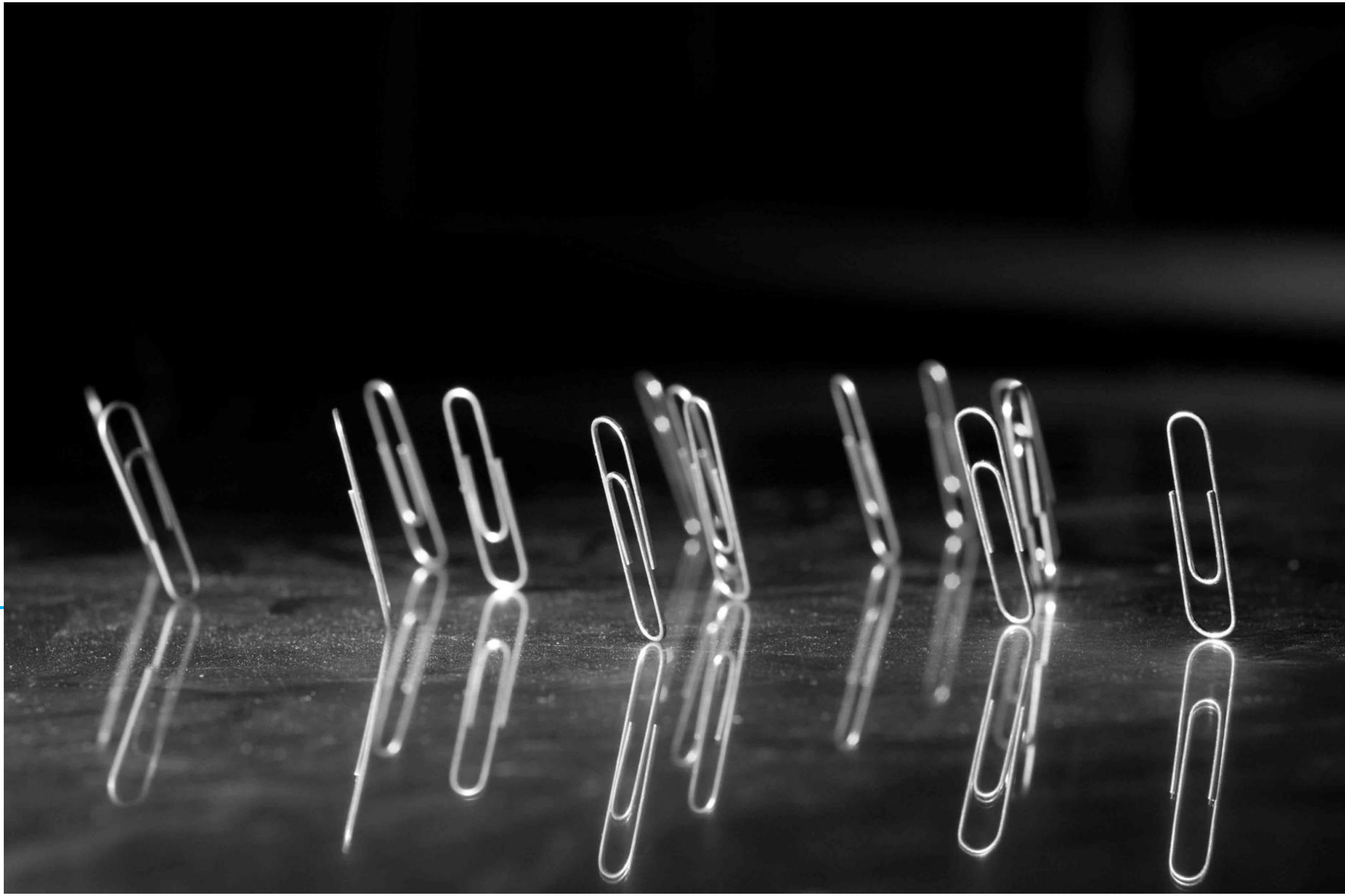
Lorentz force oscillations

Ramona Leewe,

Ken Fong,

TRIUMF



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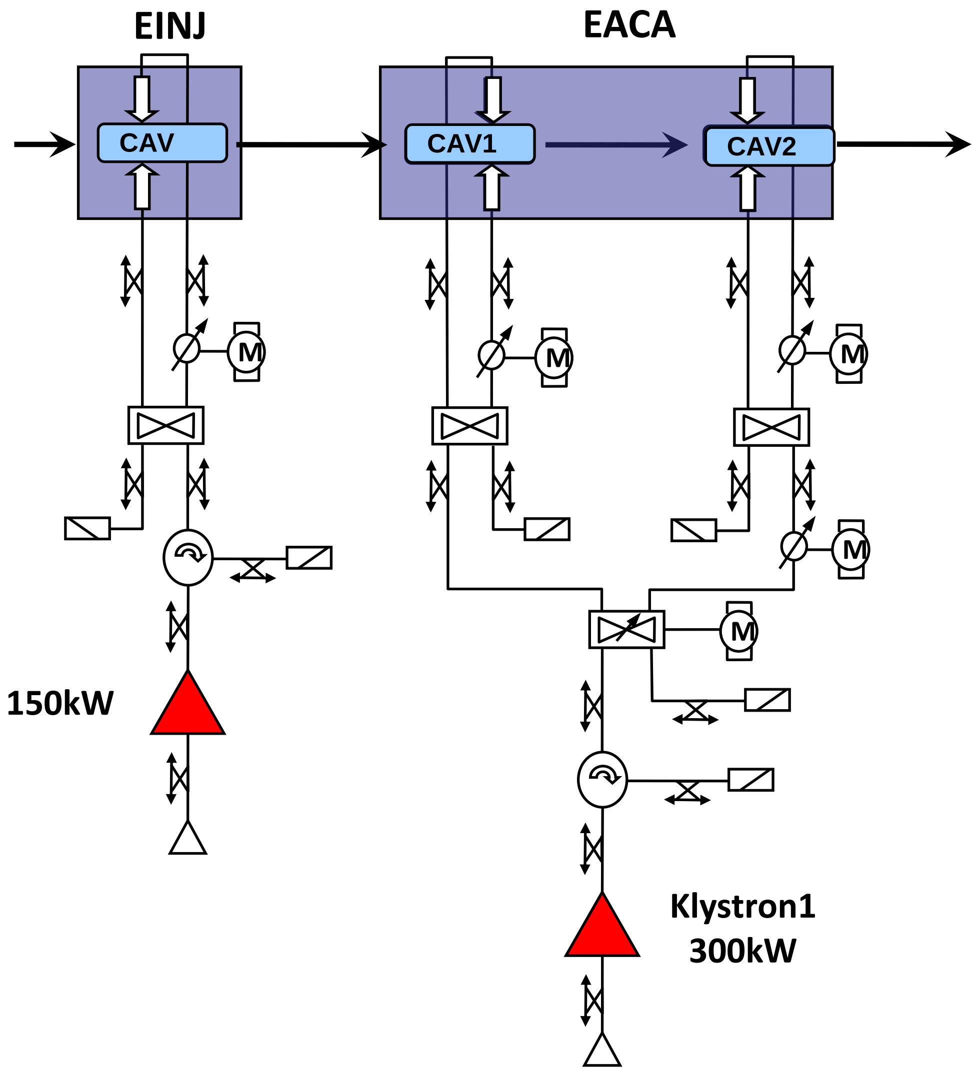
Overview

- TRIUMF’s e-Linac cavity driving configuration
- First operational experiences
- Parametric oscillations
- Lorentz oscillation problem formulation
- Nonlinear system stability analysis
- Parametric oscillation in cavity with lorentz force
- Simulations
- Conclusion

