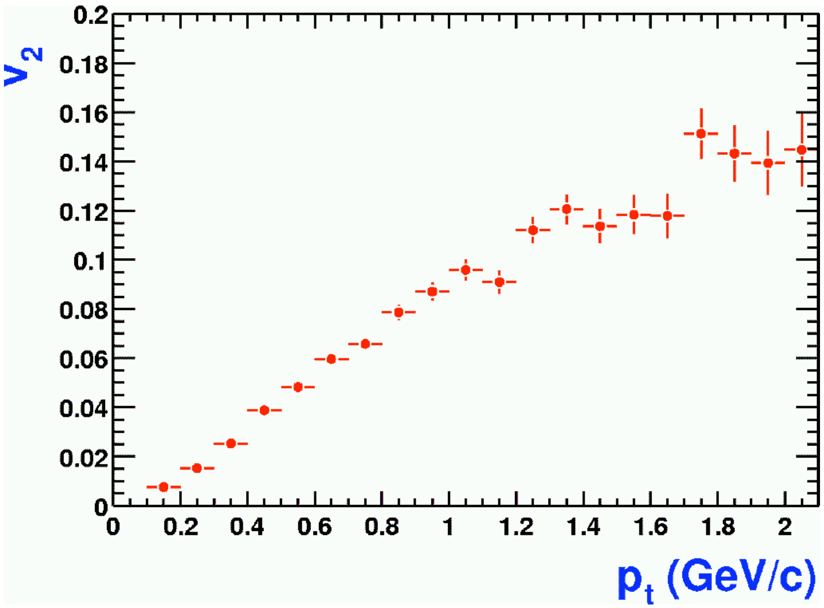
Flow Analysis Methods

Color by Roberta Weir

Exploring the secrets
of the universe

Art Poskanzer

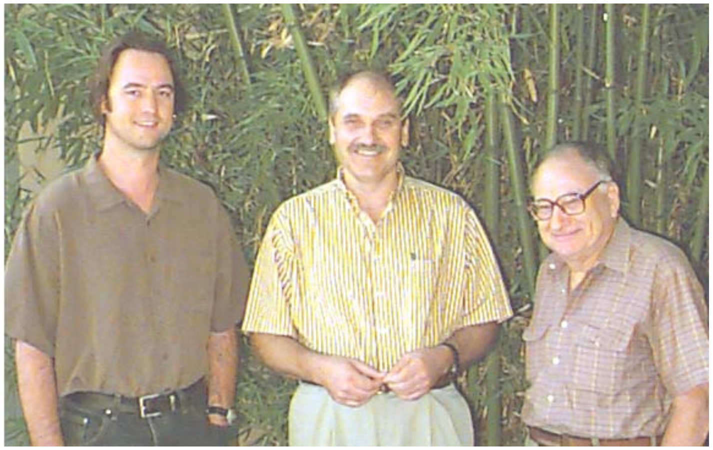
First RHIC Elliptic Flow



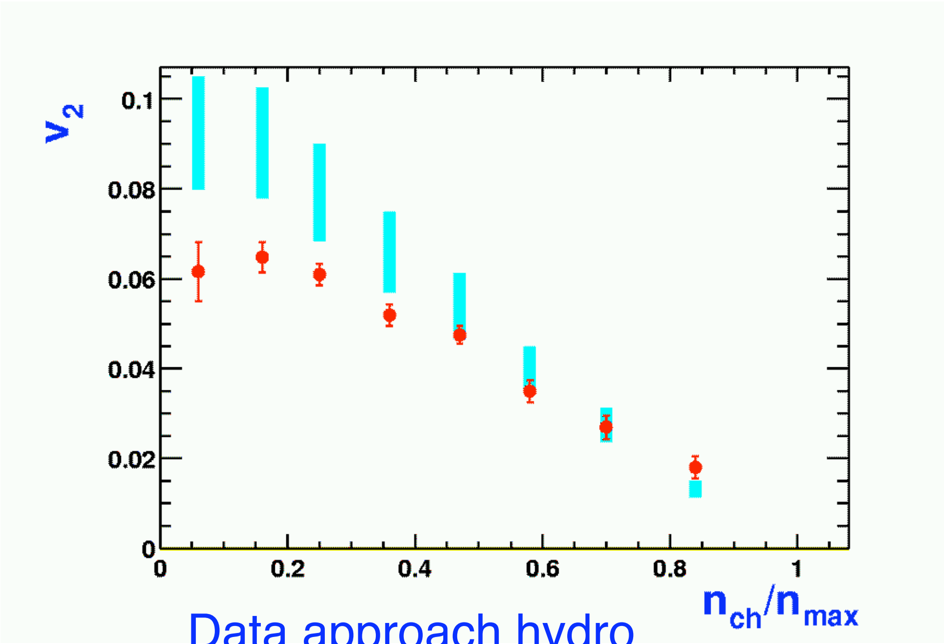
First paper from STAR

22 k events

valley / peak = 1 - 4 v_2



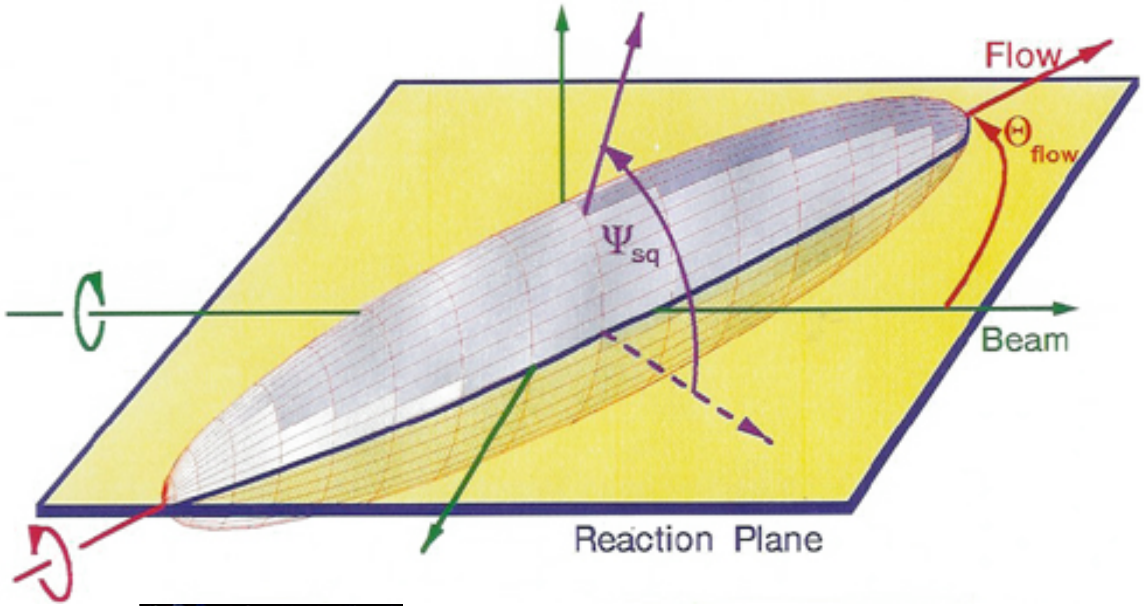
Snellings Voloshin Poskanzer



Data approach hydro
for central collisions

STAR, K.H. Ackermann et al., PRL **86**, 402 (2001)

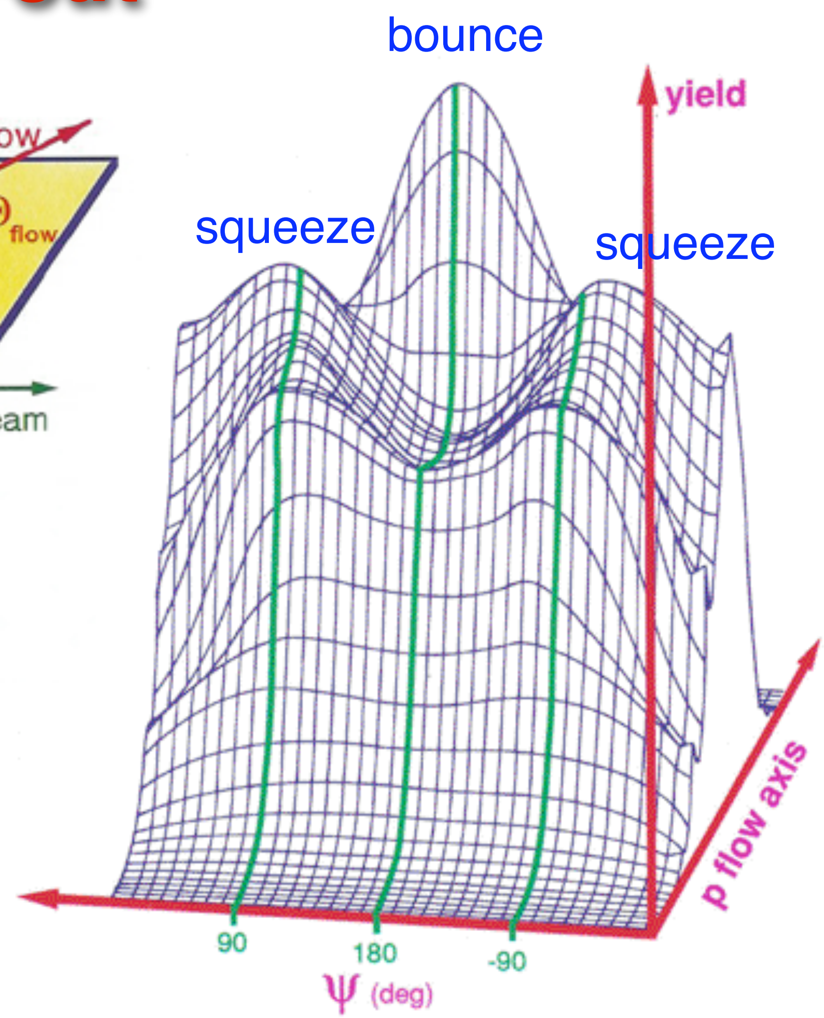
Squeeze-out



Schmidt 1986

Best ellipsoid

Negative elliptic flow



400 MeV/A Au+Au (MUL 3)

Plastic Ball, H.H. Gutbrod et al., PRC **42**, 640 (1991)
Diogene, M. Demoulins et al., Phys. Lett. **B241**, 476 (1990)
Plastic Ball, H.H. Gutbrod et al., Phys. Lett. **B216**, 267 (1989)

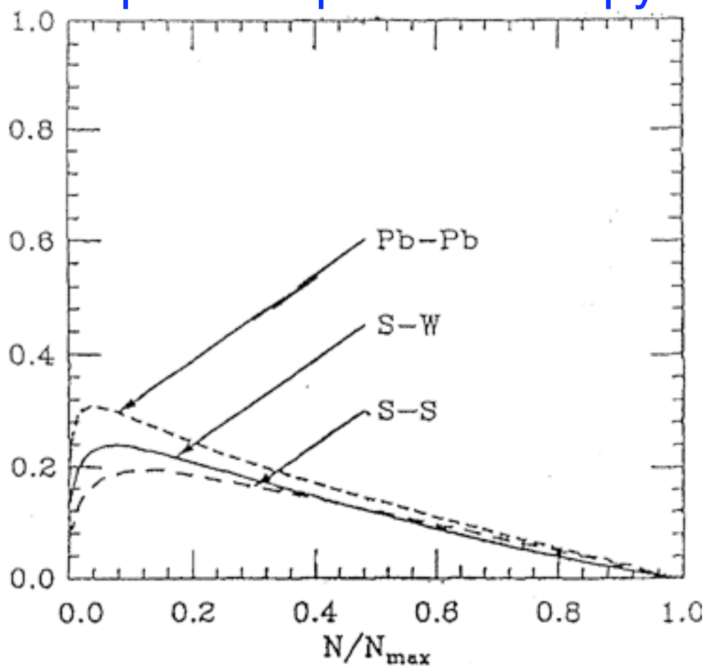
Prediction of Positive Elliptic Flow

At a meeting in Jan ‘93, Jean-Yves told me he was predicting in-plane elliptic flow at high beam energies. I responded that we had just discovered out-of-plane elliptic flow

