The Vortical Zilch

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[K.L. , M. Chernodub and A. Cortijo, to appear]

Open Problems and Opportunities in Chiral Fluids, **Santa Fe**, July 16-19, 2018

Outline

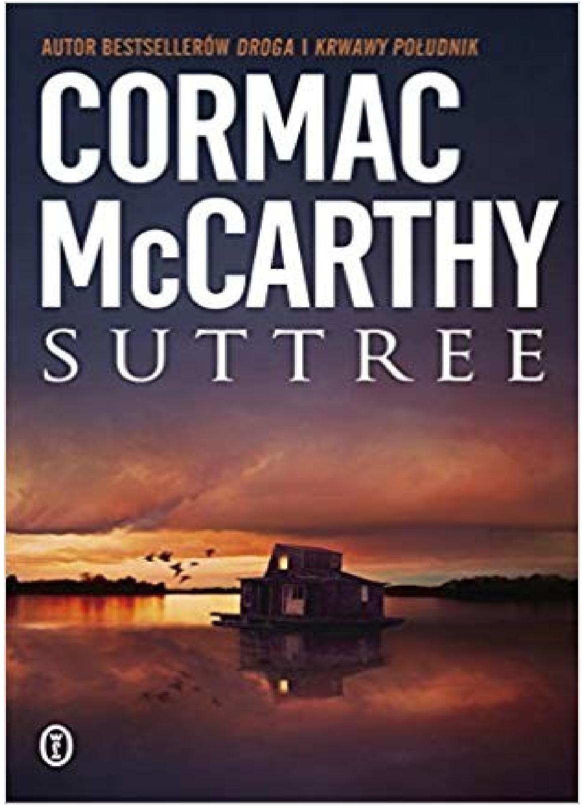
- Chiral vortical effect
- Maxwell theory
- Maxwell on a Cylinder
- The vortical Zilch
- Conclusion and Outlook

- Web search Santa Fe + Chirality ( )

DuckDuckGo

"On the right temple a mauve halfmoon. Suttree turned and lay staring at the ceiling, touching a like mark on his own left temple gently with his fingertips. The ordinary of the second son. Mirror image. Gauche carbon."

"A dextrocardiac, said the smiling doctor. Your heart's in the right place."



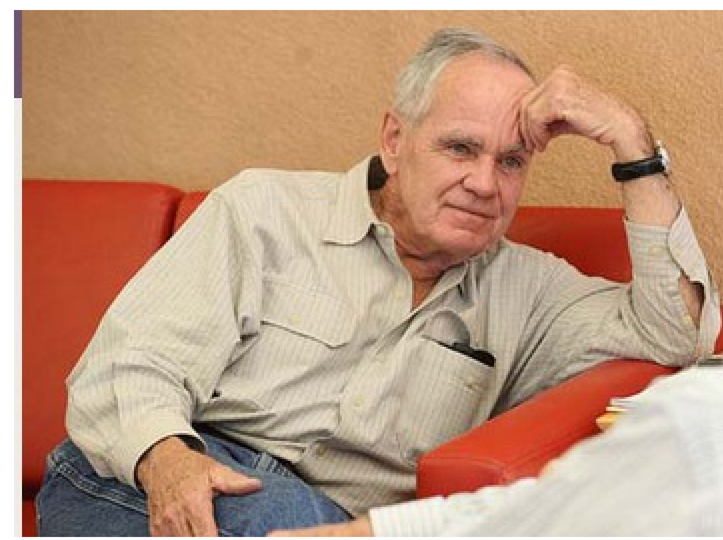
"Gray vines coiled leftward in the northern hemisphere, what winds them shapes the dogwhelk's shell."

"For now, suffice it to say that we may be in something of a **golden age of chirality**, from *Breaking Bad* to Nobel Prize-winning areas of scientific enquiry."

"Mirror Image, Asymmetry, Chirality and *Suttree*",
Bryan Giemza
Special Issue of the European Journal of American Studies: Cormac McCarthy Between Worlds

Occasionally we find that an invited guest is insane. This generally cheers us all up. We know we're on the right track.

