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Monte Carlo event generators in QCD

Abhijit Majumder

2nd JETSCAPE winter school and workshop, Texas A & M University, Jan 7 - 13, 2019,

Outlook

- Basics of Monte-Carlo methods
- MC simulation / event generation
- Event generation in QCD
- From DGLAP to full jet simulation
- Outlook to the rest of the school: jets in medium, hadronization.

What are Monte Carlo methods?

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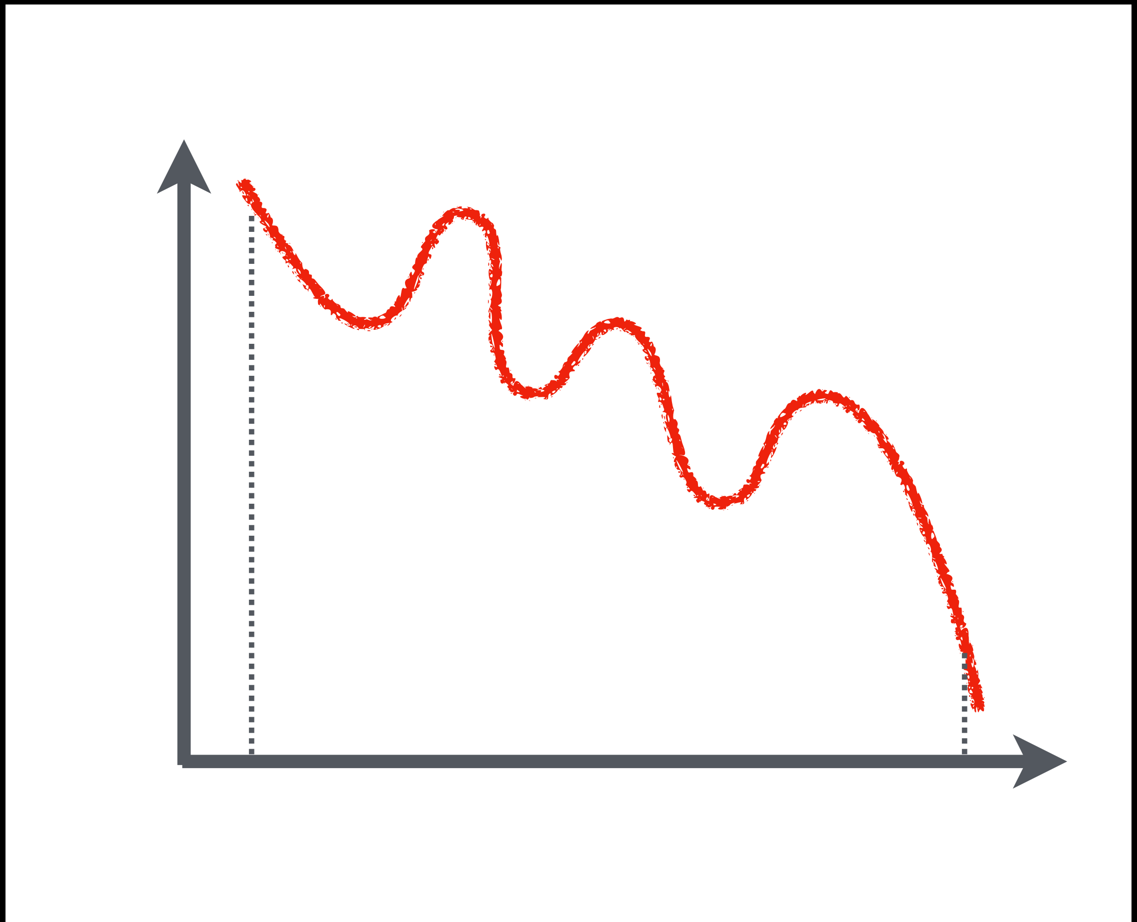
- A simulated process whose outcome is determined by a series of random numbers !

Integrating a function using "darty MC"

- Integrating a generic function
  - The trapezoid method



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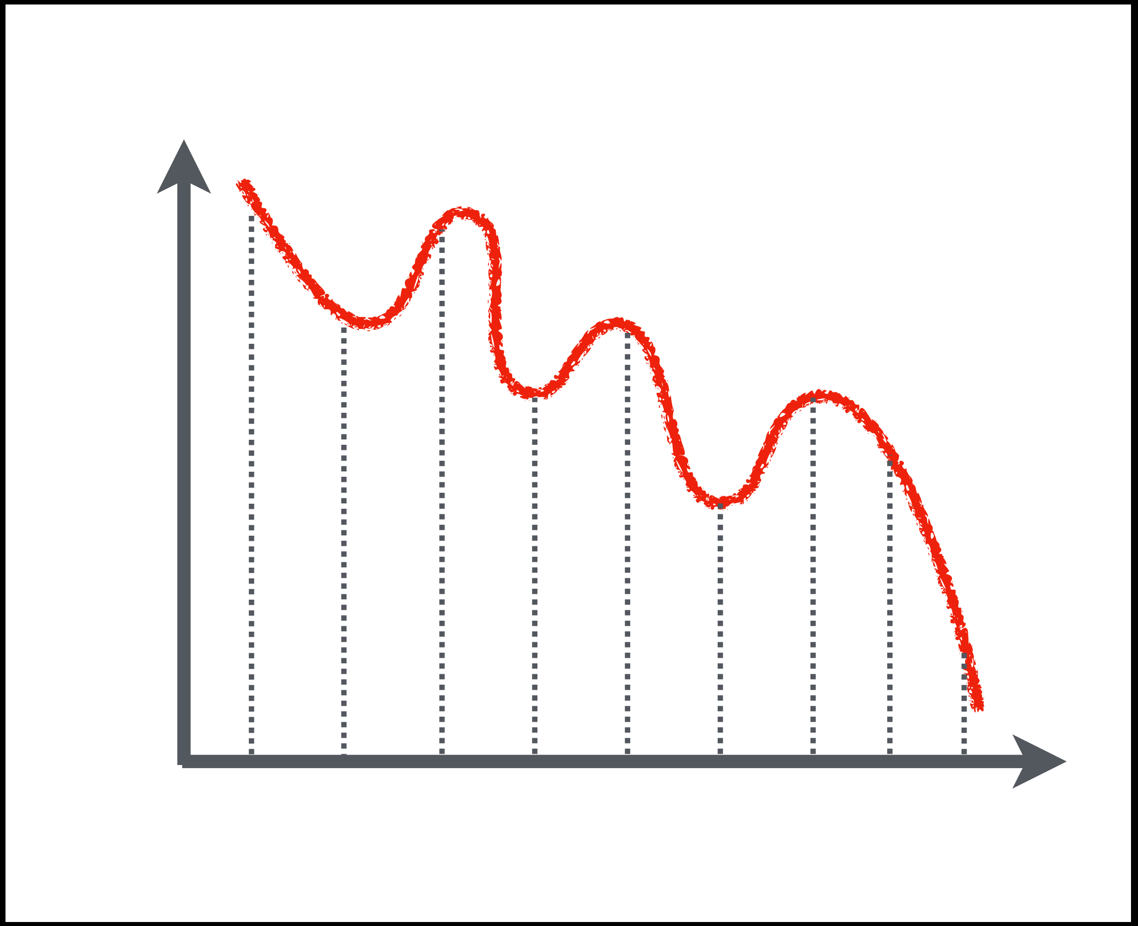

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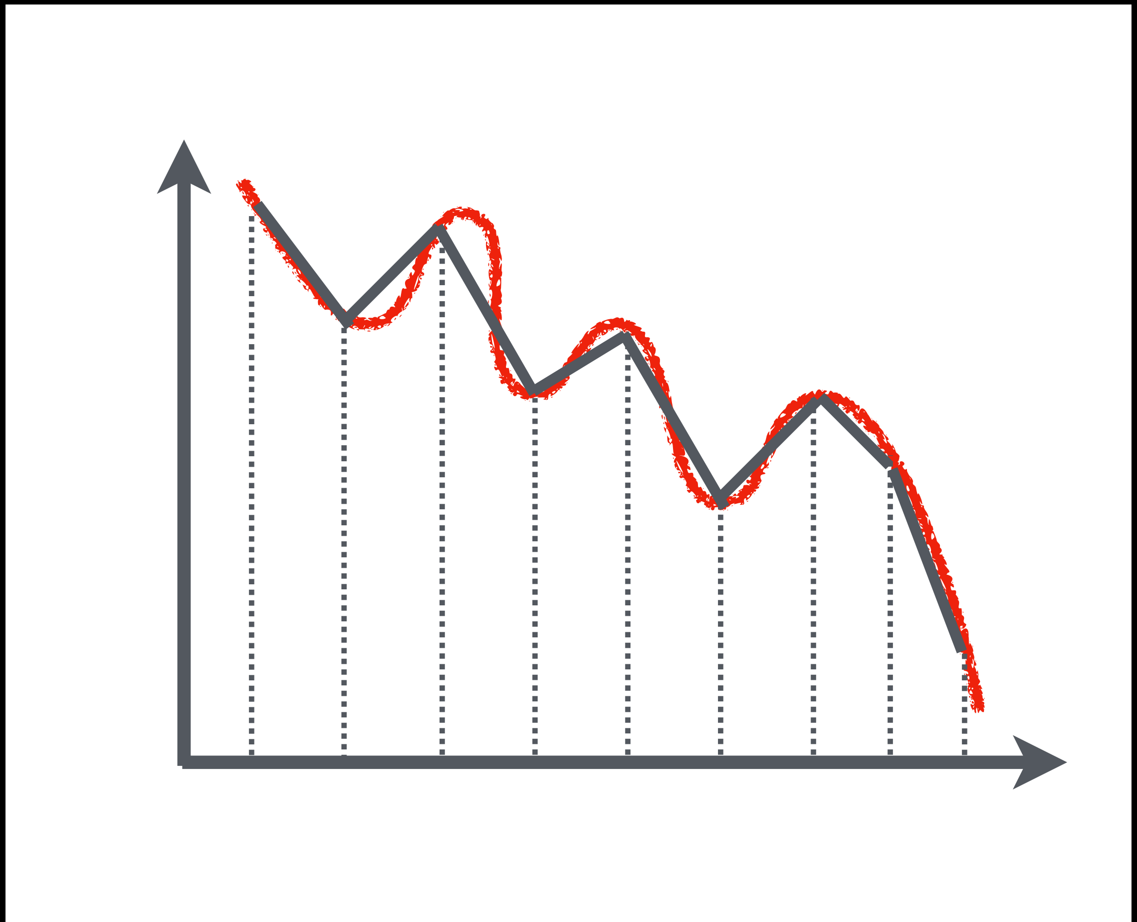

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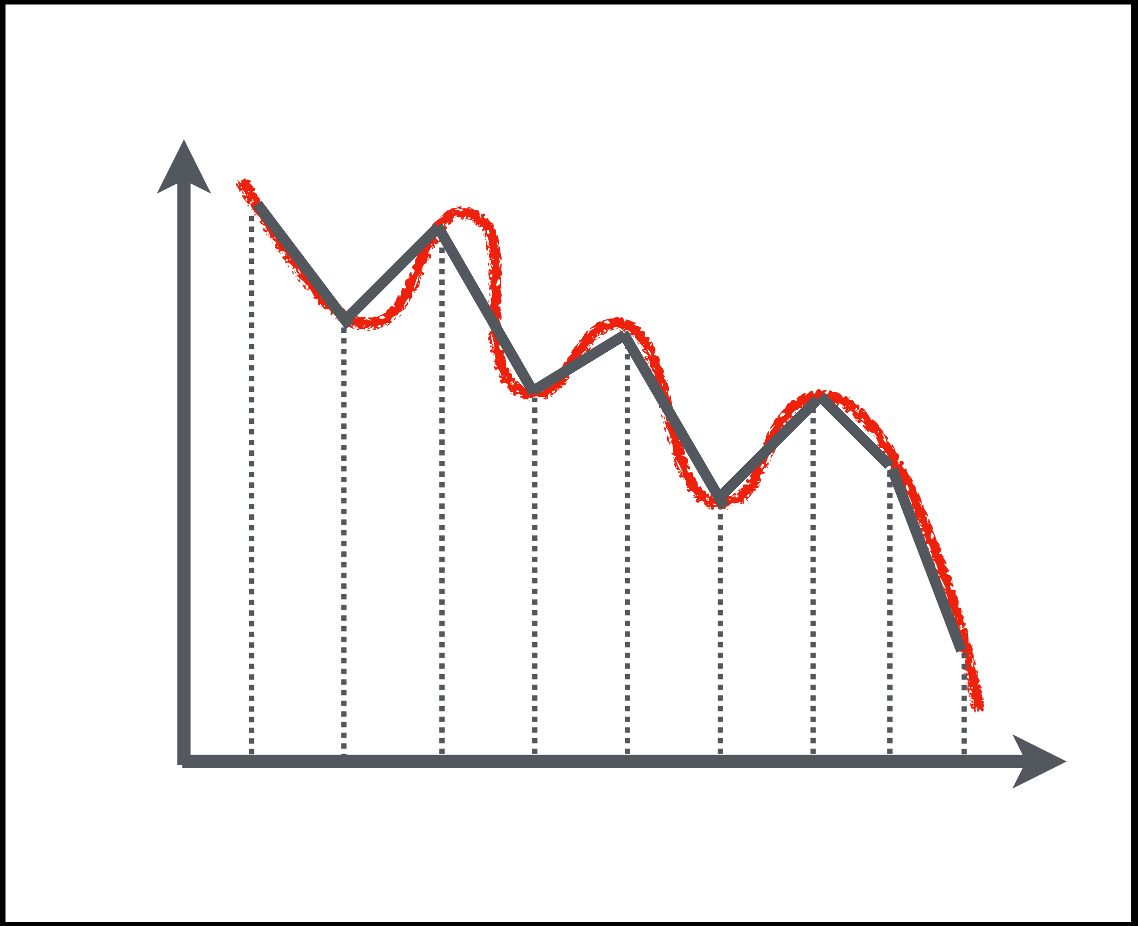

$$\int_a^b dx f(x) = \sum_{i=0}^{N-1} (x_{i+1} - x_i) \left[\frac{f(x_{i+1}) + f(x_i)}{2} \right]$$

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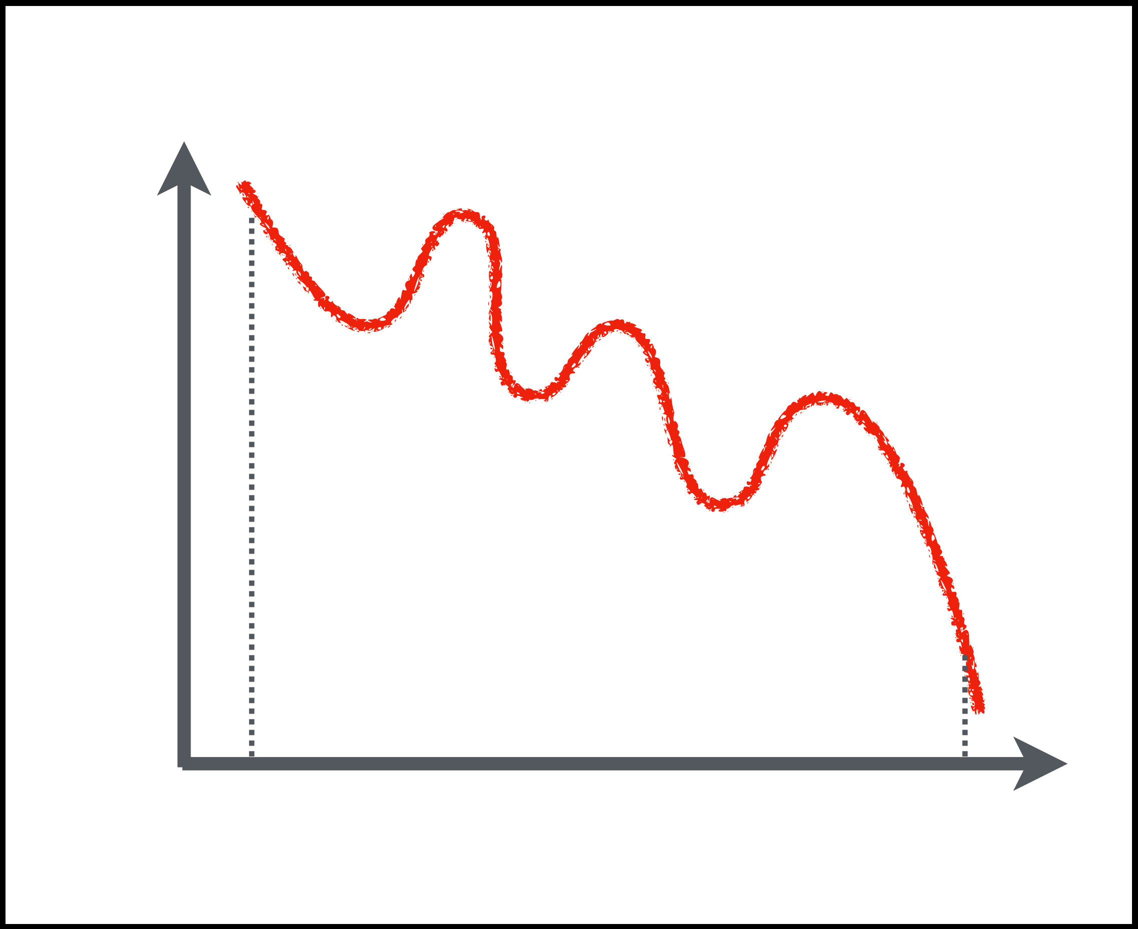

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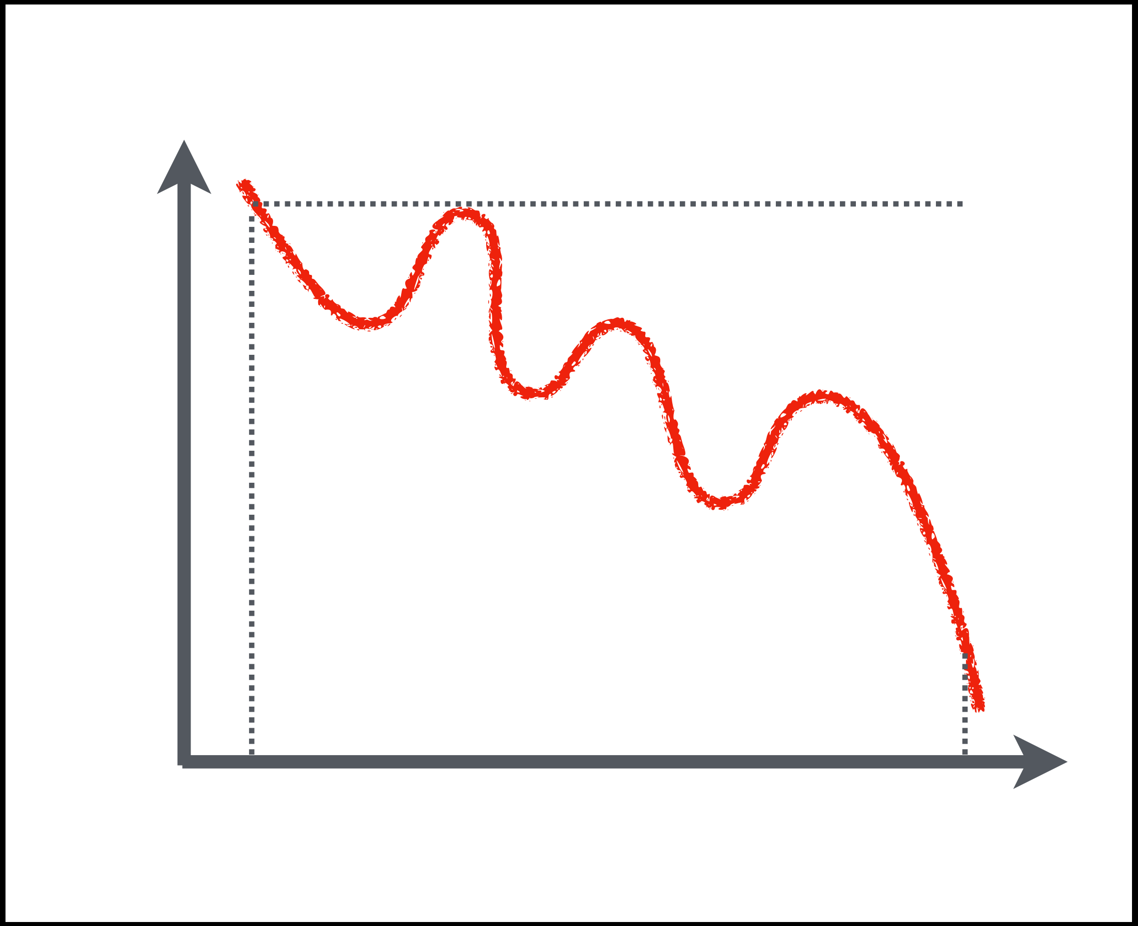

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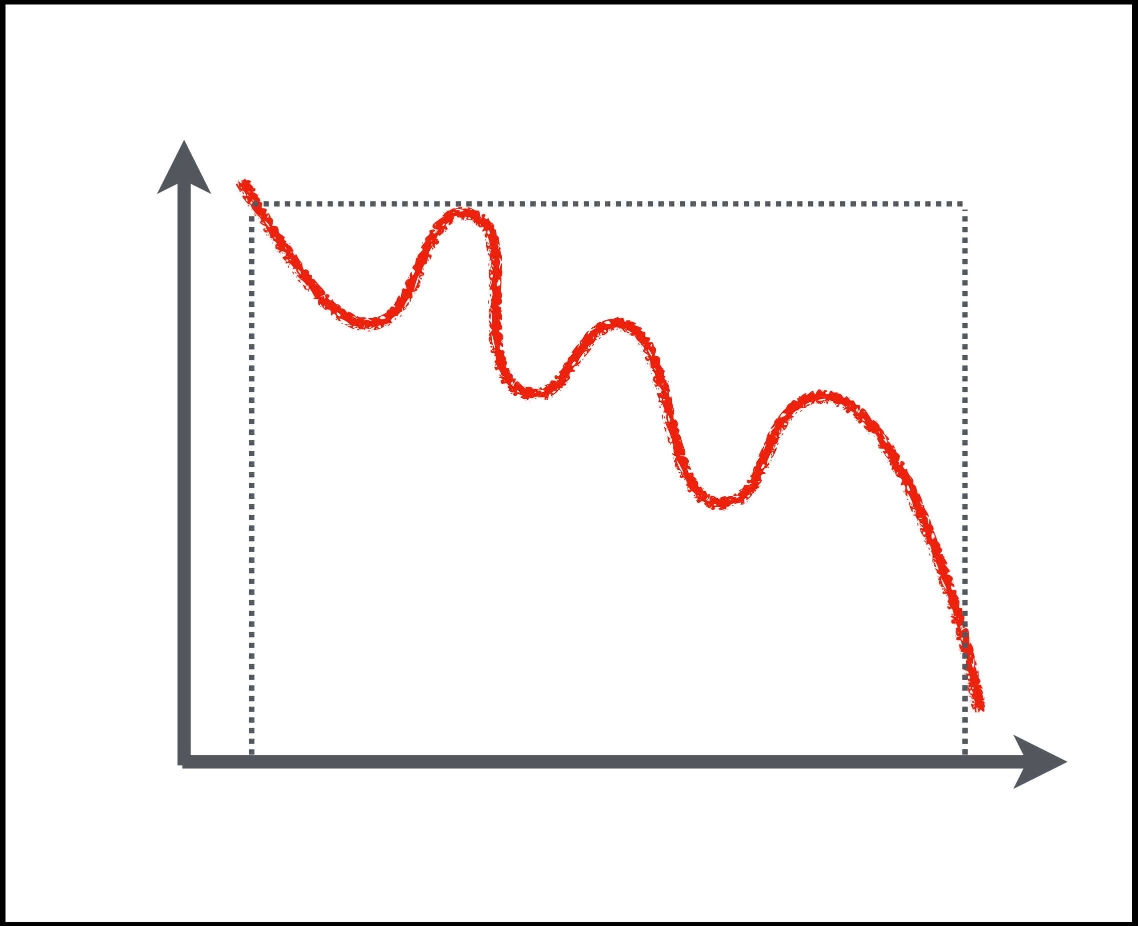

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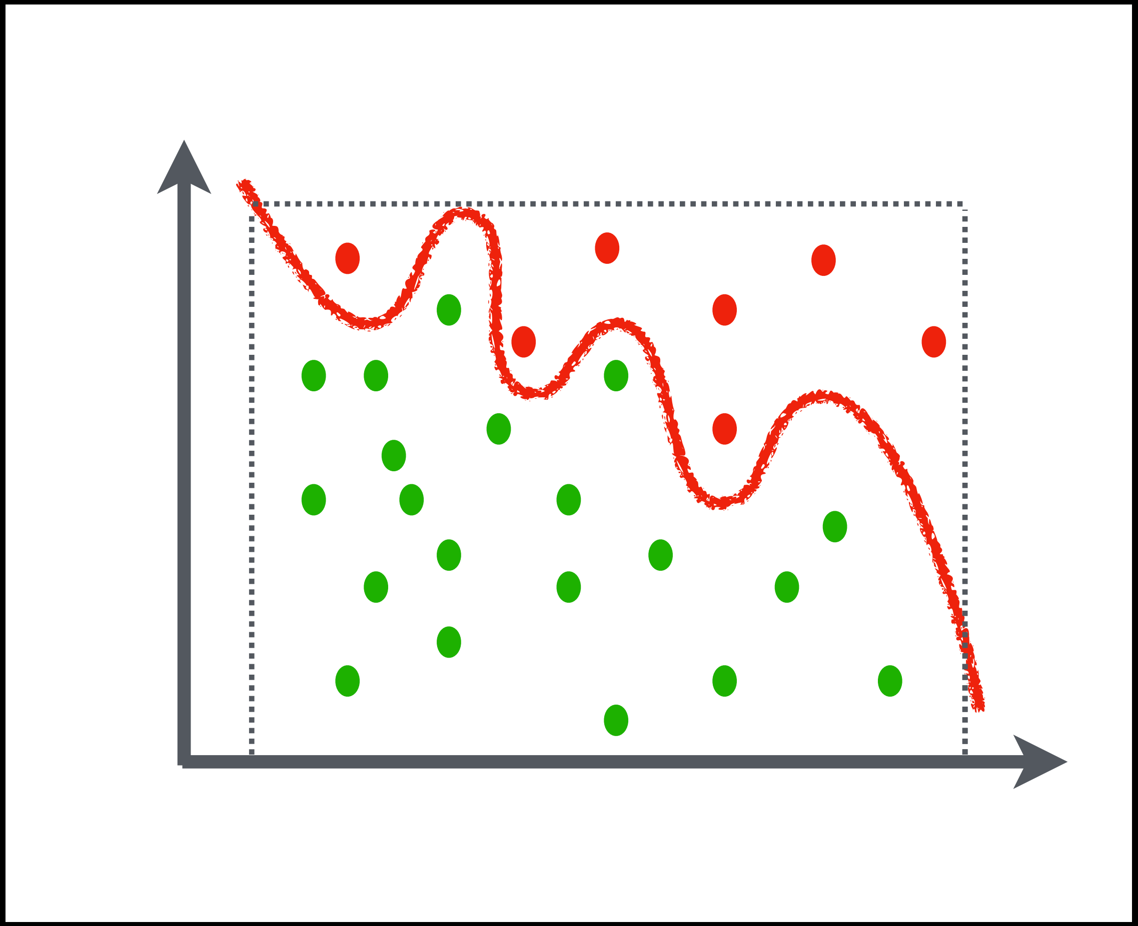

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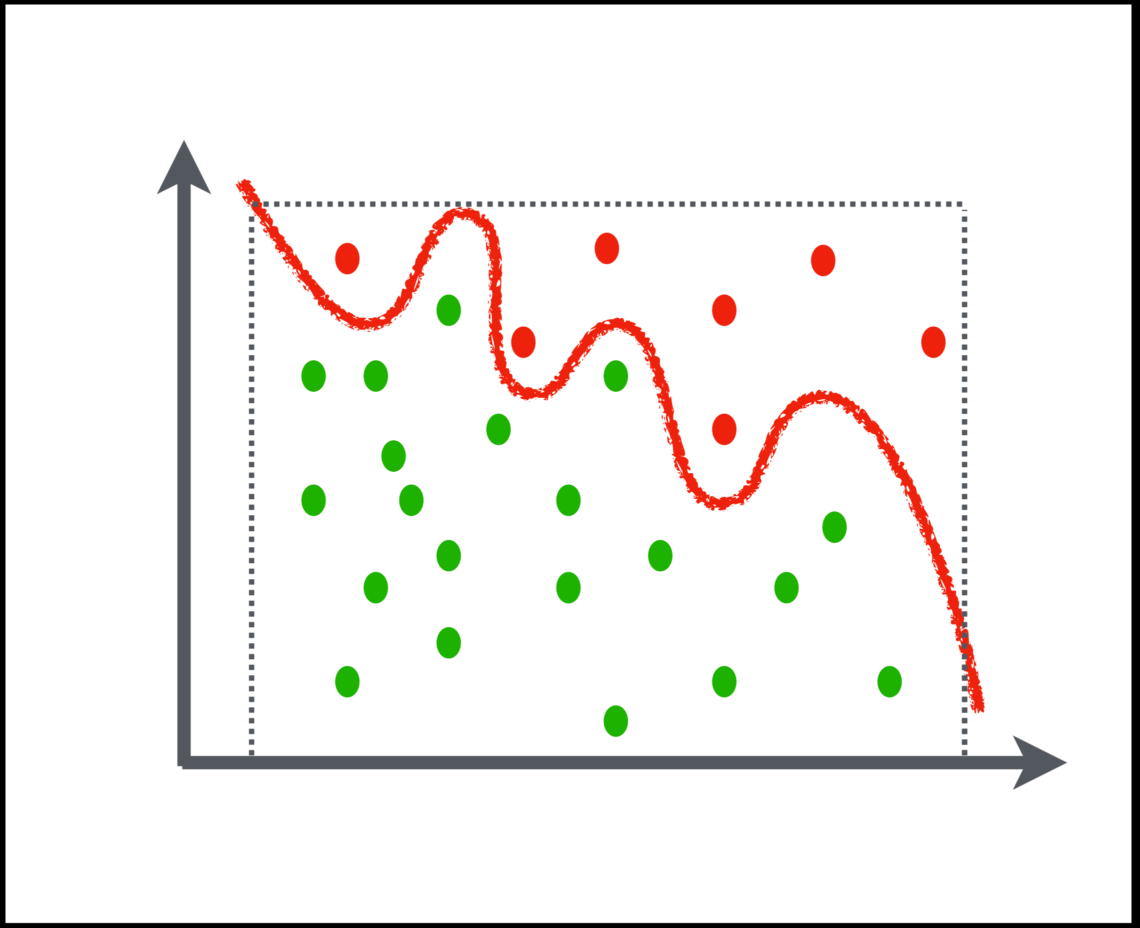

$$\int_a^b dx\, f(x) = f(a) \times (b-a) \frac{N_{green}}{N_{green} + N_{red}}$$

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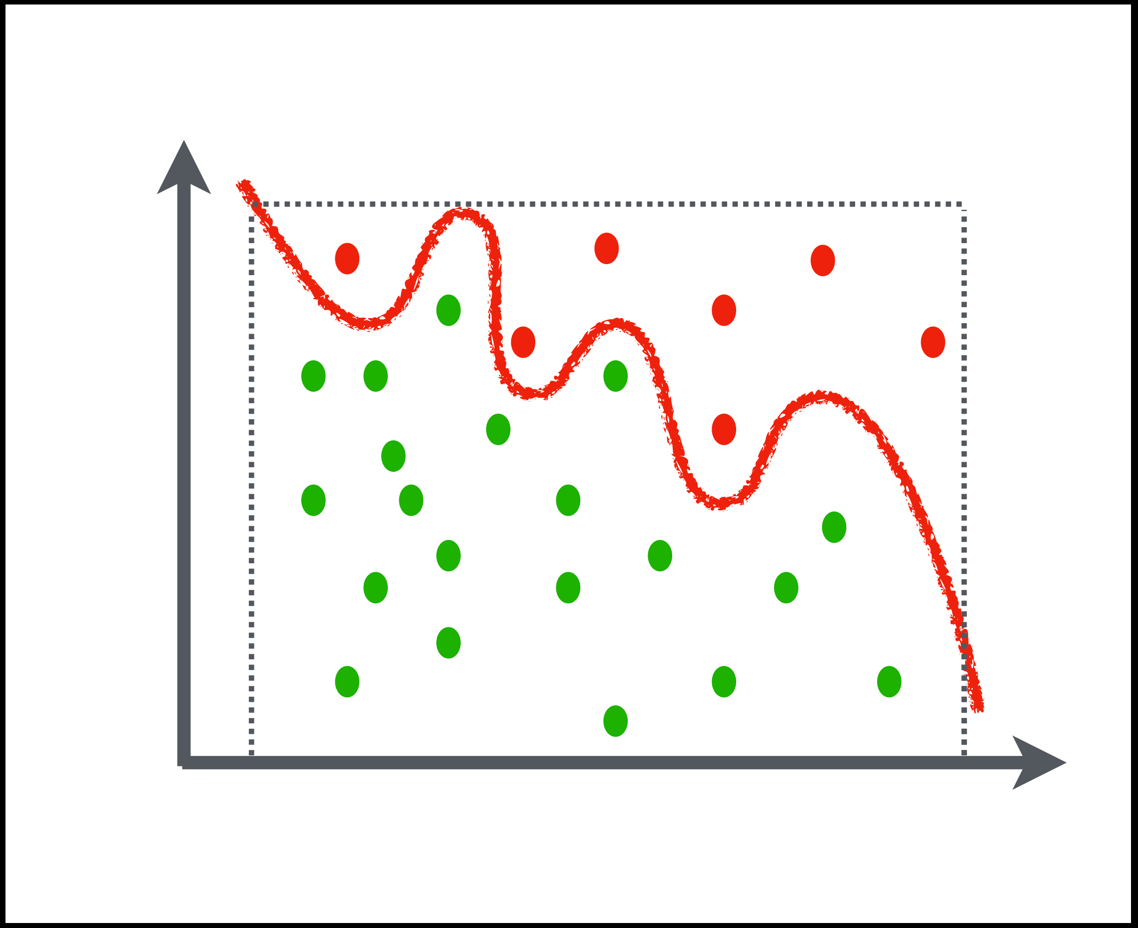

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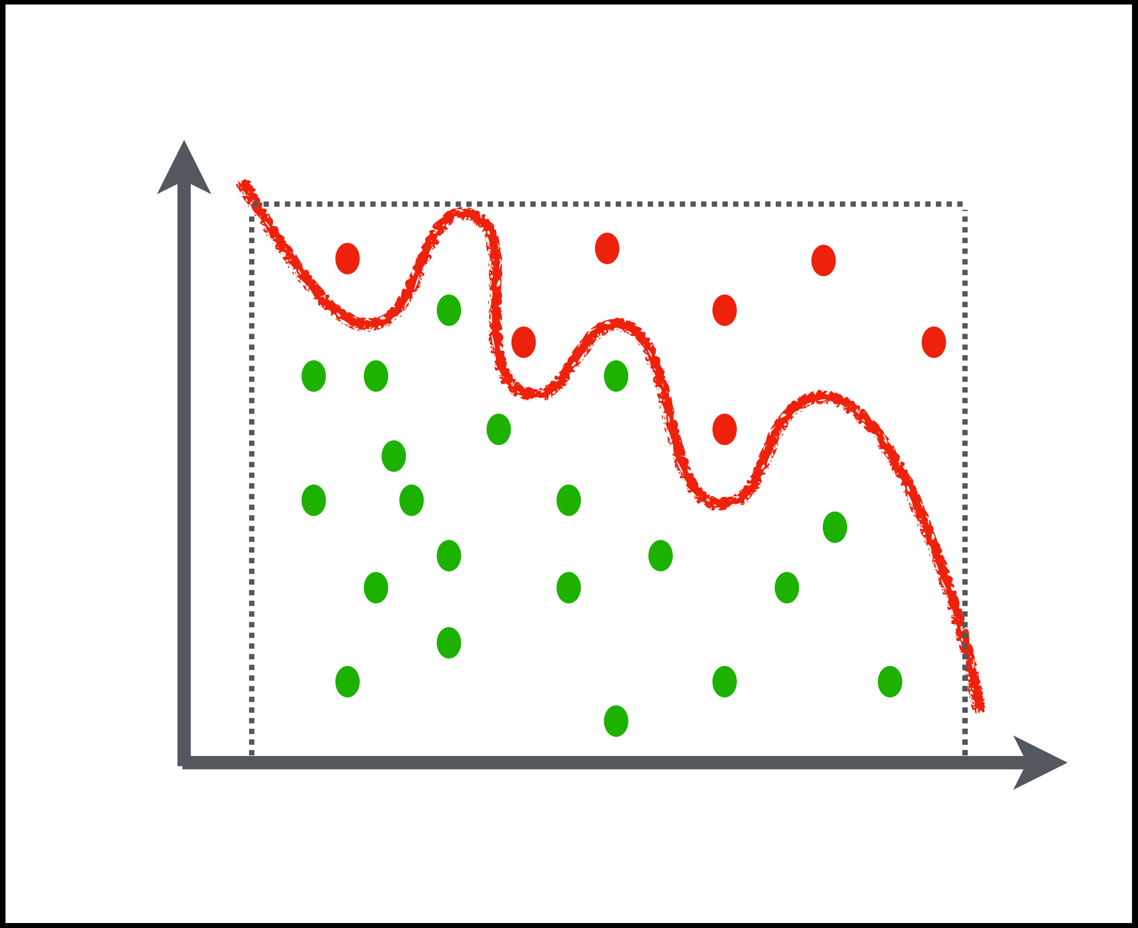

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1)What happens if maximum is not at x=a?