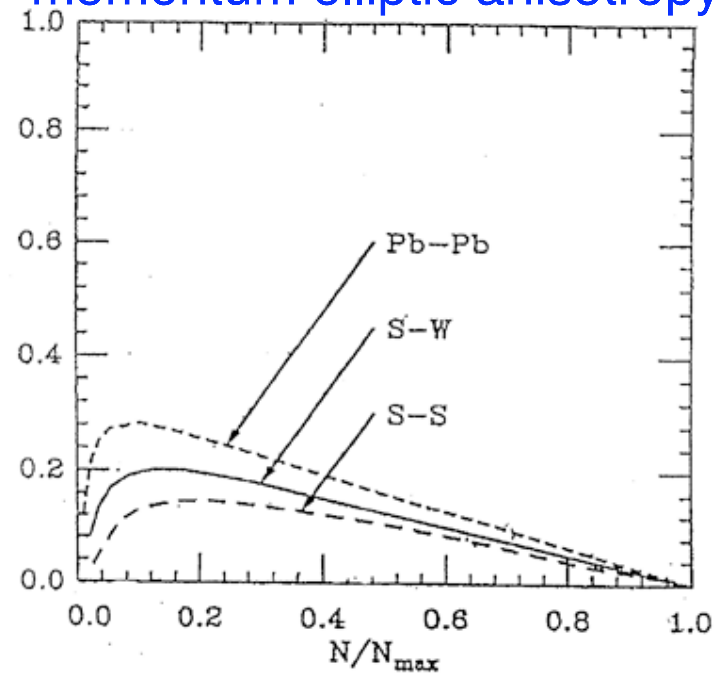
Ollitrault

2-dimensional transverse sphericity analysis: best ellipse

space elliptic anisotropy



momentum elliptic anisotropy



First Observation: E877, J. Barrette et al., PRC **55,** 1420 (1997)
J.-Y. Ollitrault, PRD **46**, 229 (1992), PRD **48**, 1132 (1993)

Transverse Momentum Analysis

Second to use the transverse plane
First to define 1st harmonic Q-vector
First to use weighting
First to use sub-events
First to remove auto-correlations

Danielewicz

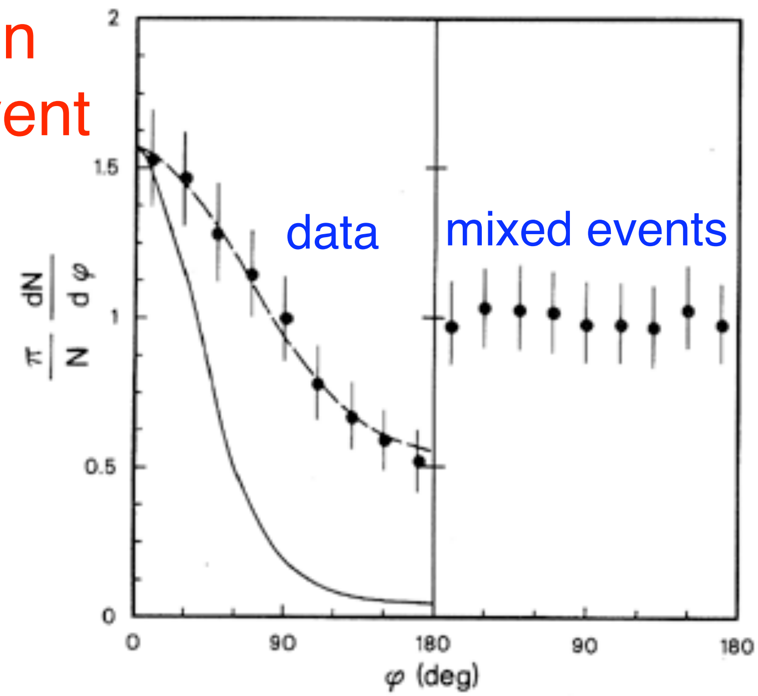
Odyniec

unit vector

$$Q_1 = \sum_{i=1}^{N} w_i u_i$$

negative in backward hemisphere

correlation of sub-event planes

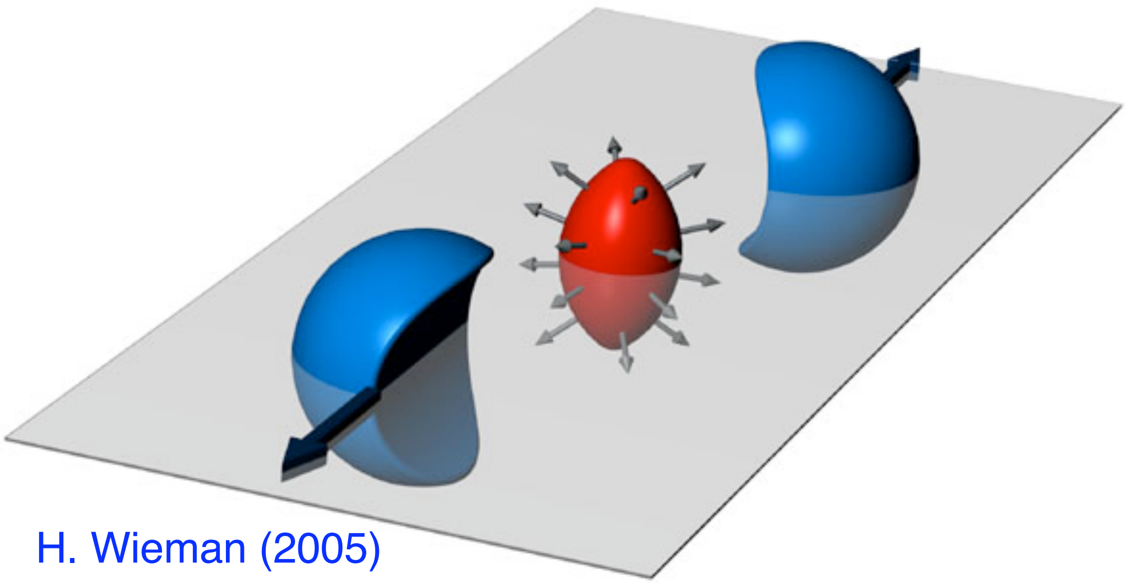


Mistake in event plane resolution

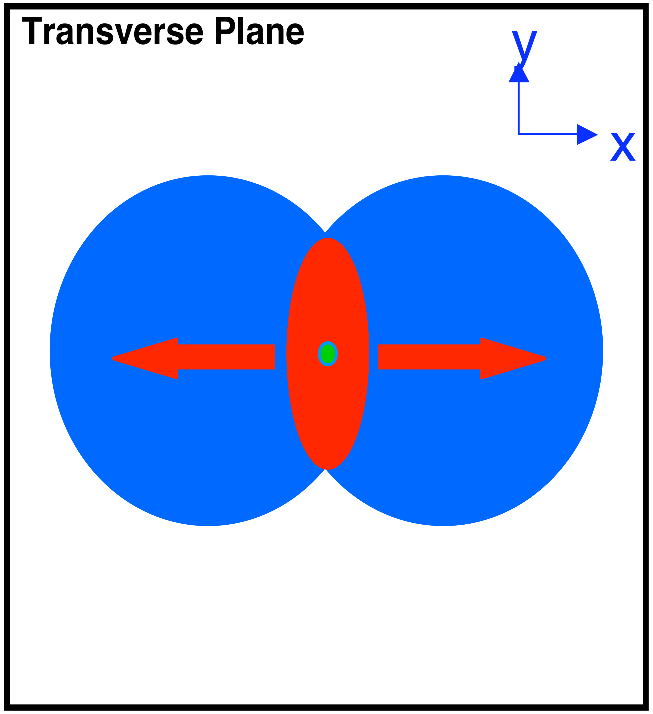
P. Danielewicz and G. Odyniec, Phys. Lett. **157B**, 146 (1985)

Transverse Plane

Anisotropic Flow as a function of rapidity



H. Wieman (2005)



around the beam axis

$$\epsilon = \frac{\langle y^2\rangle - \langle x^2\rangle}{\langle y^2\rangle + \langle x^2\rangle} \qquad S = \pi\sqrt{\langle x^2\rangle\ \langle y^2\rangle}$$

self quenching expansion
probe of early time

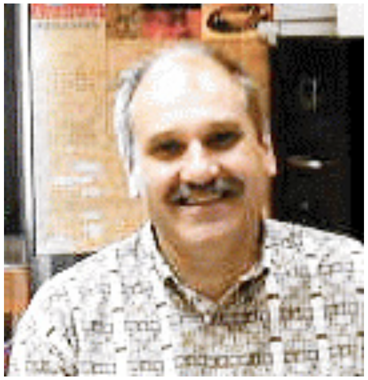
H. Sorge, PRL **78**, 2309 (1997)

Fourier Harmonics

First to use Fourier harmonics:

$$1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos[2(\phi - \Psi_{RP})] + \cdots$$

$$v_n = \langle \cos[n(\phi_i - \Psi_{RP})] \rangle$$



Voloshin

Event plane resolution correction made for each harmonic

- Unfiltered theory can be compared to experiment!
- Tremendous stimulus to theoreticians

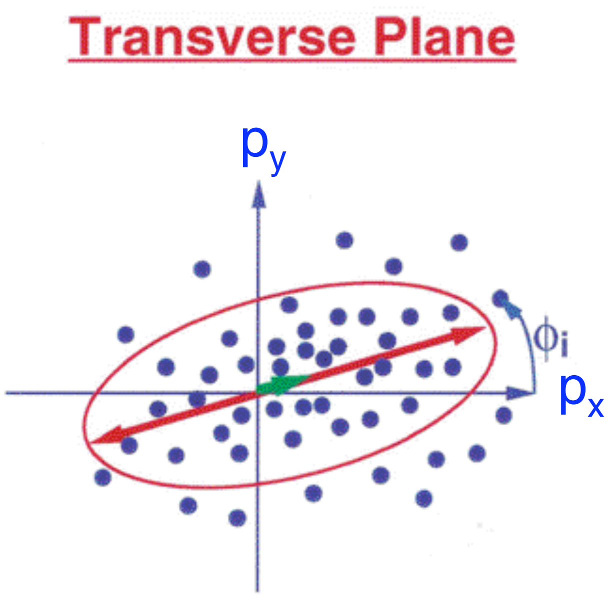
First to use mixed harmonics

First to use the terms directed and elliptic flow for v_1 and v_2

S. Voloshin and Y. Zhang, hep-ph/940782; Z. Phys. C **70**, 665 (1996)
See also, J.-Y. Ollitrault, arXiv nucl-ex/9711003 (1997)
and J.-Y. Ollitrault, Nucl. Phys. **A590**, 561c (1995)

Flow Vector

Sum of vectors of all the particles



$$\Psi_{plane} = \tan^{-1} \frac{\sum \sin(\phi_i)}{\sum \cos(\phi_i)}$$

$$2\,\Psi_{ellipse} = \tan^{-1} \frac{\sum \sin(2\phi_i)}{\sum \cos(2\phi_i)}$$

For each harmonic n:

$$Q_n \cos(n\Psi_n) = \sum [w_i \cos(n\phi_i)]$$

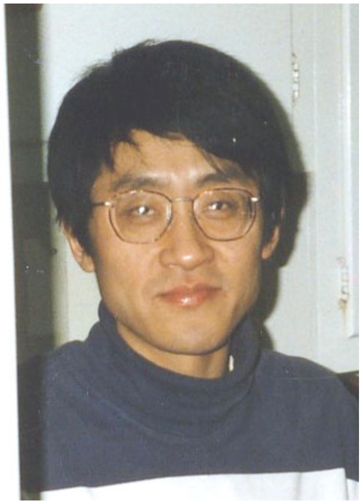
$$Q_n \sin(n\Psi_n) = \sum [w_i \sin(n\phi_i)]$$

Q is a 2 vector

w_i negative in backward hemisphere for odd harmonics

for n=1:

$$Q_1 = \sum_{i=1}^{N} w_i u_i$$



Yingchao Zhang

S. Voloshin and Y. Zhang, Z. Phys. C **70**, 665 (1996)

Standard Event Plane Method I

- Define 2 independent groups of particles
  - random subs — most affected by non-flow
  - charge subs — like-sign less sensitive to neutral resonance decays
  - η subs — suppresses short-range correlations; FTPC even better
- Flatten event plane azimuthal distributions in lab
  - Both sub-events and full event Q-vectors

A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Flattening Methods I

To remove "Acceptance Correlations" flatten the azimuthal distribution of the event plane

- **phi weighting** - in constructing the Q-vector one weights with the inverse of the azimuthal distribution of the particles averaged over many events

$$w_i = \frac{1}{\langle N(\phi_i) \rangle}$$

- **recentering** - from each event Q-vector one subtracts the Q-vector averaged over many events

$$X \mathrel{-}= \langle X \rangle \; ; \; Y \mathrel{-}= \langle Y \rangle$$

- **shifting** - one fits the non-flat azimuthal distribution of the Q-vector angles with a Fourier expansion and calculates the shifts necessary to force a flat distribution

$$n\Delta\Psi_n = \sum_{i=1}^{i_{max}} \frac{2}{i}\left(-\langle \sin(in\Psi_n) \rangle \cos(in\Psi_n) + \langle \cos(in\Psi_n) \rangle \sin(in\Psi_n)\right)$$

A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Flattening Methods II

Mixed events is not recommended because it has no advantage over phi weights from the same event

Shifting is good as a second method when either phi weights or recentering does not produce a flat distribution (e.g. FTPC)

For elliptic flow, only the **second** harmonic component of the flattened distribution needs to be small !

