About the shape of the Photon Transfer Curve of CCD sensors

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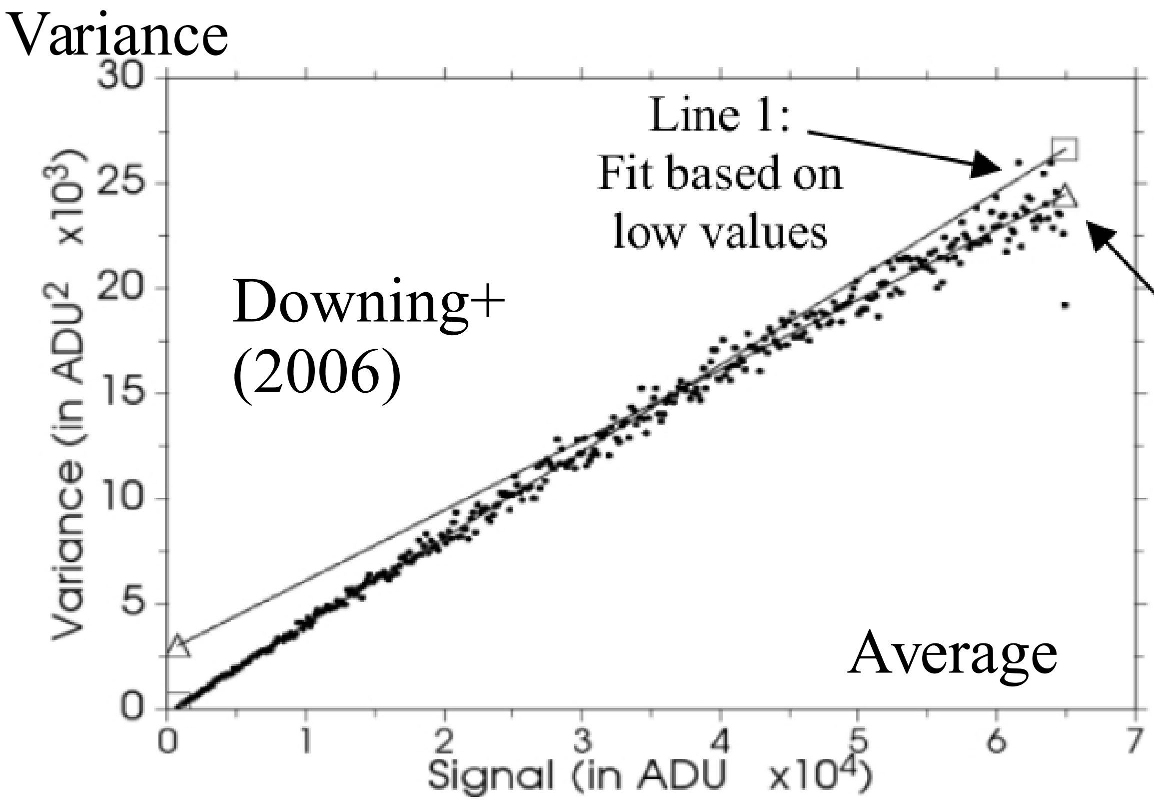
The PTC shape

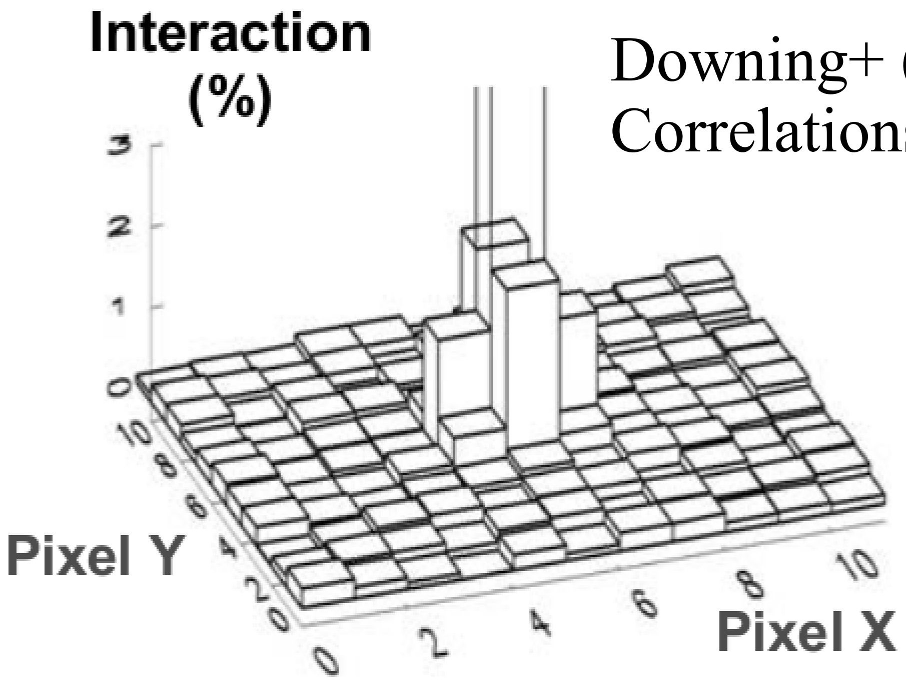


e2v CCD44-82

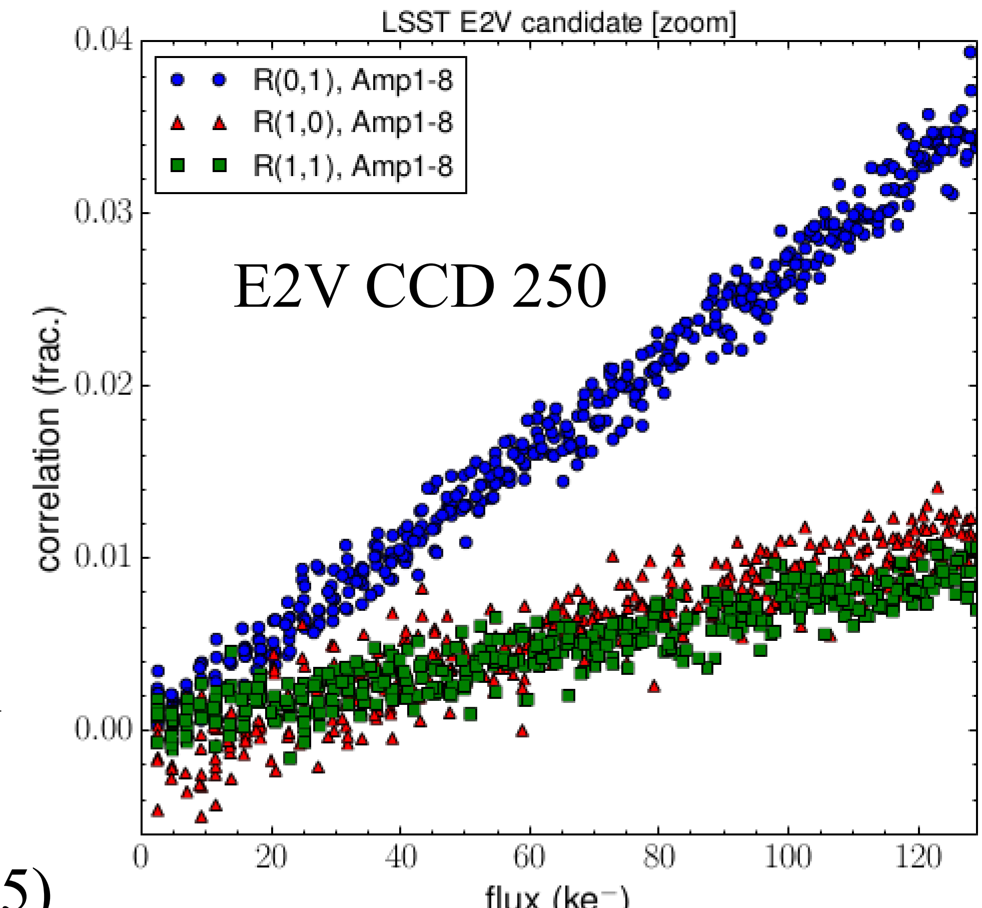
The variance of flat fields is **not** proportional to their average

- Not due to non-linearity of the video chain
- Present on all tested sensors
- Associated to covariances of neighboring pixels

Covariances/Correlations



Downing+ (2006)
Correlations at 90 ke



E2V CCD 250

Correlations increase roughly linearly, and can reach a few %.

