

Quarkonium

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References:

- Brambilla et.al., Eur.Phys.J. C71 (2011) 1534 (arXiv:1010.5827)
- Andronic et.al., Eur.Phys.J. C76 (2016) 107 (arXiv:1506.03981)
- Aarts et.al., Eur.Phys.J.A53 (2017) 93 (arXiv:1612.08032)

Outline

- Quarkonium: from vacuum to relativistic heavy-ion collisions
- A selection of experimental quarkonium results in HIC
- Selected theory topics for quarkonium in HIC
- Summary

From vacuum to HIC

Quarkonium in vacuum

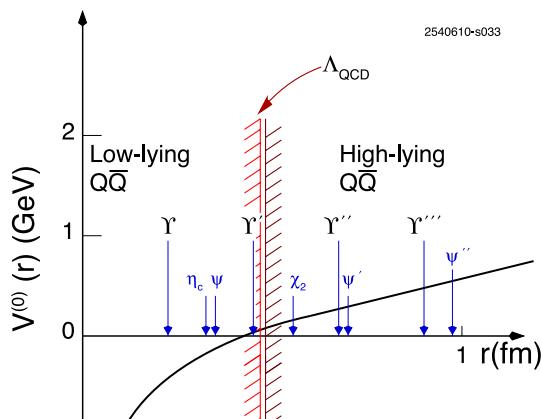
- **QCD laboratory** of long lived & non-relativistic $b\bar{b}$ or $c\bar{c}$ bound states:
decay only into three gluons due to OZI rule: keV widths, significant decay into dileptons

$$\begin{aligned} m^{J/\psi} &= 3.096 \text{ GeV}, \\ m^{\Upsilon} &= 9.460 \text{ GeV} \end{aligned}$$

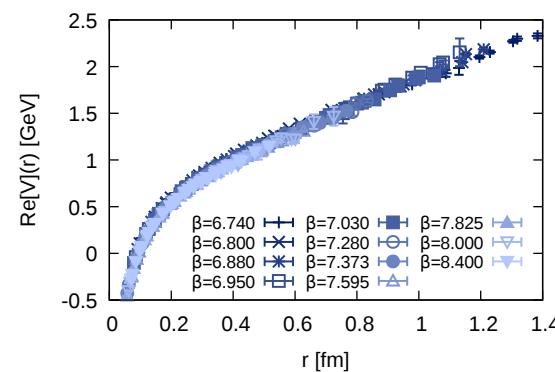


$\Upsilon(3S_1)$ $X_b(3P_{012})$ $J/\psi(3S_1)$ $\Upsilon(3S_{1(n=2)})$ $\Upsilon(3S_{1(n=3)})$ $\Psi'(3S_{1(n=2)})$

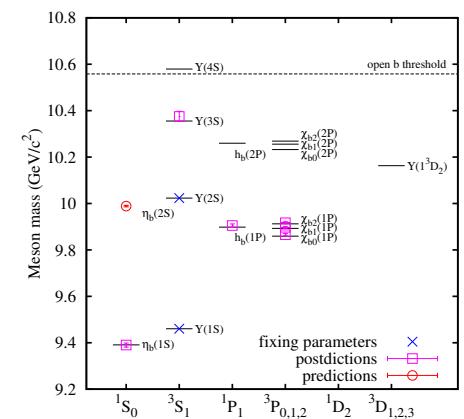
- Intrinsic separation of scales: $m_Q \gg \Lambda_{\text{QCD}}$ & $m_Q \gg m_Q v \sim |\mathbf{p}_{\text{rel}}| \gg m_Q v^2 \sim E_{\text{bind}}$
- Quantum numbers in the naïve quark picture: $1/2 \otimes 1/2 = 0 \oplus 1 \quad 3 \otimes \bar{3} = 0 \oplus 8$
- Below threshold good understanding: Cornell potential & direct lattice QCD studies



$$V_{\text{Cornell}}(r) = -\alpha_s(r)/r + \sigma r + c$$



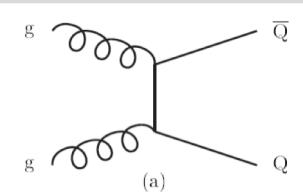
Lattice QCD potential at $T=0$



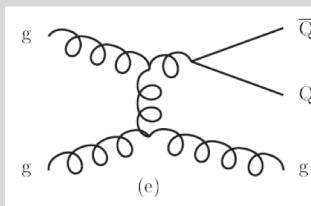
Lattice QCD spectrum

Vacuum Quarkonium production I

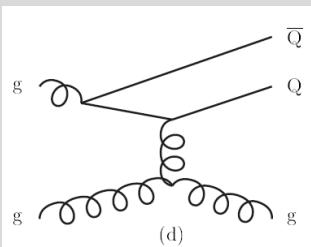
Partonic production processes



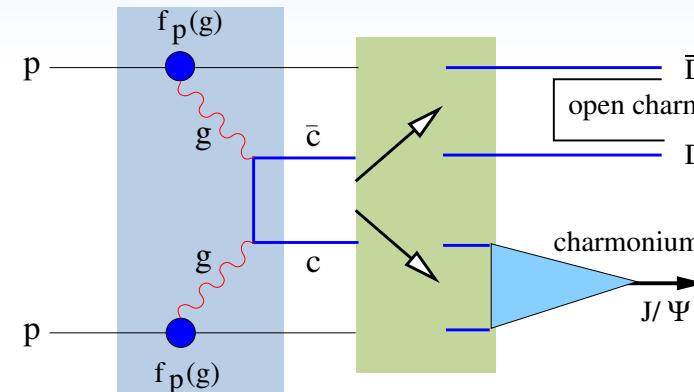
flavor production (LO)



gluon splitting (NLO)



flavor excitation (NLO)



■ Hadron collider: learn about QCD from quarkonium production

$$d\sigma^{pp \rightarrow c\bar{c} + X} \stackrel{Q \gg \Lambda_{QCD}}{\approx} \sum_{n, X'} f_g^p(Q^2) \otimes f_g^p(Q^2) \otimes d\sigma_n^{gg \rightarrow c\bar{c} + X'}$$

$c\bar{c}$ production cross section

parton distribution functions
(nonperturbative, from DIS e-p)

perturbative cross section

■ From $c\bar{c}$ pair to bound state

$$d\sigma^{pp \rightarrow J/\psi + X} \approx d\sigma^{pp \rightarrow c\bar{c} + X} \otimes (\kappa_1 \langle 0 | \mathcal{O}_8(^1S_0) | 0 \rangle + \kappa_2 \langle 0 | \mathcal{O}_8(^1P_0) | 0 \rangle + \dots)$$

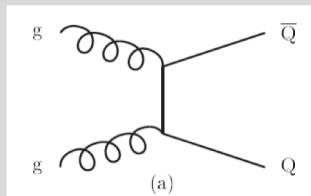
J/ψ production cross section

cc production cross section

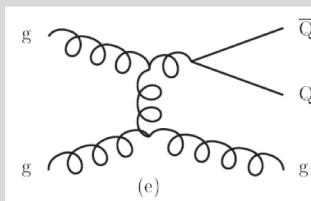
non-perturbative matrix elements for transition to bound state

Vacuum Quarkonium production II

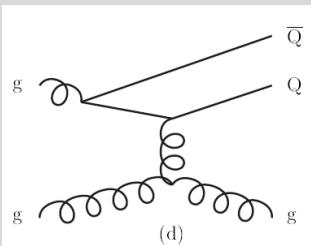
Partonic production processes



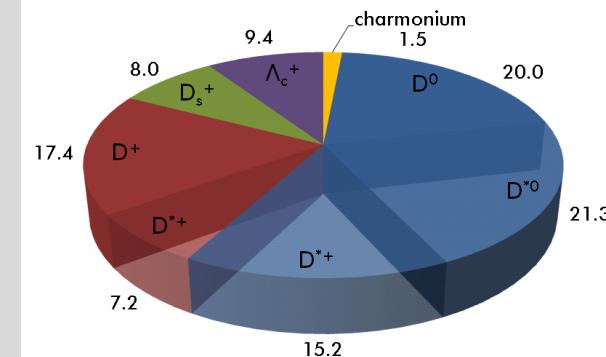
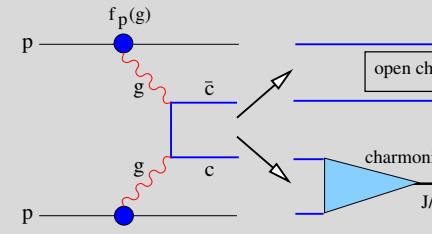
flavor production (LO)



gluon splitting (NLO)



flavor excitation (NLO)



plot by J. Mercado

Inclusive J/ Ψ

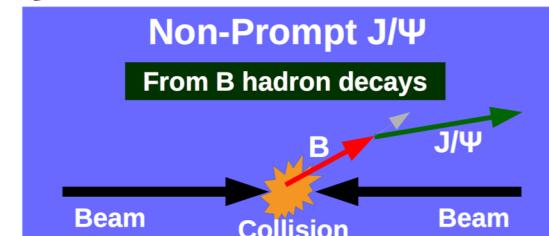
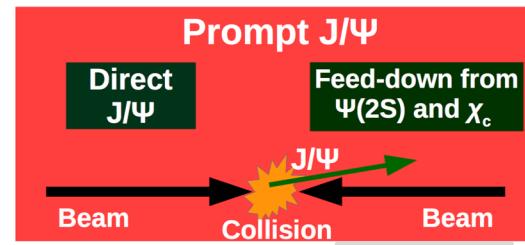
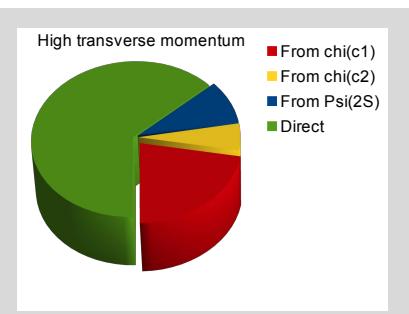
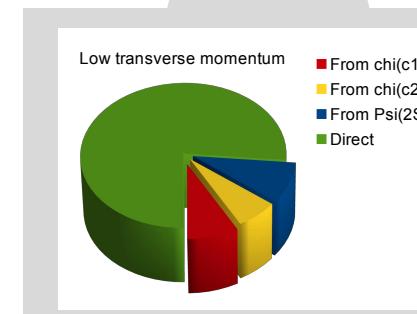