Standard Event Plane Method II

- **Correlate subevent planes** $\langle \cos[n(\Psi_n^a - \Psi_n^b)] \rangle$
- **Calculate subevent plane resolution**

$$\mathrm{resSub} \equiv \langle \cos(n(\Psi_n^a - \Psi_{RP})) \rangle = \sqrt{\langle \cos(n(\Psi_n^a - \Psi_n^b)) \rangle}$$

- **Calculate event plane resolution**

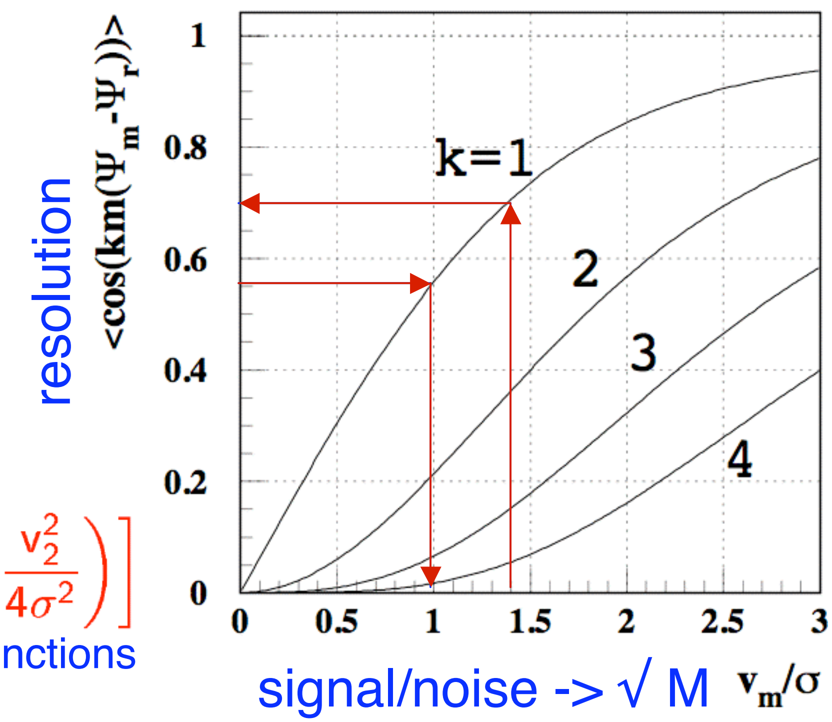
$$\mathrm{res} \equiv \langle \cos(n(\Psi_n - \Psi_{RP})) \rangle \le \sqrt{2}\,\mathrm{resSub}$$

k = 2 for mixed harmonics
k > 2 not yet used

For k = 1:

$$\langle \cos[2(\Psi_2 - \Psi_r)] \rangle = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{v_2}{\sigma} e^{-\frac{v_2^2}{4\sigma^2}} \left[I_0\left(\frac{v_2^2}{4\sigma^2}\right) + I_1\left(\frac{v_2^2}{4\sigma^2}\right) \right]$$

modified Bessel functions



A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Standard Event Plane Method III

- **Correlate particles with the event plane**
  - **But the event plane not containing the particle**
  - **(no autocorrelations)**

$$v_n^{obs}(\eta, p_t) = \langle \cos[n(\phi - \Psi_n)] \rangle$$

- **Correct for the event plane resolution**

$$v_n(\eta, p_t) = v_n^{obs} / res$$

- **Average over p_t, η, or both (with yield weighting)**
  - **$v_2(\eta)$ — may need p_t extrapolation**
  - **$v_2(p_t)$ — for specified η range**
  - **v_2 vs. centrality — called integrated v_2**

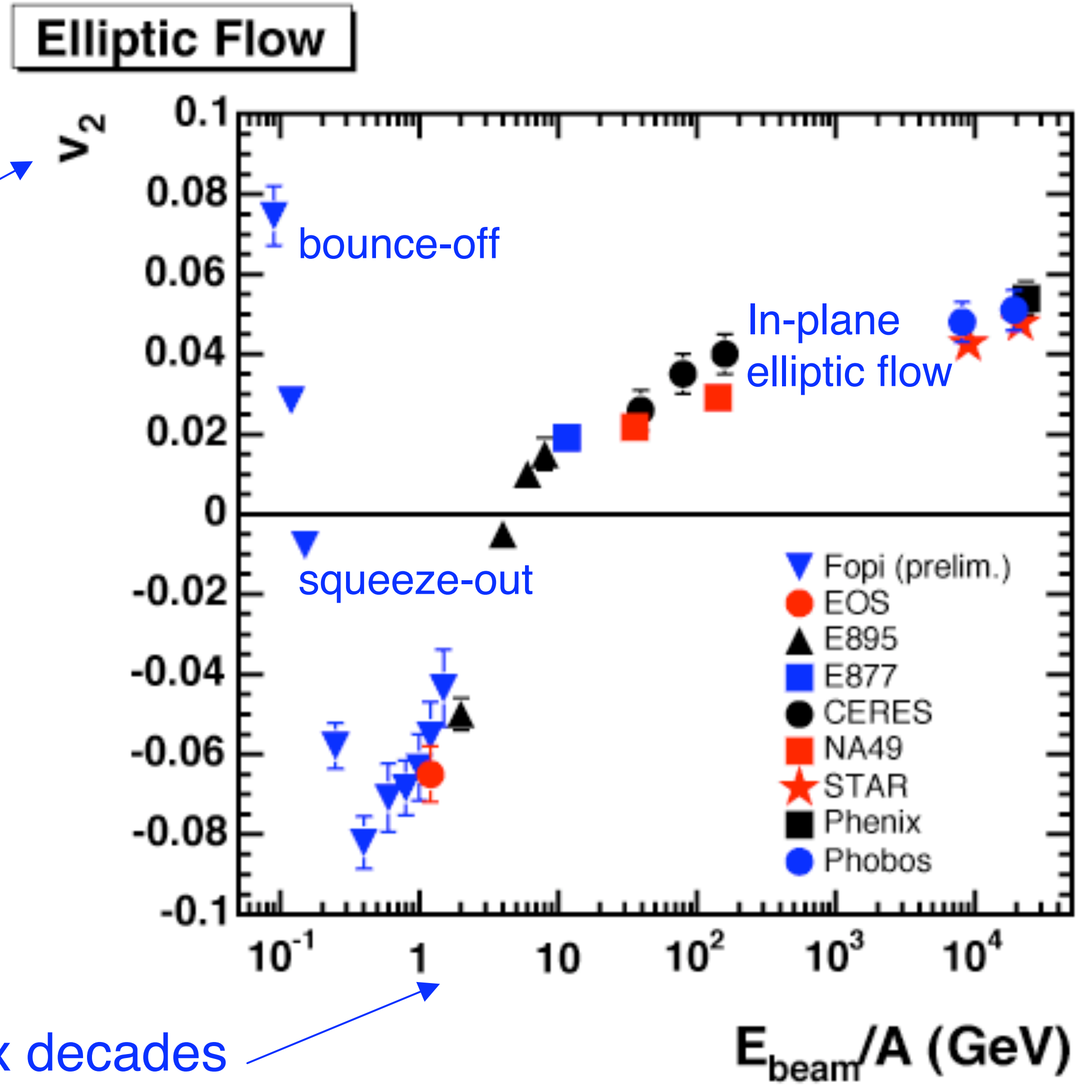
StFlowAnalysisMaker
in STAR cvs library

A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Elliptic Flow vs. Beam Energy

25% most central
mid-rapidity

all v_2



six decades

Wetzler 2004

Pair-wise Correlations

$$\frac{dN^{pairs}}{d\Delta\phi} \propto \left(1 + \sum_{n=1}^{\infty} 2v_n^2 \cos(n\Delta\phi)\right)$$

acceptance correlations
removed by mixed events

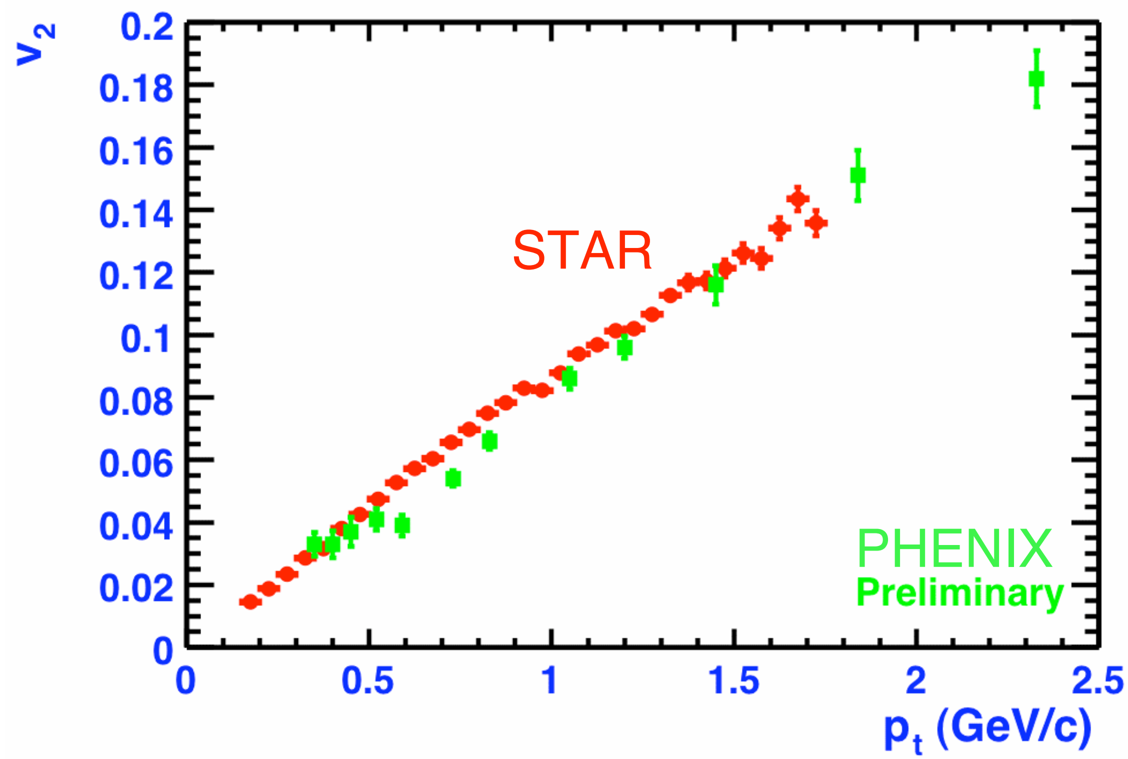
$$v_n^2 = \langle \cos[n(\phi_1 - \phi_2)] \rangle$$

