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# Fine Freq Resolution TF Measurements of a Cavity System to Characterize High-Q Mechanical Resonances

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TF Measurments for High-Q Resonances

26 October 2018

MRCW18

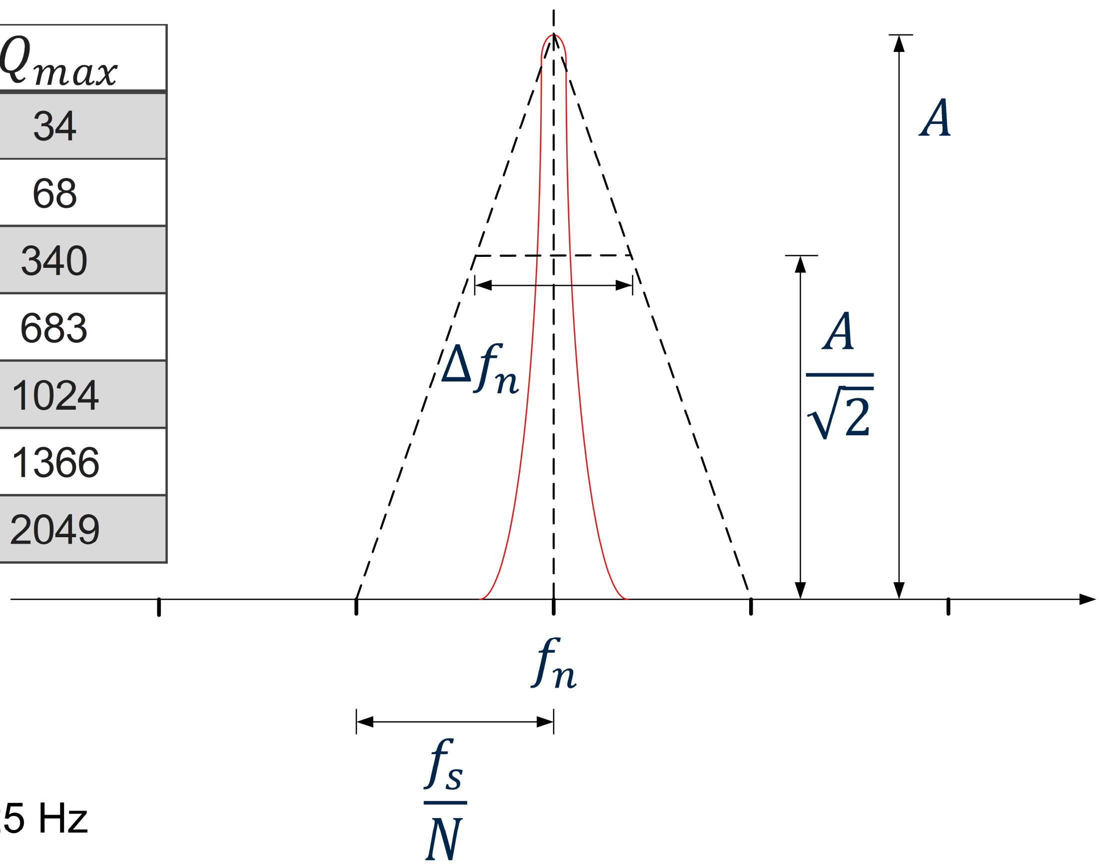
# Measurement Goals

- Use higher resolution measurements to capture high Q resonances
- Obtain frequency response function based on input output time series data
- Use System Identification Tools to create a system model
- Apply controllability and observability tests to see if system is controllable

# Frequency resolution vs Detectable Resonance Q

$$Q_{max} = \frac{f_n}{\Delta f_n} = \frac{N f_n}{2 f_s (1 - 1/\sqrt{2})}$$

| $f_n$ | $Q_{max}$ |
|---|---|
| 5 | 34 |
| 10 | 68 |
| 50 | 340 |
| 100 | 683 |
| 150 | 1024 |
| 200 | 1366 |
| 300 | 2049 |