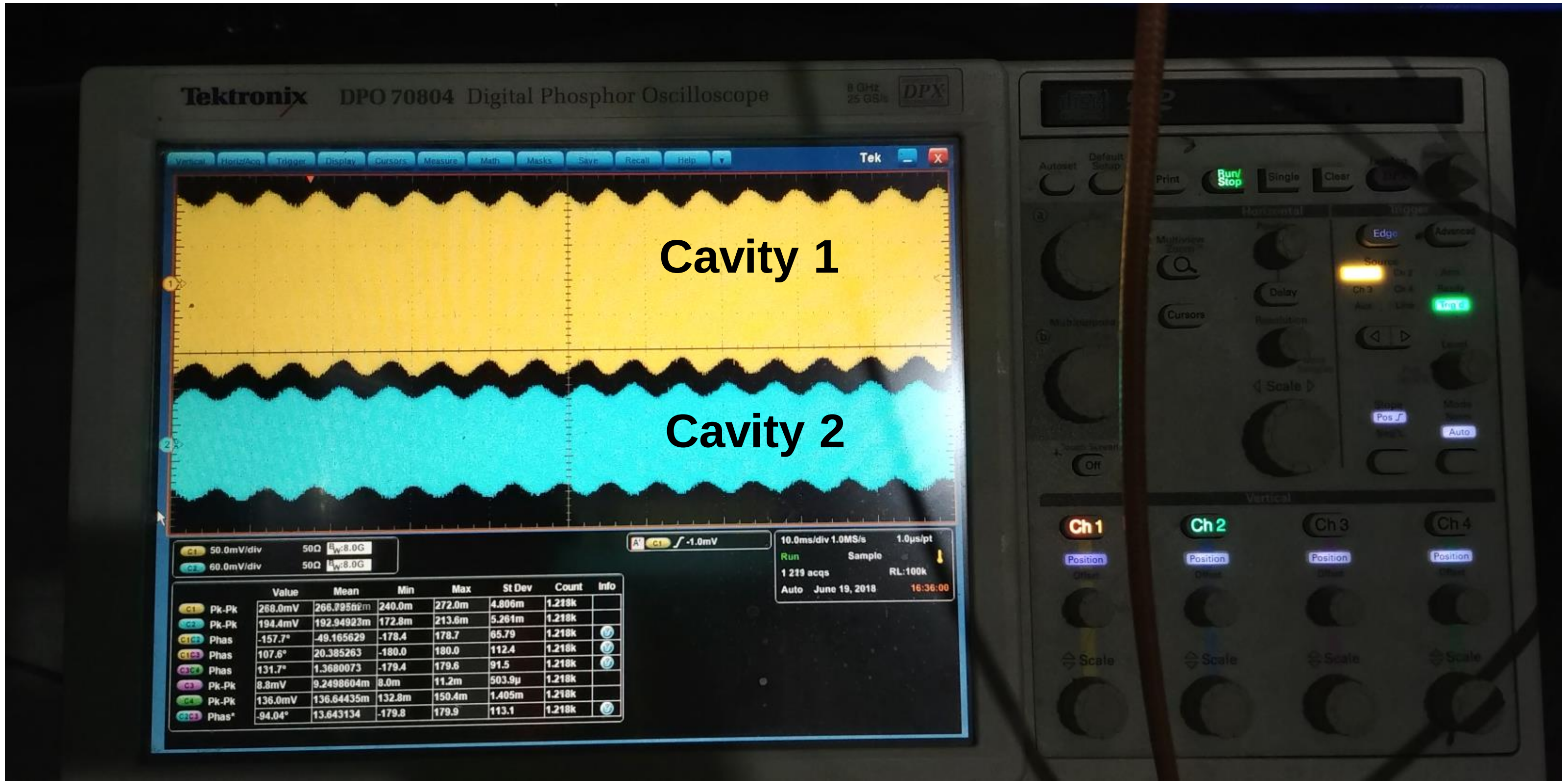
TRIUMF’s e-linac driving configuration



- TRIUMF’s e-Linac acceleration cryomudule, consists of 2 9 cell cavities and is operated with a single klystron in CW mode and vector sum control.