square

two different particles

no event plane

Keane 2003



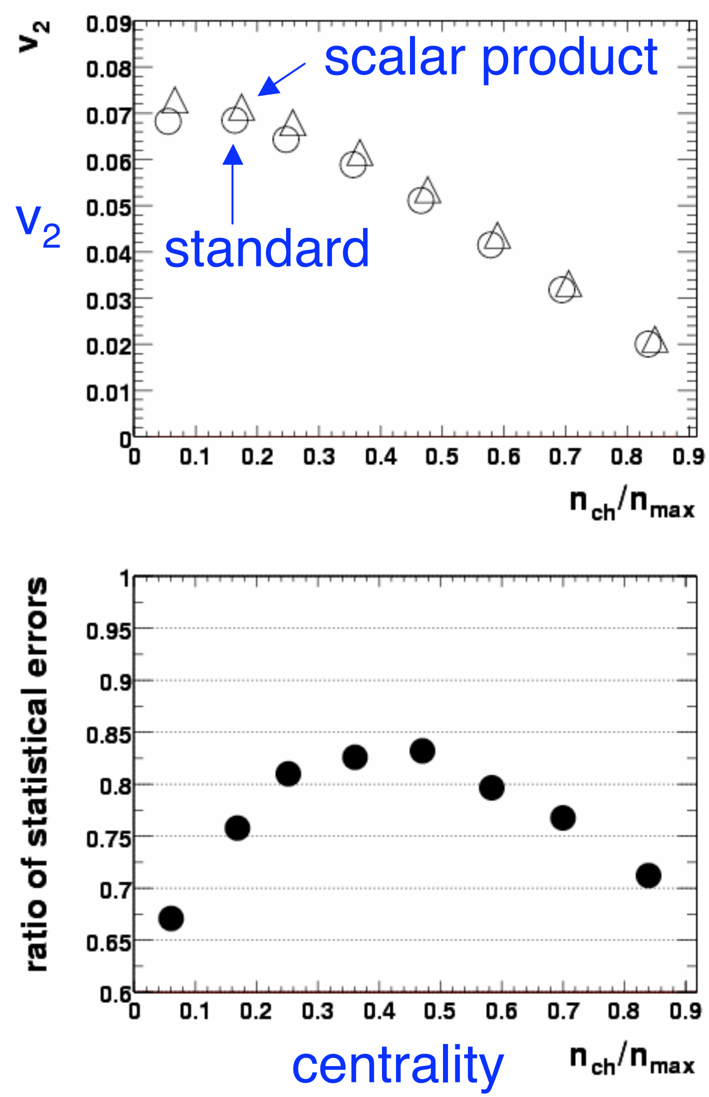
Streamer Chamber, S. Wang et al., PRC **44**, 1091 (1991)
PHENIX, K. Adcox et al., PRL **89**, 21301 (2002)

Scalar Product

correlation of particles with Q-vector

$$v_n(\eta, p_t) = \frac{\langle Q_n u^*_{n,i}(\eta, p_t)\rangle}{2\sqrt{\langle Q^a_n Q^{b*}_n\rangle}}$$

resolution



similar to standard method
but weighted by the length
of the Q-vector

$$\frac{\text{error scalar product}}{\text{error standard}}$$

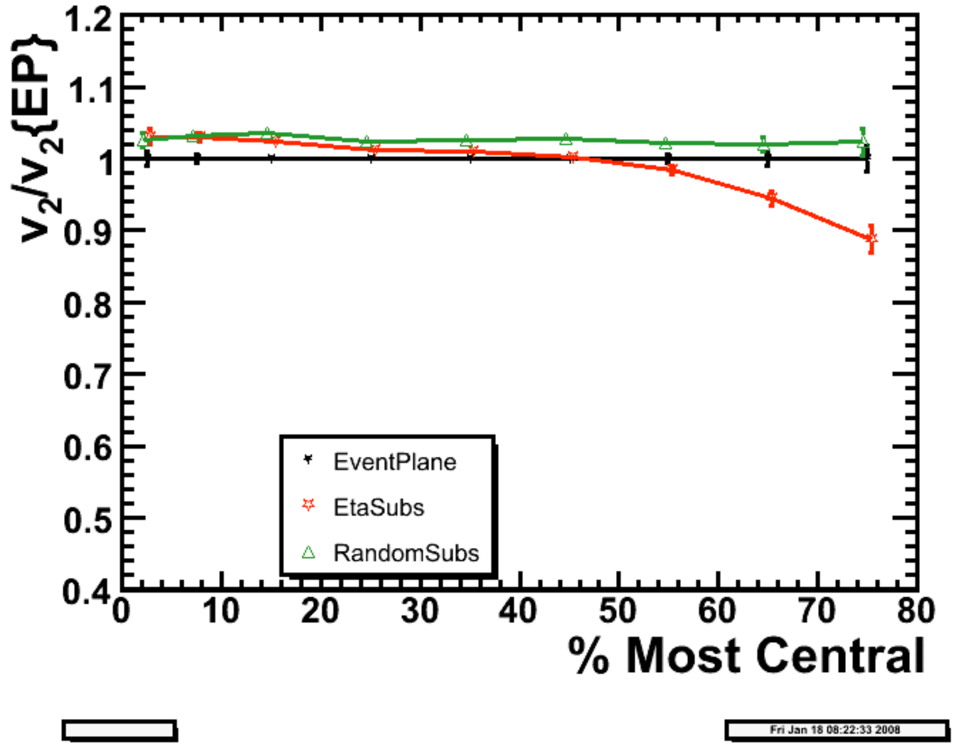
STAR, C. Adler et al., PRC **66**, 034904 (2002)

η-Subs

η-subs similar to standard method
but the event plane is from the opposite hemisphere

Large η gap reduces non-flow due to short-range correlations

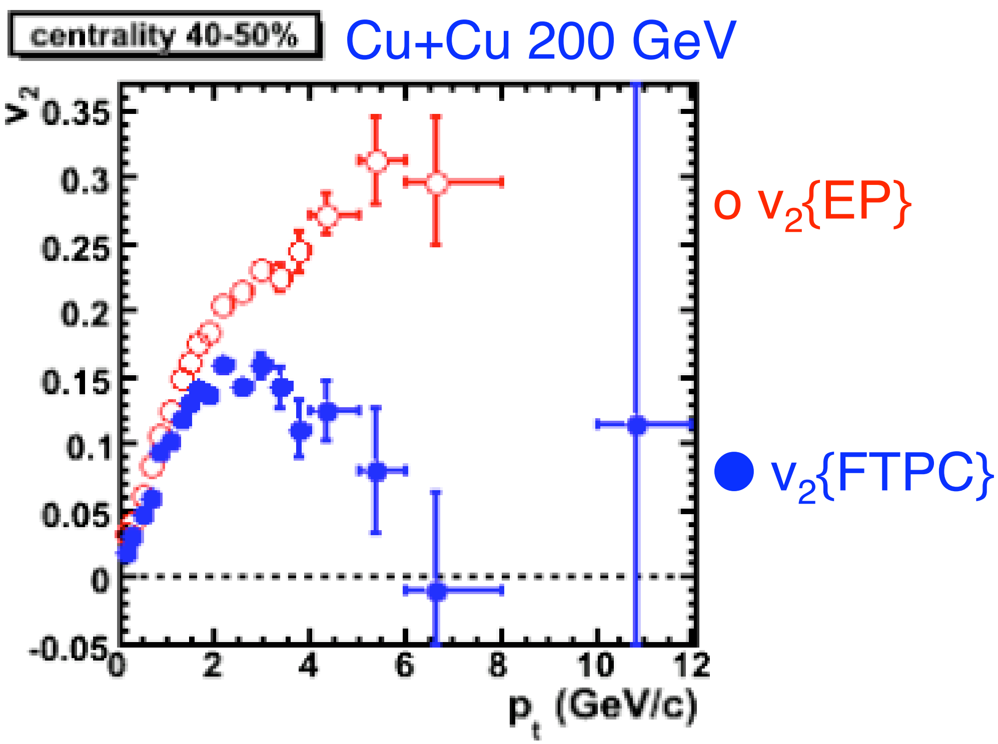
but only for the
peripheral collisions



FTPC

FTPC similar to standard method
but the event plane is from the FTPCs

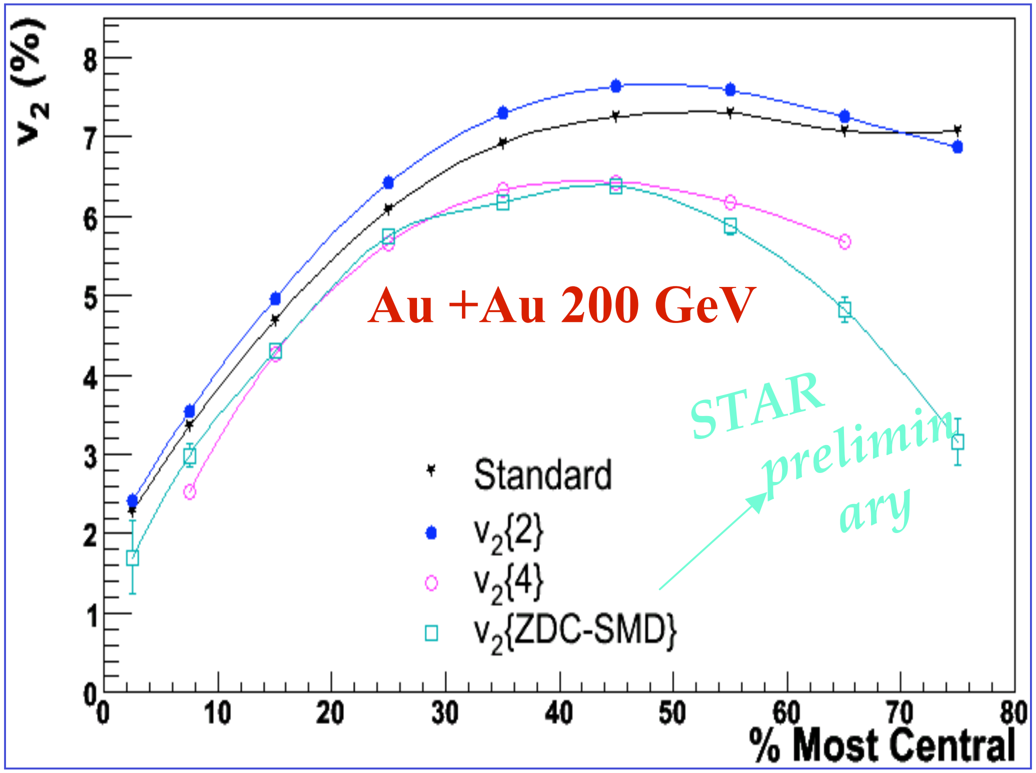
Larger η gap reduces non-flow due to short-range correlations

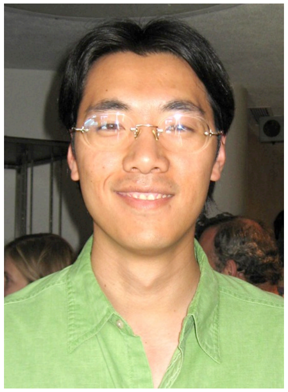


STAR preliminary

ZDC-SMD

Still larger η gap reduces non-flow due to short-range correlations





$v_2\{ZDC\text{-}SMD\}$ similar to $v_2\{4\}$

G. Wang, Quark Matter (2005)

Cumulants I

Four-particle correlation subtracts 2-particle nonflow

$$\langle u_{n,1} u_{n,2} u^*_{n,3} u^*_{n,4} \rangle = v_n^4 + 4 v_n^2 \sigma^2_{dyn} + 2\sigma^4_{dyn}$$

$$\langle u_{n,1} u^*_{n,2} \rangle = v_n^2 + \sigma^2_{dyn} \quad = v_2\{2\}^2$$

fourth power

$$C\{4\} \equiv \langle u_{n,1} u_{n,2} u^*_{n,3} u^*_{n,4} \rangle - 2 \langle u_{n,1} u^*_{n,2} \rangle^2 = -v_n\{4\}^4$$

$\sigma^2_{dyn} = v_2\{2\}^2 - v_2\{4\}^2$ is non-flow $\sigma^2_{dyn} = \delta_2 + 2\sigma^2_{v2}$

Generating function:

$$G_n(z) = \prod_{j=1}^{M} \left(1 + \frac{z^* u_{n,j} + z u^*_{n,j}}{M}\right)$$

$$M \cdot \left(\langle G_n(z) \rangle^{1/M} - 1\right) = \sum_k \frac{|z|^{2k}}{(k!)^2} C\{2k\}$$