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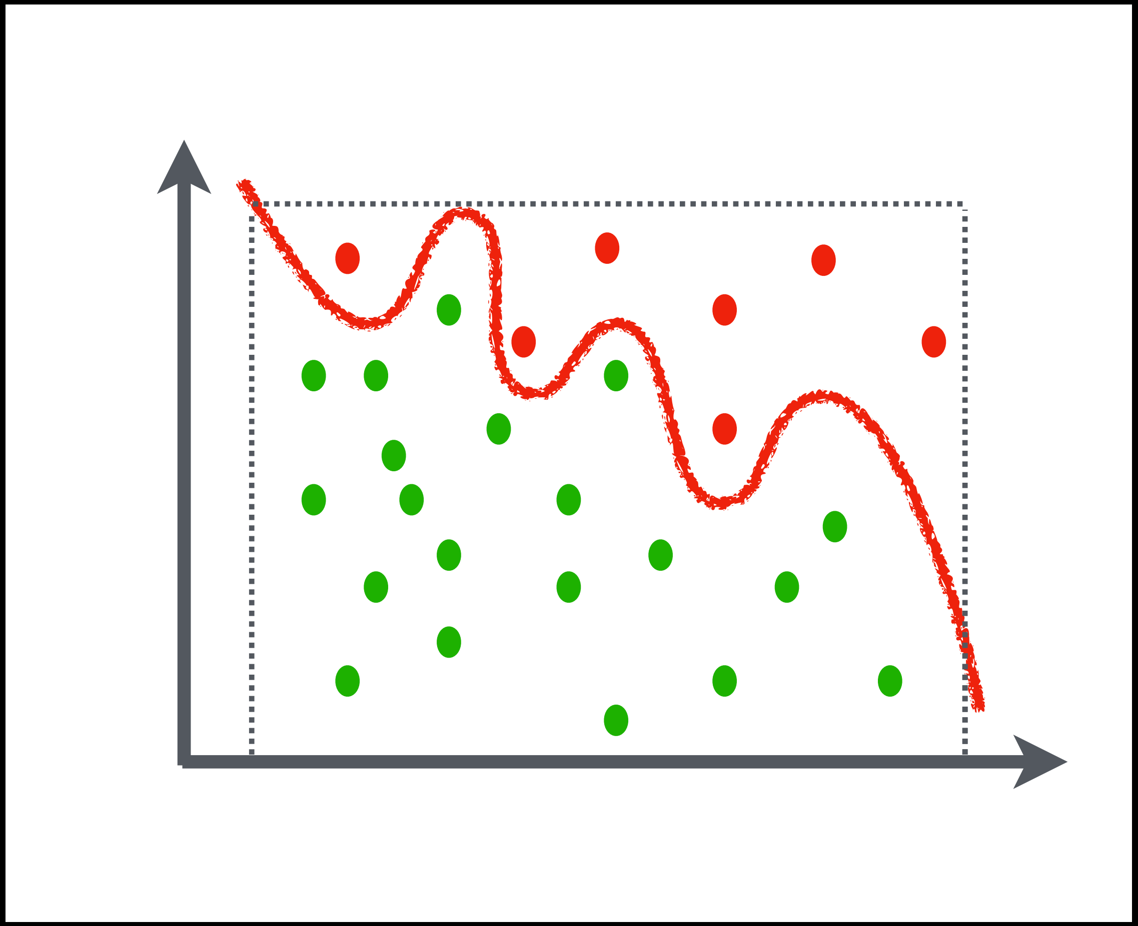

Questions:
1)What happens if maximum is not at x=a?
2) What if function is not positive?

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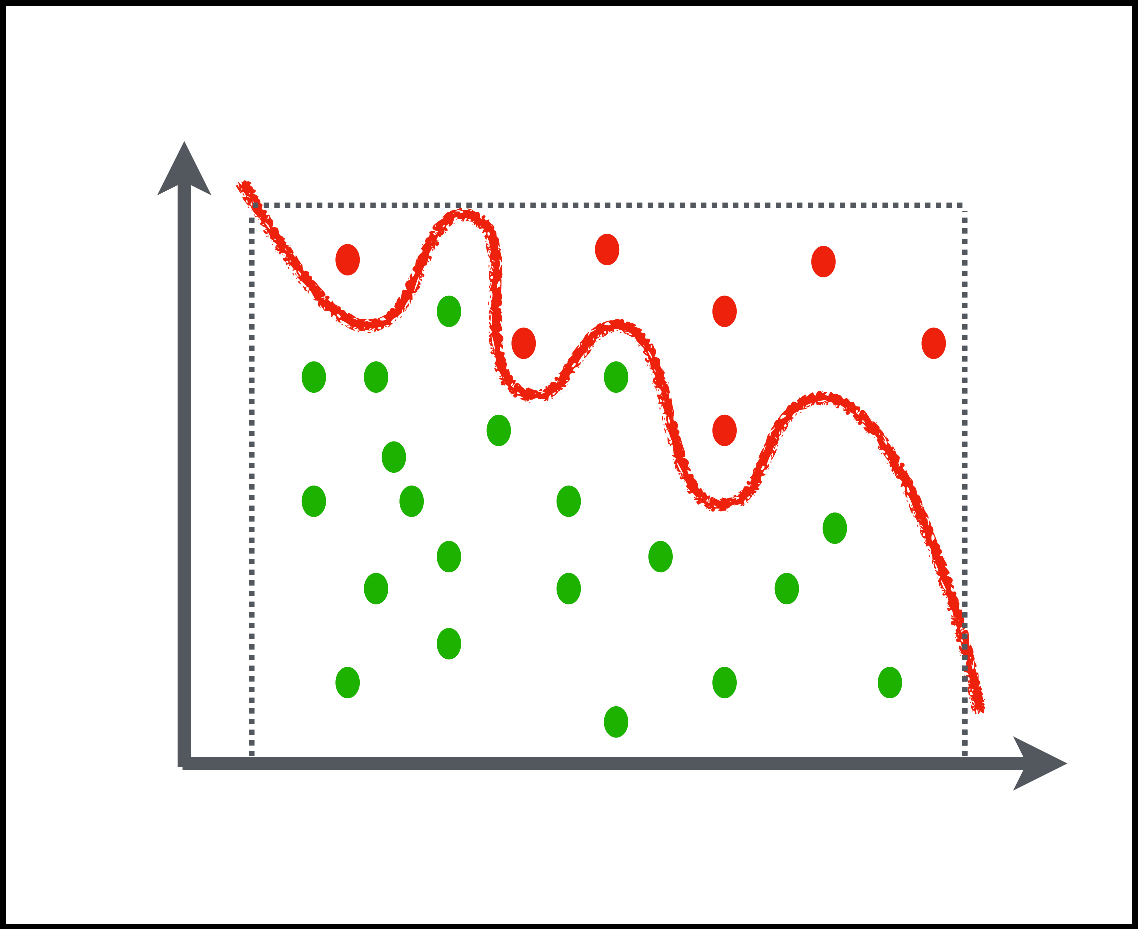

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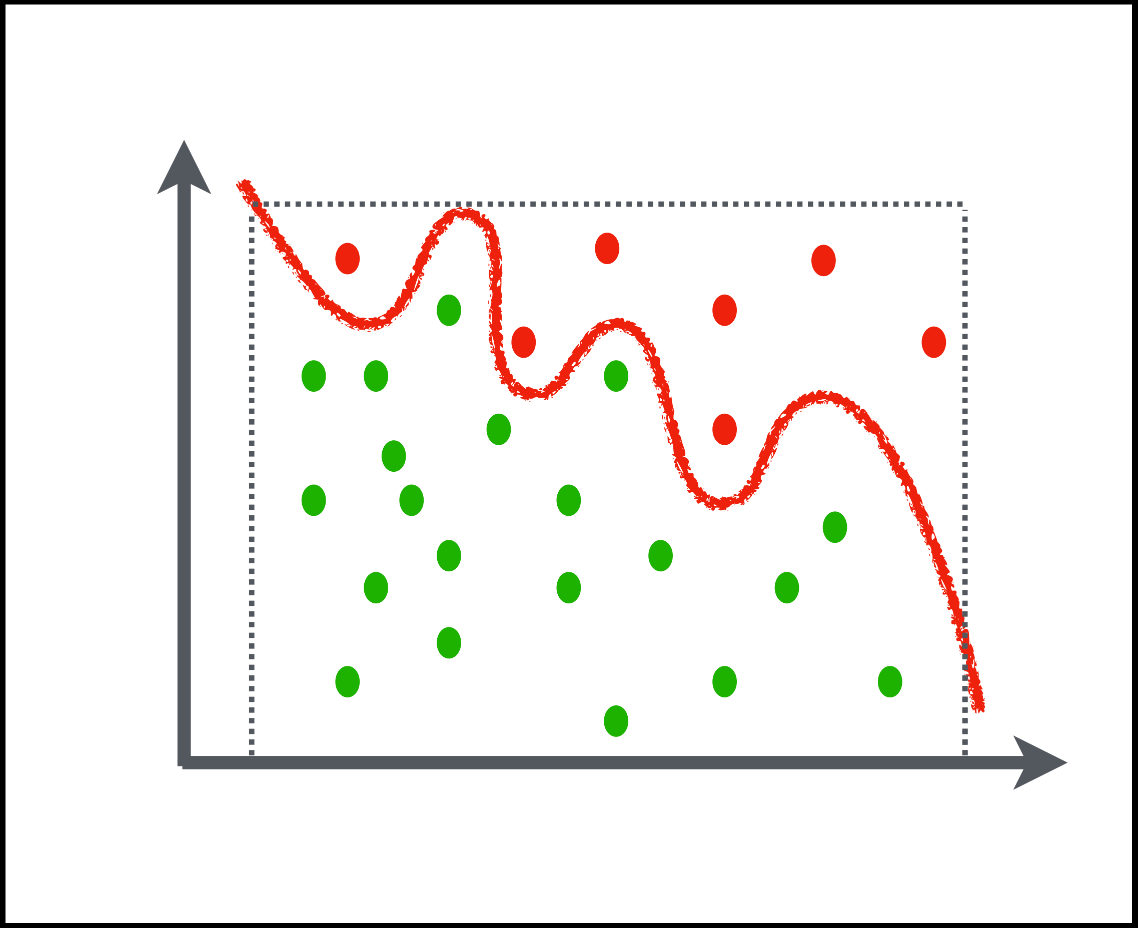

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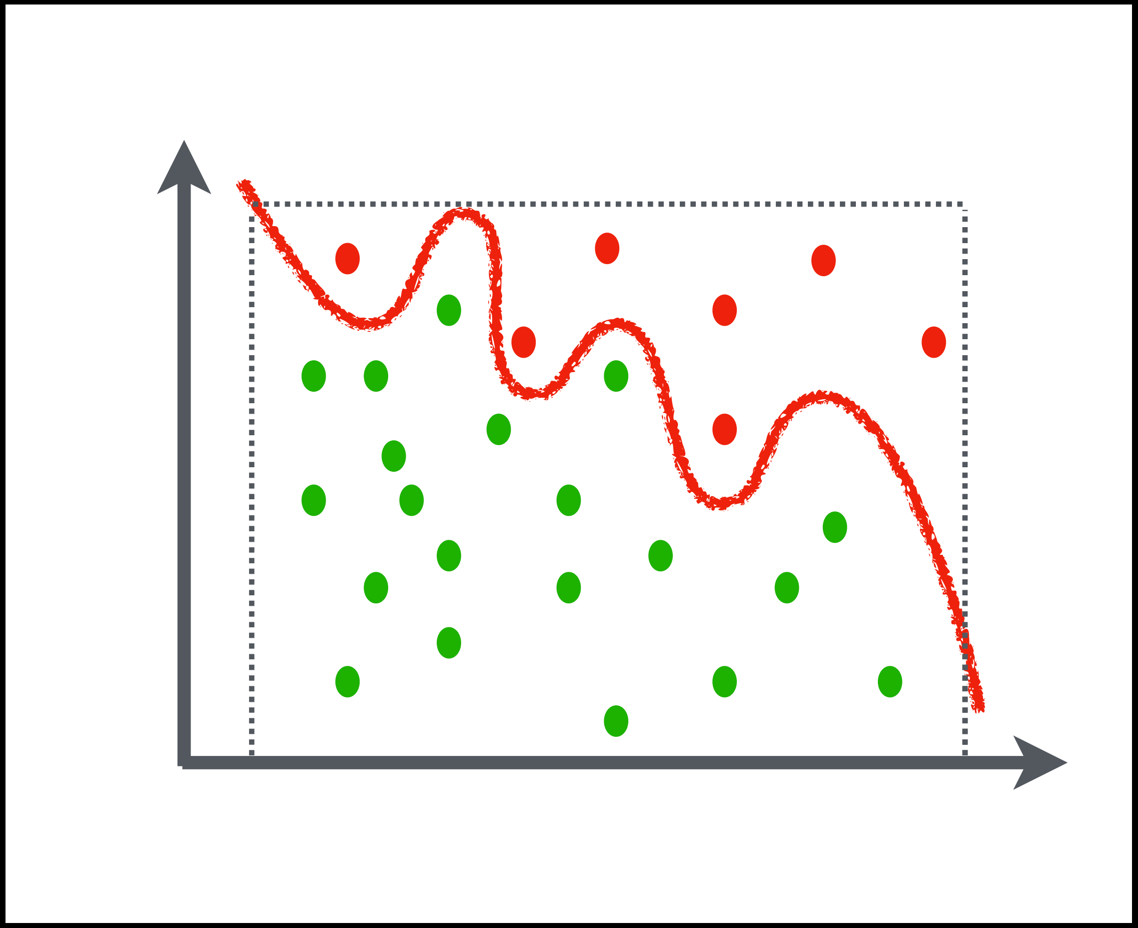

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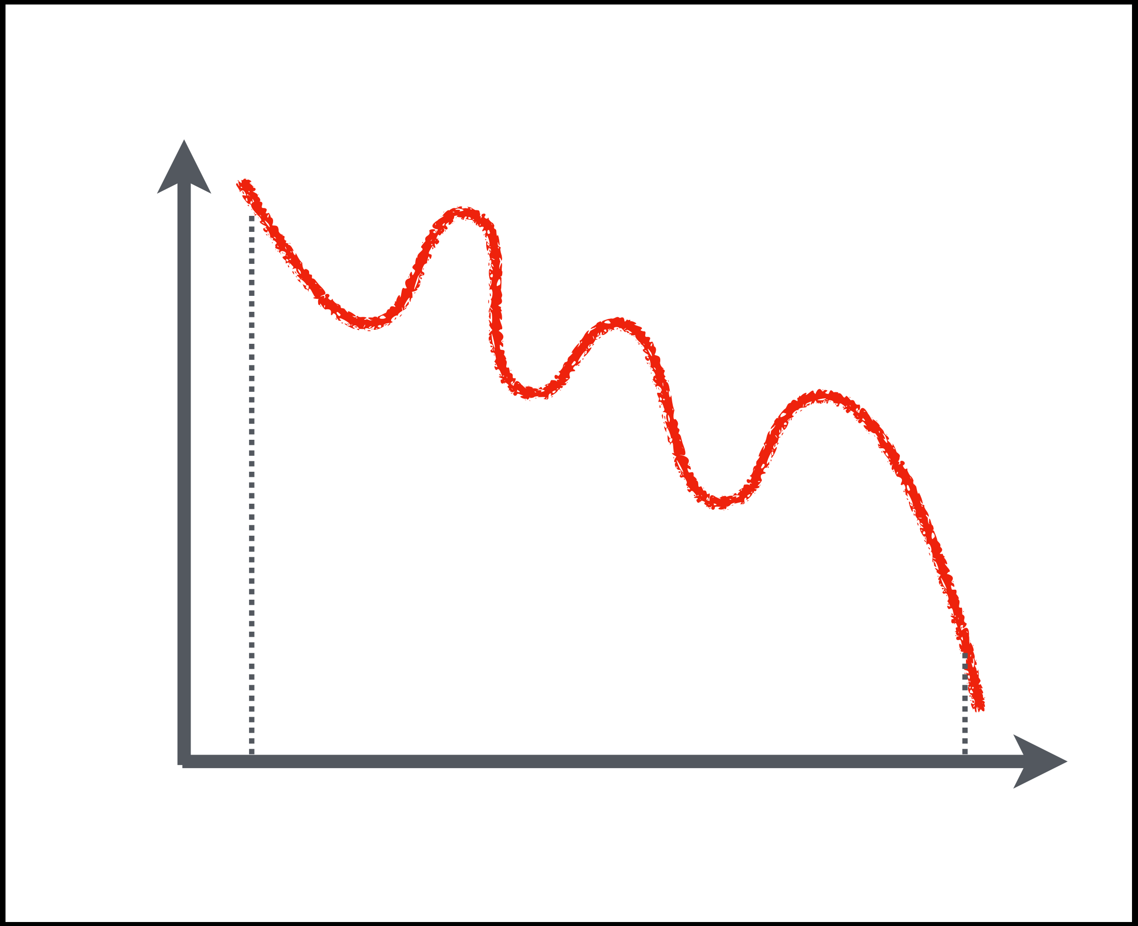
Assignment: Calculate π using MC methods

From integrating to sampling distributions

- Say f(x) is positive definite
- A probability distribution for x
- What is <x> , <x²> , … or < g(x) >?



$$\langle x^n \rangle = \frac{\int_a^b dx x^n f(x)}{\int_a^b dx f(x)} = \frac{1}{N_{green}} \sum_i^{N_{green}} (x_i)^n$$

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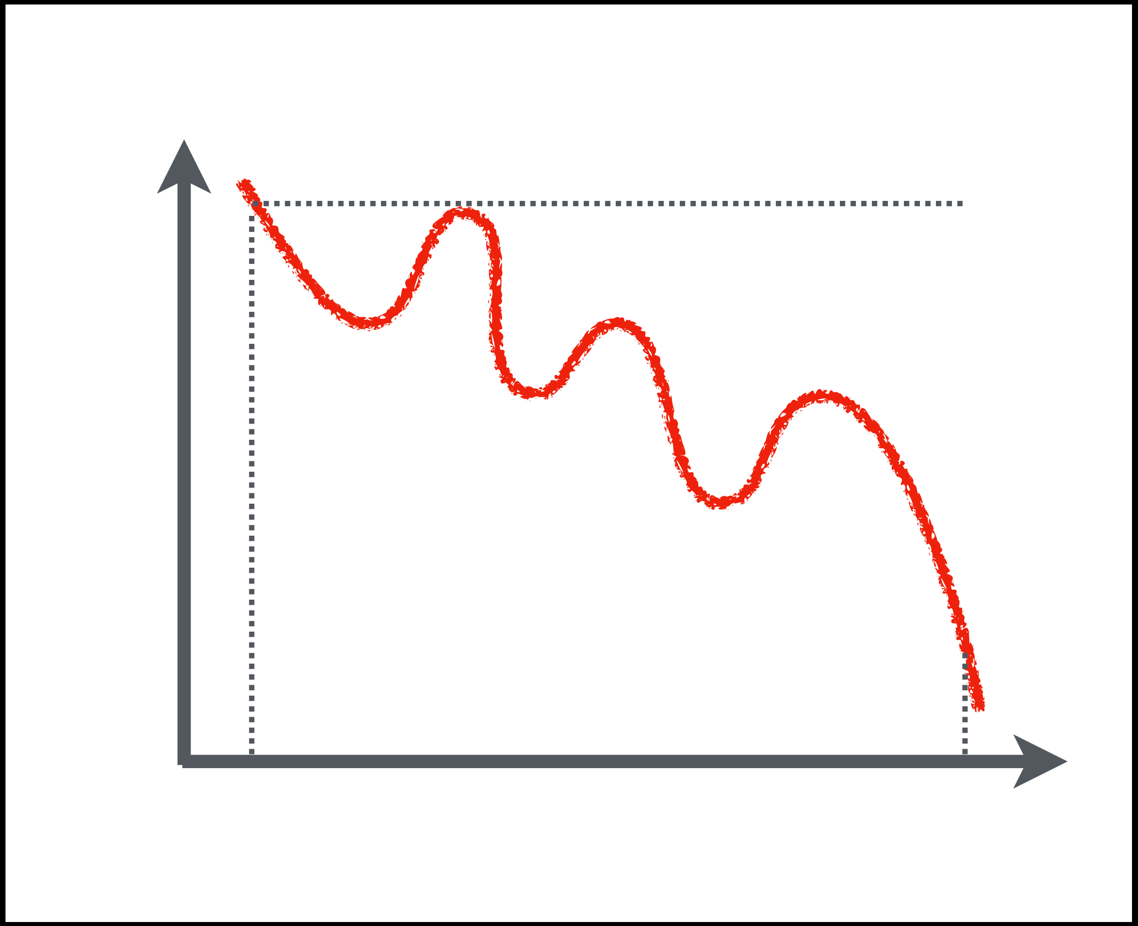

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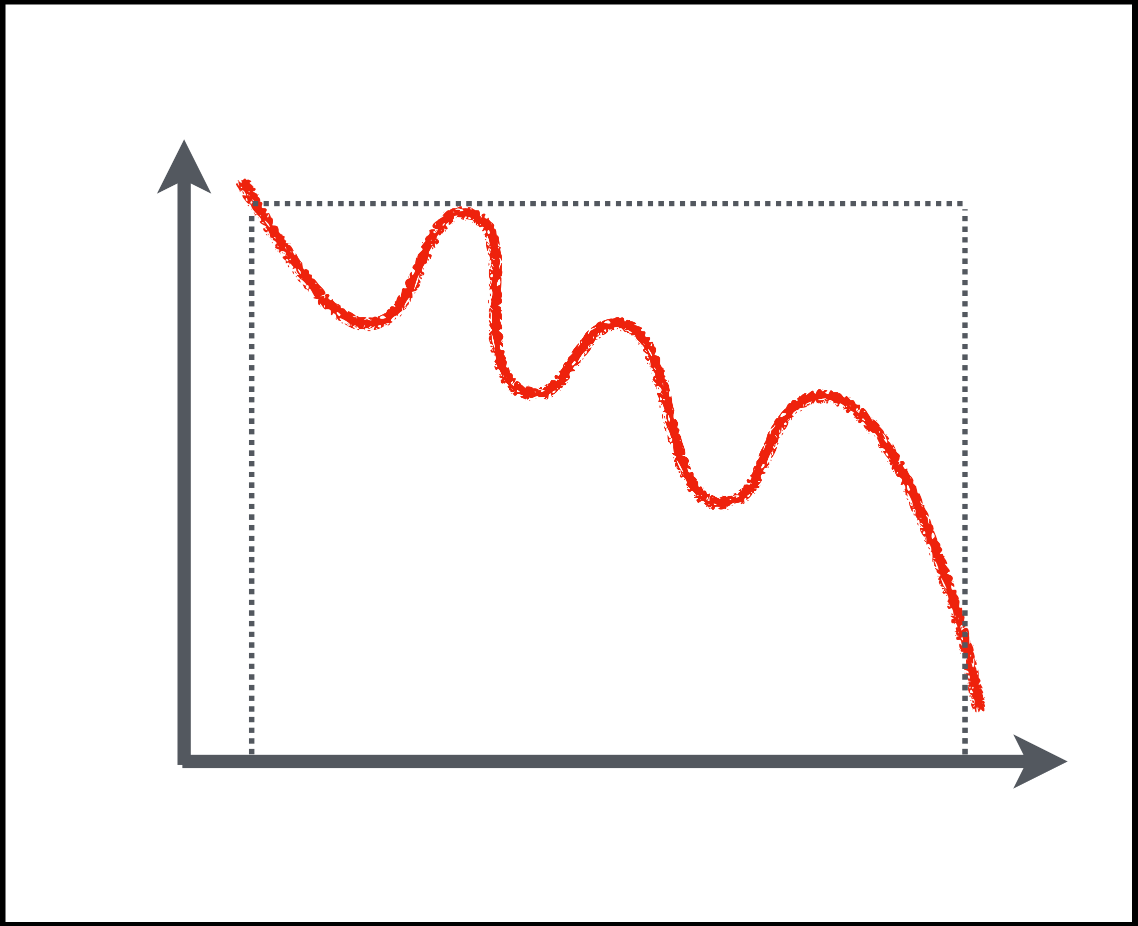

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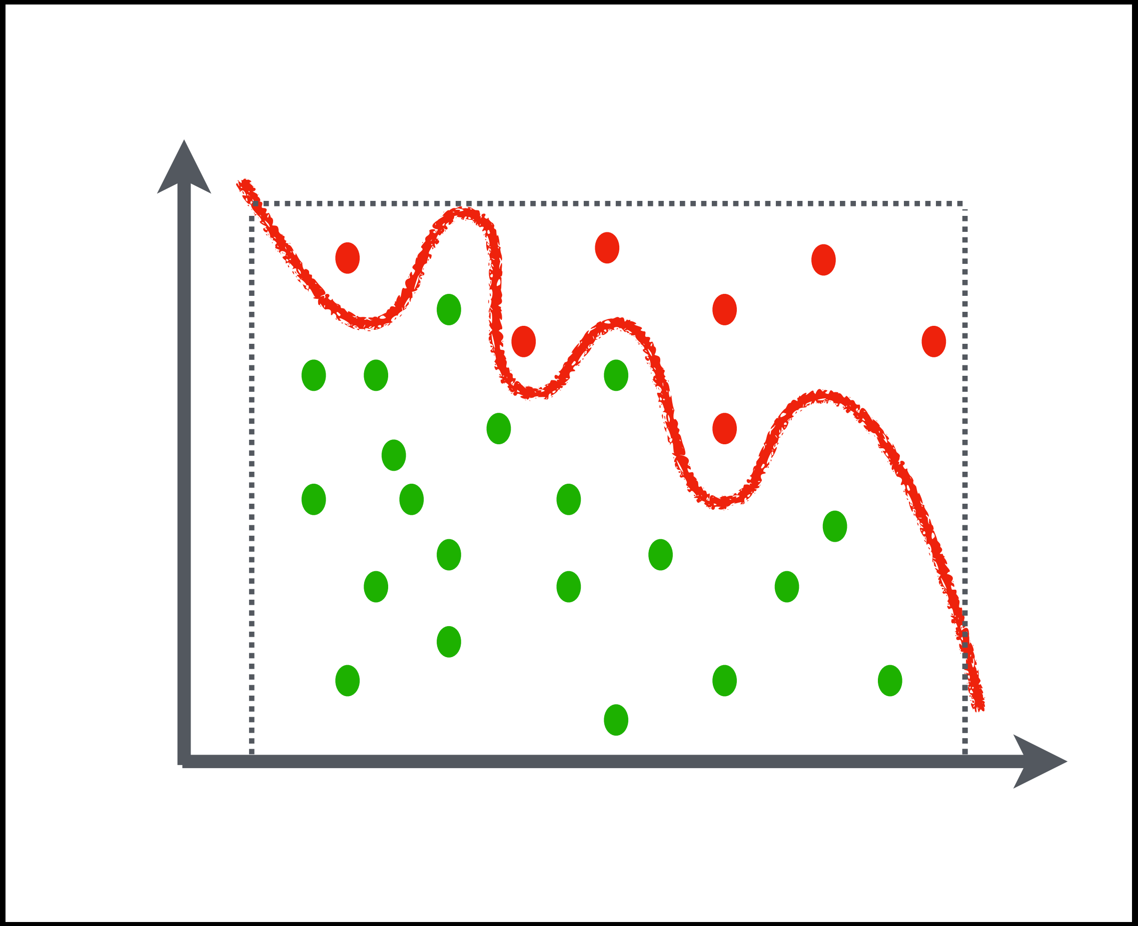

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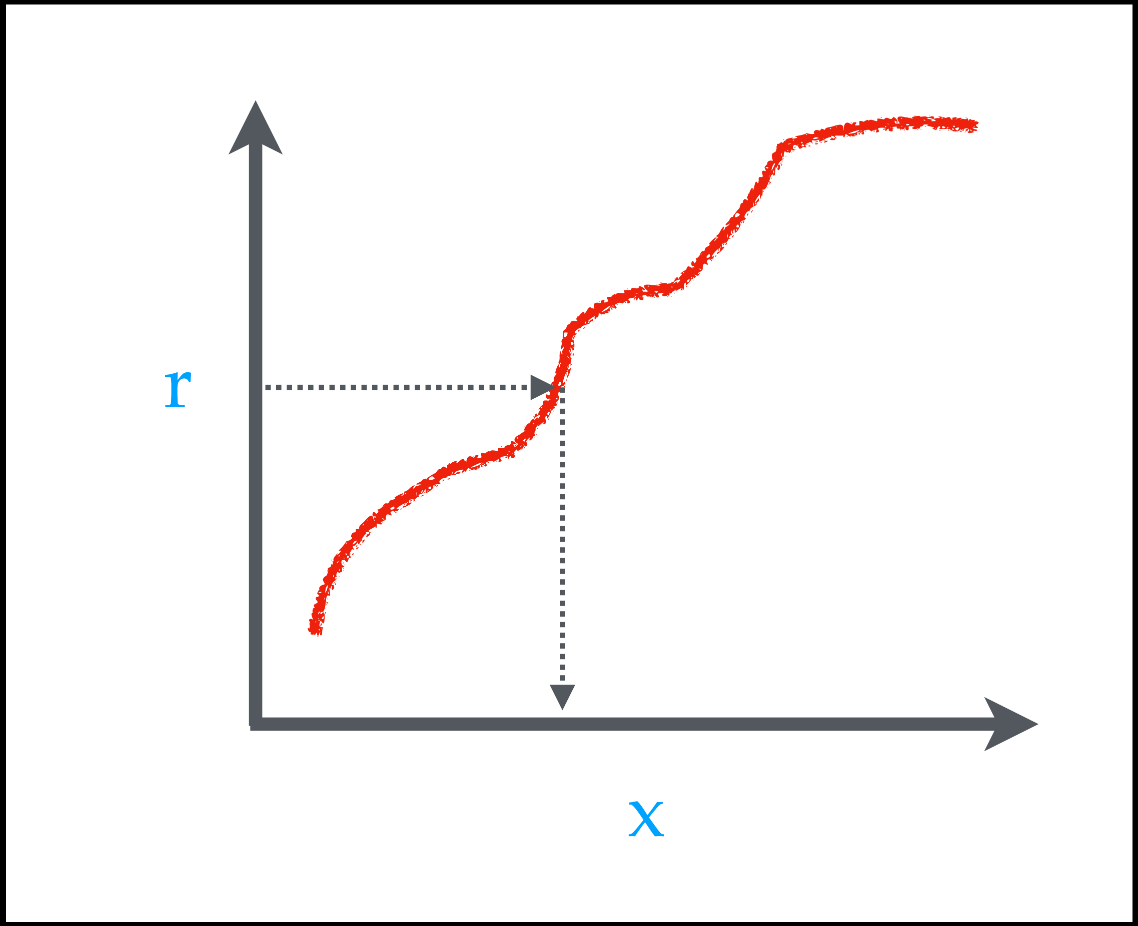
Sample integrals of distributions