Operational experience

- Amplitude oscillation in both cavities (operational gradient dependent)



Time to grow oscillations $\approx 6 - 10\ seconds$

Parametric oscillations

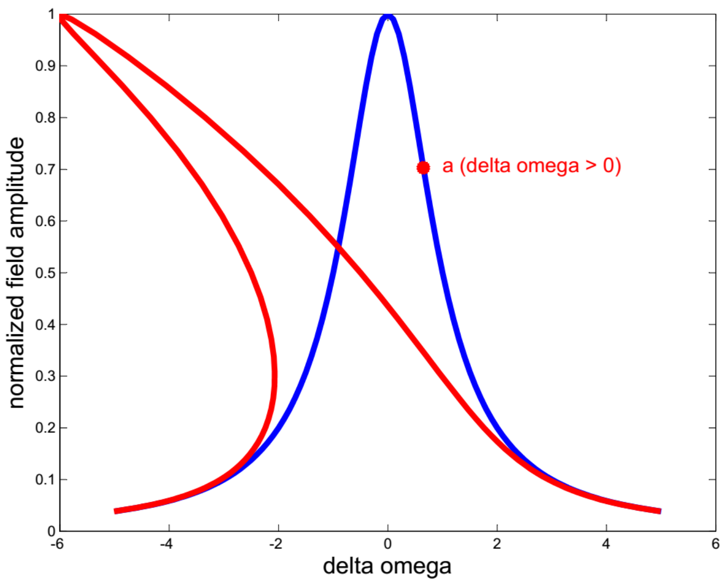
- “A parametric oscillator is a driven harmonic oscillator in which the oscillations are driven by varying some parameter of the system frequency, typically different from the natural frequency of the oscillator”, *Wikipedia*

- Examples:
  - Kid on a swing
  - Roll instabilities of ships

Lorentz oscillation problem formulation

- **Lorentz force is driving a cavity as spring mass system**
  - $m\ddot{x} + c\dot{x} + kx = F$
  - $F = -K_L V_{acc}^2$
  - $V_{acc} = \gamma V_g \cos(\varphi)$
  - $\varphi = tan^{-1}(\tau\Delta\omega)$
  - $\Delta\omega = bx$

$K_L = lorentz\ constant$
$\gamma = coupling\ factor$
$V_g = generator\ voltage$
$\varphi = detuning\ angle$
$\tau = cavity\ time\ constant$
$b = cavity\ sensitivity$



- **Equation of motion:**

$$m\ddot{x} + c\dot{x} + kx = -K_L\gamma^2 V_g^2\{\cos^2(tan^{-1}(\tau b(a+x)) - cos^2(tan^{-1}(\tau ba))\}$$

Lorentz force at 'a'

Nonlinear system

Nonlinear system stability analysis

- System is said to be stable in the sense of Lyapunov if
  - $V(x) > 0$, $V(x)$ *Lyapunov function candidate*
  - $\dot{V}(x) < 0$

- With $x_1 = x, x_2 = \dot{x}$

$$m\ddot{x} + c\dot{x} + kx = \Lambda(x)$$
$$\Lambda(x) = Lorentz\ force$$

  - $V = \frac{k}{2}x_1^2 + \frac{1}{2}mx_2^2$
  - $\dot{V} = \Lambda(x_1)x_1 - cx_2^2$

Only stable if $\Lambda(x_1)x_1 < cx_2^2$

Stability depends on the input power and the cavity spring constant

Parametric oscillations in cavity with lorentz force

Lorentz force oscillation differential equation

- $m\ddot{x} + c\dot{x} + kx = K_L V_g^2 \gamma^2 cos^2\left(tan^{-1}\left(\tau b(a+x)\right)\right) cos^2(tan^{-1}(\tau ba))(2a+x)x$