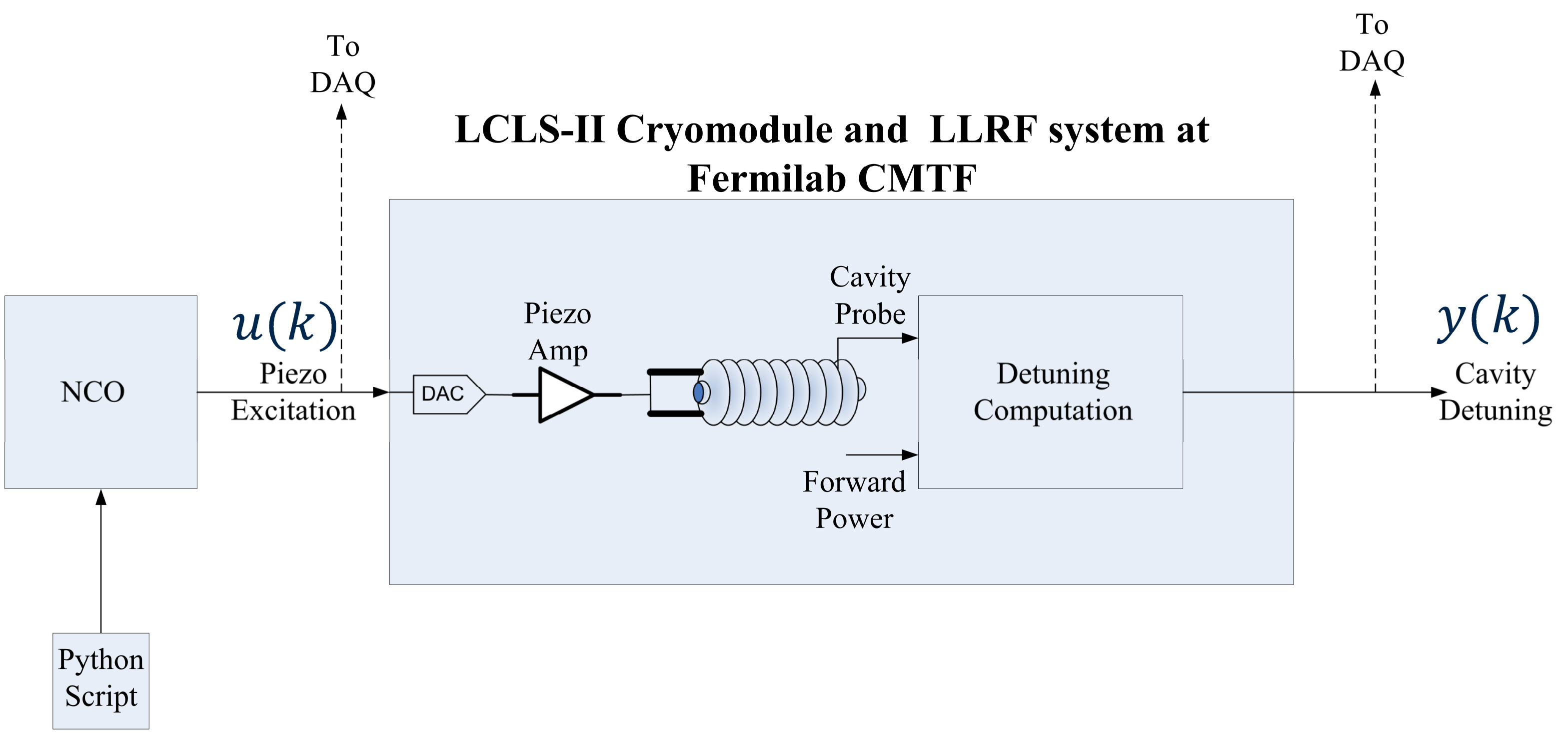



$f_s$ =2 kHz

N = 8000

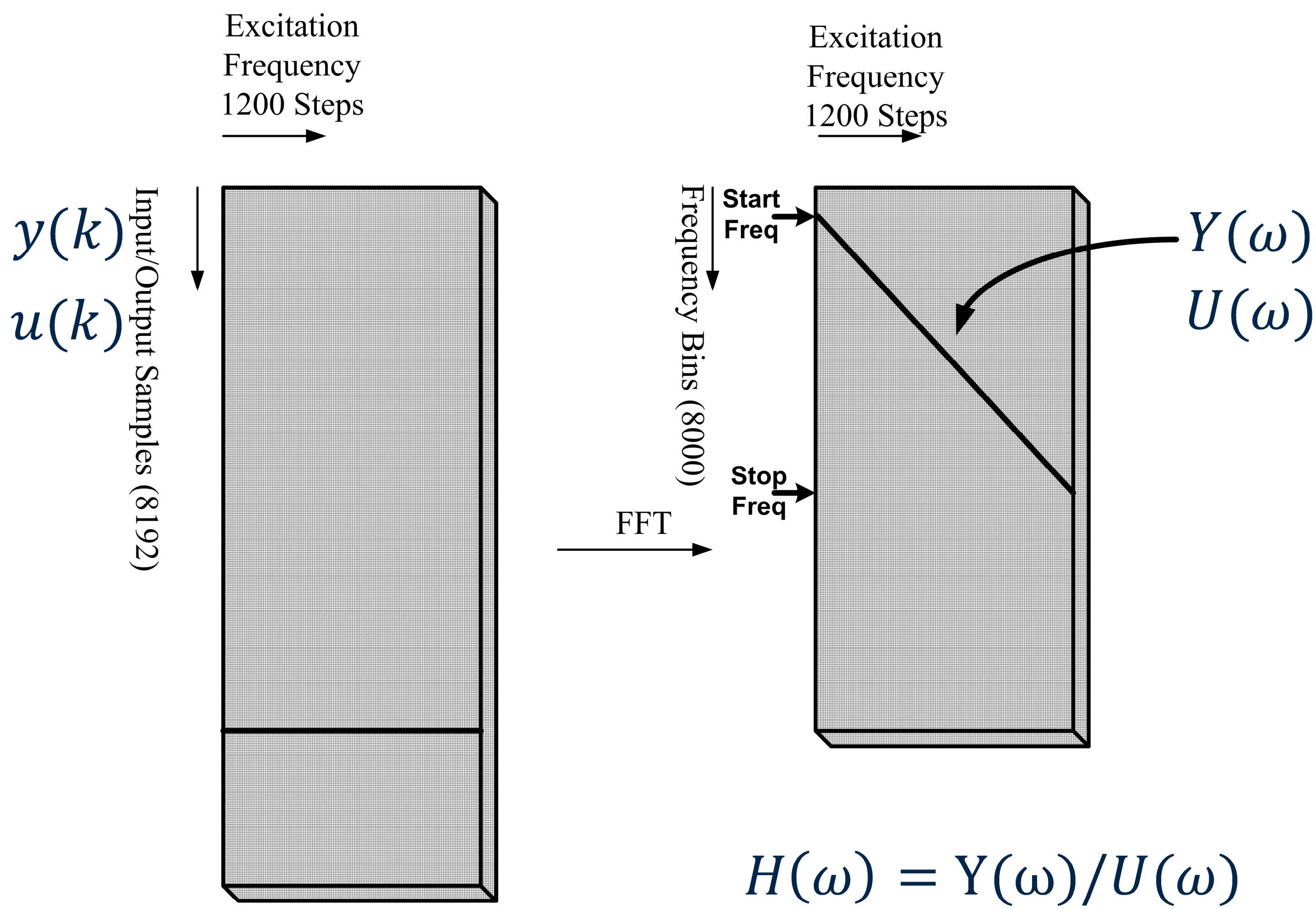