- In most cases, sampling the function is inefficient
- Better to sample the integral of the function
- Generate a random number *r*, and find an *x*, such that

$$r = \frac{\int_a^x dy\, f(y)}{\int_a^b dy\, f(y)}$$



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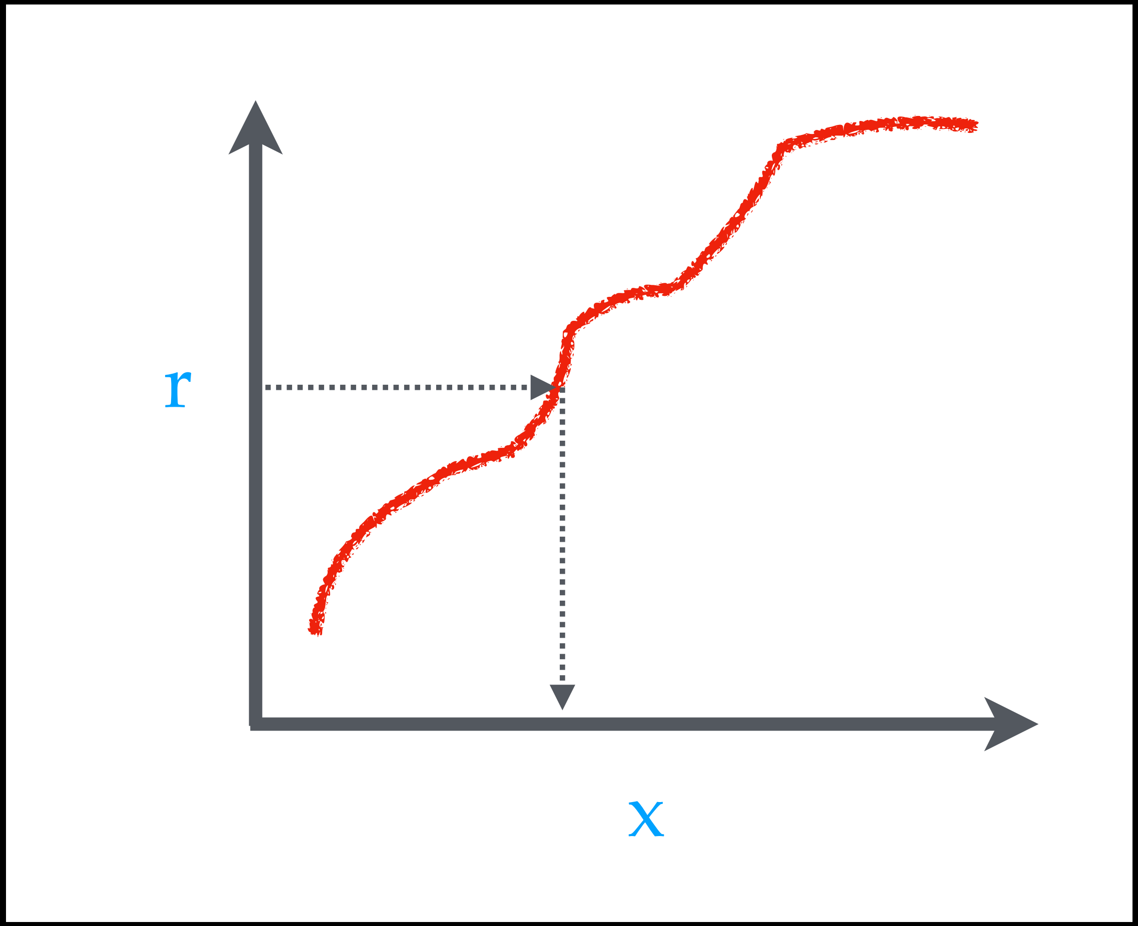
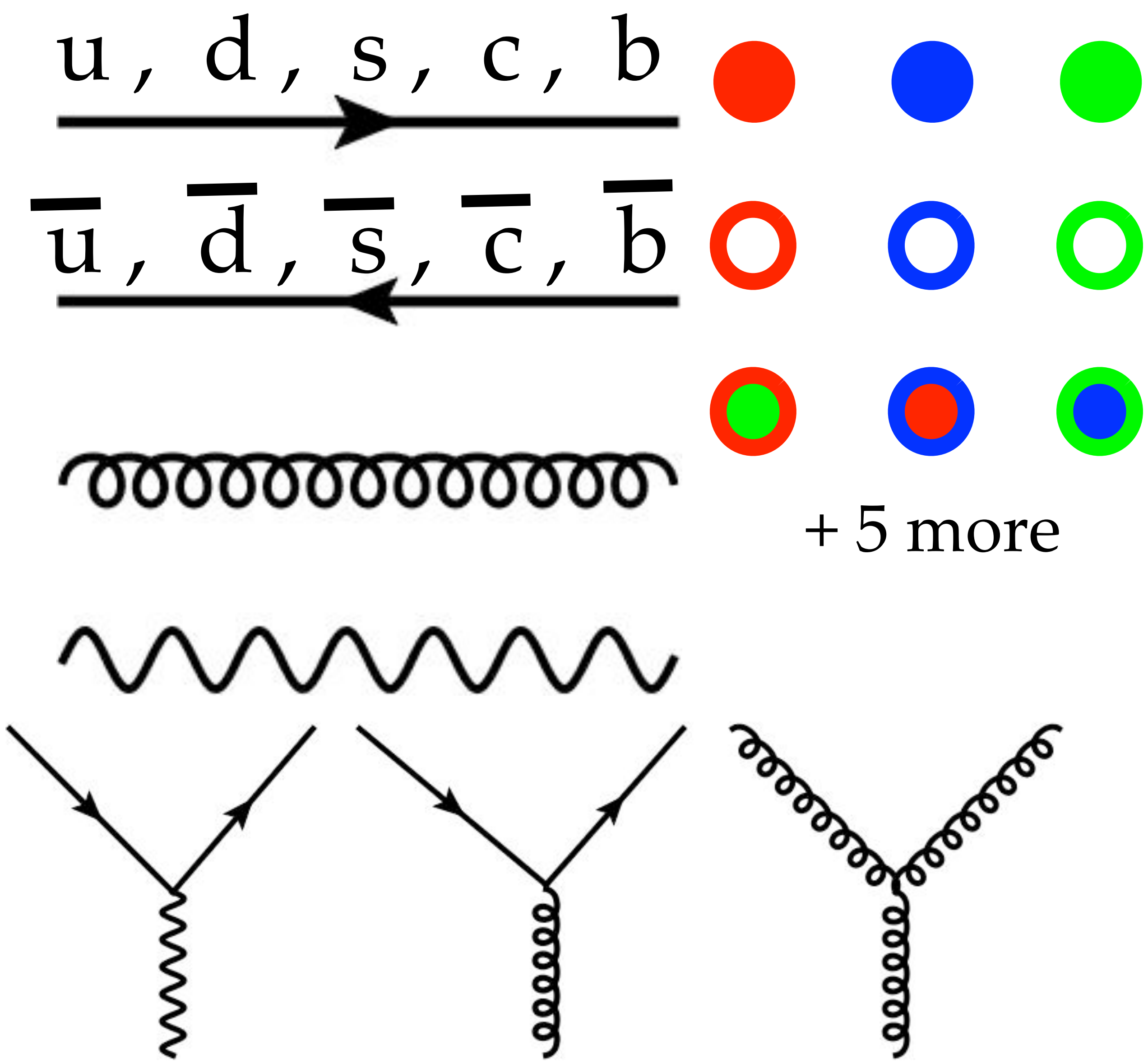


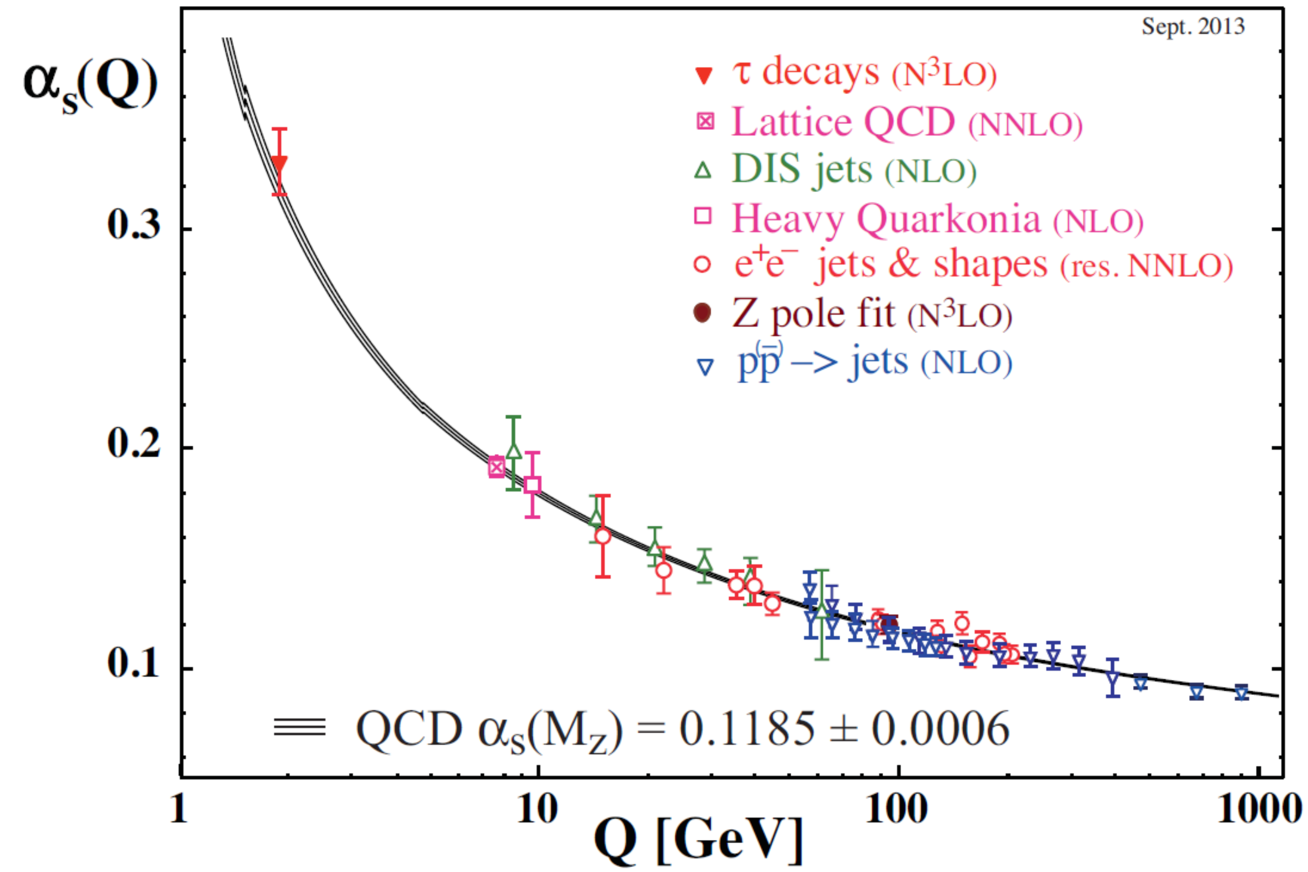
Assignment: How does this work?

Event generation

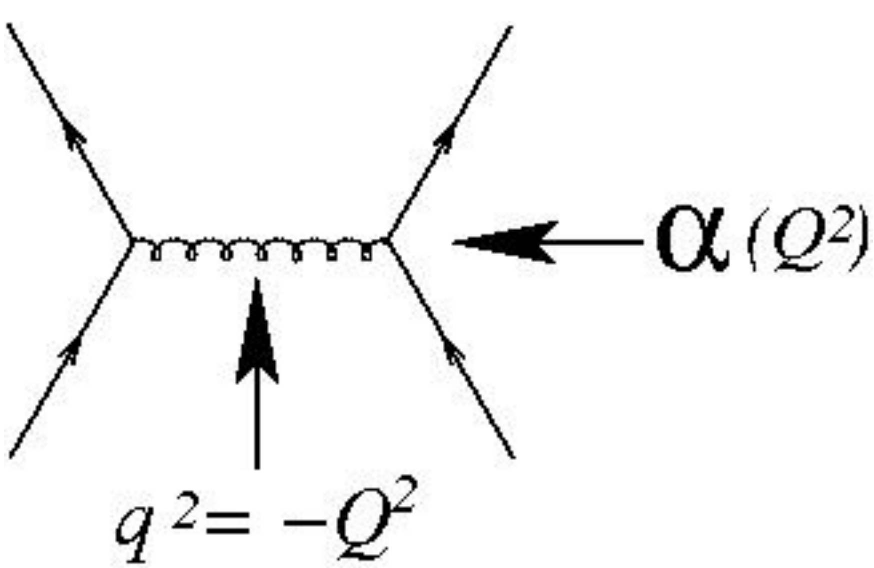
- Any stochastic process can be experimentally measured over multiple events
- Stochastic -> randomness: from thermal or quantum fluctuations
- Theoretically simulated by sampling a probability distribution
- Compare statistical “averages” of various quantities

Quarks, Gluons, and the QCD coupling





coupling gets weaker at large momentum transfer

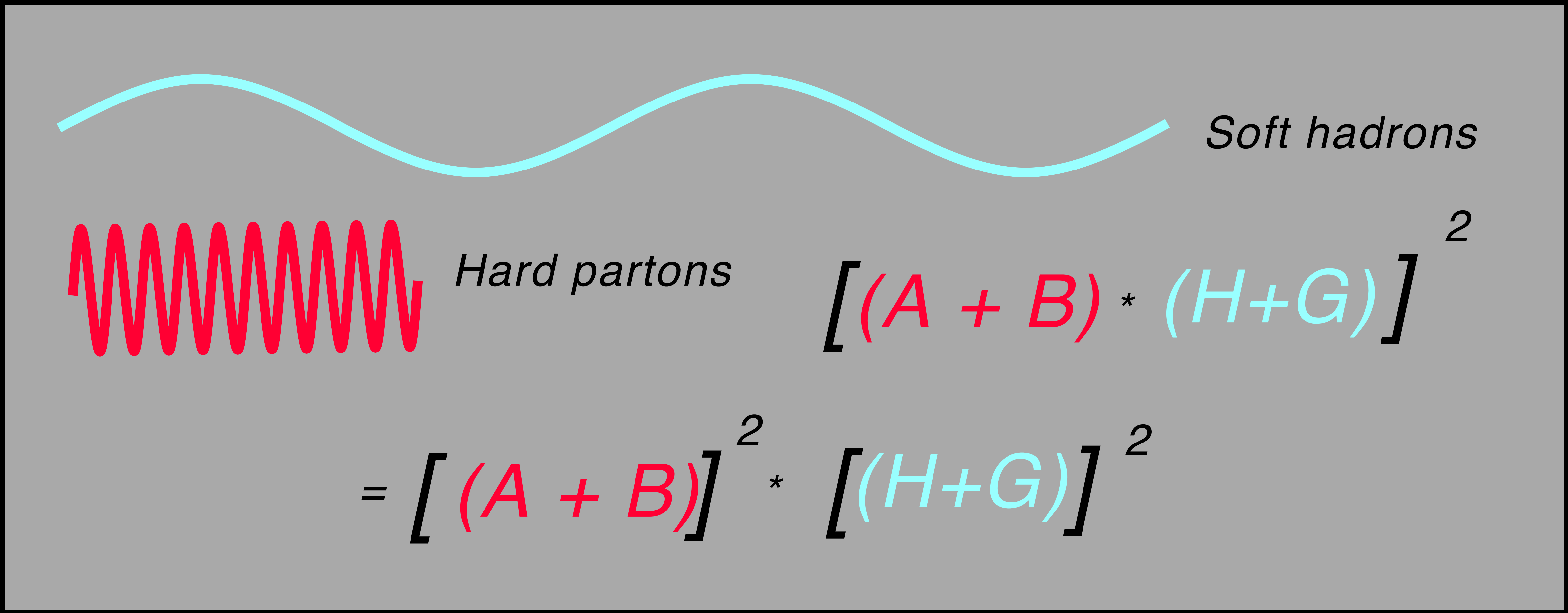


$$\alpha_S(Q^2) = \frac{12\pi}{(11N_c - 2n_f)\log\left[\frac{Q^2}{\Lambda^2_{QCD}}\right]}$$

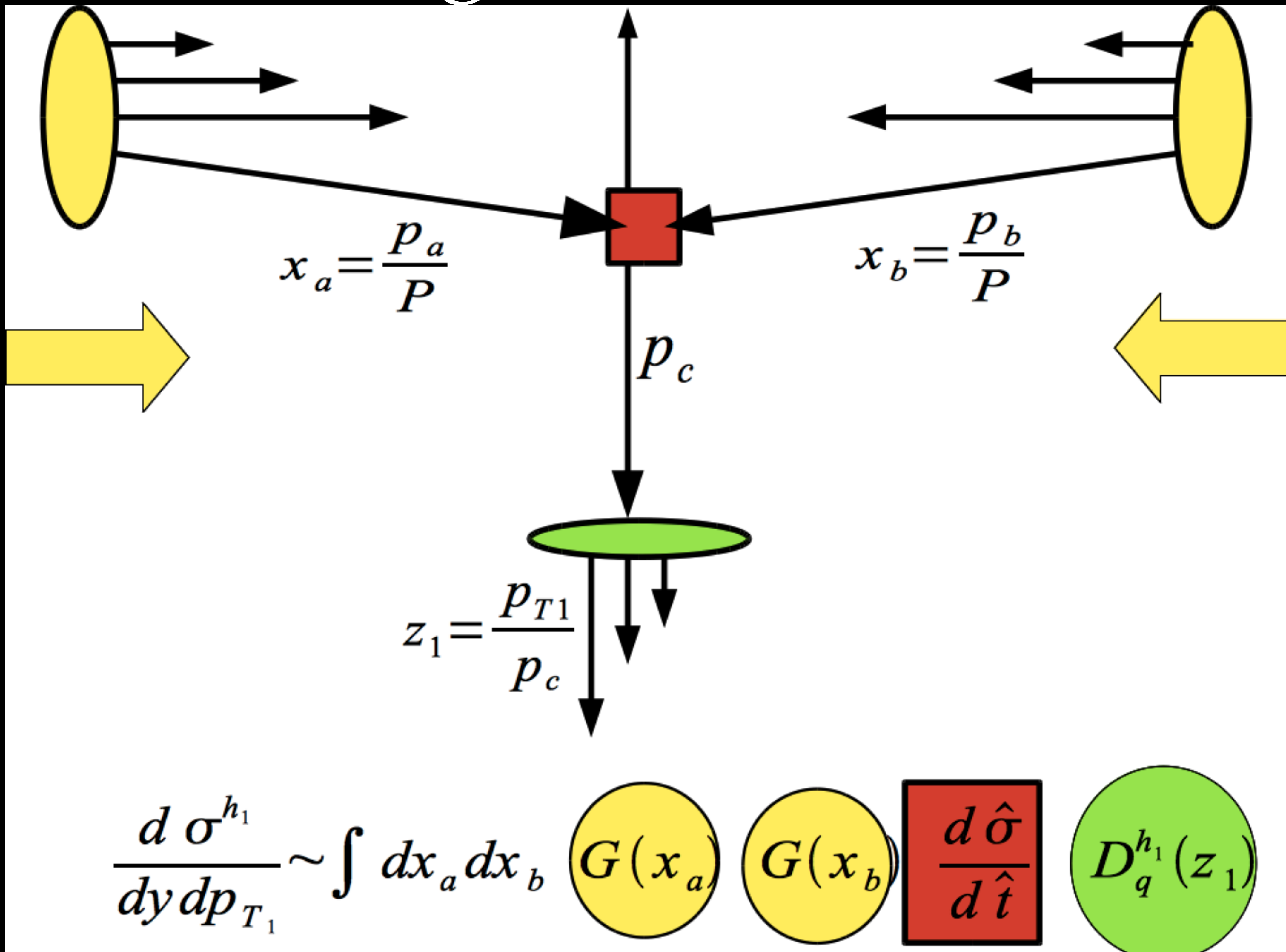
Factorization

Any hard process has a range of scales up to the hard scale

Observation: there should be minimal interference between hard and soft processes



At leading order this means



How about those higher order corrections

Collins Soper Sterman

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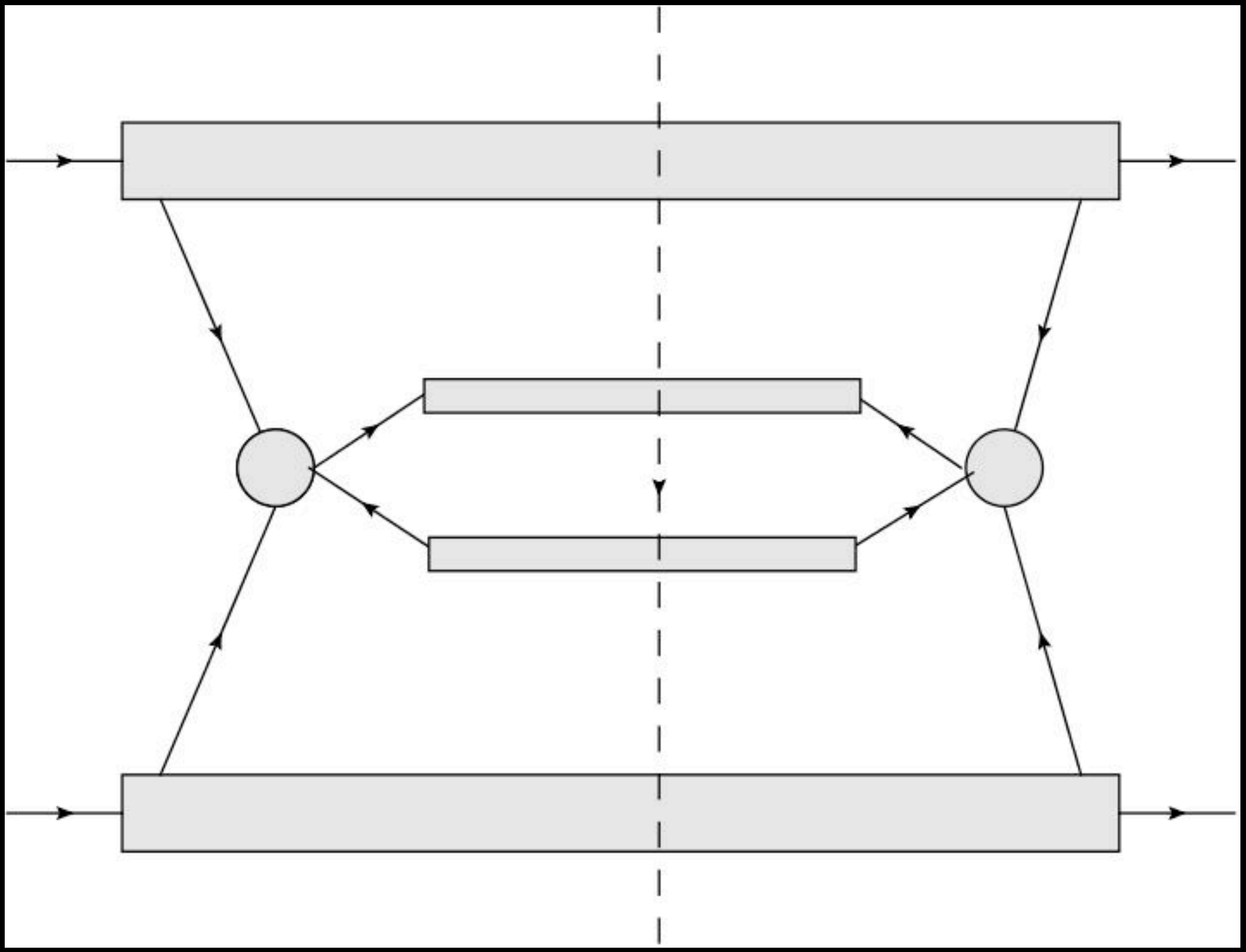

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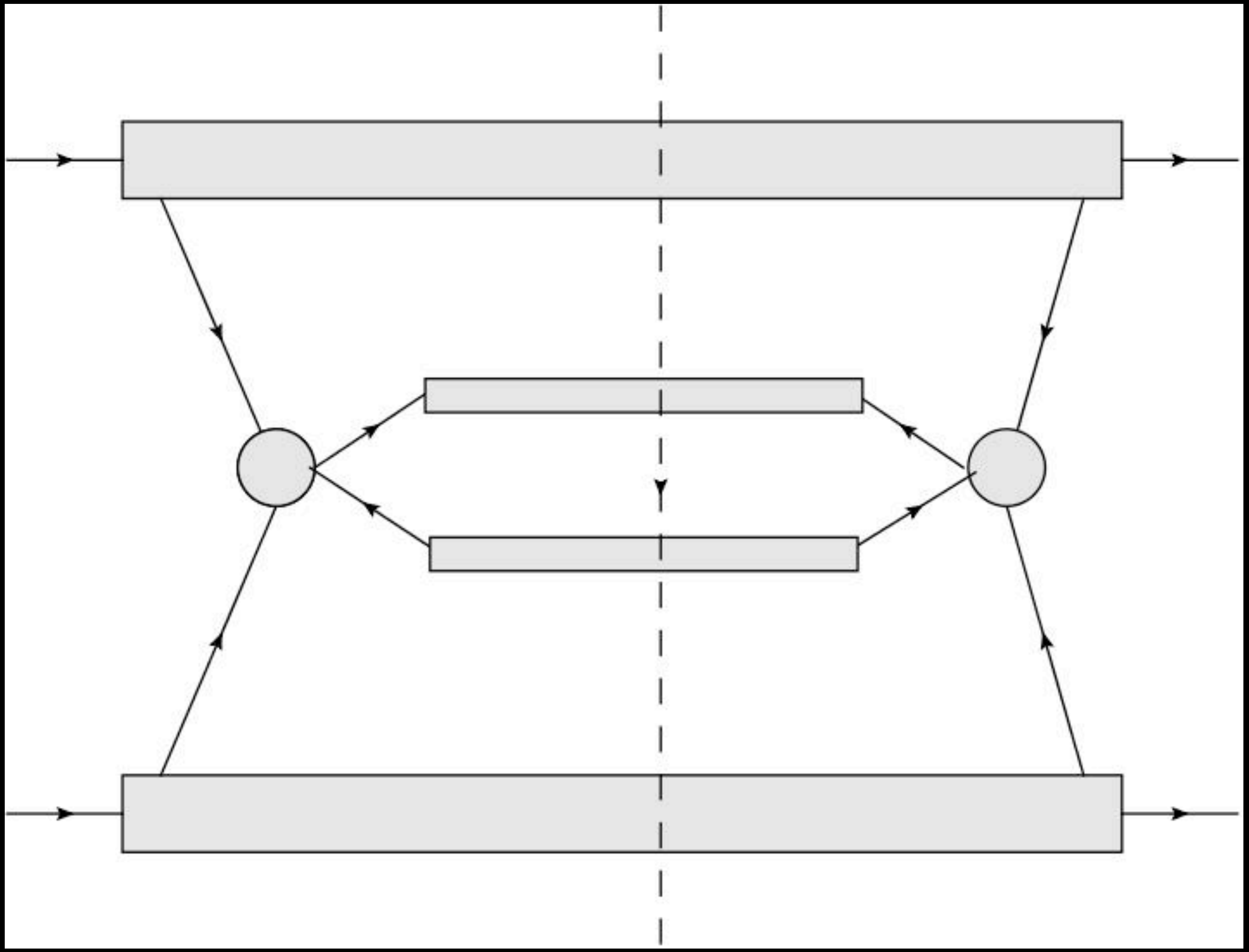

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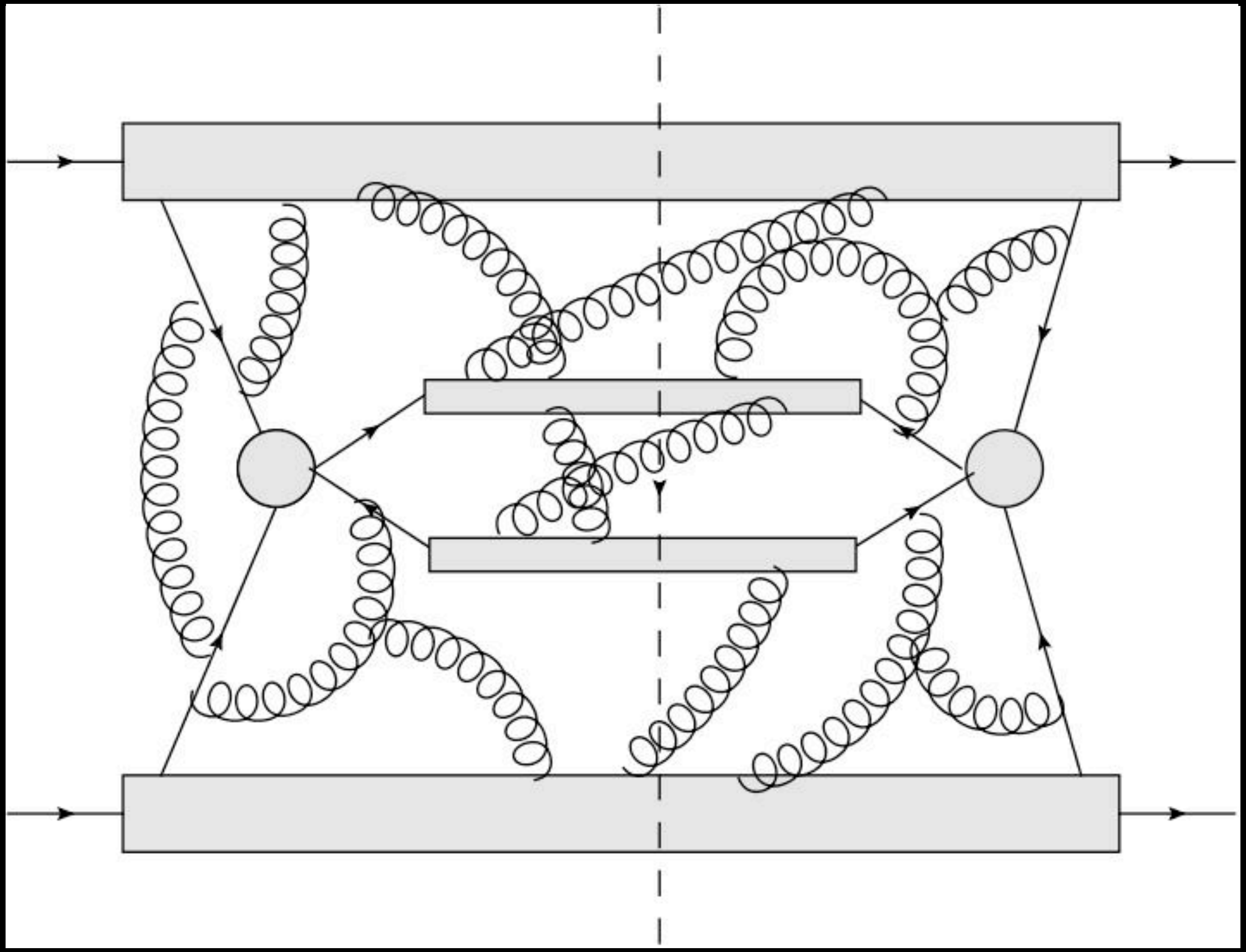

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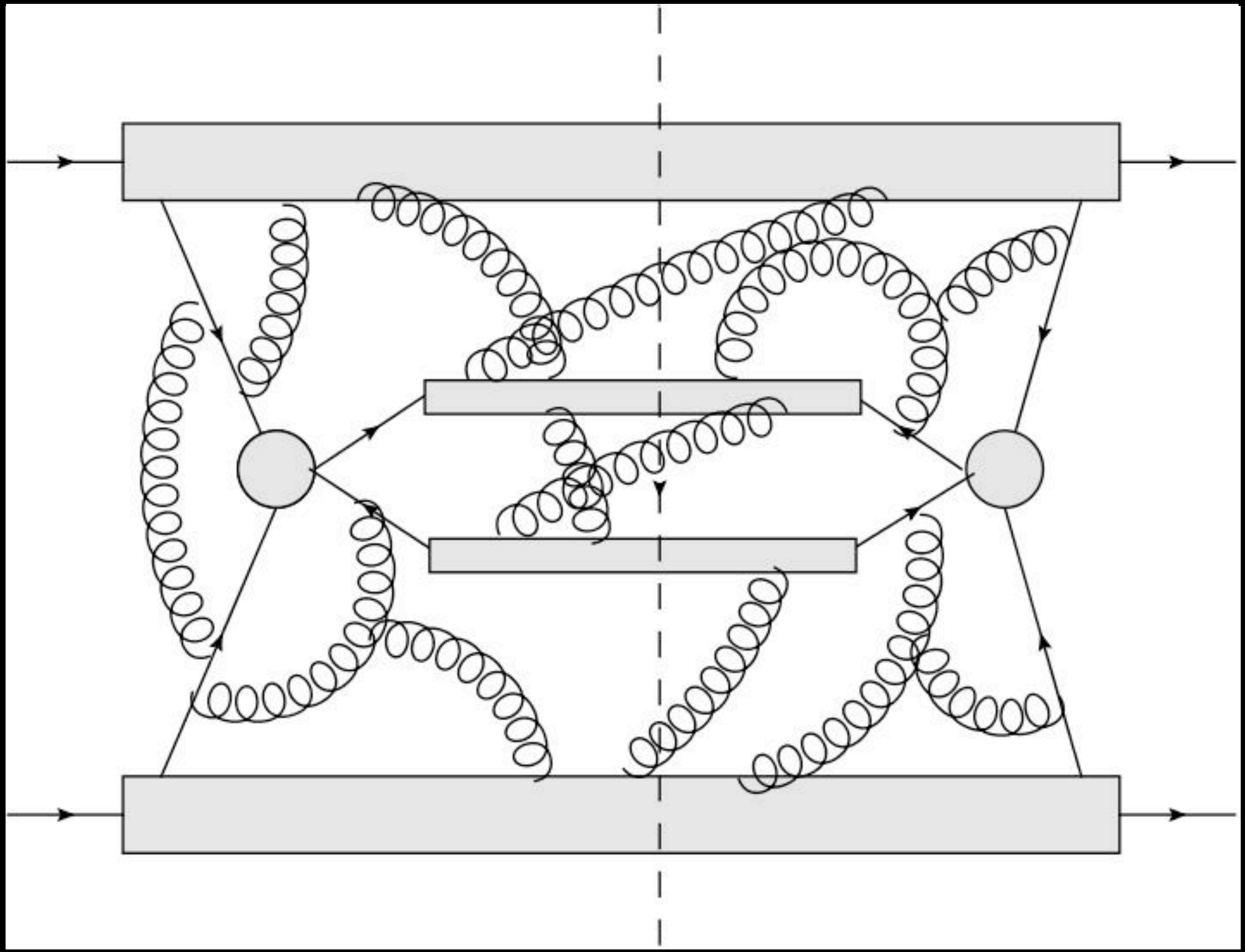

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introduce higher orders

identify pinched contours

retain leading twist only

Sum over final states,
Eikonalize!