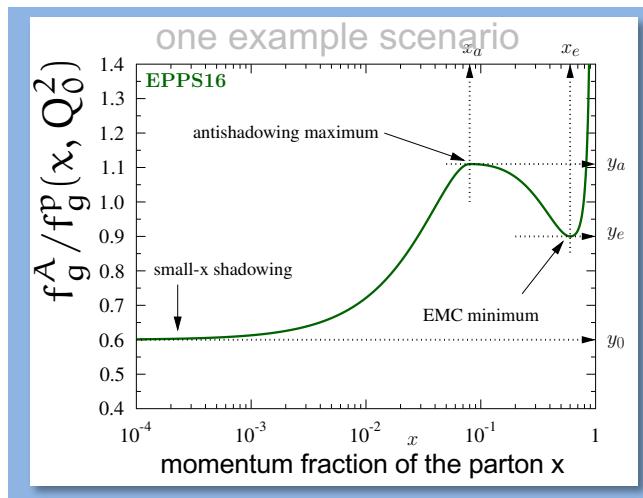
figure by A. Stähli



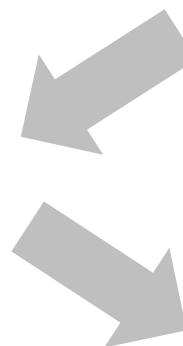
Quarkonium and CNM effects

- Let's start making things a bit more complicated: production involving a nucleus

$$d\sigma^{pA \rightarrow c\bar{c}+X} \stackrel{Q \gg \Lambda_{QCD}}{\approx} \sum_{i,j,X'} f_g^p(Q^2) \otimes f_g^A(Q^2) \otimes d\sigma^{gg \rightarrow c\bar{c}+X'}$$



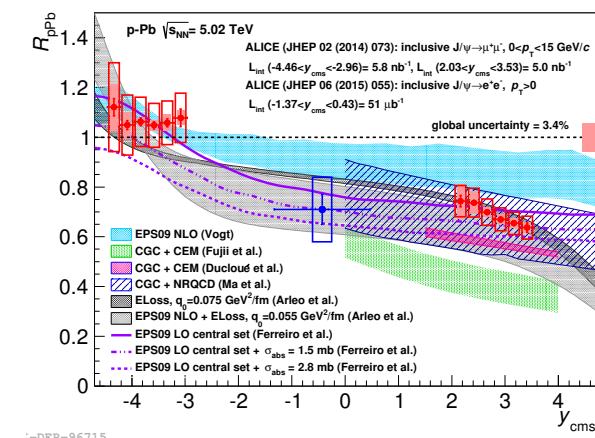
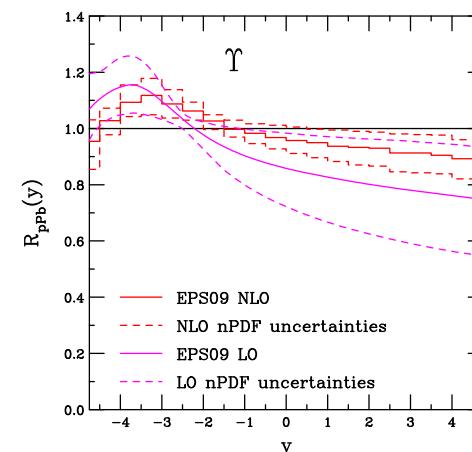
besides nPDF further
effects: nuclear absorption
comovers ...



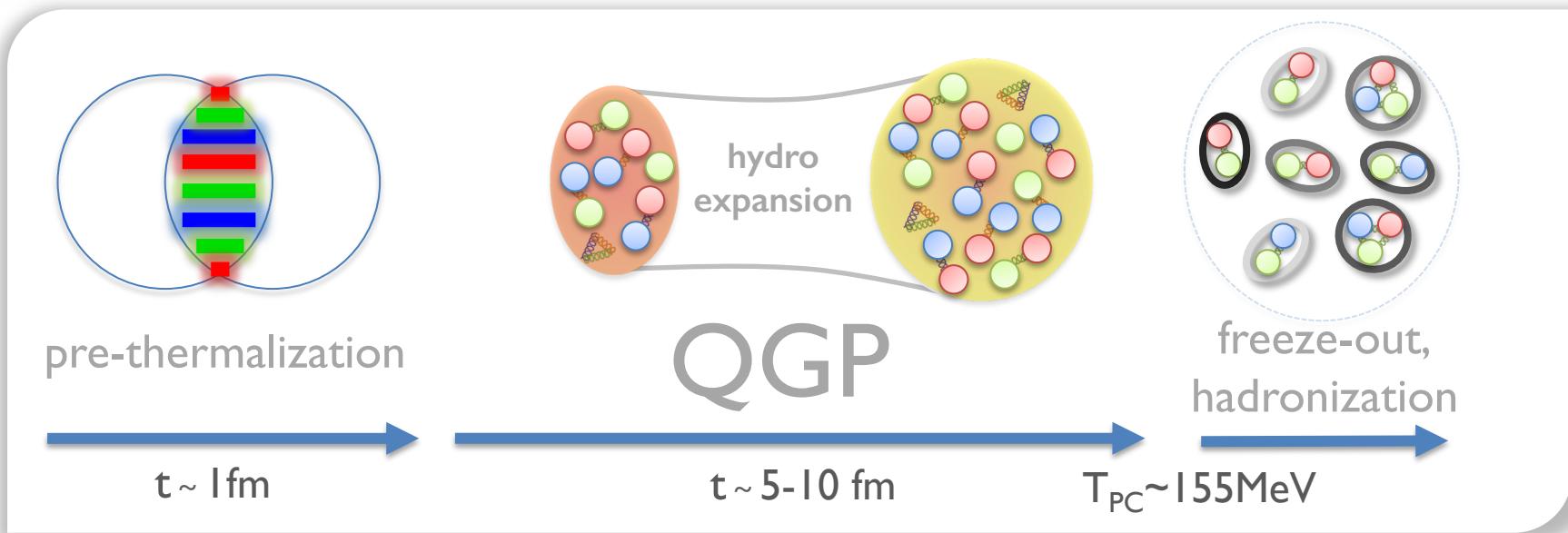
Nuclear modification factor

$$R_{xA}(p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{dN_{xA}/dp_T}{dN_{pp}/dp_T}$$

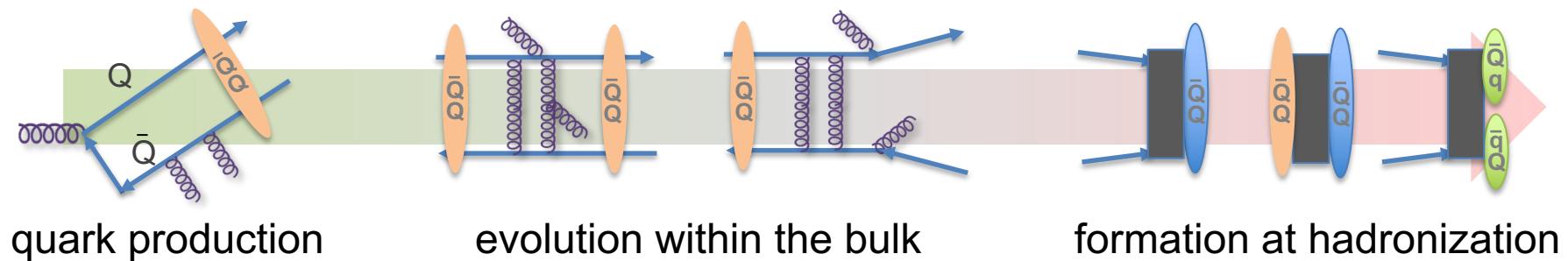
$\langle N_{coll} \rangle$ number of binary collisions



Onward to Nucleus-Nucleus



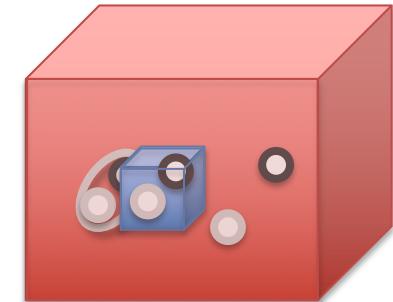
Quarkonium in a heavy-ion collision



Quarkonium in AA

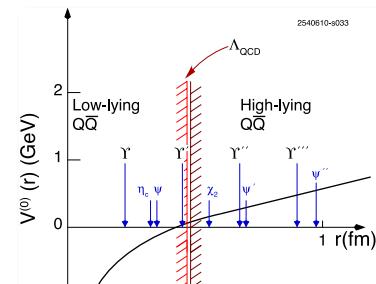
- Presence of QGP is new ingredient (QCD laboratory at $T>0$)

- how do $Q\bar{Q}$ pairs or bound quarkonium equilibrate?
- what happens to the thermalized two-body system

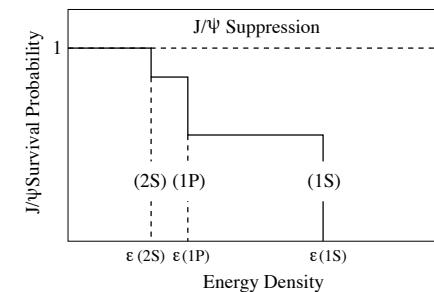
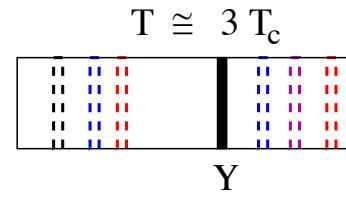
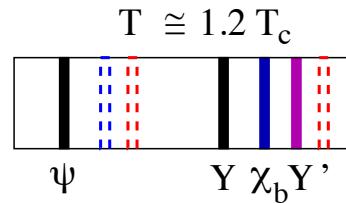
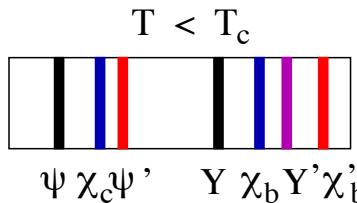


- Matsui & Satz' proposal: presence of QGP prohibits J/ψ formation

- q's & g's **screen** external test charge: what happens to $V_{cornell}$ at $T>0$?
- QCD screening properties in terms of a **Debye mass** $m_D=1/r_D$?