Freq Step = 0.25 Hz

# Test Setup



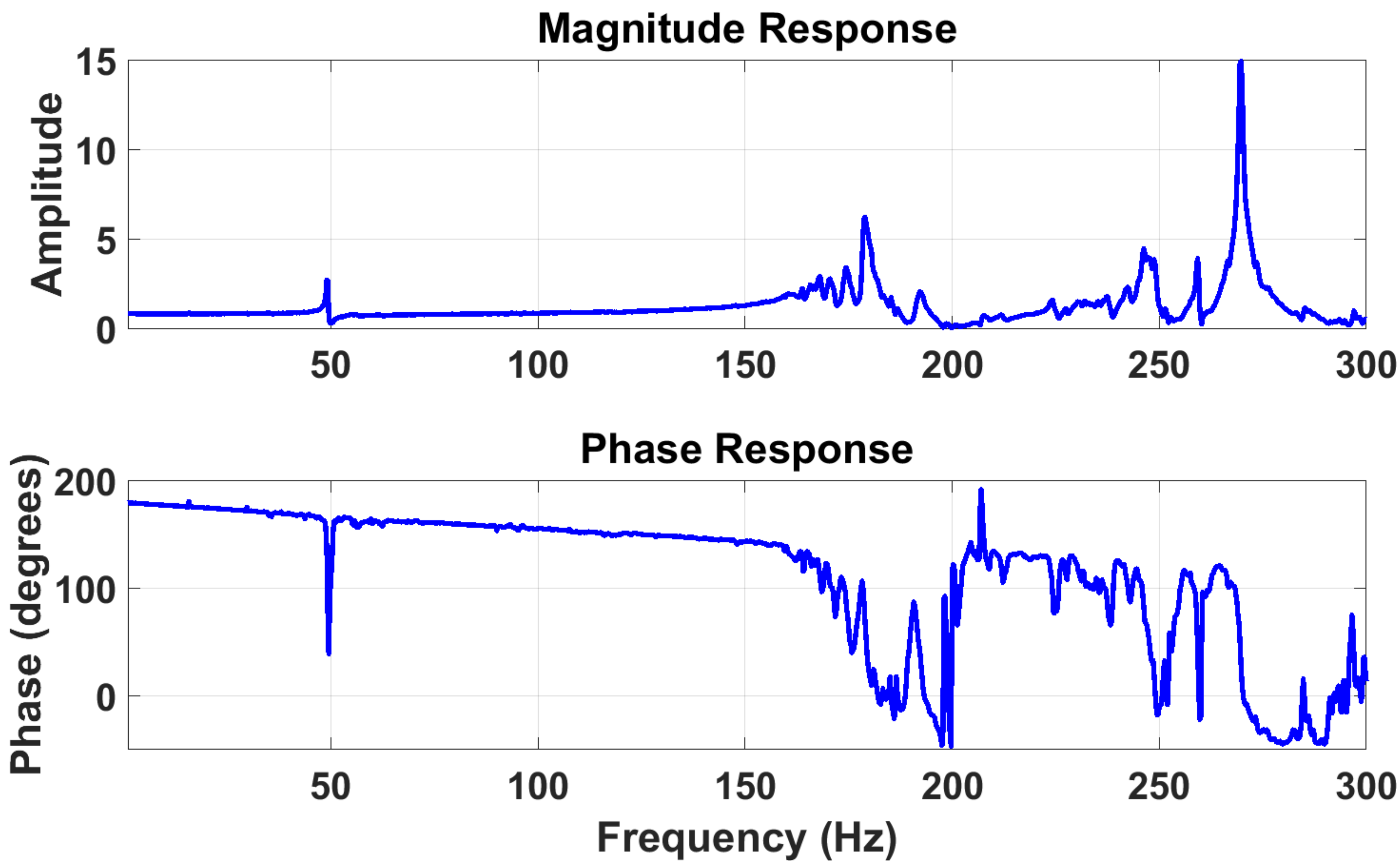

Start Frequency = 1 Hz
Stop Frequency = 300 Hz
Frequency Step Size = 0.25 Hz
Dwell Time ~ 6 sec

