Quark and Gluon Contributions to Proton Mass and Spin

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Center for Frontiers
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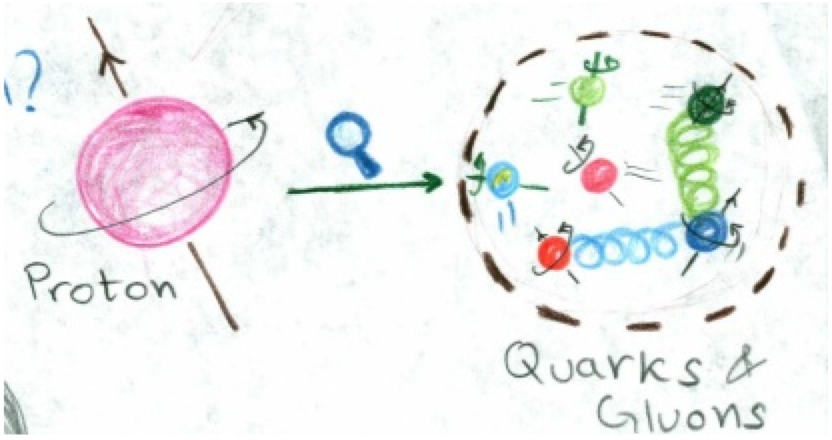
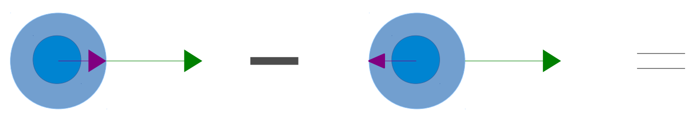


Outline

- Partonic Orbital Angular Momentum
- Wandzura Wilczek and genuine twist three contributions to twist three GPDs
- QCD trace anomaly and the mass of the proton
- Measuring these in experiment

Proton Spin Crisis

 $g_1^P(x)$ Quark Spin Contribution

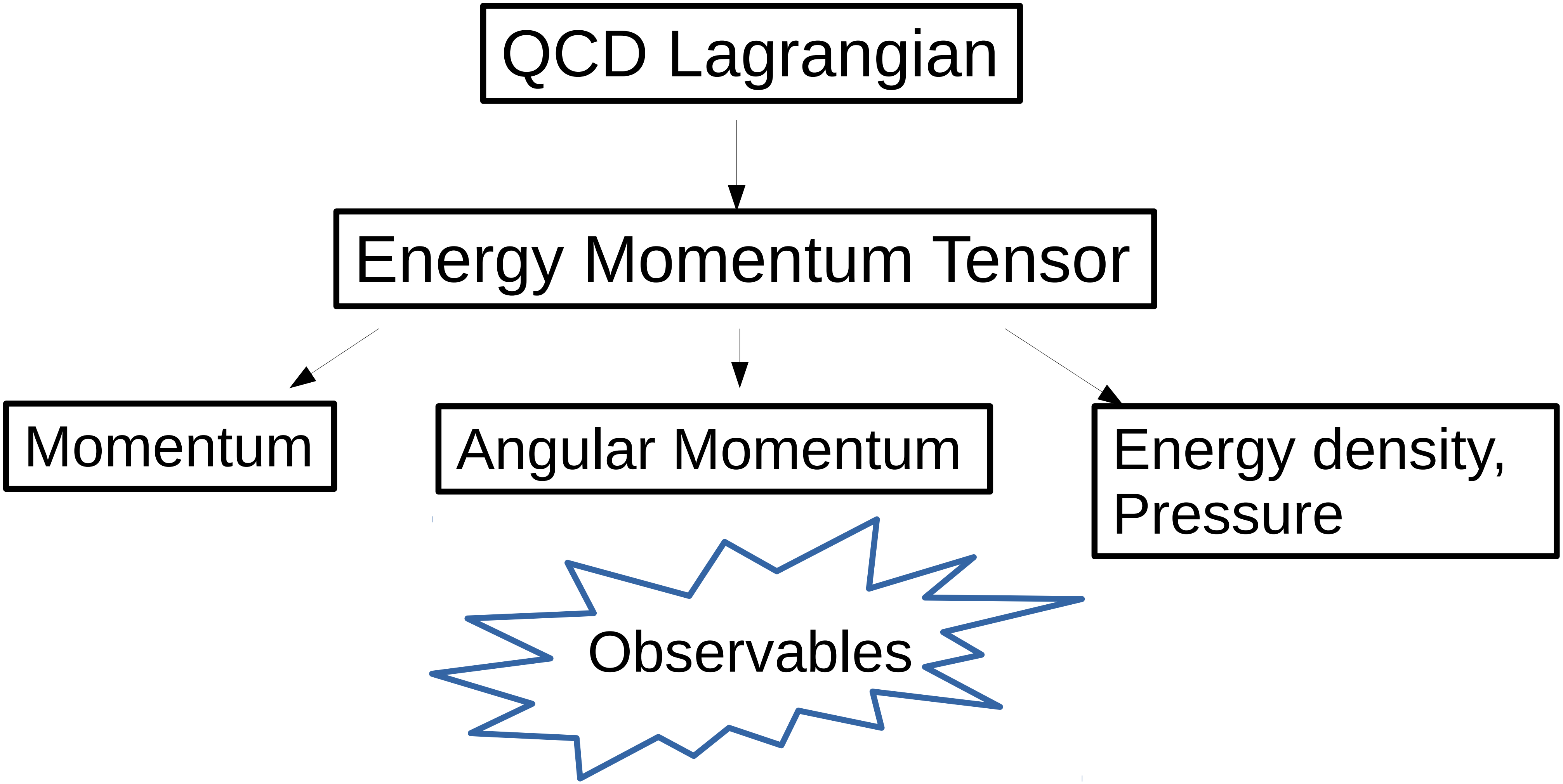
$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in 1980s to be small, present values about 30% of total !!