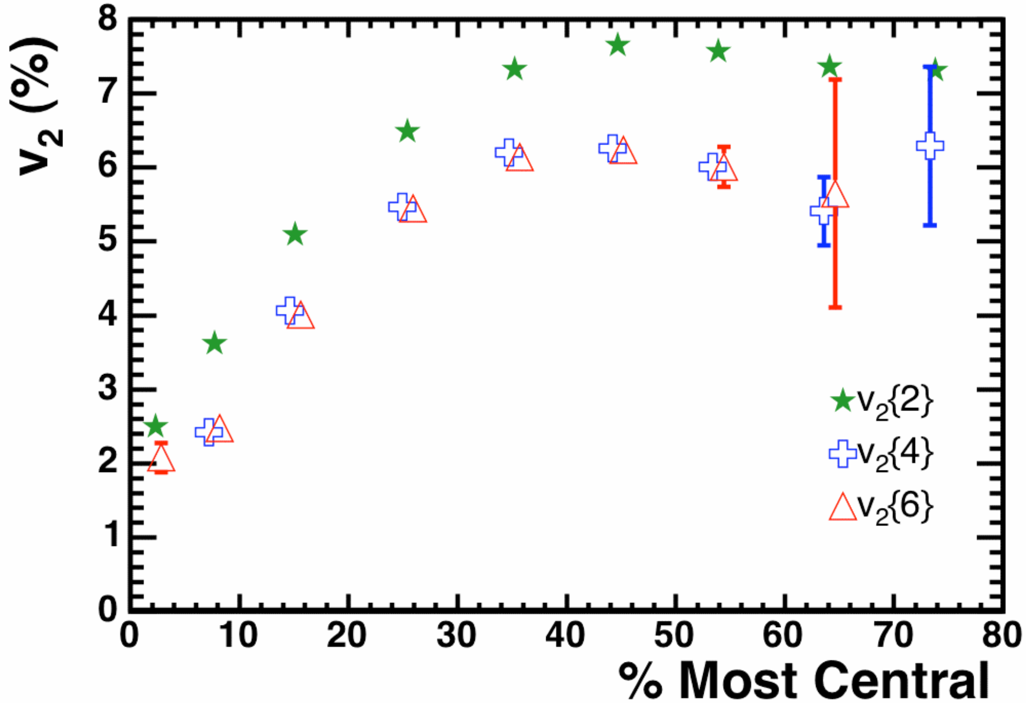
C{4} term of fit

Can be calculated directly from

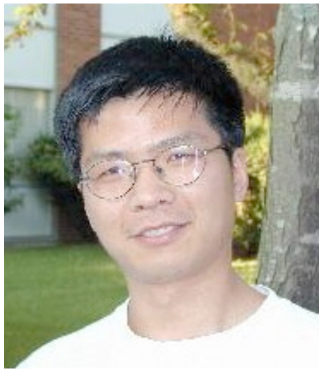
$\langle Q_n Q_n^* \rangle, \langle Q_n^2 Q_n^{*2} \rangle, \langle Q_{2n} Q_{2n}^* \rangle, \langle Q_n^2 Q_{2n}^* \rangle$ Voloshin (2002)

N. Borghini, P.M. Dinh, and J.-Y. Ollitrault, PRC **64**, 054901 (2001)
STAR, C. Adler et al., PRC **66**, 034904 (2002)

Cumulants II



$v_2\{6\}$ no better than $v_2\{4\}$



Tang

STAR, J. Adams et al., PRC **72**, 014904 (2005)

q-dist Method I

reduced flow vector:

$$q_n = Q_n/\sqrt{M}$$

multiplicity

flow vector

Bessel-Gaussian distribution of q:

$$\frac{dP}{q_n dq_n} = \frac{1}{\sigma_n^2} e^{-\frac{v_n^2 M + q_n^2}{2\sigma_n^2}} I_0\left(\frac{q_n v_n \sqrt{M}}{\sigma_n^2}\right)$$

modified Bessel function

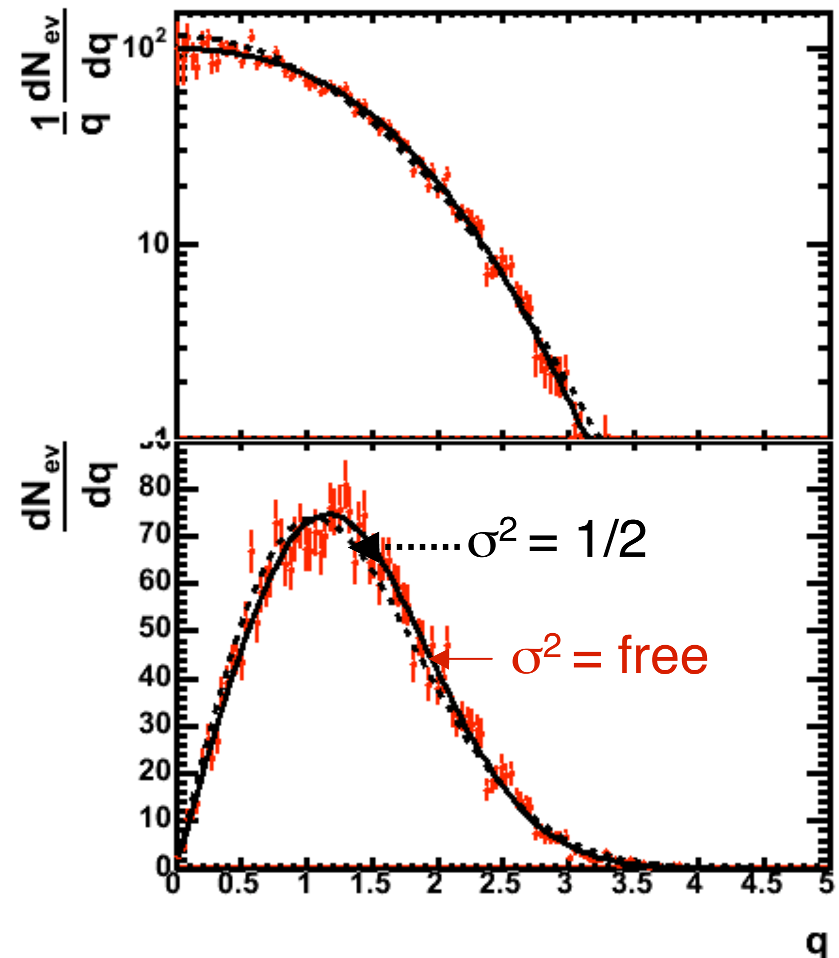
shifted out by v_n^2

$\sigma_n^2 = 1/2$ from statistical effects

broadened by non-flow

and v_2 fluctuations

no event plane

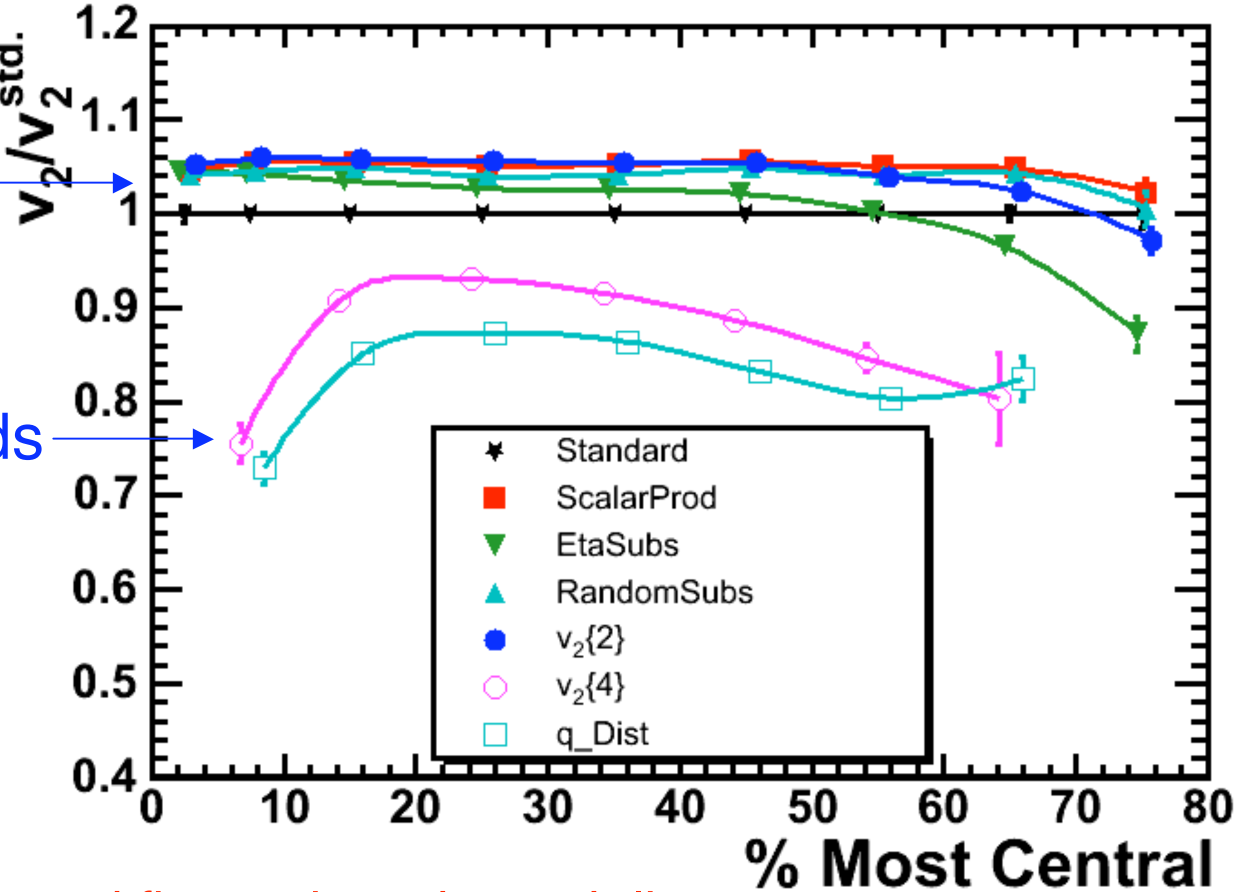


Methods Comparison (2005)

Ratio to the Standard Method:

2-part. methods

multi-part. methods



Because of nonflow and fluctuations the truth lies between the lower band and the mean of the two bands

STAR, J. Adams et al., PRC **72**, 014904 (2005)

Lee-Yang Zeros Method I

All-particle correlation subtracts nonflow to all orders

Sum Generating Function:

- Flow vector projection on arbitrary lab angle, θ

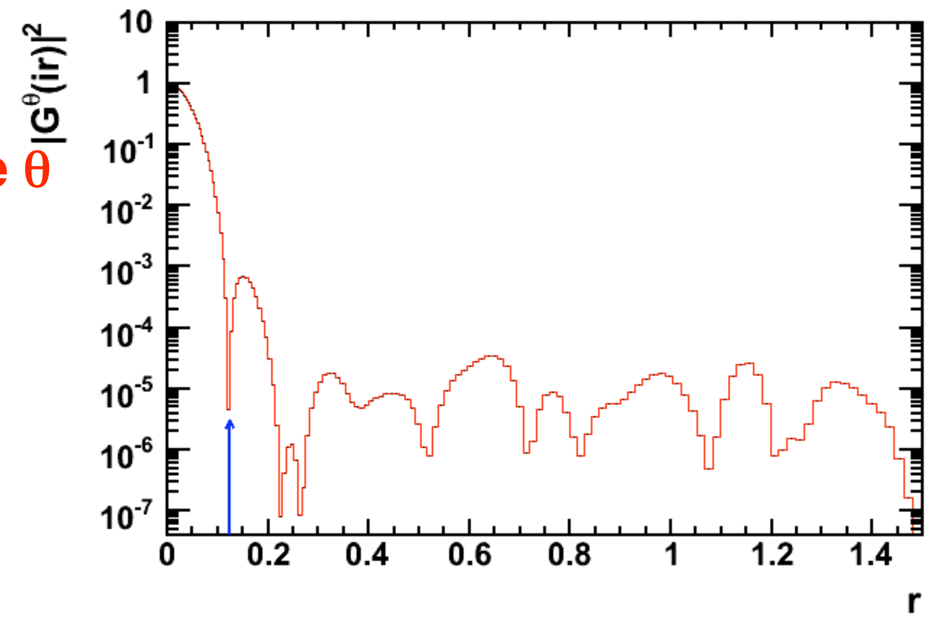
$$\mathbf{Q}_{\mathbf{n}}^{\theta} = \sum_{j=1}^{N} w_j \cos(n(\phi_j - \theta))$$