# Transfer Function Computation

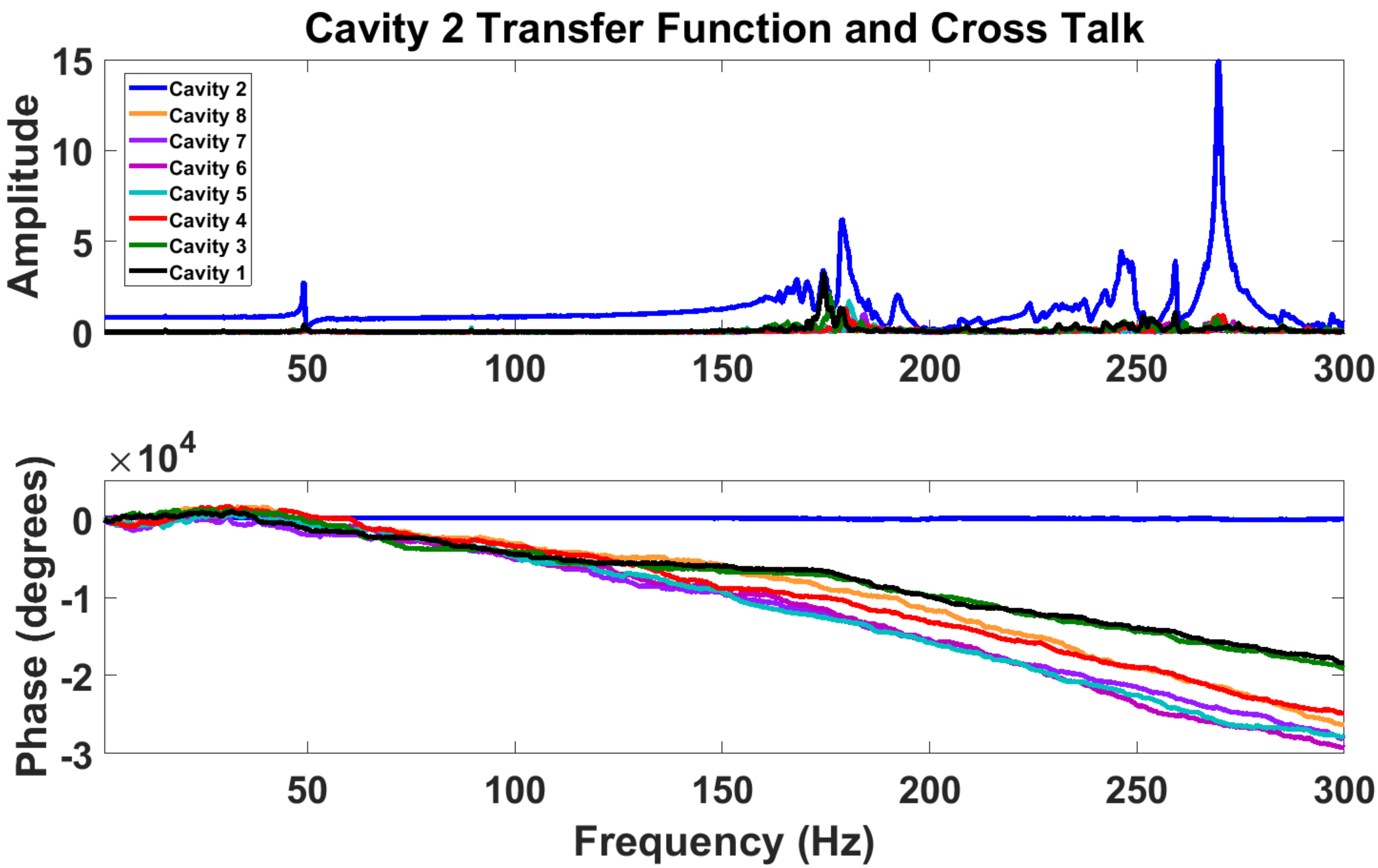



$$H(\omega) = \mathrm{Y}(\omega)/U(\omega)$$

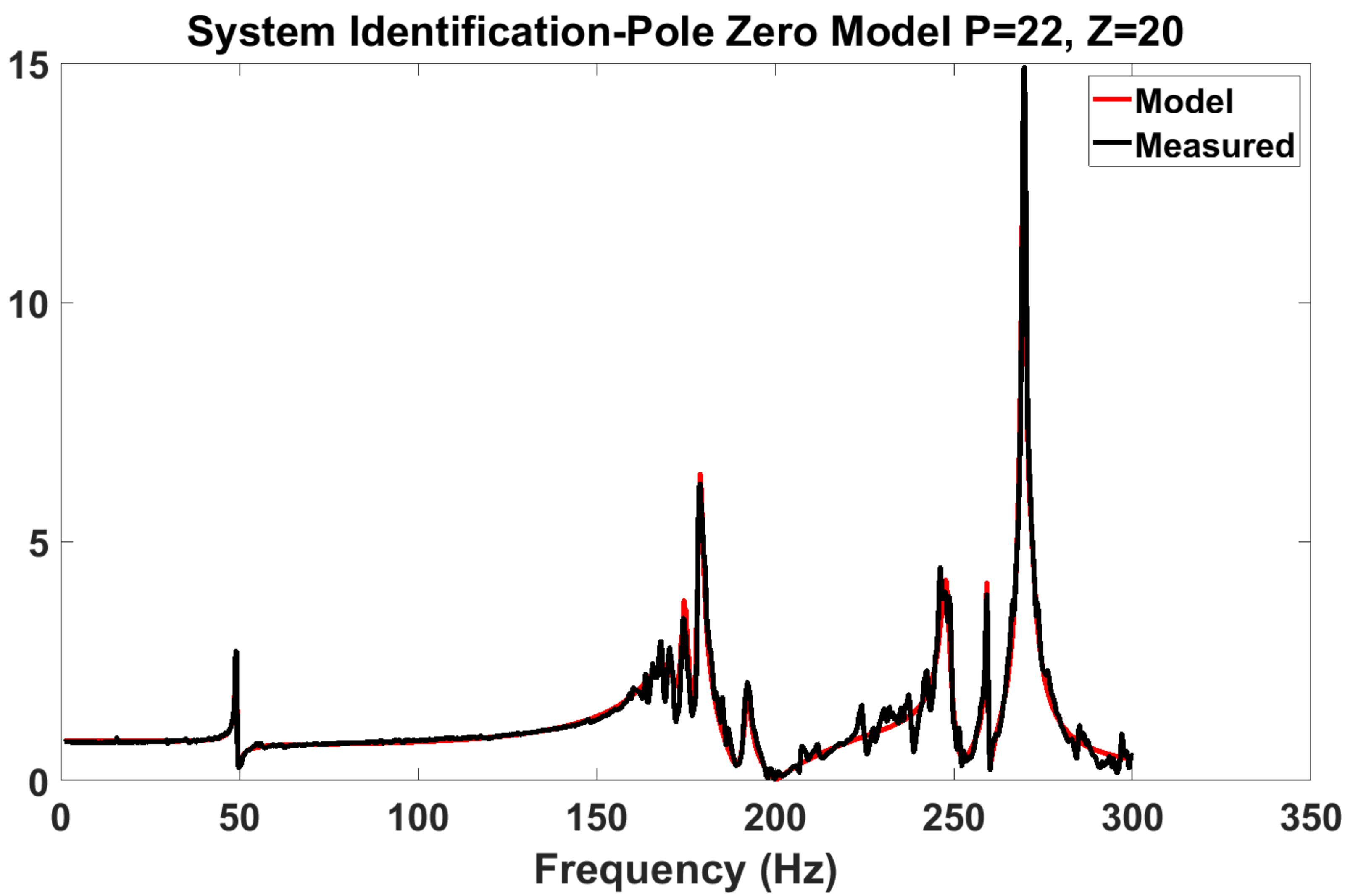
# Measured Frequency Response



# Cross Talk Frequency Response



# Model Fit using System Identification



# State Space Model

$$x(k+1) = Ax(k) + Bu(k), \qquad A \rightarrow n \times n, B \rightarrow n \times r$$

$$y(k) = Cx(k) + Du(k), \qquad C \rightarrow m \times n, D \rightarrow m \times r$$

- A SISO system has $r, m = 1$
- Eigen values of A represent the poles of the system
- The imaginary part represents resonant frequencies $f_n$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 49 | 172 | 174 | 180 | 192 | 223 | 248 | 259 | 270 | 271 |

# System Controllability/Observability

$$P = [B \;\vdots\; AB \;\vdots\; A^2B \;\vdots\; \cdots \;\vdots\; A^{n-1}B] \;, \qquad P \to n \times rn$$

$$Q = [\, C^T \;\vdots\; A^T C^T \;\vdots\; (A^T)^2 C^T \;\vdots\; \cdots \;\vdots\; (A^T)^{n-1} C^T ] \;, \qquad Q \to n \times mn$$

- System is completely controllable if and only if the rank of P is n
- If a system is controllable, it can be driven from an initial state x(0) to zero in a finite number of steps N.

- System is completely observable if and only if the rank of Q is n
- If a system is observable, its state x(0) can be determined from the outputs y[0,N] where N is finite

# Example

$$A = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathrm{C} = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Eigen values of A are -1, -3 and -5, the state equations can be represented in the canonical form with a co-ordinate transformation $q = Mx$ , the state equations can be written as

