Exclusive diffractive processes in high energy *eA* collisions

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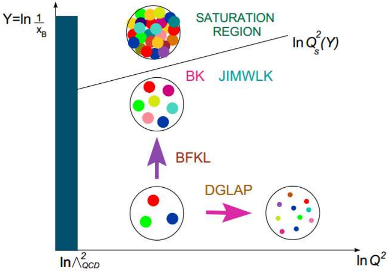
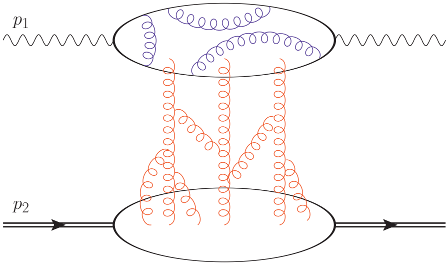
The shockwave description of the Color Glass Condensate



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

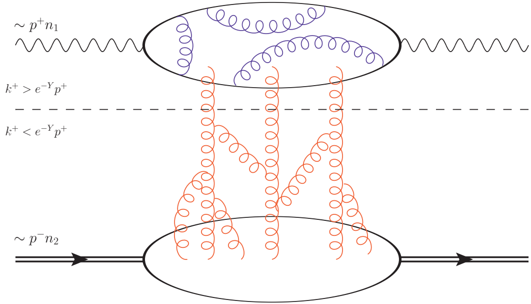
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

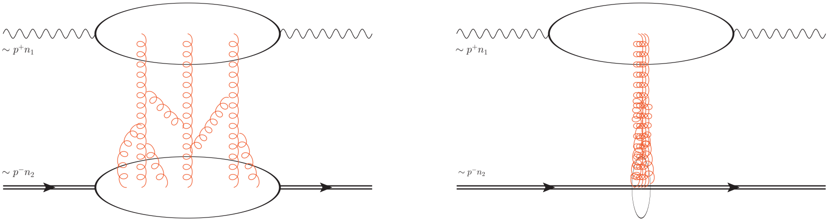


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$$e^\eta = e^{-Y} \ll 1$$

Large longitudinal boost to the projectile frame

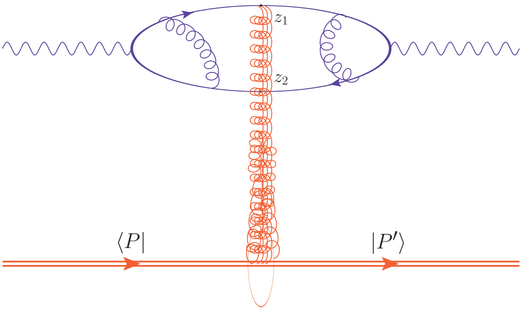


Large longitudinal boost $\Lambda \propto \sqrt{s}\; b^\mu(x) \rightarrow b^-(x)\, n_2^\mu \simeq \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu$
(*Shockwave* approximation)

Multiple interactions with the target can be resummed into path-ordered Wilson lines attached to each parton crossing lightcone time 0:

$$\tilde{U}^\eta(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})} U^\eta_{\vec{z}}, \quad U_i^\eta = U^\eta_{\vec{z}_i} = P e^{ig \int b_\eta^-(z_i^+, \vec{z}_i)\, dz_i^+}$$

Factorized picture



Factorized amplitude

$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

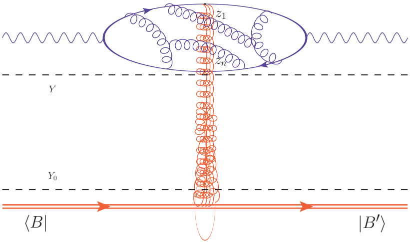
Mean field approximation (large N_C)
$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



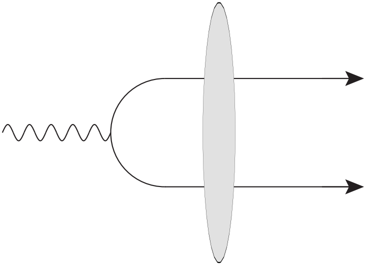
$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n) \times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction probes the $b_\perp$-dependent, off-diagonal part of the non-perturbative scattering amplitude

