

Three-Particle Correlation

Analysis Note

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1 Multi-particle Correlations

2 Global Momentum Conservation

For two particle correlation :

$$C(p_1, p_2) = \frac{f_c(p_1, p_2)}{f_c(p_1)f_c(p_2)} - 1 = -\frac{2p_1p_2\cos(\phi_1 - \phi_2)}{N\langle p_T^2 \rangle} \quad (1)$$

For three particle correlation :

$$C(p_1, p_2, p_3) = \frac{f_c(p_1, p_2, p_3)}{f_c(p_1)f_c(p_2)f_c(p_3)} - 1 = -\frac{2}{N\langle p_T^2 \rangle} [p_1p_2\cos(\phi_1 - \phi_2) + p_2p_3\cos(\phi_2 - \phi_3) + p_1p_3\cos(\phi_1 - \phi_3)] \quad (2)$$

Taking :

$$C = -\frac{2}{N\langle p_T^2 \rangle} \quad (3)$$

2.1 C_{11}

Momentum correction term for two particle correlator :

$$C_{11}^{MC} = C \int p_1p_2\cos(\phi_1 - \phi_2)\cos(\phi_1 - \phi_2)d\phi_1d\phi_2 \quad (4)$$

$$= C \int p_1p_2 \frac{1 + \cos(2(\phi_1 - \phi_2))}{2} d\phi_1d\phi_2 \quad (5)$$

(6)

If there is flow in the system, then :

$$\frac{dN}{d\phi} = \frac{1}{2\pi} \left[1 + \sum_n 2v_n \cos n(\phi - \Psi_{RP}) \right] \quad (7)$$

To keep the calculation simple, lets take $\Psi_{RP} = 0$ and n from 1 to 4.

$$\frac{dN}{d\phi} \approx \frac{1}{2\pi} [1 + 2\{v_1\cos(\phi) + v_2\cos(2\phi) + v_3\cos(3\phi) + v_4\cos(4\phi)\}] \quad (8)$$

The normalised momentum correction term is :

$$C_{11}^{MC} = \frac{C \int p_1p_2 \frac{1 + \cos(2(\phi_1 - \phi_2))}{2} \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} d\phi_1 d\phi_2}{\int \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} d\phi_1 d\phi_2} \quad (9)$$

$$= C \frac{p_1p_2}{2} (1 + v_2^2) \quad (10)$$

2.2 C_{112}

Momentum correction term for three particle correlator C_{112} :

$$\begin{aligned}
C_{112}^{MC} &= C \int [p_1 p_2 \cos(\phi_1 - \phi_2) + p_2 p_3 \cos(\phi_2 - \phi_3) + p_1 p_3 \cos(\phi_1 - \phi_3)] \\
&\quad \cos(\phi_1 + \phi_2 - 2\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&= C \int p_1 p_2 \cos(\phi_1 - \phi_2) \cos(\phi_1 + \phi_2 - 2\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + C \int p_2 p_3 \cos(\phi_2 - \phi_3) \cos(\phi_1 + \phi_2 - 2\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + C \int p_1 p_3 \cos(\phi_1 - \phi_3) \cos(\phi_1 + \phi_2 - 2\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&= \frac{C}{2} \int p_1 p_2 (\cos 2(\phi_1 - \phi_3) + \cos 2(\phi_2 - \phi_3)) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + \frac{C}{2} \int p_2 p_3 (\cos(\phi_1 + 2\phi_2 - 3\phi_3) + \cos(\phi_1 - \phi_3)) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + \frac{C}{2} \int p_1 p_3 (\cos(2\phi_1 + \phi_2 - 3\phi_3) + \cos(\phi_2 - \phi_3)) d\phi_1 d\phi_2 d\phi_3 \\
&= \frac{C}{J} p_1 p_2 \cos 2(\phi_1 - \phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + C \int p_1 p_3 (\cos(2\phi_1 + \phi_2 - 3\phi_3) + \cos(\phi_2 - \phi_3)) d\phi_1 d\phi_2 d\phi_3
\end{aligned}$$