- Generating function for one θ

$$\mathbf{G}_{\mathbf{n}}^{\theta}(\mathbf{r}) = \left| \left\langle e^{irQ_n^{\theta}} \right\rangle \right|$$

- Average over θ to remove acceptance effects

$$v_2 = \frac{2.4}{M \left\langle r_0^{\theta} \right\rangle}$$



First minimum of $|G|^2$ determines r_0^{θ}

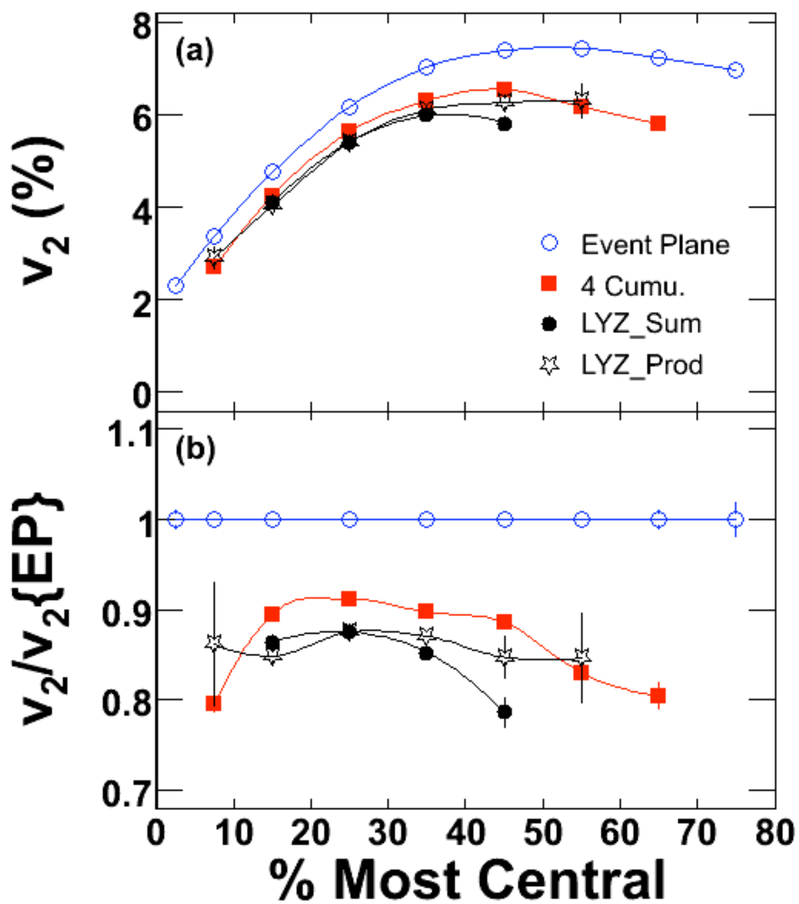
Product Generating Function:

- Better for mixed harmonics but slower

R.S. Bhalerao, N. Borghini, and J.-Y. Ollitrault, Nucl. Phys. A **727**, 373 (2003)

STAR, B.I. Abelev et al, PRC, to be submitted (2008)

Lee-Yang Zeros Method II



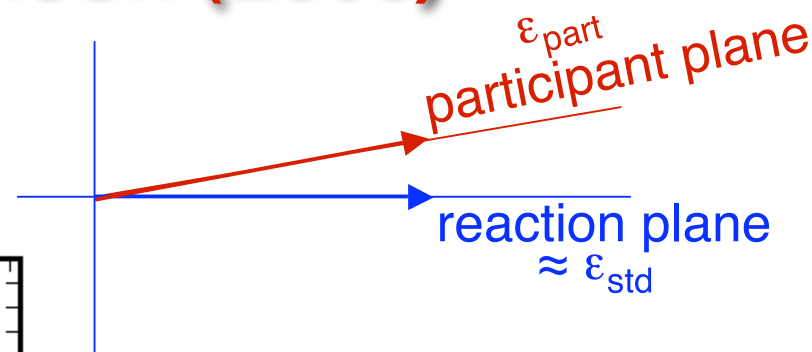
Sergei's Bessel Transform method is a simplified version of LYZ sum

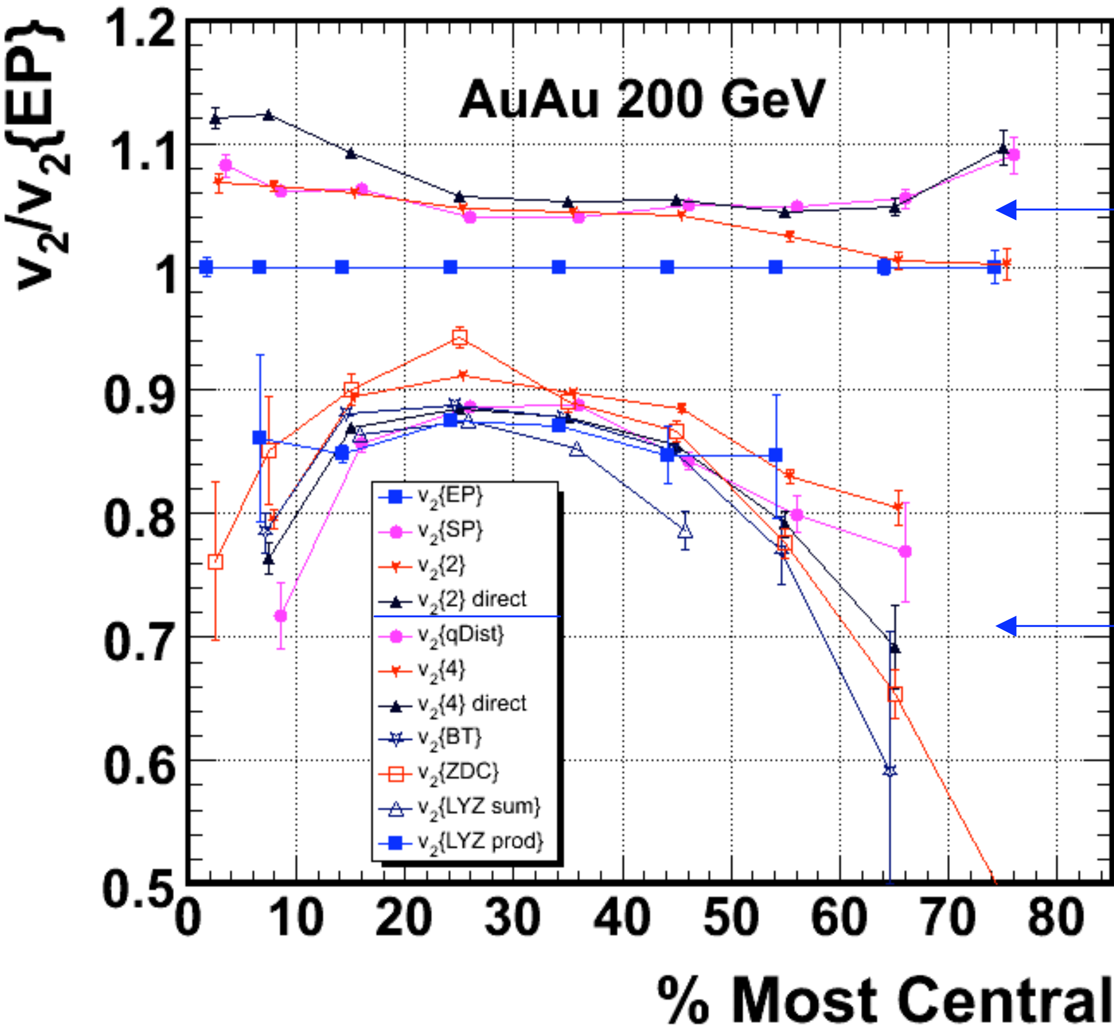
Sum and Prod agree
Both slightly lower than $v_2\{4\}$

STAR, B.I. Abelev et al., PRC, to be submitted (2008)

Methods Comparison (2008)

v in the participant plane always
greater than v in the reaction plane





2-part. methods

multi-part. methods

very preliminary

q-Dist with Nonflow and Fluctuations I

$$\frac{d^2N}{q_n dq_n d(\Delta\phi)} = \frac{1}{2\pi\sigma_x\sigma_y} e^{\left[-\frac{(q_n\cos(n\Delta\phi)-\sqrt{M}v_n)^2}{2\sigma_x^2} - \frac{q_n^2\sin^2(n\Delta\phi)}{2\sigma_y^2}\right]}$$

$$\sigma_{n,x}^2 = \frac{1}{2}[1 + v_{2n} - 2v_n^2 + (M-1)(\delta_n + 2\sigma_{vn}^2)]$$

$$\sigma_{n,y}^2 = \frac{1}{2}[1 - v_{2n} + (M-1)(\delta_n + 2\sigma_{vn}^2)]$$

QM06 left out M in front of δ

$$\sigma_{dyn}^2 = \delta_2 + 2\sigma_{v2}^2$$

non-flow fluctuations

fluctuations broaden

non-flow correlations broaden because there are effectively fewer independent particles

$$\sigma_q^2 = \frac{1}{2}(1 + M\,\sigma_{dyn}^2)$$

because σ_x close to σ_y

integrate over φ by expansion 2 ways: (for n = 2)

$$\frac{dN}{dq_2} = \frac{q_2}{\sigma_q^2} e^{-\frac{q_2^2+Mv_2^2}{2\sigma_q^2}} I_0\left(\frac{q_2 v_2\sqrt{M}}{\sigma_q^2}\right) + \cdots$$

leading term the same

higher terms different

they involve the difference between σ_x and σ_y

Voloshin and Sorensen (2007)

q-Dist with Nonflow and Fluctuations II

Paul sets $\sigma_{v2} = 0$ for the integration and then smears with σ_{v2}

$$\frac{dN}{q_2 dq_2}(\langle v_2 \rangle, \sigma_{v2}) = \int dv_2\, f(v_2 - \langle v_2 \rangle, \sigma_{v2}) \frac{dN}{q_2 dq_2}$$

$$f = \frac{1}{\sqrt{2\pi\sigma_{v2}^2}} e^{-\frac{(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v2}^2}}$$

Gaussian along PP, but not restricted

Sergei sets $\sigma_{v2x} = \sigma_{v2y}$ before integration,
assuming a 2D Gaussian for the fluctuations

Both depend only on $\sigma_{dyn}^2 = \delta_2 + 2\sigma_{v2}^2$

and thus can not separate δ_2 from σ_{v2}

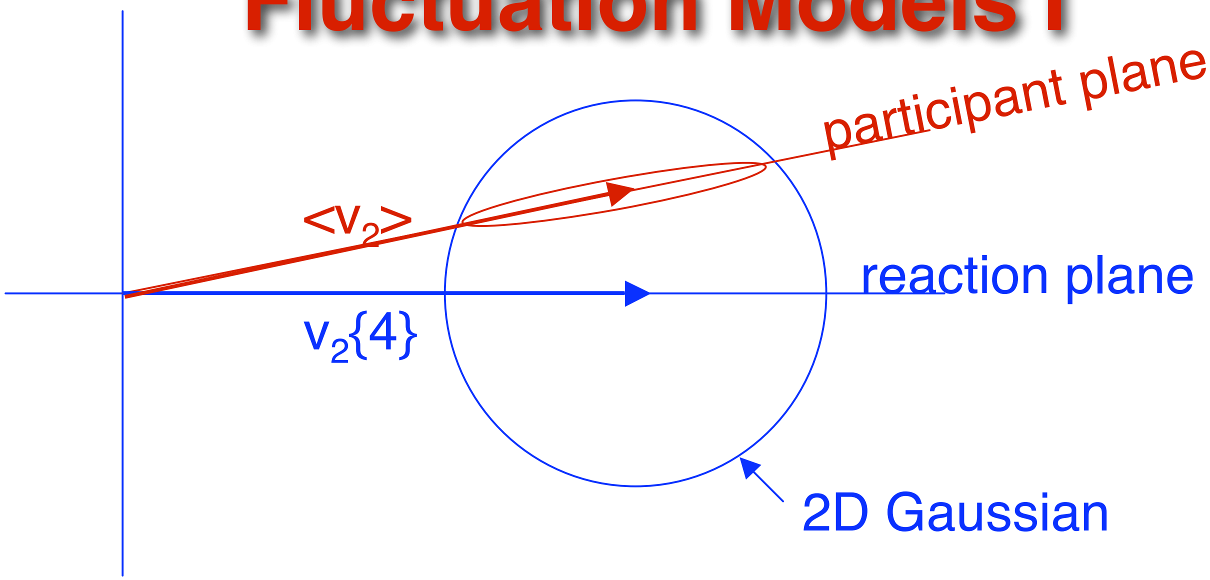
A upper limit on δ_2 gives lower limit on σ_{v2}
or, arrange to have δ_2 small, as PHOBOS does with an η-gap

