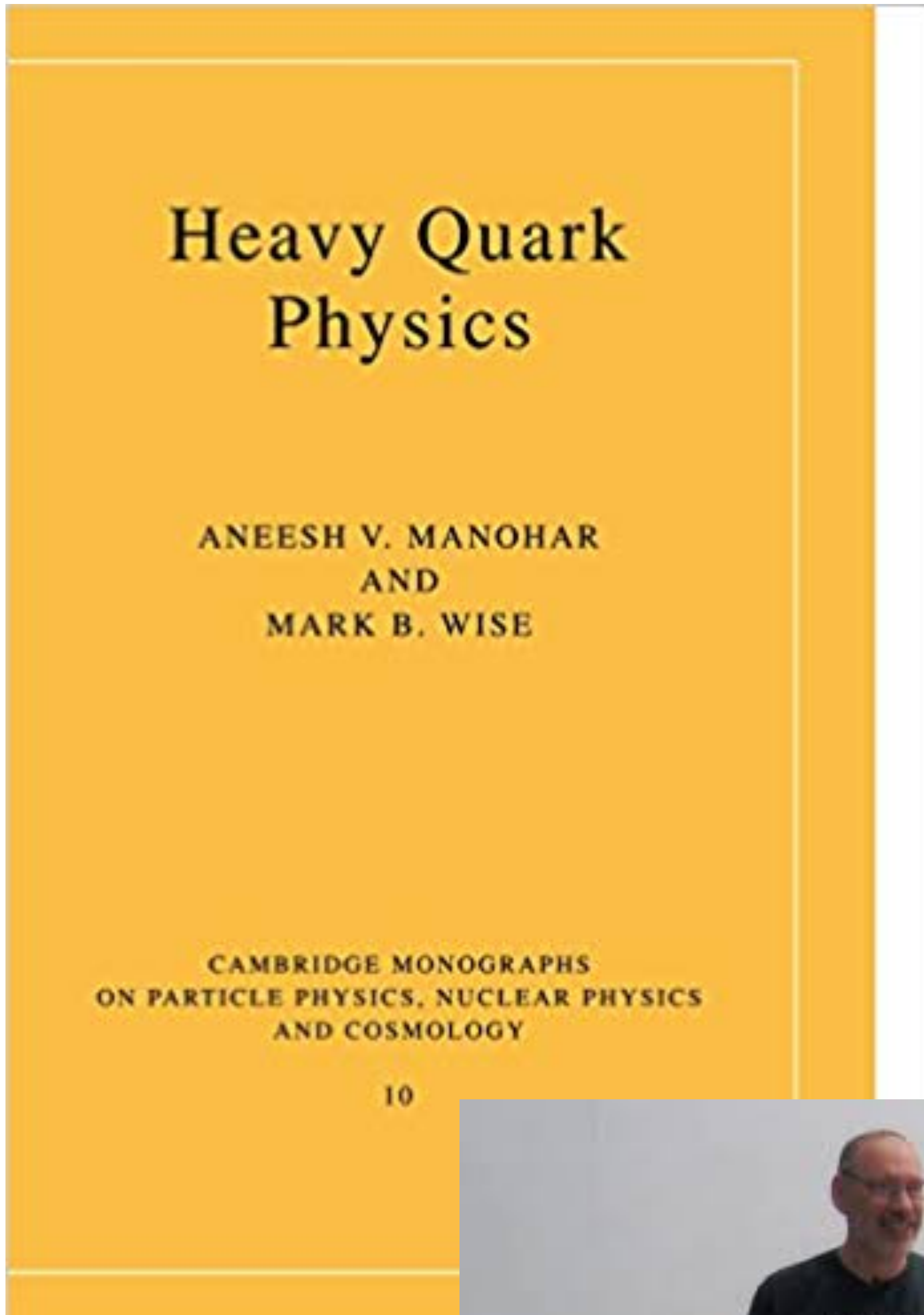
Gravitational waves and QCD

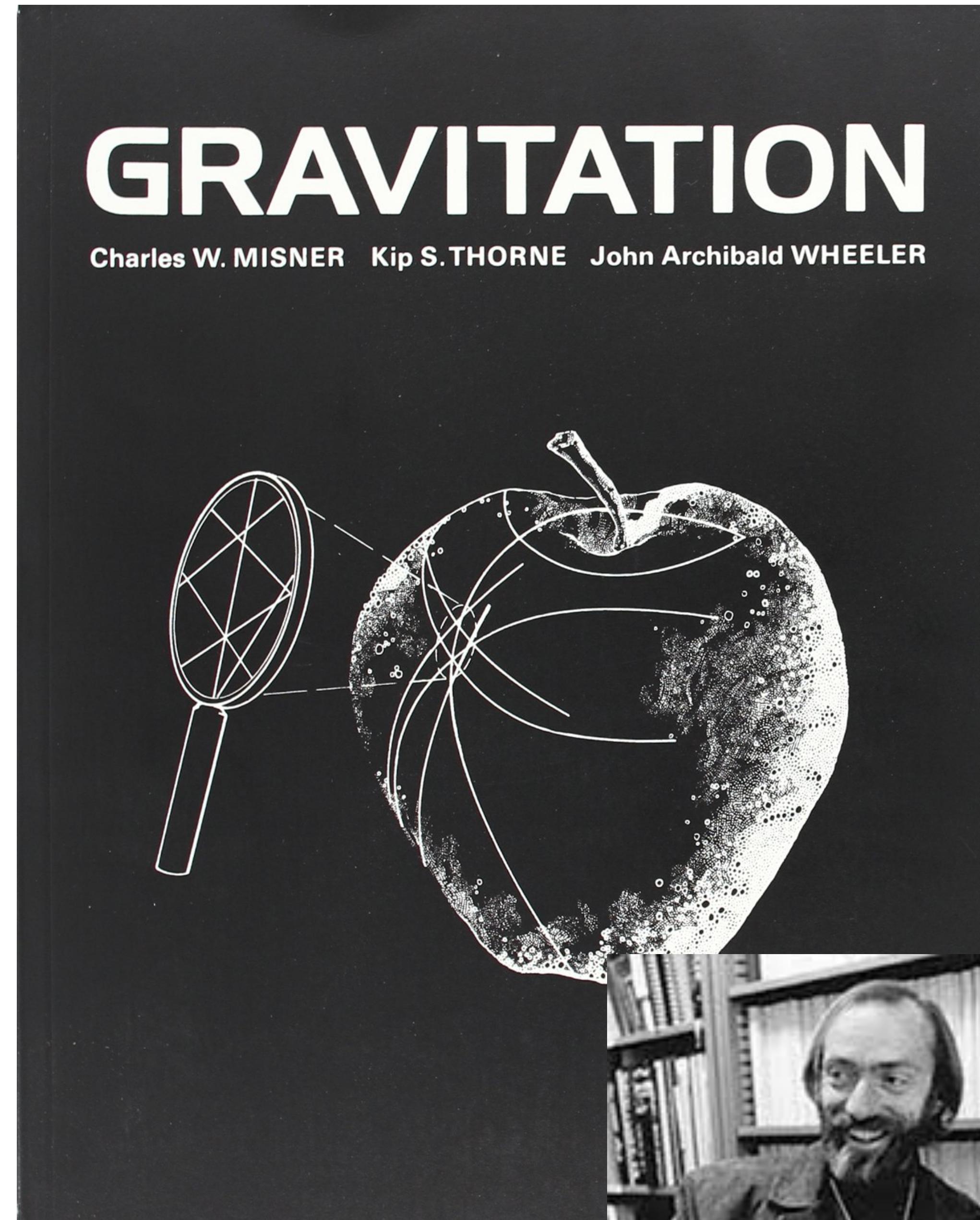
Walter Goldberger
Yale

or…

QCD **meets** **GR**



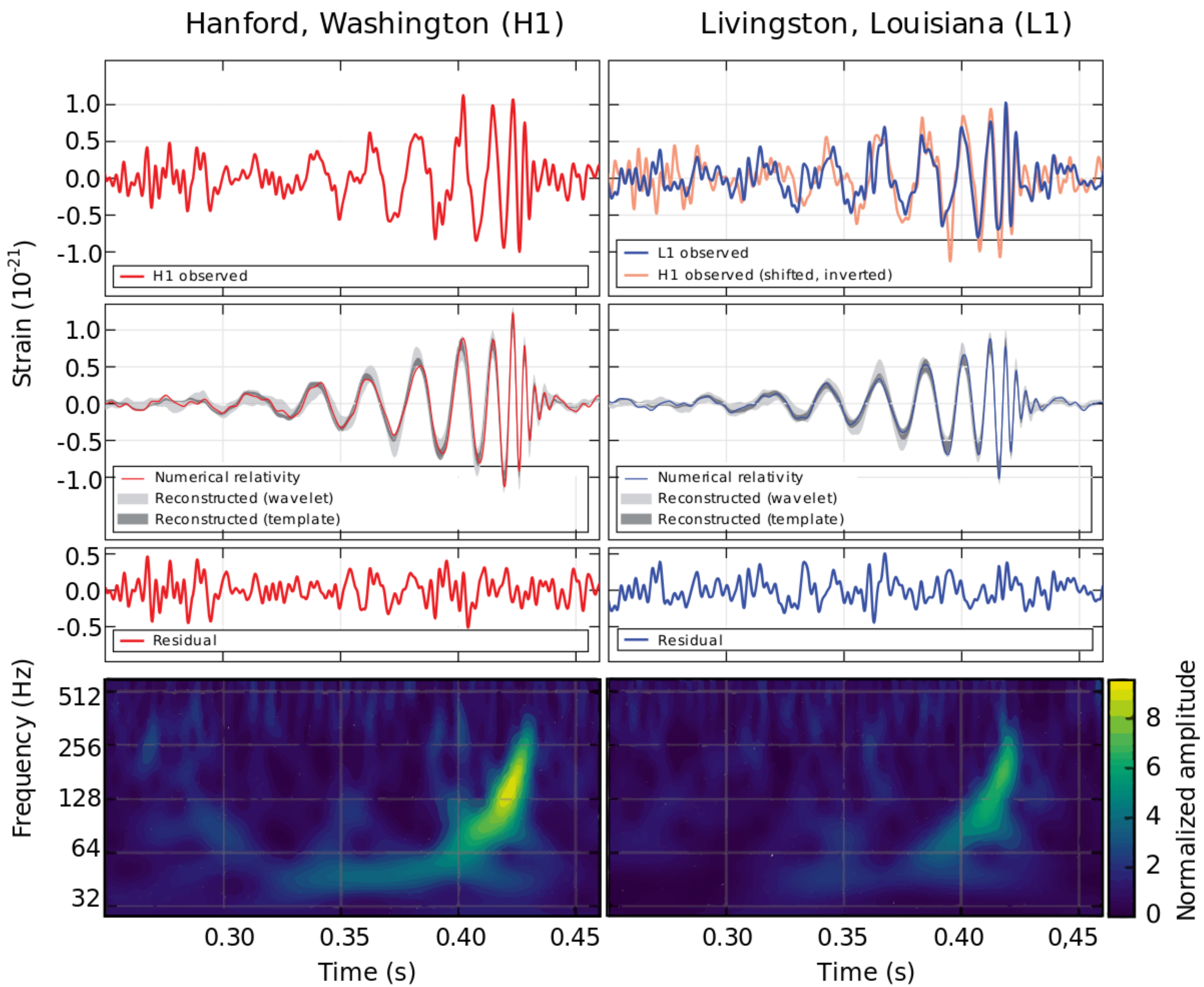
(M. Wise)



(K. Thorne)

The modern era of gravitational wave astrophysics started on September 2015:

$\mathrm{SNR} = 24$



Consistent with gravitational waves from the merger of binary black holes at $d_L \simeq 440\mathrm{MPc}$

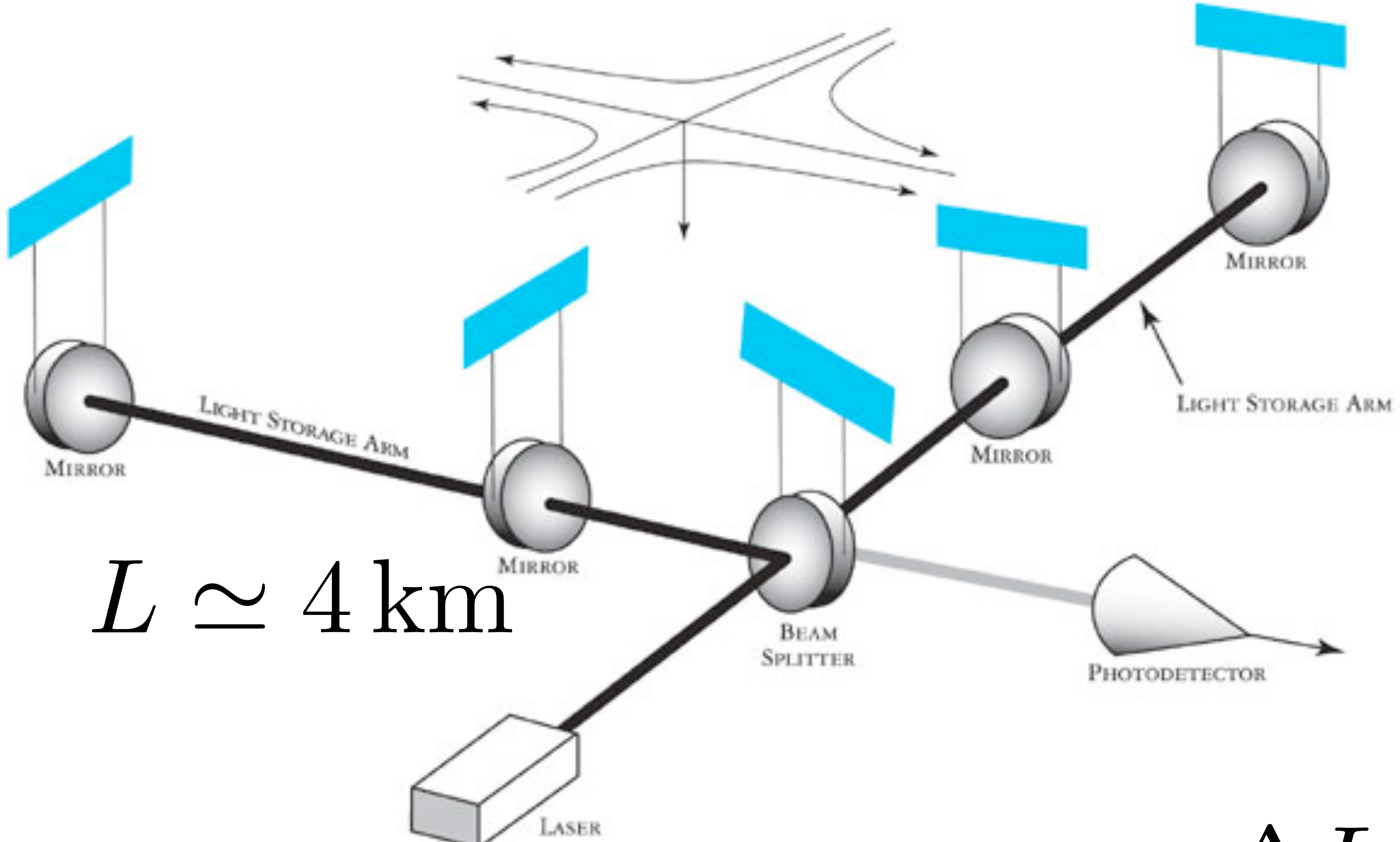
$$m_1(35.4m_\odot) + m_2(29.8m_\odot) \to m^*(62.2m_\odot)$$

corresponding to radii~90 km and separation 350 km.

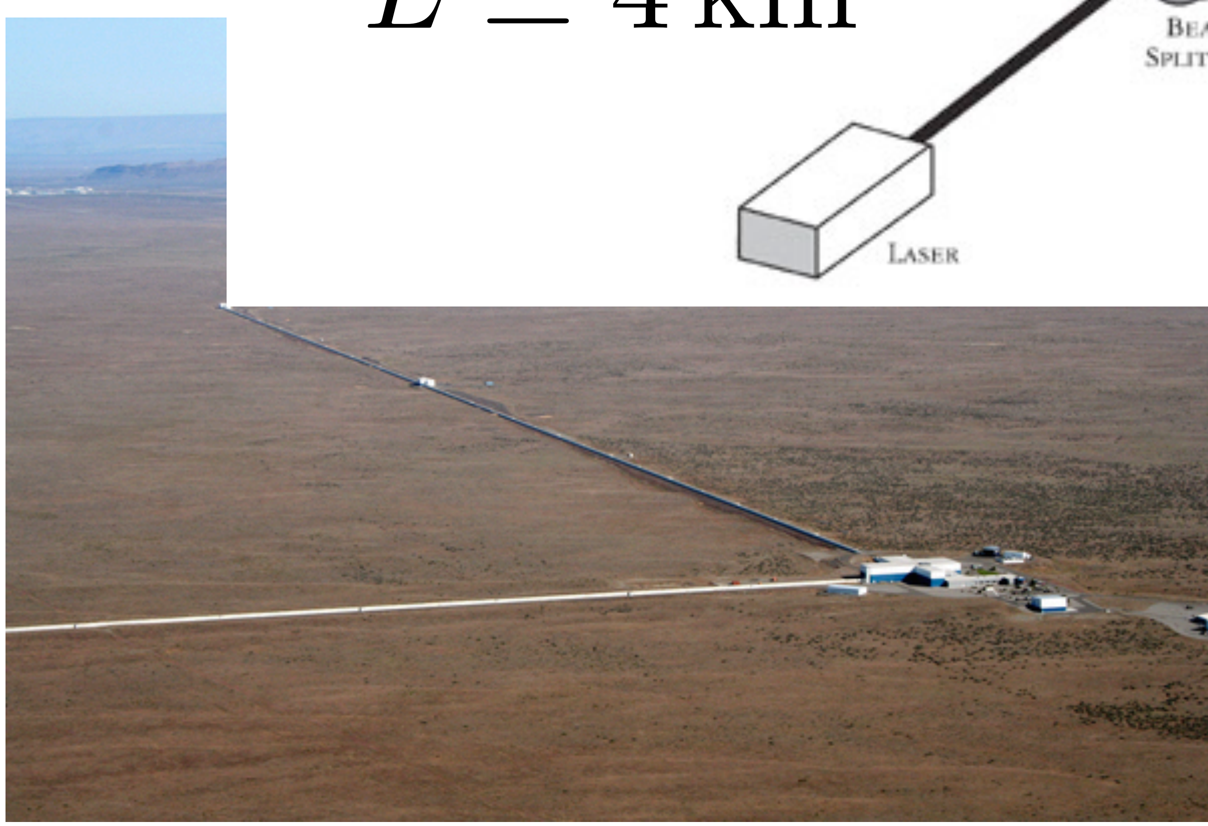
Advanced LIGO

Michelson interferometers with Fabry-Perot cavities.