In the last step the last two integrals are symmetrized and also in the first integral the two terms are symmetrised. Correlations having three terms in the correction will have small contribution and can be neglected. Keeping only the correlations having two terms :

$$C_{112}^{MC} \approx C \int p_1 p_2 (\cos 2(\phi_1 - \phi_3)) d\phi_1 d\phi_2 d\phi_3 \quad (11)$$

$$+ C \int p_1 p_3 \cos(\phi_2 - \phi_3) d\phi_1 d\phi_2 d\phi_3 \quad (12)$$

$$(13)$$

Now addressing flow in the system :

$$\frac{dN}{d\phi} \approx \frac{1}{2\pi} [1 + 2\{v_1 \cos(\phi) + v_2 \cos(2\phi) + v_3 \cos(3\phi) + v_4 \cos(4\phi)\}]$$

The normalised momentum correction term is :

$$C_{112}^{MC} \approx \frac{C \int p_1 p_2 (\cos 2(\phi_1 - \phi_3)) \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3} d\phi_1 d\phi_2 d\phi_3}{\int \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} d\phi_1 d\phi_2} + \frac{C \int p_1 p_3 \cos(\phi_2 - \phi_3) \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3} d\phi_1 d\phi_2 d\phi_3}{\int \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3} d\phi_1 d\phi_2 d\phi_3} \quad (14)$$

$$= C [p_1 p_2 v_2^2 + p_1 p_3 v_1^2] \quad (15)$$

2.3 C_{123}

Momentum correction term for three particle correlator C_{123} :

$$\begin{aligned}
C_{123}^{MC} &= C \int [p_1 p_2 \cos(\phi_1 - \phi_2) + p_2 p_3 \cos(\phi_2 - \phi_3) + p_1 p_3 \cos(\phi_1 - \phi_3)] \\
&\quad \cos(\phi_1 + 2\phi_2 - 3\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&= C \int p_1 p_2 \cos(\phi_1 - \phi_2) \cos(\phi_1 + 2\phi_2 - 3\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + C \int p_2 p_3 \cos(\phi_2 - \phi_3) \cos(\phi_1 + 2\phi_2 - 3\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + C \int p_1 p_3 \cos(\phi_1 - \phi_3) \cos(\phi_1 + 2\phi_2 - 3\phi_3) d\phi_1 d\phi_2 d\phi_3 \\
&= \frac{C}{2} \int p_1 p_2 (\cos(2\phi_1 + \phi_2 - 3\phi_3) + \cos(3(\phi_2 - \phi_3))) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + \frac{C}{2} \int p_2 p_3 (\cos(\phi_1 + 3\phi_2 - 4\phi_3) + \cos(\phi_1 + \phi_2 - 2\phi_3)) d\phi_1 d\phi_2 d\phi_3 \\
&\quad + \frac{C}{2} \int p_1 p_3 (\cos(2\phi_1 + 2\phi_2 - 4\phi_3) + \cos(2(\phi_2 - \phi_3))) d\phi_1 d\phi_2 d\phi_3
\end{aligned}$$

Keeping only the correlations having two terms :

$$C_{123}^{MC} \approx \frac{C}{2} \int p_1 p_2 \cos 3(\phi_2 - \phi_3) d\phi_1 d\phi_2 d\phi_3 \quad (16)$$

$$+ \frac{C}{2} \int p_1 p_3 \cos 2(\phi_2 - \phi_3) d\phi_1 d\phi_2 d\phi_3 \quad (17)$$

$$(18)$$

Now addressing flow in the system :

$$\frac{dN}{d\phi} \approx \frac{1}{2\pi} [1 + 2\{v_1 \cos(\phi) + v_2 \cos(2\phi) + v_3 \cos(3\phi) + v_4 \cos(4\phi)\}]$$

The normalised momentum correction term is :

$$C_{123}^{MC} \approx \frac{\frac{C}{2} \int p_1 p_2 (\cos 3(\phi_2 - \phi_3) \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3}) d\phi_1 d\phi_2 d\phi_3}{\int \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} d\phi_1 d\phi_2} + \frac{\frac{C}{2} \int p_1 p_3 (\cos 2(\phi_2 - \phi_3) \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3}) d\phi_1 d\phi_2 d\phi_3}{\int \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \frac{dN}{d\phi_3} d\phi_1 d\phi_2 d\phi_3} \quad (19)$$

$$= \frac{C}{2} [p_1 p_2 v_3^2 + p_1 p_3 v_2^2] \quad (20)$$