Operating Principles Santa Fe Institute
Cormac McCarthy



CVE

$$\boxed{\vec{J}_a = \left(\frac{d_{abc}}{8\pi^2}\mu_b\mu_c + \frac{b_a}{24}T^2\right) 2\vec{\omega}}$$

$$d_{abc} = \sum_r q_a^r q_b^r q_c^r - \sum_l q_a^l q_b^l q_c^l$$

Chiral anomaly

$$b_a = \sum_r q_a^r - \sum_l q_a^l$$

Gravitational chiral anomaly

- Chemical potential – chiral anomaly
- Temperature – gravitational anomaly (perturbative vs. global ?)

[Vilenkin], [Kharzeev, Zhitntsky], [K.L., Megias, Pena-Benitez],[Haack, Erdmenger, Kaminski, Yarom], [Banerjee, Bhattacharya, Bhattacharyya, Loganayagam, Surowka],[K.L., Megias, Melgar, Pena-Benitez], [Jensen, Loganayagam, Yarom], [Golkar, Son], [Golkar, Sethi], [Glorioso,Liu,Rajagopal],[Stone, Kim]

Maxwell Theory

$$dF = d * F = 0$$

- 1986: [Dolgov, Kriplovich, Vainstein, Zhakharov]

$$J^\mu = \epsilon^{\mu\nu\rho\lambda} A_\nu F_{\rho\lambda}$$

- “Magnetic” Helicity

$$\partial_\mu J^\mu = F\tilde{F}$$

- Quantum

$$\partial_\mu J^\mu = F\tilde{F} + \frac{1}{48\pi^2} R\tilde{R}$$

- 2017: [Agullo, del Rio, Navarro-Salas]

$$J^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda} \left(A_\nu F_{\rho\lambda} - C_\nu \tilde{F}_{\rho\lambda} \right)$$

- “Optical” Helicity

$$\partial_\mu J^\mu = 0 \qquad dC = \tilde{F}$$

- Quantum

$$\partial_\mu J^\mu = \frac{1}{24\pi^2} R\tilde{R}$$

- Conserved current = Symmetry
- 1976 [Deser, Teitelboim] : ***electric-magnetic duality***

Maxwell Theory

$$J^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\left(A_\nu F_{\rho\lambda} - C_\nu \tilde{F}_{\rho\lambda}\right)$$

$$\partial_\mu J^\mu = \frac{1}{24\pi^2} R\tilde{R}$$

- Gravitational Anomaly in electric-magnetic duality ?
- Physical significance not clear (to me)
- Non-gauge invariant current
- Non-local current (simultaneous presence of A and C)
- Anomaly is by itself the divergence of Chern-Simons term

- Helical vortical effect
- [Avkhadiev, Sadofyev], [Yamamoto], [Zyuzin]

$$\vec{J}_{\text{Helicity}} = \frac{1}{12} T^2 \vec{\omega}$$

Maxwell Theory on a Cylinder

- Rigid rotation is impossible in unbounded space
- Study the theory on a cylinder
- Insure that tangential velocity < 1

- Boundary conditions:
- Energy and Angular momentum conserved

- No radial momentum on boundary
- No $\rho\varphi$ stress on boundary

- Perfect conductor:

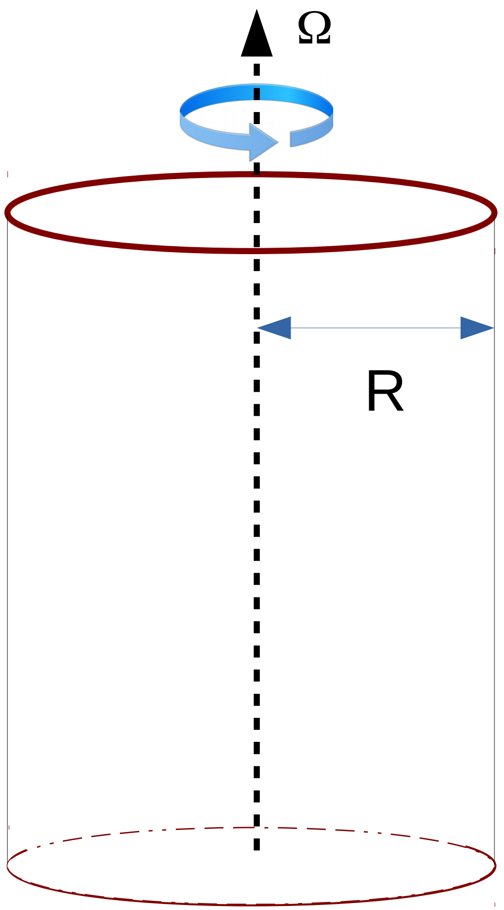
$$B_\rho = 0 \quad , \quad E_z = E_\varphi = 0$$

- Perfect dual conductor:

$$E_\rho = 0 \quad , \quad B_z = B_\varphi = 0$$

- Limit R $\rightarrow \infty$ natural boundary conditions

$$\int_0^\infty \sqrt{\rho}\,|\Phi| d\rho < \infty$$



Maxwell Theory on a Cylinder

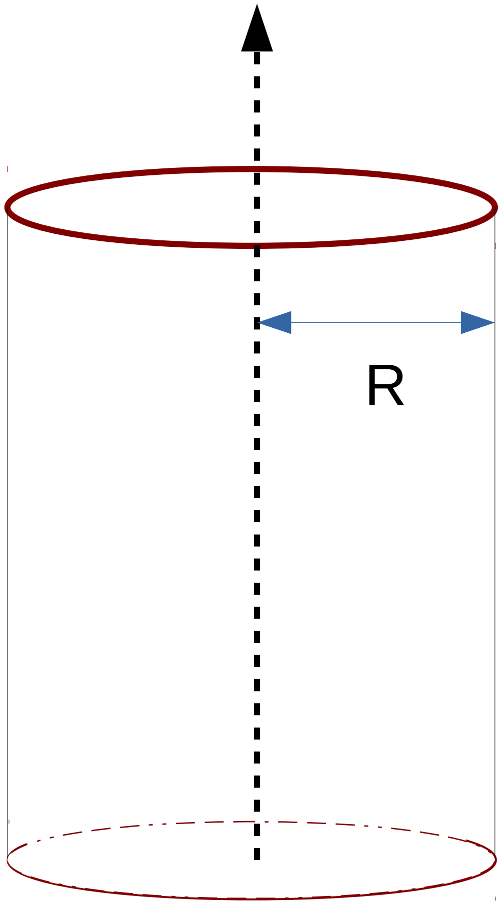
- Radiation gauge: $\vec{\nabla}\cdot\vec{A}=0 \quad , \quad A_0=0$

- Eigenmodes: $\vec{A}^{(\lambda)}(t,\rho,\varphi,z)=\vec{a}^{(\lambda)}(\rho)e^{-i\omega t+ikz+im\varphi}$

- Transverse Electric: $\vec{a}^{(\mathrm{TE})}(\rho)=\begin{pmatrix}\frac{mf_{\mathrm{TE}}(\rho)}{i\rho}\\ \frac{\partial f_{\mathrm{TE}}(\rho)}{\partial\rho}\\ 0\end{pmatrix}$

- Transverse Magnetic: $\vec{a}^{(\mathrm{TM})}(\rho)=\begin{pmatrix}\frac{k_z}{i\omega}\frac{\partial f_{\mathrm{TM}}(\rho)}{\partial\rho}\\ \frac{mk_z}{\omega}\frac{f_{\mathrm{TM}}(\rho)}{\rho}\\ -\frac{k_\perp^2}{\omega}f_{\mathrm{TM}}(\rho)\end{pmatrix}$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f_\lambda}{\partial\rho}\right)-\frac{m^2}{\rho^2}f_\lambda+k_\perp^2 f_\lambda=0$$