Guyonnet+(2015)

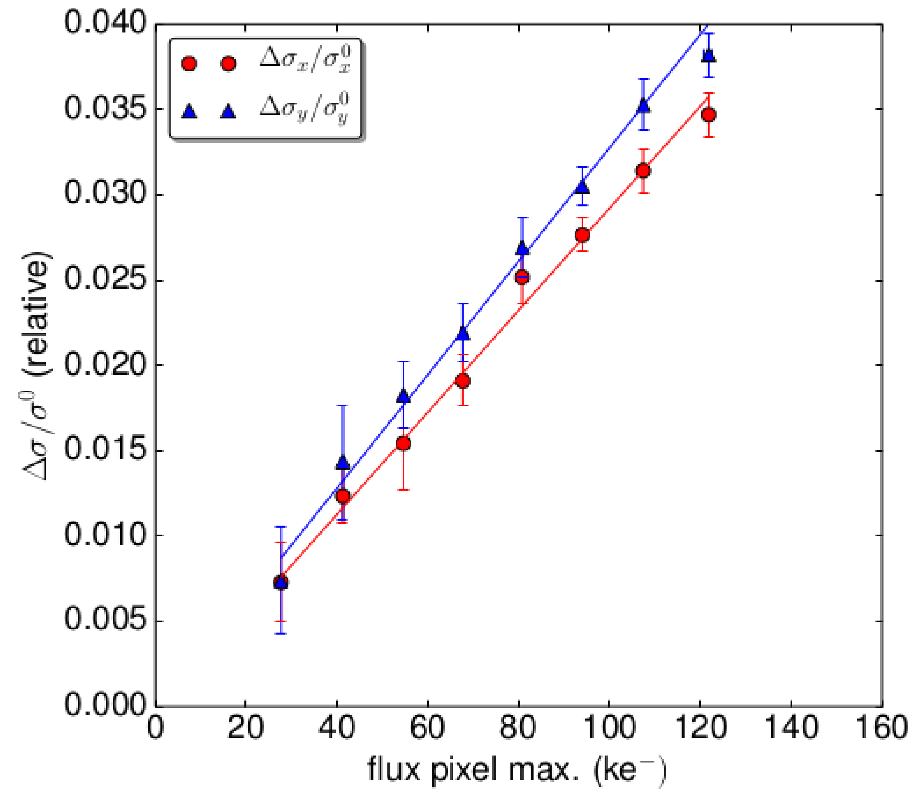
Interpretation

- For variances not to add up, incoming charges have to be sensitive to what happened earlier.
- Electrostatic forces can do that
- They can also perturb "structured" images

Brighter Fatter

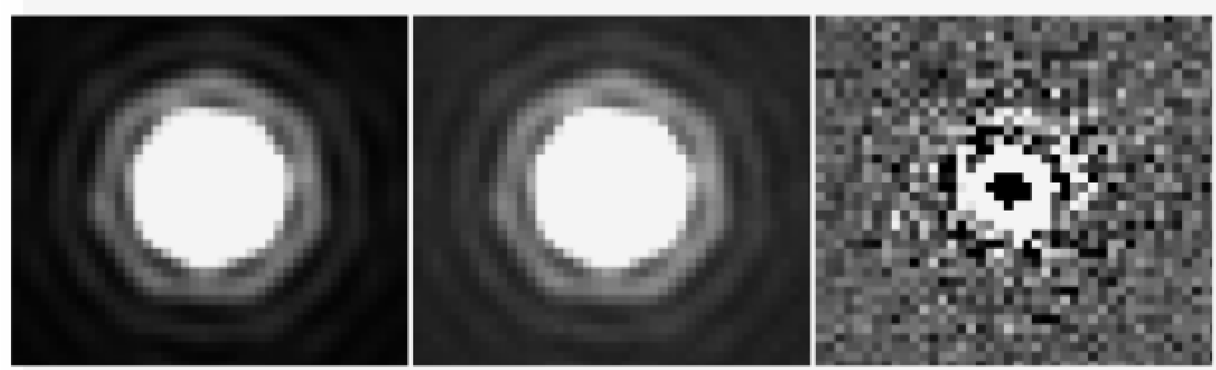
Spot sizes increase with total (or peak) flux.
In an anisotropic way.

The size of the effect varies with chip type and operating voltages



(a) LSST - E2V 250 - Spots 550 nm

Guyonnet+ (2015)

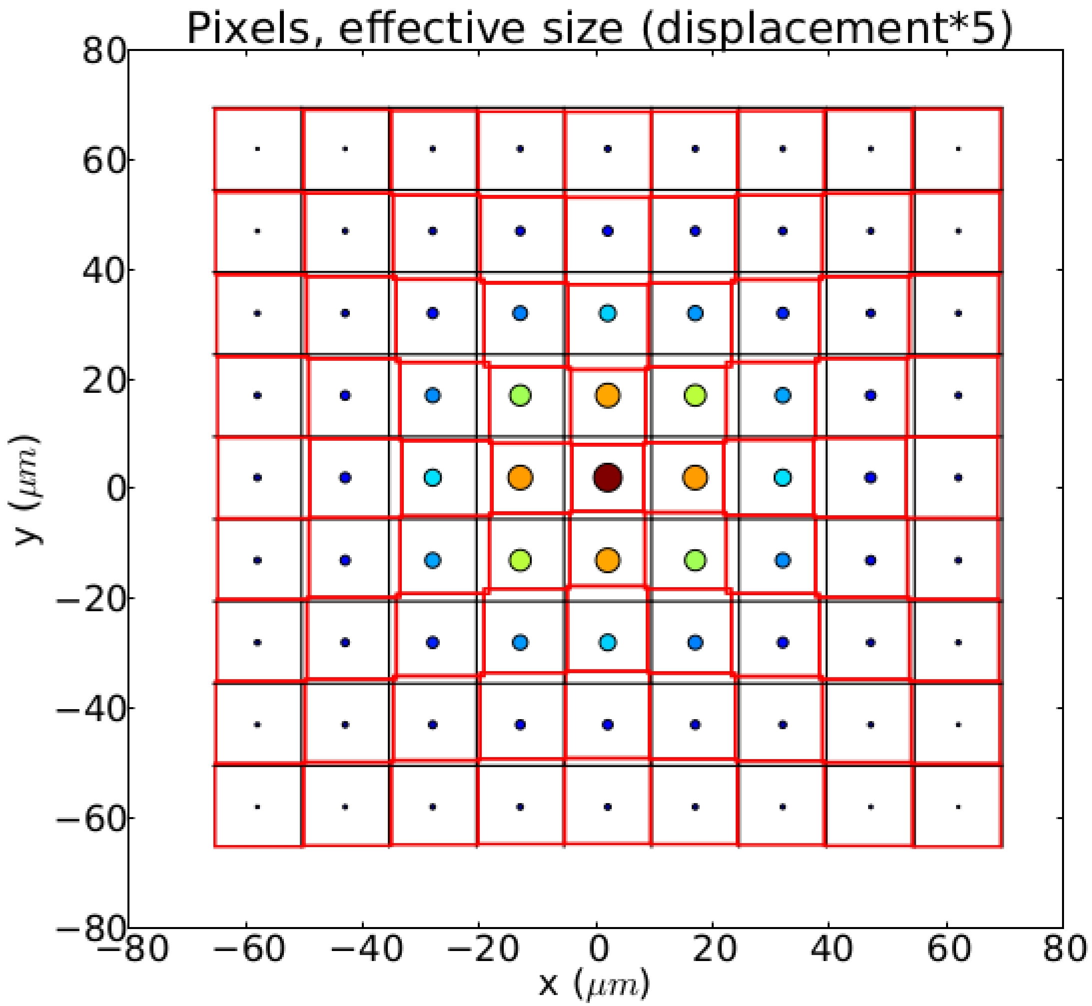


(a) 200-s exposures (b) 20-s exposures (c) subtraction (a)-(b)

Star shapes do not evolve with flux, but pixel shapes do

Gaussian star
Rms = 1.6 pixel
Peak = 100 ke

(Guyonnet+ 2015)



Summary of evidences

- The size of the effects (BF & flat field correlations) is compatible with electrostatic effects within the sensor (Laige+17)
- The chromaticity of the effects is weak if not undetectable
- Flat-field correlations are roughly linear with flux
- PTC is essentially never linear.