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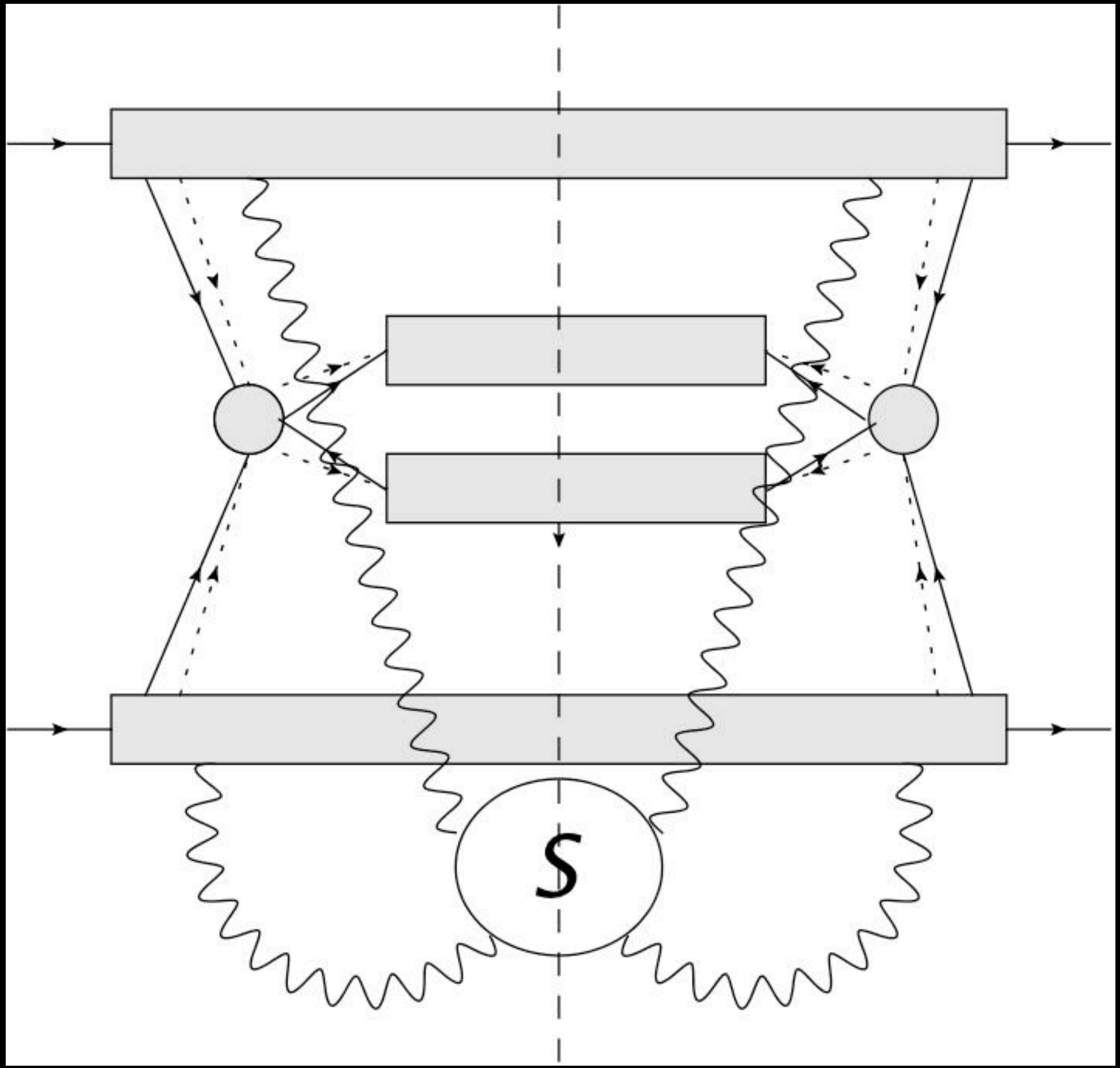

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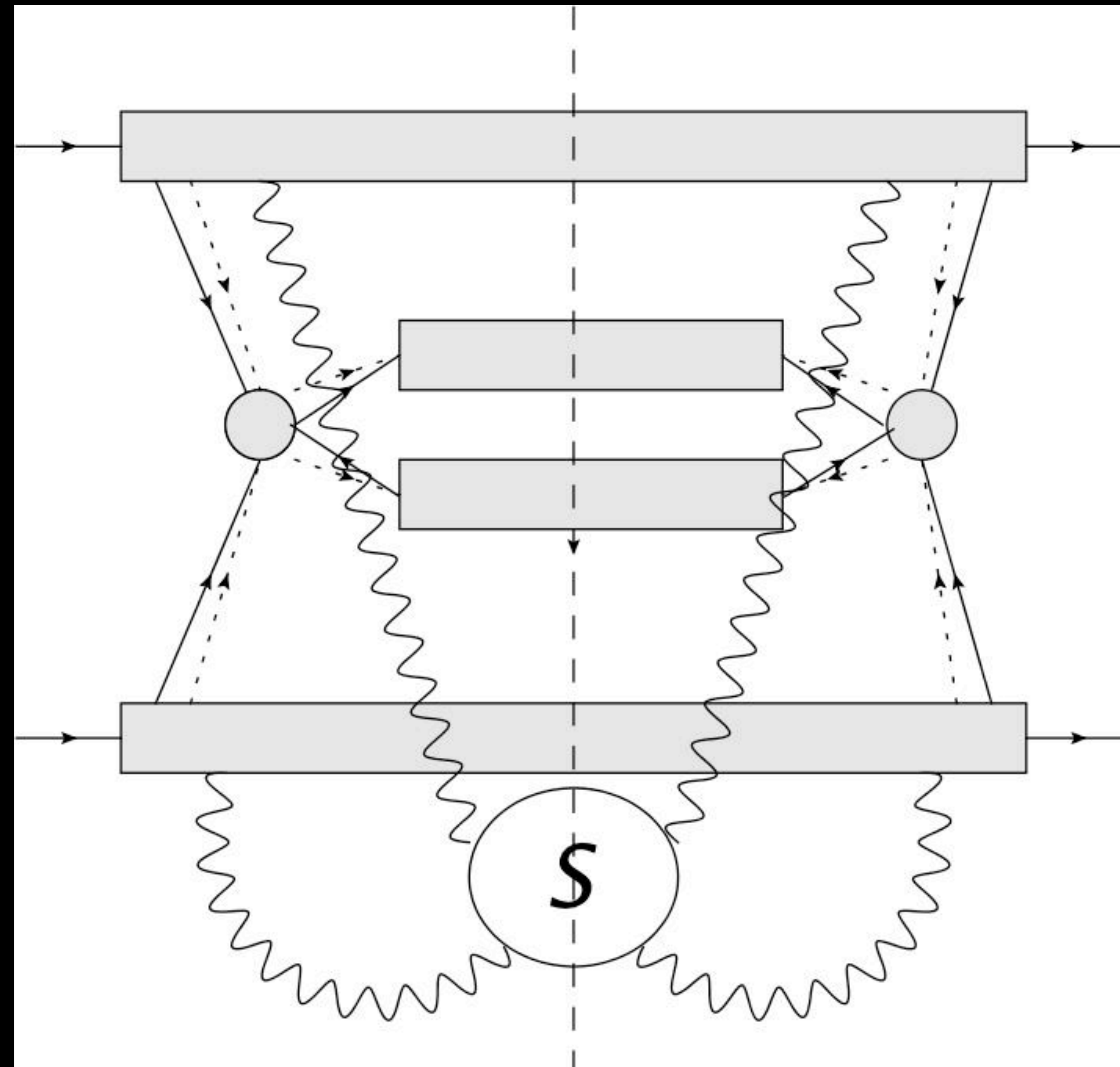


$$\frac{d\sigma}{dz} \propto \int dx_1 dx_2 G(x_1) G(x_2) \frac{d\hat{\sigma}}{d\hat{t}} D(z) + C \frac{\Lambda^2}{Q^2}$$

Collins Soper Sterman

Factorization makes everything systematic

PDFs and FFs have universal operator definitions

Measure in one Expt. use in another
(intrinsic property of a proton)

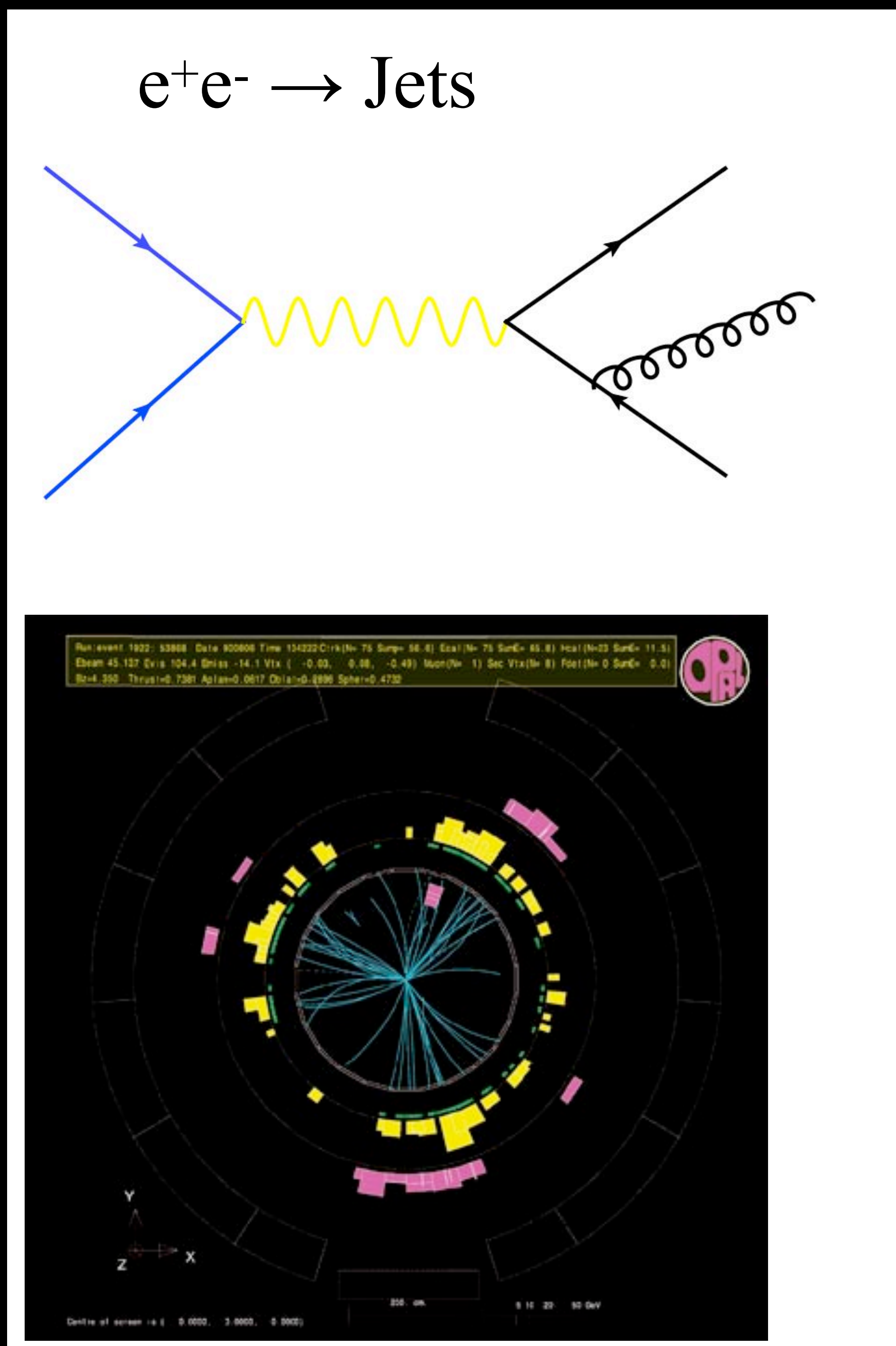
$$G(x) = \int \frac{dy^-}{2\pi} \langle P|\bar{\psi}(y^-)\frac{\gamma^+}{2P^+}\psi|P\rangle e^{-ixP^+y^-}$$

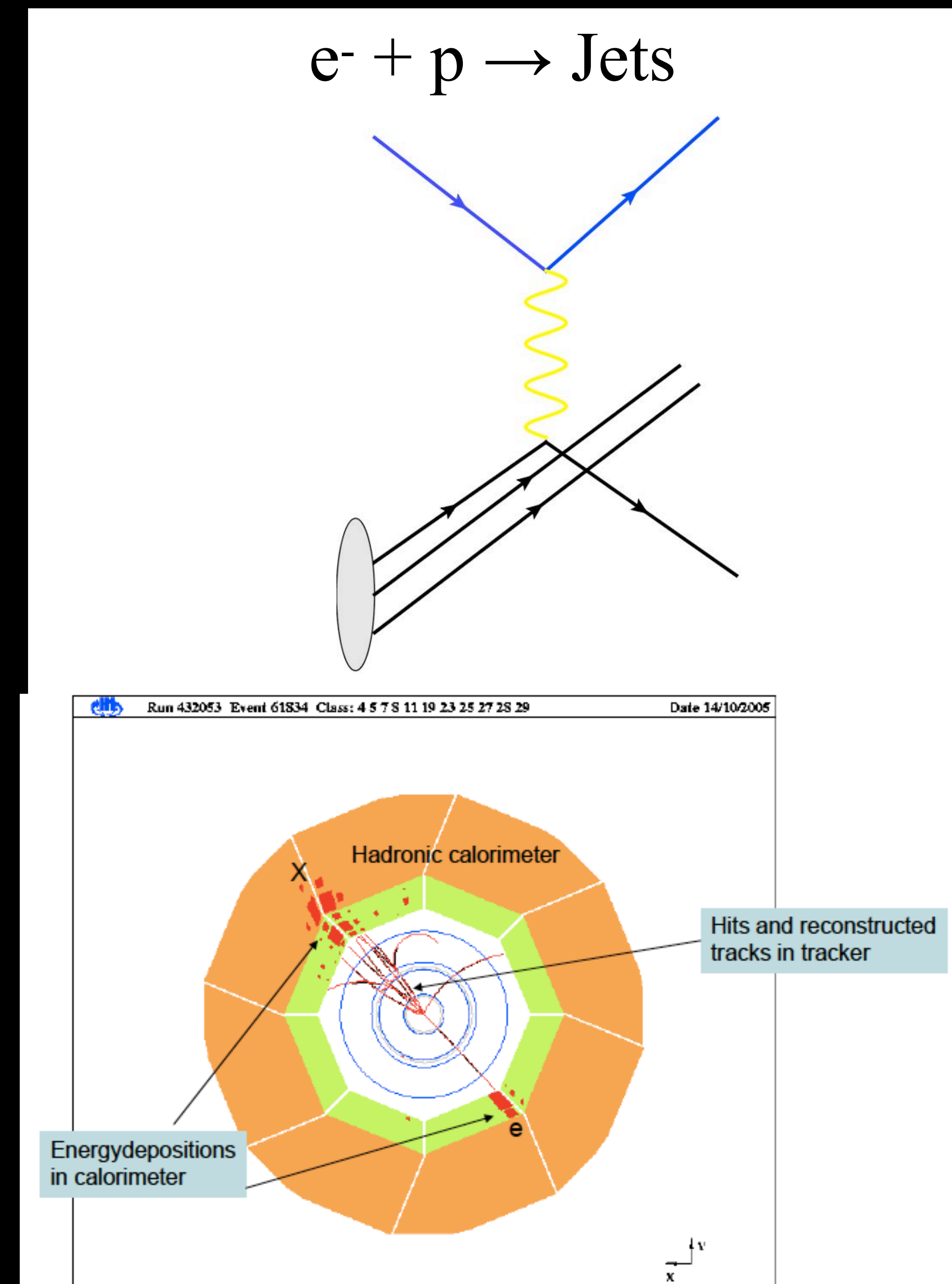
$$D(z) = \sum_X \frac{z^3}{2} \int \frac{dy^-}{2\pi} \langle P_h X|\bar{\psi}(y^-)|0\rangle \frac{\gamma^+}{2P_h^+} \langle 0|\psi|P_h X\rangle e^{-i\frac{P_h^+}{z}y^-}$$

A controlled expansion in coupling

$$d\sigma = d\sigma_0 + \alpha_S d\sigma_1 + \alpha_S^2 d\sigma_2 + \ldots$$

Can measure PDF in DIS, FF in $e^+ e^-$
Universality





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By their definition, G(x) and D(z)

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Is this true?

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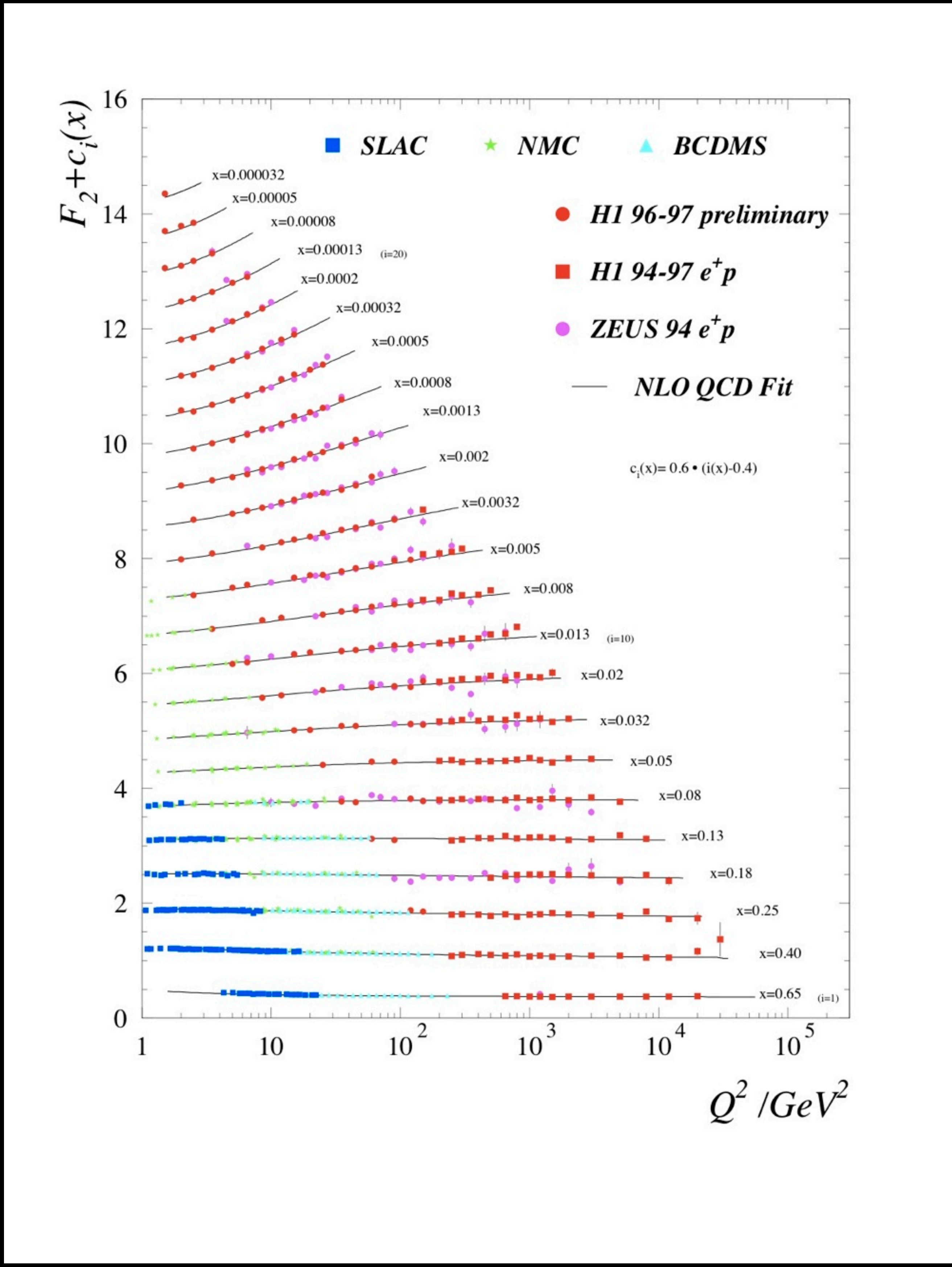

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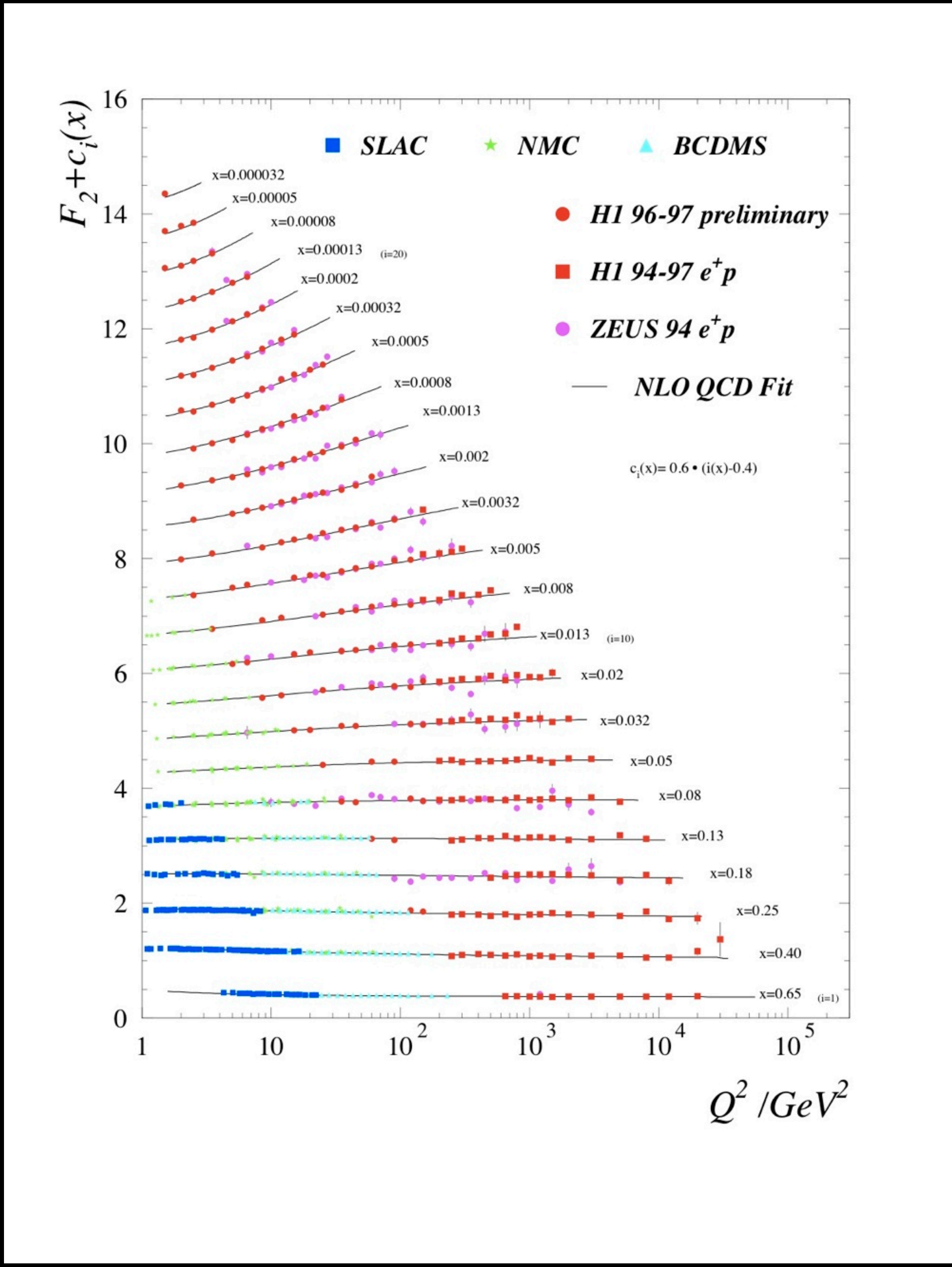
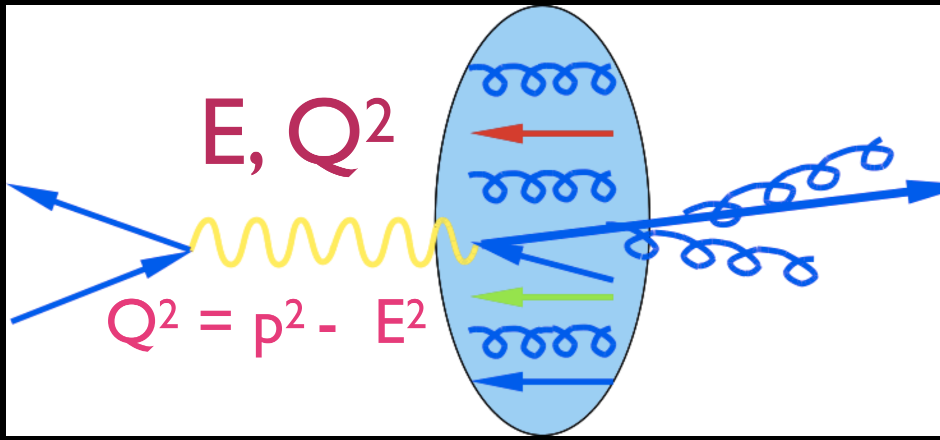
Why does the G(x) change?

higher order corrections!



What does this change in G(x) mean?

Increasing energy E = boosting nucleon
Increasing Q^2 = getting closer to nucleon



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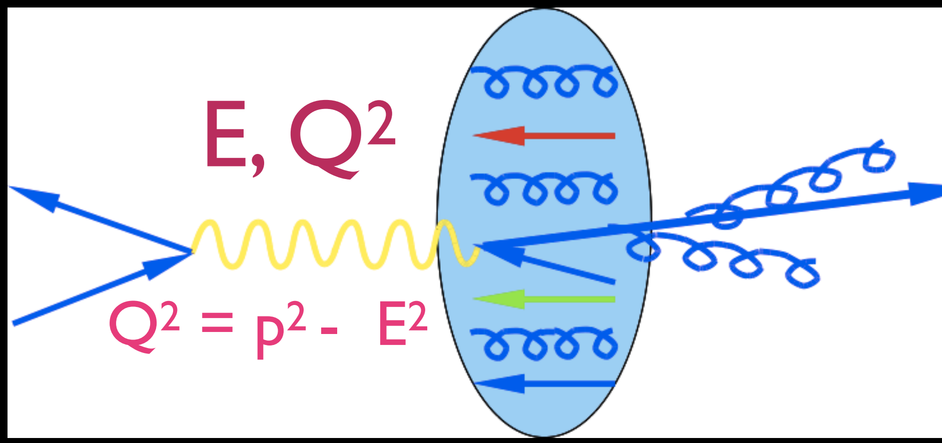

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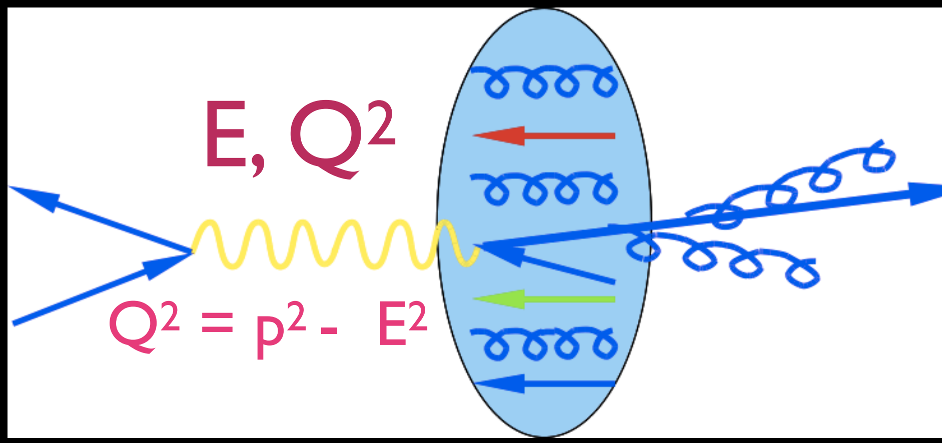

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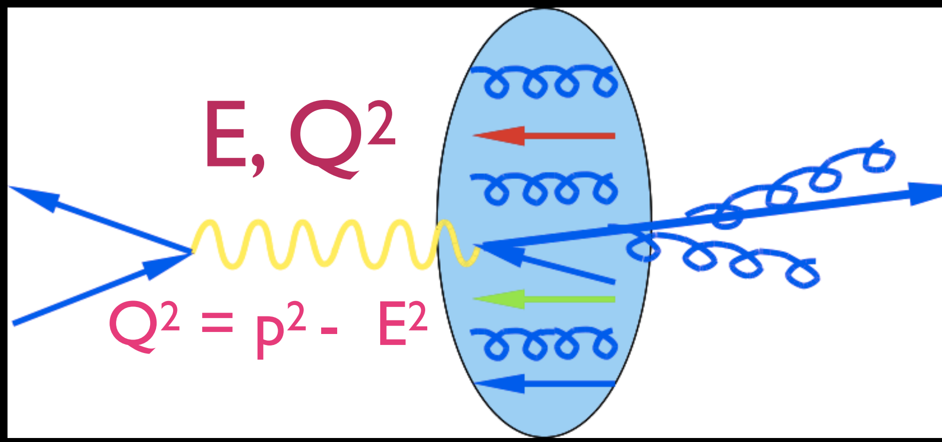

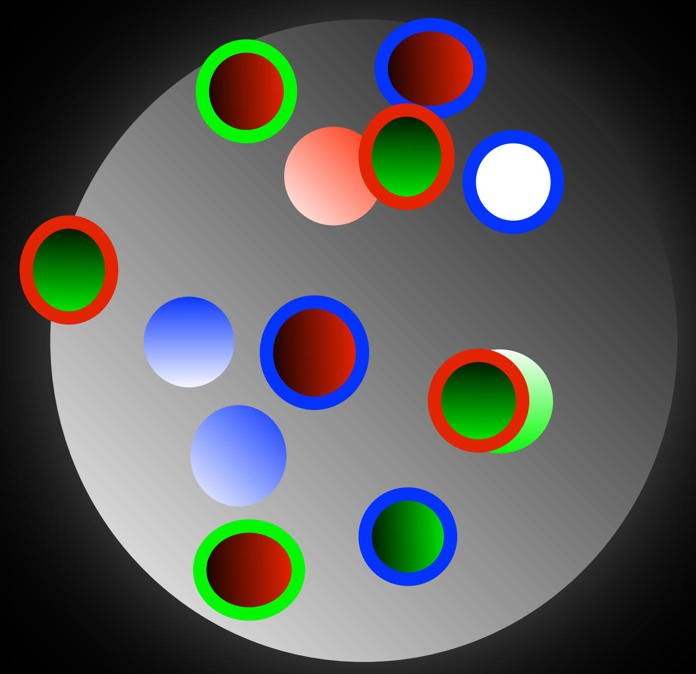

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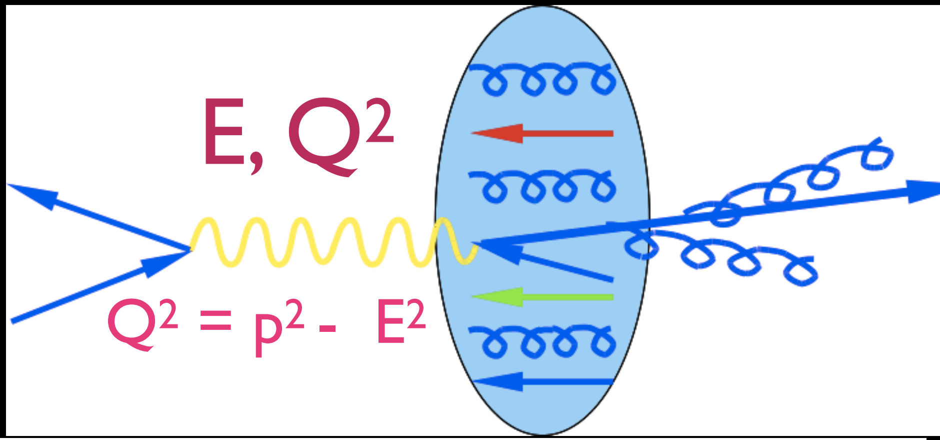

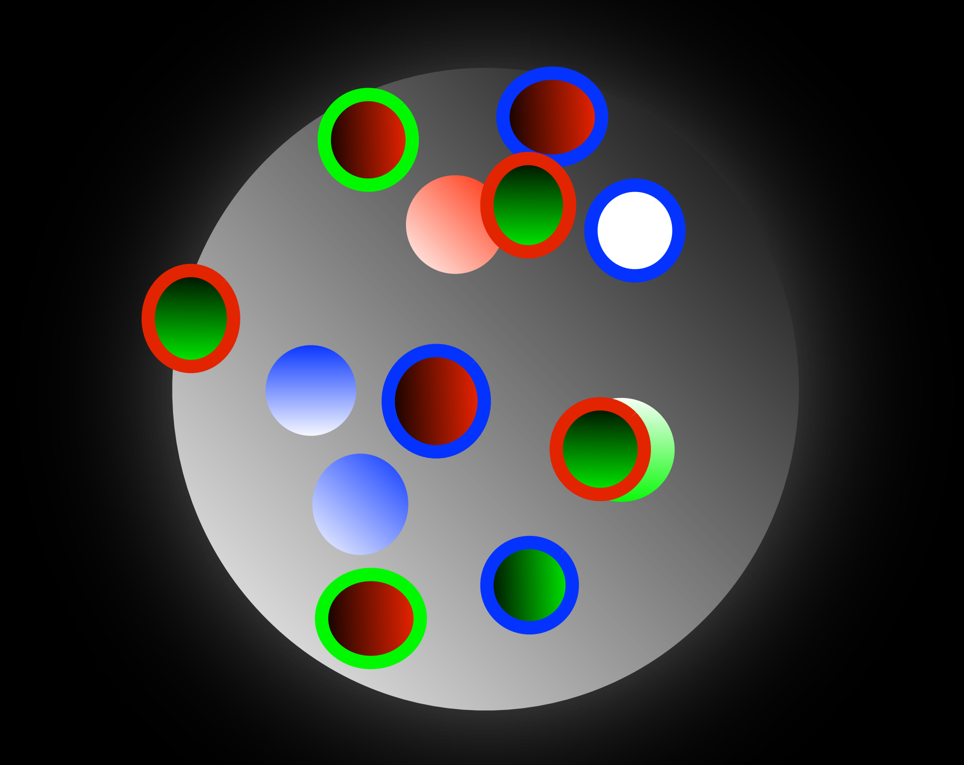

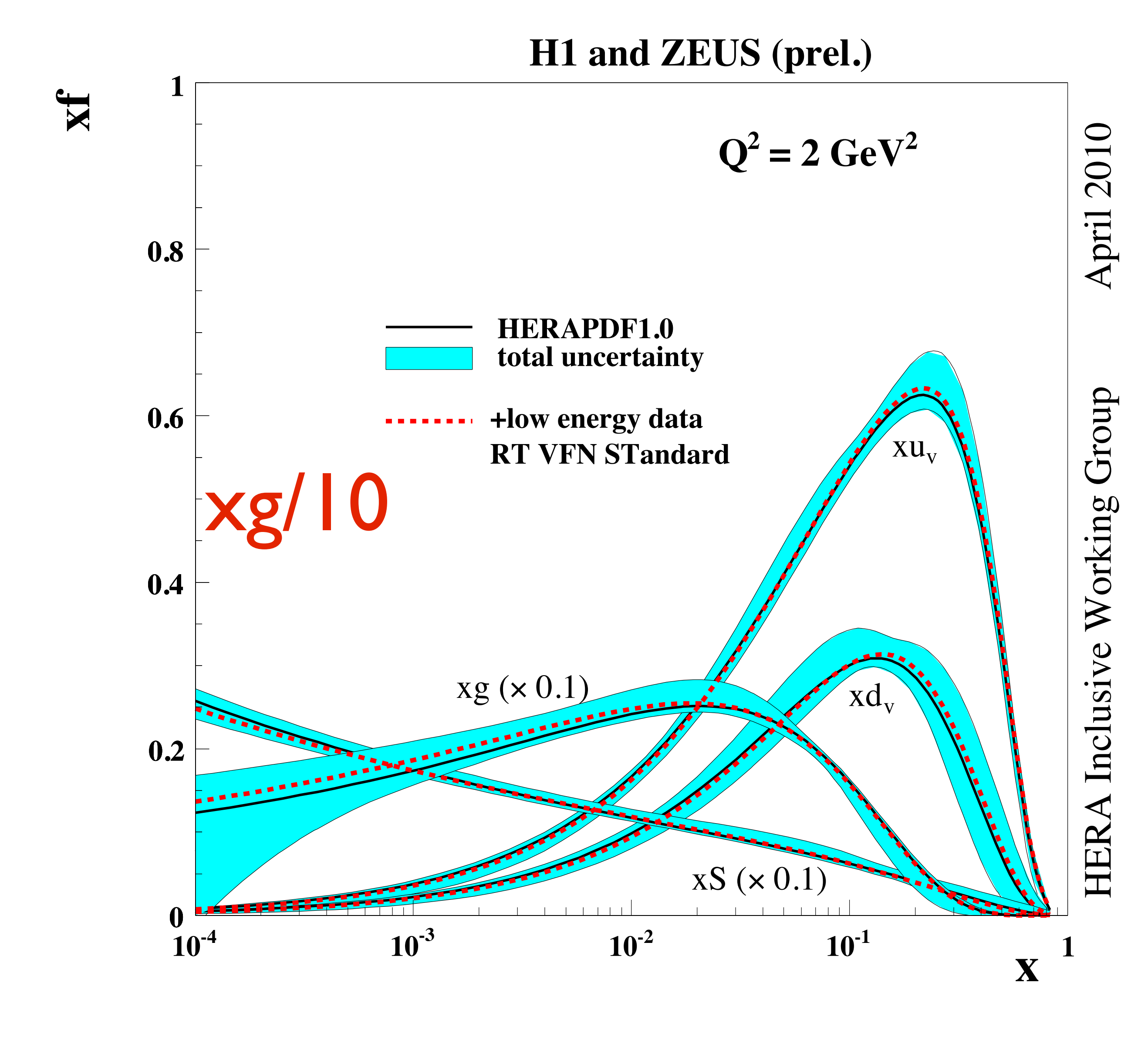

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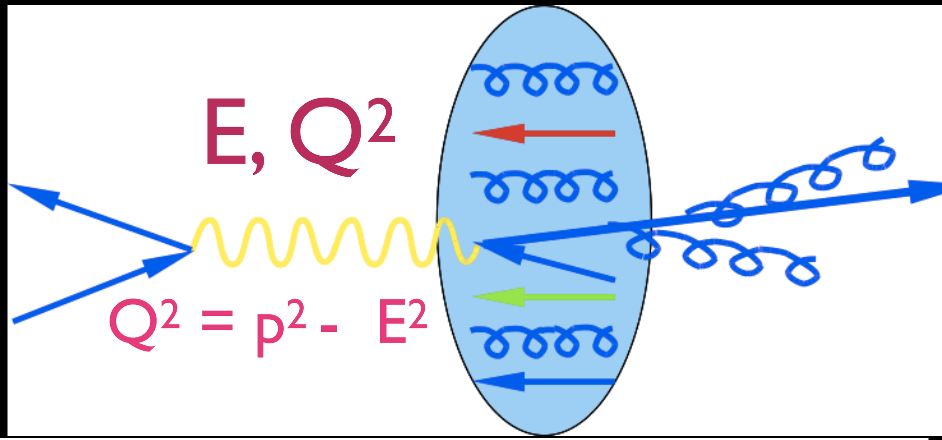