- Bessel functions: $f_\lambda=C_\lambda J_m(k_\perp\rho)$

Maxwell Theory on a Cylinder

- Boundary conditions: $J'_m(k_\perp R) = 0, \quad \begin{pmatrix} \lambda \\ \mathrm{BC} \end{pmatrix} = \begin{pmatrix} \mathrm{TE} \\ E \end{pmatrix} \& \begin{pmatrix} \mathrm{TM} \\ M \end{pmatrix}$

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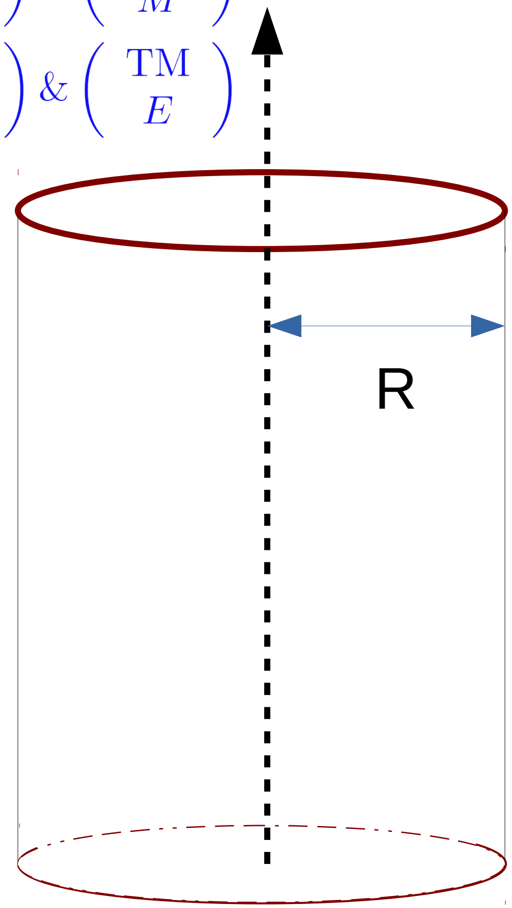


- Radial quantum numbers: $k_\perp = \frac{\kappa_{ml}}{R} \quad , \quad k_\perp = \frac{\kappa'_{ml}}{R}$

- Normalization: $$C_\lambda = \frac{R}{\sqrt{(\kappa'_{ml})^2 - m^2}|J_m(\kappa'_{ml})|}$$

  $$C_\lambda = \frac{R}{\kappa_{ml}|J'_{m+1}(\kappa_{ml})|}$$

- Quantum numbers: $\{J\} = k_z \in \mathbb{R}, \quad m \in \mathbb{Z}, \quad l \in \mathbb{N}$

  $$\omega_J = \sqrt{k_z^2 + k_\perp^2}$$

- Field operator: $$\vec{A} = \sum_{J,\lambda} \frac{1}{\sqrt{2\omega_J}} \left(\vec{A}_J^{(\lambda)} \alpha_J^{(\lambda)} + \vec{A}_J^{(\lambda)*} \alpha_J^{(\lambda)\dagger} \right)$$

Maxwell Theory in unbounded Domain

- New basis: $$\vec{A}_J^{(\pm)} = \vec{A}_J^{(\mathrm{TE})} \pm \vec{A}_J^{(\mathrm{TM})}$$

- Duality eigenstates (circularly polarized): $$\vec{\nabla} \times \vec{A}_J^{(\pm)} = \pm\omega \vec{A}_J^{(\pm)}$$

$$\{J\} = k_z \in \mathbb{R}, \quad m \in \mathbb{Z}, \quad k_\perp \in \mathbb{R}_+$$

$$\omega = \sqrt{k_z^2 + k_\perp^2}$$

- Complex field operator: $$\vec{A} = \sum_J \frac{\sqrt{2}}{\sqrt{\omega_J}} \left(A_J^{(+)} \alpha_J^{(+)} + A_J^{(-)*} \alpha_J^{(-)\dagger} \right)$$

EM Duality = phase rotations

- Energy and Angular momentum: $$\mathcal{H} = \sum_J \omega_J \alpha_J^{(\lambda)^\dagger} \alpha_J^{(\lambda)}$$

$$\mathcal{L}_z = \sum_J m\, \alpha_J^{(\lambda)^\dagger} \alpha_J^{(\lambda)}$$

The Zilch

Zilch: nothing at all, non-entity, a person regarded to be insignificant, ...