Exclusive diffractive dijet production

Exclusive diffractive dijet production

LO diagram for diffractive dijet production



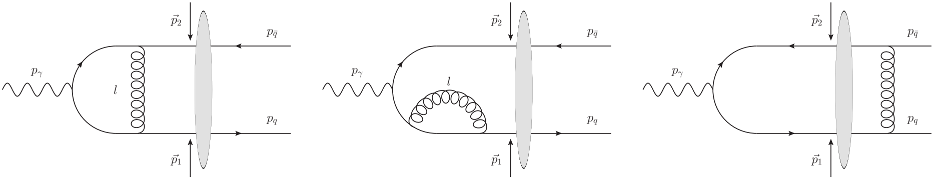
$$\mathcal{A} = \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_0(\vec{p}_1, \vec{p}_2) \times \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2\, e^{-i(\vec{p}_1\cdot\vec{z}_1)-i(\vec{p}_2\cdot\vec{z}_2)}\left[\frac{1}{N_c}\mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1\right]$$

Probes the Dipole Wigner distribution [Hatta, Xiao, Yuan]

First kind of virtual corrections

NLO dipole diagrams



$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Second kind of virtual corrections

NLO double dipole corrections

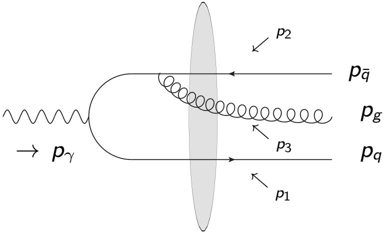
$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3\, \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

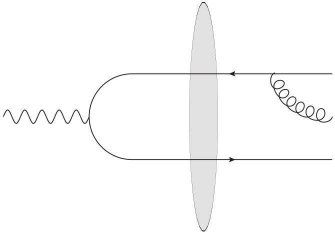
$$\times[\Phi'_{V1}(\vec{p}_1, \vec{p}_2)C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+\Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

Real corrections

Real dipole and double dipole corrections





$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$

$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle$$

Divergences

Divergences

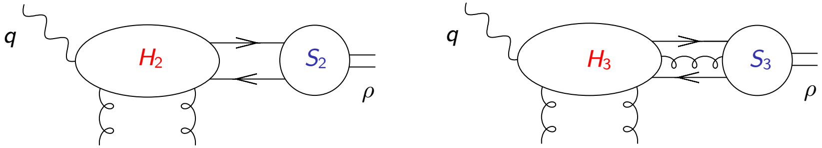
- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Cancels between real and virtual corrections, along with renormalization
- Soft and collinear divergence
  - Removed via a jet algorithm

We thus built a finite NLO exclusive diffractive cross section with saturation effects

Exclusive diffractive light vector meson production

Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator



$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x)\psi_j^\beta(0) \right| 0 \right\rangle \qquad \int_{x_1,x_2} (H_3^\mu(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x_1)A_\mu^c(x_2)\psi_j^\beta(0) \right| 0 \right\rangle$$

H and S are by convolution and by summation over spinor and color indices

Once factorization in the t channel is done, now factorize in the s channel with collinear factorization: expand the impact factor in powers of the hard scale

Twist 2

Collinear factorization at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\langle \rho \left| \bar{\psi}(z)\gamma^{\mu}\psi(0) \right| 0 \rangle \rightarrow p^{\mu} f_{\rho} \int_0^1 dx e^{ix(p\cdot z)} \varphi_1(x)$$

- Leading twist DA for a transversely polarized light vector meson

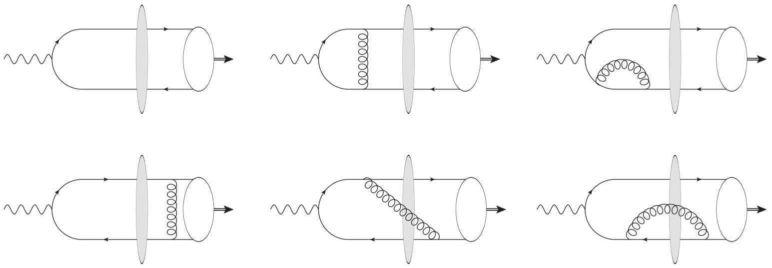
$$\langle \rho \left| \bar{\psi}(z)\sigma^{\mu\nu}\psi(0) \right| 0 \rangle \rightarrow i(p^{\mu}\varepsilon_{\rho}^{\nu} - p^{\nu}\varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_0^1 dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^* A \rightarrow \rho_T A$ starts at twist 3

