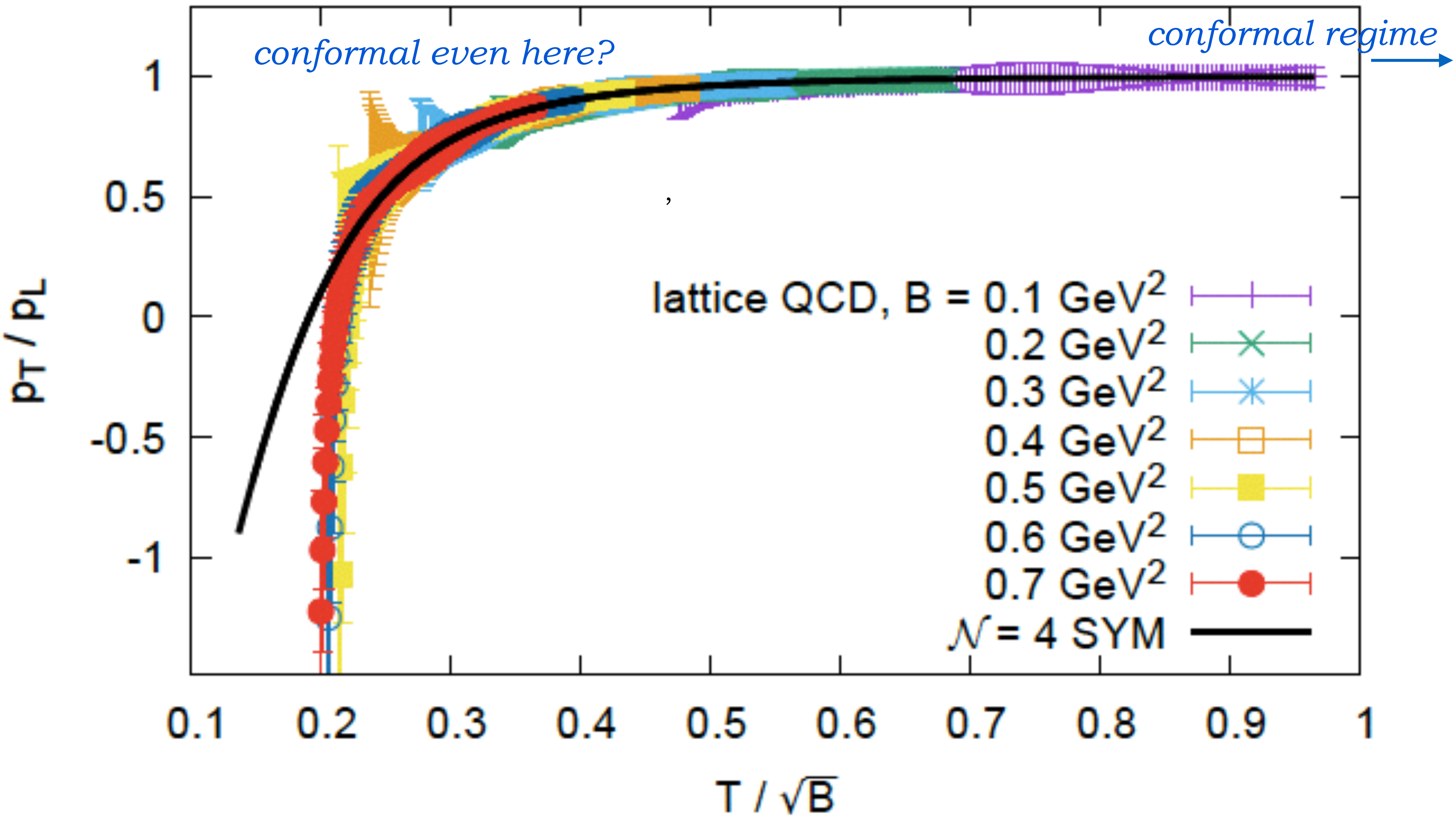
$L_{\rm T}$ **…** *transverse system size*

$L_{\rm L}$ **…** *longitudinal system size*

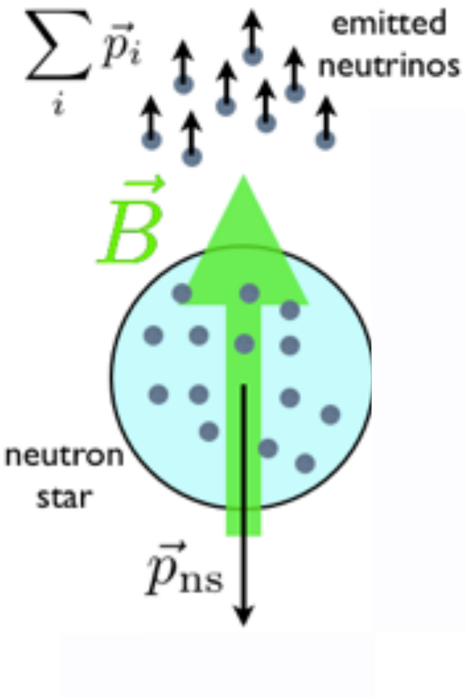
V **…** *system volume*

… and $\mathcal{N}$=4 Super-Yang-Mills theory

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Summary

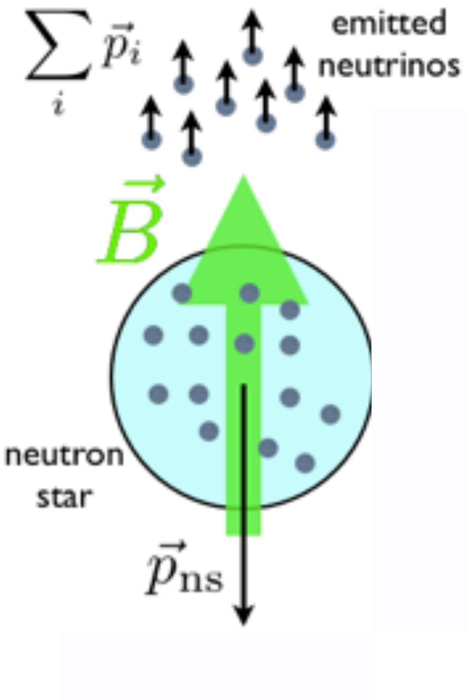


- Yes, standard (first order) **CME & CMV can lead to neutron star kicks** at $\boxed{B \sim \mathcal{O}(\partial)}$
- **equilibrium** (zeroth order) **chiral transport** at $\boxed{B \sim \mathcal{O}(1)}$
- **“polarization” framework exists** (equilibrium hydro)
- polarization and thermodynamic **effects important** (holo models)

- outlook: derived order zero CME (and CVE); then explicitly in holographic model, along with other (non)equilibrium transport coefficients; e.g. shear and bulk viscosities heavily modified!
- outlook: far-from equilibrium transport and correlators

  [Cartwright, Kaminski; to appear] see my ***talk at HoloQuark2018***
- outlook: implications of this treatment for (“polarization” of) QGP

Summary



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  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]

- outlook: far-from equilibrium transport and correlators
  [Cartwright, Kaminski; to appear] see my ***talk at HoloQuark2018***

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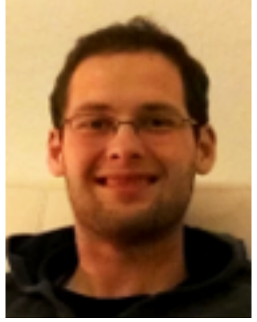
Collaborators

Perimeter, Canada
Juan Hernandez

Friedrich-Schiller University of Jena, Germany


Prof. Dr. Martin Ammon


Dr. Julian Leiber


Sebastian Grieninger

Regensburg University, Germany


Prof. Dr. Andreas Schäfer

Goethe University, Frankfurt, Germany


Dr. Gergely Endrödi

University of Washington, Seattle, USA


Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA


Roshan Koirala


Markus Garbiso


Dr. Jackson Wu

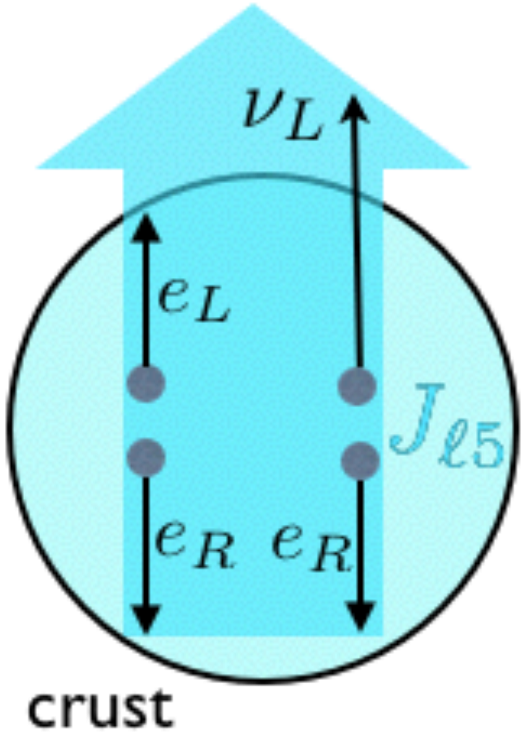

Casey Cartwright

APPENDIX

APPENDIX: Consider a pathological case

$$J_\ell \approx 0$$

$$J_{\ell,5} \approx 1\mathrm{MeV}^3$$



Is there any net momentum?