- 1966 [Lipkin] :

$$\vec{H} = \vec{\nabla} \times \vec{B} \qquad \zeta = \vec{H} \cdot \vec{B} + \vec{G} \cdot \vec{E}$$

$$\vec{G} = \vec{\nabla} \times \vec{E} \qquad \vec{J}_\zeta = \vec{E} \times \vec{H} + \vec{G} \times \vec{B}$$

$$\dot{\zeta} + \vec{\nabla} \cdot \vec{J}_\zeta = 0$$

- 1967 [Kibble] :

$$\vec{H}_s = \Delta^{(-s)} \vec{\nabla} \times \vec{B}$$

$$\vec{G}_s = \Delta^{(-s)} \vec{\nabla} \times \vec{E}$$

- Infinite tower of “infra” and “super” Zilches!
- Formally s=1 is optical Helicity

$$h = \vec{A} \cdot \vec{B} + \vec{C} \cdot \vec{E}$$

$$\vec{J}_h = \vec{E} \times \vec{A} + \vec{C} \times \vec{B}$$

- Covariant formulation: higher spin currents

The Zilch

- Physical meaning in unbounded domain

$$Q_h = \int d^3x\, h = \sum_J \left(\alpha^{(+)\dagger}\alpha^{(+)} - \alpha^{(-)\dagger}\alpha^{(-)}\right)$$

$$Q_\zeta = \int d^3x\, \zeta = \sum_J \omega_J^2 \left(\alpha^{(+)\dagger}\alpha^{(+)} - \alpha^{(-)\dagger}\alpha^{(-)}\right)$$

- Optical helicity is not gauge invariant local density

- Zilch measures difference between right and left and is local and gauge invariant

- Zilch determines asymmetry in interaction with chiral molecules (“*optical chirality*”)

[Tang, Cohen] PRL 2010

Rotation

- Density operator $$\rho = \frac{1}{Z} e^{-\beta(H - \Omega \mathcal{L}_z)}$$

- Thermal Vev: $$\langle \mathcal{O} \rangle = \sum_J n_B(T, \Omega) \langle J | \mathcal{O} | J \rangle$$

$$n_B(T, \Omega) = \frac{1}{e^{\frac{\omega_J - \Omega m}{T}} - 1}$$

$$\langle J | J^z_{h,\zeta} | J \rangle = m(\omega_J)^{2-2s} \left(1 + \frac{k^2}{\omega_J^2}\right) \frac{f_J f_J'}{\rho}$$

- Total derivative = no contribution from Dirichlet BCs to total current!

- Only Neumann BCs give net current !

Rotation

- In unbounded domain: study current at the axis

$$\langle J_h^z(0)\rangle = \frac{T^3}{3\pi^2}\int_0^\infty dx x^2 \left[\frac{1}{e^{x-\Omega/T}-1} - \frac{1}{e^{x+\Omega/T}-1}\right]$$

- Integrals are not defined

- Analytic continuation (Jonquiere inversion relations)

(Fermions: M. Stone's talk)

$$\langle J_h(0)\rangle = \frac{2T^2}{9}\Omega \pm i\frac{T}{3\pi}\Omega^2 - \frac{1}{9\pi^2}\Omega^3\,,$$