Livingston,LA



$$L \simeq 4\,\mathrm{km}$$



Hanford, WA

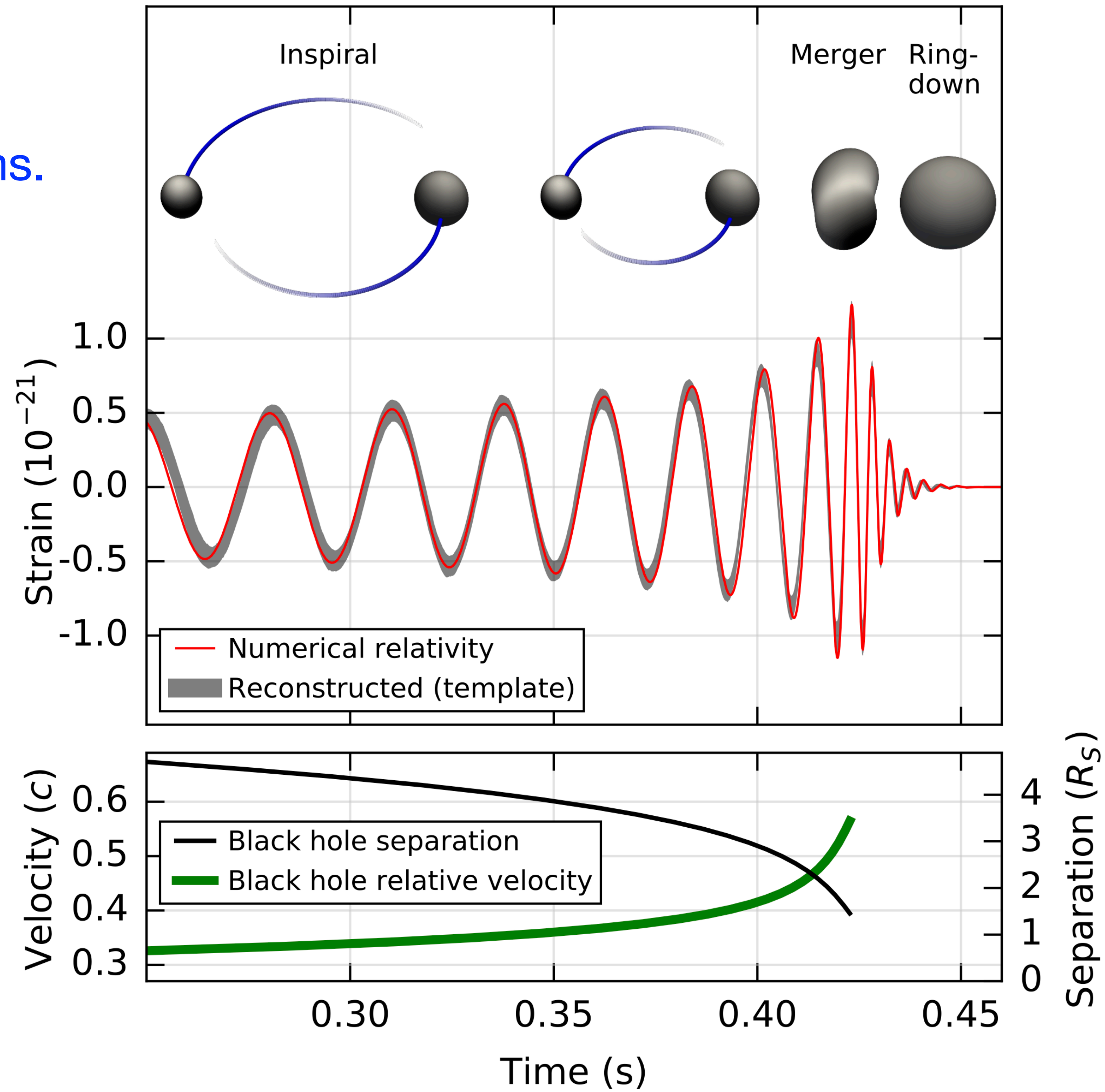
$$\frac{\Delta L}{L} \sim h \sim 10^{-21}$$

$$P_{laser} = 20\ \mathrm{W}$$

Compact binary mergers

Most promising GW events at LIGO consist of mergers of binary black holes or neutron stars with masses $m_{NS} \sim \mathcal{O}(1) m_\odot \qquad m_{BH} \sim \mathcal{O}(10) m_\odot$.

There are three qualitatively different regimes in the evolution of these systems.



Calculating the gravitational waveform

$$h_{\mu\nu}(r \to \infty) = g_{\mu\nu} - \eta_{\mu\nu} \ll 1$$

in each phase requires different theoretical tools…

The Inspiral Phase

This is the early part of the signal as the binary enters the LIGO band. Consists of well separated orbits and slow ("adiabatic") time evolution, with velocities $v/c \ll 1$

To a good approx, gravitational dynamics becomes linear, and Einstein's theory reduces to a simple wave equation:

$$\left(-\frac{1}{c^2}\partial_t^2 + \nabla^2\right) h_{\mu\nu}(x) = -16\pi G_N T_{\mu\nu}(x)$$

(energy-momentum of sources)

The binary constituents can be regarded as point particles, with nearly Newtonian gravitational dynamics:

Radiation field:

$$h_{ij}^{TT} = \frac{4G_N}{R}\frac{d^2}{dt^2} I_{ij}^{TT}$$

Quadrupole moment (inertia tensor)

dist. to source

Radiated power:

$$\frac{dE}{dt} = -\frac{G_N}{5}\langle \dddot{I}_{ij} \dddot{I}_{ij} \rangle$$

valid for arbitrarily strong (e.g black hole) but non-relativistic sources.

Compact binary in a circular orbit:

$$h = |h_{\mu\nu}| \sim \frac{m^{5/3}\Omega^{2/3}}{R} \sim \frac{G_N E_{int}}{R}$$

Use $\Omega = v/r \sim 2\pi\nu_{GW}$ and $v^2 \sim G_N m/r$ to find:

Orbital radius:

$$r(10\mathrm{Hz}) \sim 300\mathrm{km}\left(\frac{m}{m_\odot}\right)^{1/3} \longrightarrow r(500\mathrm{Hz}) \sim 20\mathrm{km}\left(\frac{m}{m_\odot}\right)^{1/3}$$

Orbital velocity:

$$v(10\mathrm{Hz}) \sim 0.06\left(\frac{m}{m_\odot}\right)^{1/3} \longrightarrow v(500\mathrm{Hz}) \sim 0.2\left(\frac{m}{m_\odot}\right)^{1/3}$$

GW strain:

$$h \sim \frac{\Delta L}{L} \sim 10^{-22}\left(\frac{50\mathrm{MPc}}{R}\right)\left(\frac{m}{m_\odot}\right)^{5/3}\left(\frac{\nu}{500\mathrm{Hz}}\right)^{2/3}$$

for neutron stars as they sweep the LIGO frequency band .

Dynamics:

Time evolution at a semi-quantitative level through energy balance

$$\frac{d}{dt}E = \frac{d}{dt}\left(-\frac{1}{2}mv^2\right) = -P_{GW} = -\frac{32}{5}G_N^{-1}v^{10}$$ (circular orbit)

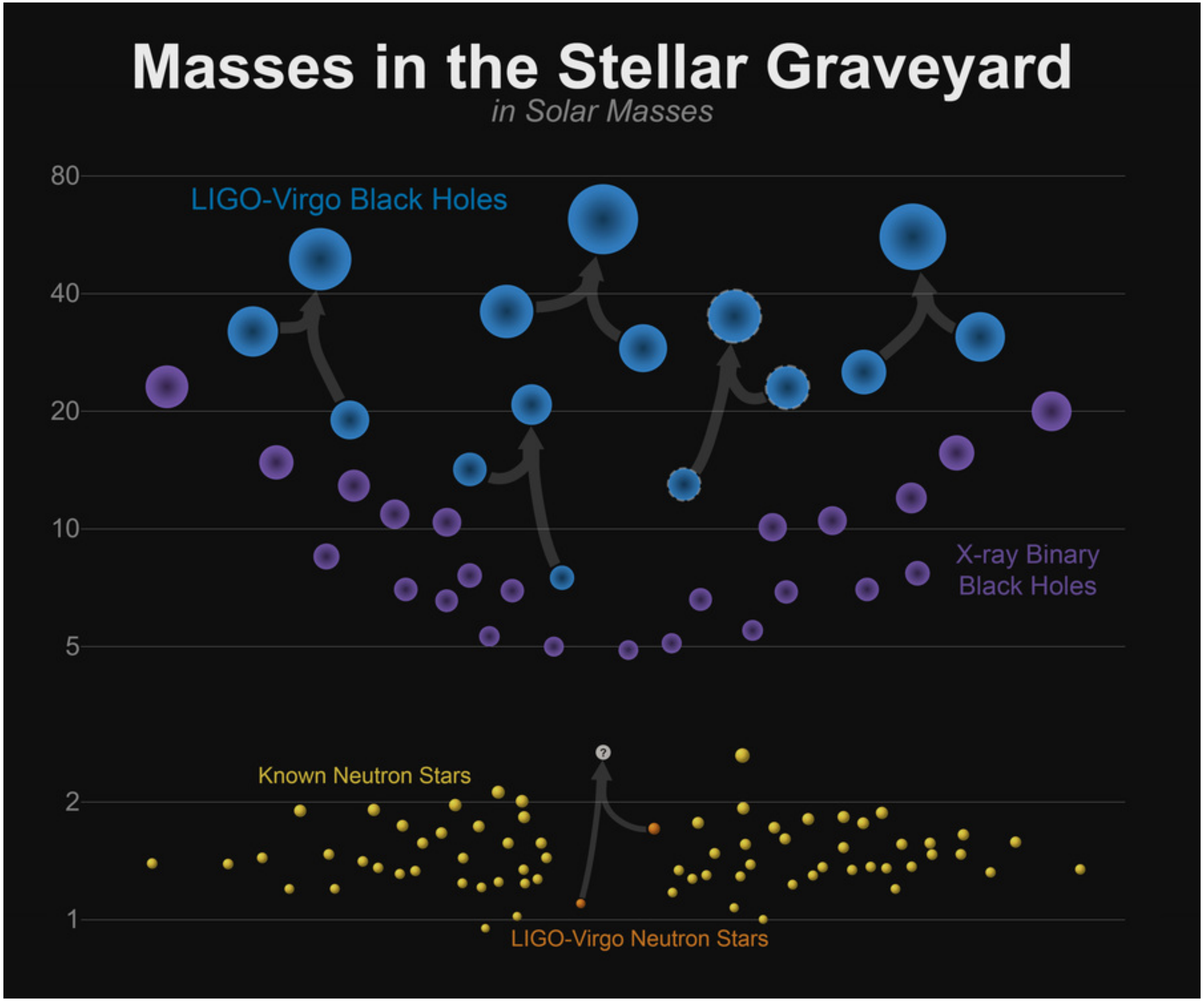
→

Signal duration: $$\Delta t = \frac{5}{512}(G_N M)\left(\frac{1}{v_i^8} - \frac{1}{v_f^8}\right) \sim 3\text{min}\left(\frac{m}{m_\odot}\right)^{-8/3}$$

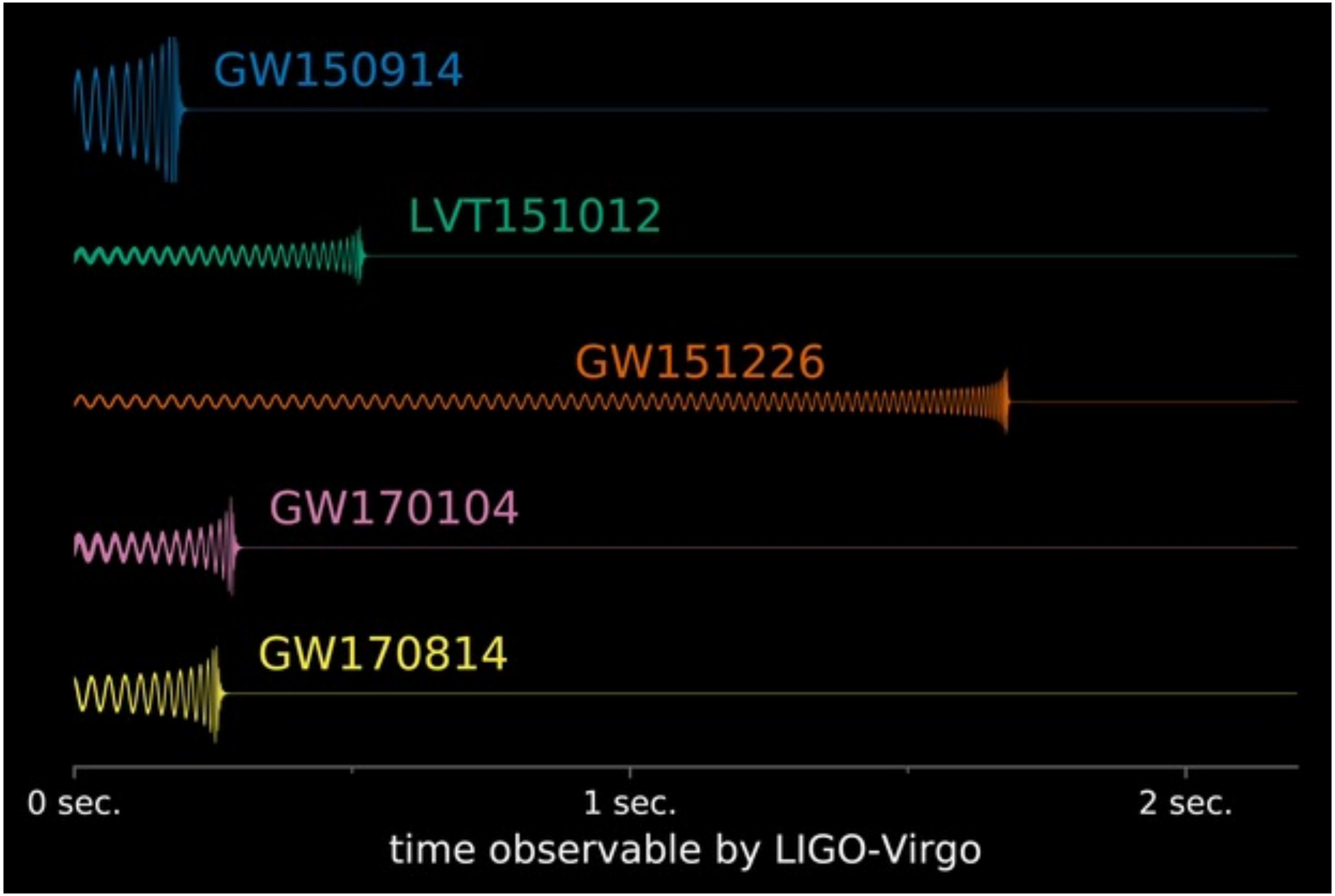
GW phase: $$\Phi = \int_{t_i}^{t_f}\omega(t)dt = \frac{1}{32}\left(\frac{1}{v_i^5} - \frac{1}{v_f^5}\right) \sim 10^4\text{radians}\left(\frac{m}{m_\odot}\right)^{-5/3}$$

corresponding to $\sim 10^4 - 10^5$ orbital cycles in the LIGO band.