$$\dot{q} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} \mathrm{q} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{u}, \qquad y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathrm{q},$$

Mode 1 is not controllable and mode 3 is not observable

# Controllability/Observability Test

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15.39539 | -7.21418 | 4.364227 | -3.81035 | 2.542323 | -2.67694 | 2.261203 | -1.5379 | 1.66352 | -1.36785 | 0.728496 | -0.031 | -0.96989 | 1.296271 | -1.09175 | 0.68342 | -0.66065 | 0.493051 | -0.27727 | 0.22277 | -0.11461 | 0.057019 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.25 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.25 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.125 | 0 |

$$P = [B \;\vdots\; AB \;\vdots\; A^2B \;\vdots\; \cdots \;\vdots\; A^{21}B]\ , \qquad P \to 22 \times 22$$

$$Q = [\,C^T \vdots\; A^T C^T \vdots\; (A^T)^2 C^T \;\vdots\; \cdots \;\vdots\; (A^T)^{21} C^T]\ , \qquad Q \to 22 \times 22$$

$$Rank(P) = 22, \qquad System\ Controllable$$

$$Rank(Q) = 14, \qquad System\ not\ Observable$$

# Conclusions

- Piezo actuator in the longitudinal axis cannot directly cancel out vibrations excited in the transverse axis
- Even if transverse vibrational modes are uncontrollable, the detuning that they create may be counteracted by the longitudinal piezo actuator
- The controllability tests on the model extracted indicates that it should be possible to build a stable state feedback system for this model.
- Controller design and simulation needs to be done to validate the model and evaluate the performance