How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $$S = S_{\text{QCD}}(e, B) + S_{\text{EM}}(e, B)$$

QCD action coupled to external magnetic field (through covariant derivative)

action for external magnetic field; not included in code (not part of the dynamics)

Electric charge is renormalization scale dependent: $$e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log\frac{\mu}{\Lambda}, \quad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\text{H}}^2 + c_B |B|$$

How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $$S = S_{\mathrm{QCD}}(e, B) + S_{\mathrm{EM}}(e, B)$$

QCD action coupled to external magnetic field (through covariant derivative)

action for external magnetic field; not included in code (not part of the dynamics)

Free energy: $$F = -T \log \mathcal{Z}[S]$$

$$= F_{\mathrm{QCD}}(e, B) + F_{\mathrm{EM}}(e, B) \qquad F_{\mathrm{EM}}(e, B) = -V \frac{B^2}{2e^2}$$

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How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]

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action for external magnetic field; not included in code (not part of the dynamics)

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$$\boxed{p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial \boxed{F_{\mathrm{QCD}}(e,B)}}{\partial L_{\mathrm{T}}}}$$

this free energy is renormalization scale dependent

hence this pressure is renormalization scale dependent

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How to compare QCD to Super-Yang-Mills

SYM action: $$S = S_{\mathrm{SYM}}(e, \mathcal{B}) + S_{\mathrm{EM}}(e, \mathcal{B})$$

SYM field content: fermions, scalar particles, vector field

SYM properties: conformal symmetry, supersymmetry, …

SYM appears to be entirely different from QCD!

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Strategy:

- compare thermodynamic quantities (macroscopic / effective); e.g. pressure
- match divergencies in the two theories, i.e. match beta functions
- measure magnetic fields in “same units”
- compare two theories at same renormalization scale

SYM magnetic field $\mathcal{B}$ vs. QCD magnetic field B: $$B = \boxed{\xi}\mathcal{B}$$

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]

[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

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Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^\mu\rangle = (\rho,\, 0,\, 0,\, p_1)\;.$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^\mu_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

➡ agrees in form with strong *B* thermodynamics from EFT

EFT result II: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle,\, \langle T^{\mu\nu}\, J^{\alpha}\rangle,\, \langle J^{\mu}\, T^{\alpha\beta}\rangle,\, \langle J^{\mu}\, J^{\alpha}\rangle$:

spin 1 modes under SO(2) rotations around *B*

$$\omega = \qquad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
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$$\omega = \mp\frac{Bn_0}{\underline{\epsilon_0+P_0}} - ik^2\frac{\eta}{\epsilon_0+P_0} + \underline{k\frac{Bn_0\xi_3}{(\epsilon_0+P_0)^2} - \frac{iB^2\sigma}{\epsilon_0+P_0}}$$

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$$\omega_0 = \underline{v_0\,k} - iD_0\,k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = \underline{v_+\,k - i\Gamma_+\,k^2} + \mathcal{O}(\partial^3)$$

$$\omega_- = \underline{v_-\,k - i\Gamma_-\,k^2} + \mathcal{O}(\partial^3)$$

former sound modes

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former sound modes

➡ **a chiral magnetic wave**

[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]

[Abbasi et al.; PLB (2016)]

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former sound modes

$$w_0 = \epsilon_0 + P_0$$

$$\mathfrak{s}_0 = s_0/n_0$$

$$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$$

$$c_s^2 = (\partial P/\partial \epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\, \frac{w_0\, \sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\, \frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3CT_0\mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] \qquad v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

$$+ B\, \frac{1 - c_s^2}{w_0}\, \xi_V^{(0)}\,,$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument

[Son,Surowka; PRL (2009)]

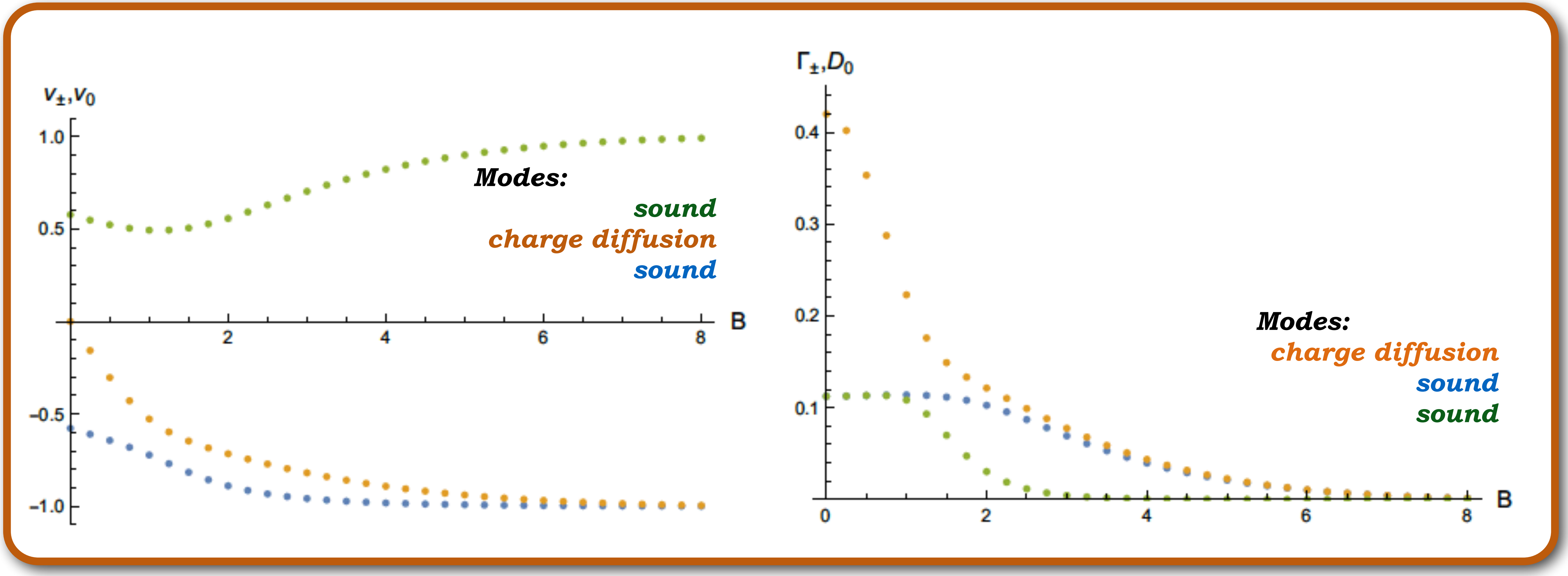
[Neiman,Oz; JHEP (2010)]

Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in “agreement” with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

the speed of light **and without attenuation**



confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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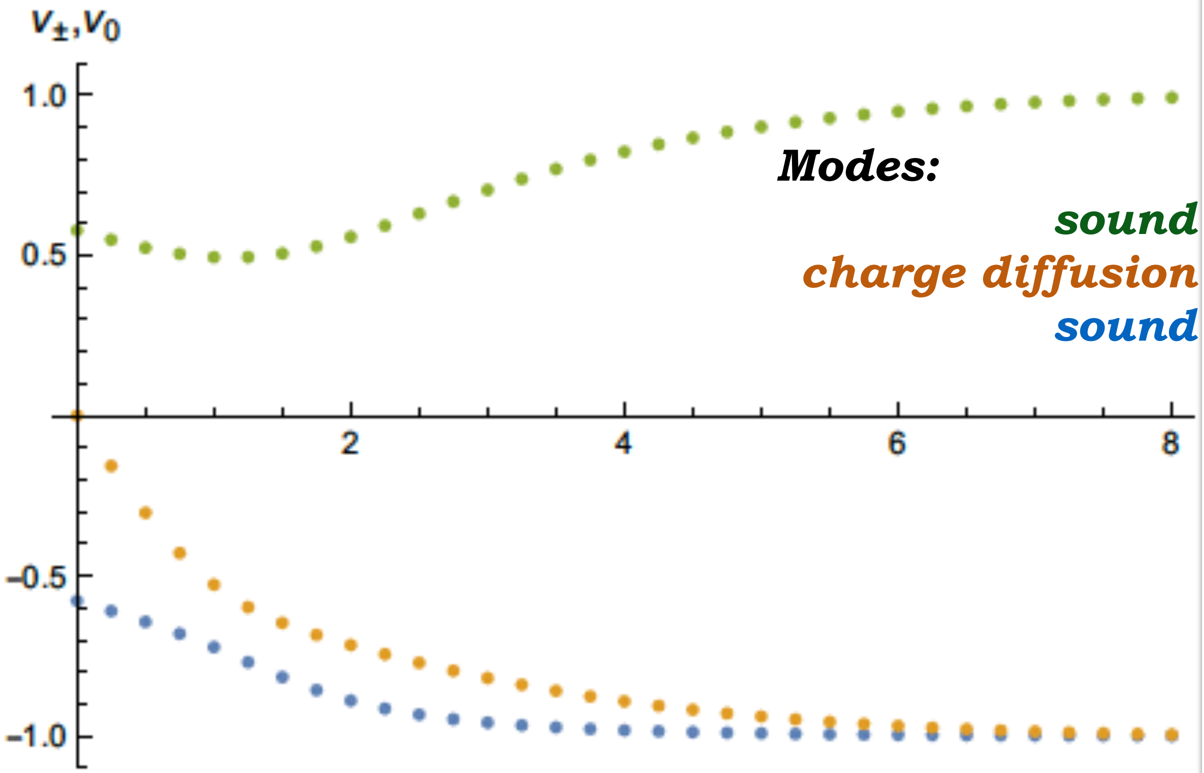