What LIGO has seen so far in the O1 and O2 runs (blue and orange)



Simulated waveforms corresponding to 5+1 detected GW signals in O1+O2:

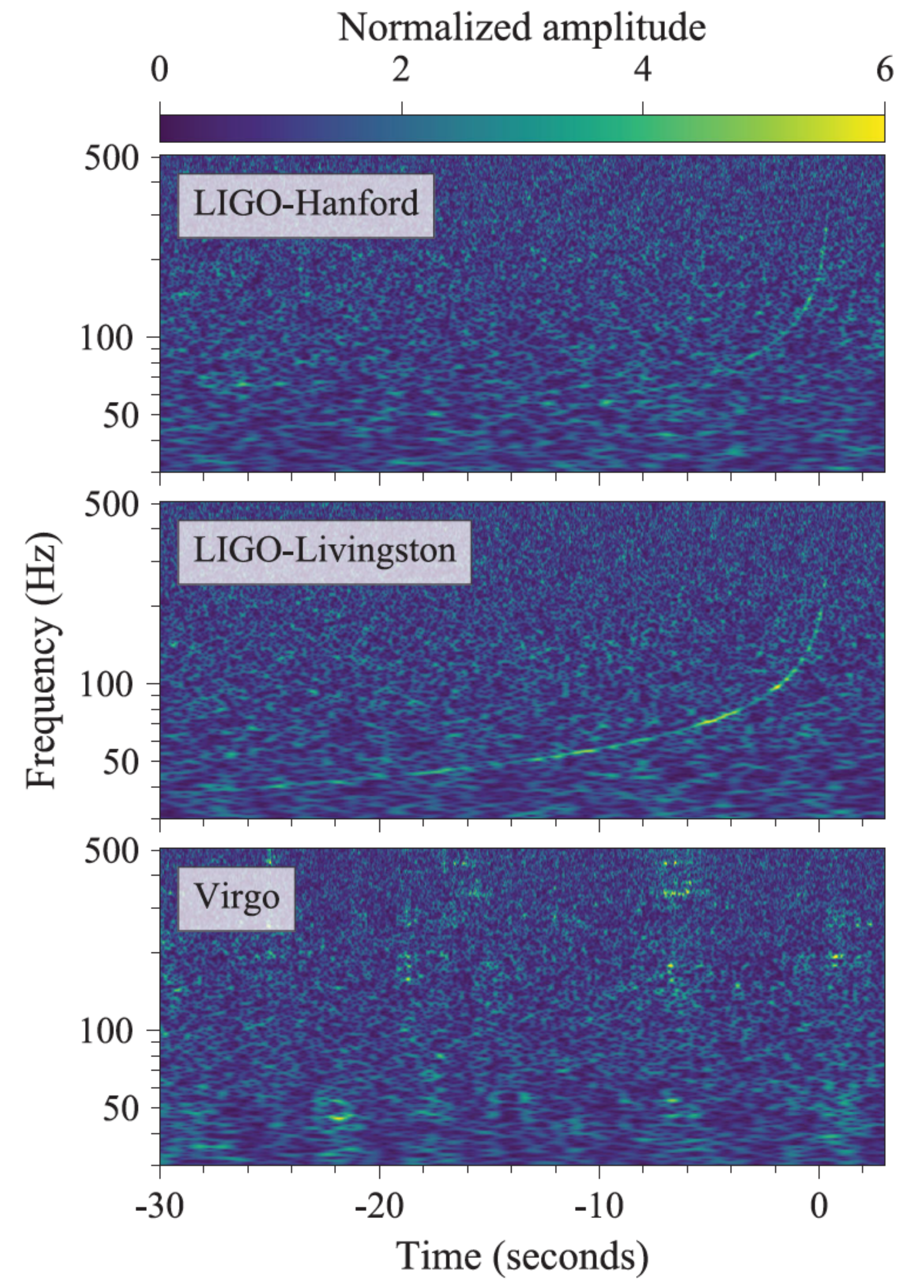


Not included: GW170608

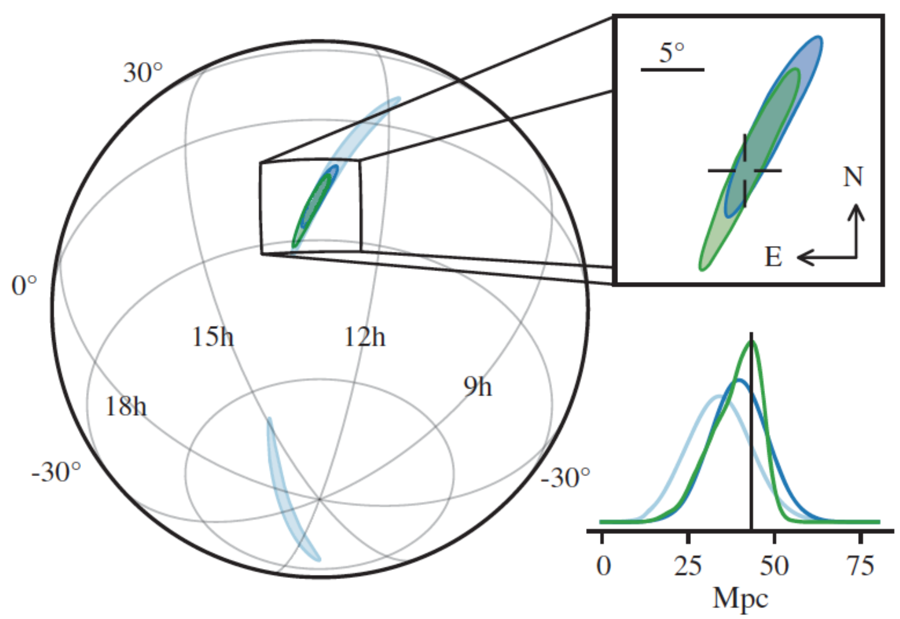
GW170817

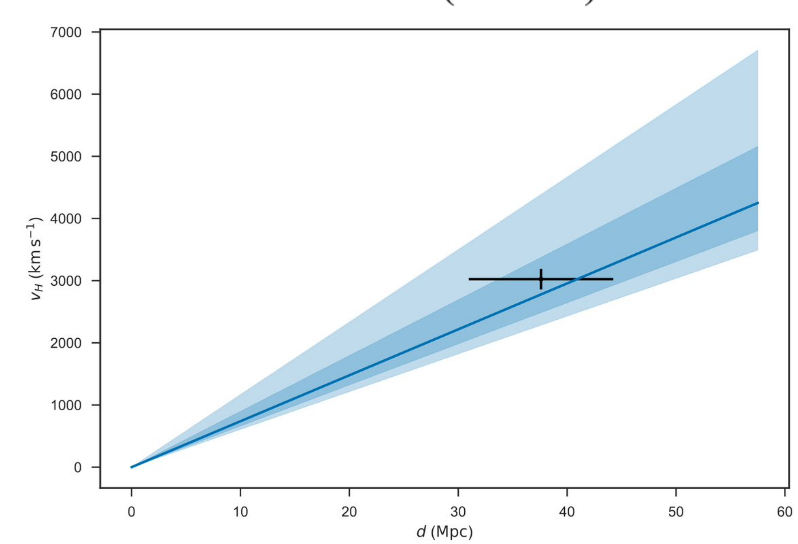
Strongest signal so far w/ SNR~32.4.
Corresponds to NS/NS at $z \sim 0.01$

	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–$1.60\ M_\odot$	1.36–$2.26\ M_\odot$
Secondary mass m_2	1.17–$1.36\ M_\odot$	0.86–$1.36\ M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400



Companion $\gamma-$ray burst 1.7 s after coalescence allowed source localization within NGC4993.





Hubble diagram from GW "standard siren" $H_0 = 70.0^{+12.0}_{-8.0} \mathrm{km s^{-1} Mpc^{-1}}$

The need for precision in the inspiral phase

During the adiabatic inspiral phase, the binaries are non relativistic. The quantity

$$v/c \ll 1$$

serves as a small expansion parameter that organizes the gravitational dynamics in this regime. From the quadrupole radiation formula, the GW phase is

$$\Phi = \int_{t_i}^{t_f} \omega(t)dt = \frac{1}{32}\left(\frac{1}{v_i^5} - \frac{1}{v_f^5}\right) \sim 10^4 \text{radians} \left(\frac{m}{m_\odot}\right)^{-5/3}$$

Matched filtering to the LIGO data requires a waveform template that is phase coherent to $\sim \mathcal{O}(1)$ orbital cycles. Including GR corrections to the phase

$$\Phi = \frac{v^{-5}}{32}\left[\mathcal{O}(1) + \mathcal{O}(v^2) + \cdots \mathcal{O}(v^5)\right]$$

suggests that the theoretical prediction should be accurate to at least $(v/c)^5 \ll 1$

A more careful estimate by Cutler et al, PRL (1993) show that phase coherence between waveform templates and the GW data requires theoretical predictions that are accurate to

$$\boxed{\text{Sensitivity} = \quad (v/c)^7}$$

"Precision gravity"

With higher order corrections motivated by matching analytical and numerical GR…

This means that the inspiral phase carries a wealth of information about the binary system. One expects to obtain

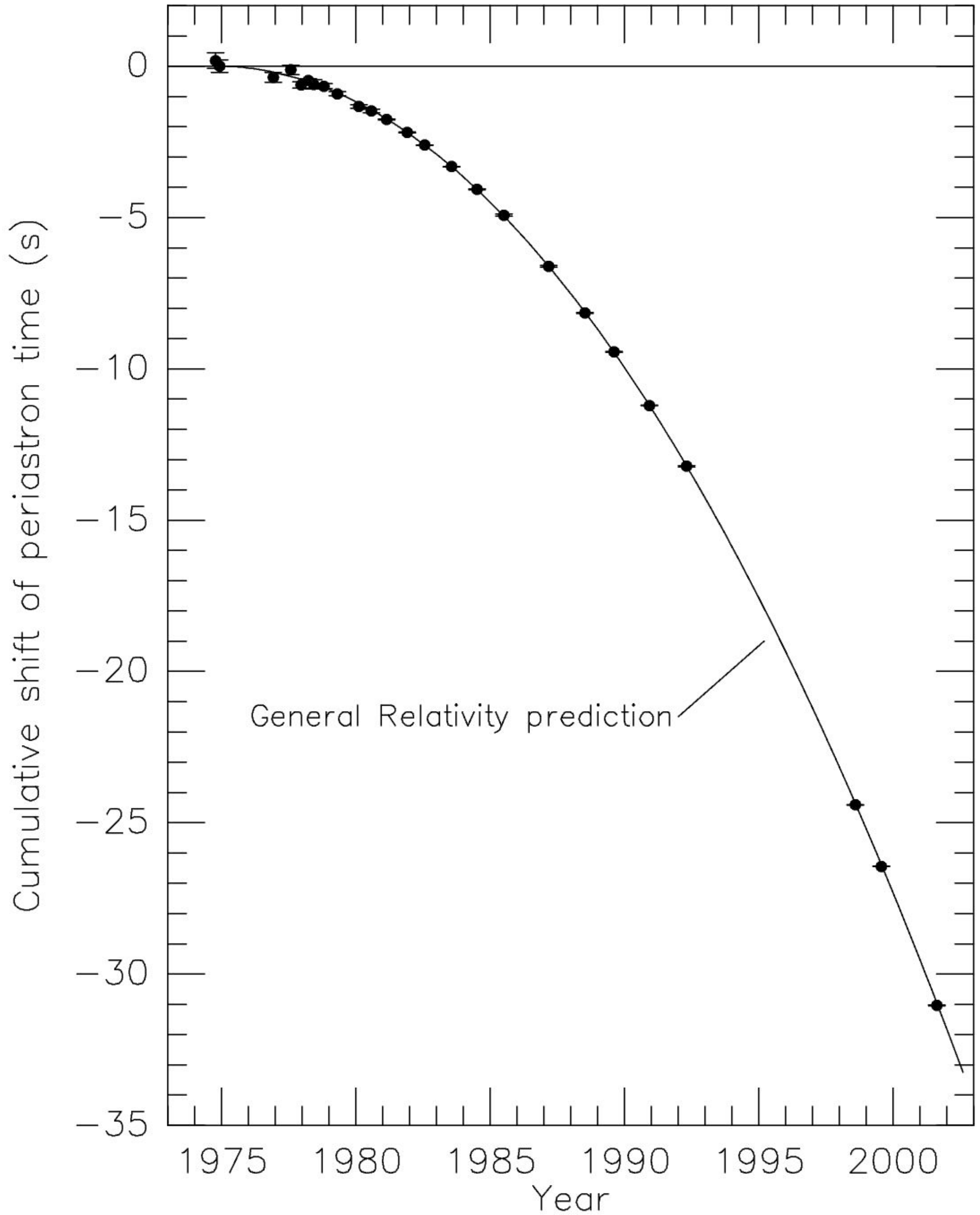
1. Accurate measurements of masses, spins, and distances for compact binaries out to $d_L \sim 1000\text{Mpc}$

2. Stringent tests of classical GR and constraints on "new physics"

3. Dynamics of BH horizons, neutron star EoS…

Hulse-Taylor pulsar (1974): a case study

This is a binary NS/NS system with $v/c \sim 10^{-3}$ and orbital radius $\sim 1R_\odot$



$$\frac{\dot{P}_{exp}}{\dot{P}_{GR}} = 1.0013 \pm 0.0021$$

Parameter	Symbol (units)	Value
(i) "Physical" parameters		
Right Ascension	α	$19^{\mathrm{h}}15^{\mathrm{m}}27.^{\mathrm{s}}99999(2)$
Declination	δ	$16°06'27.''4034(4)$
Pulsar Period	P_{p} (ms)	59.0299983444181(5)
Derivative of Period	$\dot{P}_{\mathrm{p}}$	$8.62713(8) \times 10^{-18}$
(ii) "Keplerian" parameters		
Projected semimajor axis	$a_{\mathrm{p}} \sin i$ (s)	2.341774(1)
Eccentricity	e	0.6171338(4)
Orbital Period	P_{b} (day)	0.322997462727(5)
Longitude of periastron	ω_0 (°)	226.57518(4)
Julian date of periastron	T_0 (MJD)	46443.99588317(3)
(iii) "Post-Keplerian" parameters		
Mean rate of periastron advance	$\langle\dot{\omega}\rangle$ (° yr^{-1})	4.226595(5)
Redshift/time dilation	γ' (ms)	4.2919(8)
Orbital period derivative	$\dot{P}_{\mathrm{b}}$ (10^{-12})	−2.4184(9)

$$m_p = 1.4414(2)m_\odot$$

$$m_c = 1.3867(2)m_\odot$$

Post-Newtonian Theory

Blanchet et al. PRL (1995)
Damour et al PLB (2001)
Blanchet et al PRL (2004)
Goldberger and Rothstein PRD (2006)

The starting point for the velocity expansion of the binary system is Einstein's equations recast as a non-linear wave equation for a spin-2 field propagating in flat space

(eg Weinberg 1972)

$$(-\partial_t^2 + \nabla^2) h^{\mu\nu}(x) = -16\pi G_N \tilde{T}^{\mu\nu}(x) \qquad (h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu})$$

where the source term on the RHS is the energy-momentum of matter and of gravity itself

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + T_g^{\mu\nu}$$

(matter source)

(grav. energy-momentum)

$\sim h\partial^2 h + h^2\partial^2 h + \cdots$

which is conserved, $\partial_\mu \tilde{T}^{\mu\nu} = 0$, by Einstein's equations. The GW waveform at $r \to \infty$ is

$$h_{ij}(t, \vec{x}) = \frac{4G_N}{r} \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{T}^{ij}(k)$$

w/ $\tilde{T}^{\mu\nu}(k) = \int d^4x e^{ik\cdot x} \tilde{T}^{\mu\nu}(x)$ $\qquad k^\mu = \omega(1, \vec{n} = \frac{\vec{x}}{r})$ $\qquad k^2 = 0$ ("on-shell" wavevector)

In the post-Newtonian limit, the sources can be treated as a point particles. All the effects of finite size (tidal, etc) can be described model-independently by an “effective Lagrangian”

Goldberger and Rothstein PRD (2006)

$$\tilde{T}^{\mu\nu} = T^{\mu\nu}_{pp} + T^{\mu\nu}_{g}$$

$$S_{pp} = -m \int d\tau + c \int d\tau R^2_{\mu\nu\alpha\beta} + \cdots$$

(spins, higher gradients, multipoles)

Containing in general an infinite number of terms suppressed by powers of $r_s/\lambda \ll 1$.
The coefficients encode the tidal response to an ext. gravitational field. E.g

$c \propto$ “Love number”

depends on (non-gravitational) microphysics:

$$c_{NS} \sim mR^4$$

Flanagan and Hinderer, 2008; depends on NS EoS

$$c_{BH,d=4} = 0$$

(Damour et al; Poisson et al;Kol+Smolkin 2010)

In practice computing higher order terms in perturbation theory $(v \ll 1)$ is difficult for two reasons:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

Many physically relevant scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \gg r \gg \lambda$$

all correlated with the perturbative expansion parameter

$$r \sim r_g / v^2 \qquad \lambda \sim r/v \sim r_g / v^3$$

We (Goldberger+ Rothstein, 2006) found that these challenges can be ameliorated by employing some 20th century tools from quantum field theory:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

Organize the expansion in terms of Feynman diagrams

Many physically relevant scales

Treat each scale separately, by constructing a tower of gravity Effective Field Theories

$$h_{\mu\nu} = h_{\mu\nu}^{potential} + h_{\mu\nu}^{rad}$$

Feynman Diagrams from Graviton Effective Field Theory

In order to simplify the classical calculations, I will use a field theory of gravitons:

Graviton EFT (GREFT)= Low Energy Quantum Gravity

Feynman, DeWitt, Weinberg (1960's); 't Hooft+Veltman (1970s)

This is a Lorentz invariant theory of spin 2 particles interacting with matter. The linear coupling to matter is

$$\mathcal{L}_{int} = \frac{h_{\mu\nu}}{M_{Pl}} T^{\mu\nu}$$

(analogous to EM interactions $\mathcal{L}_{int} = A_\mu J^\mu$). This theory makes sense at energies

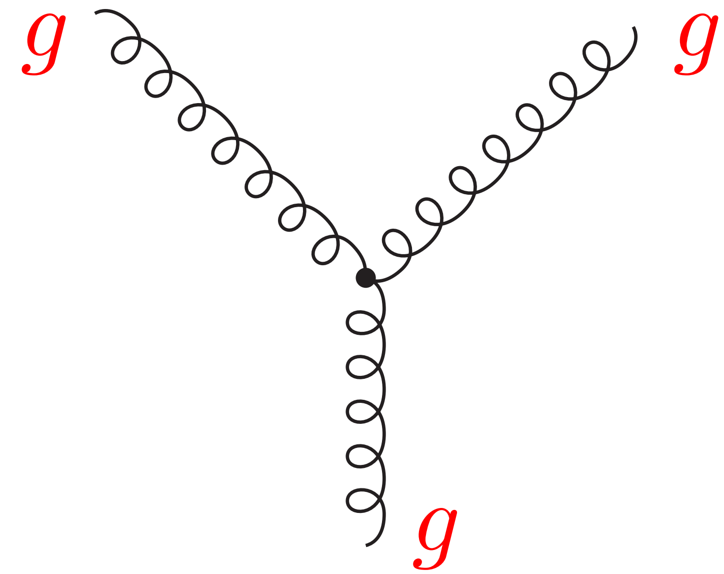
$$E < M_{Pl} = 1/\sqrt{32\pi G_N} \sim 10^{19} \mathrm{GeV}$$

so it is an "effective field theory" with limited predictive power.