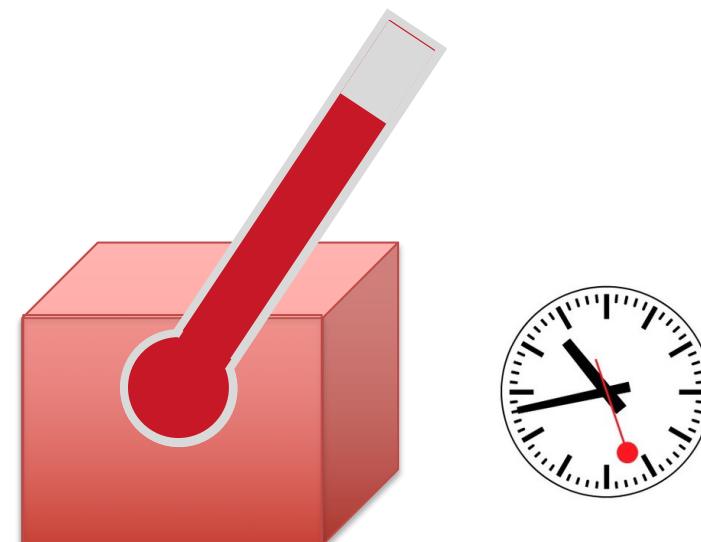
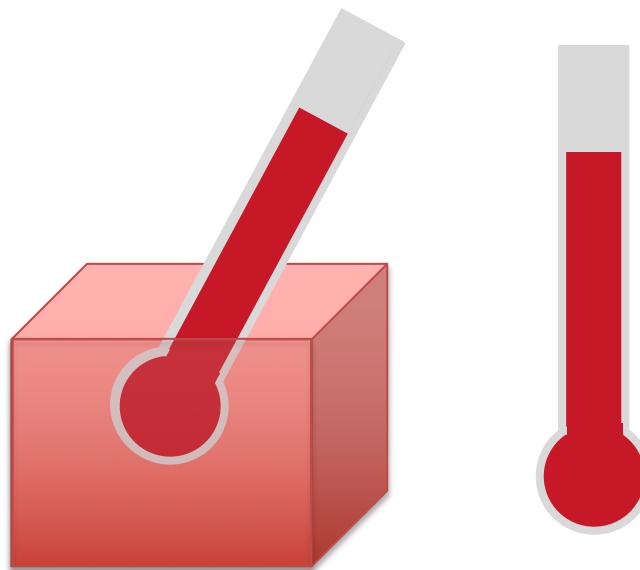
- For multiple species: **sequential melting** according to vacuum binding energy



- can we use quarkonium to determine the **temperature of the QGP**?

The thermometer metaphor

- Remember, temperature can be measured in different ways: **static & dynamic**
- Both strategies rely on a small test-system which only slightly perturbs the bath



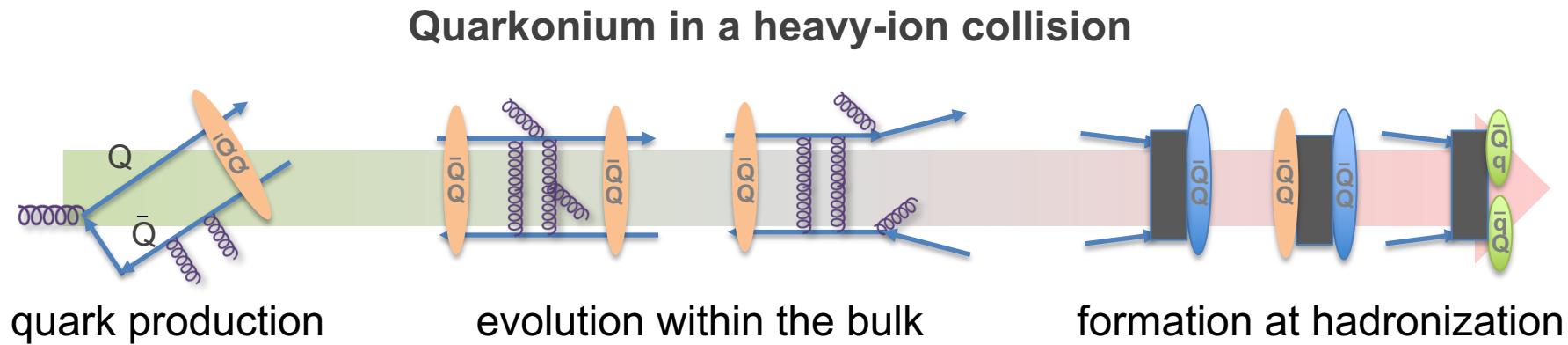
- Bring test-system into contact until equilibrated: investigate its internal state

intuitive for charm, since more easily equilibrated with bulk

- Bring test-system into contact: observe its dynamical approach towards equilibrium

intuitive for bottom, since scale separation well pronounced

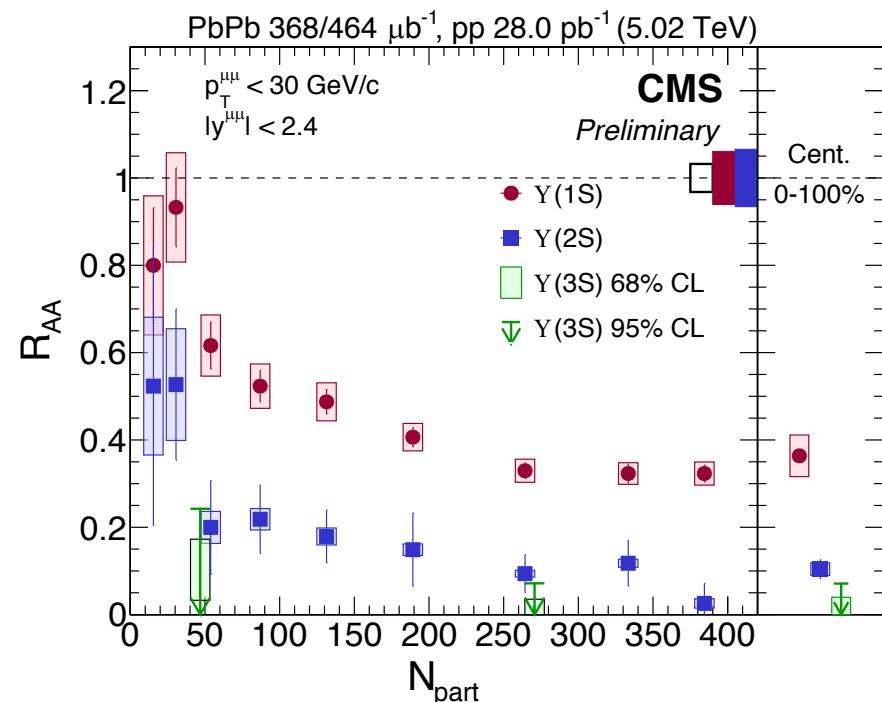
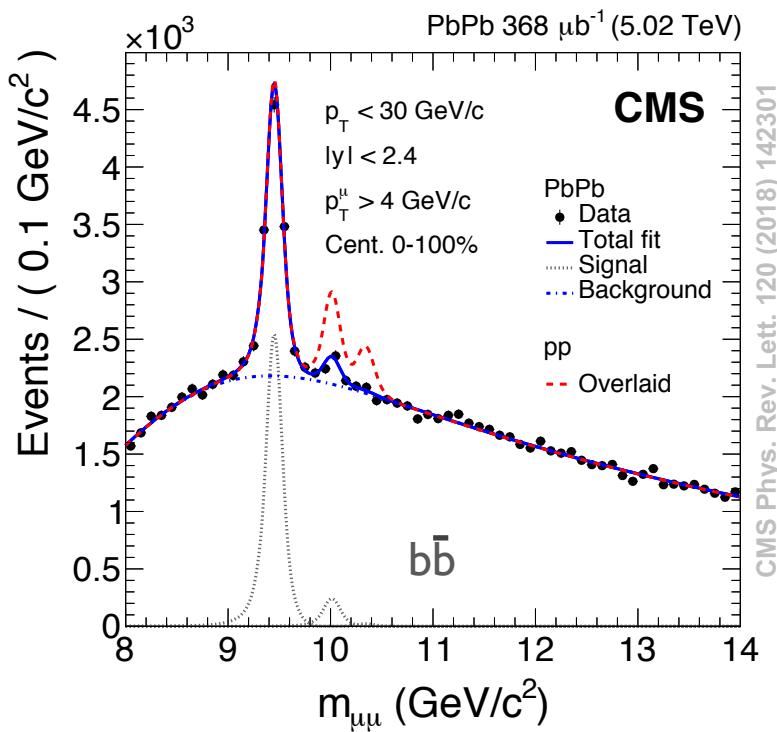
Conclusion of section I



- Quarkonium in HIC is a **multifaceted QCD laboratory**
- We can learn about properties of **nucleons** (nPDF), **dynamics of QCD** at high energy densities **in and out-of equilibrium** and the nature of **confinement** via hadronization

Selected experimental results

Bottomonium at LHC I

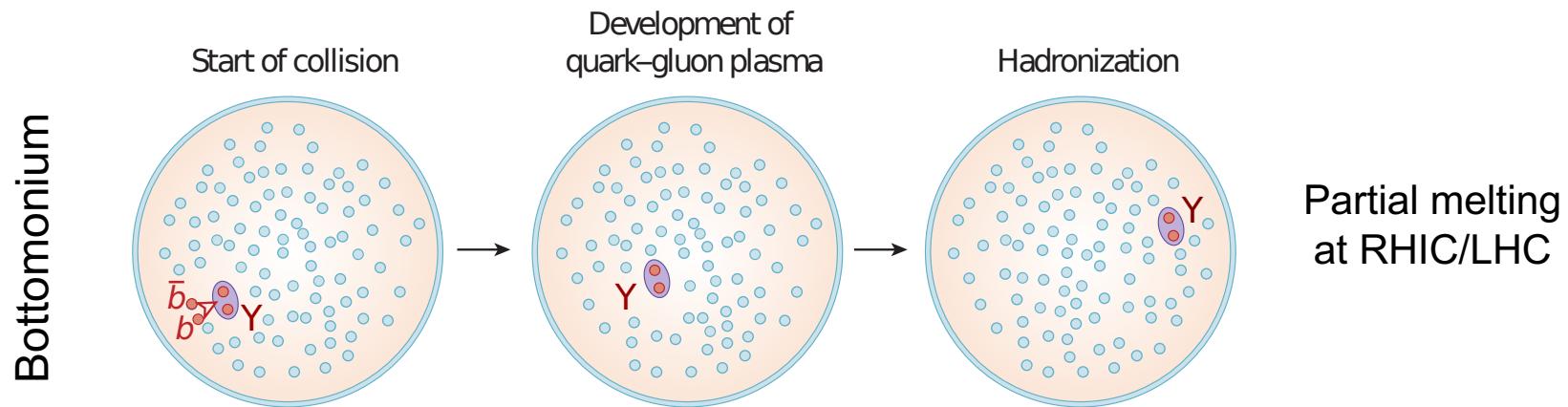
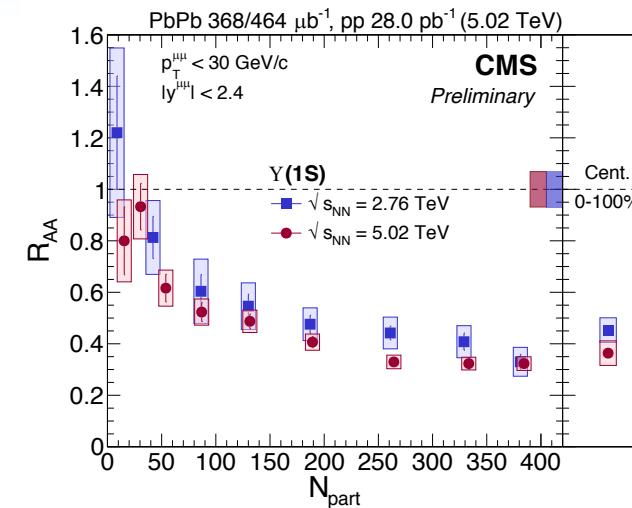


- Decay into dimuons reveals relative suppression of excited states beyond CNM
- Suppression increases with N_{part} , consistent with sequential suppression picture

Bottomonium at LHC II

- Suppression increases with beam energy.