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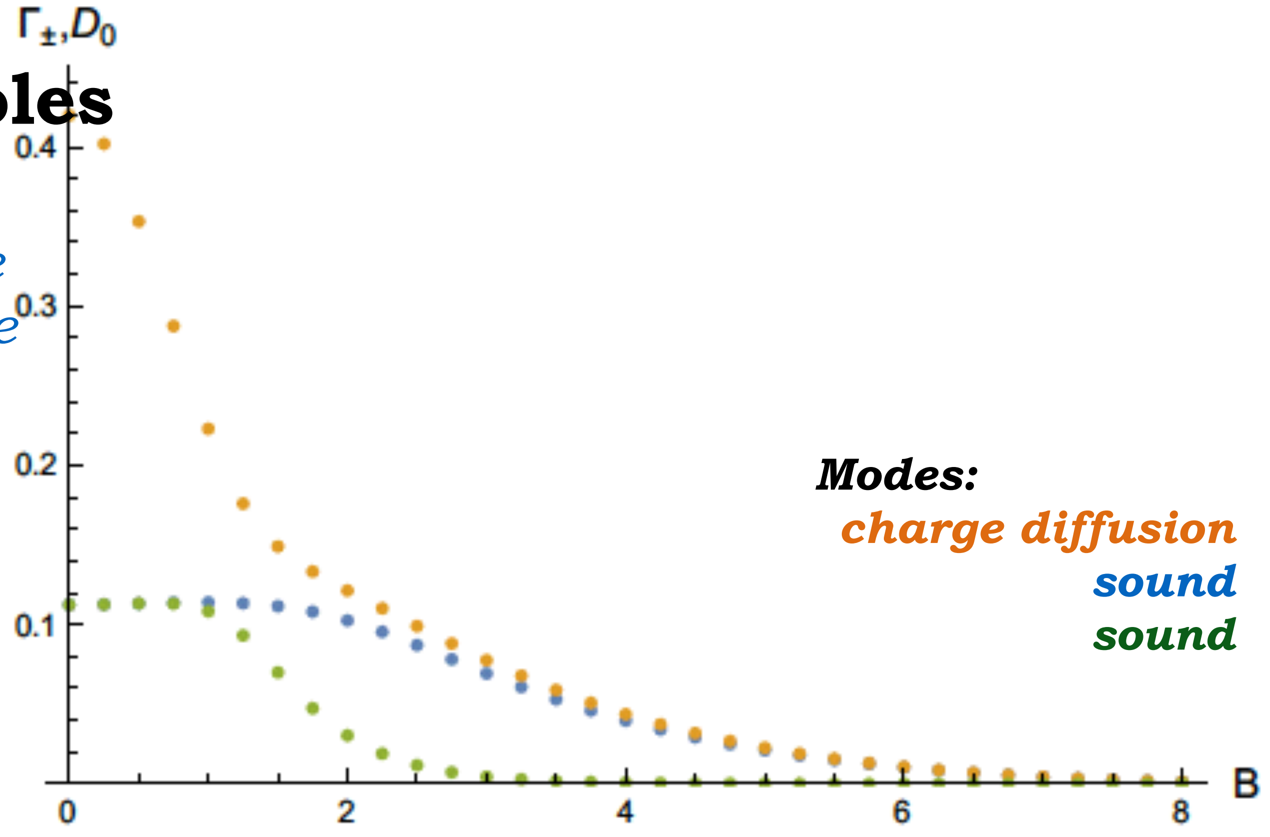

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Method summary: holography & hydrodynamics

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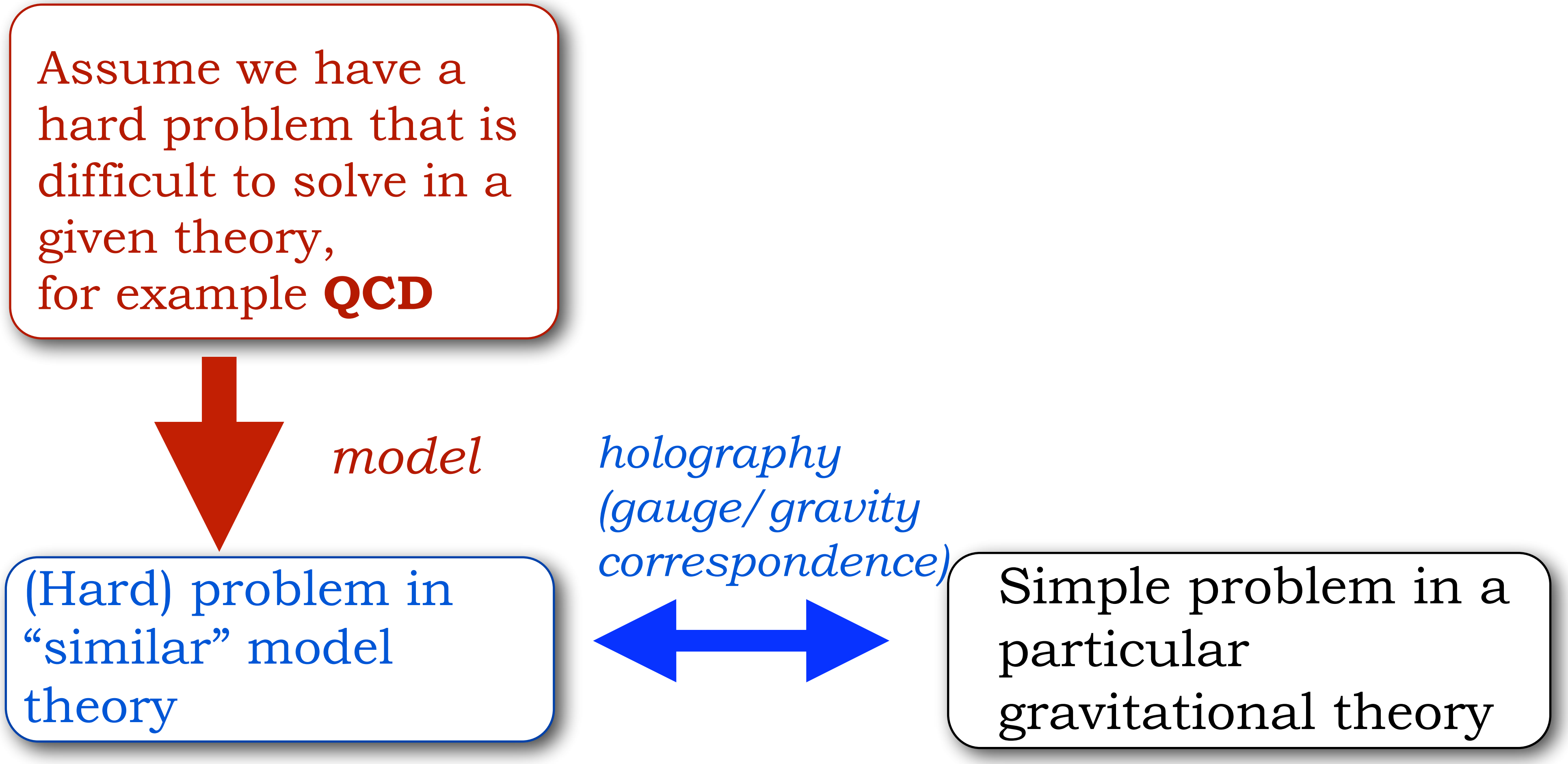