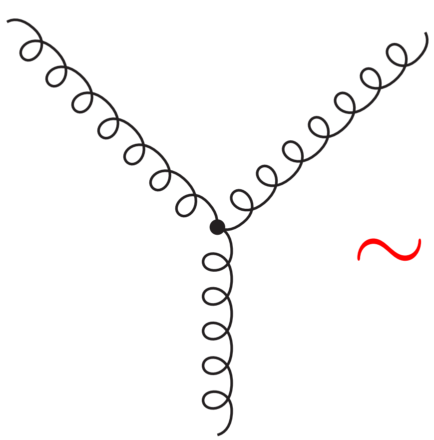
(*see chiral perturbation theory=pion EFT, in QCD)

Because the graviton couples to energy-momentum it must couple to itself:

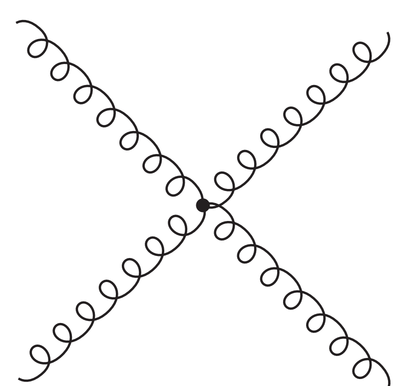
$$\mathcal{L}_{int} = \frac{h_{\mu\nu}}{M_{Pl}} T^{\mu\nu}$$



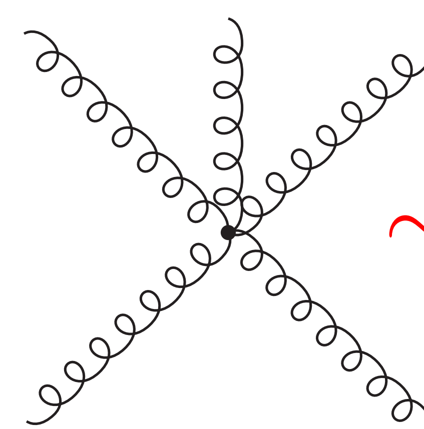
Together with gauge invariance, this implies that there are is a tower of self-interactions



$$\sim \frac{1}{M_{Pl}}$$



$$\sim \frac{1}{M_{Pl}^2}$$



$$\sim \frac{1}{M_{Pl}^3}$$

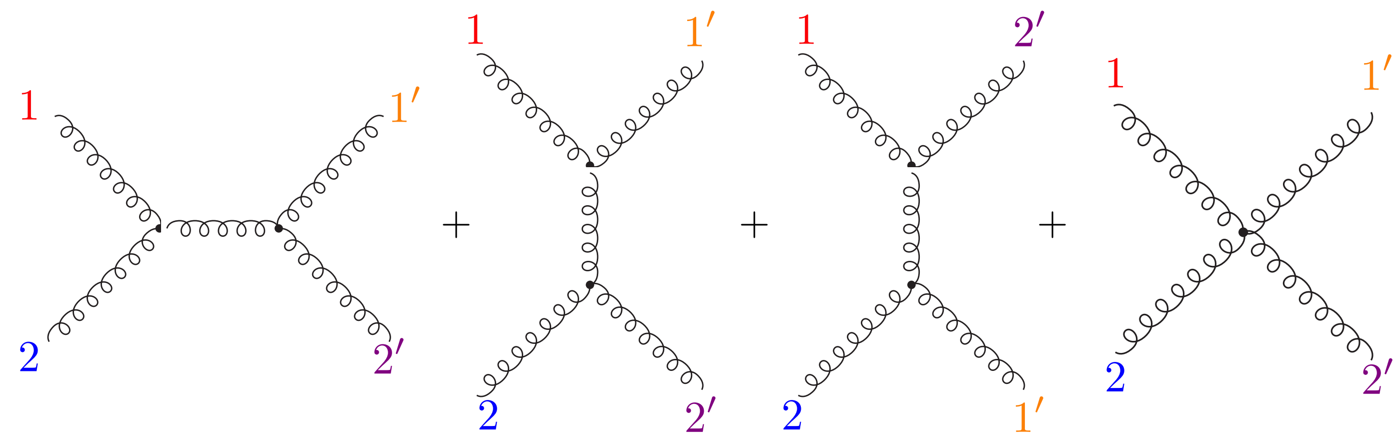
This is similar to QCD, except that in gravity, the # of self interactions is infinite.

Even though "GREFT" cannot be used to make sense of quantum gravity phenomena at energies $E \sim M_{Pl}$, it does make unambiguous low energy predictions. E.g.:

Graviton scattering:

DeWitt, 1967.

$\mathcal{A} =$

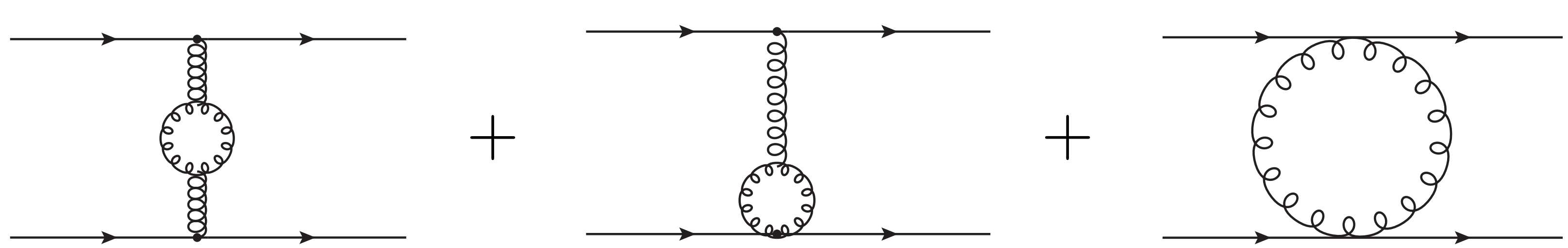


$$\Longrightarrow \quad \frac{d\sigma}{d\Omega} = 4G_N^2 E_{cm}^2 \left(\frac{32 + \cos^{12}\frac{\theta}{2}}{\sin^4 \frac{\theta}{2}} \right) + \mathcal{O}(G_N^3 E^4) \quad \sim 10^{-117}\text{barn} \left(\frac{1\text{km}}{\lambda}\right)^2 !!$$

$\mathcal{O}(\hbar)$ corrections to Newton potential:

Donoghue et al 2002, Khriplovich et al 2002

$V(r)_{\hbar^1} =$



$$= -\frac{41}{10\pi}\frac{G_N \hbar}{r^2 c^3} \cdot \frac{G_N m_1 m_2}{r} \qquad \sim 10^{-64} \left(\frac{1\text{mm}}{r}\right)^2 \times \frac{G_N m_1 m_2}{r}$$

This theory also makes predictions that are less academic in nature. In particular, it has applications to the inflation paradigm for early universe cosmology

Tensor modes:

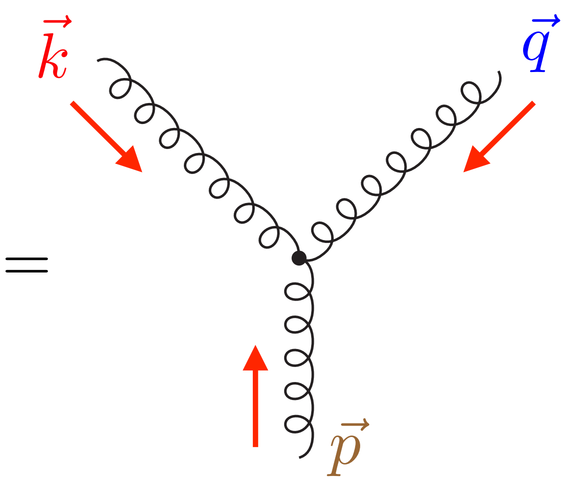
L. Abbott and M. B. Wise (1984)

$$\langle h_{\vec{k}} h_{\vec{q}} \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{q}) \cdot \frac{1}{\pi^2} \frac{H_\star(k)^2}{M_{Pl}^2} \qquad \sim 10^{-7} \left(\frac{H_\star(k)}{10^{16}\mathrm{GeV}} \right)^2$$

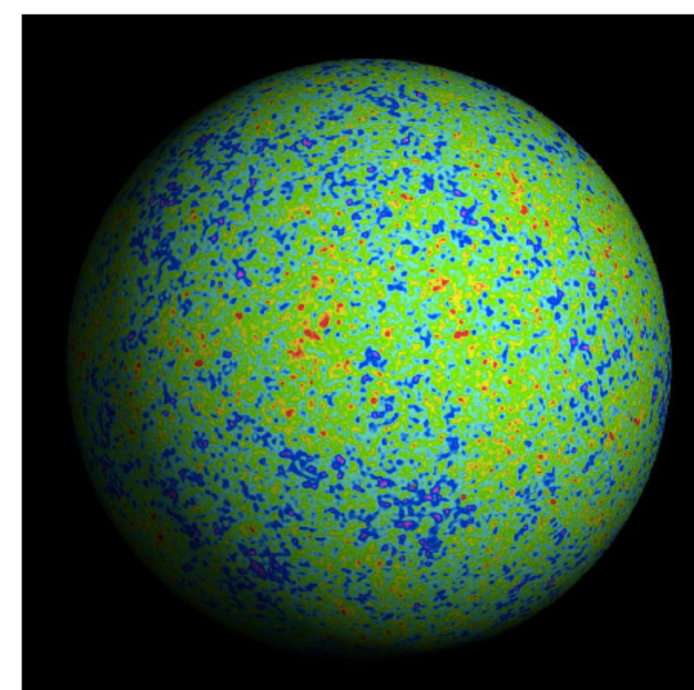
Scalar and tensor "non-Gaussianites"

J. Maldacena (2001)

$$\langle h_{\vec{k}} h_{\vec{q}} h_{\vec{p}} \rangle =$$



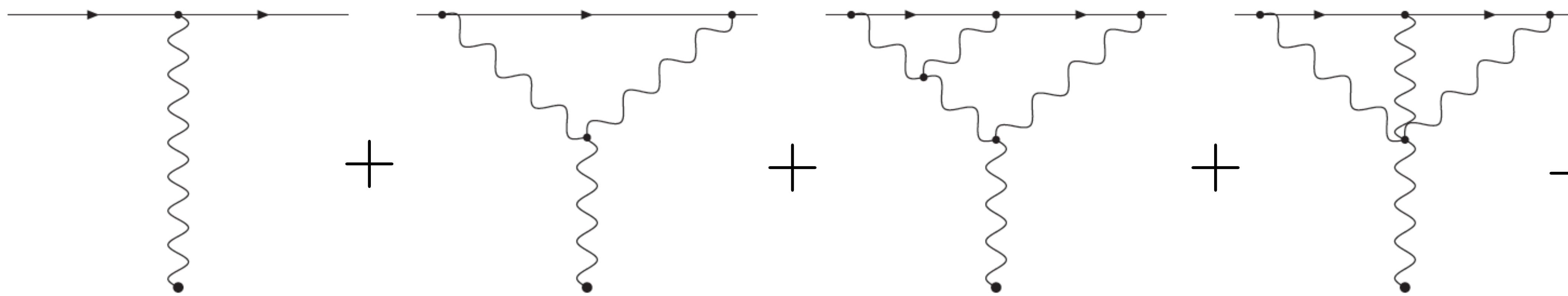
These predictions are relevant for estimates of B-mode polarization effects in the CMB…



As first shown by Duff (1973) the Feynman diagrams of GREFT also reproduce classical solutions in GR:

(NOT A PROPAGATOR!)

$h_{\mu\nu} =$



$+\cdots$

$$g_{00} = 1 - \frac{2G_N m}{r} + 2\left(\frac{G_N m}{r}\right)^2 + 2\left(\frac{G_N m}{r}\right)^3 + \cdots$$

$$g_{ij} = -\delta_{ij}\left[1 + \frac{2G_N m}{r} + 5\left(\frac{G_N m}{r}\right)^2 - \frac{2}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots \right.$$

$$\left. + \frac{x_i x_j}{r^2}\left[7\left(\frac{G_N m}{r}\right)^2 - \frac{4}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots\right]\right]$$

("deDonder coordinates")

w/ $\mathcal{O}(1/r^3)$ "two-loop" terms computed first by Goldberger and Rothstein (2006).

Scale Separation:

The Lorentz covariant GREFT is not optimal to the bound state problem.   There are two distinct kinematic regions relevant to the non-relativistic limit

Potential Exchange: $$k^\mu \sim (v/r, 1/r)$$

Radiation: $$k^\mu \sim (v/r, v/r)$$

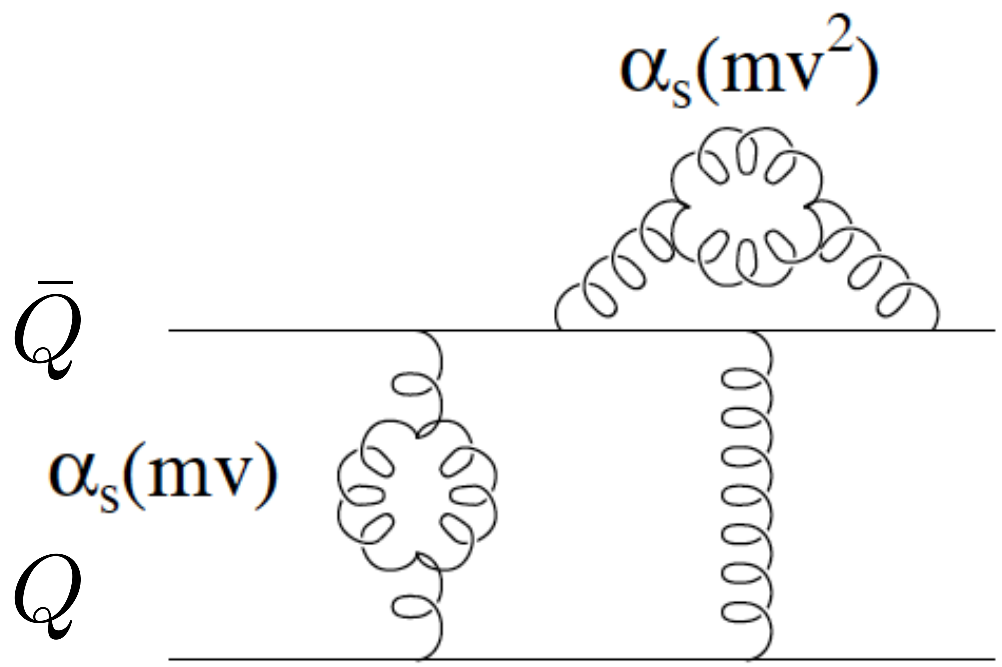
It becomes convenient to reformulate the GREFT by splitting up the fields into modes with non-overlapping support in momentum space:

$$h_{\mu\nu} = h_{\mu\nu}^{potential} + h_{\mu\nu}^{rad}$$

The basic idea is that the potentials are short distance modes, and can be "integrated out". This technique is borrowed from EFTs for heavy mass bound states in QED ($e^+e^-, \mu^+\mu^-$) and in QCD ($Q\bar{Q} = c\bar{c}, b\bar{b}, t\bar{t}$):

"NRQED" "NRQCD"

(Bodwin et al (1994); Luke, Manohar, Rothstein (1997))



(Manohar and Stewart)