BF Correction or handling schemes

- Measure correlations/covariances
- Constrain some (crude) model of electrostatic influences
- Compute how much charge was deflected and put it back where it belongs:
  - Guyonnet et al (2015)
  - Gruen et al (2015)
  - Coulton et al (2018)

Limitations

- All approaches assume that pixel boundary shifts are proportional to source charges.
  - This is just an hypothesis, visit Andy’s poster.
- All approaches assume that the slope of correlations encodes the relative change of pixel area
  - This is just Taylor
- Covariances are tricky to measure, and polluted by extra contributions…
  - To be detected and removed

Dynamics (in flat fields)

- Incoming currents are affected by stored charges

$$\dot{Q}_{00} = I[1 + \sum_{kl} a_{kl} Q_{kl}]$$

Average current per pixel

By how much a stored charge alters a pixel area (at lag k,l).

- There is here a linearity hypothesis : pixel boundaries shift by amounts proportional to the cause (the stored charge).

From interaction to covariances

$$\dot{Q}_{00} = I[1 + \sum_{kl} a_{kl} Q_{kl}]$$

$$\sum_{kl} a_{kl} = 0$$

Charge conservation.
Sum runs over positive and negative lags

Time evolution of covariances :

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I \sum_{kl} a_{kl} C_{i-k,j-l}$$

Poisson variance per unit time

For a = 0, we get Poisson: $C_{00}(t) = V_I t$

Solution of the differential equation (1)

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I\sum_{kl} a_{kl}C_{i-k,j-l}$$

$$\dot{\boldsymbol{C}} = \boldsymbol{\delta}V_I + 2I\boldsymbol{C}\otimes\boldsymbol{a}$$

Fourier space
$$\dot{\tilde{\boldsymbol{C}}} = V_I + 2I\tilde{\boldsymbol{a}}\tilde{\boldsymbol{C}}$$

Solution
$$\tilde{\boldsymbol{C}}(t) = \frac{V_I}{2I\tilde{\boldsymbol{a}}}\left[e^{2I\tilde{\boldsymbol{a}}t} - 1\right]$$

Taylor
$$\tilde{\boldsymbol{C}}(t) = V_I t\left[1 + I\tilde{\boldsymbol{a}}t + \frac{2}{3}(I\tilde{\boldsymbol{a}}t)^2 + \frac{1}{3}(I\tilde{\boldsymbol{a}}t)^3 + \cdots\right] \qquad \mu \equiv It$$

$$\tilde{\boldsymbol{C}}(\mu) = V\left[1 + \tilde{\boldsymbol{a}}\mu + \frac{2}{3}(\tilde{\boldsymbol{a}}\mu)^2 + \frac{1}{3}(\tilde{\boldsymbol{a}}\mu)^3 + \cdots\right] \qquad V \equiv V_I t$$

Solution of the differential equation (2)

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I\sum_{kl} a_{kl}C_{i-k,j-l}$$

Taylor

$$\tilde{\boldsymbol{C}}(t) = V_I t\left[1 + I\tilde{\boldsymbol{a}}t + \frac{2}{3}(I\tilde{\boldsymbol{a}}t)^2 + \frac{1}{3}(I\tilde{\boldsymbol{a}}t)^3 + \cdots\right]$$

$$\tilde{\boldsymbol{C}}(\mu) = V\left[1 + \tilde{\boldsymbol{a}}\mu + \frac{2}{3}(\tilde{\boldsymbol{a}}\mu)^2 + \frac{1}{3}(\tilde{\boldsymbol{a}}\mu)^3 + \cdots\right]$$

$$\mu \equiv It$$

$$V \equiv V_I t$$

Direct space

$$C(\mu) = V\left[\delta_{i0}\delta_{j0} + a\mu + \frac{2}{3}TF^{-1}[(\tilde{a})^2]\mu^2 + \ldots\right]$$

Noise terms

$$C_{ij}(\mu) = \frac{\mu}{g}\left[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a\otimes a]_{ij}\mu^2 + \frac{1}{3}[a\otimes a\otimes a]_{ij}\mu^3 + \ldots\right] + n_{ij}/g^2$$

Solution

$$C_{ij}(\mu) = \frac{\mu}{g}\left[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a]_{ij}\mu^2 + \frac{1}{3}[a \otimes a \otimes a]_{ij}\mu^3 + ...\right] + n_{ij}/g^2$$

- Beyond second order, all curves are “mixed” (in direct space): every lag involves all “a” values.
- For the PTC a fair approximation is that a_{00} dominates (and is negative) and :

$$C_{00} = \frac{1}{2g^2 a_{00}}\left[\exp\left(2a_{00}\mu g\right) - 1\right] + n_{00}/g^2$$

Questioning the linearity assumption

$$\dot{Q}_{00} = I(1 + \sum_{kl} a_{kl}(1 + b_{kl} * I * t)Q_{kl})$$

Linearity violation
"Next to Leading Order" terms

$$C_{ij}(\mu) = \frac{\mu}{g}[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a + ab]_{ij}\mu^2 + \frac{1}{6}(2a \otimes a \otimes a + 5a \otimes ab)_{ij}\mu^3 + ...] + n_{ij}/g^2$$

Poisson's revenge

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I\sum_{kl} a_{kl}C_{i-k,j-l}$$