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model

(Hard) problem in "similar" model theory

holography (gauge/gravity correspondence)

Simple problem in a particular gravitational theory

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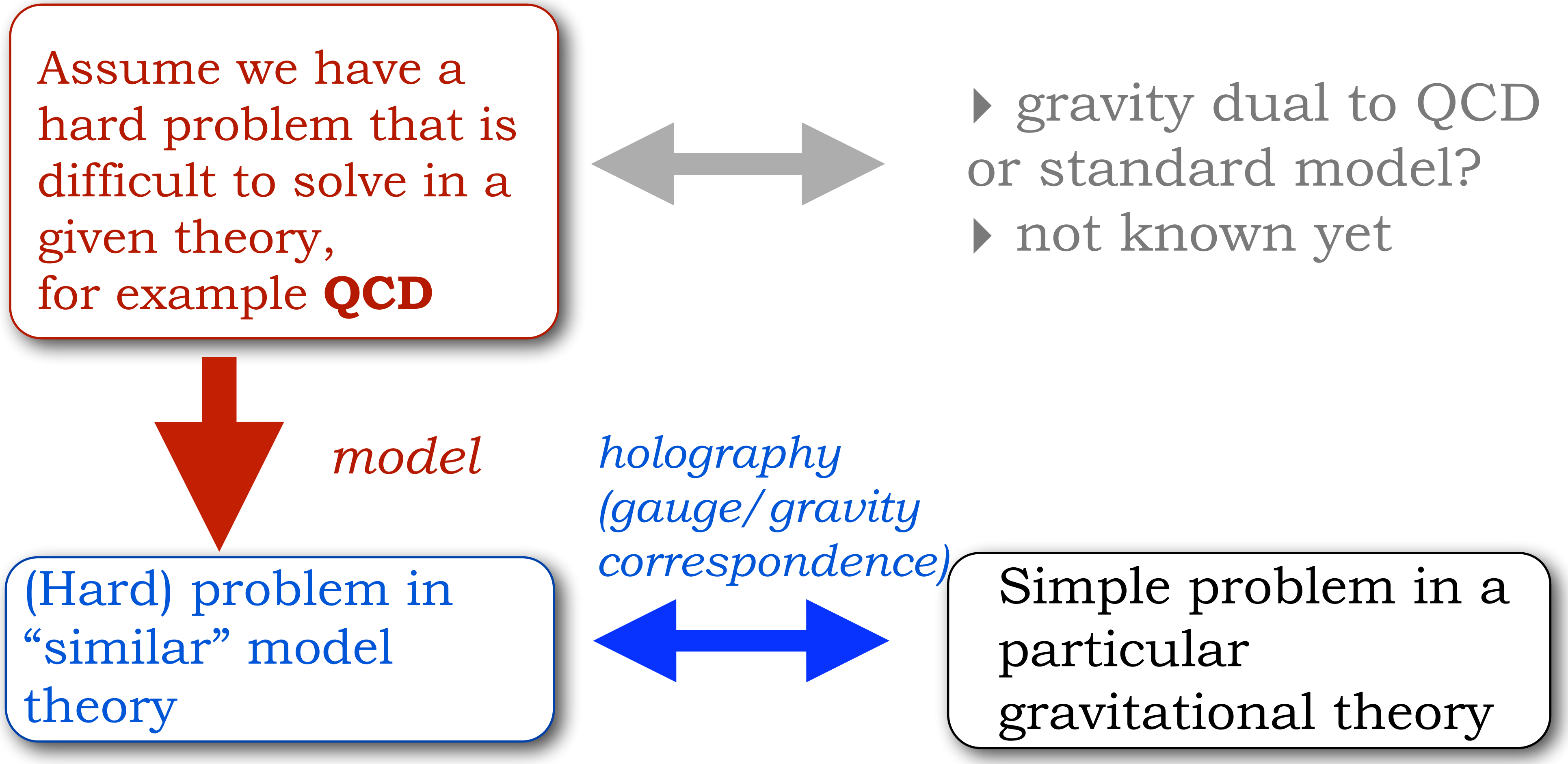

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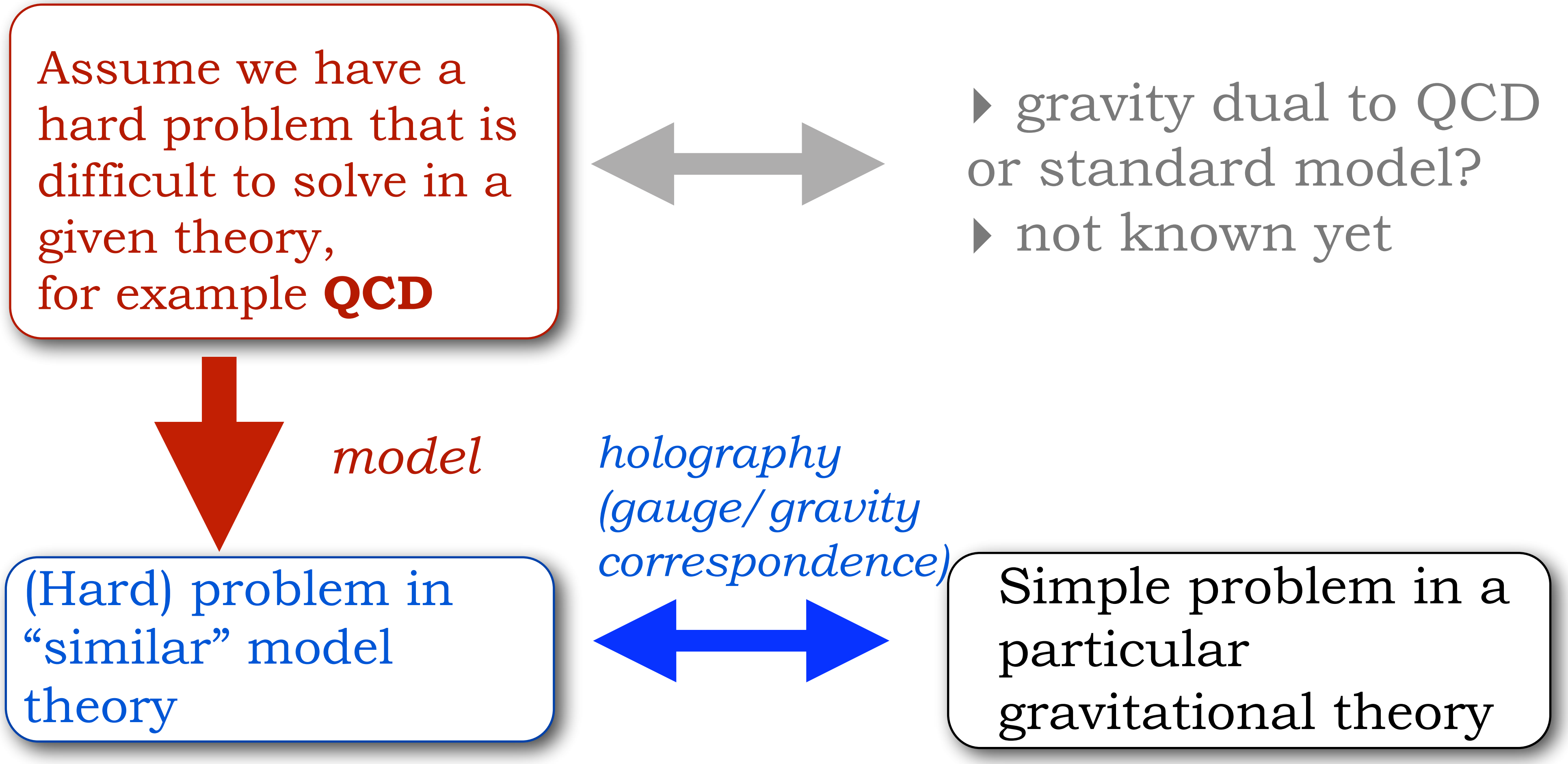