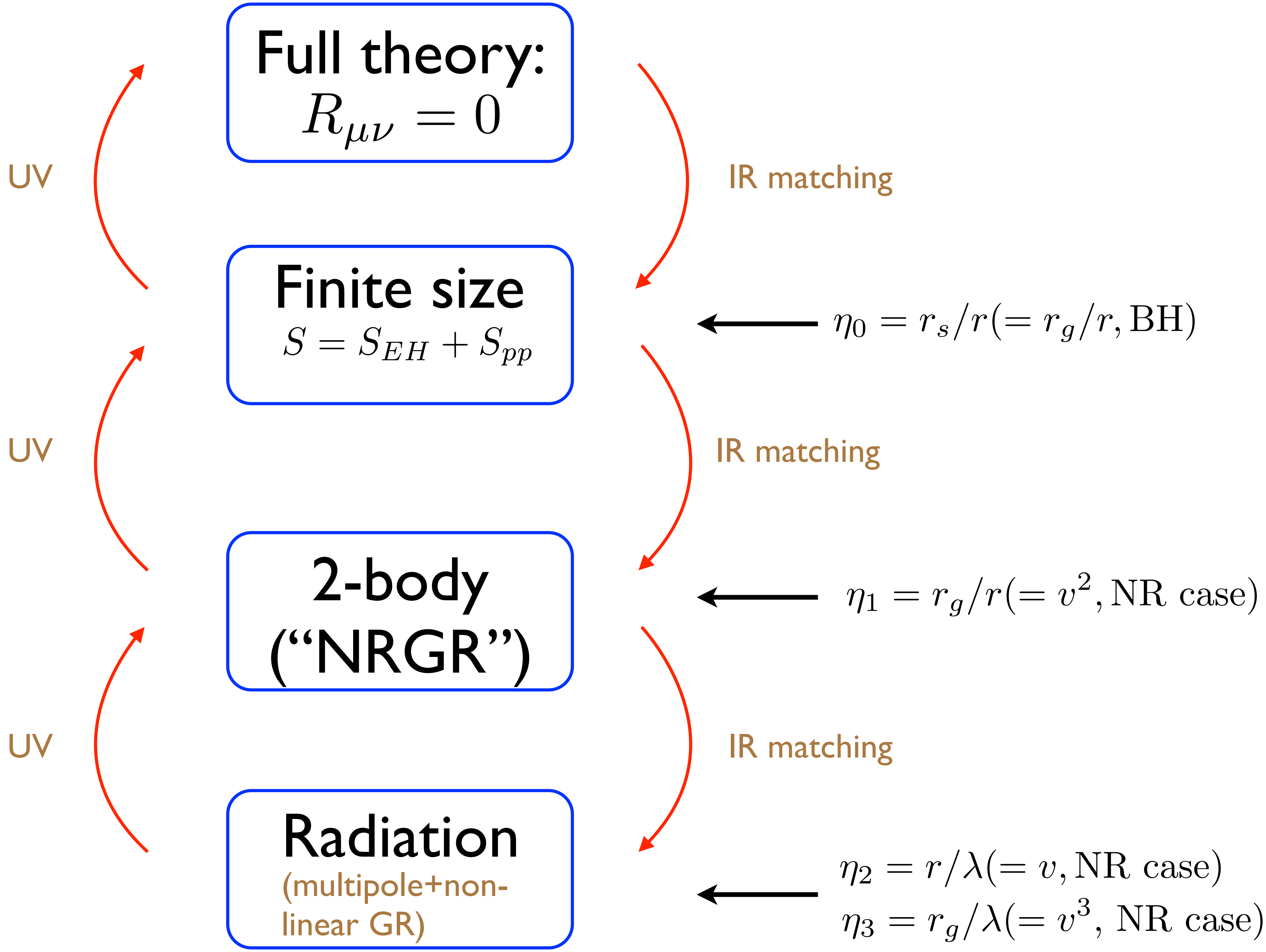
Double expansion in $\Lambda_{QCD}/m_Q \ll 1$ and $v \sim \alpha_s(m_Q) \ll 1$

$$A_\mu = A_\mu^{potential} + A_\mu^{rad}$$

We found in gravity a similar splitting is useful, with

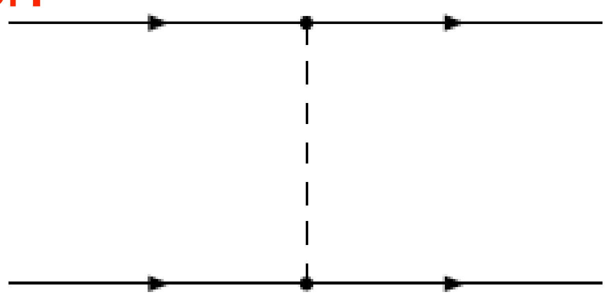
$$\boxed{\begin{array}{c} Q, \bar{Q} \to BH \\ A_\mu \to h_{\mu\nu} \end{array}}$$

Tower of gravity EFTs:



Independent EFTs with distinct expansion parameter coincide in PN limit.
UV divergence in EFT_{i+1} corresponds to IR effect in EFT_i

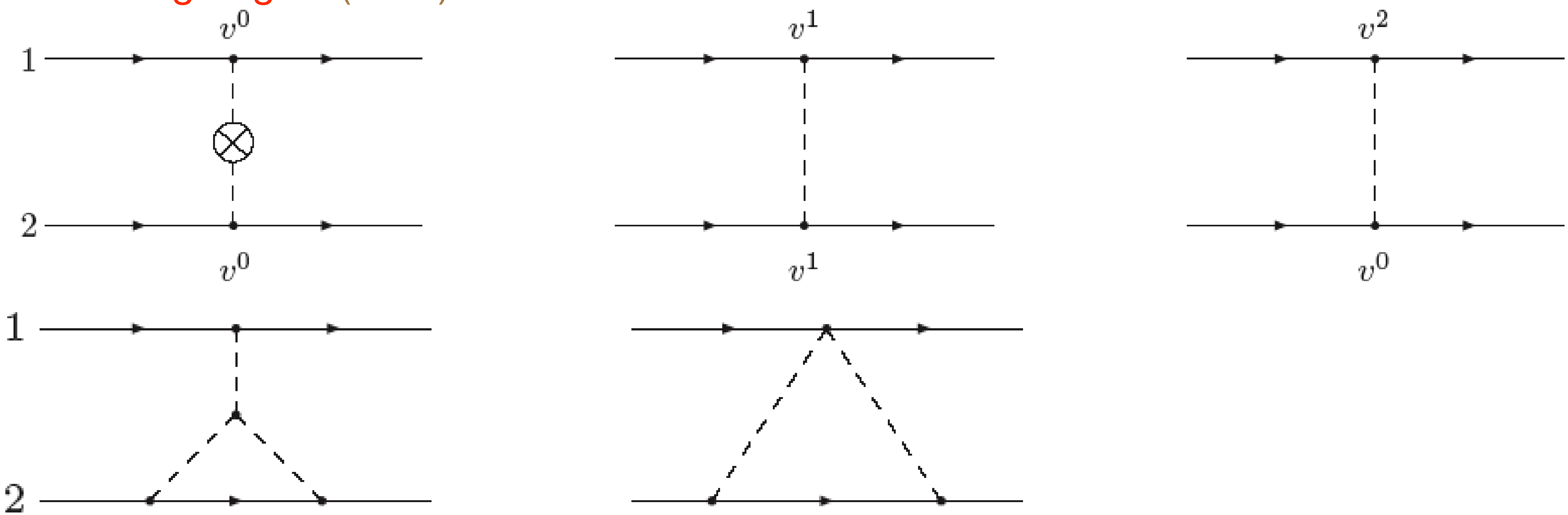
Leading order: Newton (1687)



(a)

$$L = \frac{1}{2}\sum_a m_a \vec{v}_a^{\,2} + \frac{G_N m_1 m_2}{r}$$

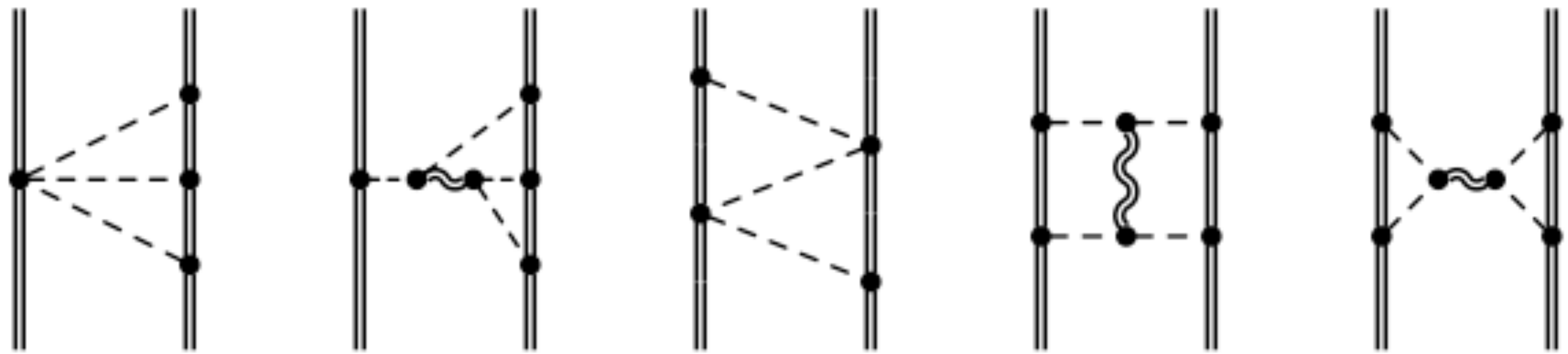
Next-to-leading (1PN): Einstein-Infeld Hoffman Lagrangian (1938)



$$L_{EIH} = \frac{1}{8}\sum_a m_a \vec{v}_a^{\,4} + \frac{G_N m_1 m_2}{2r}\left[3(\vec{v}_1^{\,2} + \vec{v}_2^{\,2}) - 7\vec{v}_1\cdot\vec{v}_2 - (\vec{v}_1\cdot n)(\vec{v}_1\cdot n)\right]$$

$$-\frac{G_N^2 m_1 m_2}{2r^2}$$

2PN (1981-2002): Some of the diagrams are (Gilmore+Ross, PRD 2008)



$$L_{2PN} = \frac{m_1 \mathbf{v}_1^6}{16}$$
$$+\frac{Gm_1m_2}{r}\left(\frac{7}{8}\mathbf{v}_1^4 - \frac{5}{4}\mathbf{v}_1^2\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{4}\mathbf{v}_1^2\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 + \frac{3}{16}\mathbf{v}_1^2\mathbf{v}_2^2 + \frac{1}{8}(\mathbf{v}_1\cdot\mathbf{v}_2)^2\right.$$
$$\left. - \frac{1}{8}\mathbf{v}_1^2(\mathbf{n}\cdot\mathbf{v}_2)^2 + \frac{3}{4}\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{3}{16}(\mathbf{n}\cdot\mathbf{v}_1)^2(\mathbf{n}\cdot\mathbf{v}_2)^2\right)$$
$$+Gm_1m_2\left(\frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}\mathbf{v}_2^2 + \frac{3}{2}\mathbf{a}_1\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 - \frac{7}{4}\mathbf{a}_1\cdot\mathbf{v}_2\mathbf{n}\cdot\mathbf{v}_2 - \frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}(\mathbf{n}\cdot\mathbf{v}_2)^2\right)$$
$$+Gm_1m_2r\left(\frac{15}{16}\mathbf{a}_1\cdot\mathbf{a}_2 - \frac{1}{16}\mathbf{a}_1\cdot\mathbf{n}\mathbf{a}_2\cdot\mathbf{n}\right)$$
$$+\frac{G^2m_1m_2^2}{r^2}\left(\frac{7}{4}\mathbf{v}_1^2 + 2\mathbf{v}_2^2 - \frac{7}{2}\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{1}{2}(\mathbf{n}\cdot\mathbf{v}_1)^2\right)$$
$$+\frac{G^3m_1m_2^3}{2r^3} + \frac{3G^3m_1^2m_2^2}{2r^3} + (1\leftrightarrow 2),$$

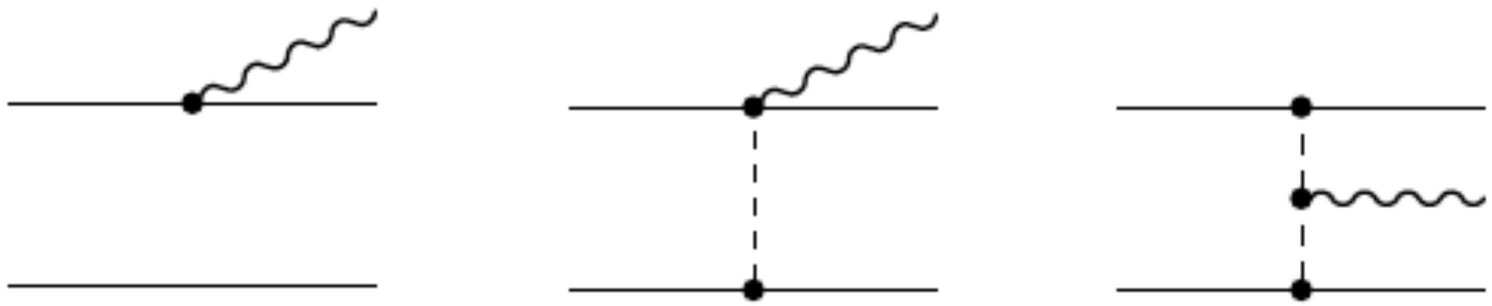
reducible to one-loop integrals via IBP:

$$\int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{1}{[(\mathbf{k}+\mathbf{p})^2]^\alpha[\mathbf{k}^2]^\beta}$$

(simplification of PT via field redefs: B. Kol+M. Smolkin, 2007-2008.)

Radiation in the two-body sector

(WG+Ross,2010)

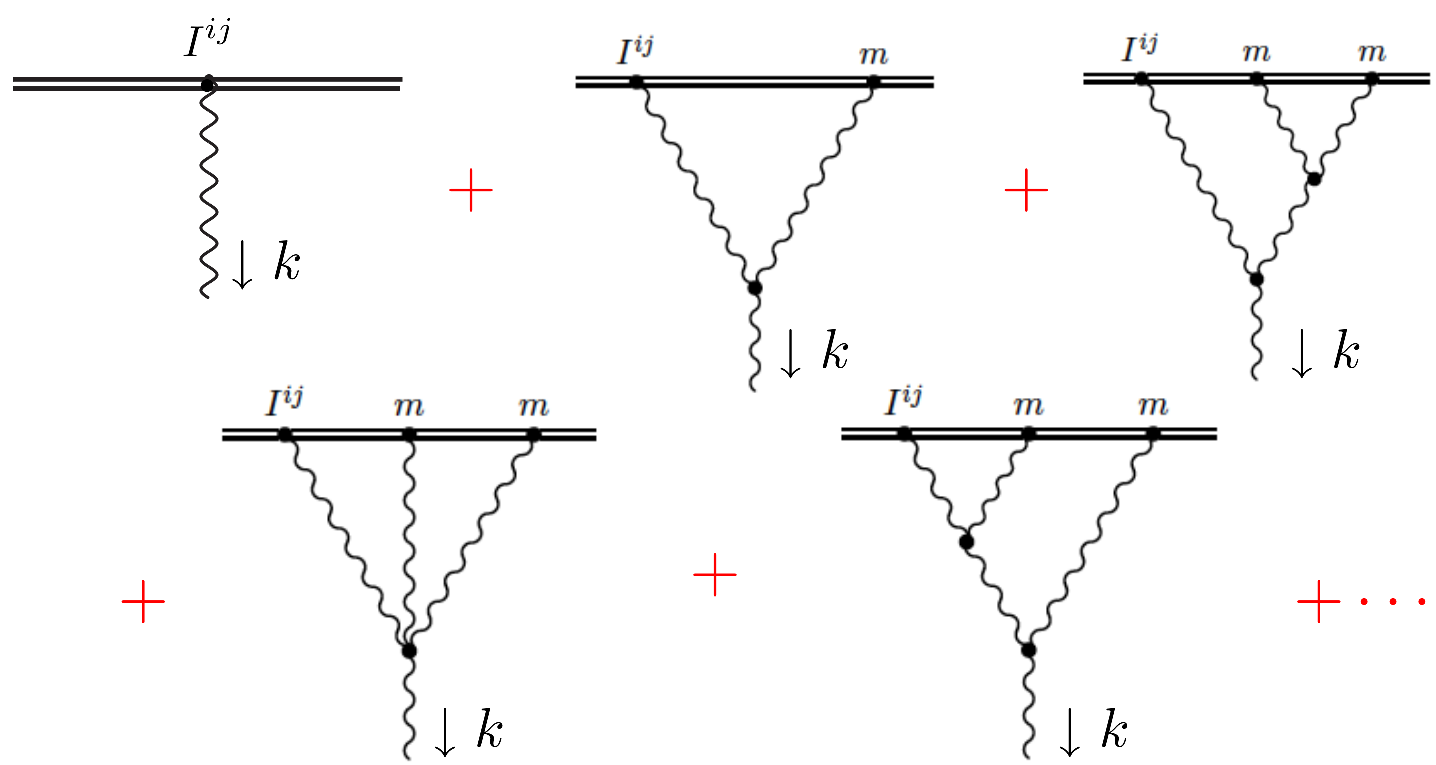


Quadrupole moment

$$I_{ij} = \sum_a m_a \mathbf{x}_a^i \mathbf{x}_a^j \left[1 + \frac{3}{2}\mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \frac{11}{42}\sum_a m_a \frac{d^2}{dt^2}(\mathbf{x_a}^2 \mathbf{x}_a^i \mathbf{x}_a^j)$$
$$-\frac{4}{3}\sum_a m_a \frac{d}{dt}(\mathbf{x_a}\cdot\mathbf{v}_a \mathbf{x}_a^i \mathbf{x}_a^j) - \text{traces} + \mathcal{O}(v^4)$$