What are other sources ?

Partonic Orbital Angular Momentum

QCD Energy Momentum Tensor



Deeply Virtual Compton Scattering, moments of GPDs etc.

GPD based definition of Angular Momentum

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x \left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

$$\vec{J}_q = \int d^3x \psi^\dagger \left[\vec{\gamma}\gamma_5 + \vec{x} \times i\vec{D}\right]\psi \qquad \vec{J}_g = \int d^3x \left(\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right)$$

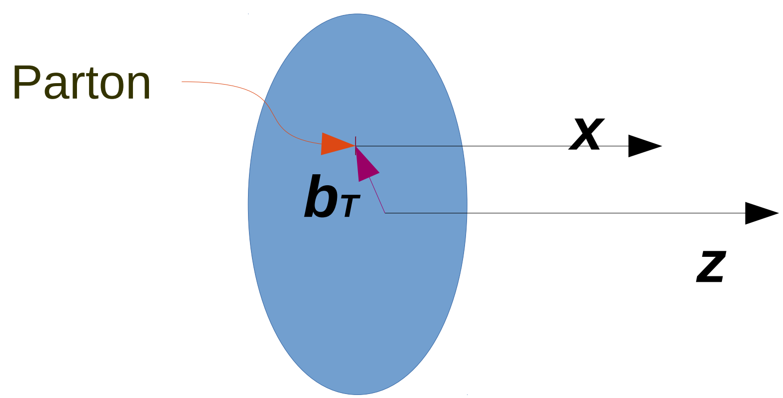
$$J_q = \frac{1}{2}\int dxx(H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

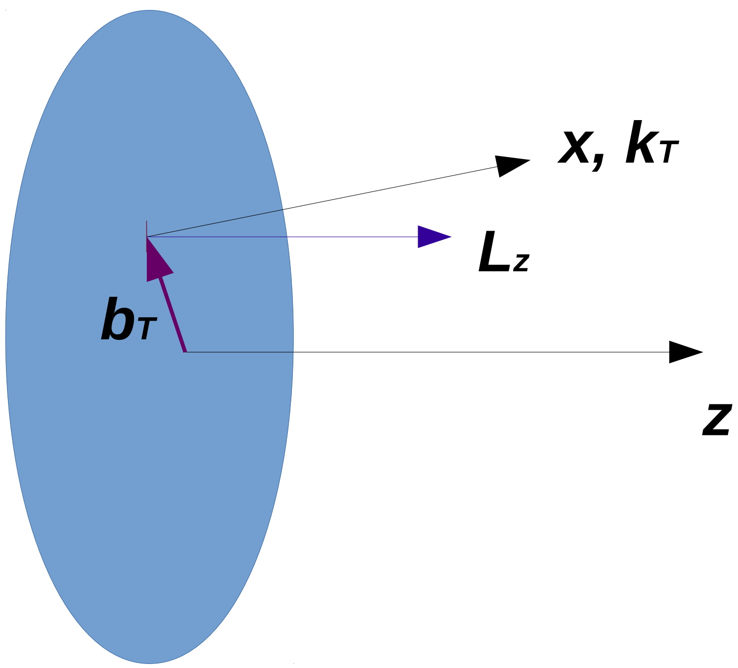
$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