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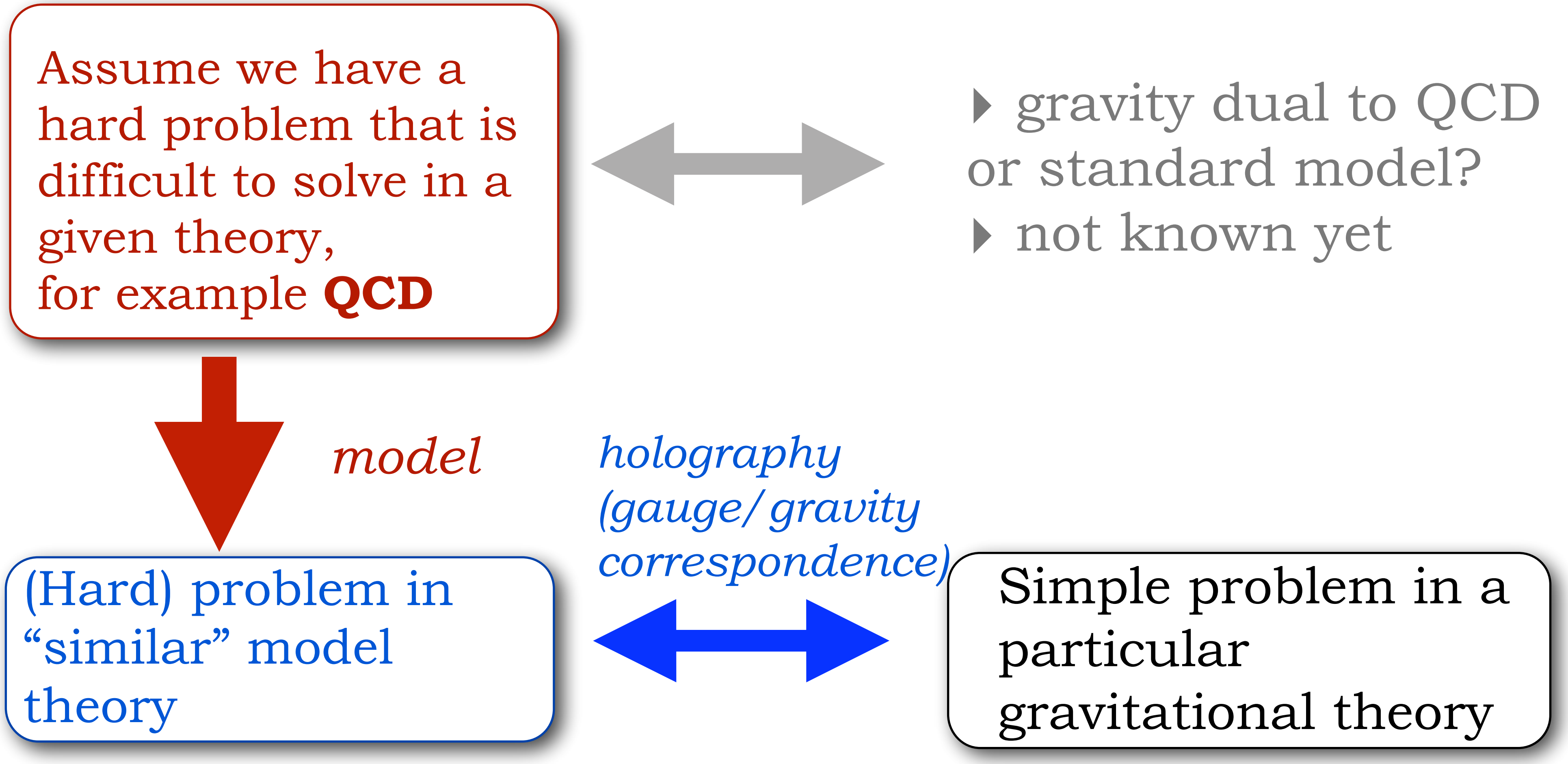

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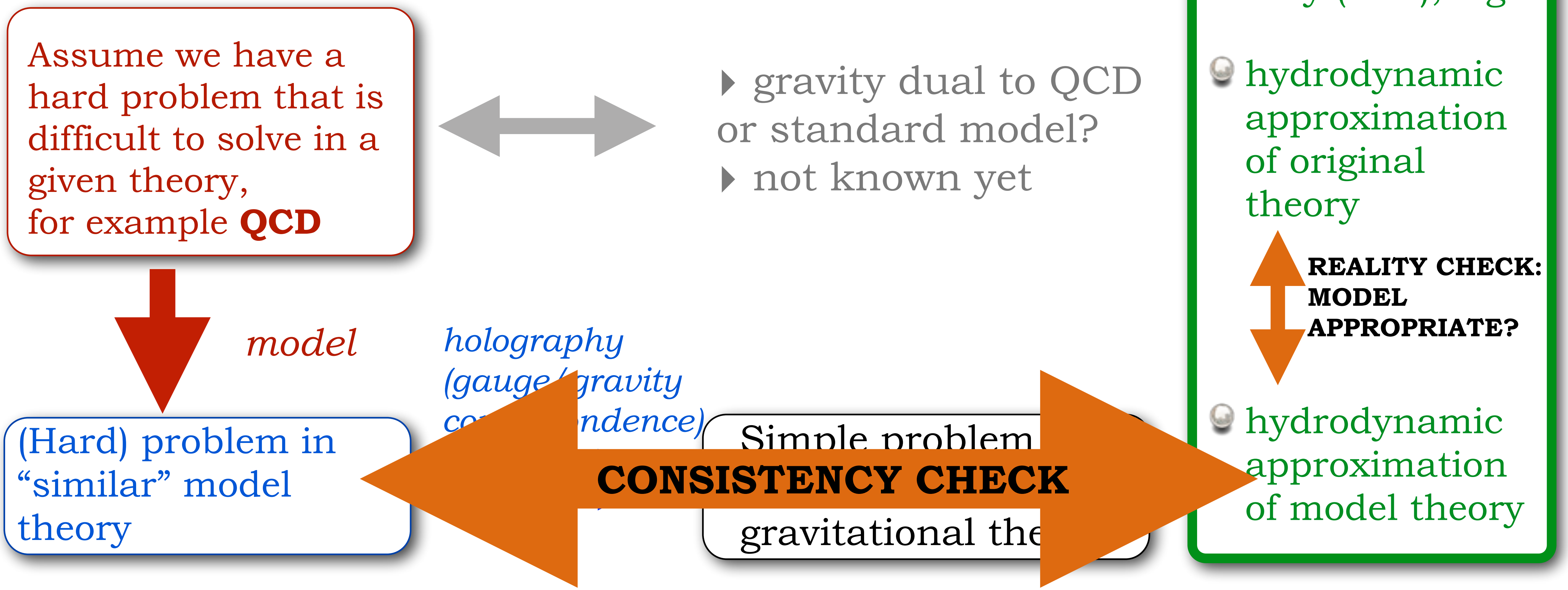

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CONSISTENCY CHECK

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REALITY CHECK: MODEL APPROPRIATE?

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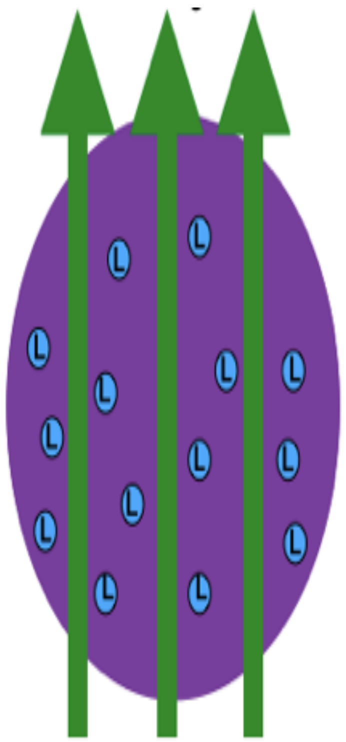
Setup

Einstein-Maxwell-Chern-Simons action:

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}(R - 2\Lambda + F_{\mu\nu}F^{\mu\nu}) + \cancel{\gamma \epsilon^{\alpha\beta\gamma\delta\eta} A_\alpha F_{\beta\gamma} F_{\delta\eta}}$$

neglect in this work



Metric ansatz:

$$\mathrm{d}s^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(t,r)^2(e^{B(r,t)}(\mathrm{d}x^2 + \mathrm{d}y^2) + e^{-2B(r,t)}\mathrm{d}z^2)$$

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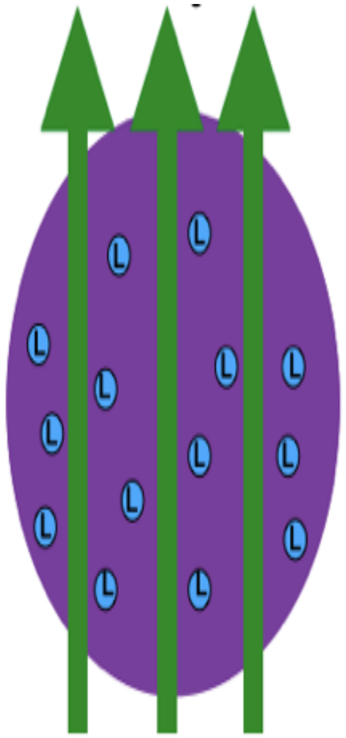