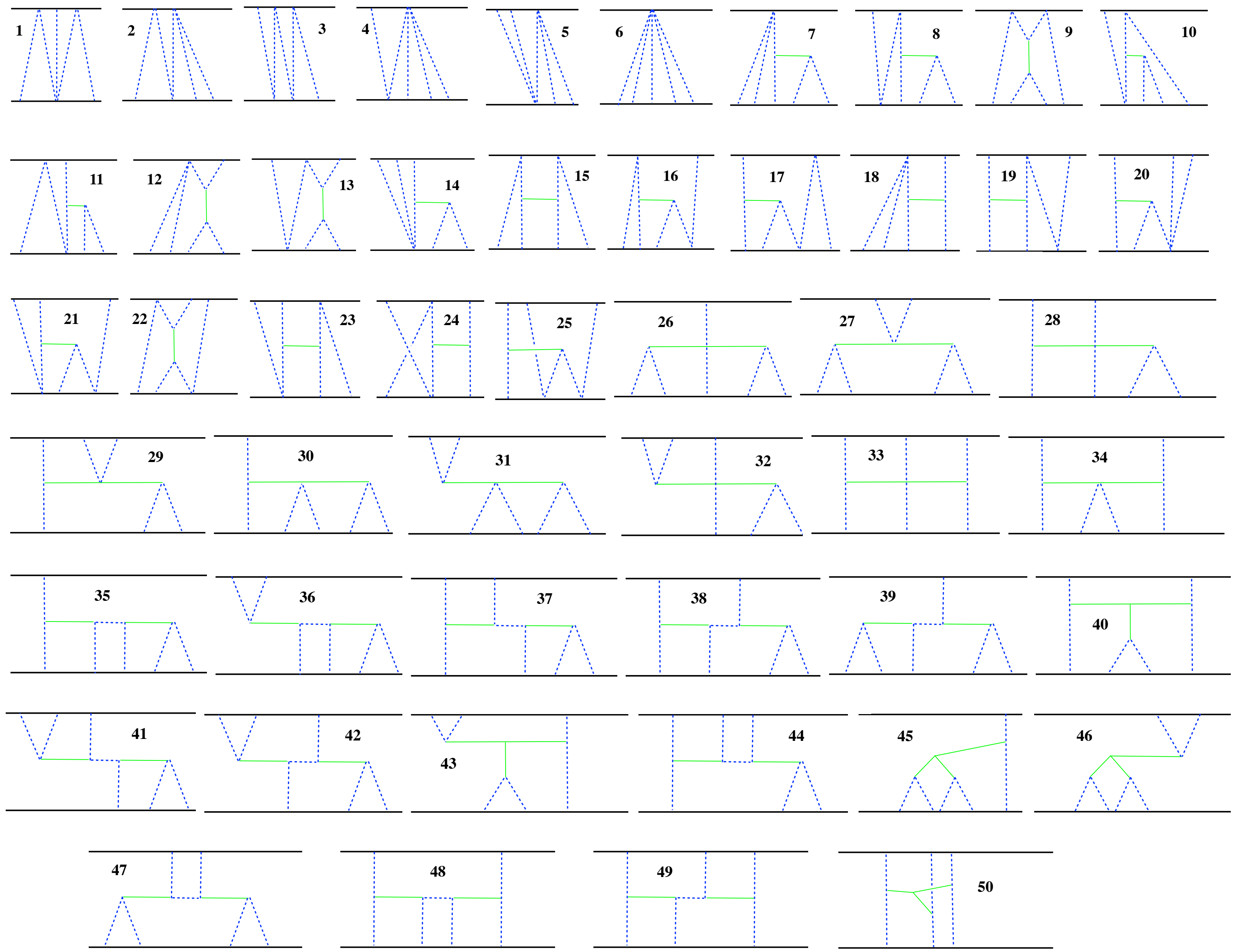
Quadrupole Radiation:

$h_{ij}^{TT} =$



$$\frac{\dot{E}_{3PN,LL}}{\dot{E}_{LO}} = -\frac{1712}{105}v^6 \ln v$$

State of the art: Potentials at 4PN (Foffa, Sturani, Mastrolia, Sturm, PRD 2017). All diagram topologies



static part of the 2-body potentials:

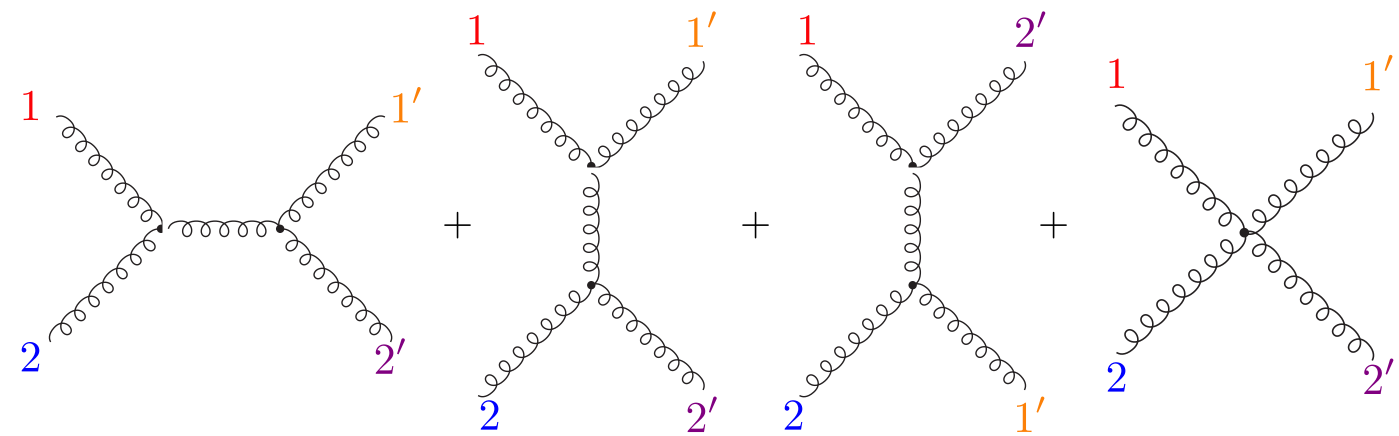
$$\sum_{a=1}^{50} \mathcal{L}_a = \frac{3}{8}\frac{G_N^5 m_1^5 m_2}{r^5} + \frac{31}{3}\frac{G_N^5 m_1^4 m_2^2}{r^5} + \frac{141}{8}\frac{G_N^5 m_1^3 m_2^3}{r^5}\,.$$

Is there an easier way?

Color-Kinematics "Duality"

A hint that there is a hidden simplicity in gravity comes already from DeWitt's 1967 result:

$\mathcal{A} =$

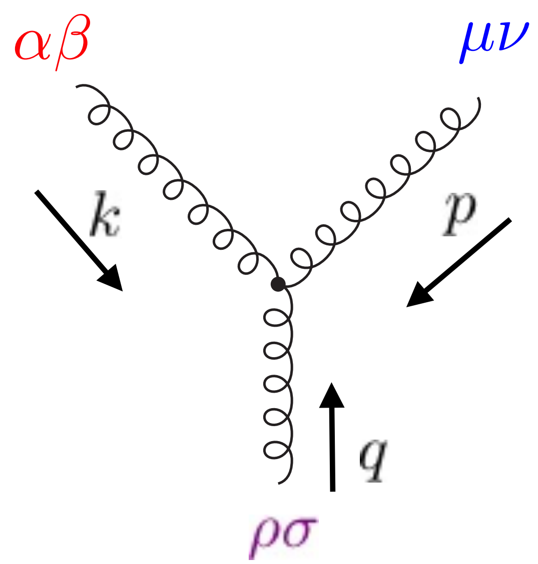


Summing the diagrams and squaring the amplitude yields a simple answer

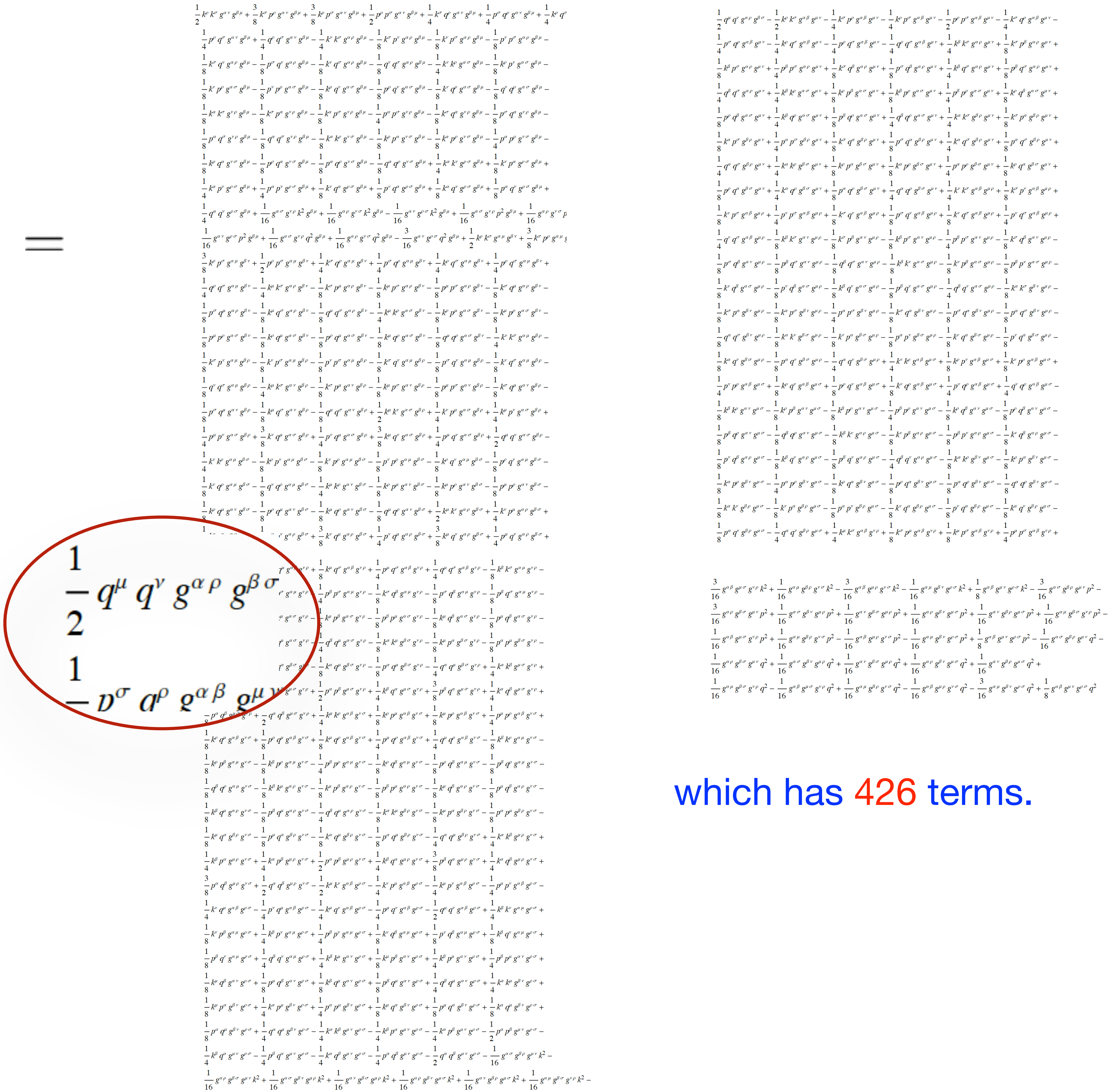
$$\frac{d\sigma}{d\Omega} = 4G_N^2 E_{cm}^2 \left(\frac{32 + \cos^{12}\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} \right)$$

However, the intermediate steps are far from simple….

E.g., the graviton triple self-interaction:



=



which has 426 terms.

deWitt, PRD 1967:

pressions. Nevertheless, a large amount of cancellation between terms still has to be dug out of the algebra, and this, combined with the fact that the final results are ridiculously simple, leads one to believe that there must be an easier way. The cross sections which one finds are

$$\frac{d\sigma_{++}}{d\Omega}=\frac{d\sigma_{--}}{d\Omega}=4G^2E^2\left[\frac{\cos^6\frac{1}{2}\theta}{\sin^2\frac{1}{2}\theta}+\frac{\sin^6\frac{1}{2}\theta}{\cos^2\frac{1}{2}\theta}+4-\tfrac{1}{2}\sin^2\theta\right]^2, \quad (3.19)$$

$$\frac{d\sigma_{+-}}{d\Omega}=4G^2E^2\frac{\cos^{12}\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}, \quad (3.20)$$

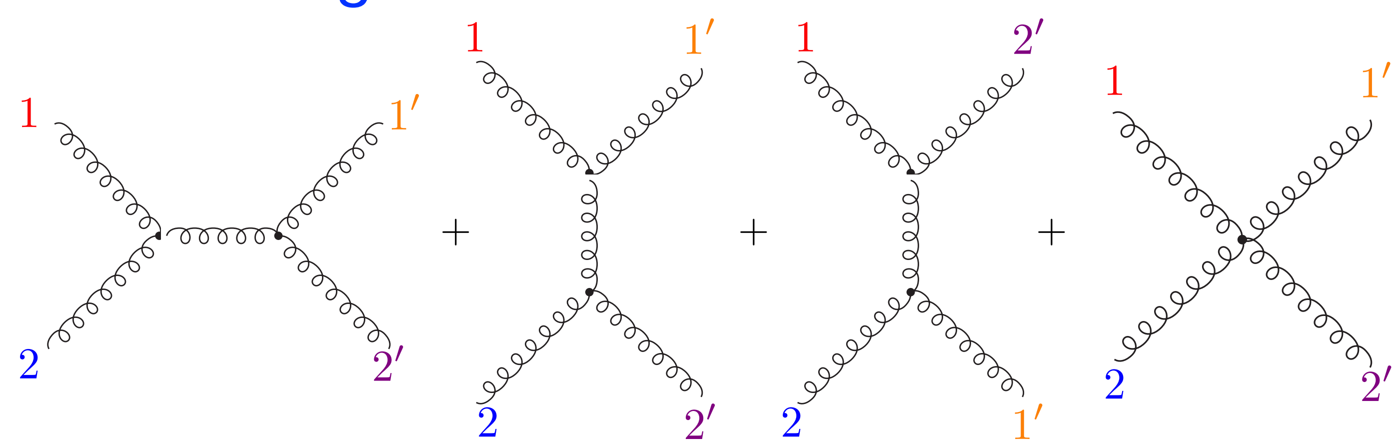
Developments in theoretical particle physics starting with Witten's twistor string theory (2004) have given insight into the "ridiculous simplicity" of results not only in perturbative gravity, but in gauge theory (eg QCD).

Bern-Carrasco-Johansen (BCJ) "duality": (2008)

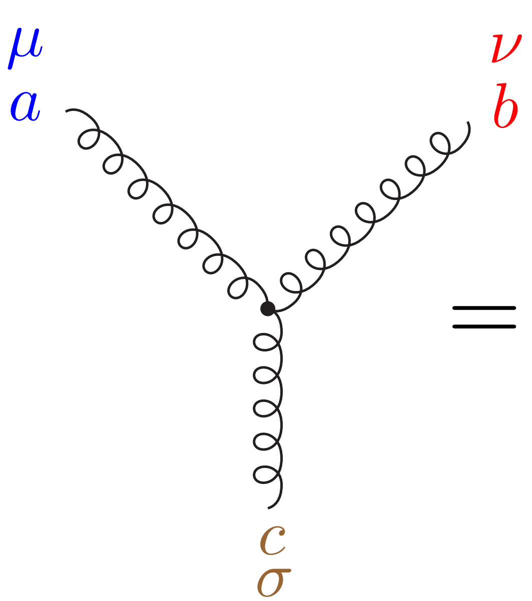
or,

"Gravity is the square of gauge theory"

To understand, look at gluon scattering in QCD:

$\mathcal{A} =$ 

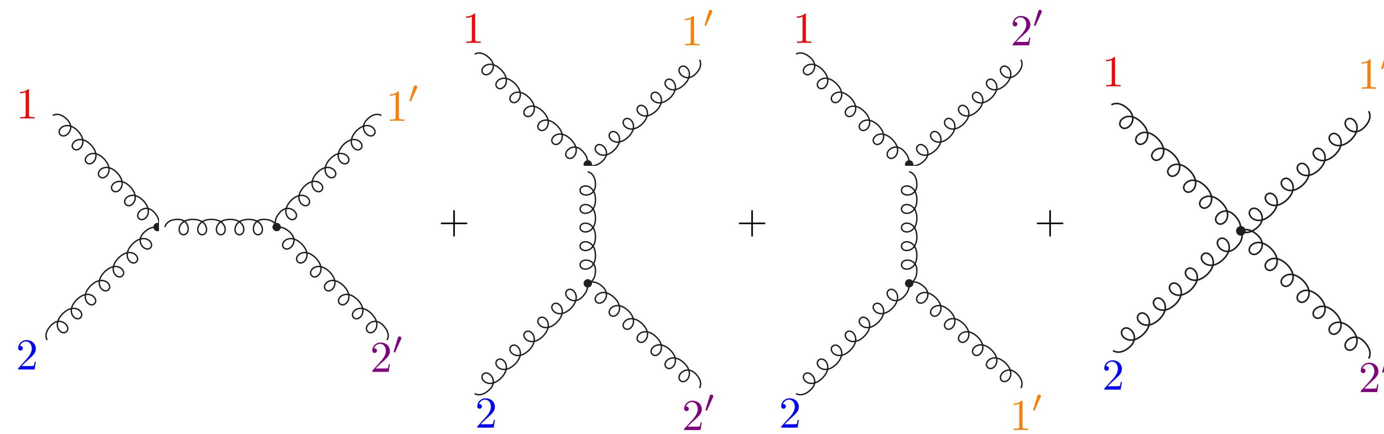
The triple gluon interaction has the structure:



$$= \underbrace{f^{abc}}_{\text{Color}} \cdot \underbrace{\left[g^{\mu\sigma}(k-q)^{\nu} + g^{\mu\nu}(p-k)^{\sigma} + g^{\nu\sigma}(q-p)^{\mu}\right]}_{\text{Kinematics}}$$

$a, b = 1, \cdots 8 =$ # of gluons

The gluon scattering amplitude in “BCJ form”

$\mathcal{A} =$ 

$$\mathcal{A} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

Here s, t, u are the Mandelstam kinematic invariants (CM energy, scattering angle):

$$s + t + u = 0$$

and:

$c_{s,t,u} =$ **Color factor** $\qquad n_{s,t,u} =$ **Kinematic factor**

with

$$c_s + c_t + c_u = 0 \qquad \textbf{(Color “Jacobi id.”)}$$

$$n_s + n_t + n_u = 0 \qquad \textbf{(Kinematic “Jacobi id.”)}$$

The BCJ “double copy”:

BCJ noticed that by applying the color-to-kinematics “duality” transformations:

$$c_s \to n_s \qquad c_t \to n_t \qquad c_u \to n_u$$

the 4-gluon amplitude maps onto

$$\mathcal{A}_4^{YM} \to \hat{\mathcal{A}}_4^{GR} = \frac{n_s n_s}{s} + \frac{n_t n_t}{t} + \frac{n_u n_u}{u}$$

This precisely matches tree-level graviton scattering in GR!