OAM Total Spin



Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z} = \mathbf{b}_T \times \mathbf{k}_T$



$$W_{\Lambda,\Lambda'}^{[\gamma^+]} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel, JHEP 0908 (2009)

The Two Definitions

- Weighted average of $b_T \times k_T$

GTMD

$$L_z = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad\qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

The Two Definitions

$F_{14}^{(1)}$

- Weighted average of $b_T \times k_T$

GTMD

$$L_z = -\int dx \boxed{\int d^2k_T \frac{k_T^2}{M^2} F_{14}}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx\, x(H_q + E_q) \qquad\qquad \frac{1}{2}\int_{-1}^{1} dx\, \tilde{H}_q$$

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

AR, Engelhardt and Liuti PRD 98 (2018)

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

Wandzura Wilczek Relations

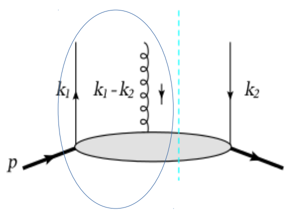
$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three
vector GPD

Twist two

Axial vector GPD
contributes to a vector
GPD

Genuine Tw 3



AR, Engelhardt and Liuti PRD 98 (2018)

Hatta and Yoshida, JHEP (1210), 2012

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

Twist three
PDF

Twist two

Genuine Tw 3

Understanding the mass decomposition of the proton

Mass decomposition of the proton

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{q,kin} + T^{\mu\nu}_{g,kin}} + T^{\mu\nu}_{m} + T^{\mu\nu}_{a}$$

Traceless

X Ji (1995)

$$M = \frac{\langle P| \int d^3\mathbf{x} T^{00}(0,\mathbf{x}) |P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$

Rest frame

$$\langle \bar{T}^{00}\rangle = 3/4M \quad \longleftarrow \quad \text{Traceless}$$

$$\langle \hat{T}^{00}\rangle = 1/4M \quad \longleftarrow \quad \text{Trace part}$$

Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

- The full energy momentum tensor is a conserved quantity and is scale independent.
- The separate contributions from the quarks and gluons on the other hand are not and do depend on the renormalization scale.

$$\langle P' | (T^{\mu\nu})_R | P \rangle =$$

$$\bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

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$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

* $$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

Trace

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

By studying the Gravitational form factors A and C̄ we will know the quark and gluon contributions to the trace anomaly separately.

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

$$\Big\downarrow \text{Trace}$$

$$T^{\mu}_{\mu} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

$$\Big\downarrow \text{Trace}$$

$$T^{\mu}_{\mu} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

$$(T^{\alpha}_{g\alpha})_R = A^R_g(\mu) + 4\bar{C}^R_g(\mu)$$

$$= \frac{1}{2M^2}\langle P|\frac{\alpha_s}{2\pi}\left(-\frac{11C_A}{6}\underline{(F^2)_R} + \frac{14C_F}{3}\underline{(m\bar{\psi}\psi)_R}\right)|P\rangle$$

gluons

$$(T^{\alpha}_{q\alpha})_R = A^R_q(\mu) + 4\bar{C}^R_q(\mu)$$

$$= \frac{1}{2M^2}\langle P|(m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}\underline{(F^2)_R} + \frac{4C_F}{3}\underline{(m\bar{\psi}\psi)_R}\right)|P\rangle$$

quarks

Y Hatta, AR, K Tanaka arxiv:1810.05116

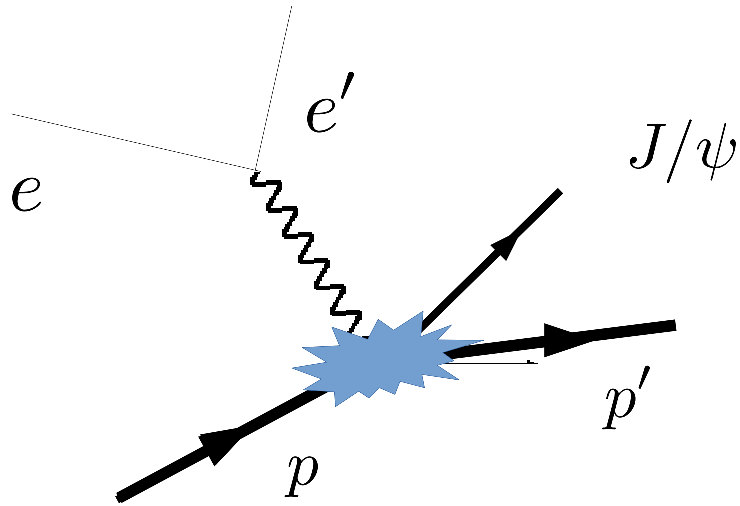
Experimental measurements

The production of a heavy quarkonium near threshold in electron-proton scattering is connected to the origin of the proton mass via the QCD trace anomaly.