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Maxwell equations are solved by:

$$\mathscr{A}(r,t) = (0, \phi(r,t), -\frac{1}{2}y\mathscr{B}, \frac{1}{2}x\mathscr{B}, 0)$$

$$-\partial_r\phi(r,t) = \mathscr{E}(r,t) = \frac{\rho(r,t)}{S(t,r)^3}$$

Einstein equations are nested:

$$\dot{f} = \partial_t f + \frac{1}{2}A\partial_r f.$$

$$S''(t,r) = -\frac{1}{2}B'(t,r)^2 S(t,r)$$

$$\dot{S}'(t,r) = \frac{\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^3} - \frac{2S'(t,r)\dot{S}(t,r)}{S(t,r)} + \frac{\rho^2}{3S(t,r)^5} + 2S(t,r)$$

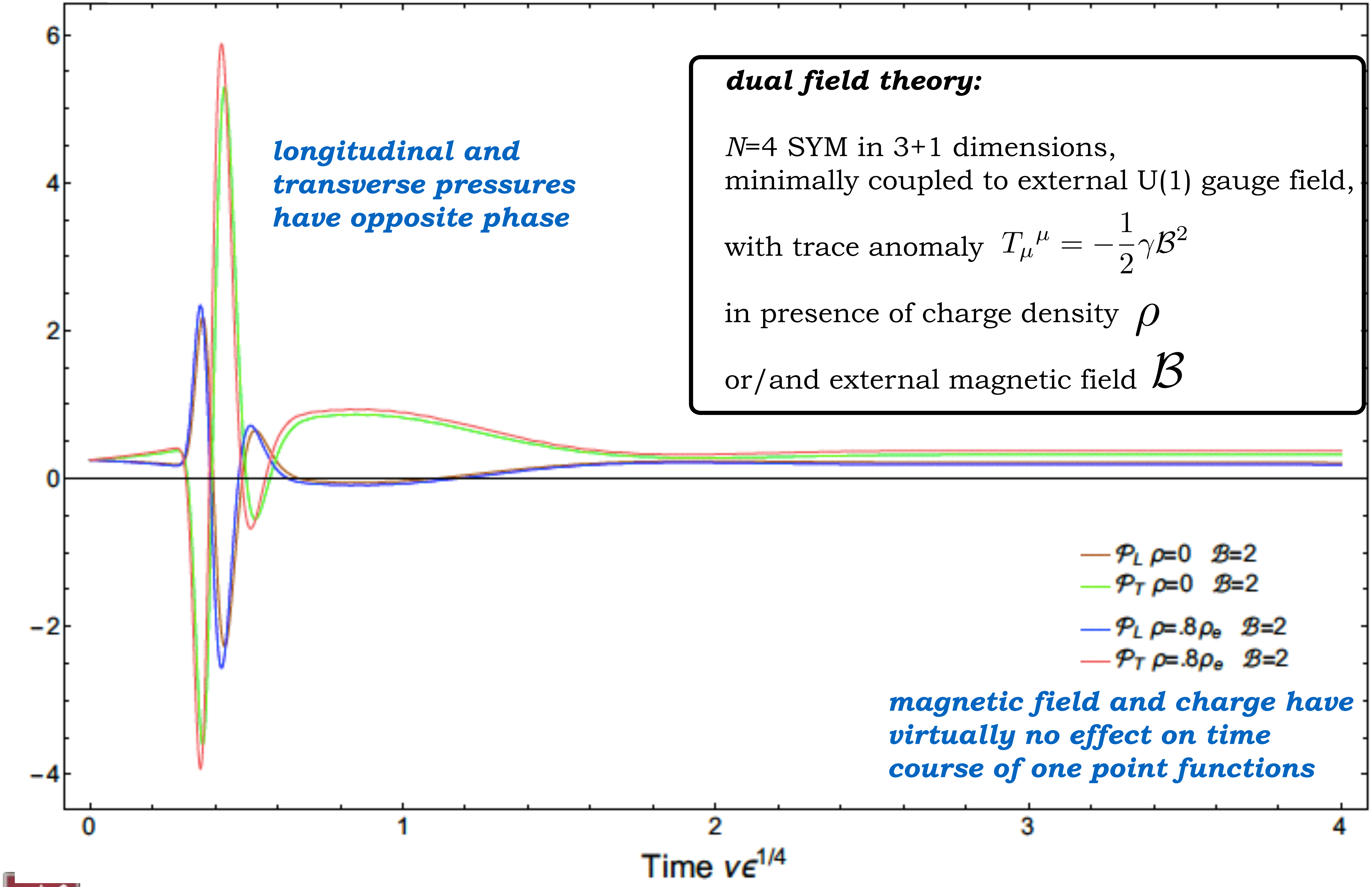
$$\dot{B}'(t,r) = -\frac{3\dot{B}(t,r)S'(t,r)}{2S(t,r)} - \frac{3B'(t,r)\dot{S}(t,r)}{2S(t,r)} + \frac{2\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4}$$

$$A''(t,r) = -3B'(t,r)\dot{B}(t,r) - \frac{10\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4} + \frac{12S'(t,r)\dot{S}(t,r)}{S(t,r)^2} - \frac{14\rho^2}{3S(t,r)^6} - 4$$

$$\ddot{S}(t,r) = \frac{1}{2}A'(t,r)\dot{S}(t,r) - \frac{1}{2}\dot{B}(t,r)^2 S(t,r).$$

Background - One Point Functions

[Fuini, Yaffe; JHEP (2015)]



longitudinal and transverse pressures have opposite phase

dual field theory:

$\mathcal{N}$=4 SYM in 3+1 dimensions,
minimally coupled to external U(1) gauge field,

with trace anomaly $T_\mu{}^\mu = -\frac{1}{2}\gamma\mathcal{B}^2$

in presence of charge density ρ

or/and external magnetic field $\mathcal{B}$

magnetic field and charge have virtually no effect on time course of one point functions

Correlations - geodesic approximation

[Balasubramanian, Ross; PRD(2000)]

Correlator as a sum over geodesics:

$$\Delta L = L - L_{\text{thermalized}}$$

$$\langle \mathcal{O}(t,\vec{x}_1)\mathcal{O}(t,\vec{x}_2)\rangle = \int \mathcal{D}\mathcal{P}e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L}$$

Geodesic length (Lagrangian):

$$L = \int \mathrm{d}\lambda\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} \qquad \Rightarrow \qquad \frac{d^2x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0$$

geodesic equation

($L = m\int \mathrm{d}\lambda\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} + \frac{q}{m}A_\mu\dot{x}^\mu$ charged probe particle)