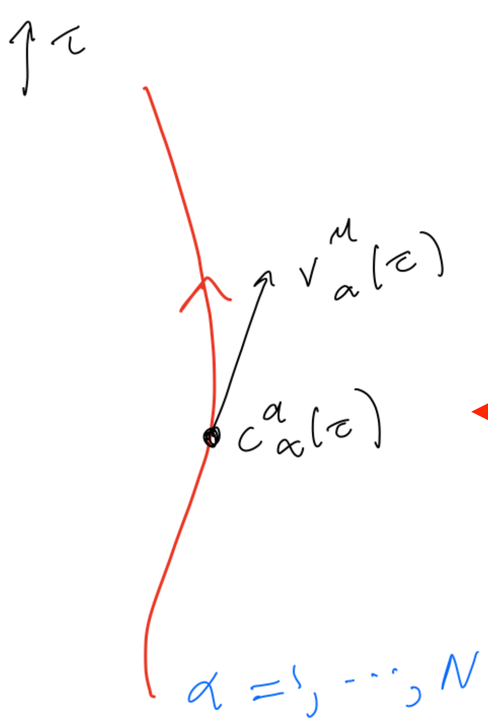
This is the easier way of doing deWitt’s calculation…

The Classical Double Copy:

Goldberger+Ridgway, PRD 2017
Goldberger+Ridgway, PRD 2018
Goldberger, Prabhu, Thompson, PRD 2017
Goldberger, Li, Prabhu, PRD 2018

Does gravity = (gauge theory)^2 provide a “much easier” way of obtaining classical GR solutions?

We’ll check this by brute force, starting with a system of “classical quarks”, interacting self-consistently through gluon exchange:



“Wilson line”

Degrees of Freedom:

$$\psi(\tau) = \begin{pmatrix} r(\tau) \\ g(\tau) \\ b(\tau) \end{pmatrix} \qquad\qquad x^{\mu}(\tau)$$

$$c^{a}(\tau) = \psi^{\dagger} T^{a} \psi \qquad\qquad v^{\mu} = \dot{x}^{\mu}$$

(Color charge) (Orbital)

Here, “classical” means that there is no pair creation $E < 2m_Q$.

This system evolves according to the classical Yang-Mills equations

$$D_\nu F_a^{\nu\mu}(x) = g J_a^\mu(x)$$

$$J_a^\mu(x) = \sum_\alpha c_a(t) v_\alpha^\mu(t) \delta^3(\vec{x} - \vec{x}_\alpha(t))$$

which are a "non-Abelian" version of Maxwell's ($D_\mu = \partial_\mu - igA_\mu$):

$$\vec{D} \cdot \vec{E}^a = J_0^a \qquad \vec{D} \times \vec{E}^a = -D_t \vec{B}^a$$

$$\vec{D} \times \vec{B}^a = \vec{J}^a + D_t \vec{E}^a \qquad \vec{D} \cdot \vec{B}^a = 0$$

Particle dynamics from current, $D_\mu J_a^\mu = 0$, and energy-momentum conservation:

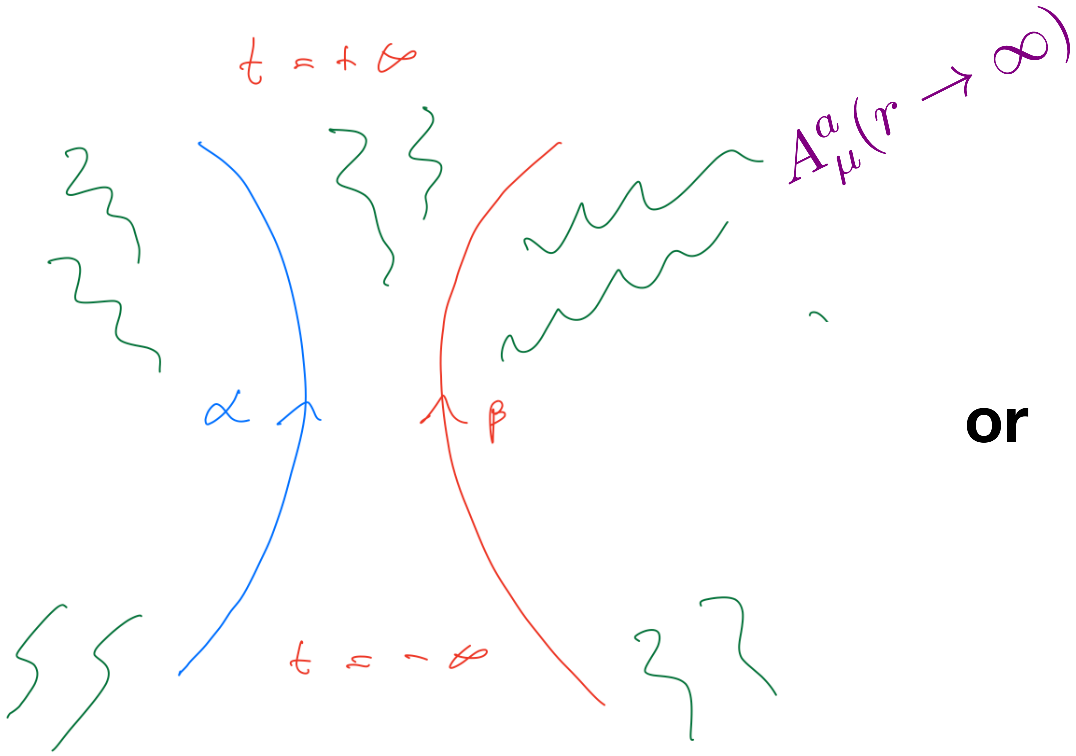
$$\frac{dp^\mu}{d\tau} = g c^a F_a^\mu{}_\nu v^\nu \longrightarrow \frac{d\vec{p}}{dt} = c_a \left(\vec{E}^a + \vec{v} \times \vec{B}^a\right)$$

(Non-Abelian Lorentz force)

$$v \cdot D c^a = 0$$

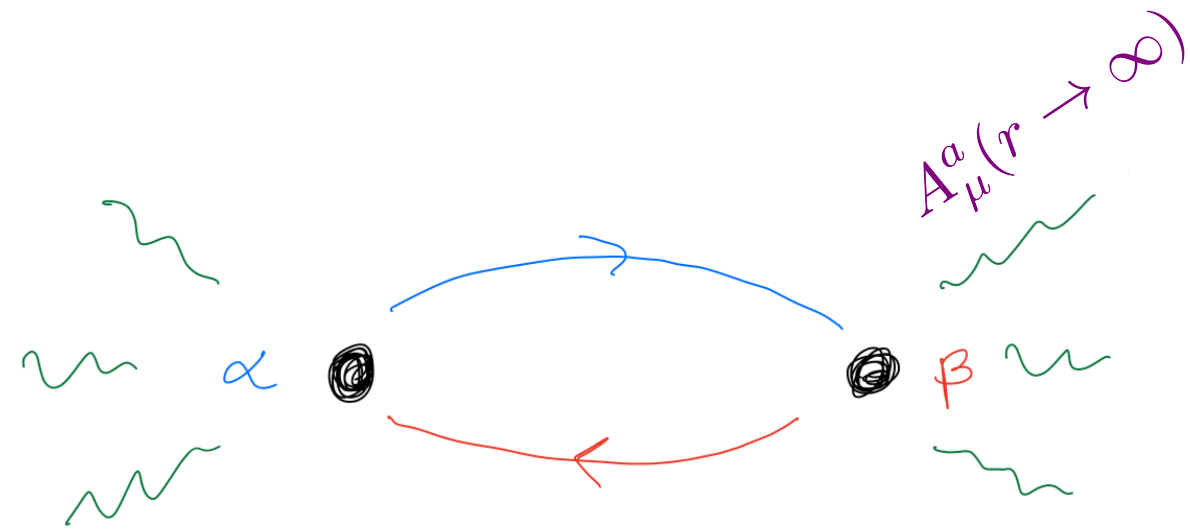
(Local conservation of color charge)

Our interest is in a collection of charges interacting self-consistently and emitting radiation out to infinity:



(classical Bremsstrahlung)

or



($Q\bar{Q}$ binary inspiral)

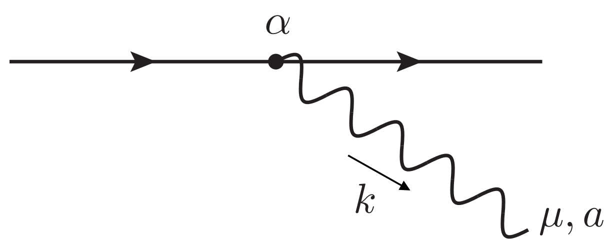
In either case, our focus is on the gluon radiation field measured by detectors at infinity:

$$\lim_{r\to\infty} A_\mu(x) = \frac{g_s}{4\pi r} \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{J}^\mu_a(k)$$

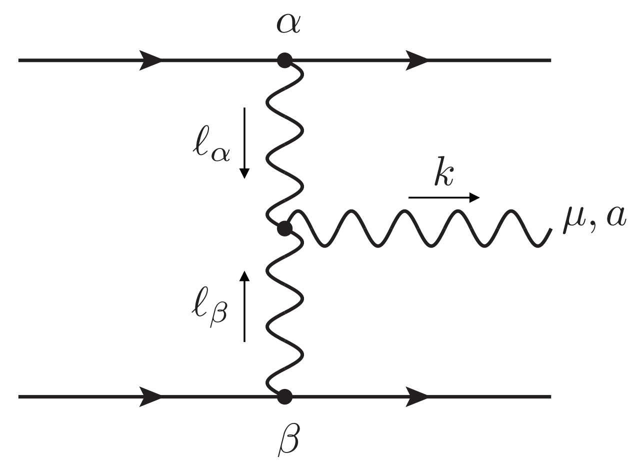
$$k^\mu = \omega(1, \frac{\vec{x}}{r})$$

The classical solution has a representation in terms of Feynman diagrams

$$\tilde{J}^{\mu}_{a}(k) =$$



$$+$$



$$+ \quad \cdots$$

and can be expressed as an integral internal gluon momenta and the particle worldlines:

$$\tilde{J}^{a\mu}(k) = g^2 \sum_{\alpha,\beta} \int_{\alpha,\beta} d\mu_{\alpha\beta}(k) \left[i f^{abc} c^b_\alpha c^c_\beta \mathcal{A}^\mu_{adj} + (c_\alpha \cdot c_\beta) \frac{c^a_\alpha}{m_\alpha} \mathcal{A}^\mu_s \right],$$

Integration measure:

$$d\mu_{\alpha\beta}(k) = d\tau_\alpha d\tau_\beta \left[\frac{d^4\ell_\alpha}{(2\pi)^4} \frac{e^{i\ell_\alpha\cdot x_\alpha}}{\ell^2_\alpha} \right] \left[\frac{d^4\ell_\beta}{(2\pi)^4} \frac{e^{i\ell_\beta\cdot x_\beta}}{\ell^2_\beta} \right] \times (2\pi)^4 \delta(\ell_\alpha + \ell_\beta - k)$$

Partial amplitudes:

$$\mathcal{A}^\mu_{adj,s} = \mathcal{A}^\mu_{adj,s}(\ell_{\alpha,\beta}, v_{\alpha,\beta}, k)$$

Checks:

(1) $k_\mu \tilde{J}^\mu_a = 0$ ✓

(2) classical brems. (Gyulassy+McLerran, 1997) ✓

(3) NR limit: ✓

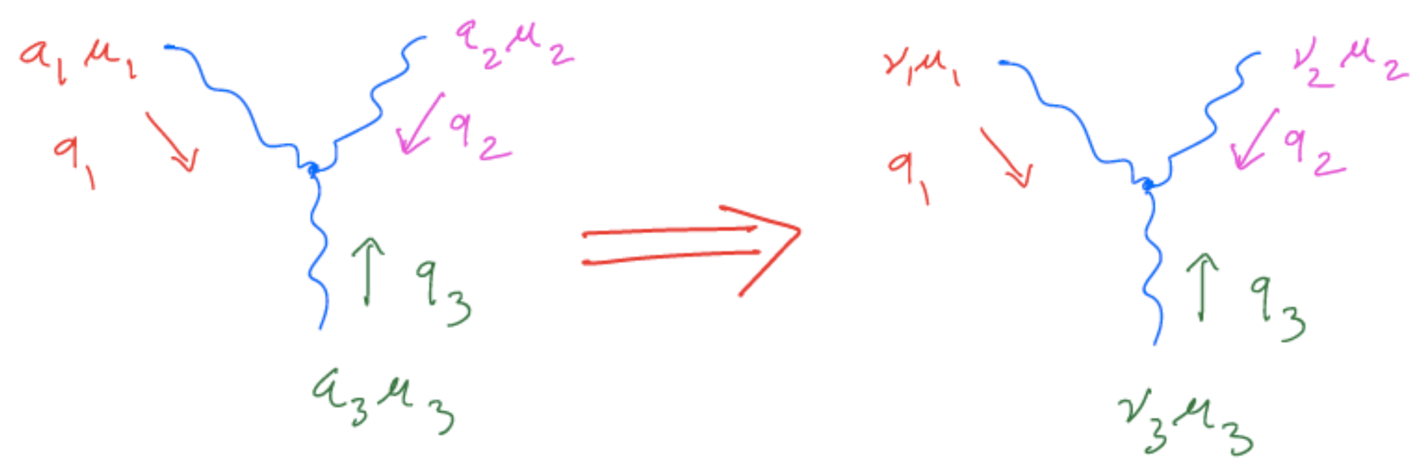
$$\vec{J}^a(x) = \delta^3(\vec{x})\dot{\vec{p}}^a(t) \qquad \vec{p}^a(t) = \sum_\alpha c^a_\alpha \vec{x}_\alpha$$

(Electric color dipole radiation)

Color-to-kinematics map:

Inspired by BCJ, we make the following formal replacements to the gauge theory result

$$\boxed{c^a_\alpha(\tau) \to i p^\mu_\alpha(\tau)}$$



$$i f^{a_1 a_2 a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = -\frac{1}{2}\left[\eta^{\nu_1\nu_3}(q_1 - q_3)^{\nu_2} + \eta^{\nu_1\nu_2}(q_2 - q_1)^{\nu_3} + \eta^{\nu_2\nu_3}(q_3 - q_2)^{\nu_1}\right],$$

$$\boxed{p^\mu_\alpha(\tau) \to p^\mu_\alpha(\tau)}$$

$$\boxed{g \to \frac{1}{2 m_{Pl}^{d/2-1}}}$$

This mapping produces a gravitational source

$$\tilde{J}^{\mu}_{a}(k) \to i\tilde{T}^{\mu\nu}(k)$$

$$\boxed{\begin{aligned}\tilde{T}^{\mu\nu}(k) = \frac{1}{4m_{Pl}^{d-2}} \sum_{\alpha,\beta} m_\alpha m_\beta \int d\mu_{\alpha\beta}(k) \Bigg[\left(\frac{1}{2}(v_\alpha \cdot v_\beta)(\ell_\beta - \ell_\alpha)^\nu + (v_\beta \cdot k) v_\alpha^\nu - (v_\alpha \cdot k) v_\beta^\nu\right) \mathcal{A}^\mu_{adj} \\ -(v_\alpha \cdot v_\beta) v_\alpha^\nu \hat{\mathcal{A}}^\mu_s\Bigg]\end{aligned}}$$

that is a consistent perturbative solution of Einstein's equations of gravity:

$$\tilde{T}^{\mu\nu}(k) = \tilde{T}^{\nu\mu}(k) \qquad\qquad k_\mu \tilde{T}^{\mu\nu}(k) = 0$$

It yields spin-2 gravitational waveform:

$$h_\pm(t, \vec{n}) = \frac{4G_N}{r} \int \frac{d\omega}{2\pi} e^{-i\omega t} \epsilon^{*\mu\nu}_{\pm}(k) \tilde{T}_{\mu\nu}(k),$$

Who ordered that?