Sum rule : $\sum_{kl} a_{kl} = 0$

$$\sum_{ij}\dot{C}_{ij} = V_I + 2I\sum_{ij}\sum_{kl} a_{kl}C_{i-k,j-l}$$

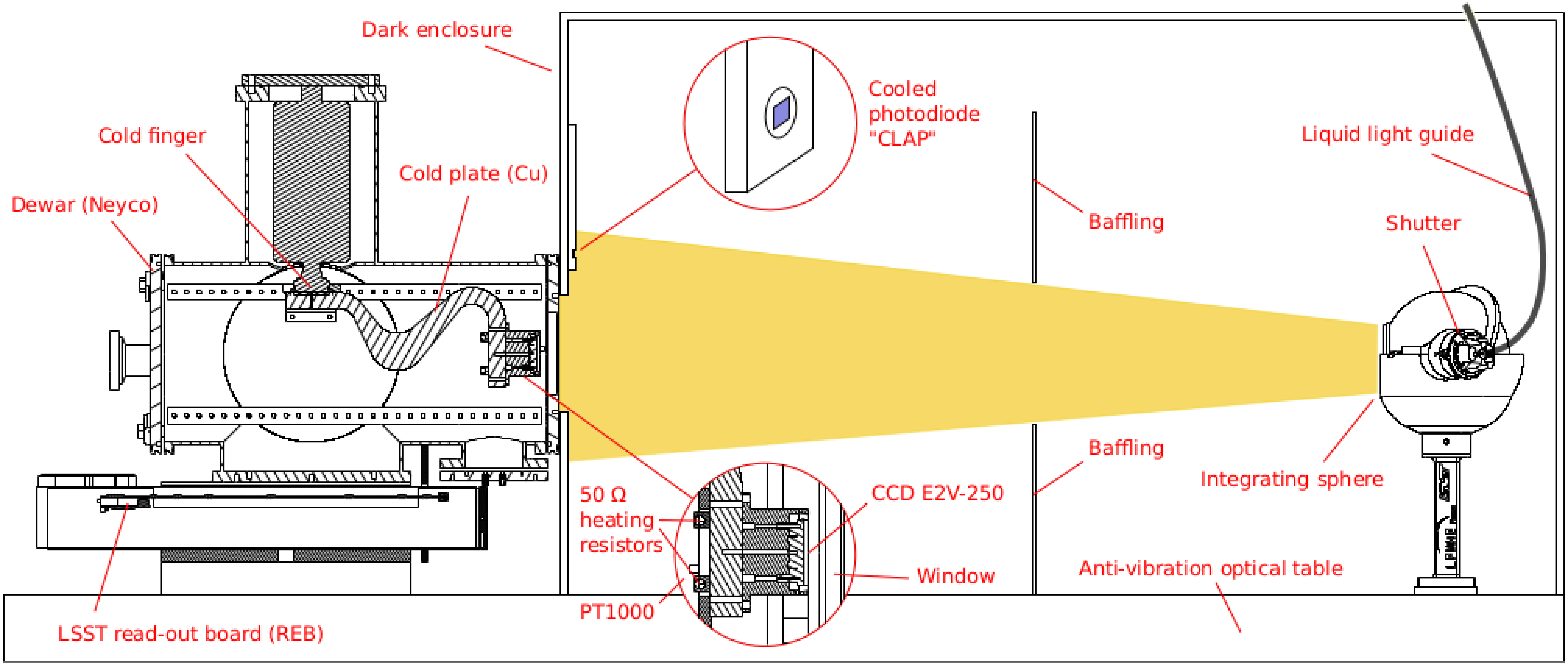
$$= V_I + 2I\sum_{ij} a_{ij}\sum_{kl} C_{kl}$$

$$= V_I$$

If one sums variance and covariances, the Poisson behavior is recovered.

Data Analysis (E2V CCD 250)

- 1000 flat fields pairs at 0< mu <10^5 electrons

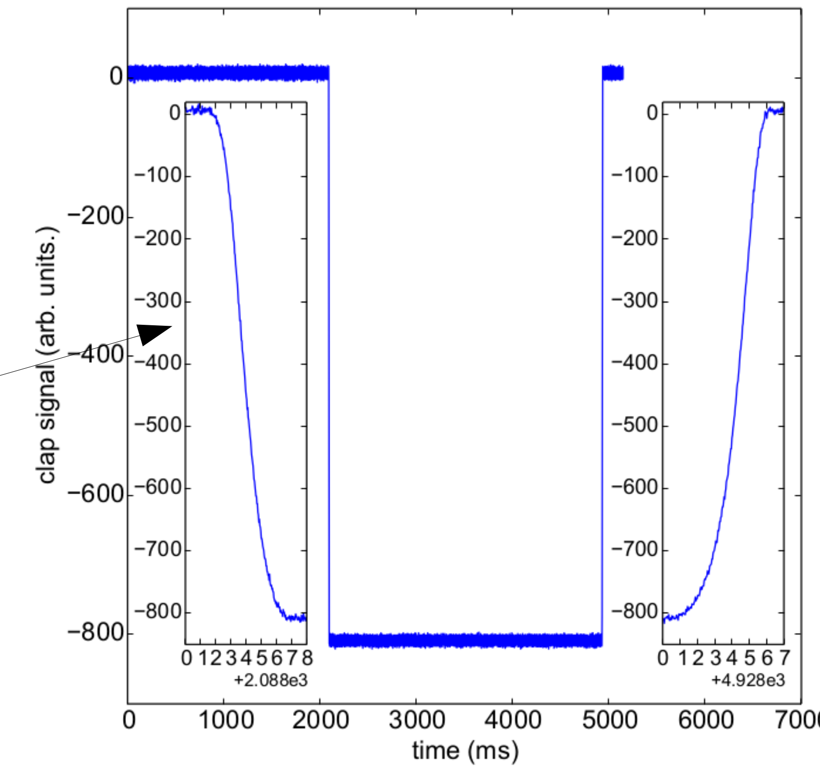


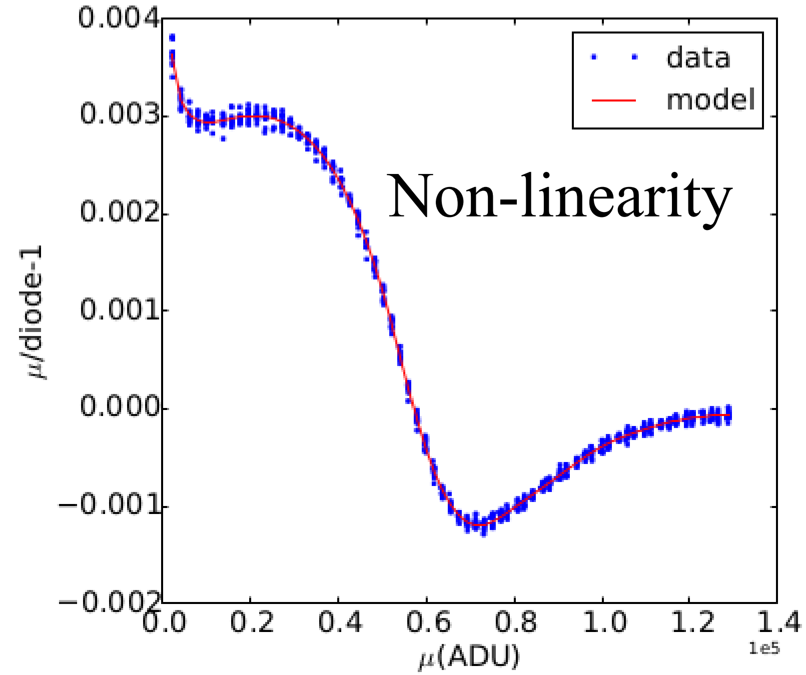
We first have to correct for: non-linearity & deferred charges