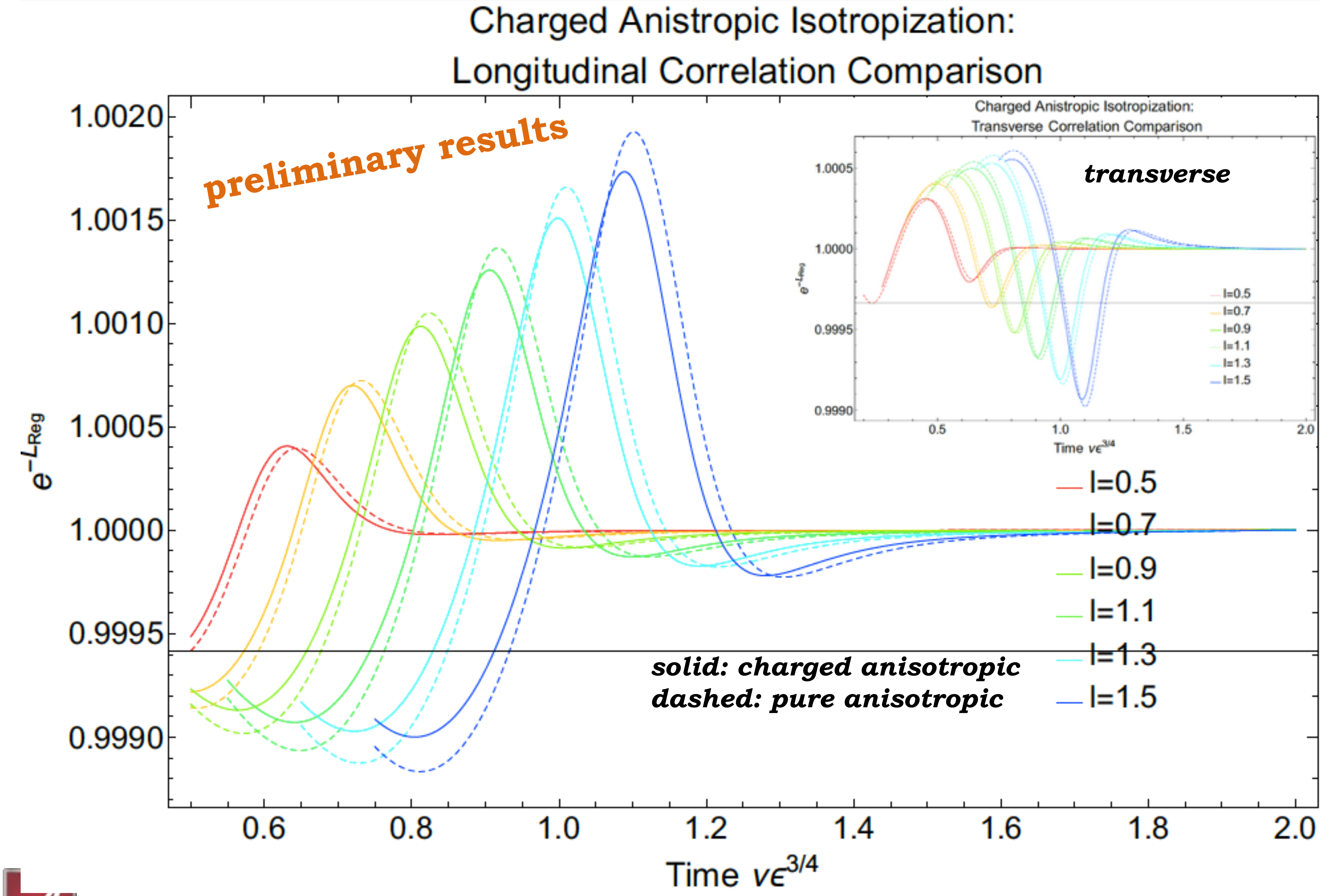
Lorentz force term

Numerical implementation - relaxation method:

[Ecker, Grumiller, Stricker; JHEP (2015)]

1. Generate the dynamic background
2. Generate interpolations of the metric functions
3. Discretize the geodesic equations using a relaxation scheme
4. Approximate the proper length using a Riemann sum

Correlations - finite charge, zero B

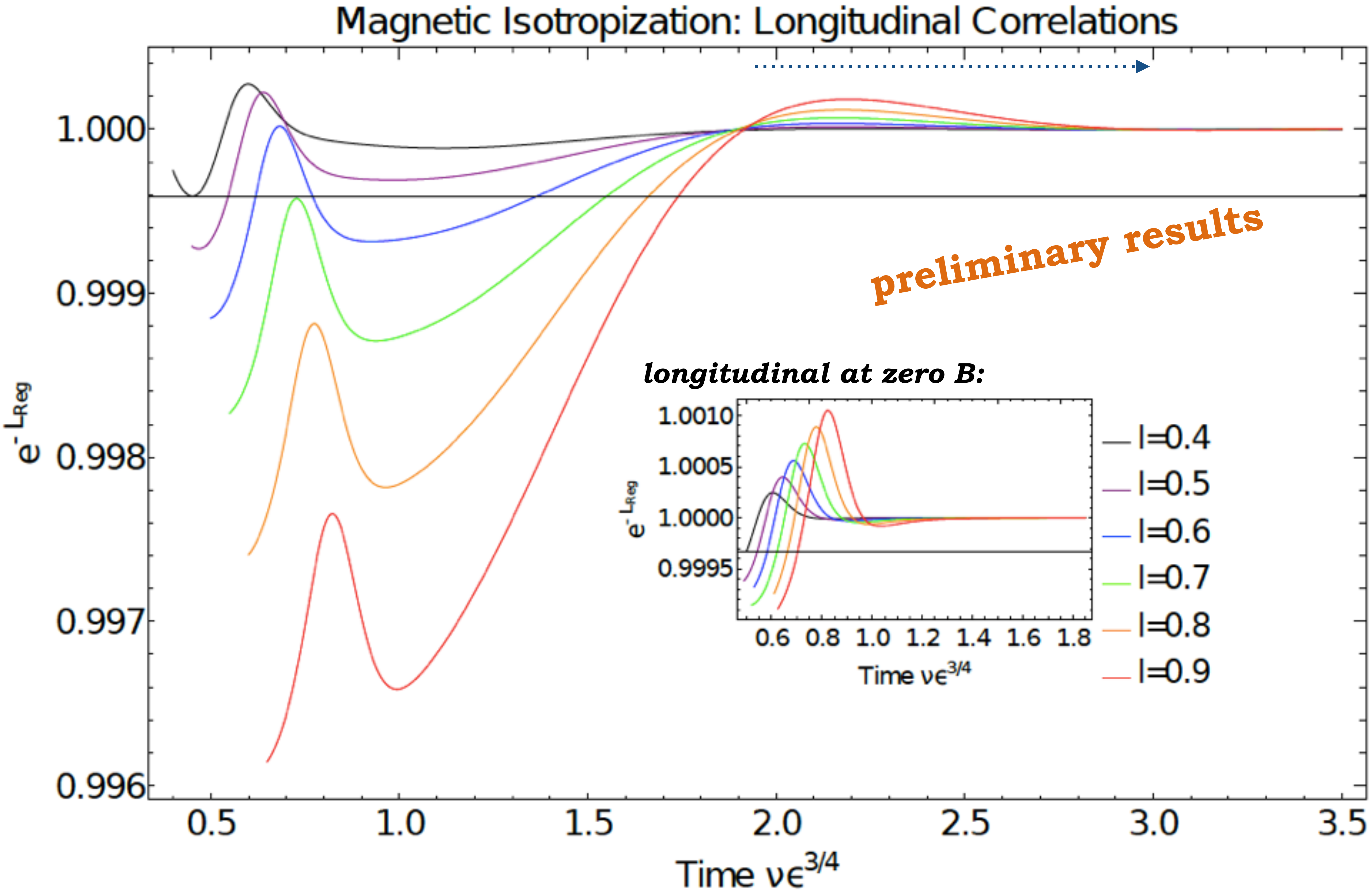


preliminary results

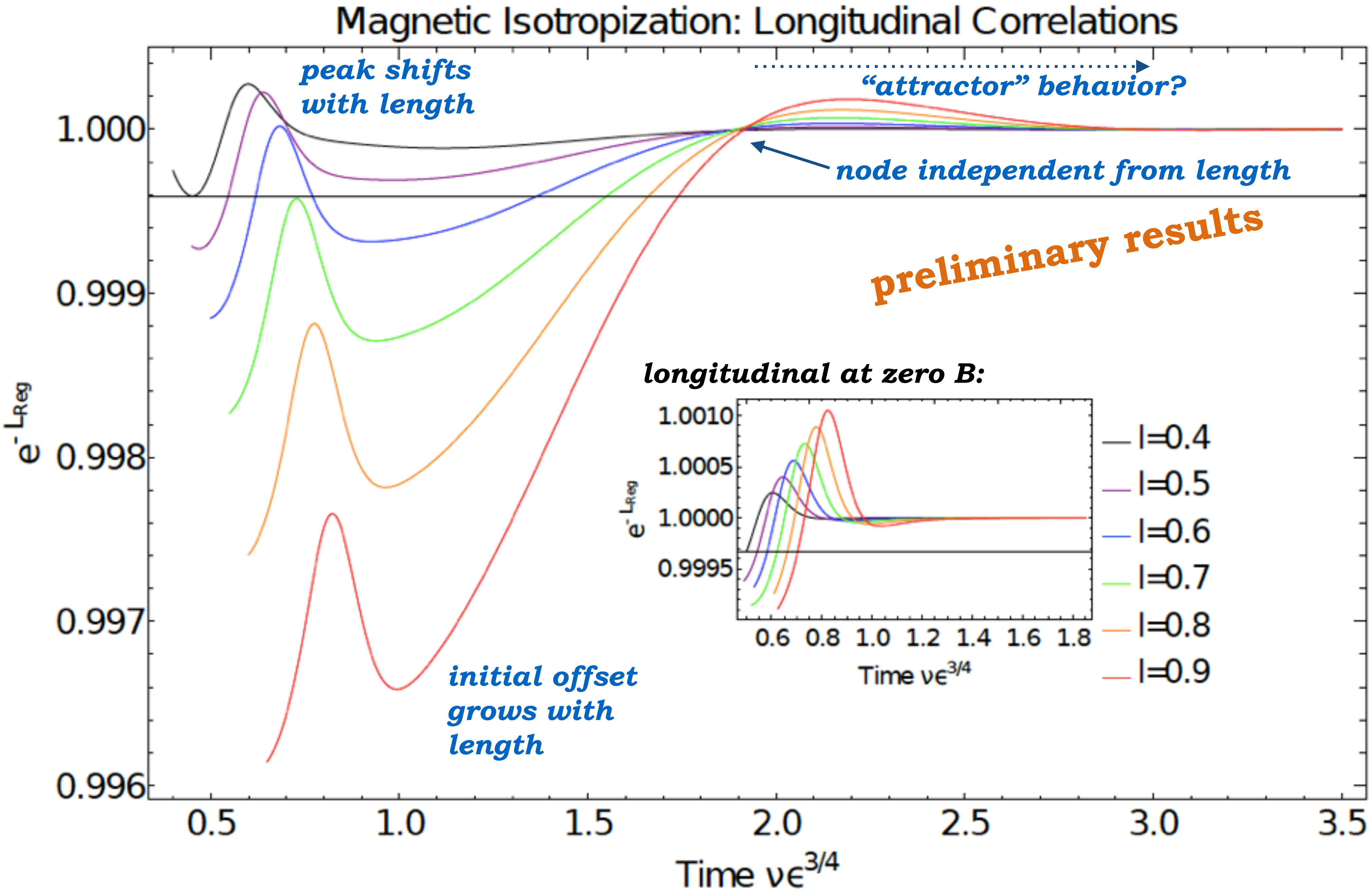
transverse

solid: charged anisotropic
dashed: pure anisotropic

Correlations - zero charge, finite B



Correlations - zero charge, finite B



peak shifts with length

"attractor" behavior?