No more info from Cumulants since

$$\sigma_{dyn}^2 = v_2\{2\}^2 - v_2\{4\}^2$$

Voloshin and Sorensen (2007)

Fluctuation Models I



Paul: 1D Gaussian along participant axis

gives $\langle v_2 \rangle$ and σ_{v2} directly

$$v_2\{4\} \simeq \langle v_2 \rangle^2 - \sigma_{v2}^2$$

Sergei: 2D Gaussian in reaction plane

gives Bessel-Gaussian in v_0 and σ along participant axis:

$$v_2\{4\} = v_0$$

If Paul uses Bessel-Gaussian he gets same result as Sergei

$v_2\{4\}$ is v_2 along the reaction plane axis

also, $v_2\{4\}$ is insensitive to fluctuations

S.A. Voloshin, A.M. Poskanzer, A. Tang, G. Wang, Phys. Lett. B, **659**, 537 (2008)

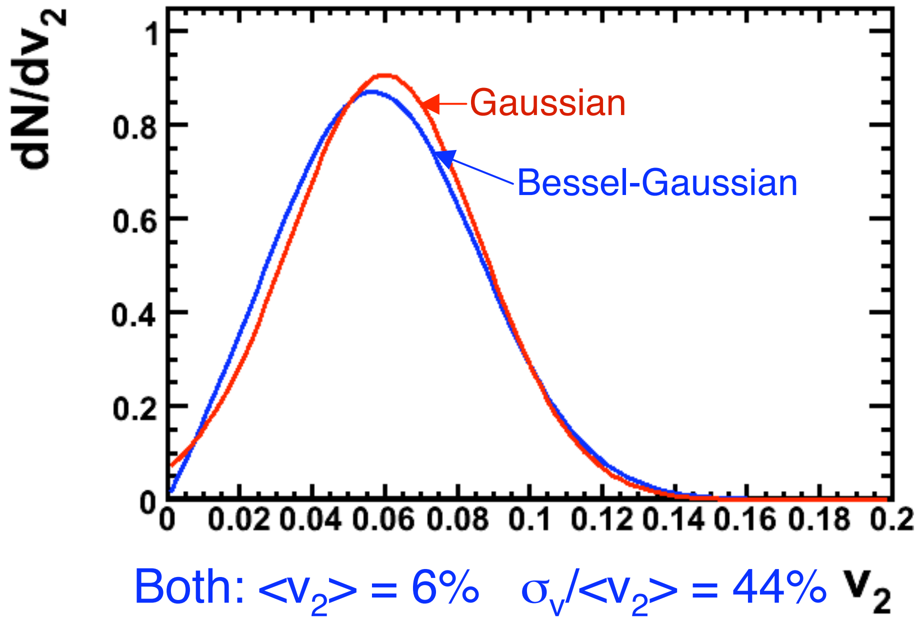
Fluctuation Models II

Paul's 1D Gaussian along PP

$$f = \frac{1}{\sqrt{2\pi\sigma_{v2}^2}} e^{-\frac{(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v2}^2}}$$

Sergei's 2D Gaussian in RP

$$\frac{dN}{dv_2} = \frac{v_2}{\sigma^2} e^{-\frac{v_2^2 + v_0^2}{2\sigma^2}} I_0\left(\frac{v_2\, v_0}{\sigma^2}\right)$$



Both: $\langle v_2 \rangle = 6\%$ $\sigma_v/\langle v_2 \rangle = 44\%$

But Sergei must calculate $\langle v_2 \rangle$ and σ_{v2} along the participant axis:

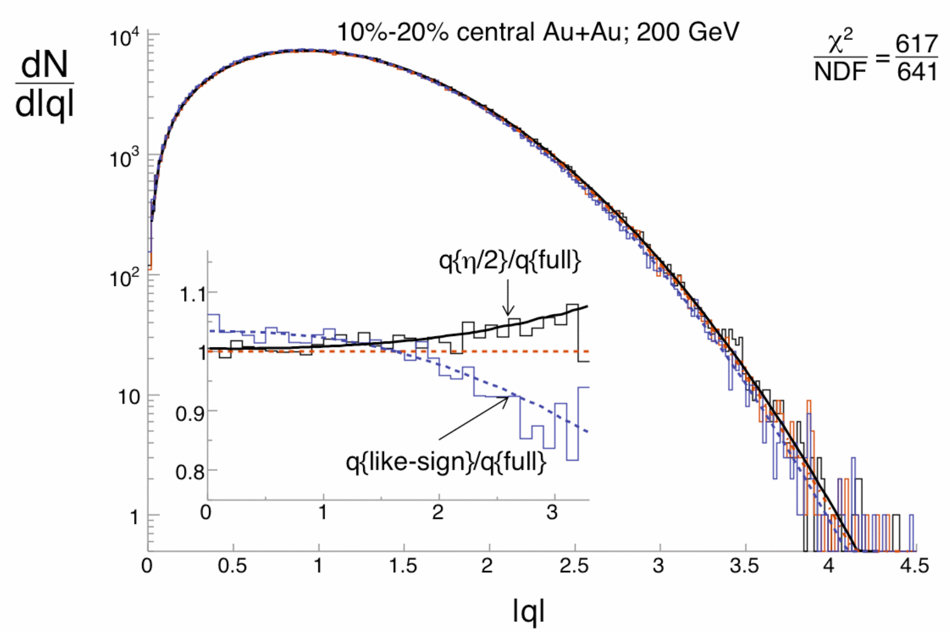
$$\langle v \rangle = \frac{1}{2\sigma} e^{-\frac{v_0^2}{4\sigma^2}} \sqrt{\frac{\pi}{2}} \left[(2\sigma^2 + v_0^2) I_0\left(\frac{v_0^2}{4\sigma^2}\right) + v_0^2 I_1\left(\frac{v_0^2}{4\sigma^2}\right) \right]$$

$$\langle v^2 \rangle = v_0^2 + 2\sigma^2$$

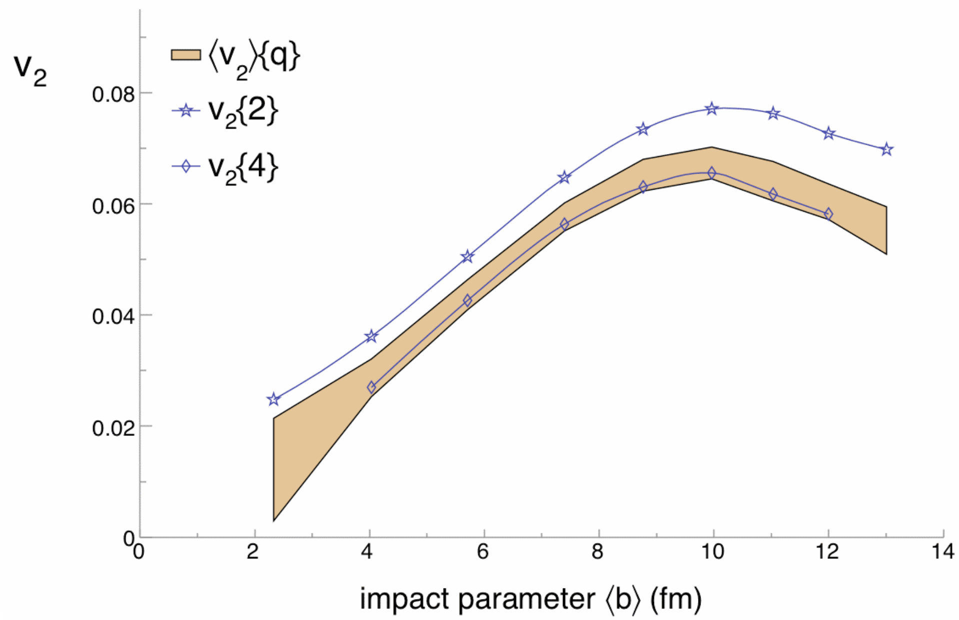
$$\sigma_v^2 = \langle v^2 \rangle - \langle v \rangle^2 = v_0^2 + 2\sigma^2 - \langle v \rangle^2$$

S.A. Voloshin, A.M. Poskanzer, A. Tang, G. Wang, Phys. Lett. B, **659**, 537 (2008)