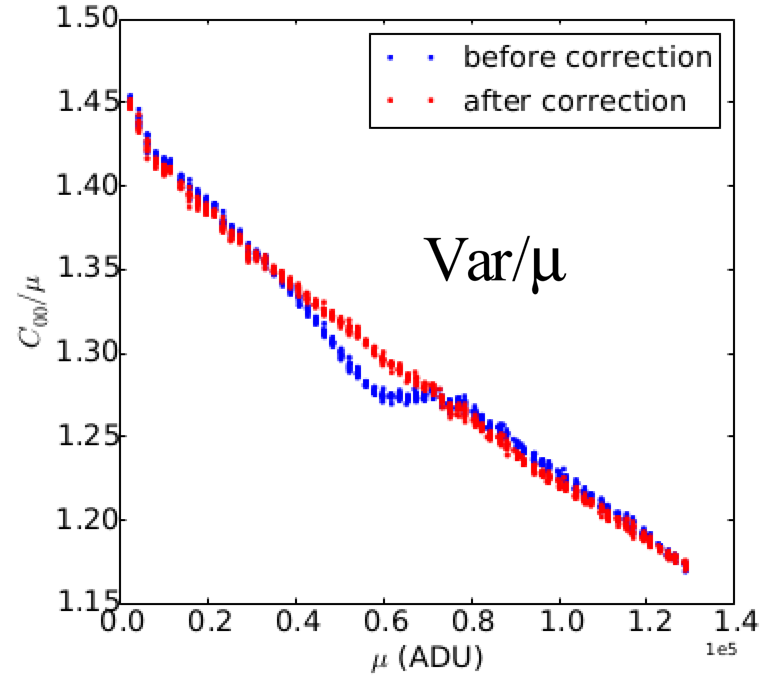
Non-linearity

The light received by the CCD is measured using an "amplified" photo-diode

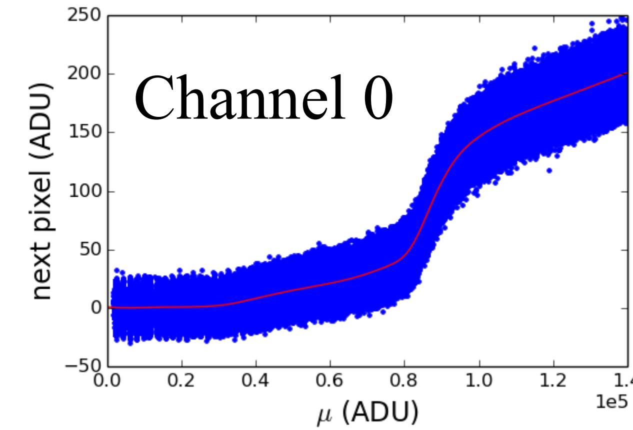
We tune the integrated charge by varying the open-shutter time



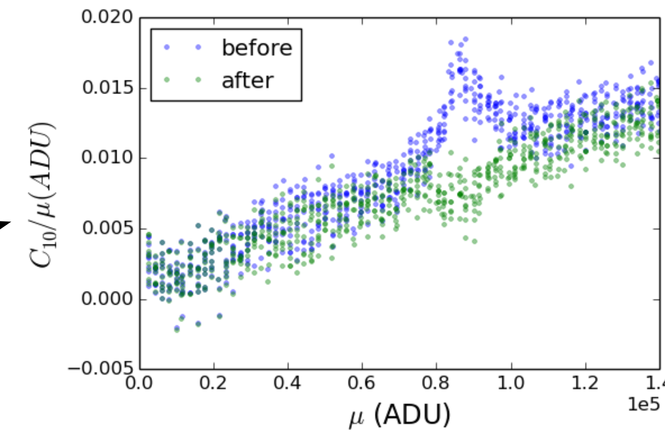




Deferred charge correction

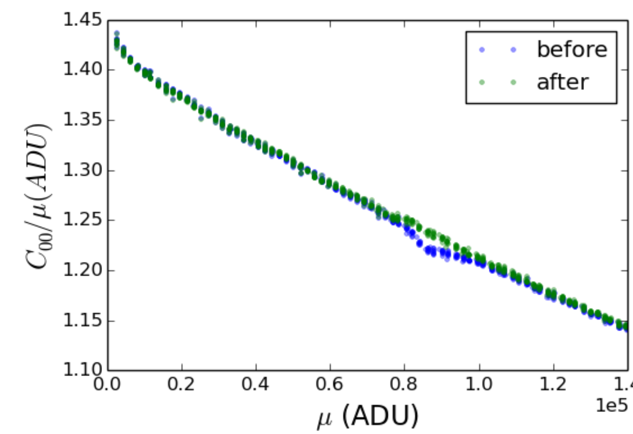


First serial overscan pixel



C_{10}/μ : nearest serial neighbor covariance

Variance/μ



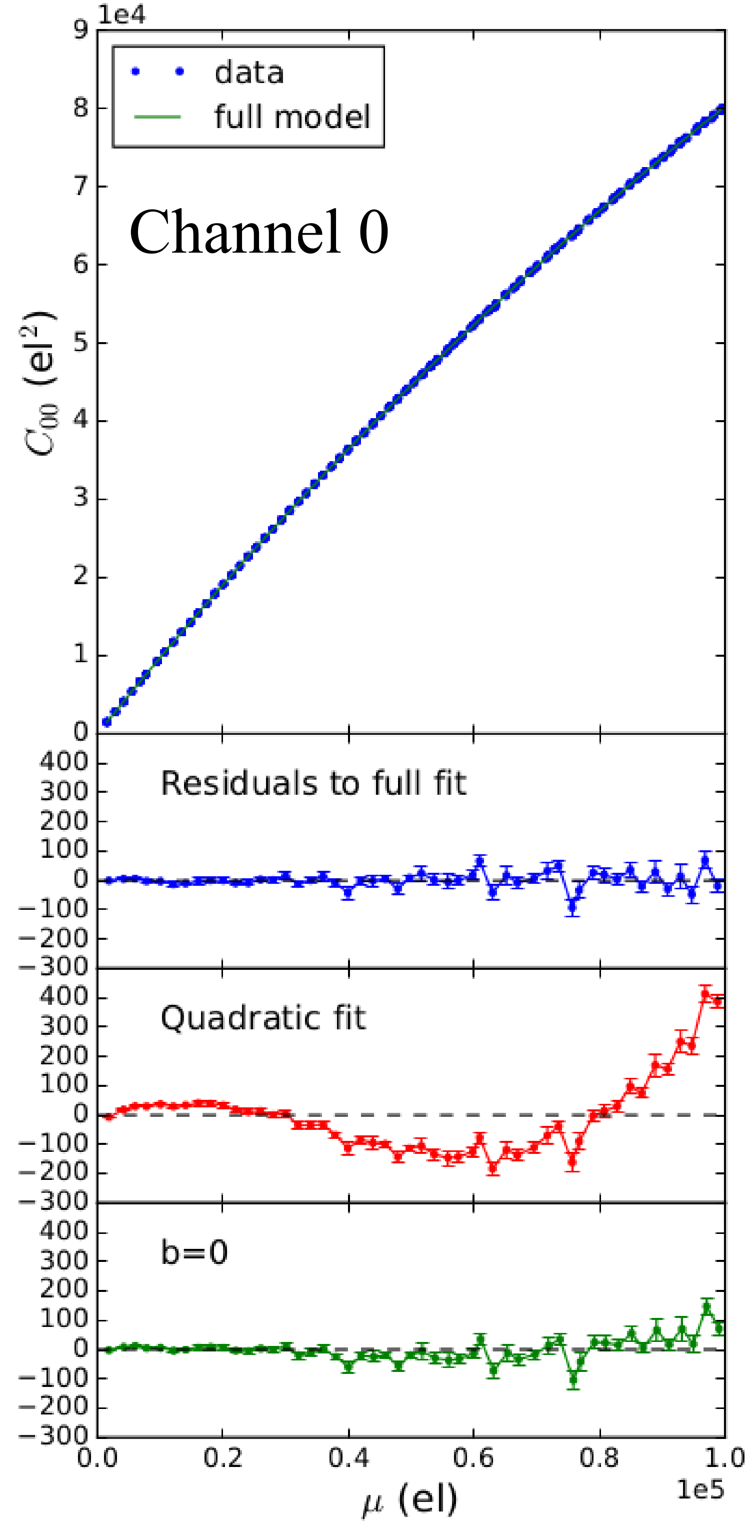
- Different for each video channel
- Small over-correction (?)
- Reduces the correlation slope (~a_{10}) by ~ 10% (for this channel)

Fit results : PTC

$$C_{ij}(\mu) = \frac{\mu}{g}[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a + ab]_{ij}\mu^2 + \frac{1}{6}(2a \otimes a \otimes a + 5a \otimes ab)_{ij}\mu^3 + ...] + n_{ij}/g^2$$