- Picture in mind:
early formation of bottomonium with
subsequent passage through QGP.



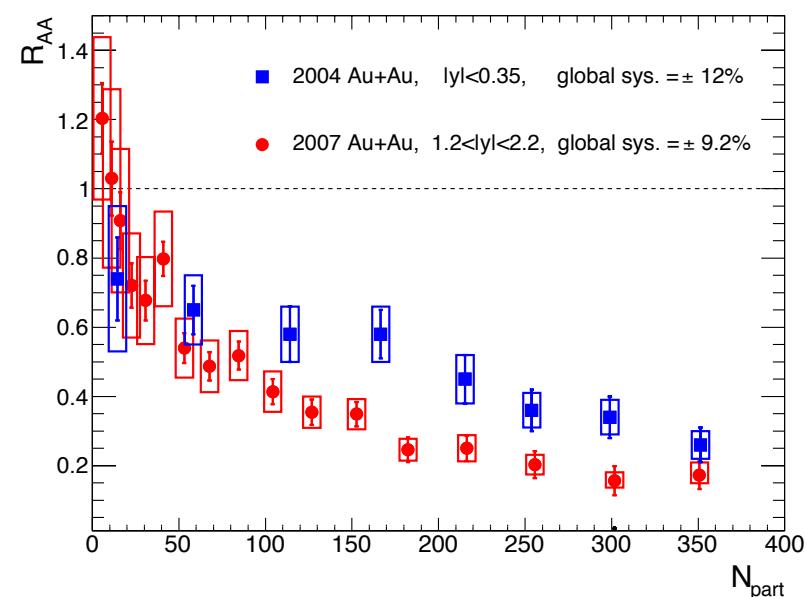
- No direct evidence of equilibration with the bulk: genuine non-equilibrium probe

Charmonium at RHIC I

- At RHIC energies similar behavior to Bottomonium at LHC.

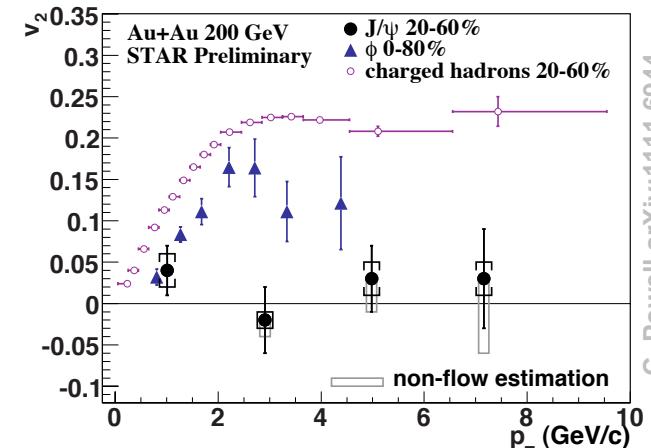
Significant suppression beyond CNM which increases with N_{part} .
- J/ ψ more strongly suppressed than Upsilon.
 ψ' more strongly suppressed than J/ ψ .

Compatible with sequential suppression.



Charmonium at RHIC II

- Again no significant sign of equilibration observed
e.g. elliptic flow consistent with zero



- Picture in mind:**
slightly later formation than Bottomonium, passage through the medium significantly affects ability to bind charm quarks into charmonium

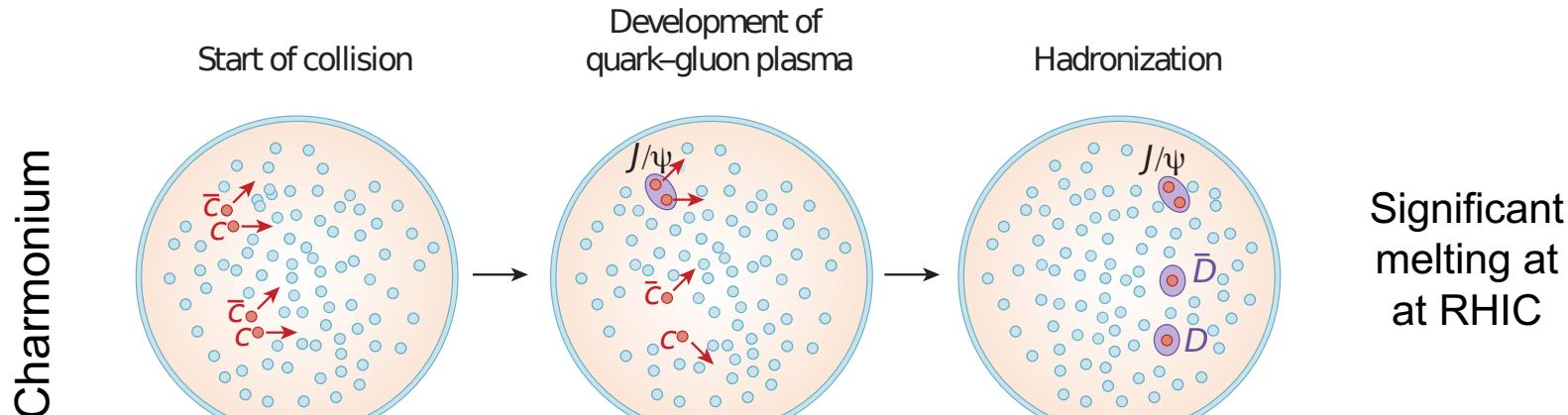
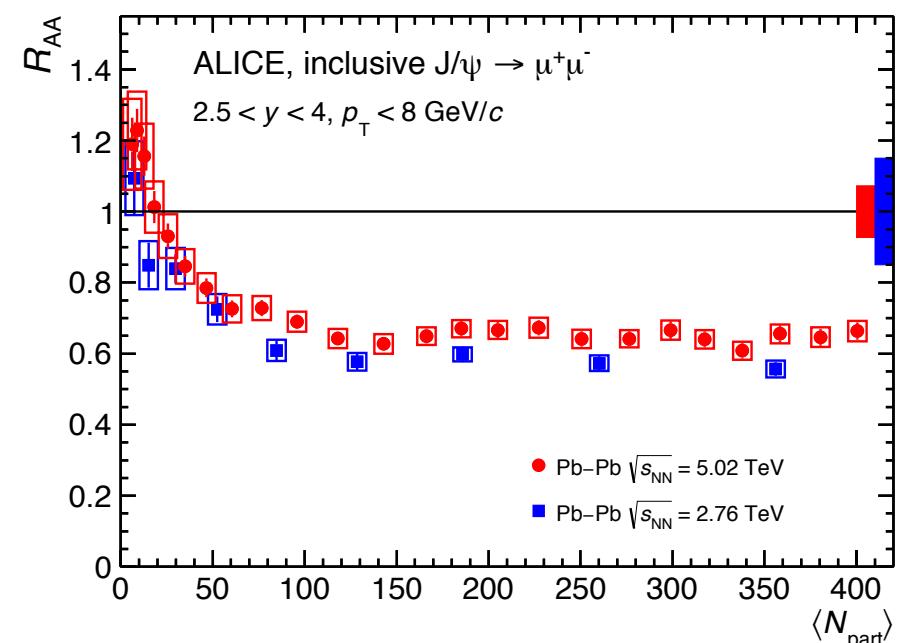
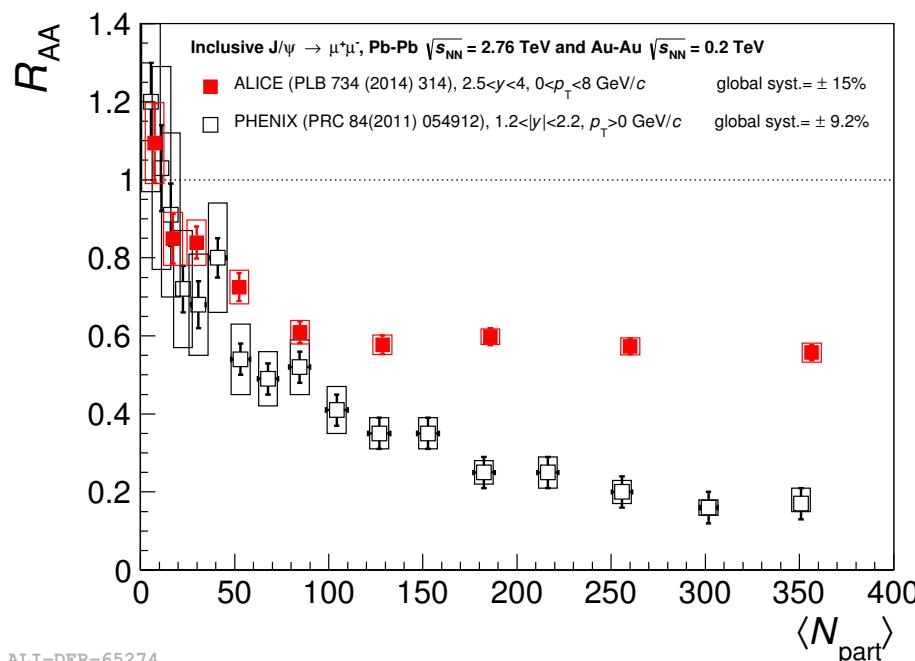


Fig. adapted from P. Braun-Munzinger, J. Stachel Nature 448 06080 (2007)