Exclusive diffractive ρ_L production:

NLO corrections to a twist 2 process

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A} = -\frac{e_V f_V \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_{\parallel}(x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \frac{d^d \vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta(x, \vec{p}_1, \vec{p}_2) + C_F \Phi_{V1}^\beta(x, \vec{p}_1, \vec{p}_2) \right) \tilde{\mathcal{U}}_{12}^\eta (2\pi)^d \delta(\vec{p}_3) \right.$$
$$\left. + \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3) \tilde{\mathcal{W}}_{123}^\eta \right]$$

Probes gluon GPDs at low x, as well as twist 2 DAs

Divergences

Divergences

- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Mostly cancel each other, but requires renormalization of the operator in the vacuum-to-meson matrix element $\rightarrow$ ERBL evolution equation for the DA

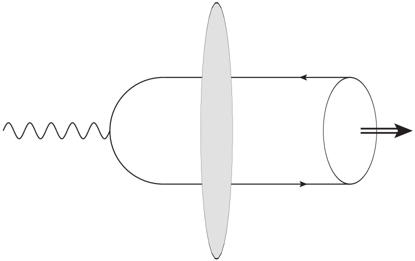
We thus built a finite NLO exclusive diffractive amplitude with saturation effects

Exclusive diffractive ρ_T production:

LO but twist 3 process

2-body diagrams

2-body contribution

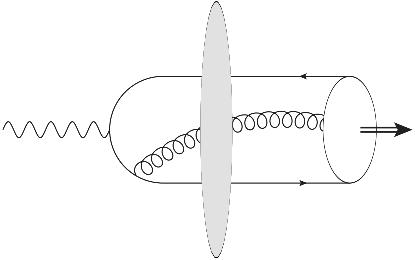


$$\int d^2\vec{z}_1 d^2\vec{z}_2 \; \Phi^{2b}_{q\bar{q}}(\vec{z}_1, \vec{z}_2)\, \mathrm{Tr}(U_1 U_2^\dagger)\, \langle \rho \,|\bar{\psi}\psi|\, 0 \rangle$$

Note that this is not the whole story. This nice and simple contribution only arises once we cancel all contributions which break QCD gauge invariance up to twist 4 corrections.

3-body contribution

Natural 3-body CGC diagram



$$\int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \; \Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right) \mathrm{Tr}[U_1 t^b U_2^\dagger t^a] U_3^{ab} \left\langle \rho \left| \bar{\psi} g A \psi \right| 0 \right\rangle$$

Double-dipole term even at tree level $\Rightarrow$ Great sensitivity to saturation

Note that this is not the whole story. This nice and simple contribution only arises once we cancel all contributions which break QCD gauge invariance up to twist 4 corrections.

Divergences

Divergences and issues?

- No divergence. No end point singularity which would break factorization in a pure collinear framework. The mixed CGC/collinear framework gets rid of *s*-channel factorization breaking.
- QCD gauge invariance is restored up to twist 4 terms
- Presence of a double dipole term at LO: enhanced saturation effects?
- In the Wandzura-Wilczek approximation, it will be easy get the NLO corrections to this twist 3 process and no end point singularity is to be expected

We thus built a finite twist 3 exclusive diffractive amplitude with saturation effects

Conclusion

- We provided the full computation of the $\gamma^{(*)} \to JetJet$ and $\gamma^*_{L,T} \to \rho_L$ impact factors at NLO accuracy, and the twist 3 impact factors for $\gamma^*_{L,T} \to \rho_T$ in the shockwave framework.
- Our results are perfectly finite, even for photoproduction (at large t for ρ)
- The computation can be adapted for twist 3 NLO production in the Wandzura-Wilczek approximation, removing factorization breaking end-point singularities even at NLO for a process which would not factorize in a full collinear factorization scheme
- Exclusive diffractive processes are perfectly suited for precision saturation physics and gluon tomography with $b_\perp$ dependence at the EIC. Dijet production probes the dipole Wigner distribution, ρ meson production probes gluon GPDs at small x.