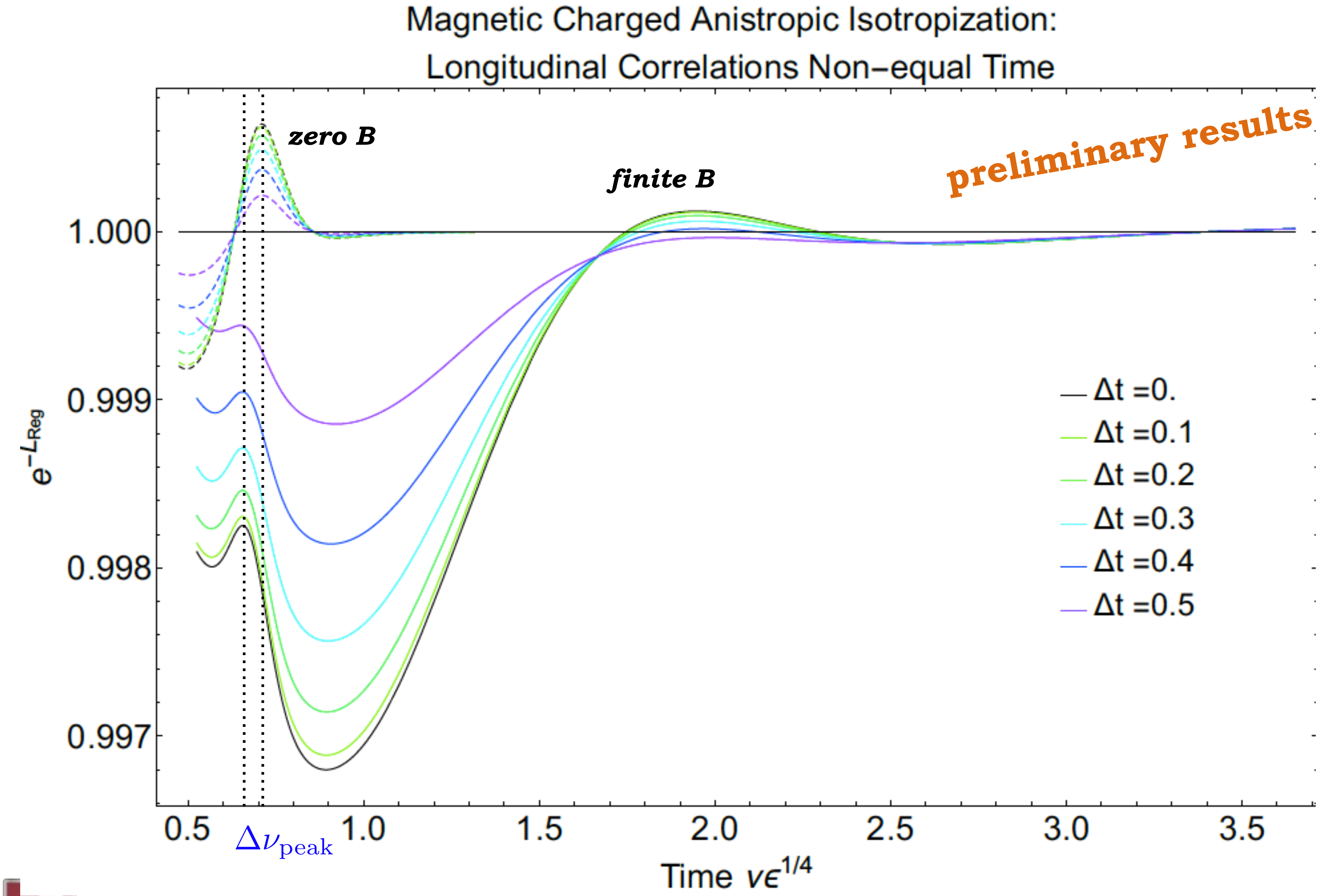
node independent from length

preliminary results

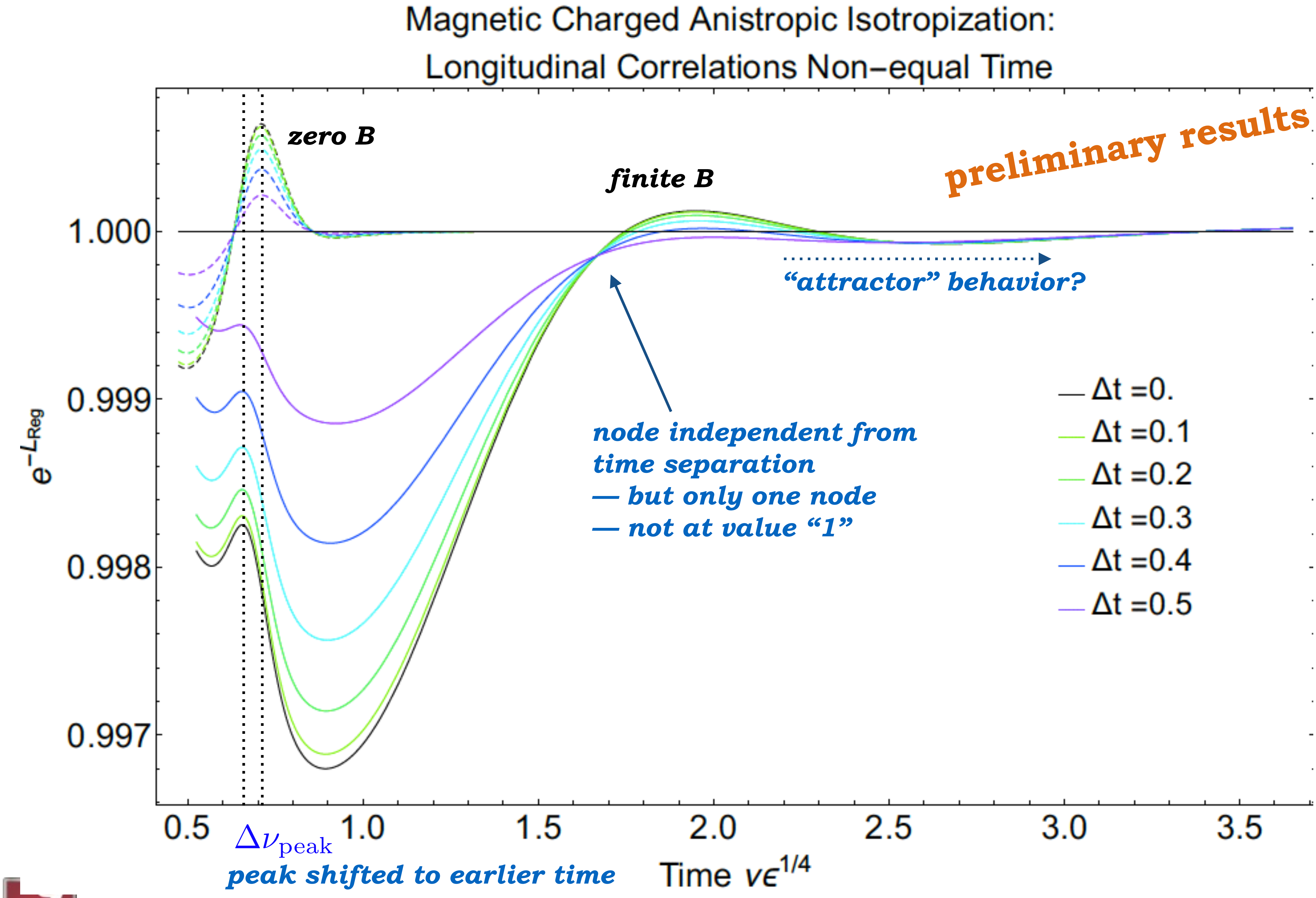
longitudinal at zero B:

initial offset grows with length

Correlations - finite charge, finite B



Correlations - finite charge, finite B



zero B

finite B

preliminary results

"attractor" behavior?

node independent from time separation
— but only one node
— not at value "1"

$\Delta\nu_{\mathrm{peak}}$

peak shifted to earlier time