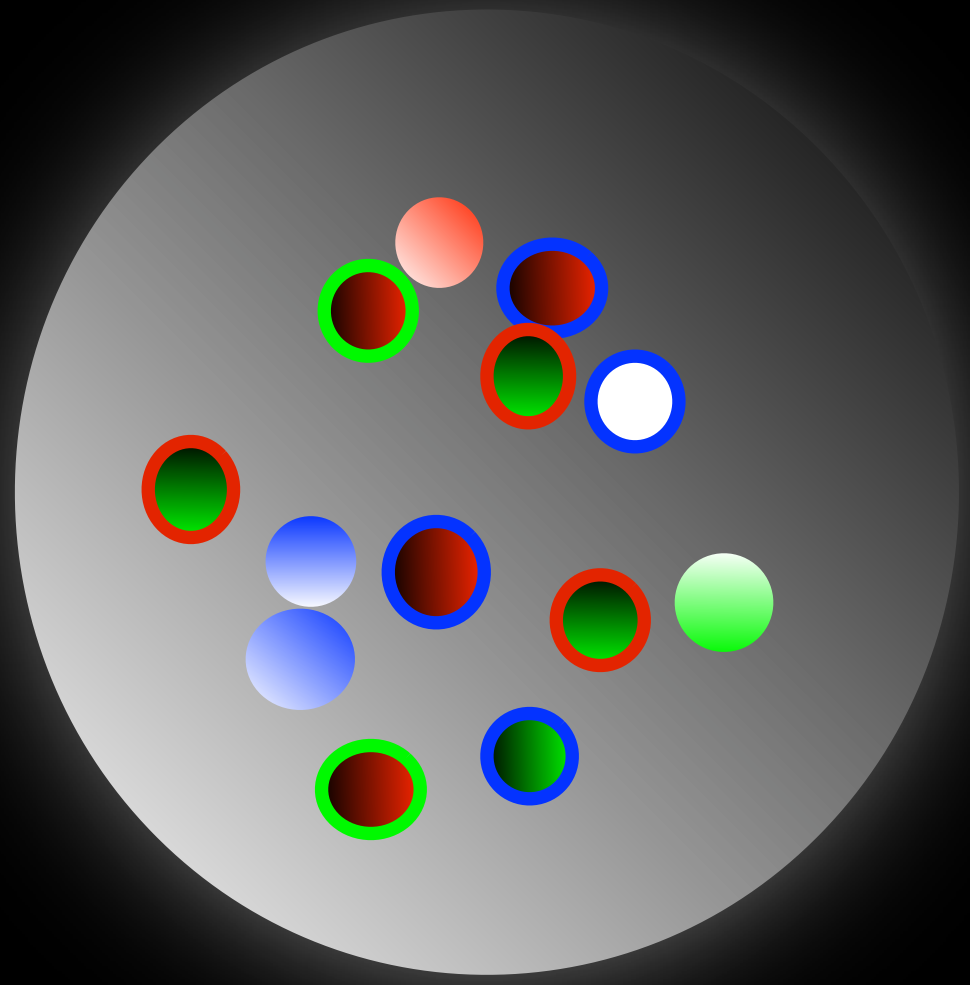

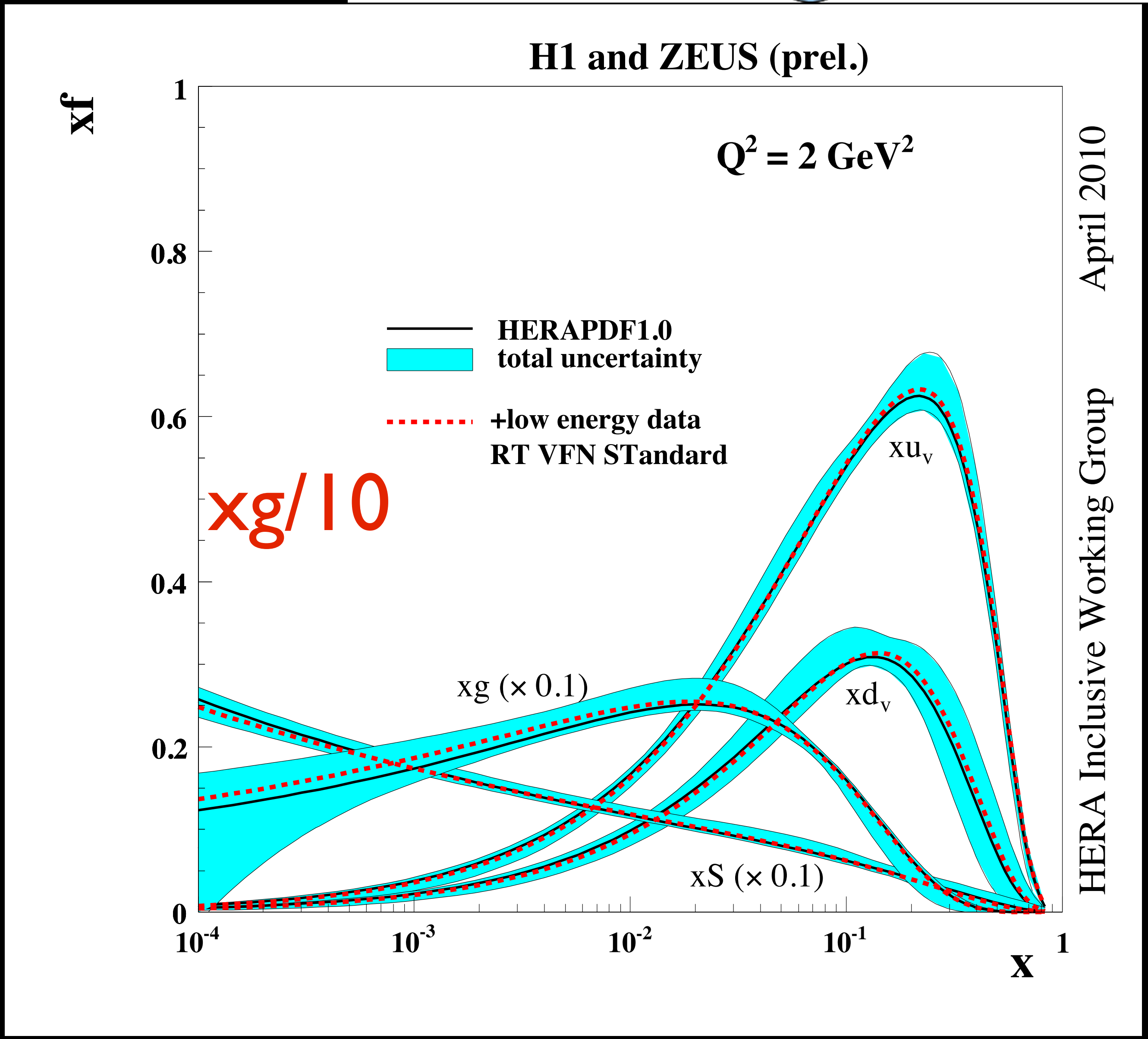

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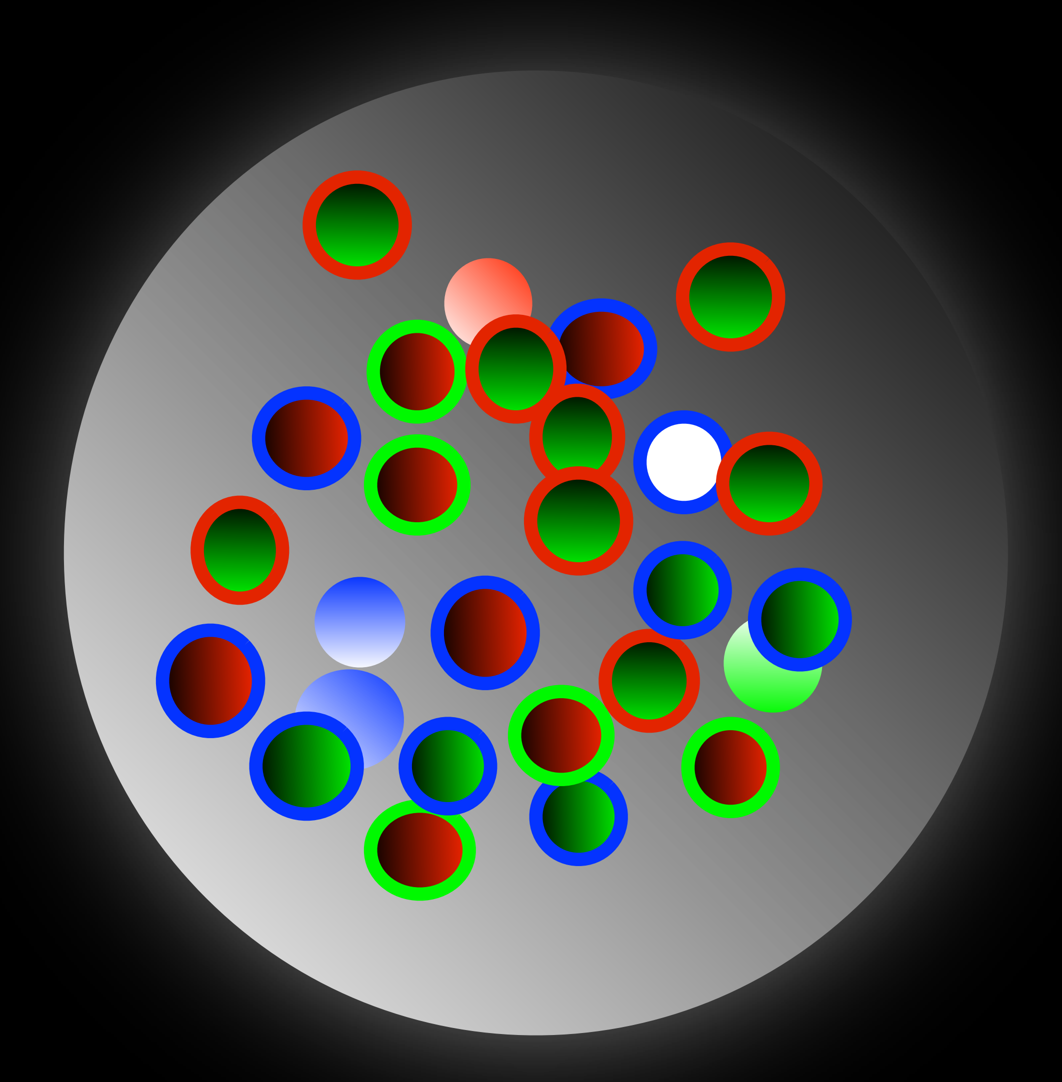

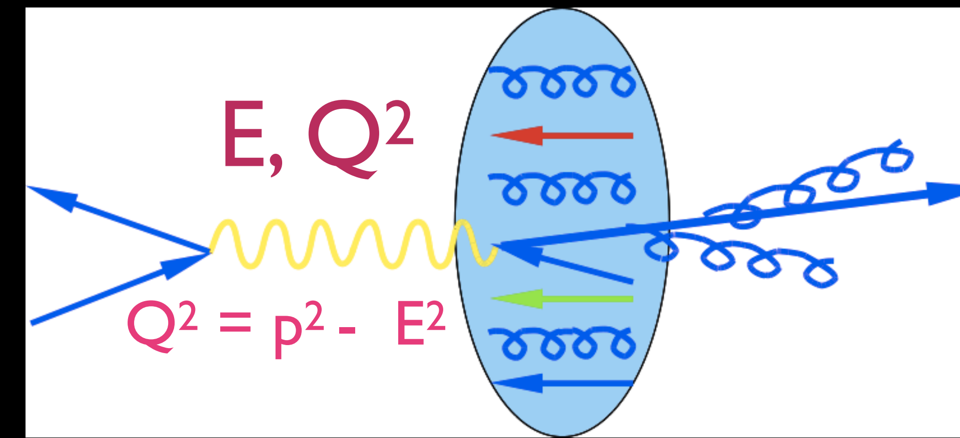

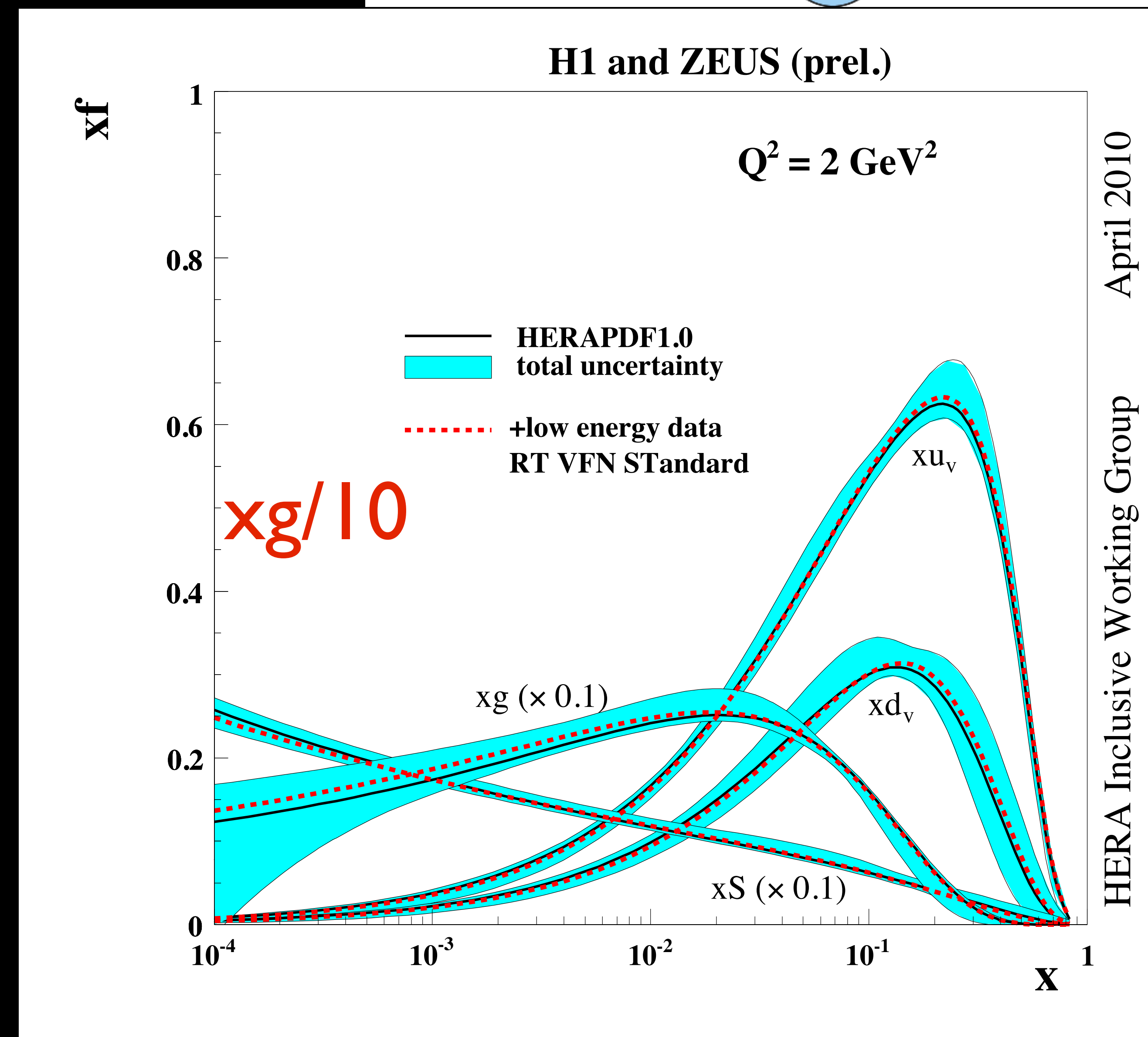

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Increasing energy E = boosting nucleon
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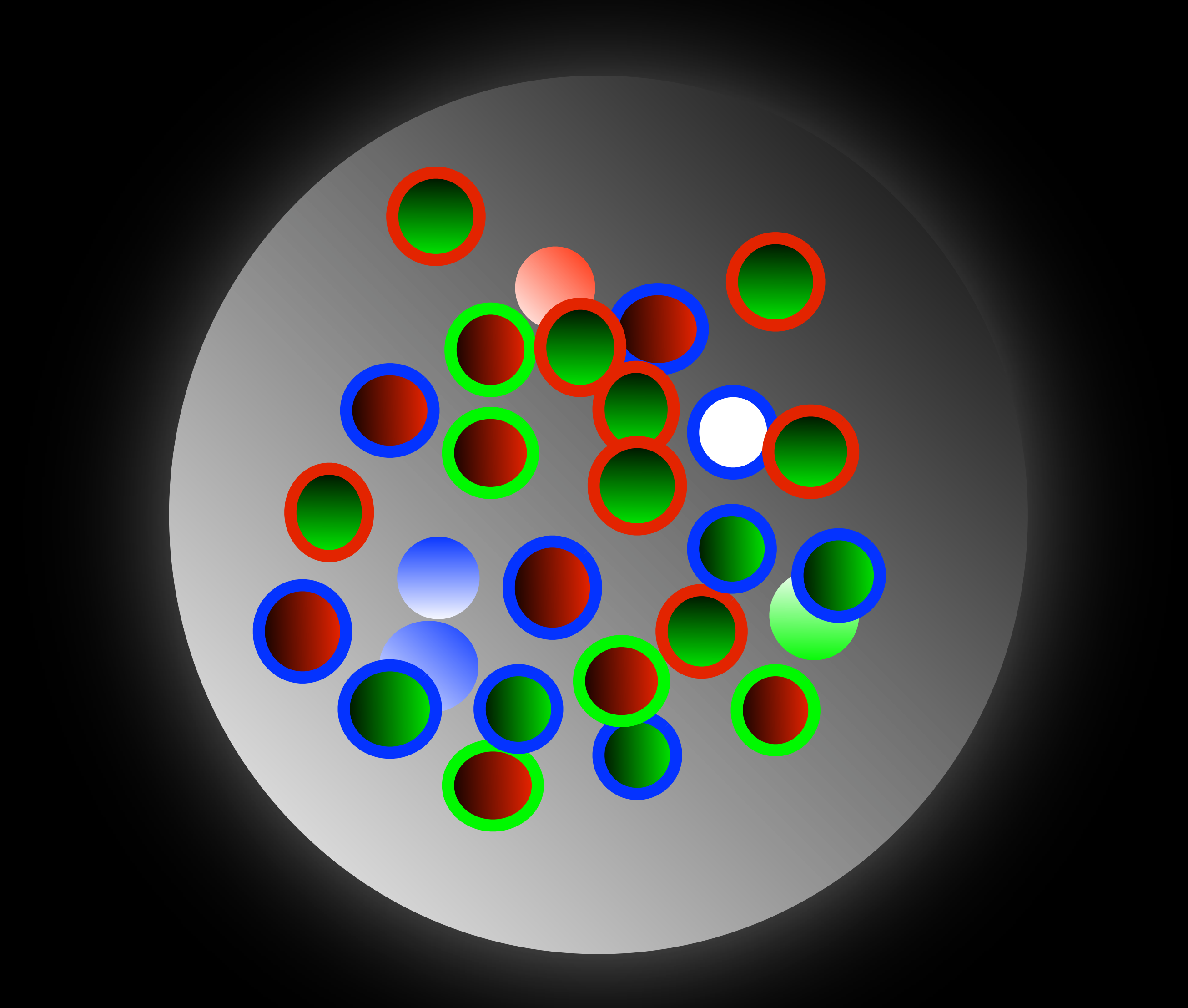
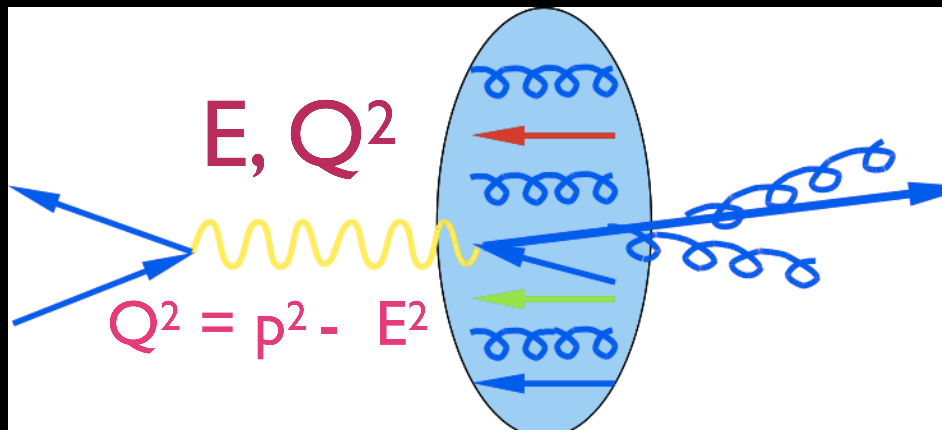
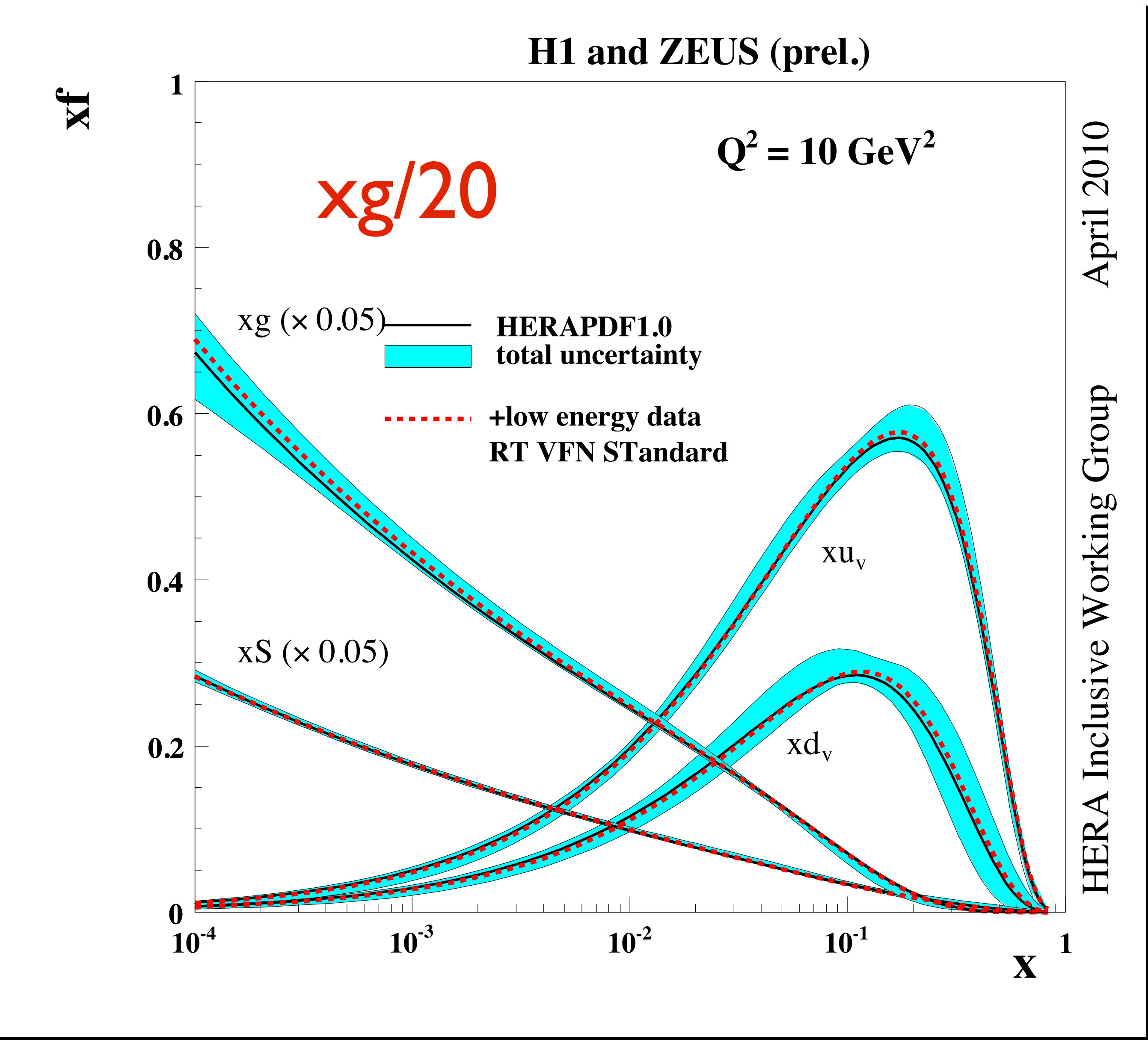


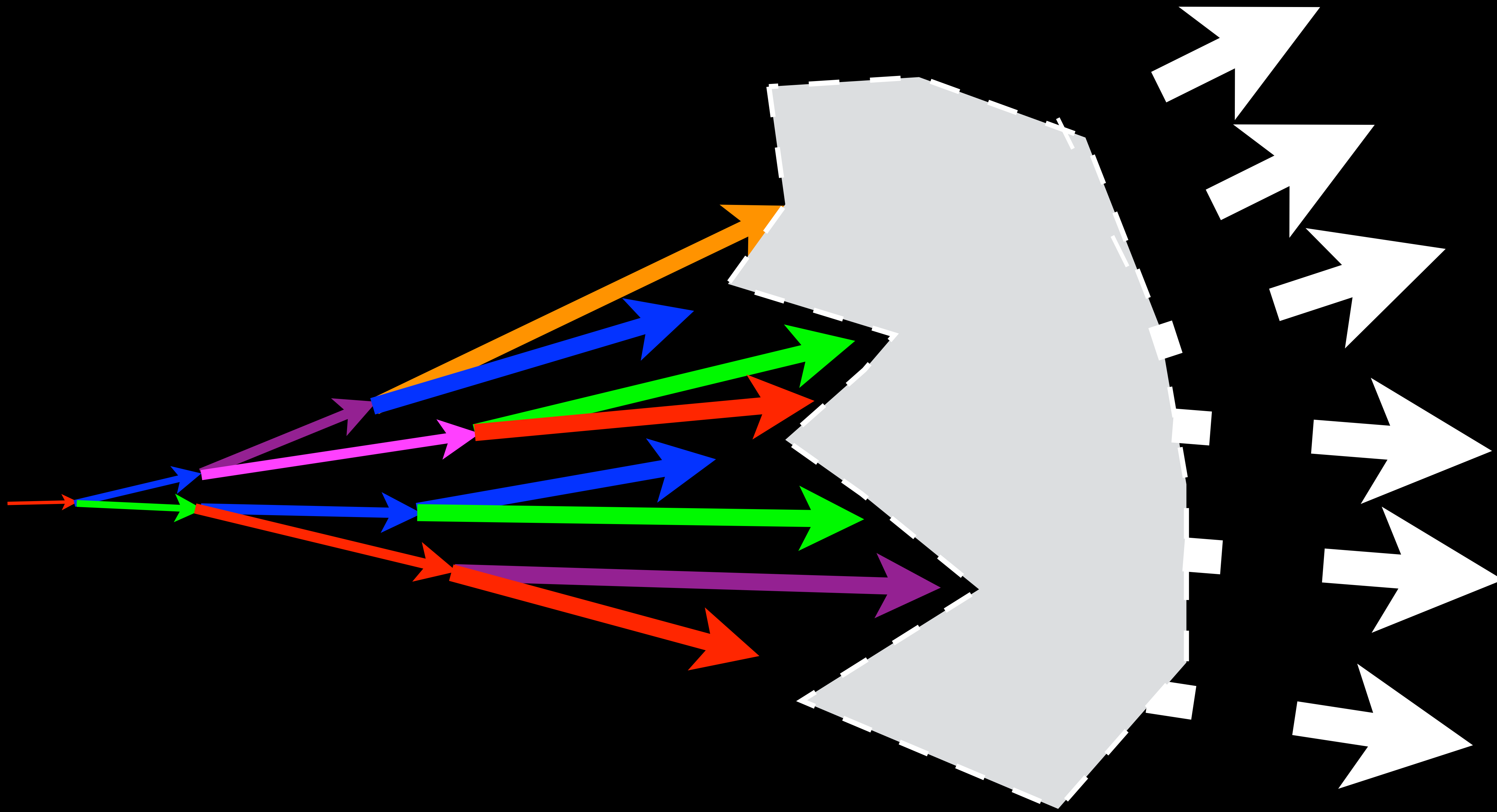




Fragmentation functions also evolve

Hard partons start out as small objects, which then grow as more splits happen

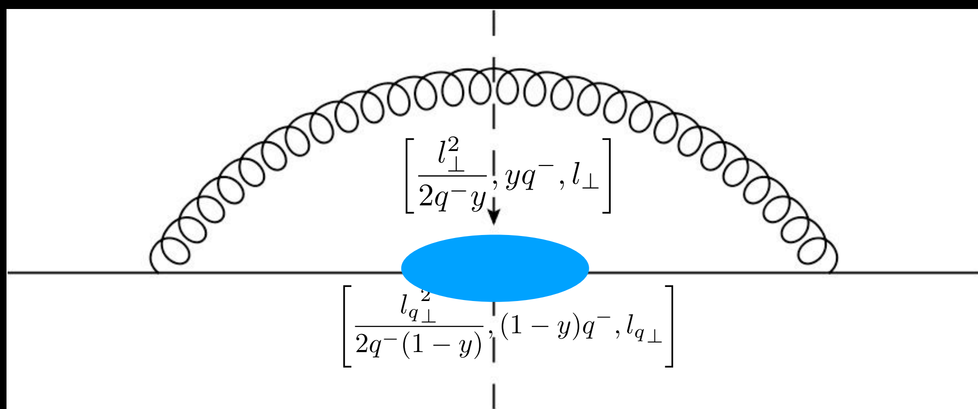
Fragmentation functions also evolve



Hard partons start out as small objects, which then grow as more splits happen

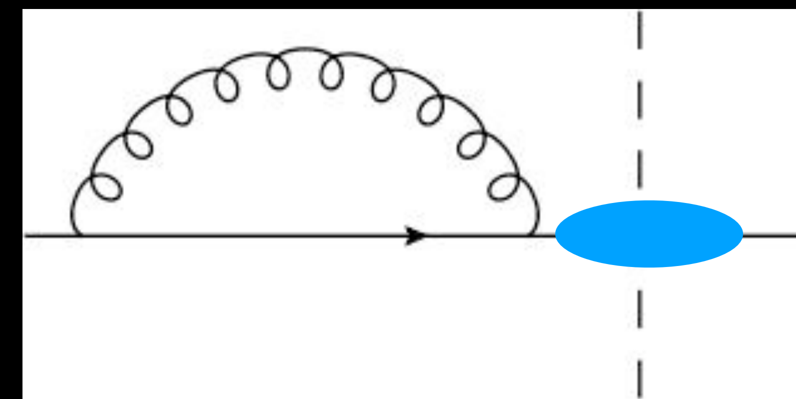
Calculate the probability of a split

Real emission



+

Virtual emission



$$\int_{\mu_0^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \frac{\alpha_S}{2\pi} \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}\right) - \int_{\mu_0^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \frac{\alpha_S}{2\pi} D(z) \int_0^1 dy P(y)$$

Increase of probability, from a new process

Decrease of the probability of nothing happening

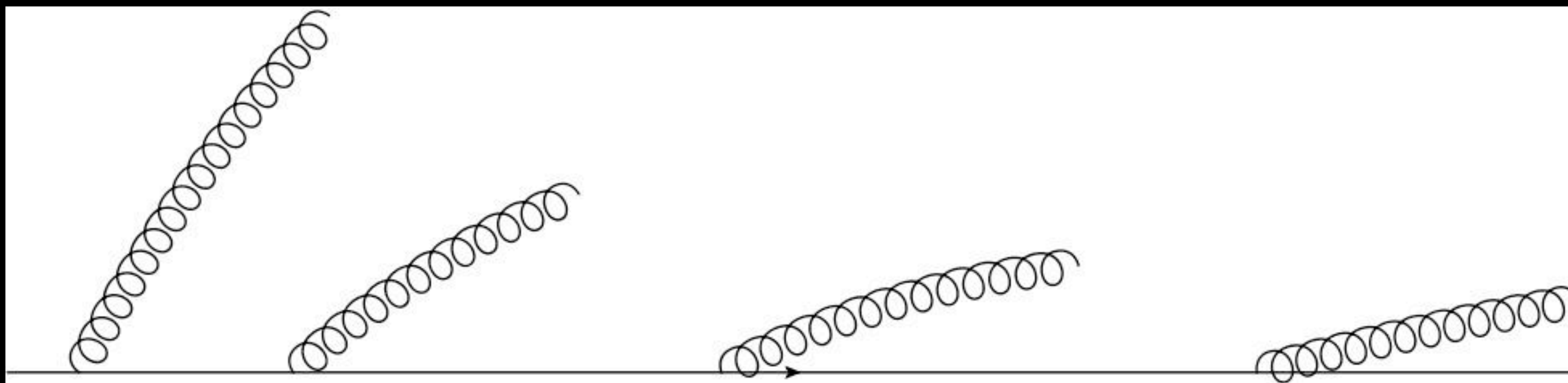
P(y) is singular at y = 1

Both terms together are not

$$= \int_{\mu_0^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \frac{\alpha_S}{2\pi} \int_z^1 \frac{dy}{y} P(y)_+ D\left(\frac{z}{y}\right)$$

Multiple emissions by iteration

Leading term is one with ordered radiations



$$\frac{\alpha_s}{2\pi}\int_{\mu_0^2}^{\mu^2}\frac{dl_{0,\perp}^2}{l_{0,\perp}^2}\int P(y_0)\frac{\alpha_s}{2\pi}\int_{\mu_0^2}^{l_{0,\perp}^2}\frac{dl_{1,\perp}^2}{l_{1,\perp}^2}\int P(y_1)\frac{\alpha_s}{2\pi}\int_{\mu_0^2}^{l_{1,\perp}^2}\frac{dl_{2,\perp}^2}{l_{2,\perp}^2}\int P(y_2)\cdots\frac{D(z/(y_0y_1\ldots))}{y_0y_1\ldots}$$

The effect of an arbitrary number of emissions (the singular or leading log term) can be obtained using an integro-differential equation called the DGLAP equation

$$\frac{dD(z,\mu^2)}{dlog\mu^2} = \frac{\alpha_S}{2\pi}\int_z^1\frac{dy}{y}P_{q\to q}(y)D\left(\frac{z}{y},\mu^2\right)$$

Note: large drops in perp. momentum mean large drops in angle, thus no interference between subsequent radiations

Inclusivity and the scale μ

Both G(x) and D(z) are inclusive distributions

The number of partons (hadrons) with fraction x(z) and …

$G(x,\mu^2)$ = all processes up to the scale μ^2

At leading order, any choice of scale near the hard scale is allowed

$\mu = p_T$, $p_T/2$ *etc.*

The exact value is a fitting parameter.