Charmonium at LHC I

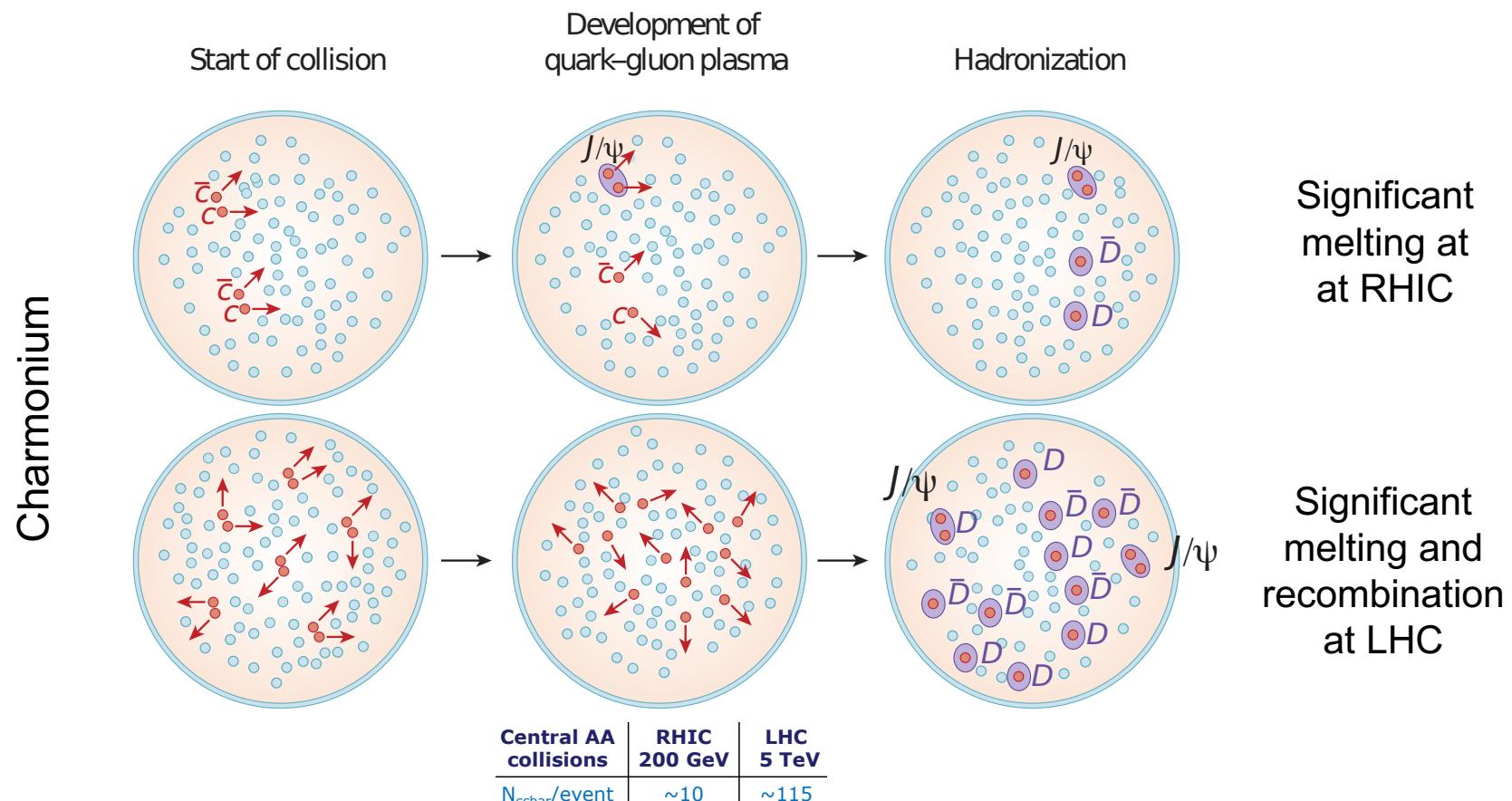
- Interesting observation early on at LHC: suppression weaker than at RHIC



- Tendency persists when increasing from 2.76 to 5.02 TeV

Charmonium at LHC II

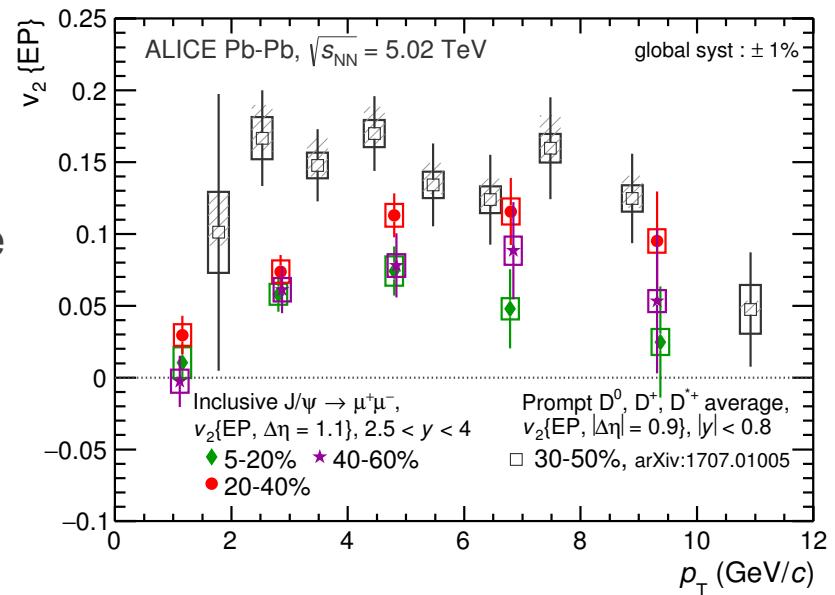
- An additional mechanism at play: **recombination** at hadronization



- Note: hadronization is one of the least understood stages of the collision

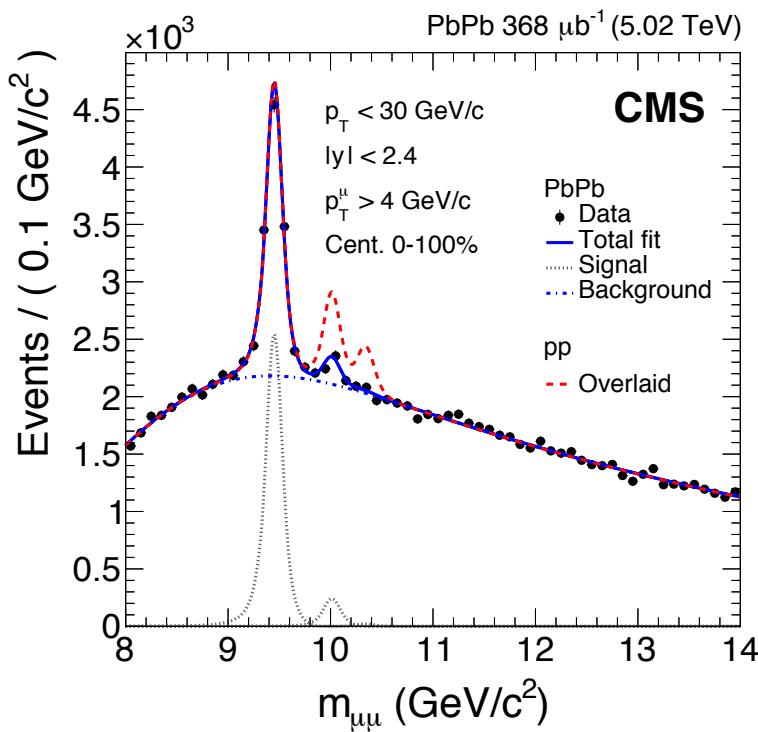
Charmonium at LHC III

- At LHC now also signs of significant kinetic equilibration of charm with the bulk: finite elliptic J/ψ flow

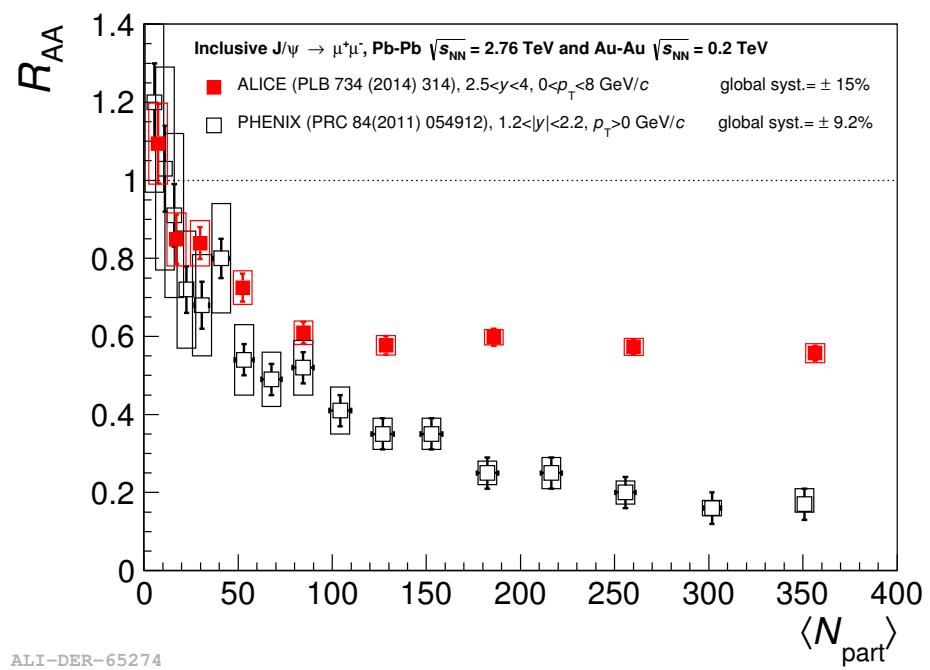


- Remember: equilibration means that we loose memory about the initial conditions over time, equilibrium is time translational invariant.