16 channel,
8x8 a_{ij} & 8x8 b_{ij}

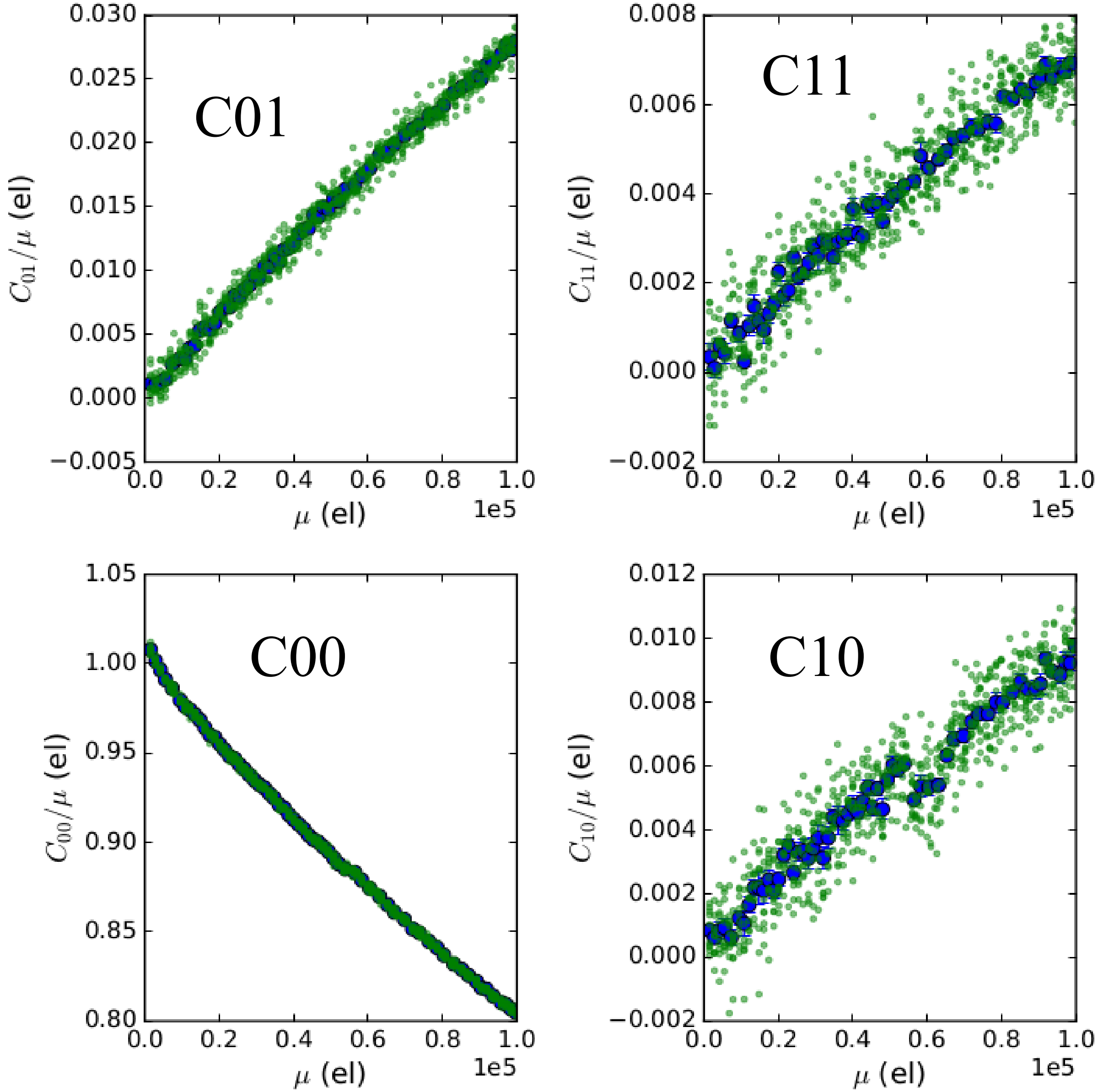
Full fit (a and b)



All channels :

	χ^2_{full}/N_{dof}	χ^2_2/N_{dof}	gain	a_{00}	RO noise
value	1.23	4.04	0.713	-2.376 10^{-6}	4.54
scatter	0.10	0.27	0.020	0.032 10^{-6}	0.43

Covariances



C01

- The curvature of C_{01}/μ can be seen by eye

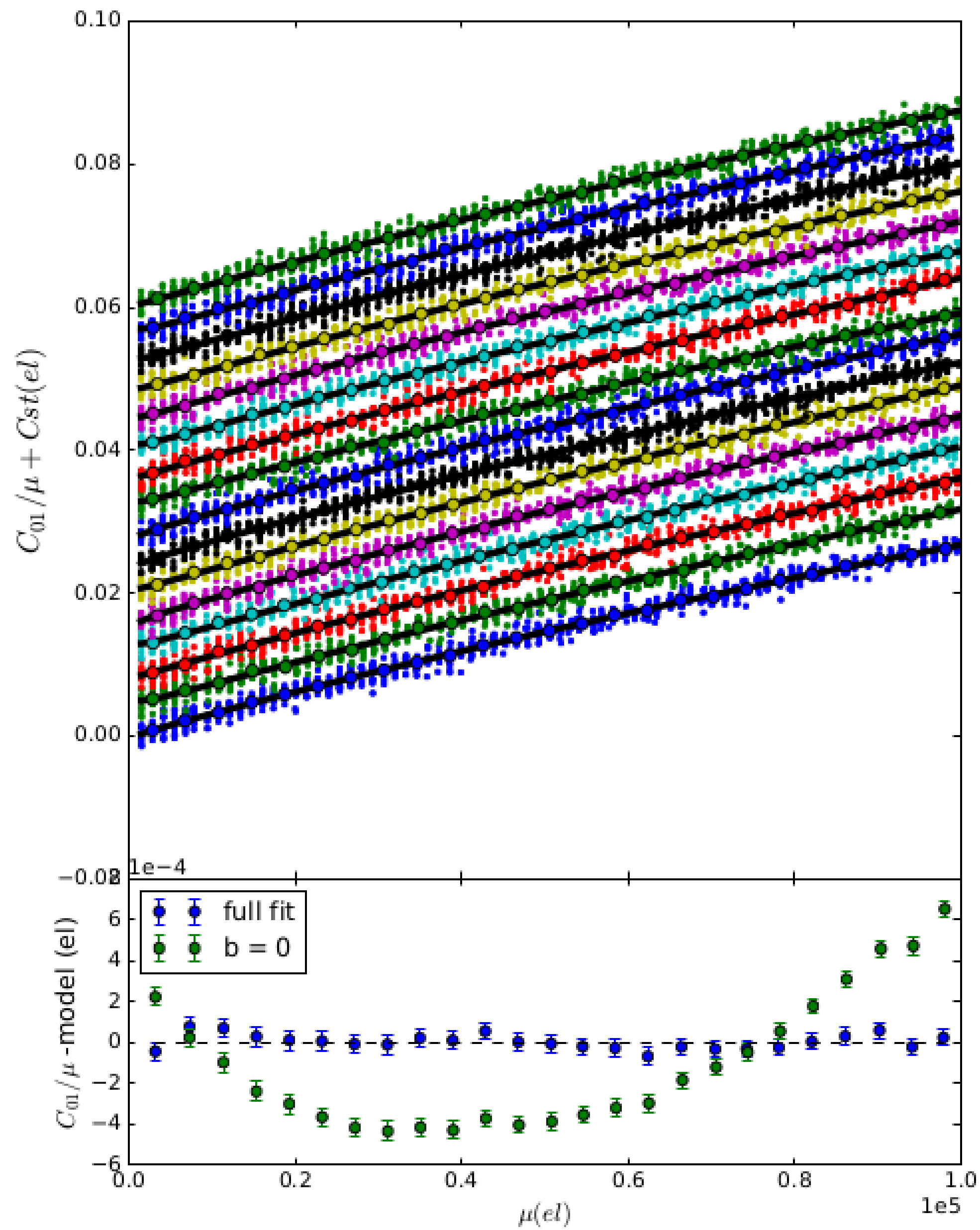
- b=0 is highly disfavored

	a_{01}	b_{01}	χ^2/N_{dof}
value	3.32e-07	1.71e-06	1.03
scatter	5.87e-09	2.87e-07	0.05