From DGLAP to Sudakov

- If event averages of single particle yields is all you want, you don't need to do MC event generation
- To simulate actual event, we have to go from an inclusive to an exclusive calculation
- We need the probability of no emission
- Then we can construct states with exactly n emissions
- The Sudakov form factor …

$\Delta(\mu_1^2,\mu_2^2)$

- Probability that a parton with virtuality = μ_1^2, will transition to a parton with lower virtuality = μ_2^2, via "unresolvable" emissions.

$$\Delta(\mu_1^2,\mu_2^2) = \Delta(\mu_1^2,\mu_3^2)\Delta(\mu_3^2,\mu_2^2)$$

$$\Delta(\mu^2,\mu^2) = 1$$

$$\Delta(\mu^2+\delta\mu^2,\mu^2) = 1 - \frac{d\mu^2}{\mu^2}\frac{\alpha_S}{2\pi}\int_{\frac{\mu_0^2}{\mu^2}}^{1-\frac{\mu_0^2}{\mu^2}} dyP(y)$$

For a small enough range $\delta\mu^2$, probability for one emission dominates
Probability of no emission = 1 - probability of one emission

$\Delta(Q^2,\mu_0{}^2)$

$$d\Delta(\mu^2,\mu_0^2) = \Delta(\mu^2+\delta\mu^2,\mu_0^2) - \Delta(\mu^2,\mu_0^2)$$

$$= -\frac{d\mu^2}{\mu^2}\left[\frac{\alpha_S}{2\pi}\int_{\frac{\mu_0^2}{\mu^2}}^{1-\frac{\mu_0^2}{\mu^2}} dyP(y)\right]\Delta(\mu^2,\mu_0^2)$$

$$\int_1^{\Delta(Q^2,\mu_0^2)}\frac{d\Delta}{\Delta} = -\int_{\mu_0^2}^{Q^2}\frac{d\mu^2}{\mu^2}\frac{\alpha_S}{2\pi}\int_{\frac{\mu_0^2}{\mu^2}}^{1-\frac{\mu_0^2}{\mu^2}} dyP(y)$$

$$\Delta(Q^2,\mu_0^2) = e^{-\int_{\mu_0^2}^{Q^2}\frac{d\mu^2}{\mu^2}\frac{\alpha_S}{2\pi}\int_{\frac{\mu_0^2}{\mu^2}}^{1-\frac{\mu_0^2}{\mu^2}} dyP(y)}$$

Sample the Sudakov to determine virtuality

- Parton starts with a maximum allowable virtuality Q^2

- The first resolvable split takes place from a parton with virtuality μ^2.

- = Probability of no resolvable splits between Q^2 and μ^2 .

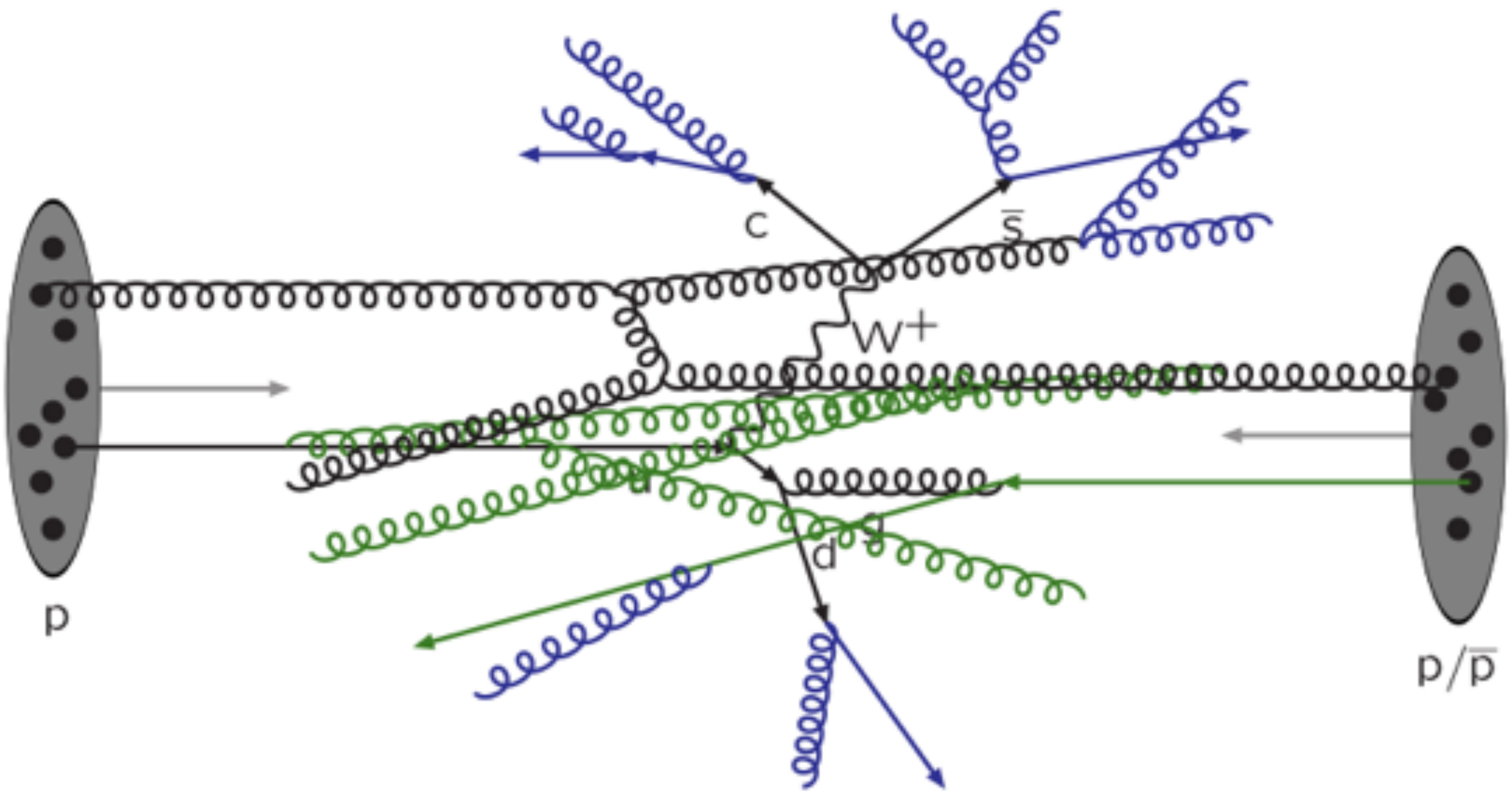
$$= \Delta(Q^2, \mu^2) = \frac{\Delta(Q^2, \mu_0^2)}{\Delta(\mu^2, \mu_0^2)} = r$$

- Then sample the splitting function to generate the momentum fraction y

- Next partons start at $y\mu$ and $(1-y)\mu$. Repeat till all partons at μ_0.

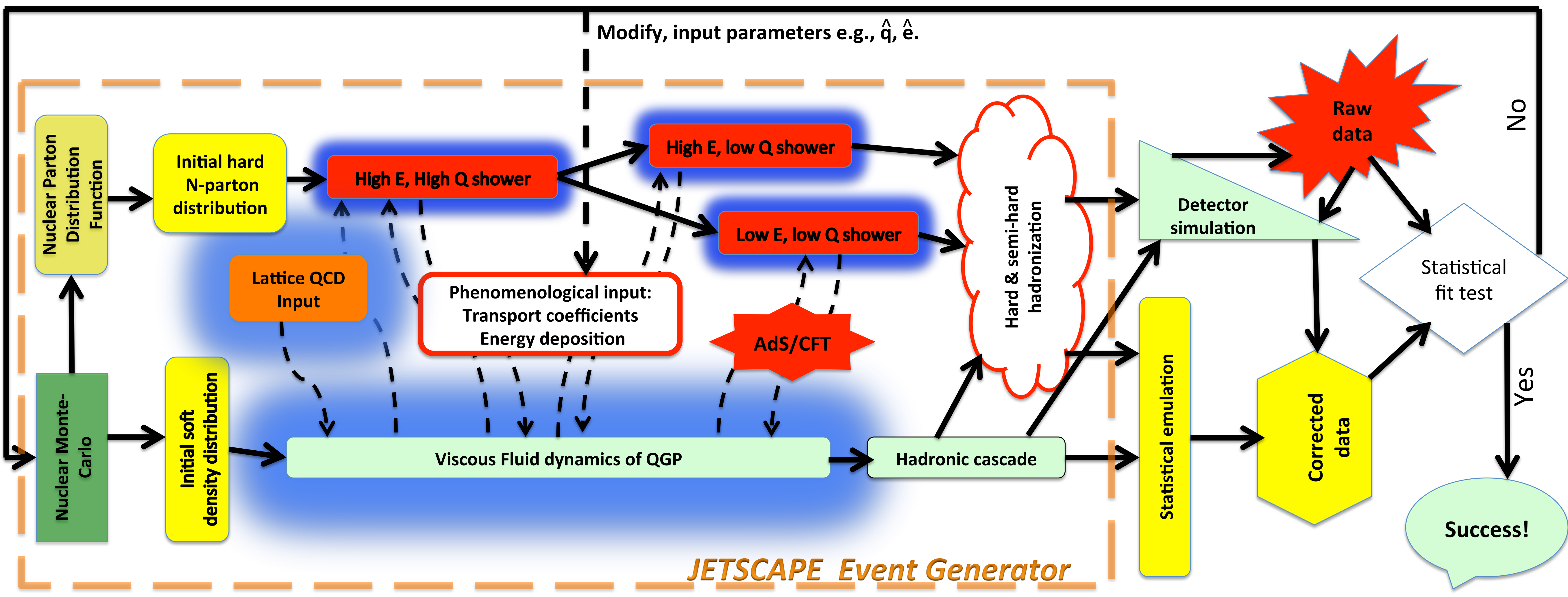
Sample initial and final states

PYTHIA generates showers (radiation) in the initial and final states
Each is an exclusive state
Initial state —> Multiple particle hard scatterings —> final state
Each is factorized and generated independently



Multiple partonic interactions (MPIs) with accompanying radiation.

What we do in JETSCAPE



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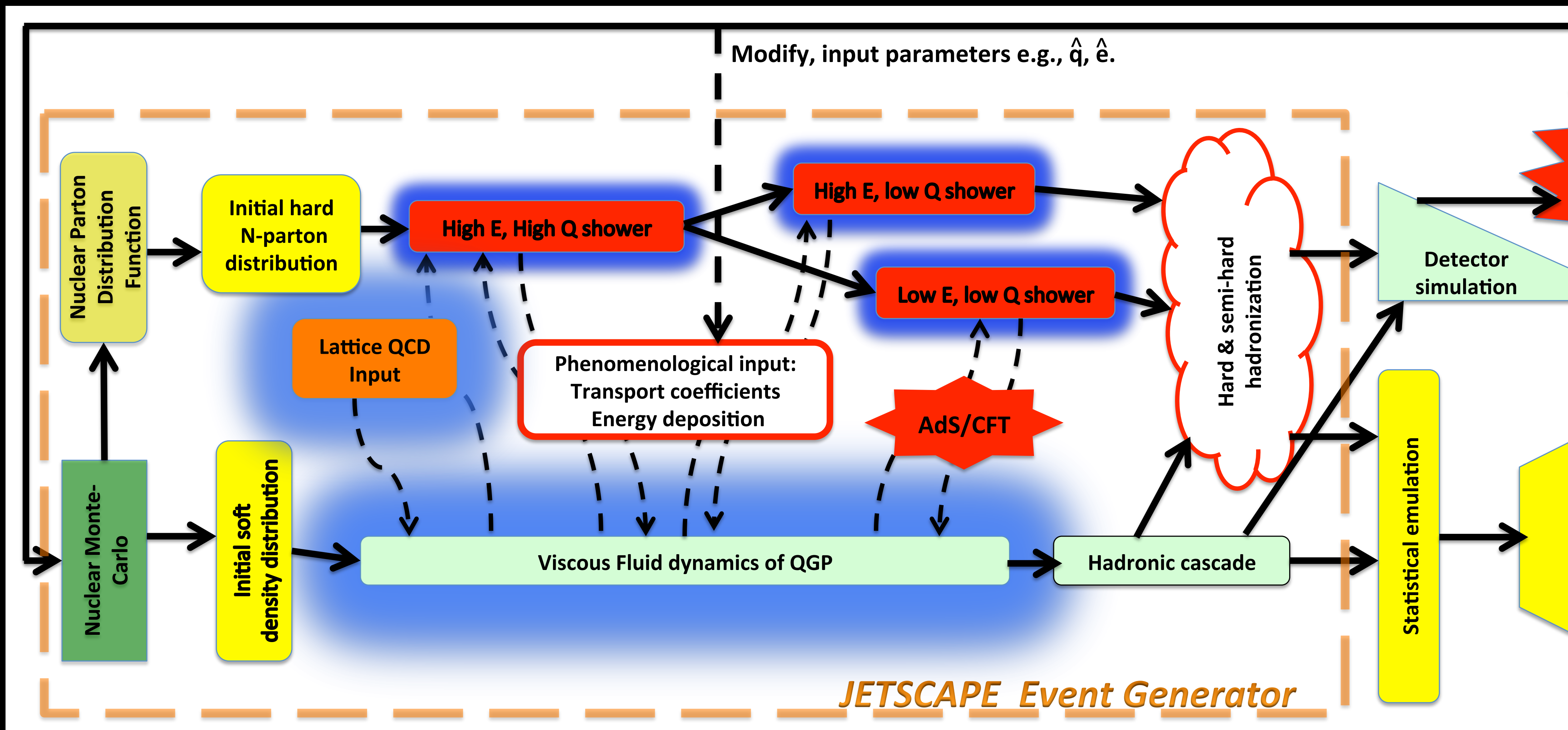

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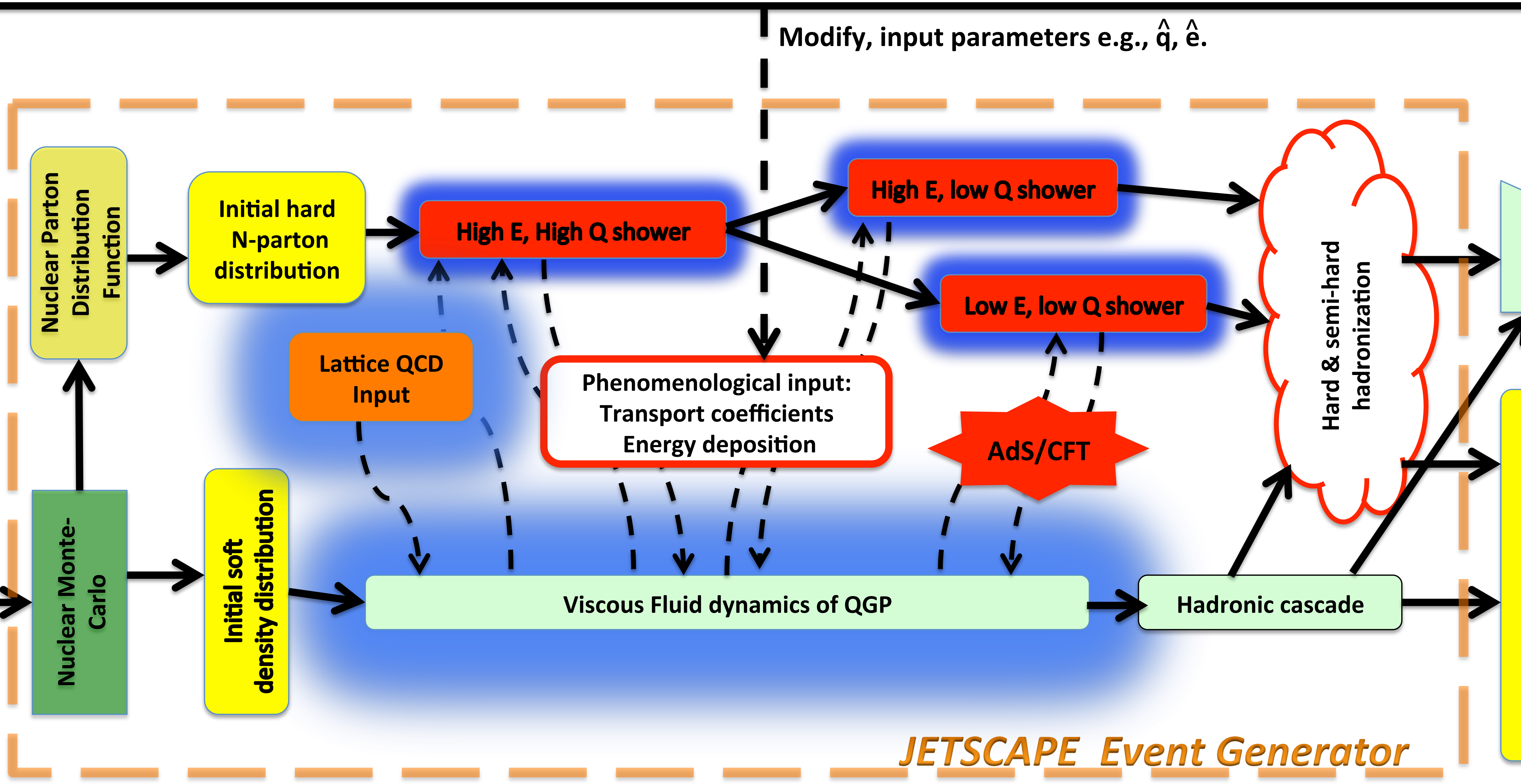

Extra focus on jets in an evolving medium

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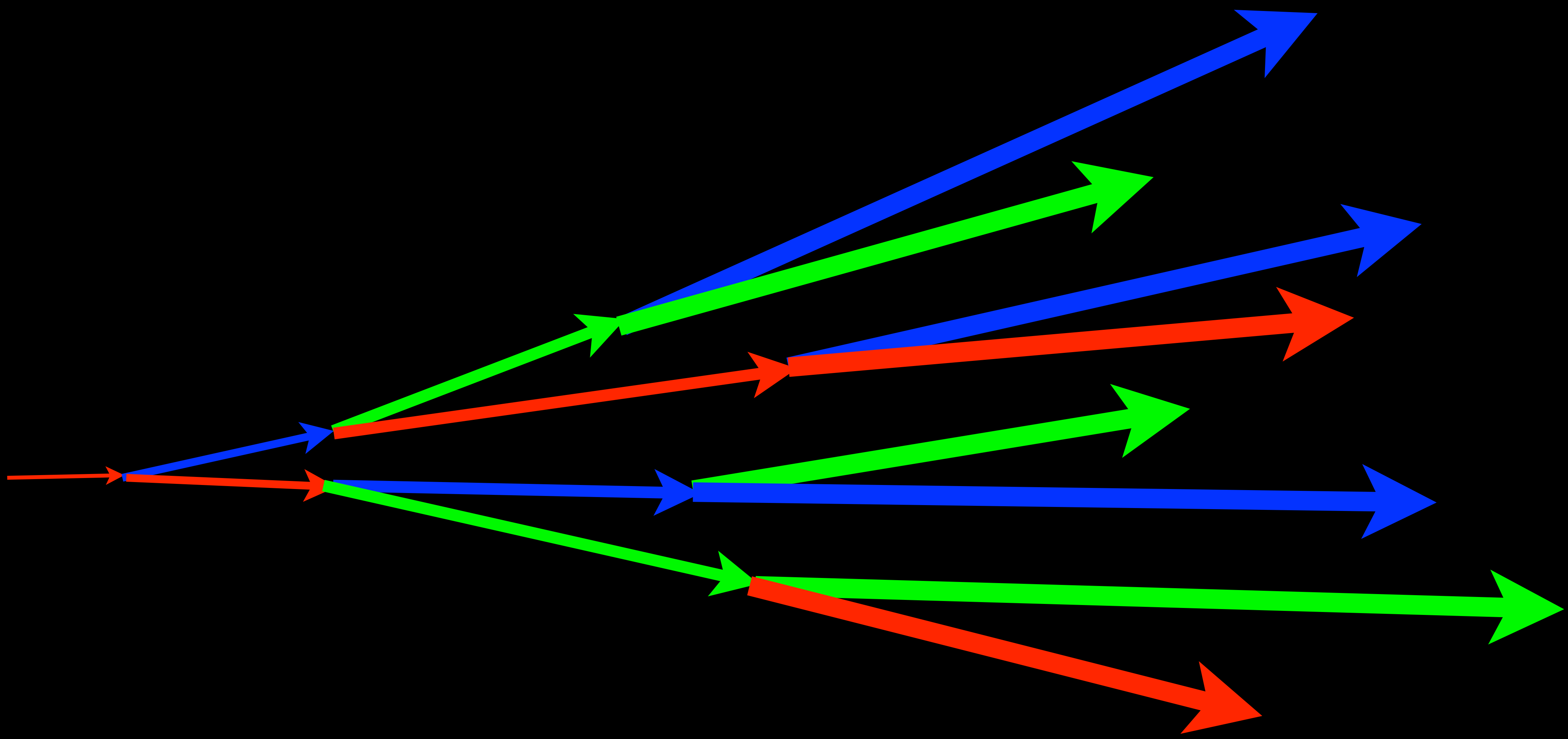

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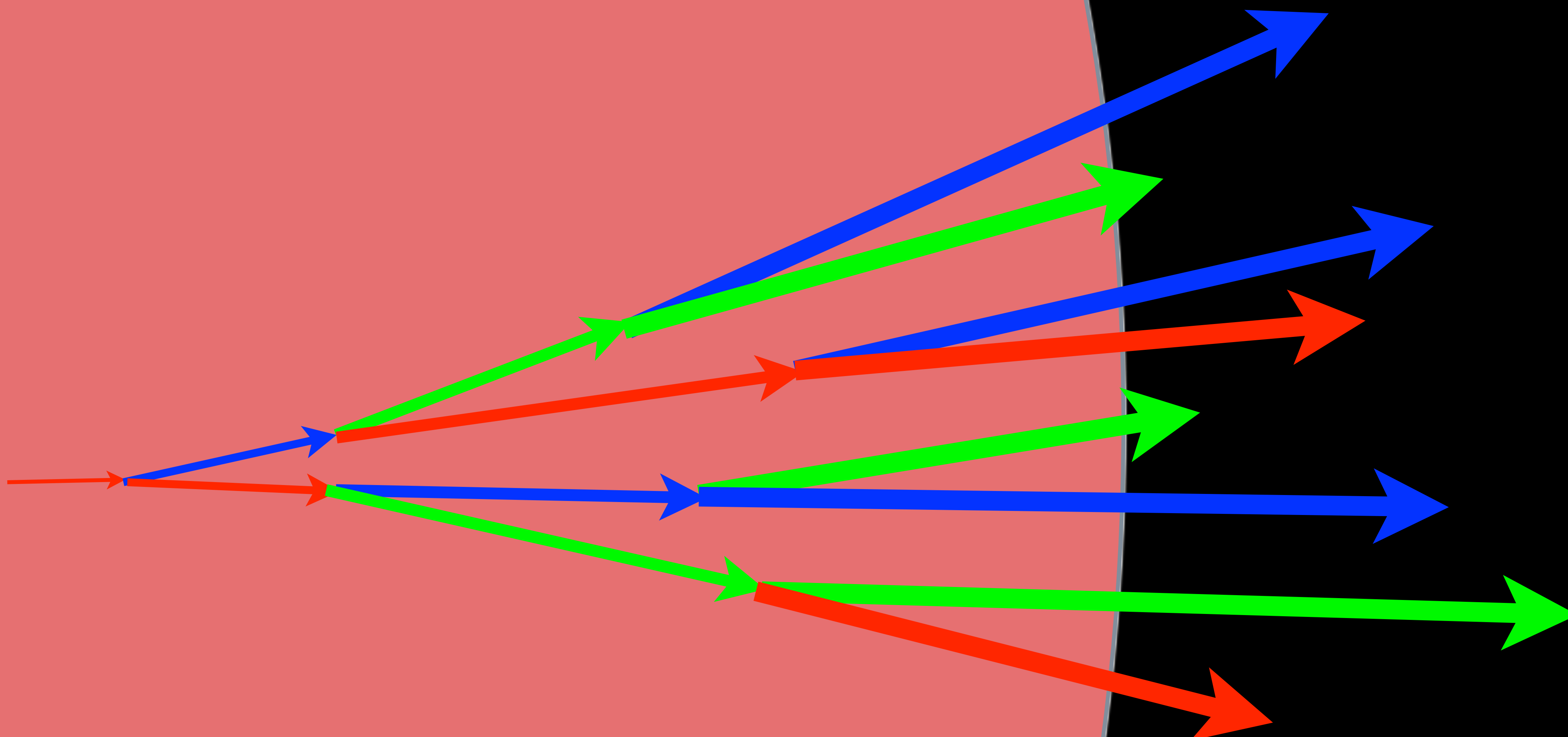

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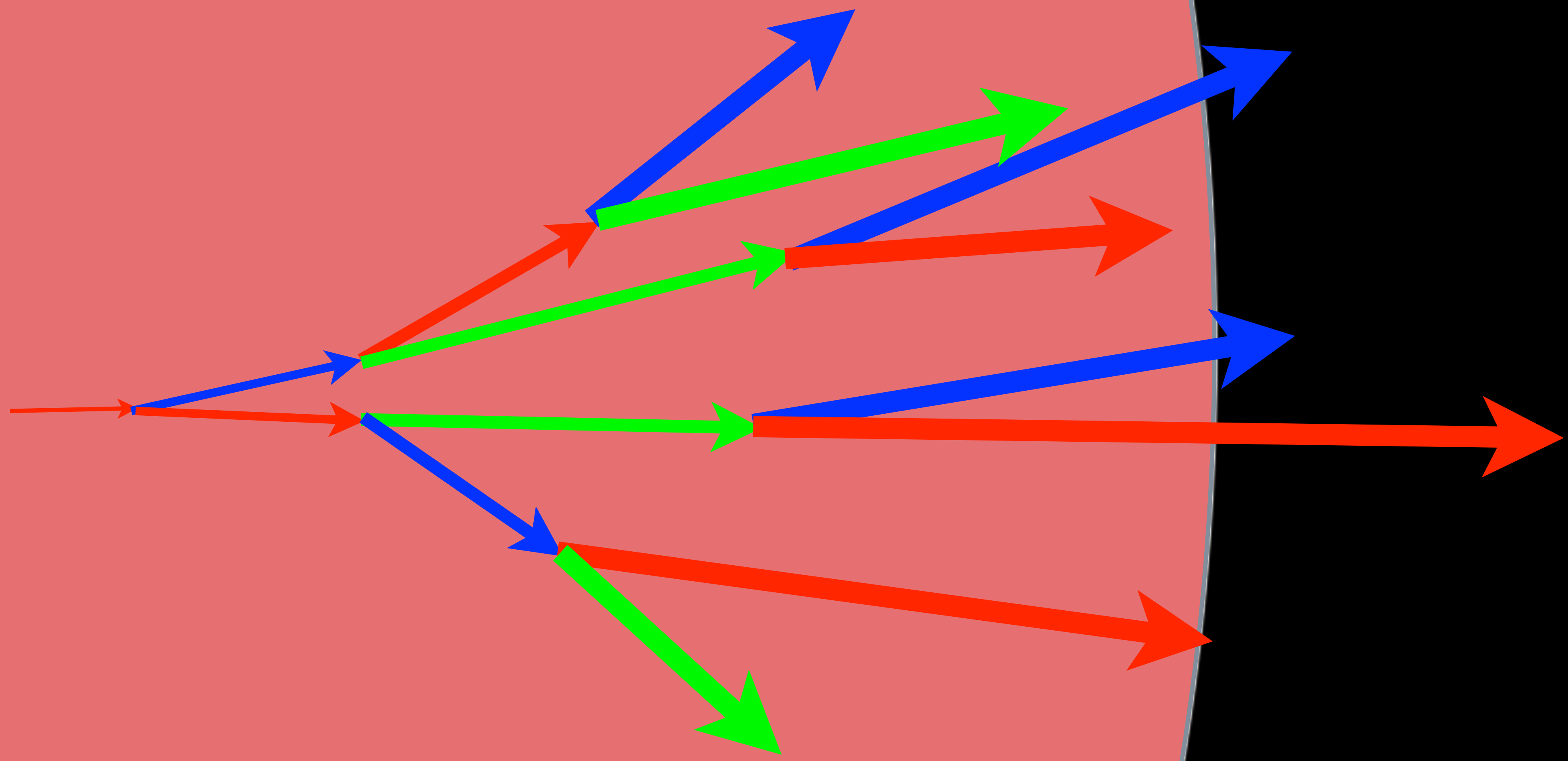

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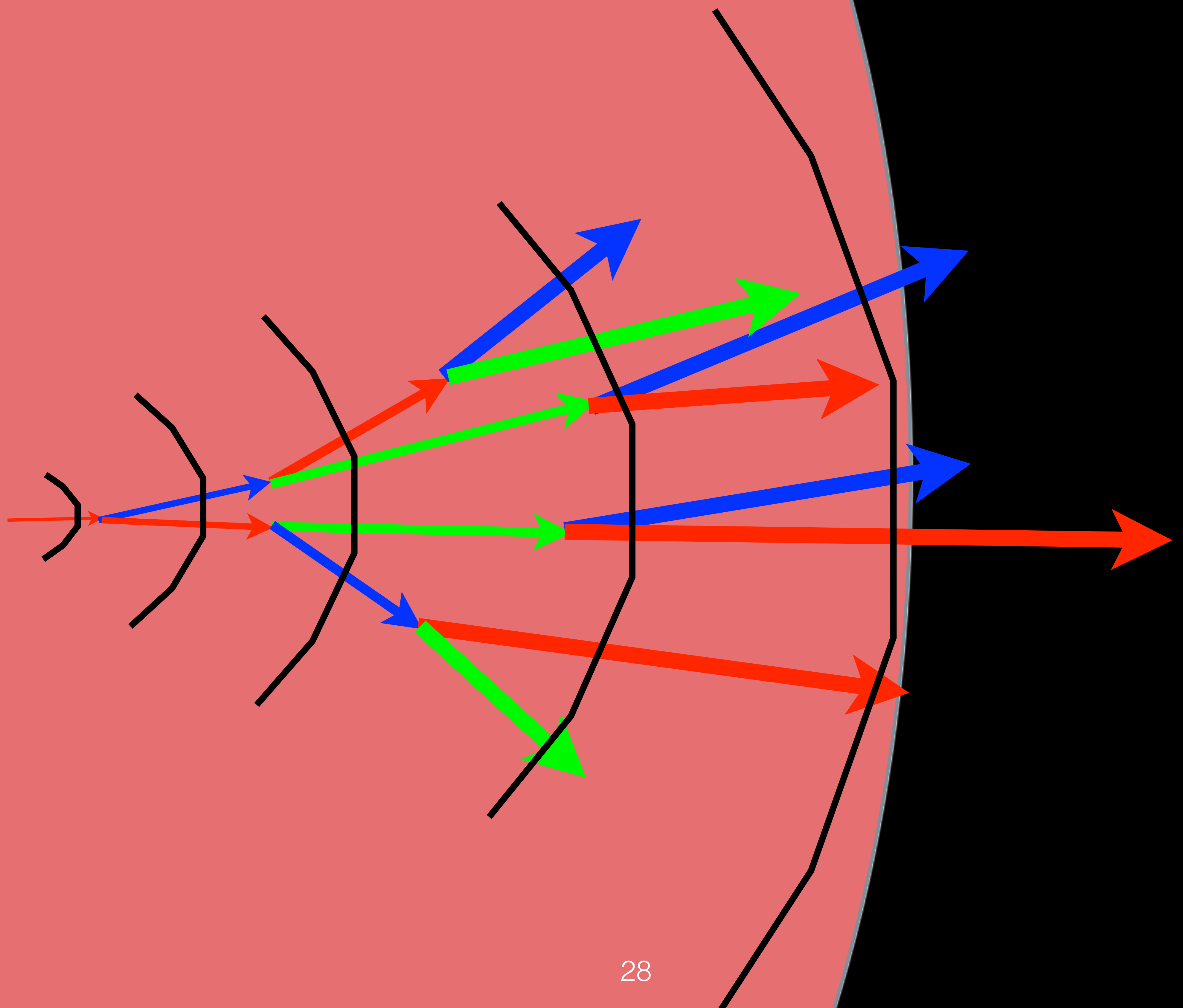

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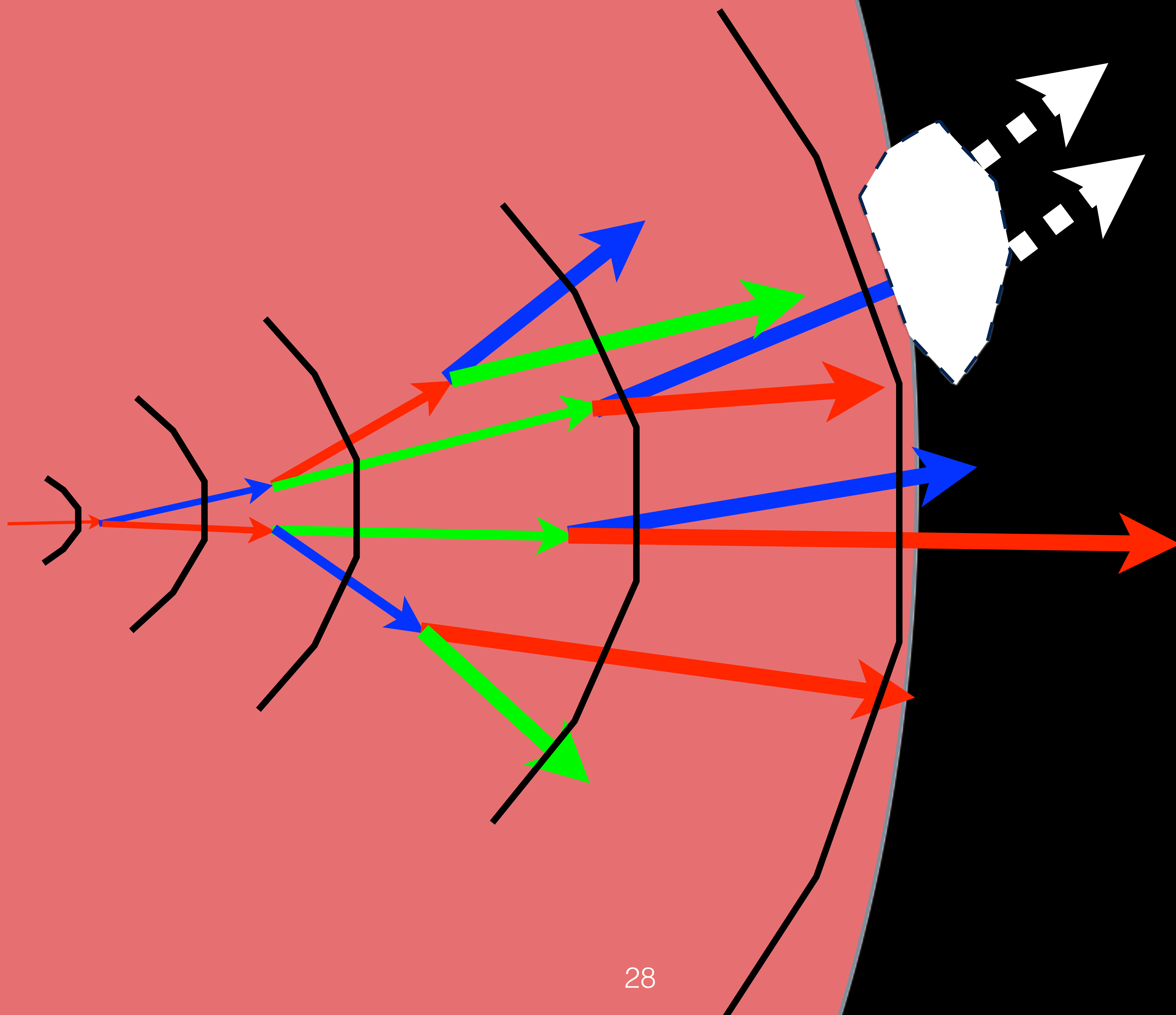