The two faces of Quarkonium at LHC



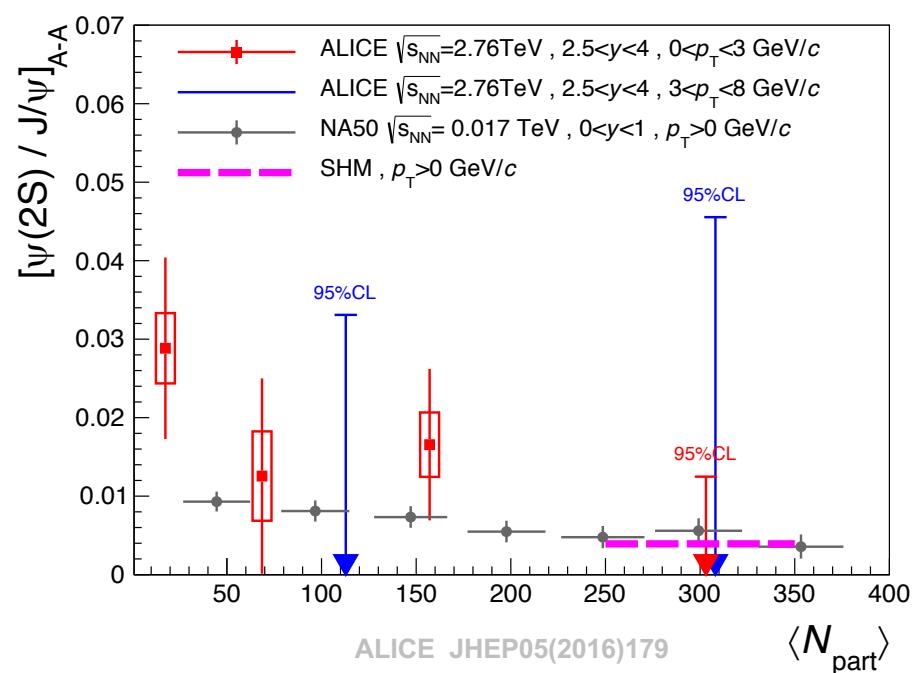
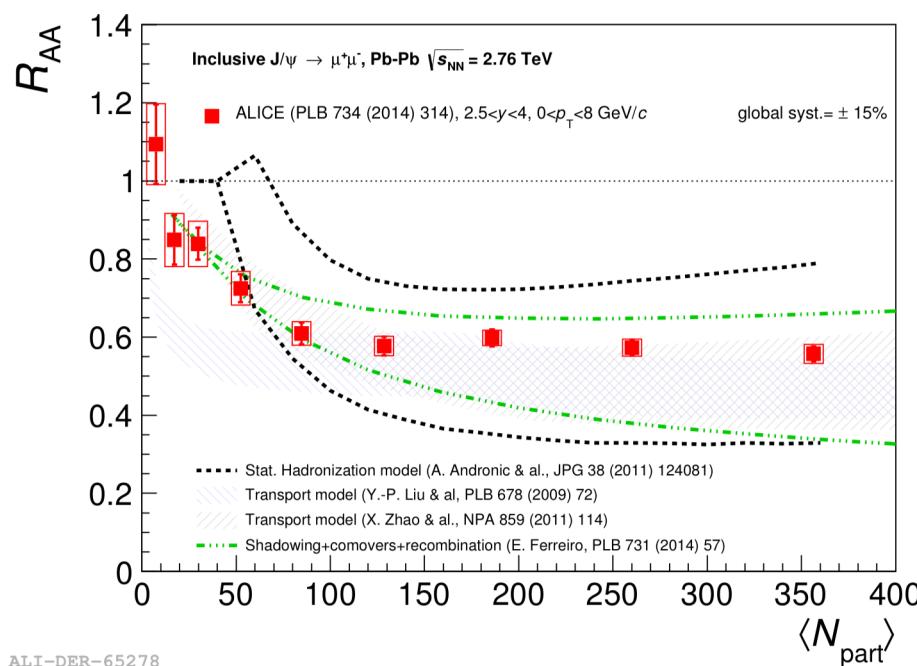
Bottomonium: a non-equilibrium probe of the whole bulk evolution



Charmonium: a partially equilibrated probe of the late stages and hadronization

Let's challenge ourselves

- Many phenomenological approaches nowadays manage to reproduce R_{AA}
- Difficulty: models may include quite different physics processes

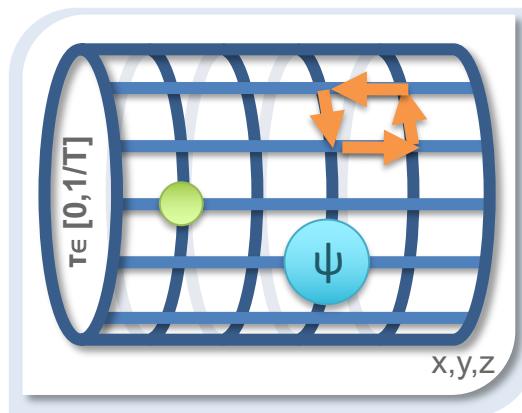


- Find new, more discriminatory observables, one example $\psi'/J/\psi$

Selected theory topics

A robust tool: Lattice QCD

- Well established non-perturbative 1st principles approach to QCD



- Gauge fields as links: $U_\mu(x) = \exp[i g \Delta x_\mu A_\mu(x)]$
- Dynamical fermions $\psi(x)$ with realistic masses
- Finite extend in **imaginary** time: $1/T = \beta = N_T a_T$

$$\langle O \rangle = \frac{1}{N} \langle O(u) \rangle_{N \rightarrow \infty} \sum_{k=1}^N \int D\bar{u} D u O(u) S_{\text{ex}}^{QCD}[u, \bar{u}, \psi, \bar{\psi}] e^{\beta S_E[u, \bar{u}, \psi, \bar{\psi}]}$$

- Successful at $T > 0$: QCD medium

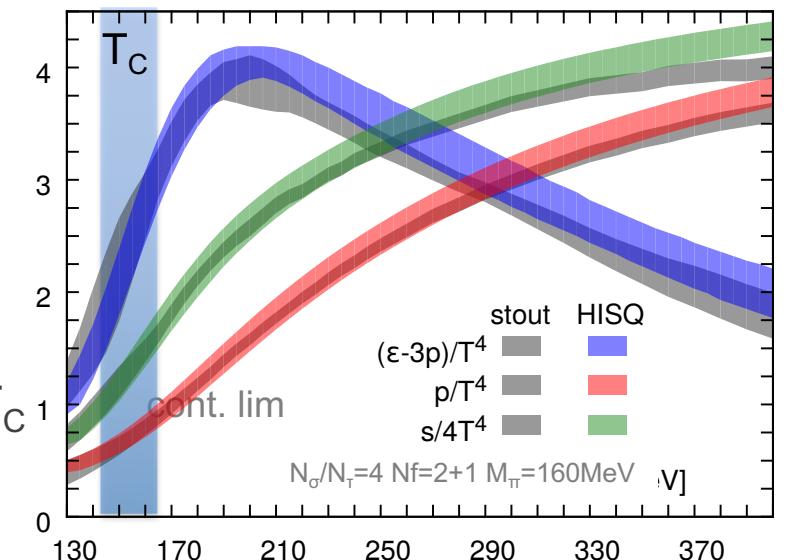
- (Pseudo)critical temperature: 154 ± 9 MeV

WB JHEP 1009 (2010) 073 - HotQCD PRD85 (2012) 054503

- Equation of state as input for hydro-dynamics

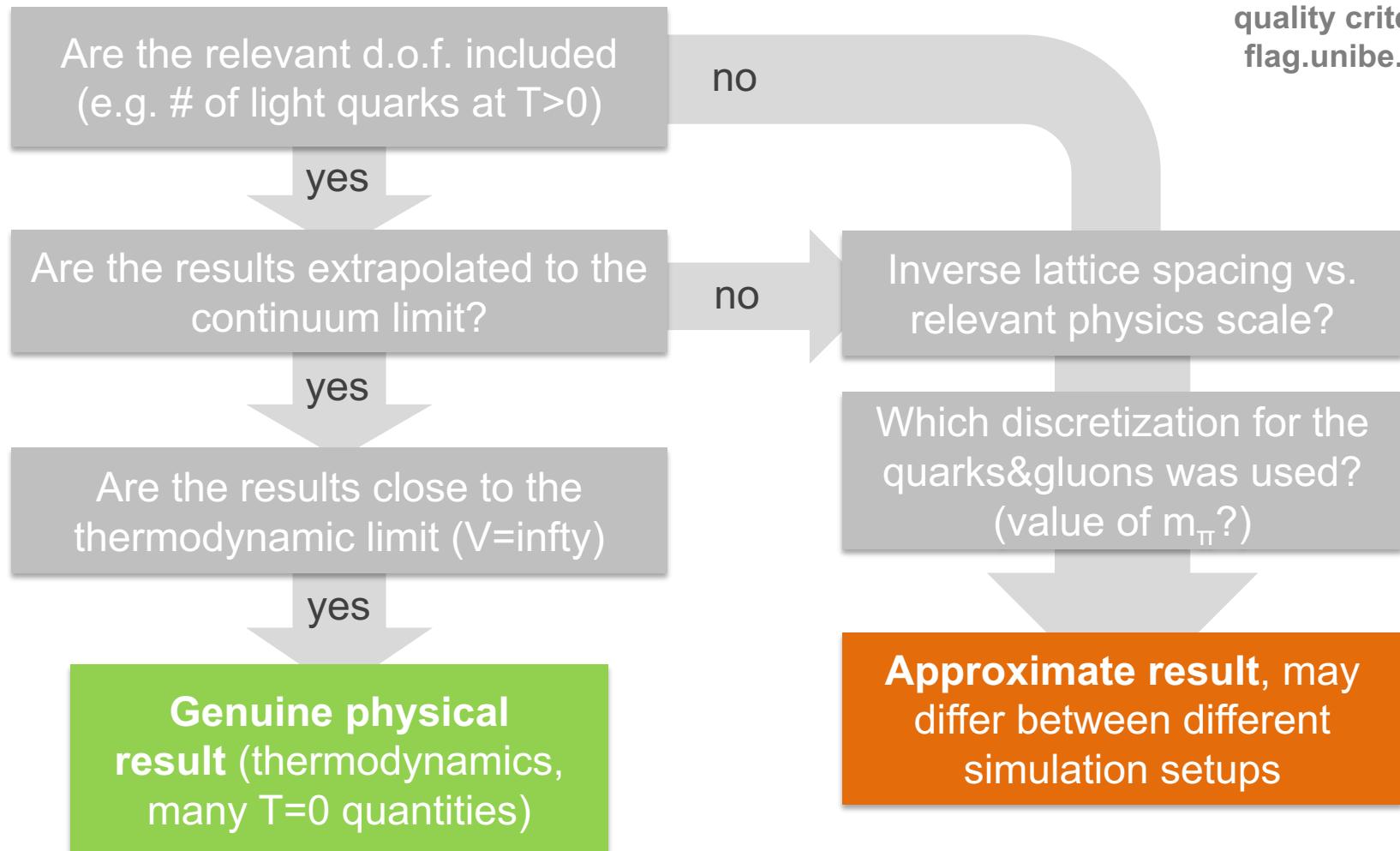
- Trace anomaly $T^{\mu\mu} = \epsilon - 3p$ strong coupling at T_C

HotQCD PRD90 (2014) 094503 - WB PLB730 (2014) 99-104



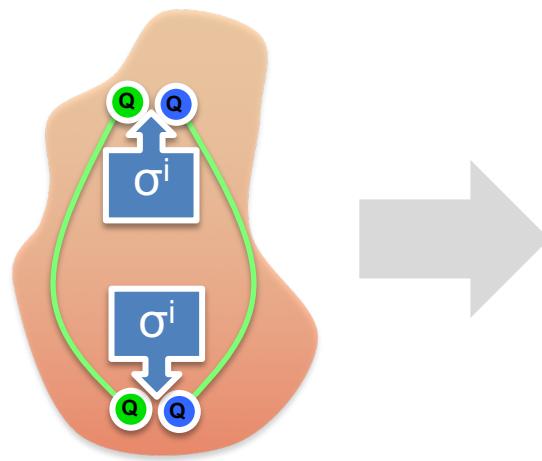
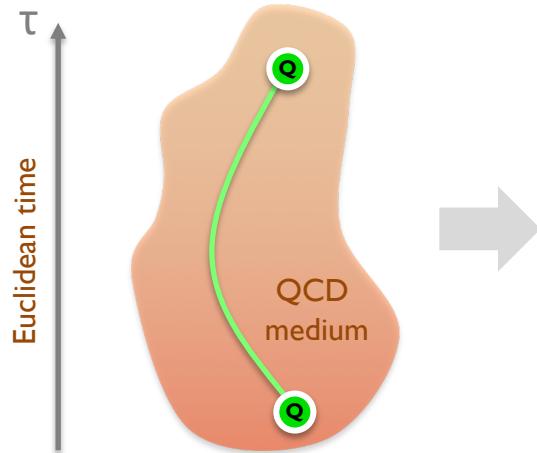
Quality control on the lattice