Scatter is twice as much as expected from shot noise

Good fits

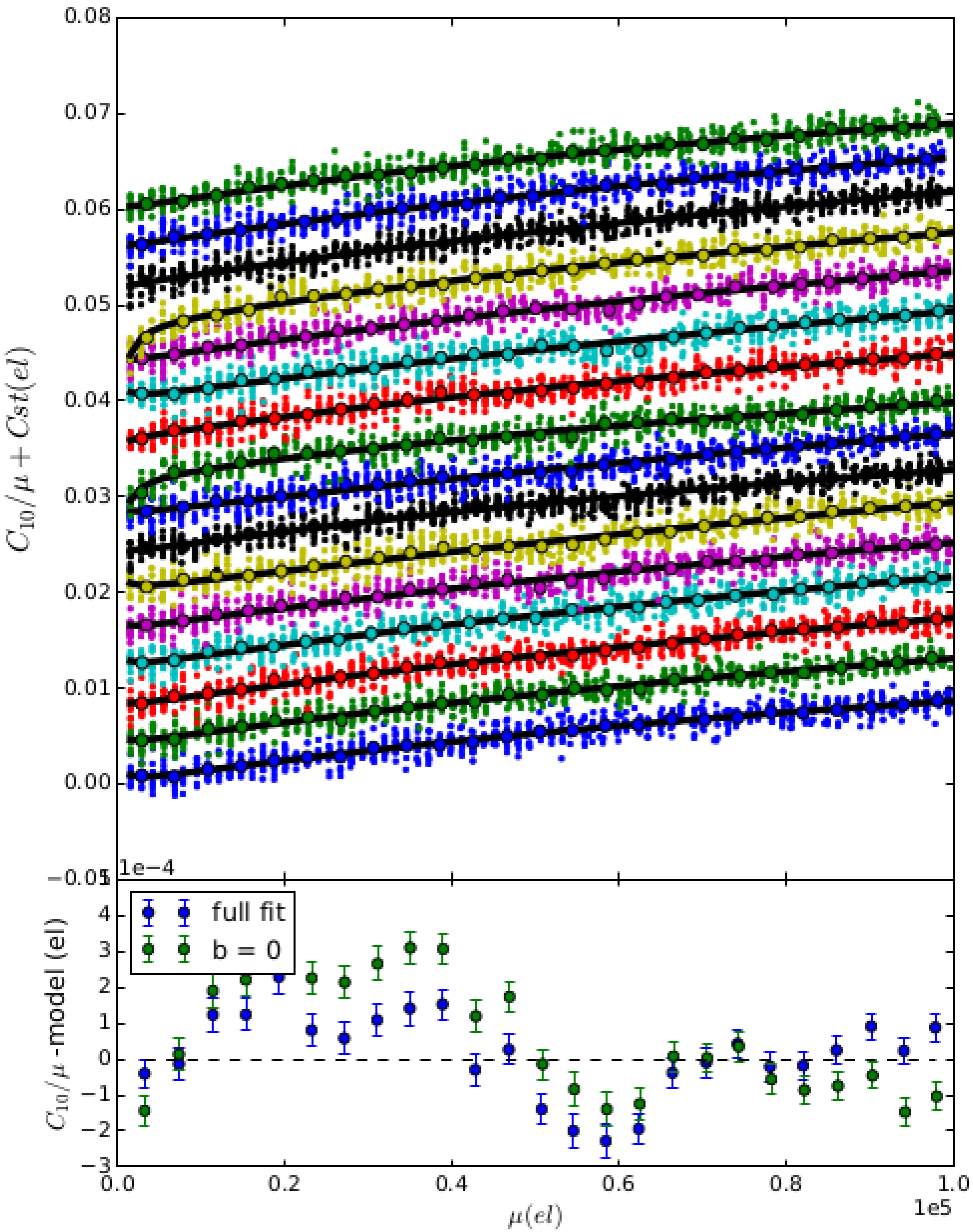
C01+ Cst for the 16 channels



C10

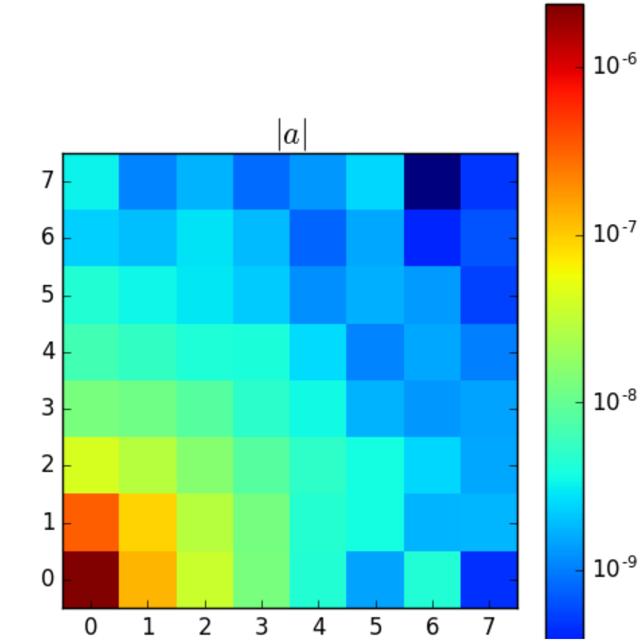
- Noisier than C01
- Much more scatter that seems real
- b =0 still disfavored but much less striking

C10+ Cst for the 16 channels



	a_{10}	b_{10}	χ^2/N_{dof}
value	1.26 10^{-7}	-1.77 10^{-6}	1.03
scatter	0.08 10^{-6}	0.97 10^{-6}	0.07