D.E Kharzeev (1995)

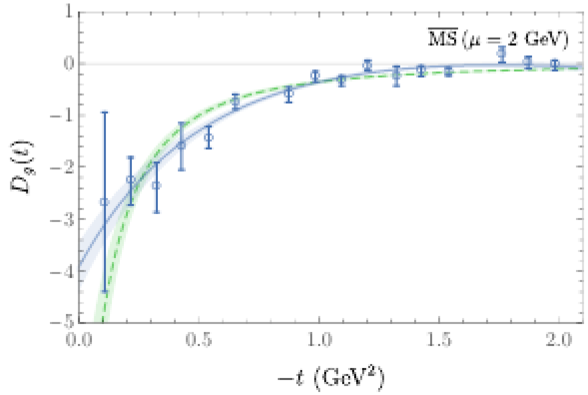
$$ep \to e'\gamma^* p \to e'p'J/\psi$$

Y Hatta, DL Yang PRD98 (2018)



In an actual experiment $p' - p \equiv t \neq 0$

Calculate cross-section using input from latest lattice QCD calculations of gluon gravitational form factors.



Detmold and Shanahan arxiv:1810:04626

Thanks!

Quark and gluon contributions to the trace anomaly

$$T^{\alpha}_{\alpha} = -2\epsilon \frac{F^2}{4} + m\bar{\psi}\psi$$

$$\Big\downarrow \text{ renormalization}$$

$$-2\epsilon \frac{F^2}{4} = \frac{\beta}{2g} F^2_R + \gamma_m (m\bar{\psi}\psi)_R$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

$$\Big\downarrow \text{ Trace}$$

$$T^{\mu}_{\mu} = (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g} F^2$$

$$T^{\alpha}_{g\alpha} = \frac{\beta}{2g} (F^2)_R + \gamma_m (m\bar{\psi}\psi)_R$$

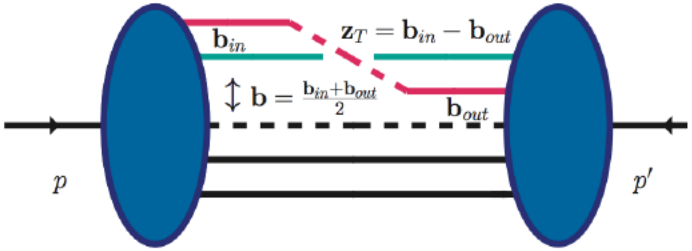
$$T^{\alpha}_{q\alpha} = m(\bar{\psi}\psi)_R$$

Equations of Motion Relations

How do we obtain these ?

$$\begin{aligned}\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D}-m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial}+g\not{A}-m)\psi(z_{out})=0,\\ \bar{\psi}(z_{in})(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma_5\mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}}-g\not{A}+m)i\sigma^{i+}\gamma_5\mathcal{U}=0\end{aligned}$$

$$b=\frac{z_{in}+z_{out}}{2}, \qquad z=z_{in}-z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta}\int dz^- d^2z_T e^{-ik\cdot z}\langle p',\Lambda'|\bar{\psi}\left[(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}}-m)\right]\psi|p,\Lambda\rangle=0$$

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3 **Twist 2** **Genuine Twist 3 (explicit gluon)**

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p',\Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p,\Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p',\Lambda'|\bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0)|p,\Lambda\rangle$$

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+ z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- The same set of As describe the whole vector sector.

γ^+

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F \right] \qquad J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{x P^2}{M^2} \right) \left(A_8^F + x A_9^F \right)$$

γ_T^i

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2} \right) A_9^F - \sigma' A_5^F - A_6^F \right]$$

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

Distribution of OAM in x !

k_T^2 moment of a twist two function

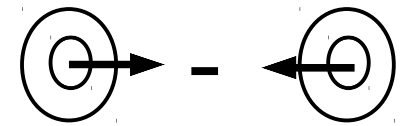
Twist three function

Generalized Lorentz Invariance Relations

Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

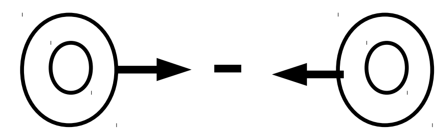


Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



Intrinsic transverse momentum

Quark gluon interactions

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T(\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3