q-Dist with Nonflow and Fluctuations III



like-sign has less non-flow



$$\langle v_2 \rangle^2 = v_2\{2\}^2 - \delta_2 - \sigma_{v2}^2$$

$$\langle v_2 \rangle^2 \simeq v_2\{4\}^2 + \sigma_{v2}^2$$

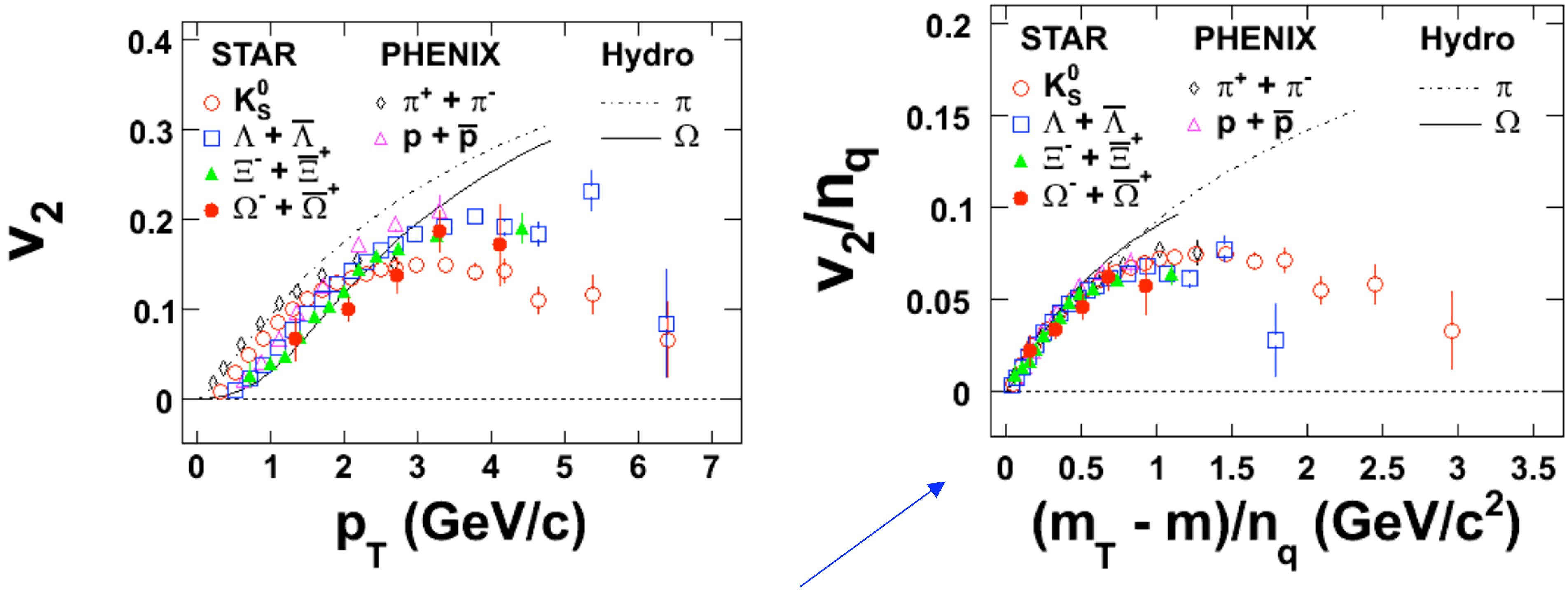
Sorensen

See Paul's talk

STAR, P. Sorensen, QM08

STAR preliminary

Particle Identification

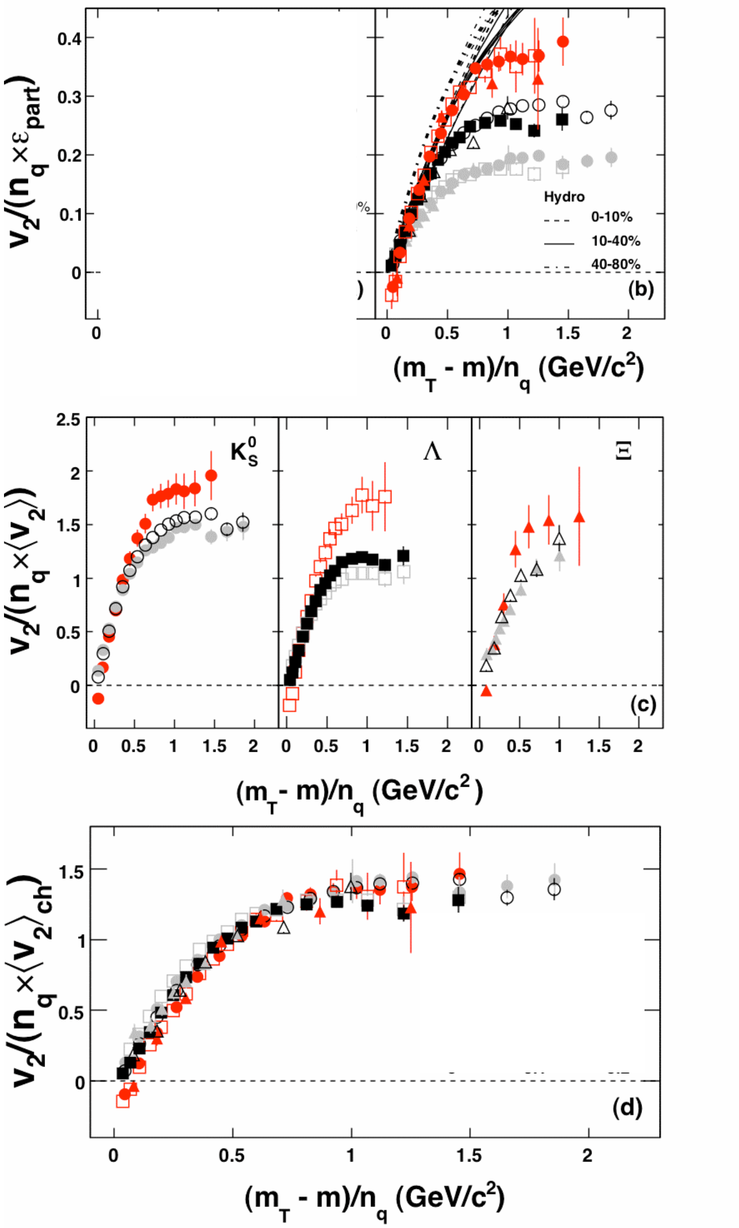


both axes scaled by number of constituent quarks

and plotted vs. trans. kinetic energy

STAR, B.I. Abelev et al., PRC, to be submitted (2008

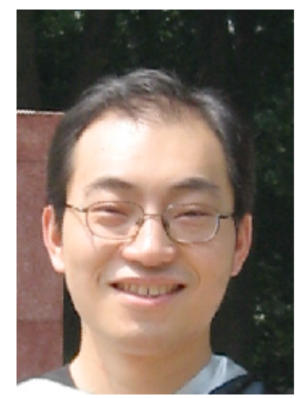
Scaling: v_2 / n_q vs. KE_t



scaled by ε_{part}

scaled by $<v_2>$
of that particle

scaled by $<v_2>$
of charged particles

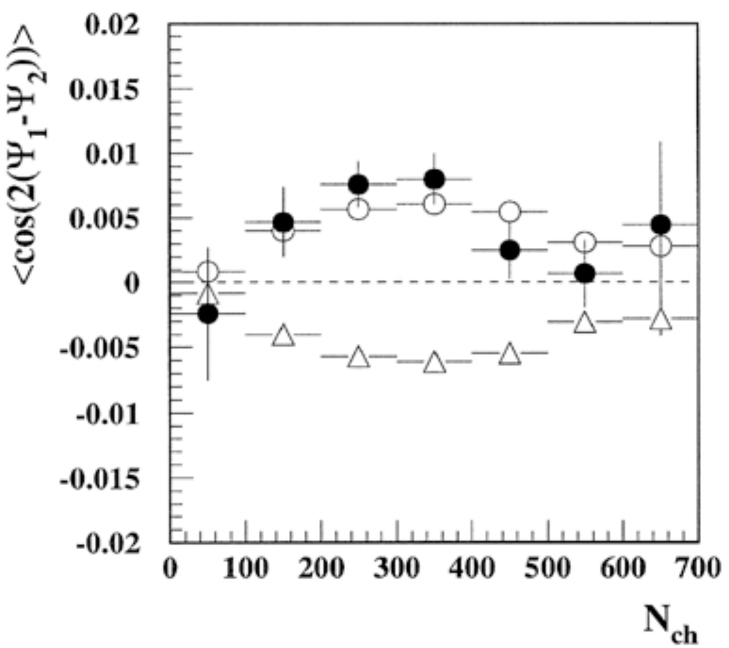


Yan Lu

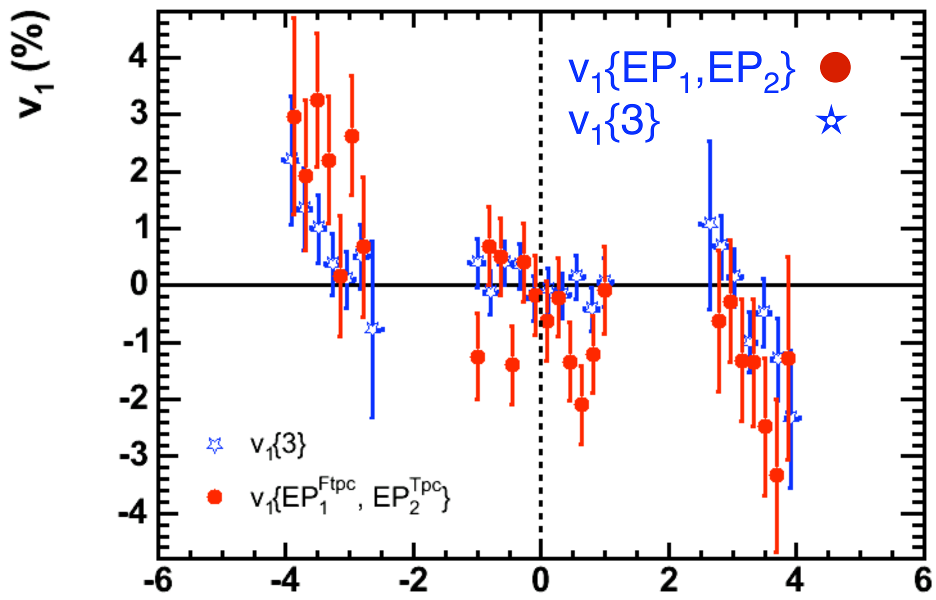
STAR, B.I. Abelev et al, PRC, to be submitted (2008)); Yan Lu thesis (2007)

Mixed Harmonics

$v_n\{EP_2\}$



CERES, S.A. Voloshin, German Physical Society meeting (1998) was the first



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Removes nonflow

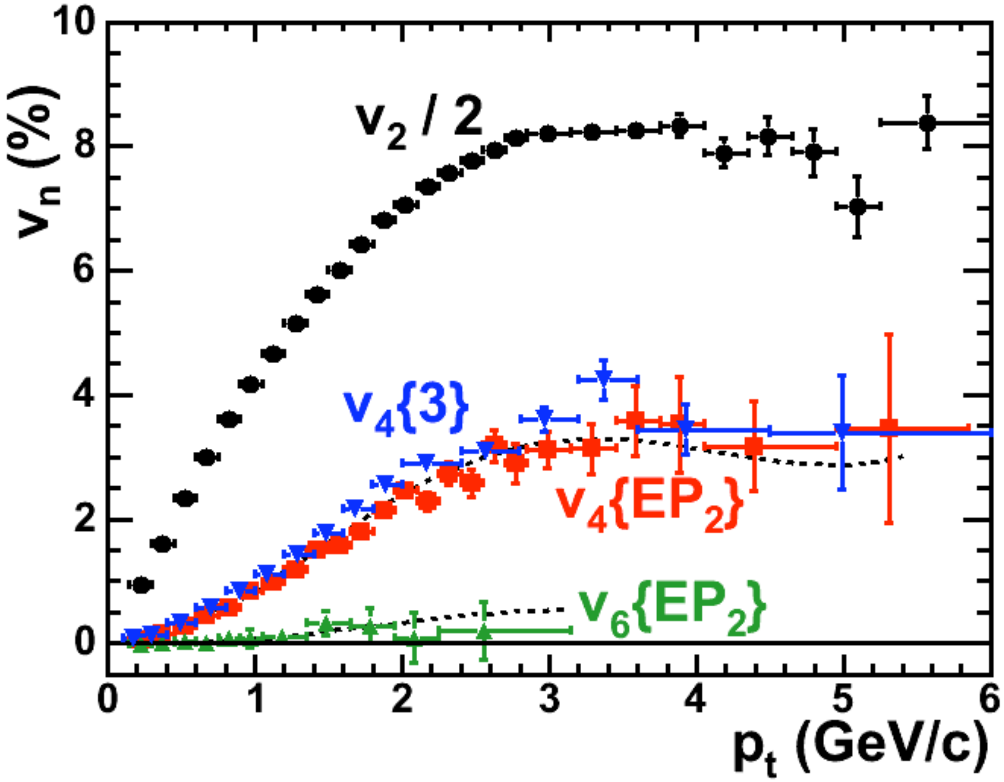
Uses best determined

2^{nd} har. event plane

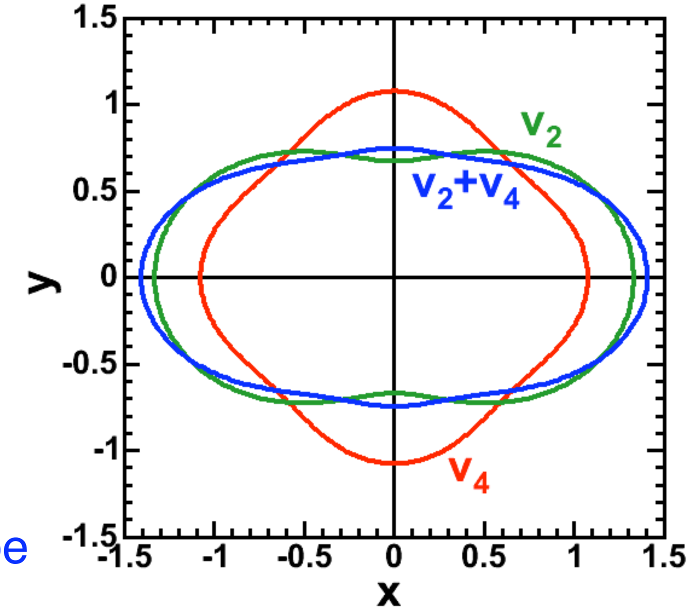
Oldenburg 2005

N. Borghini, P.M. Dinh, and J.-Y. Ollitrault, PRC, **66**, 014905 (2002)

Higher Harmonics



Kolb



$v_n \propto v_2^{n/2}$

more details of the event shape
in momentum space

J. Adams et al., PRL **92**, 062301 (2004)

Conclusions

- 25 years of flow analysis development
  - Extract parameters independent of acceptance

- Standard Method is the most efficient of statistics

- Starting with RHIC Run IV, systematics are more important than statistics
  - Separation in η of particles and plane
  - Multi-particle methods
  - Mixed harmonics
  - Separate nonflow and fluctuations