See e.g. FLAG
quality criteria
flag.unibe.ch



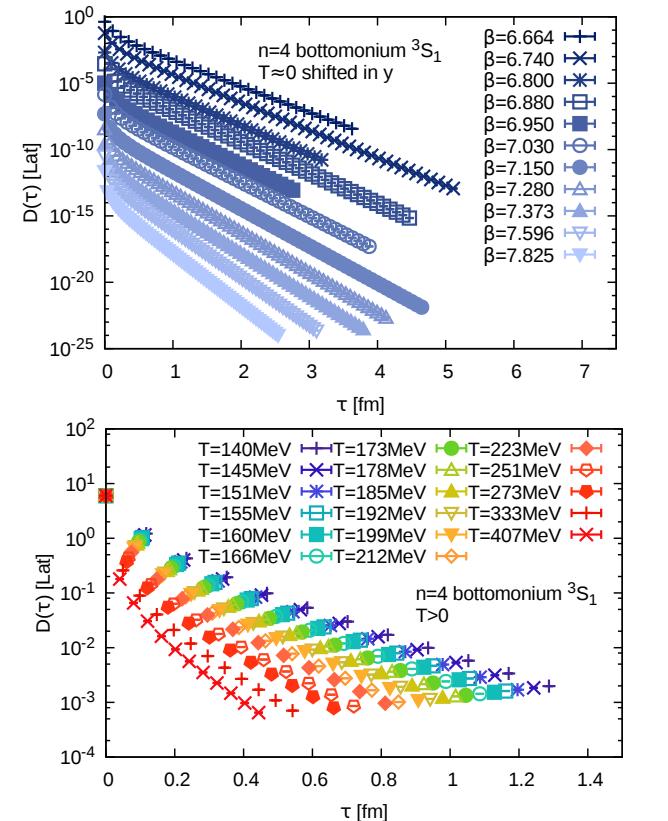
Lattice correlation functions

- Question: What happens to quarkonium when immersed in a $T>0$ medium



Need to choose appropriate discretization of heavy fermions (relativistic/non-relativistic)

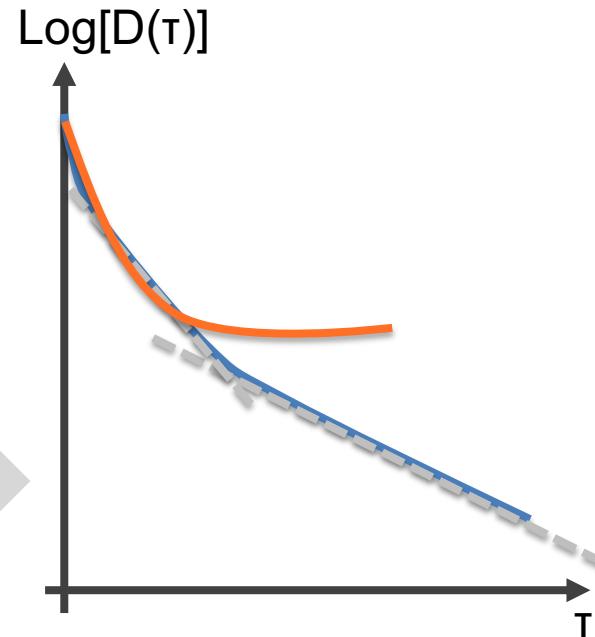
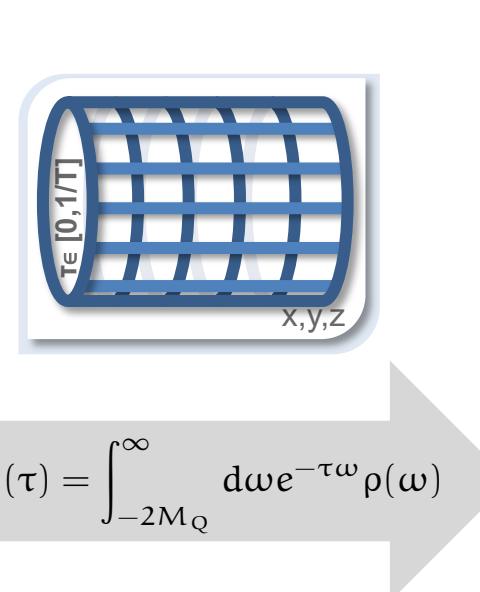
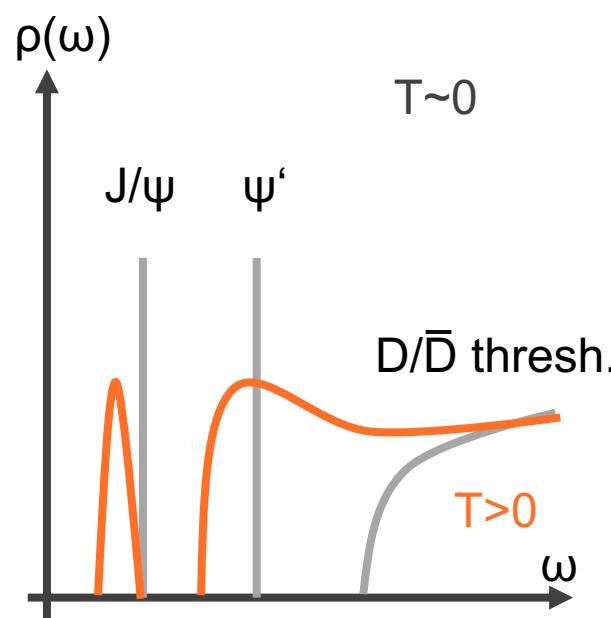
Construct correlation functions, with particular quantum numbers (discrete symmetries)



Euclidean correlation functions at $T=0$ and $T>0$

Accessing dynamical information

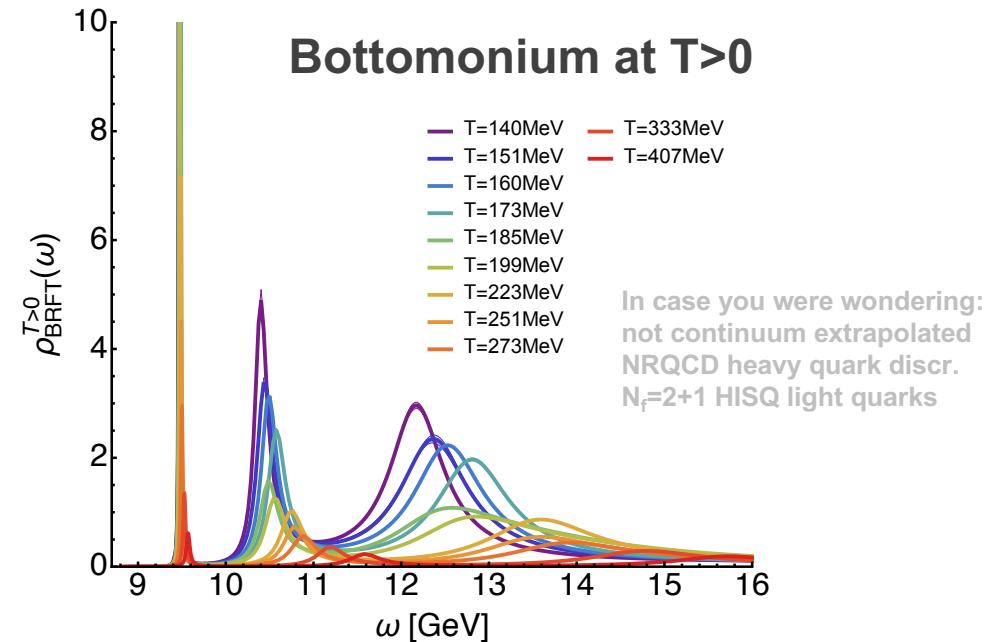
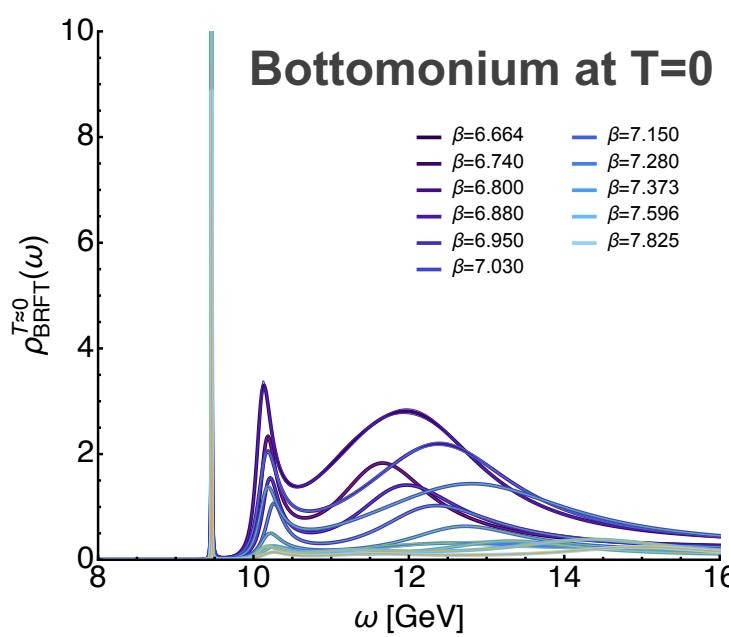
- Lattice simulations are similar to a (very) imperfect detector
- Spectral functions: representation of dimuon-emission from quarkonium decay



- Euclidean time correlations functions arise from folding of spectra
- Extraction of spectra **unfolding problem**: e.g. via Bayesian inference

Bottomonium spectral functions

- Question: What happens to quarkonium when immersed in a $T>0$ medium



- State-of-the-art: good access to ground state but excited states difficult
in-medium mass shifts and thermal broadening (medium excites the GS)
- Current efforts: improve both simulations and unfolding techniques