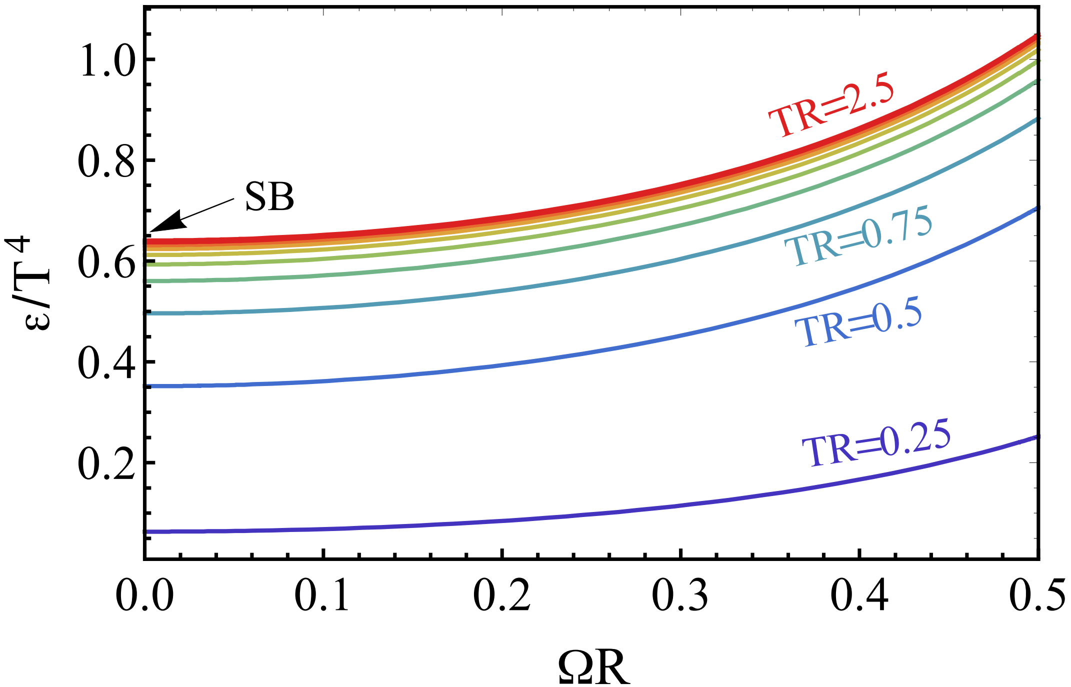
- Unphysical beyond leading order

- The vortical Zilch:

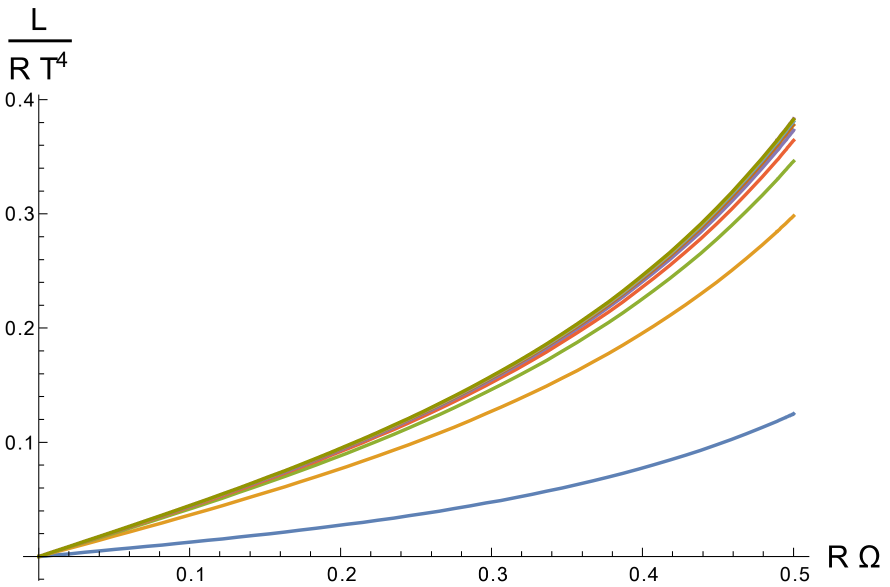
$$\boxed{\langle J_\zeta(0)\rangle = \frac{8\pi^2 T^4}{45}\Omega}$$