- $m\ddot{x} + c\dot{x} + \left(k - K_L V_g^2 \gamma^2 cos^2\left(tan^{-1}\left(\tau b(a+x)\right)\right)\cos^2(\tan^{-1}(\tau ba))(2a+x)\right)x = 0$

- Assume solution: $x = \mu\cos(\omega t)$ and substitute in nonlinear part to get an approximate solution

$$\left(k - K_L V_g^2 \gamma^2 cos^2\left(tan^{-1}\left(\tau b(a + \mu\cos(\omega t))\right)\right)\cos^2(\tan^{-1}(\tau ba))(2a + \mu\cos(\omega t))\right)$$

← Time dependent resonance frequency

General differential equation for parametric oscillation

- $\ddot{x} + \beta(t)\dot{x} + \omega^2(t)x = 0$

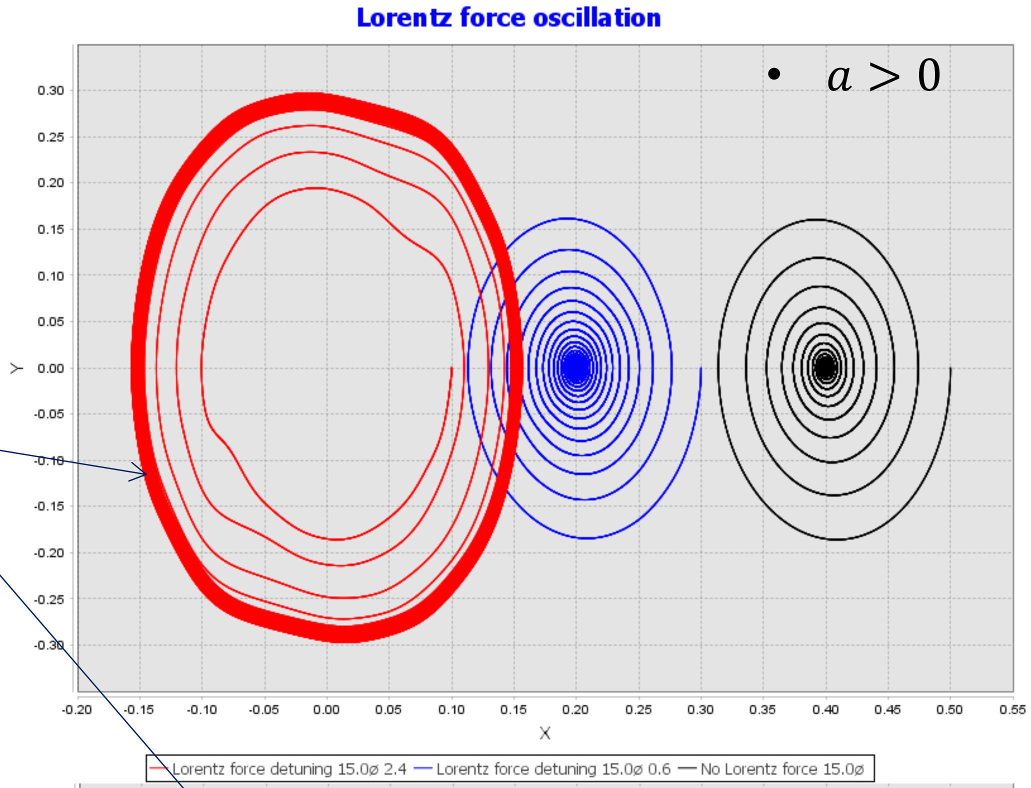
Simulation results

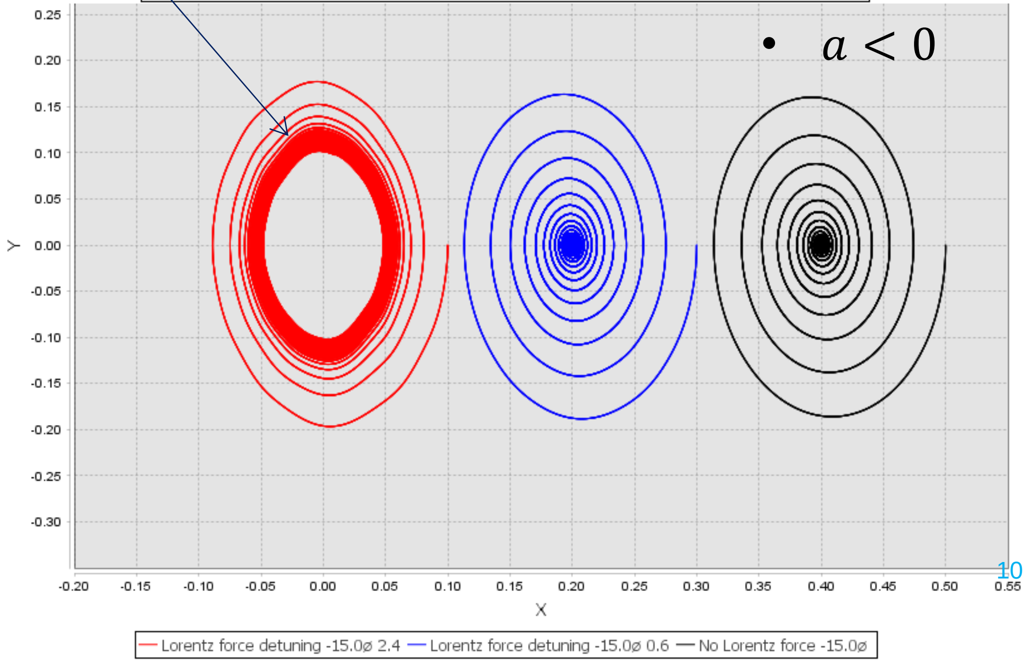
Phase portrait, $\dot{x}\ versus\ x$