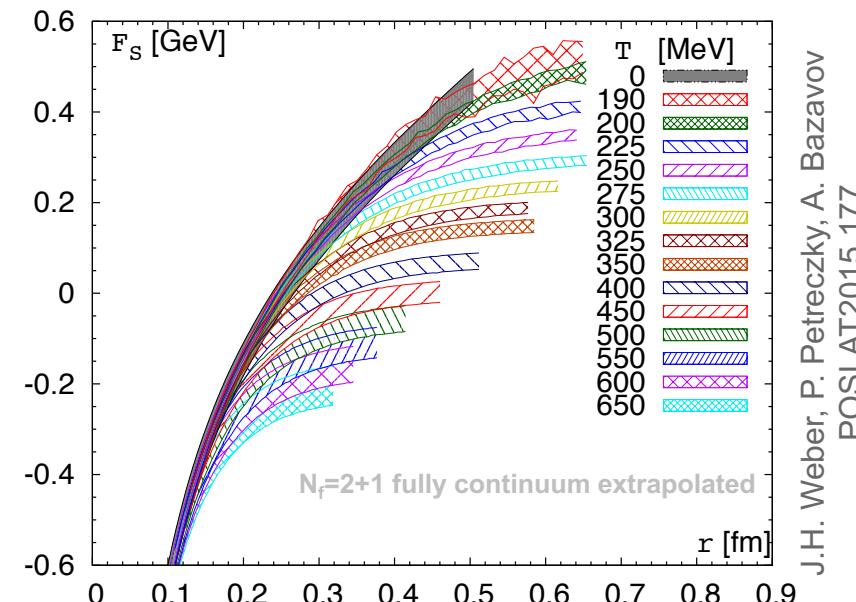
Towards a T>0 potential for Q \bar{Q}

- Since Matsui and Satz: How does the Cornell potential change at T>0?
- The historic approach: potential models at T>0:
 - Ad hoc identification of $V(r)$ with color singlet free energies in Coulomb Gauge
$$F^{(1)}(R) = -\frac{1}{\beta} \log [\langle P(R)P^\dagger(0) \rangle]$$

Nadkarni, PRD 34, (1986)

(directly computed in Euclidean time)

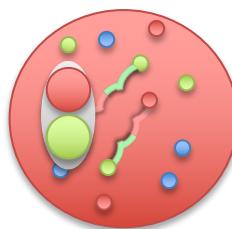
- Many other proposals:
internal energy U^1 , linear combinations
C. Y. Wong, PRC 72, (2005), H. Satz JPG 36, (2009), ...
- A QCD derived Schrödinger equation needed to define a proper potential



Effective field theory & potential

- Effective field theory $m_Q \gg \Lambda_{\text{QCD}}$ & $m_Q \gg m_Q v \sim |\mathbf{p}_{\text{rel}}| \gg m_Q v^2 \sim E_{\text{bind}}$

Relativistic thermal
field theory



QCD

Dirac fields

$$\bar{Q}(x), Q(x)$$

NRQCD

Pauli fields

$$\chi^\dagger(x), \chi(x)$$

pNRQCD

Singlet/Octet

$$\psi_S(\mathbf{R}, t), \psi_O(\mathbf{R}, t)$$

Quantum
mechanics



- Correlation functions governed by Schrödinger-like equation

$$i\partial_t \langle \psi_s(t) \psi_s(0) \rangle = \left(V^{\text{QCD}}(\mathbf{R}) + \mathcal{O}(m_Q^{-1}) + \Theta(\mathbf{R}, t) \right) \langle \psi_s(t) \psi_s(0) \rangle$$

- Matching between QCD and pNRQCD:
compare correlation functions with same physics content

$$V^{\text{QCD}}(\mathbf{R}) = \lim_{t \rightarrow \infty} \frac{i\partial_t W(\mathbf{R}, t)}{W(\mathbf{R}, t)} \in \mathbb{C}$$

- Initially surprising result: $T>0$ potential is not just screened but also $\text{Im}[V] \neq 0$
- Current efforts: understand the origin and physics implications of $\text{Im}[V]$

Open Quantum Systems

- Recent progress in understanding the real-time dynamics of quarkonium
- High energy physics inspired by a technique from condensed matter physics
 - Overall system is closed, hermitian Hamiltonian

$$\mathcal{H} = \mathcal{H}_{Q\bar{Q}} \otimes I_{\text{med}} + I_{Q\bar{Q}} \otimes \mathcal{H}_{\text{med}} + \mathcal{H}_{\text{int}}$$

$$\frac{d}{dt}\sigma(t) = \frac{1}{i\hbar} [\mathcal{H}, \sigma(t)]$$

unitary evolution: $\mathcal{H}=\mathcal{H}^\dagger$

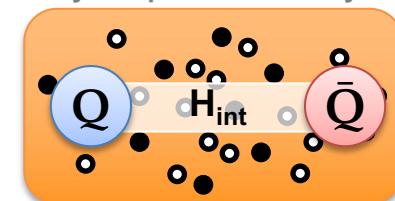
- Interested in the dynamics of the QQbar system only

$$\sigma_{Q\bar{Q}}(t, \mathbf{R}, \mathbf{R}') = \text{Tr}_{\text{med}} [\sigma(t, \mathbf{R}, \mathbf{R}')]$$

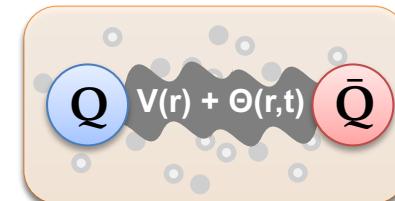
$$\frac{d}{dt}\sigma_{Q\bar{Q}}(t) = \mathcal{L}\sigma_{Q\bar{Q}}(t)$$

dynamical map

see e.g. H.-P. Breuer, F. Petruccione,
Theory of Open Quantum Systems



$$\text{Tr}_{\text{med}} [\cdot]$$



- Goal: derive dynamical map from 1st principles and implement (unravel) its dynamics

Open quantum systems

- In case of separation of timescales: Markovian master equation (Lindblad)

$$\frac{d}{dt} \sigma_{Q\bar{Q}}(t) = -i[H, \sigma] + \sum_n (2L_n \sigma_{Q\bar{Q}} L_n^\dagger - L_n^\dagger L_n \sigma_{Q\bar{Q}} - \sigma_{Q\bar{Q}} L_n^\dagger L_n)$$

- Lindblad operators L_n encode coupling to environment: induce decoherence
- Note time evolution does not have to be unitary anymore (dissipation effects can be included)

- Some current strategies for implementation

- Solve the master equation brute force
- As ensemble of stochastically evolved wavefunctions (Quantum State Diffusion)
- Approximate semi-classically with a generalized Langevin equation

- Current efforts: connect OQS with the established EFT concepts

What did not fit into 45+ minutes

Transport models:
Boltzmann equation,
rate equations

Statistical model of
hadronization:
Hadron resonance gas

Hadronic effective
descriptions in the
confined phase

AdS/CFT
approaches
to quarkonium

+ ...

Summary

- Quarkonium in HIC is a **multifaceted QCD laboratory**
- We can learn about properties of **nucleons** (nPDF), **dynamics of QCD** at high energy densities **in and out-of equilibrium** and the nature of **confinement** via hadronization
- Experiment continues to deliver copious amounts of high precision quarkonium data, providing a rich testing ground for our understanding of QCD. Ongoing work on new and more discriminatory observables
- Theory provides a multitude of paths towards understanding the questions posed by heavy-quarkonium in HIC from first principles. Connecting 1st principles to phenomenology offers lots of research opportunities

Challenge yourself: join the quarkonium quest before run3

Backup

The unfolding challenge

- Inversion of integral transform required to obtain spectra from correlators

$$D_i(\epsilon) \sum_{l=1}^{N_\omega} \int_{-2M_Q}^{\infty} \exp[-\omega_l d\omega] \rho_l^{-1} \Delta\omega \rho(\omega)$$

- N_ω parameters $\rho_l \gg N_\tau$ datapoints
- simulated D_i has finite precision

- Give meaning to problem by incorporating prior knowledge: **Bayesian approach**

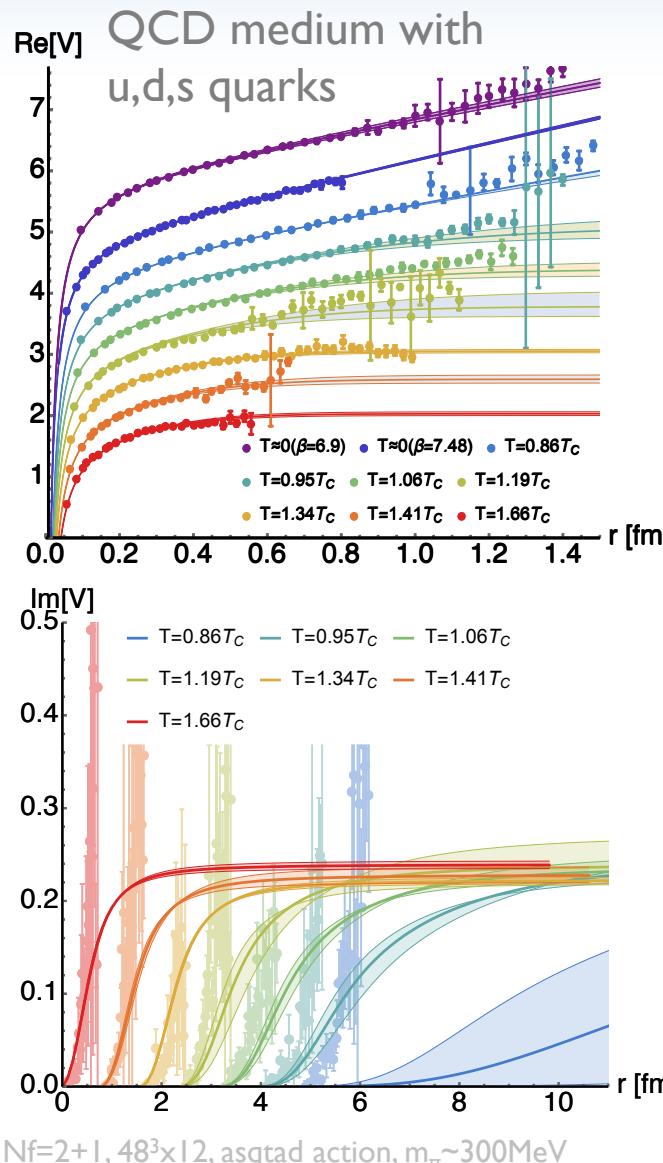
M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

- Bayes theorem: Regularize the naïve χ^2 functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D, I] \propto P[D|\rho] P[\rho|I] \rightarrow \frac{\delta P[\rho|D, I]}{\delta \rho_l} = 0$$

- Several Bayesian approaches on the market: MEM, SAI, SOR, BR method
 - Differ in the regulator functional $P[\rho|I]$ and how to find the most probable spectrum
- In Bayesian continuum limit $N_\tau \Rightarrow \infty, \Delta D/D \rightarrow 0$ different methods will agree
- Non-Bayesian competition: Backus-Gilbert, Tikhonov, Pade, Cuniberti...

T>0 static potential from the lattice



- Best estimate from N_f=2+1 lQCD at m_π~300MeV
- At T~0 Re[V] on the lattice well described by naïve Cornell ansatz: $V= -a/r + \sigma r + c$
- How to parametrize strength of screening is ambiguous: concept of Debye mass gauge dep.
- Analytic parametrization from a generalized gauss law with a single T-dep. parameter m_D