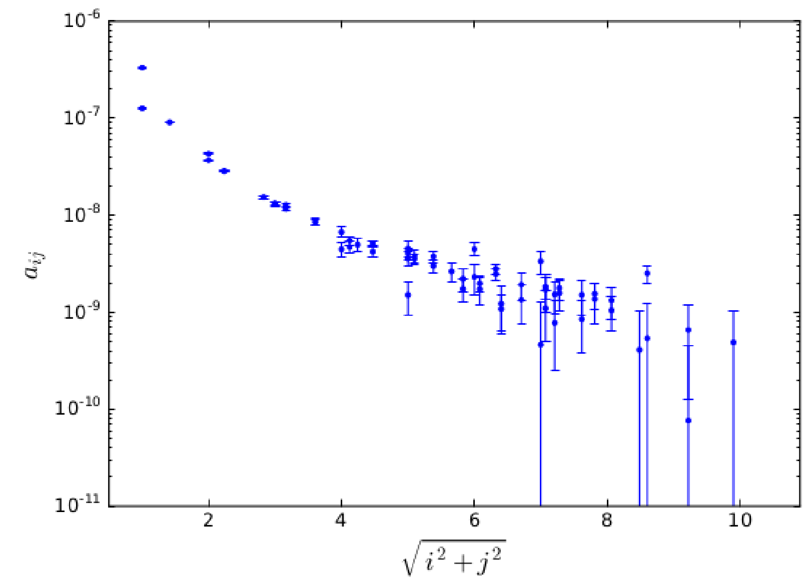
Fit results

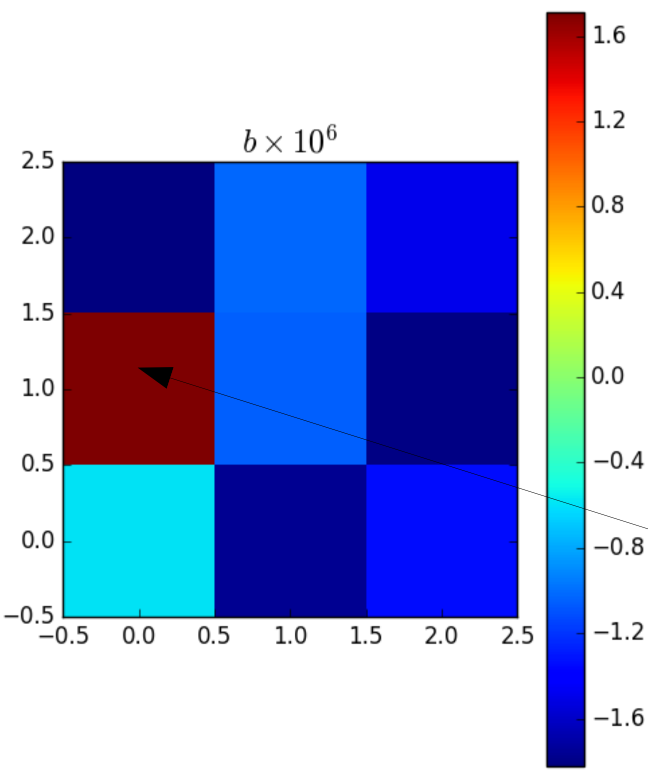


Expressed
in el^{-1}

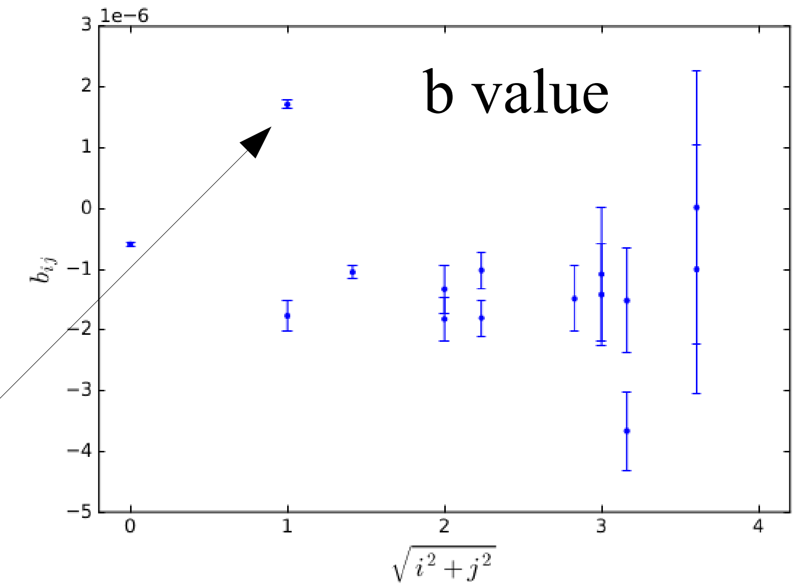
Average over
16 channels

a value vs distance





a_{00} = -2.3 10^{-6},
Σa_{ij} = -3 10^{-8},
so we miss
~1.5%



b_{10} >0
and all the others negative

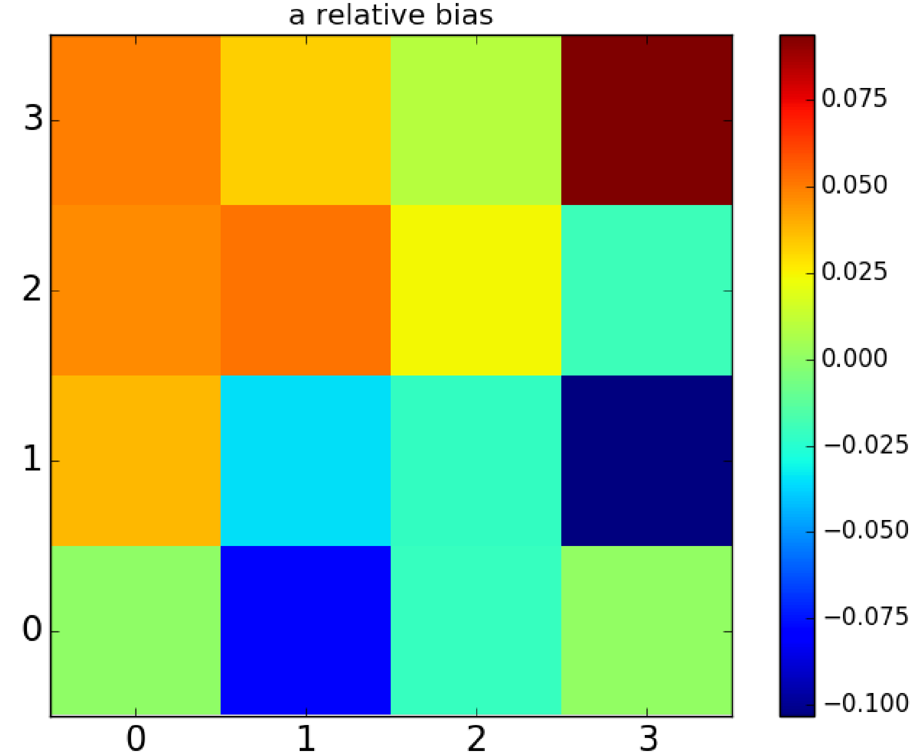
Comparison with the "standard" approach

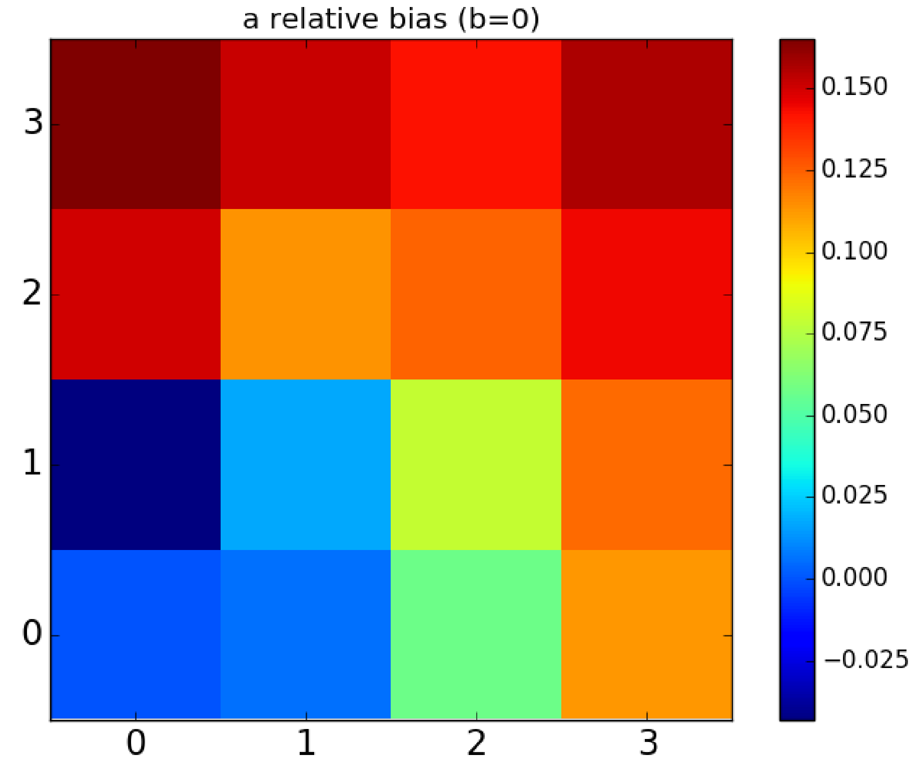
Standard way:

$$a_{ij} = \frac{C_{ij}}{C_{00}} \mu g$$

At some (high) illumination

Difference between the full fit and the simplified Way : 10% peak to peak.





Summary & conclusions

- We have developed a model for the PTC/Cov curve shapes.
- The expected shapes depend on the assumed dynamics (e.g. area alterations scale as source charges), and hence allow us to constrain the dynamics.
- There are a few potential problems (non-linearity, deferred charge) to be addressed.
- With the "standard way", systematic offsets of BF predictions by ~10% should not come as a surprise.