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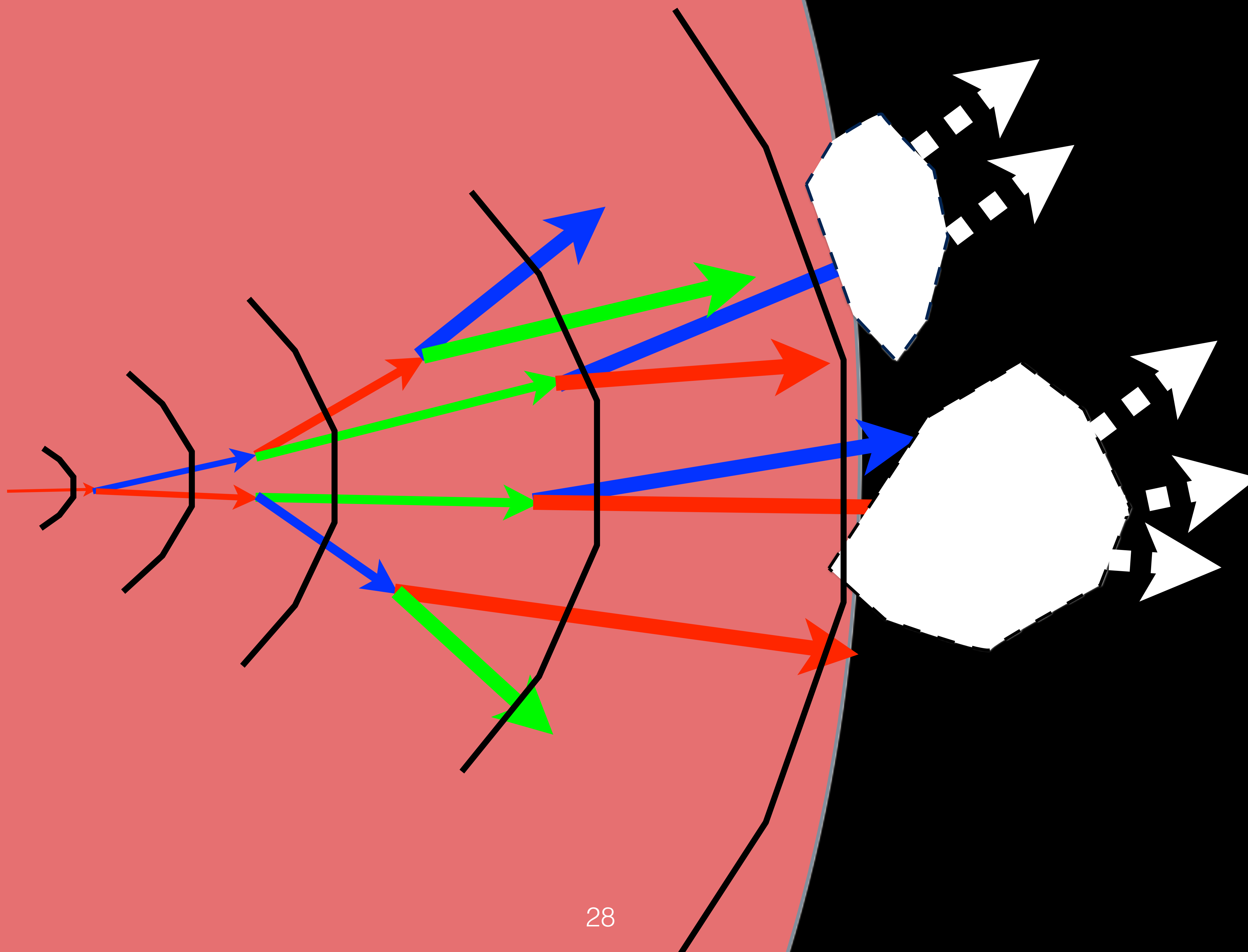
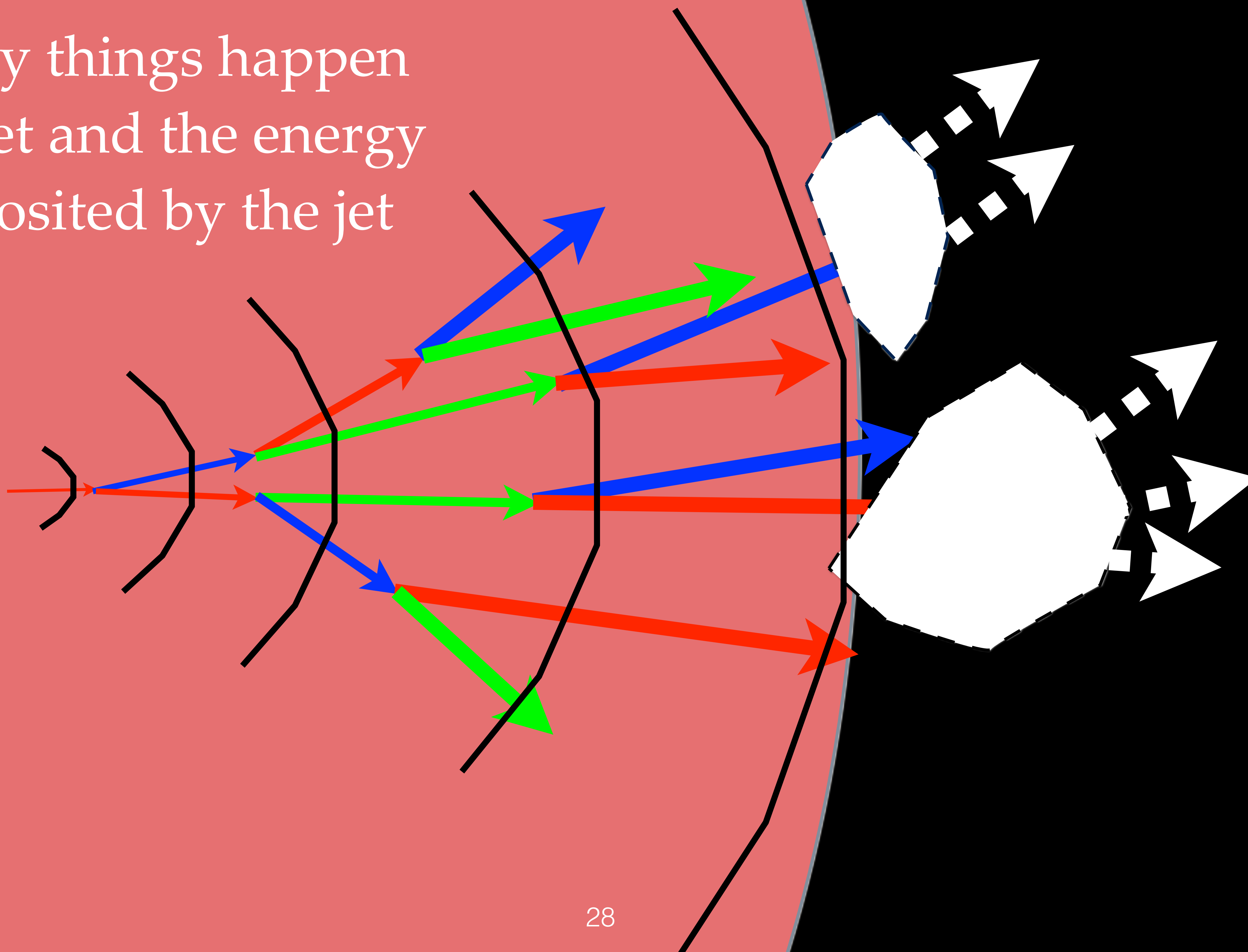


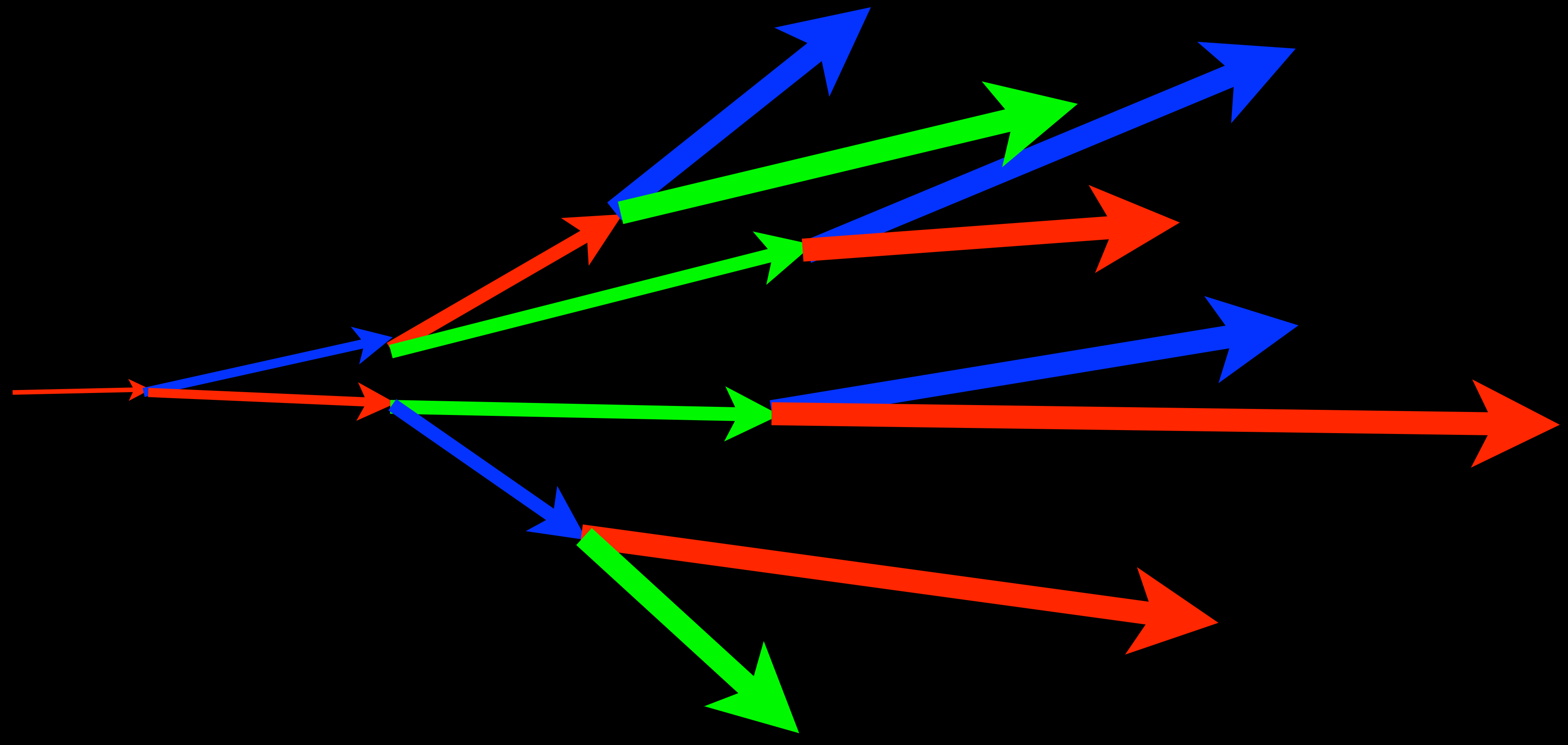
Extra focus on jets in an evolving medium

Many things happen to a jet and the energy deposited by the jet



High energy and high virtuality part of shower

- Radiation dominated regime



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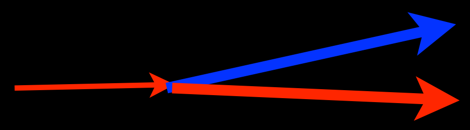

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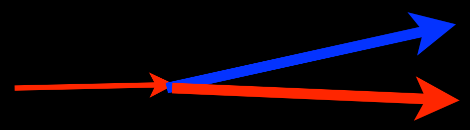

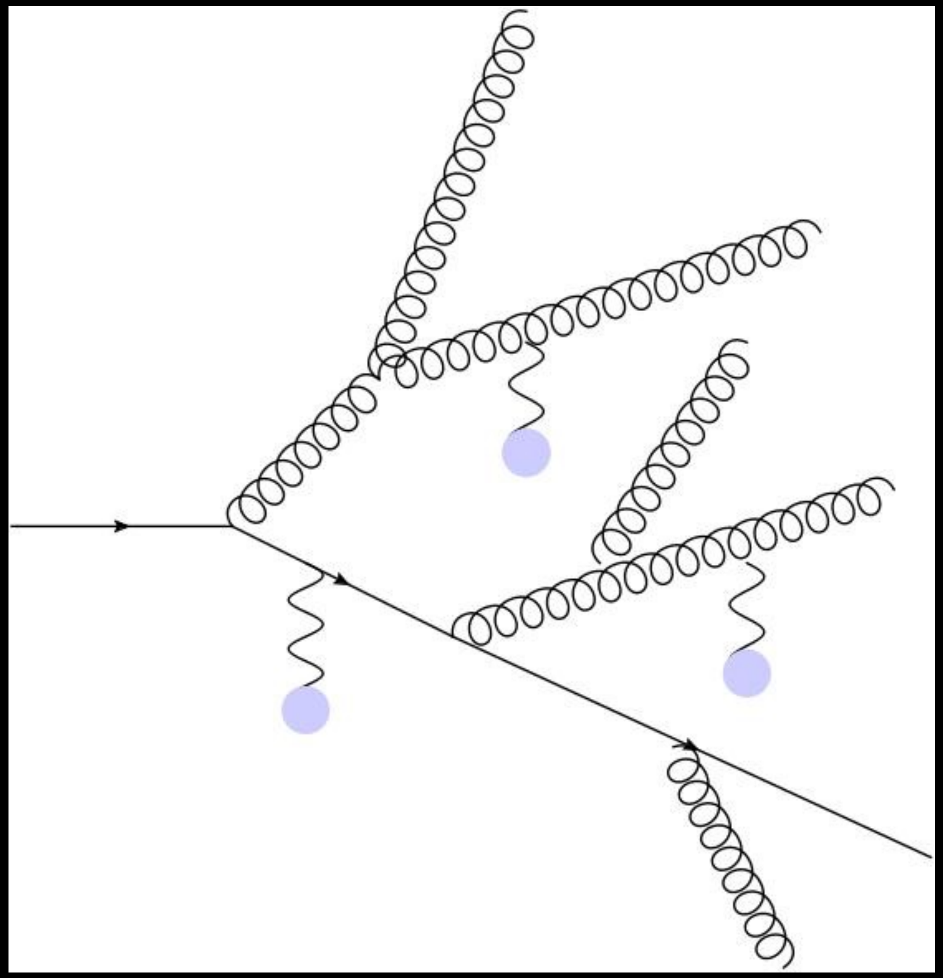

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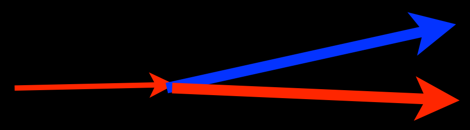
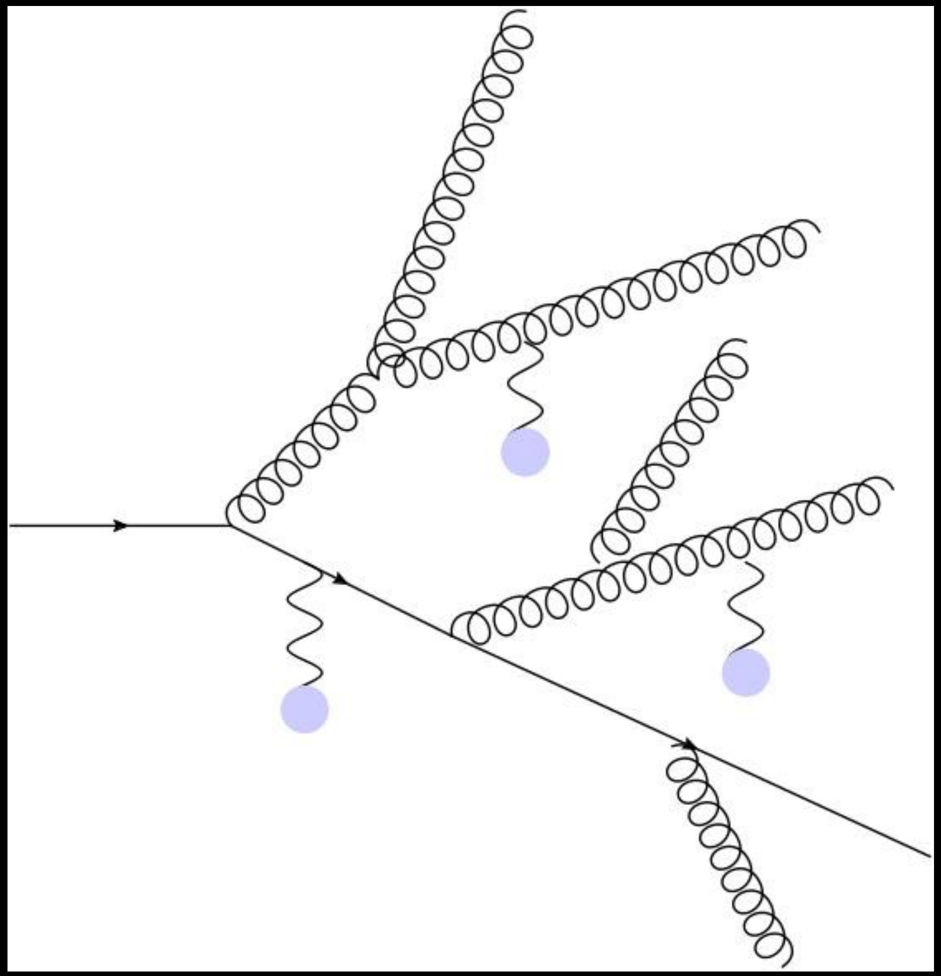




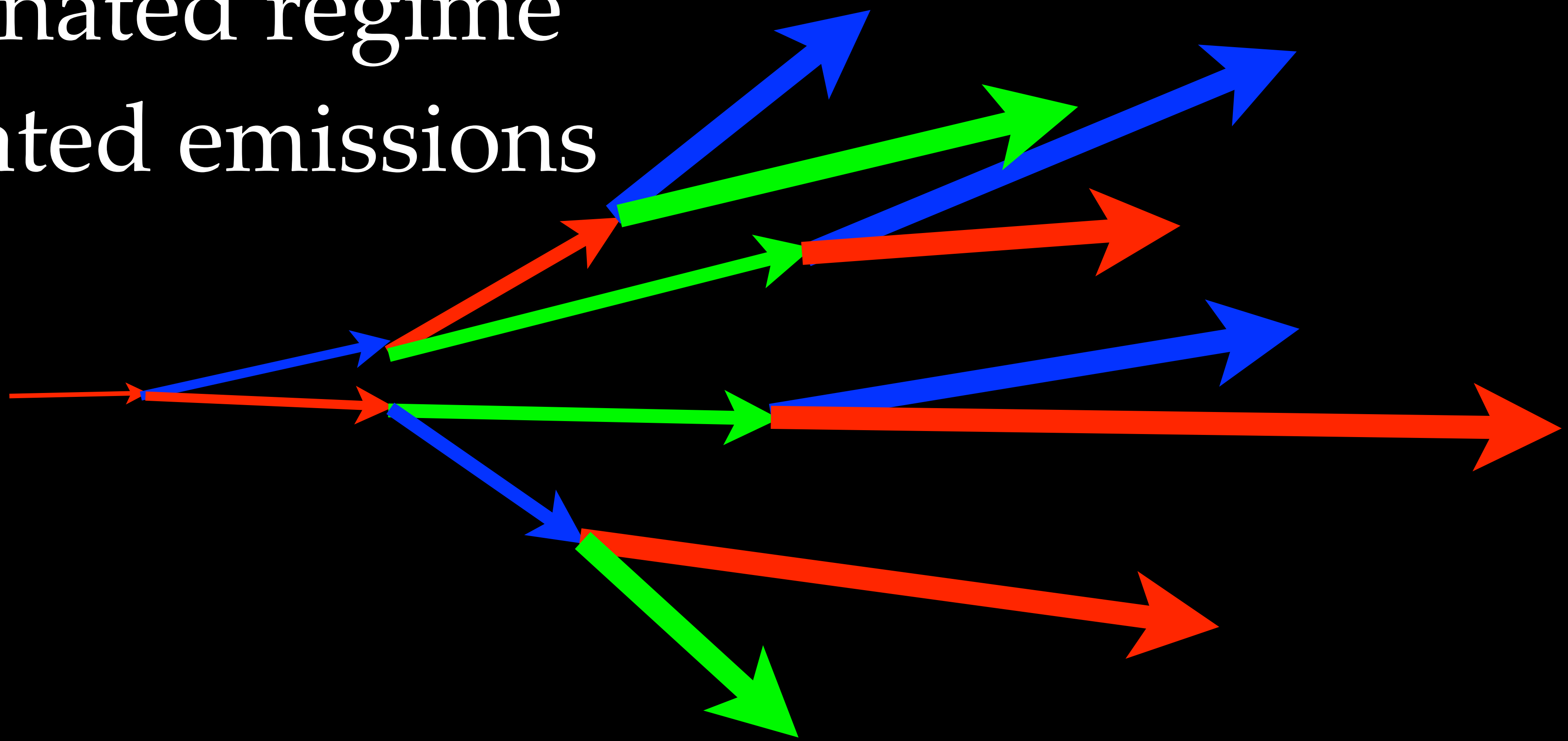
Theory: Higher Twist (X. Guo X.-N. Wang)

MC: modified Sudakov, MATTER

Low virtuality, high energy part

Scattering dominated regime

Few, time separated emissions



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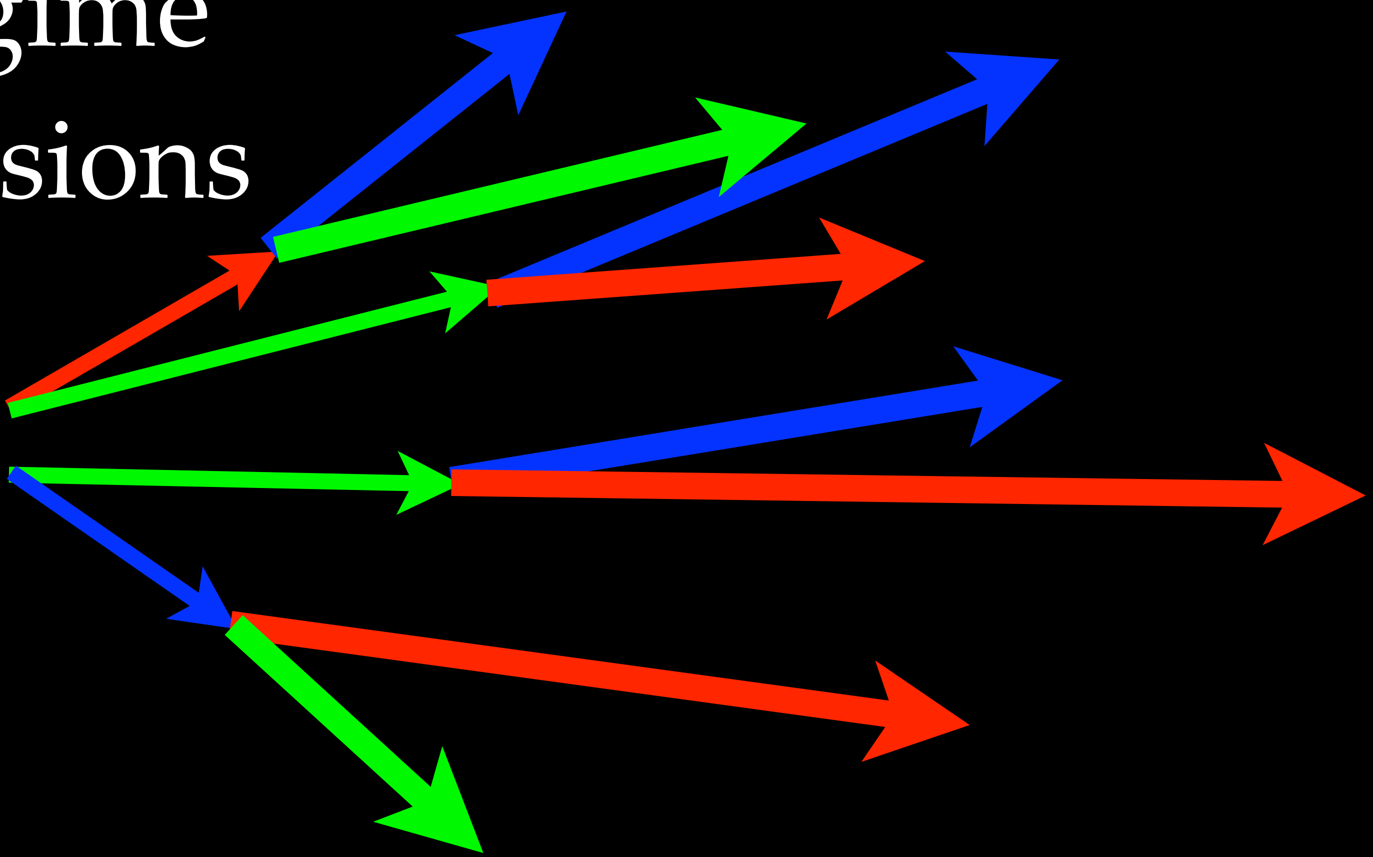

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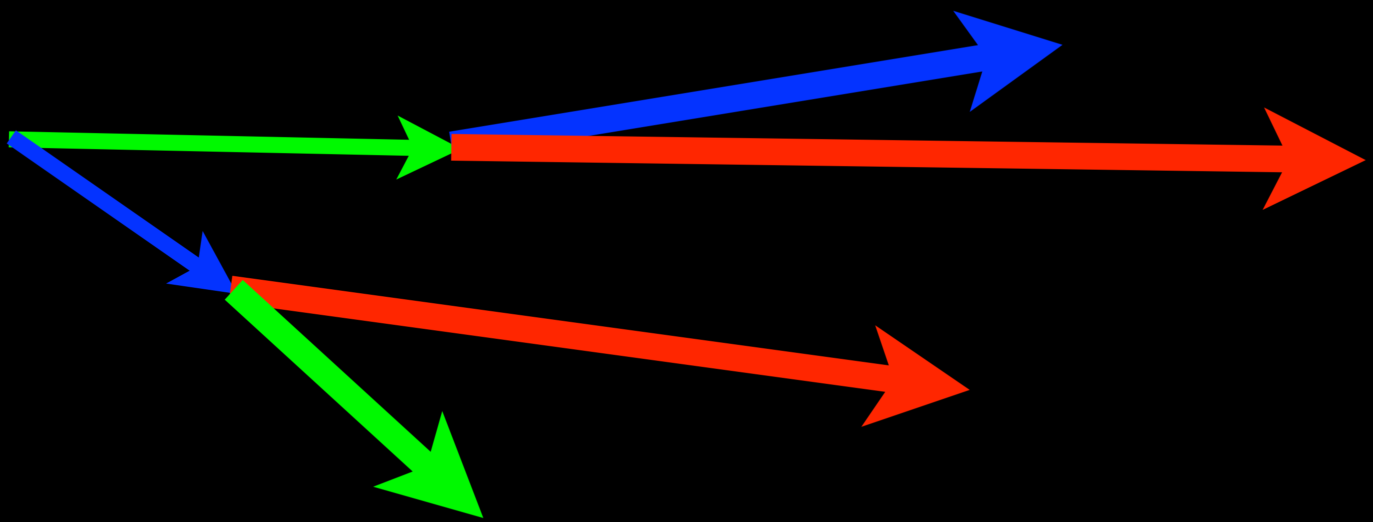

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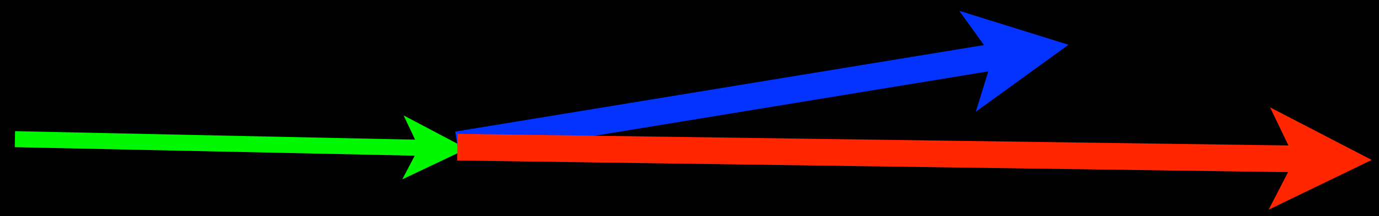

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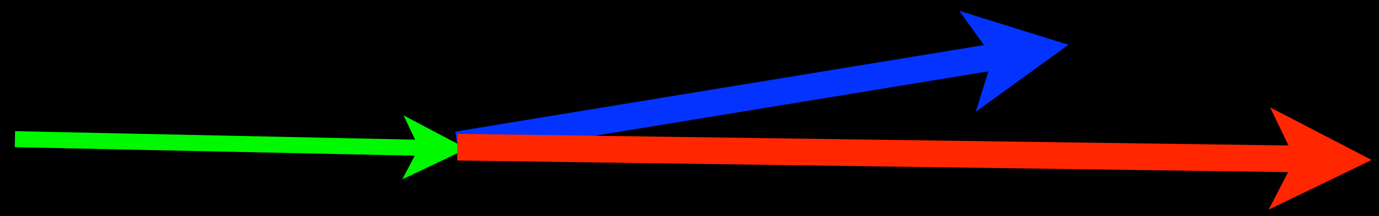

Theory: BDMPS, AMY
MC: Rate equation,
MARTINI, LBT

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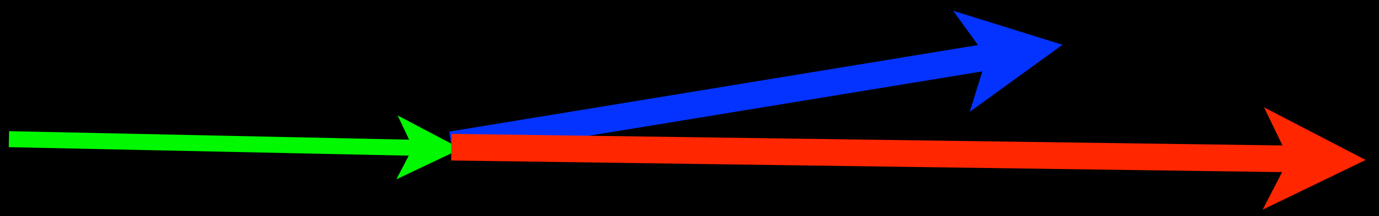
$Q^2 = q\ \tau$

τ: *lifetime of a parton*



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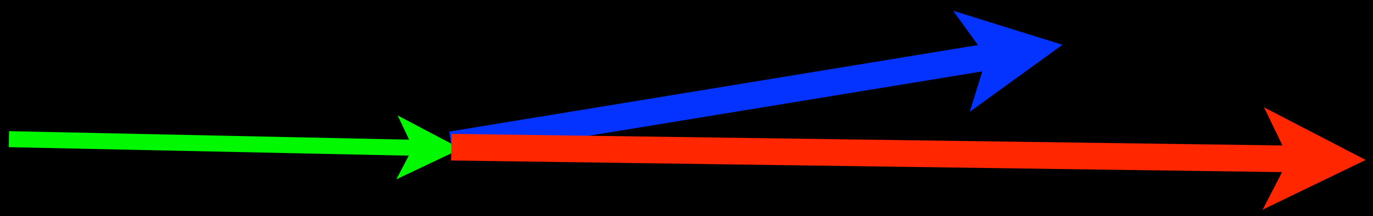

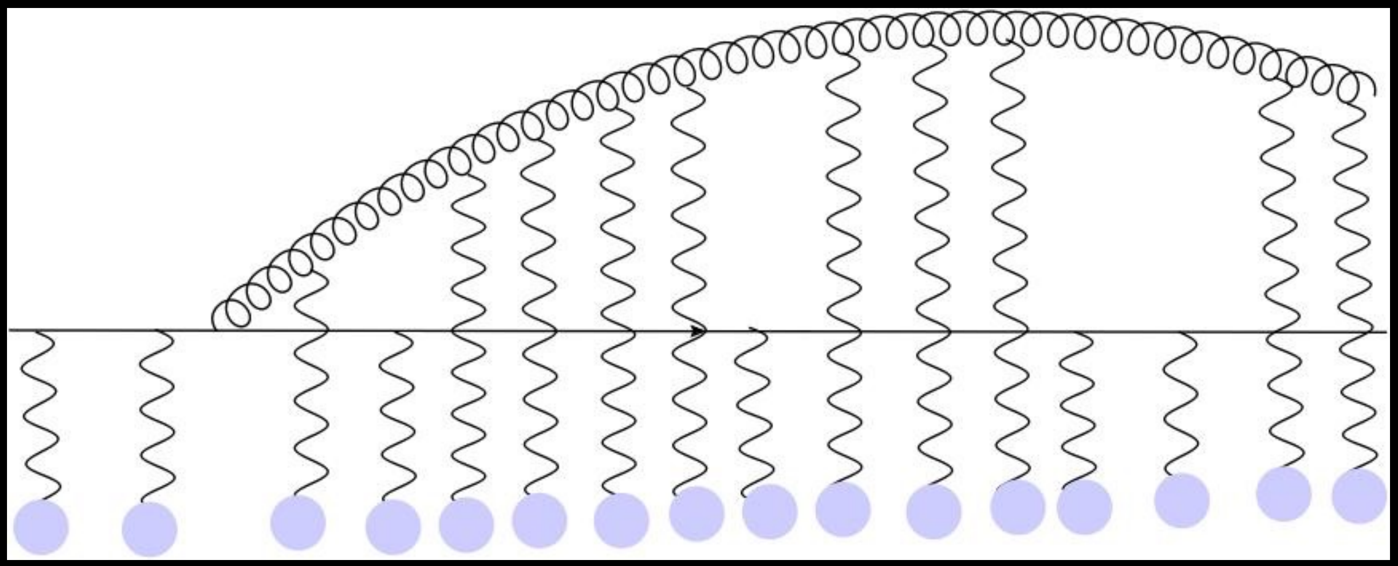

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- Many of these partons are absorbed by the medium
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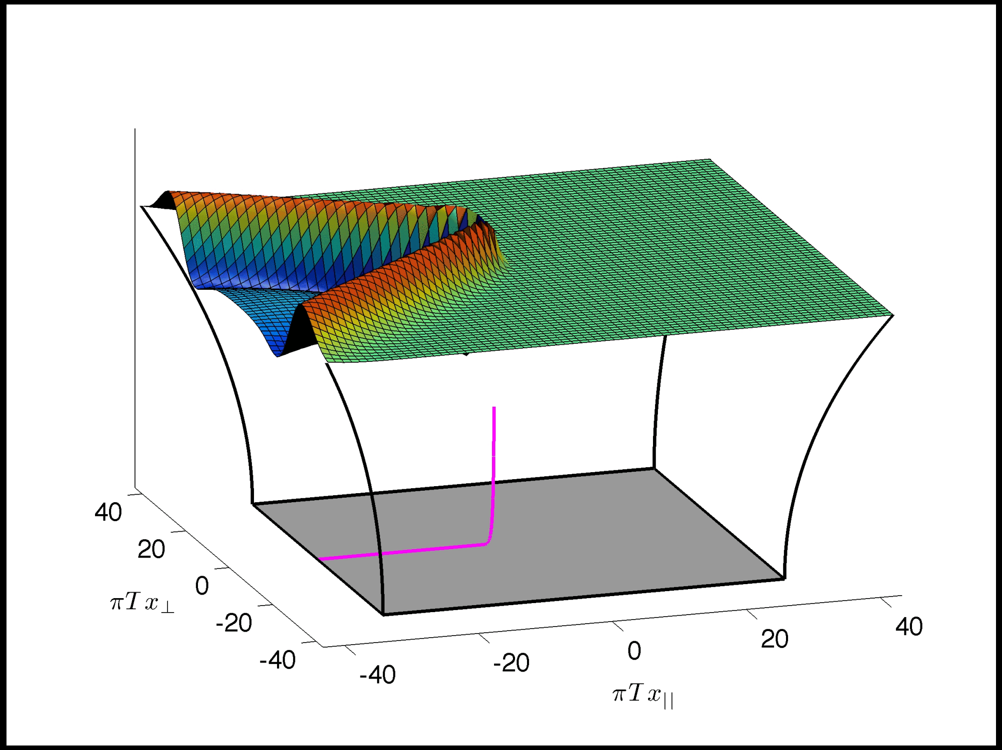