Rotation: numerics

Energy

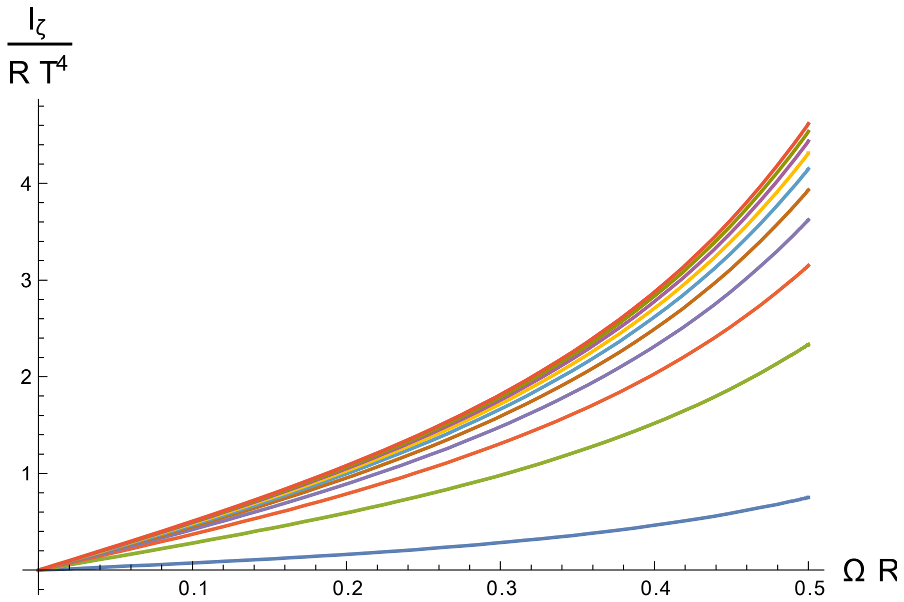


Angular Momentum

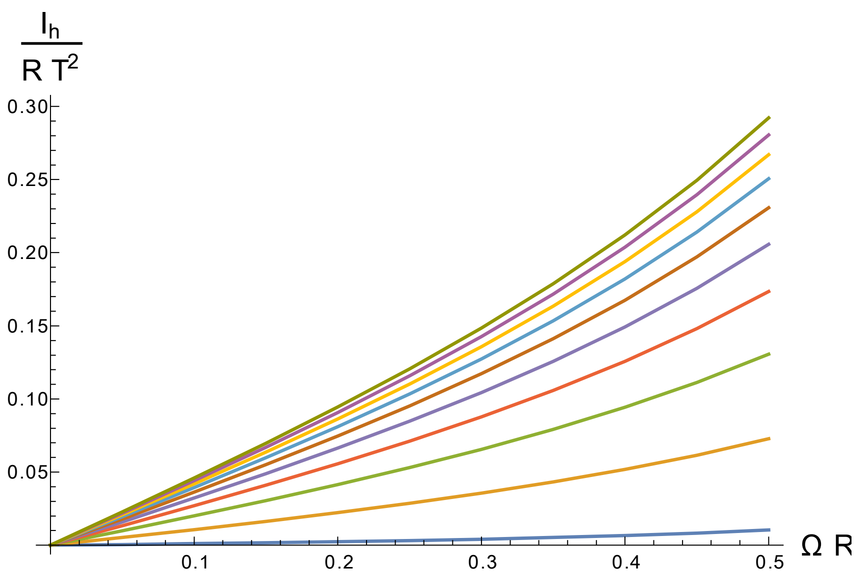


Rotation: numerics

Net Zilch current

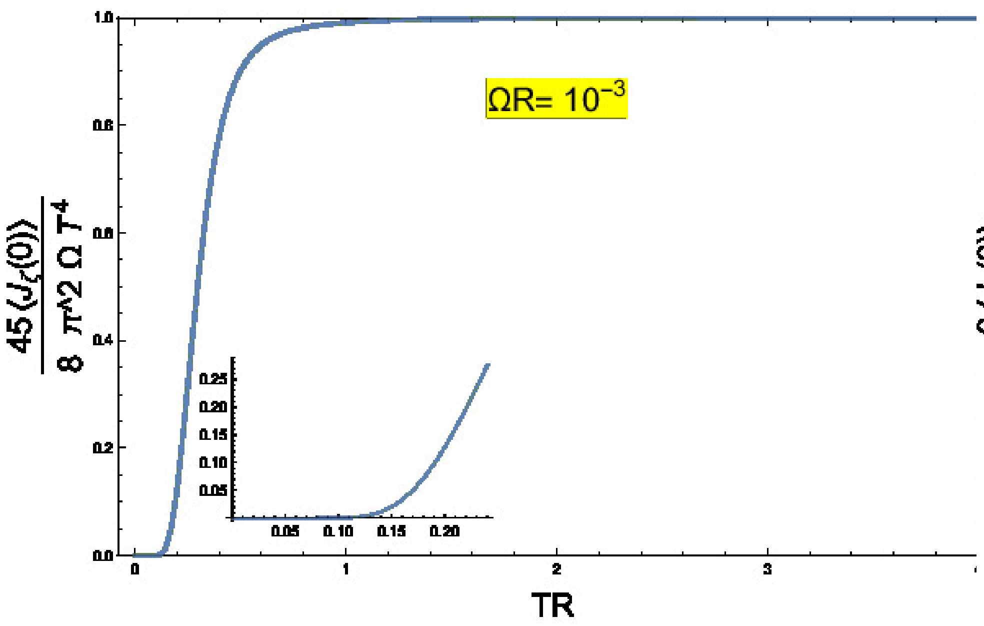


Net Helicity current

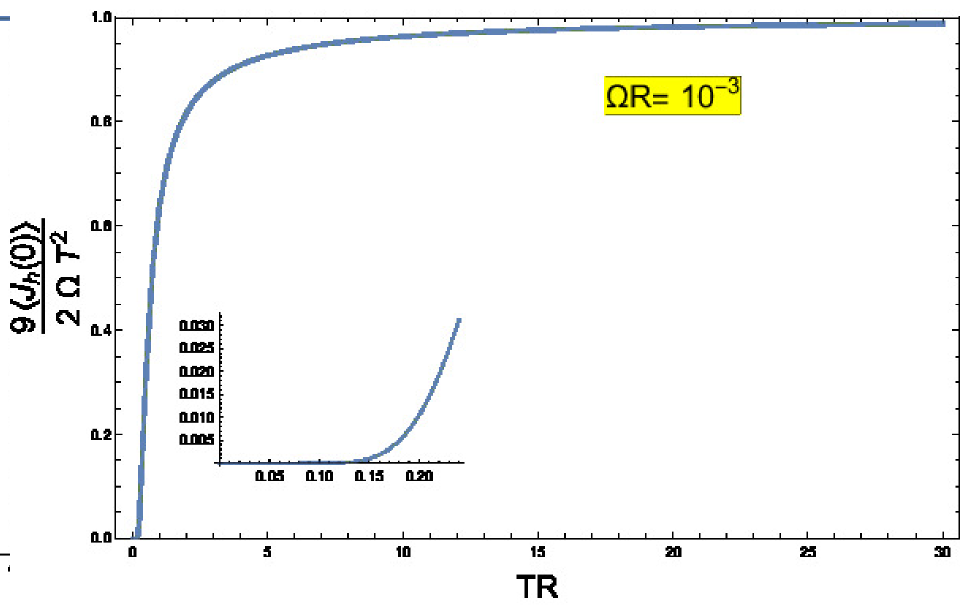


Rotation: numerics

Central Zilch current density



Central Helicity current density



Conclusion and Outlook

- Zilch is a good, gauge invariant and local measure of chirality for photons
- Rotation does induce Helicity and Zilch currents
- The picture is more complicated than for fermions
- Net currents in finite domain are only due to non-duality invariant BCs
- CVE is is still there for high T low Ω result at the center
- Questions:
    - Is there a gravitational anomaly for the Zilch?
    - Chemical potential (=gauge duality symmetry)?
    - Relation to higher spin theories?
    - Higher dimensions (e.g. 6D self-dual 3-form)?
    - Etc...