Our mapping between QCD and gravity also implies a spin-0 wave

$$\phi(t,\vec{n}) = \frac{G_N}{r}\int \frac{d\omega}{2\pi} e^{-i\omega t}\tilde{T}^{\mu}{}_{\mu}(k)$$

that is not there in Einstein's theory of gravity. Which gravity theory is it?

We (WG+Ridgway, PRD (2017)), showed that the radiation fields correspond to a scalar-tensor gravity theory known as dilaton gravity

$$S = S_g + S_{pp}$$

$$S_g = -2m_{Pl}^{d-2}\int d^dx\sqrt{g}\left[R - (d-2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

$$S_{pp} = -m\int d\tau e^{\phi} + \cdots\,,$$

This is perhaps not completely unexpected given results by Bern+Grant(1999); Scherk+Schwarz (1974) and Kawai et al (1986), as well as the BCJ mapping:

$$A_\mu \otimes A_\nu = \phi \oplus B_{\mu\nu} \oplus h_{\mu\nu}$$

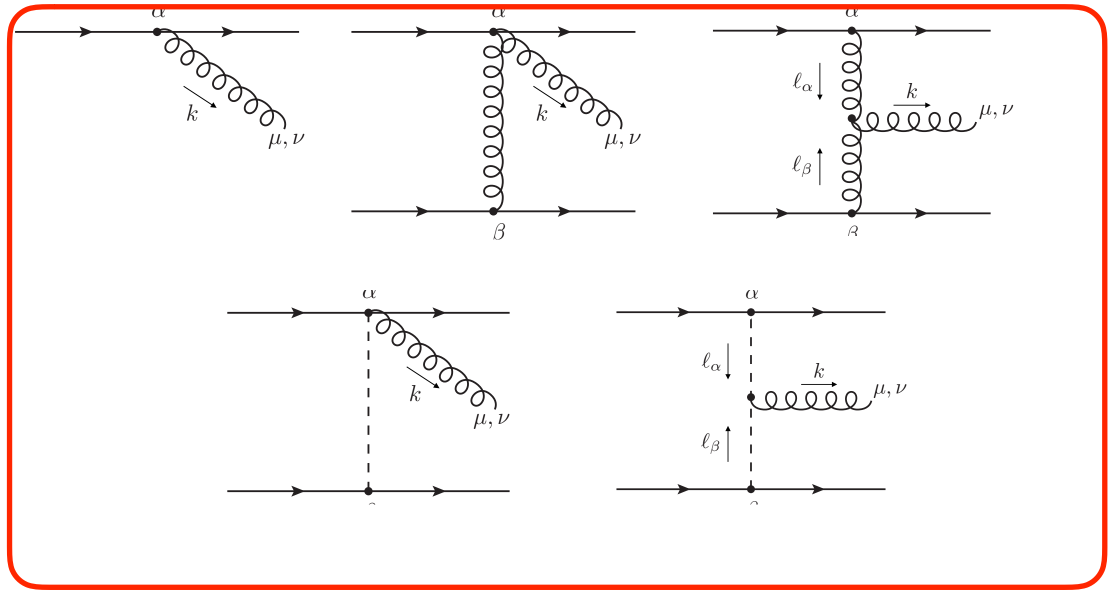
$\phi =$ dilaton

$B_{\mu\nu} = -B_{\nu\mu} =$ axion

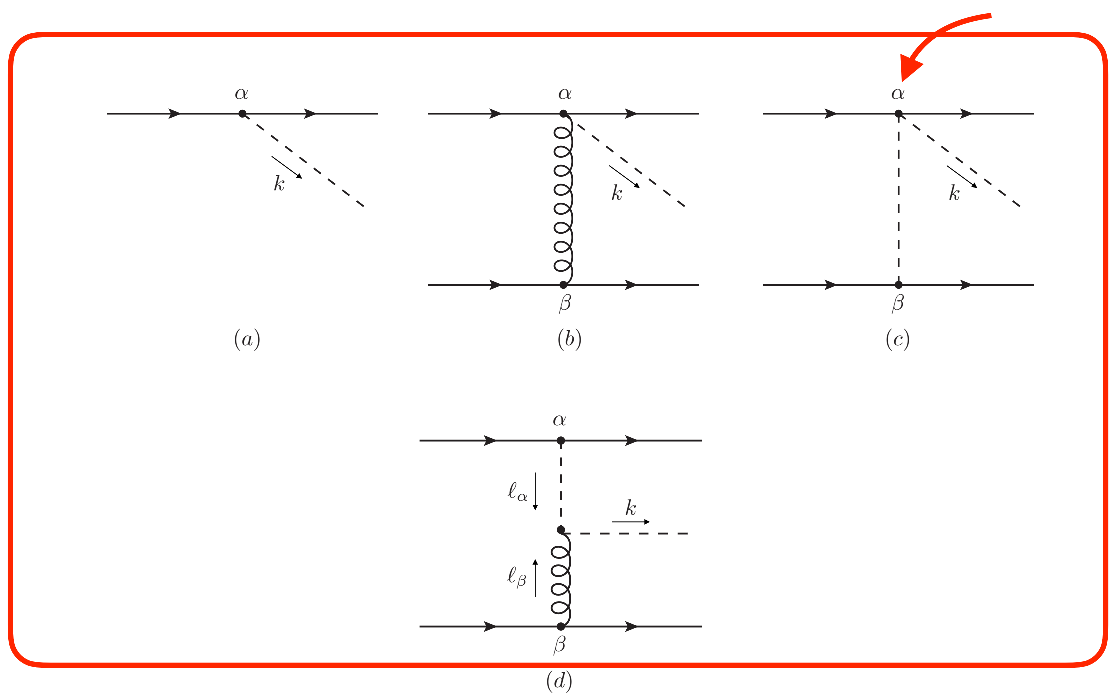
between QCD and gravity.

We verified this fact by explicit calculation in the dilaton-gravity theory:

graviton emission channel:



scalar channel:



Check: PN limit in 4D

$$Q^{ij} = \sum_\alpha m_\alpha \left(x^i_\alpha x^j_\alpha - \frac{1}{3}\delta^{ij}\vec{x}^2_\alpha \right)$$

$$h_{ij}(t,\vec{n}) = \frac{2G_N}{r}[\ddot{Q}_{ij}(t)]^{TT}$$

$$\phi(t,\vec{n}) = \frac{G_N}{r}\sum_\alpha m_\alpha \left(\vec{v}^2_\alpha - \vec{x}_\alpha \cdot \ddot{\vec{x}}_\alpha - \frac{1}{2}\frac{d^2}{dt^2}(\vec{x}_\alpha \cdot \vec{n})^2 \right),$$

agrees with (Will+Zaglauer 1989; Damour+Esposito-Farrese, 1992)

Axion radiation and particle spin

Goldberger, Li, Prabhu PRD 2018

If our classical quarks carry spin, it is possible to introduce a “chromo-magnetic” dipole interaction with the gluon field

$$S_{int} = \frac{1}{2} g_s \kappa \int d\tau c_a S_{\mu\nu} F_a^{\mu\nu}$$

This is the relativistic version of the more familiar (in 4D) magnetic dipole interaction

$$S_{int} = -\frac{g_s \kappa}{m} \int dt c_a \vec{S} \cdot \vec{B}^a$$

with Lande g-factor given by $g_L = -2\kappa$

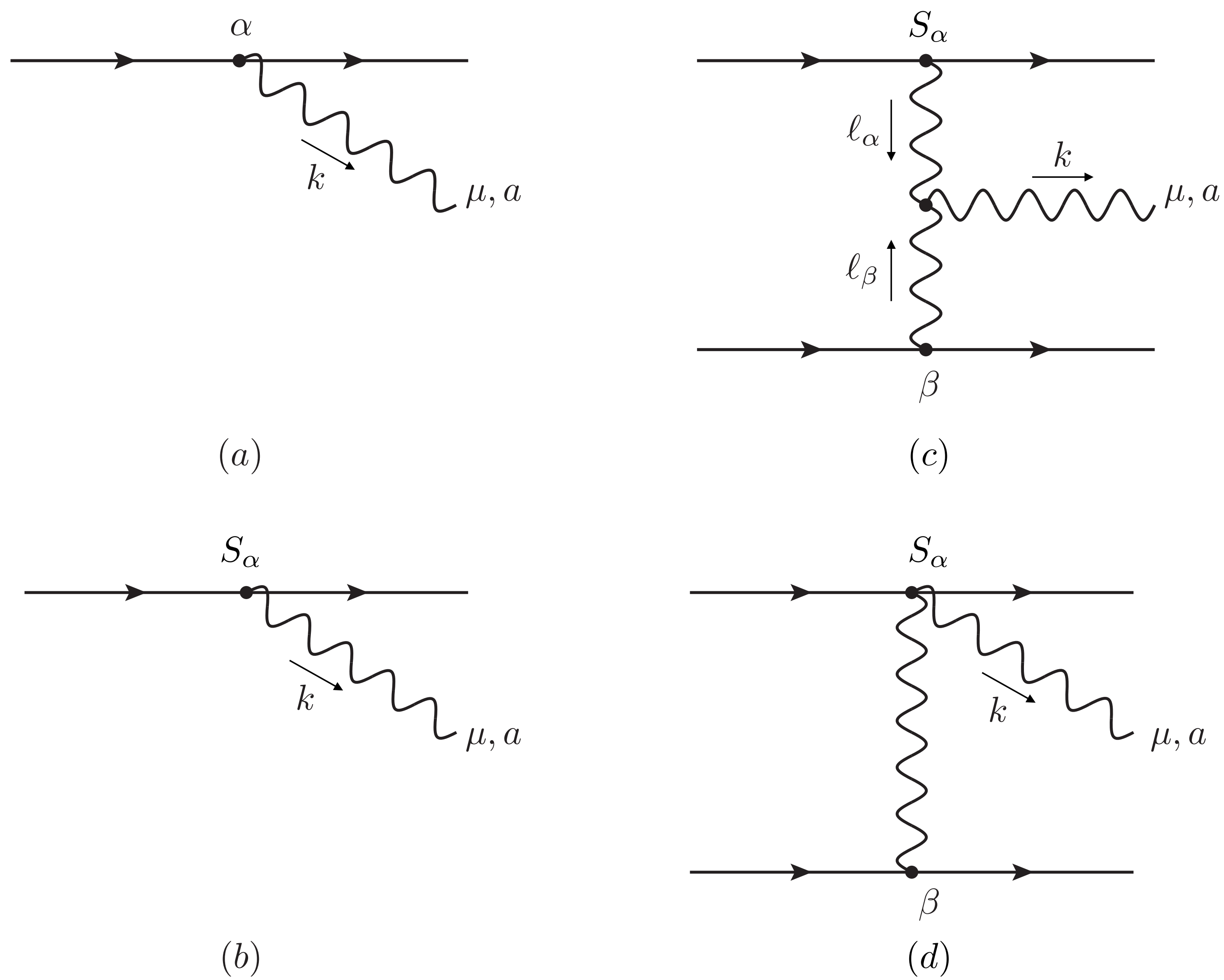
Eqns of motion from conservation laws:

$$D_\mu J^\mu_a = 0 \quad \Longrightarrow \quad (v\cdot D)c^a = \frac{i}{2} g_s \kappa [S^{\mu\nu}F_{\mu\nu}, c]^a$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \begin{aligned} \frac{d}{ds}p^\mu &= g_s c^a F^{\mu\nu}_a v_\nu - \frac{1}{2}\kappa g_s c_a S^{\alpha\beta} D^\mu F^a_{\alpha\beta} \\ \frac{d}{ds}S^{\mu\nu} &= p^\mu v^\nu - p^\nu v^\mu - 2\kappa g_s c^a F_a^{\alpha[\mu} S_\alpha{}^{\nu]} \end{aligned}$$

Then the total current at $\mathcal{O}(g^2)$, working to linear order in spin:

$$\tilde{J}^\mu_a(k) =$$



Double copy

Same formal replacement rules as before, w/. $S^{\mu\nu} \to S^{\mu\nu}$ yields a gravitational field

$$\tilde{J}^\mu_a(k) \to i\tilde{T}^{\mu\nu}(k)$$

which is again of the form

$$\tilde{T}^{\mu\nu}(k) = \frac{1}{4m_{Pl}^{d-2}} \sum_{\alpha,\beta} \int d\mu_{\alpha\beta}(k) \Big[\Big(\frac{1}{2}(p_\alpha \cdot p_\beta)(\ell_\beta - \ell_\alpha)^\nu + (p_\beta \cdot k) p^\nu_\alpha - (p_\alpha \cdot k) p^\nu_\beta \Big) \mathcal{A}^\mu_{adj}$$

$$-(p_\alpha \cdot p_\beta) p^\nu_\alpha \mathcal{A}^\mu_s \Big]$$

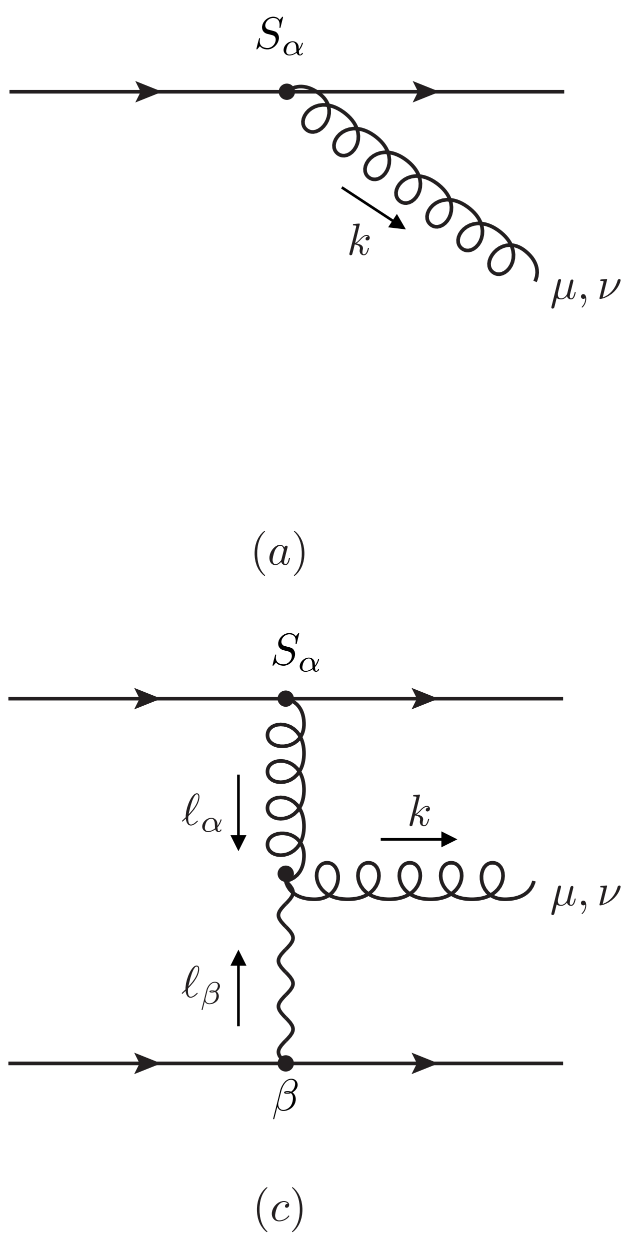
Once again, this encodes spin-2 (graviton) and spin-0 (dilaton) waves. In addition, there is radiation in the (anti-symmetric) axion channel

$$\tilde{T}^{\mu\nu}(k) - \tilde{T}^{\nu\mu}(k) \neq 0$$

So turning on spin, we are now sensitive to all the massless fields in the decomposition

$$A_\mu \otimes A_\nu = \phi \oplus B_{\mu\nu} \oplus h_{\mu\nu}$$

Focusing on axion radiation, the double copy can be used to read off the interactions of the gravitational theory:



curly=axion wavy=graviton dashed=dilaton

In the spinning case, theoretical consistency of the double-copy map imposes constraints on the form of the gravitational interactions:

$$\boxed{S_g = -2m_{Pl}^{d-2}\int d^d x\sqrt{g}\left[R-(d-2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{12}e^{-4\phi}H_{\mu\nu\sigma}H^{\mu\nu\sigma}\right]}$$

with point-particle, axion interaction:

$$\boxed{S_{pp} = \frac{1}{2}\int dx^\mu H_{\mu\nu\sigma}S^{\nu\sigma}}$$

(which reduces to 4D non-relativistic axion coupling to spin, of the form $H_{int} \propto \vec{S}\cdot\nabla a$)

This Lagrangian precisely matches the low energy limit of (closed) string theory, which contains the massless gravitational fields $(\phi, g_{\mu\nu}, B_{\mu\nu})$ (Scherk, Schwarz 1974) , i.e:

$$\boxed{(\text{Classical QCD})^2 = \text{Low energy string theory}}$$

Open questions:

- Classical correspondence at higher orders in perturbation theory? ✓ See Shen, 1806.07388.
- First principles understanding? Can it be derived from string theory?
- We see the same gravity theory emerge from color-kinematics duality as BCJ. What is the precise connection with scattering amplitudes
- Can we efficiently remove the extra gravitational modes in order to recover pure GR solutions? (see D. O'Connell et. al, 1711.03901; Johansson+Ochirov 1407.4772)

Summary and conclusions

- We are entering a golden era of GW astrophysics.
- EFT methods from QCD play an important role in compact binary inspirals.
- Color-to-kinematics relates “simple” pQCD calculation to hard (string) gravity one.