Twist 2

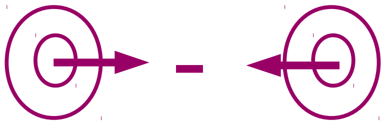
Genuine Twist 3 (explicit gluon)

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx \underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

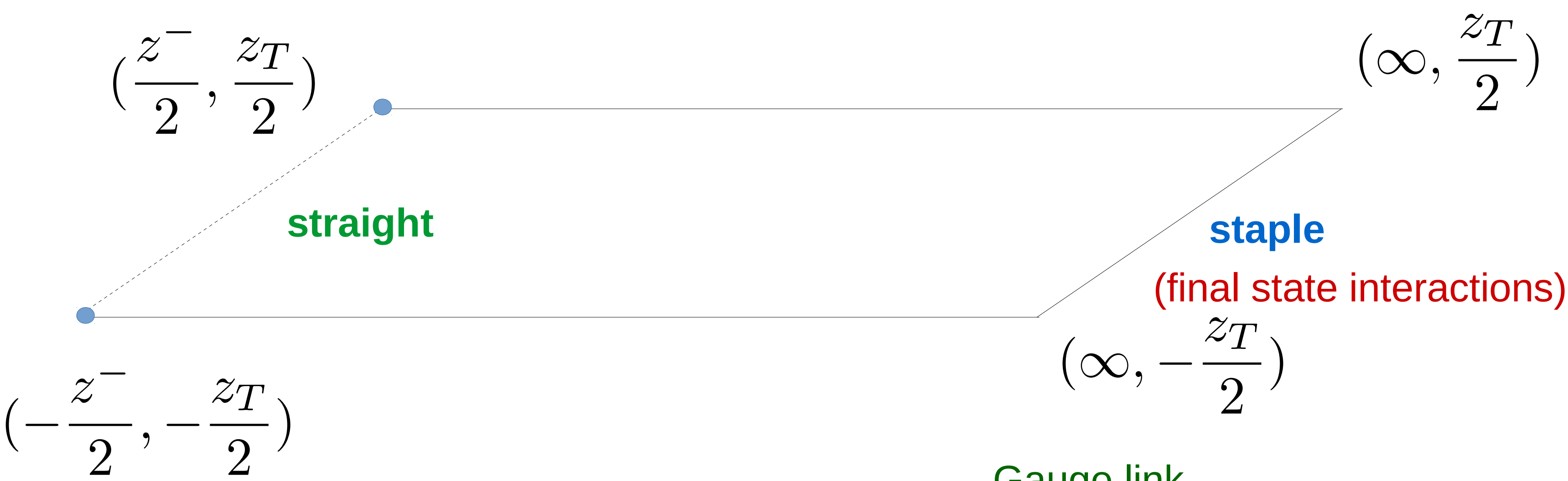
mass term

$$\int dx \, \underline{x}^2 \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)}$$
$$- \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx \, x \int d^2 k_T \, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p, \Lambda\rangle$$

Staple gauge link



Gauge link

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Non local operator

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E + \underline{\mathcal{A}_{F_{14}}}$$

LIR violating term

$$\begin{aligned}\mathcal{A}_{F_{14}}(x) &\equiv v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}+xA_{12})+A_{14}\right.\\ &+ \left.\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8}{\partial(k\cdot v)}+x\frac{\partial A_9}{\partial(k\cdot v)}\right)\right]\\ &= \frac{dF_{14}^{(1)}}{dx}-\left.\frac{dF_{14}^{(1)}}{dx}\right|_{v=0} \qquad (1)\end{aligned}$$

$$F_{14}^{(1)}-\left.F_{14}^{(1)}\right|_{v=0}=\mathcal{M}_{F_{14}}-\mathcal{M}_{F_{14}}|_{v=0} \quad (1)$$

$$\mathcal{A}_{F_{14}}(x)=\frac{d}{dx}\left(\mathcal{M}_{F_{14}}-\mathcal{M}_{F_{14}}|_{v=0}\right) \quad (1)$$

$$-\int dx\left.\left(F_{14}^{(1)}-\left.F_{14}^{(1)}\right|_{v=0}\right)\right|_{\Delta_T=0} = \qquad (1)$$

$$-\frac{\partial}{\partial\Delta^i}i\epsilon^{ij}gv^-\frac{1}{2P^+}\int_0^1 ds\,\left.\langle p',+|\bar{\psi}(0)\gamma^+U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,+\rangle\right|_{\Delta_T=0},$$