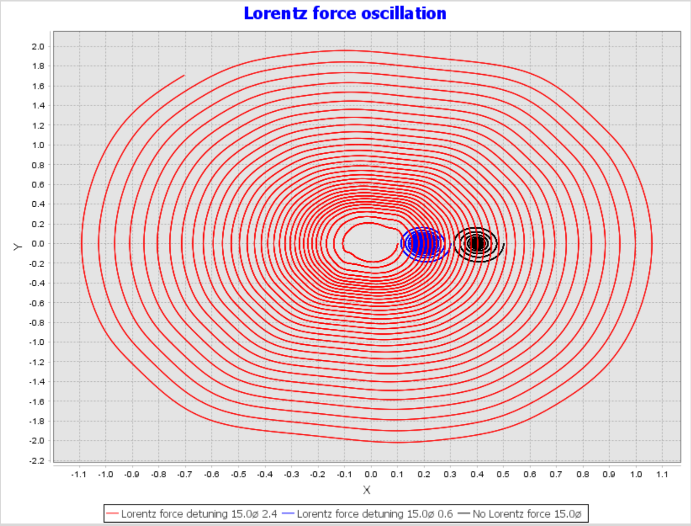
- Field amplitude = V_g
- Field amplitude = $\frac{V_g}{2}$
- No field

Existence of limit cycles

Stability is field dependent
$\Lambda(x_1)x_2 < cx_2^2$







- Field amplitude slightly increased

Conclusion and lookout

- Theory of Lorentz force has been analyzed
  - Stability conditions for stable operation are established
- Simulated system results are in agreement with system experience
  - Increasing oscillation amplitude can only be observed at high gradients
  - Stable for $\Delta\omega < 0$, unstable for $\Delta\omega > 0$

- Next steps: Analyze double cavity system in vector sum control

Thank you for your consideration!