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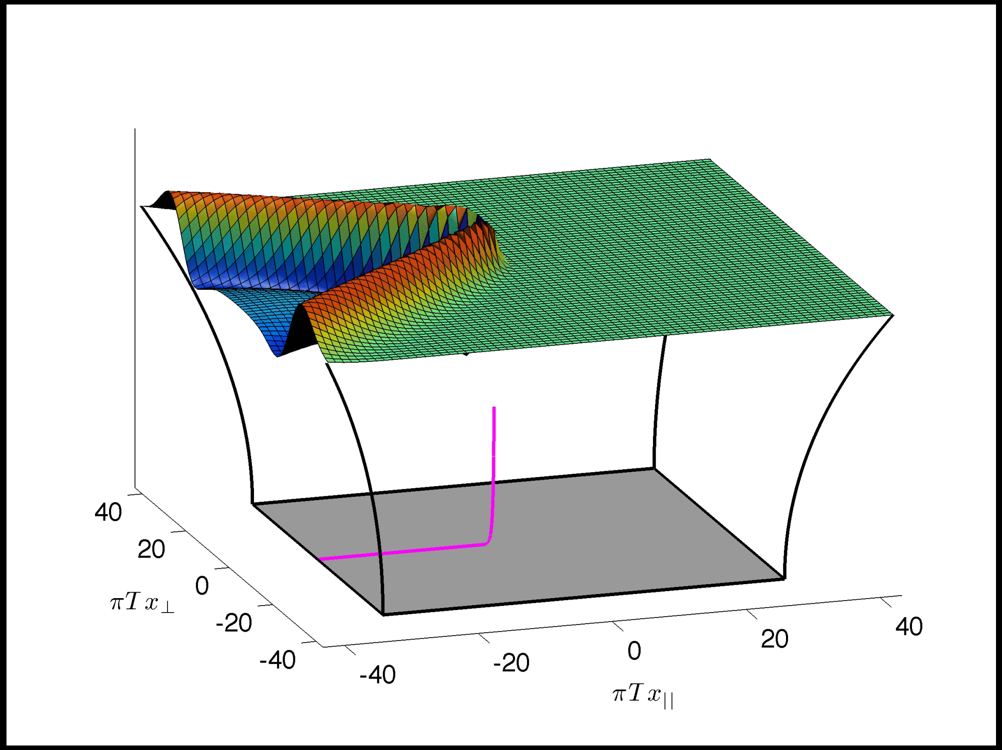
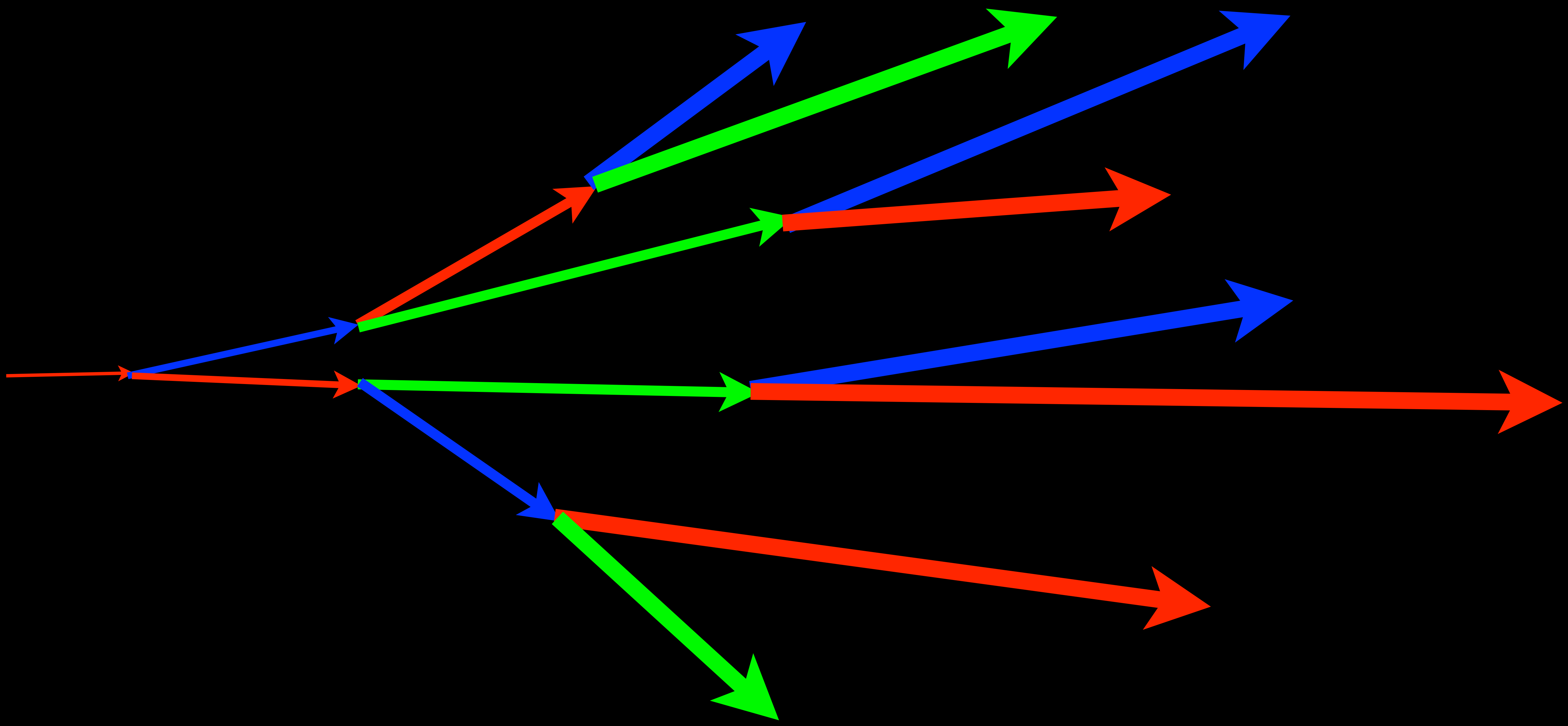


P. Chesler, W. Horowitz J. Casalderrey-Solana, G. Milhano, D. Pablos, K. Rajagopal

Grand picture (leading hadrons)



In a static brick

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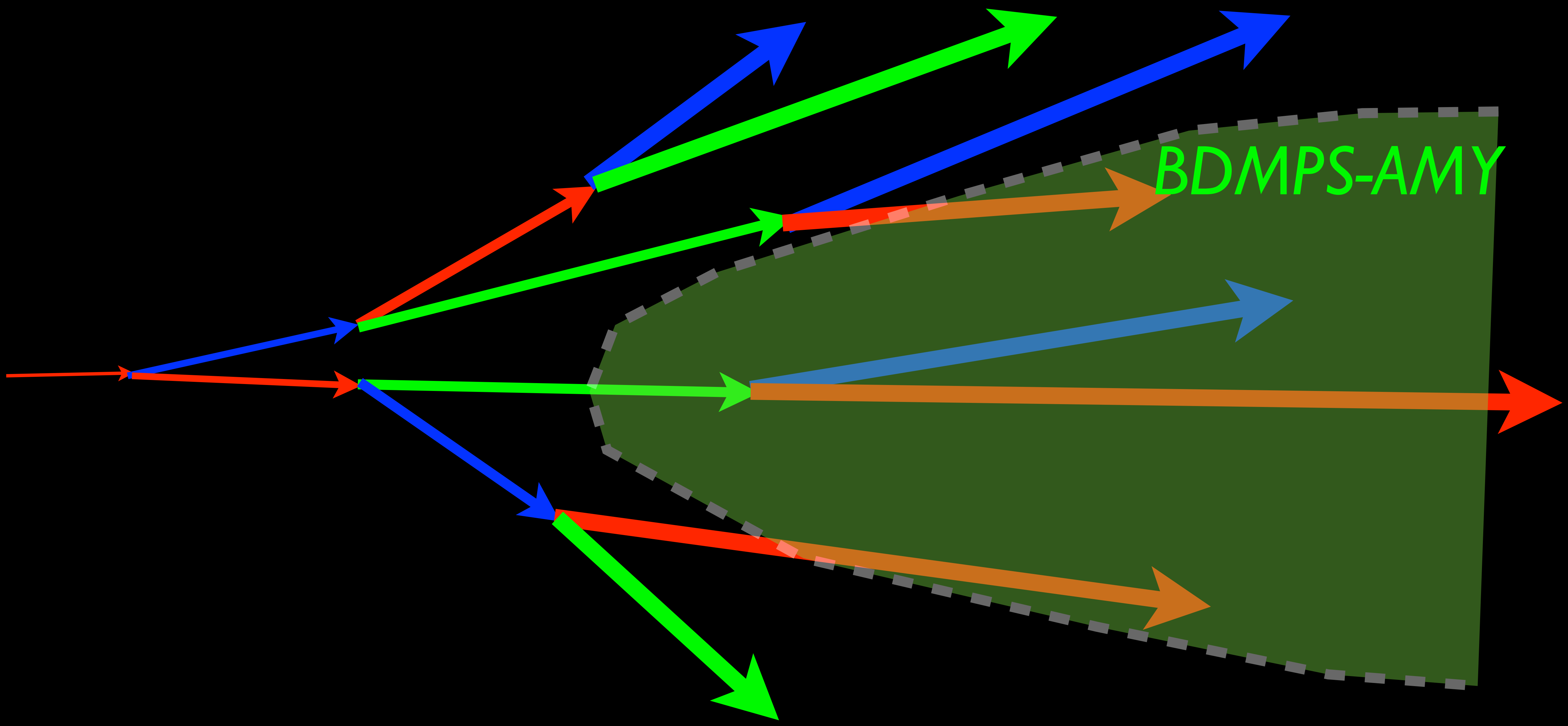

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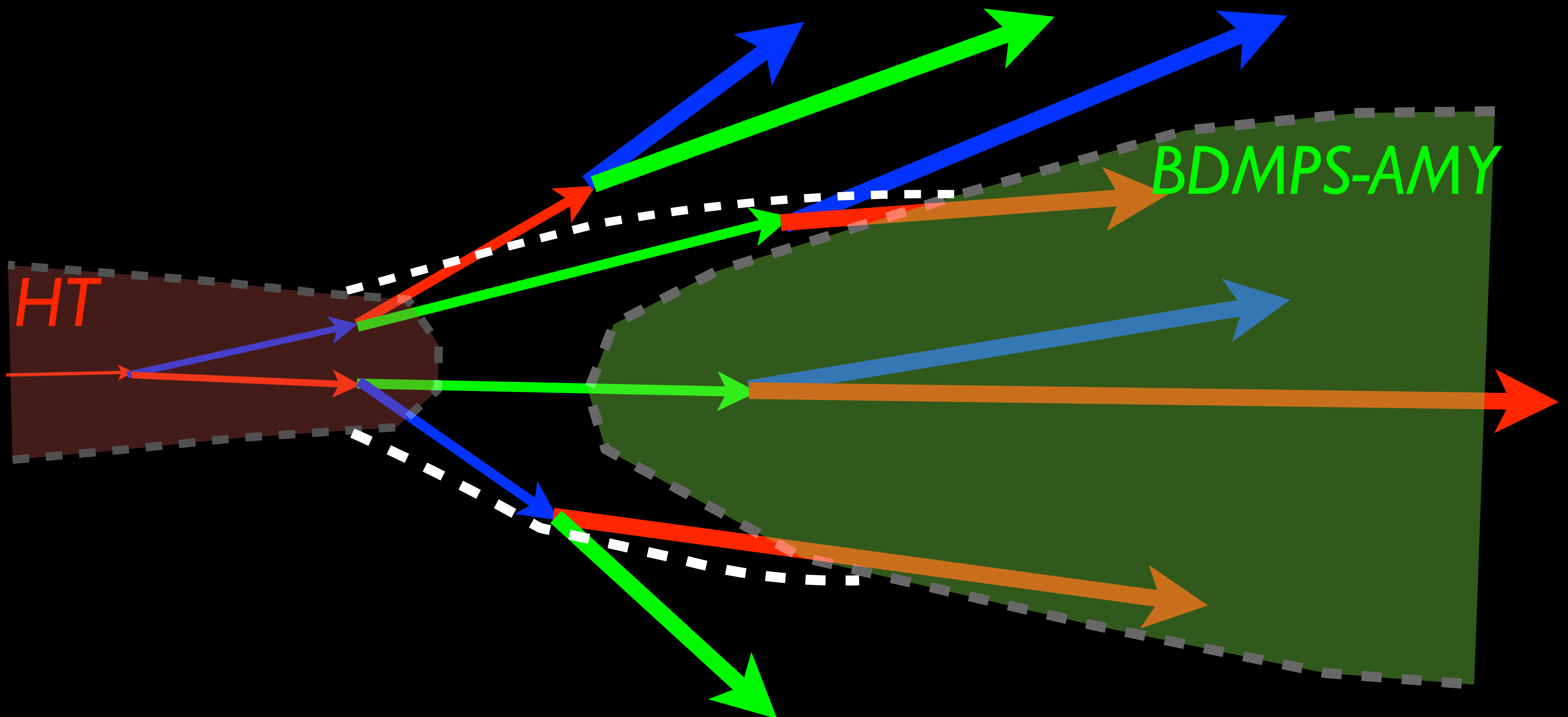

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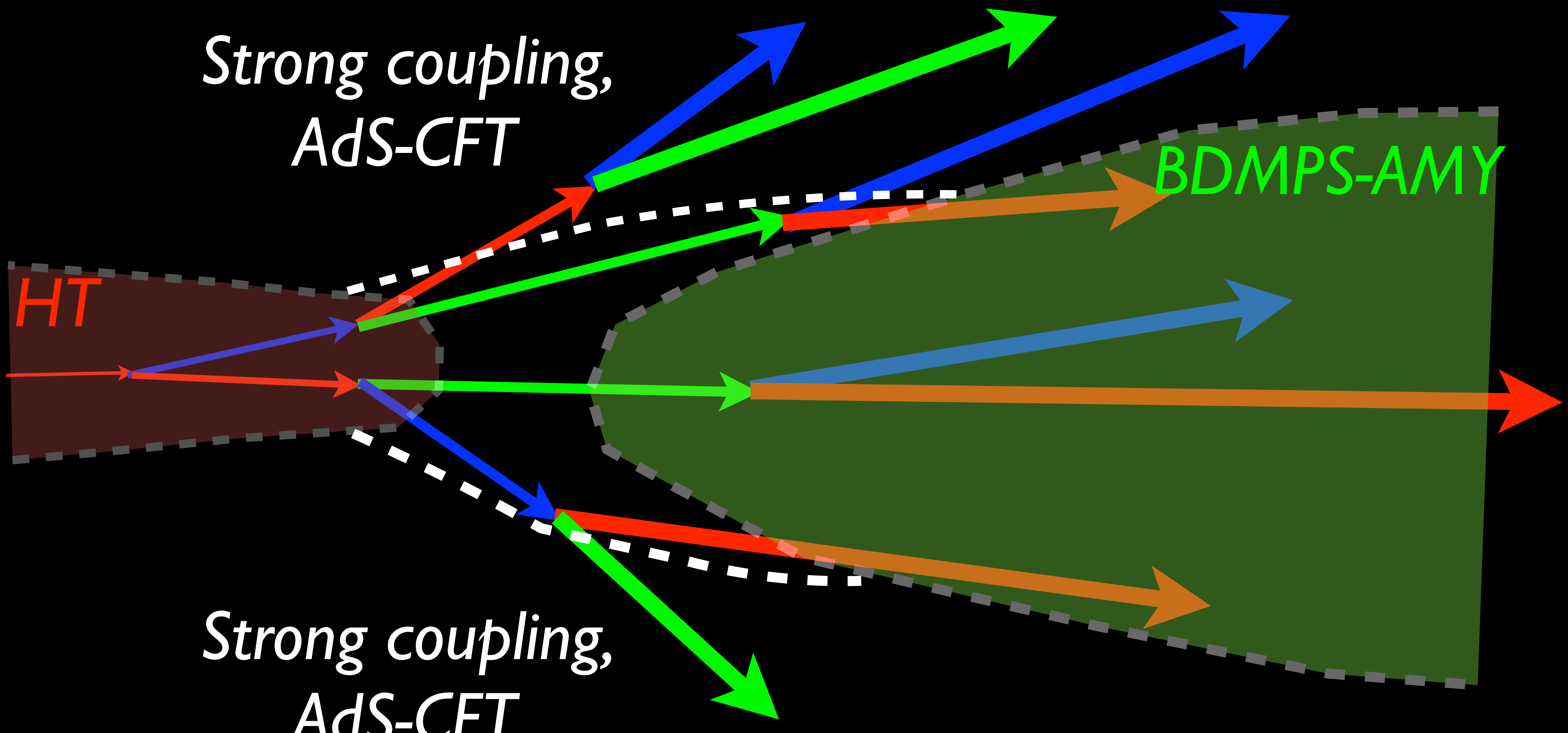

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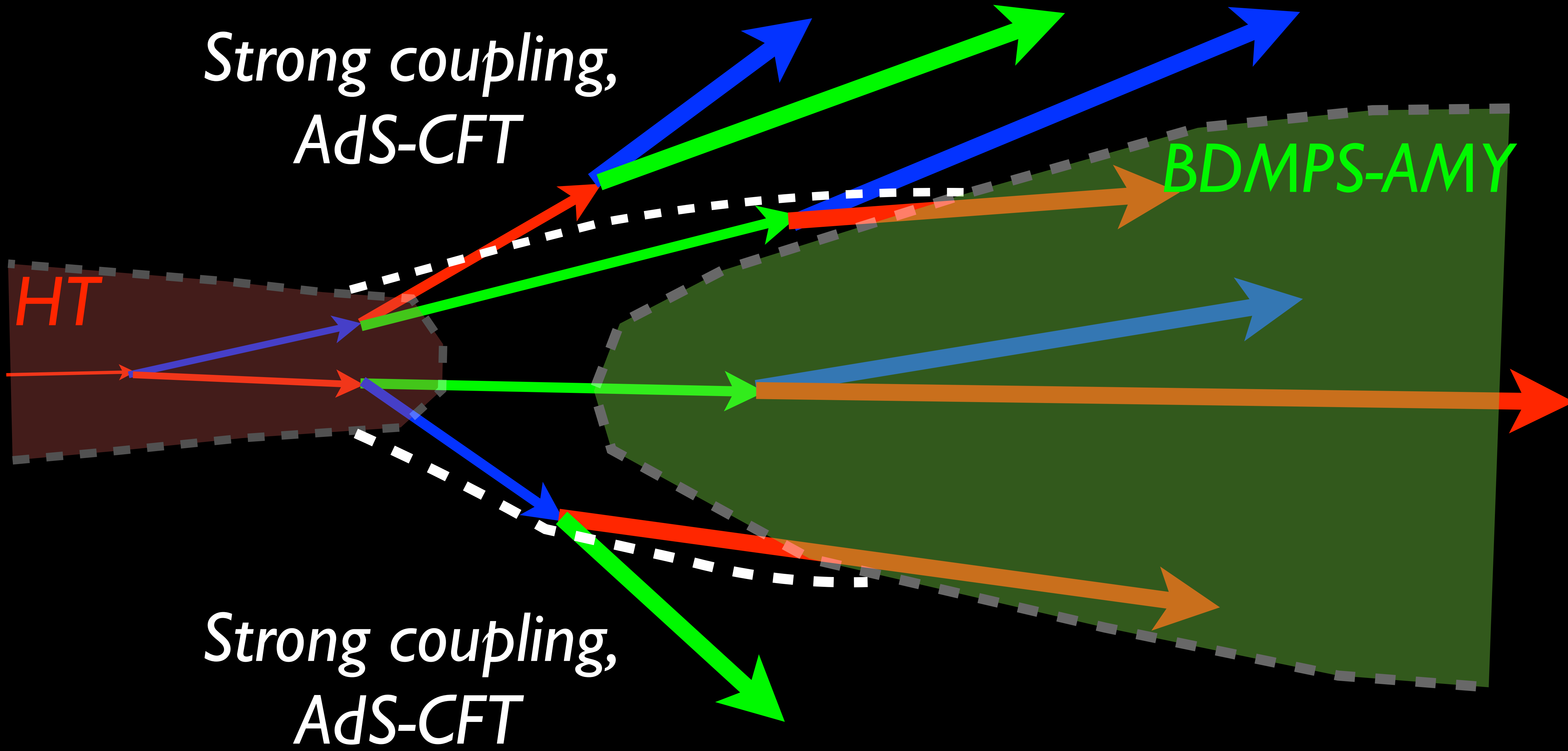
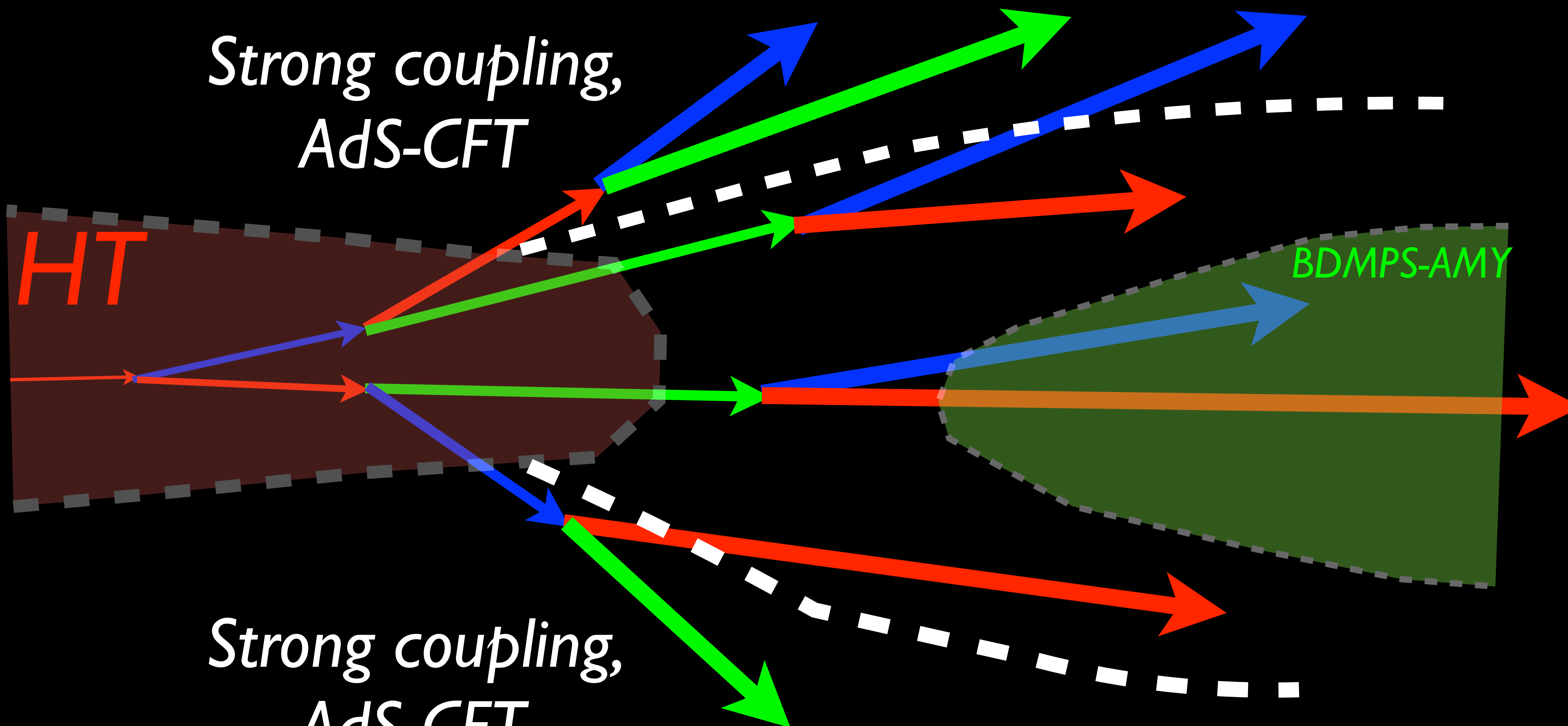


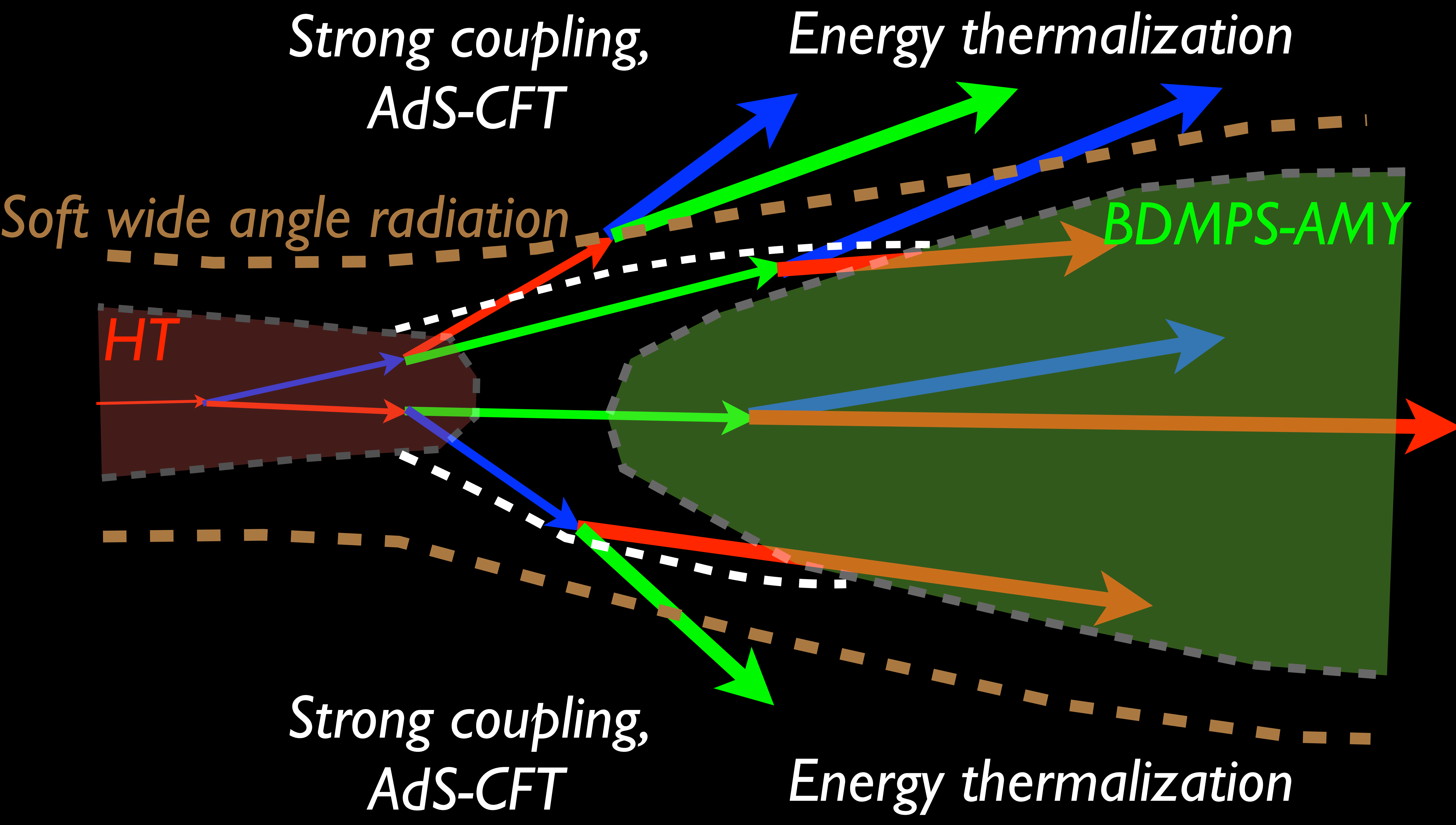
In an expanding QGP

Grand picture (leading hadrons)

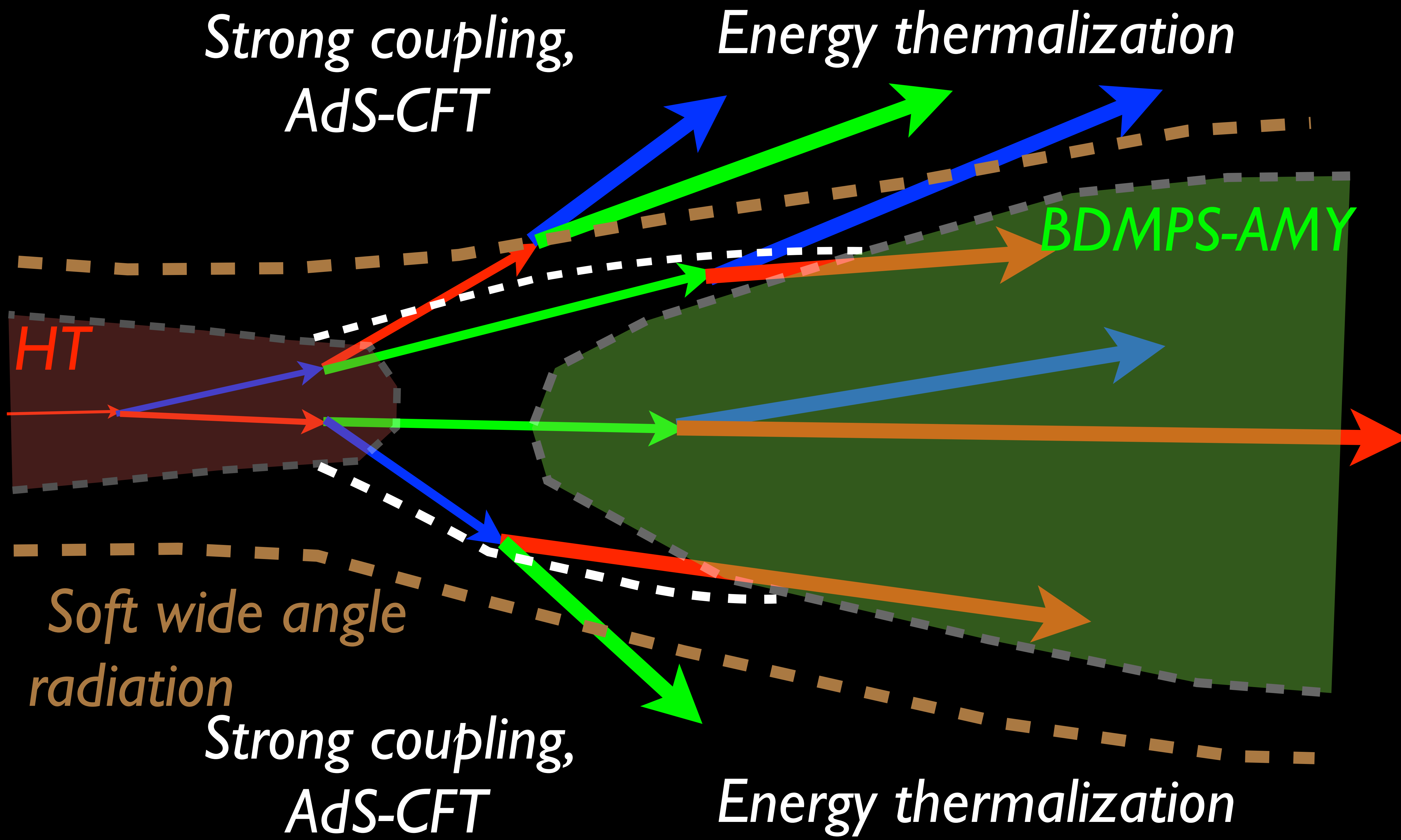


In an expanding QGP

Energy deposition-thermalization



Everything changes with scale in jet quenching



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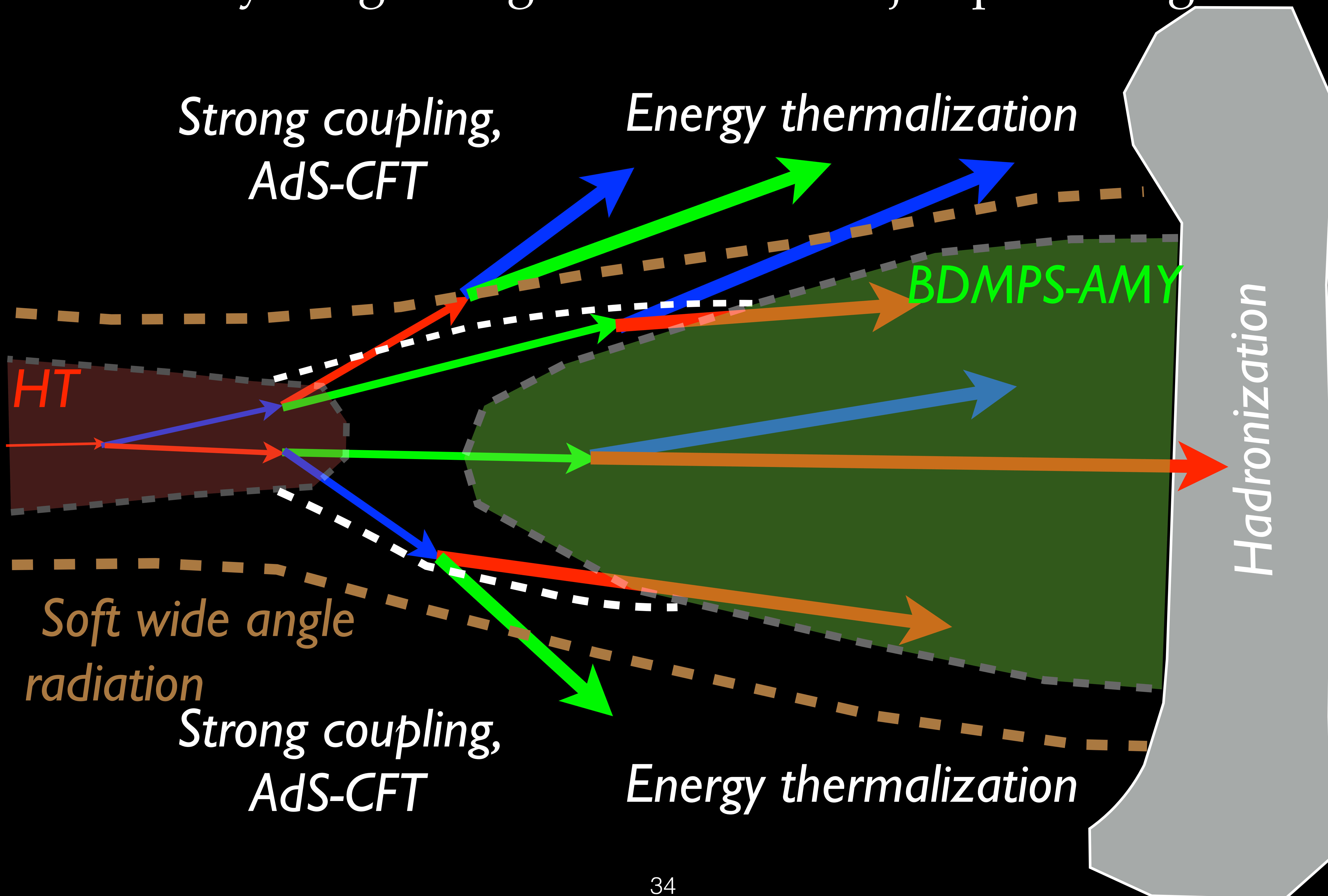
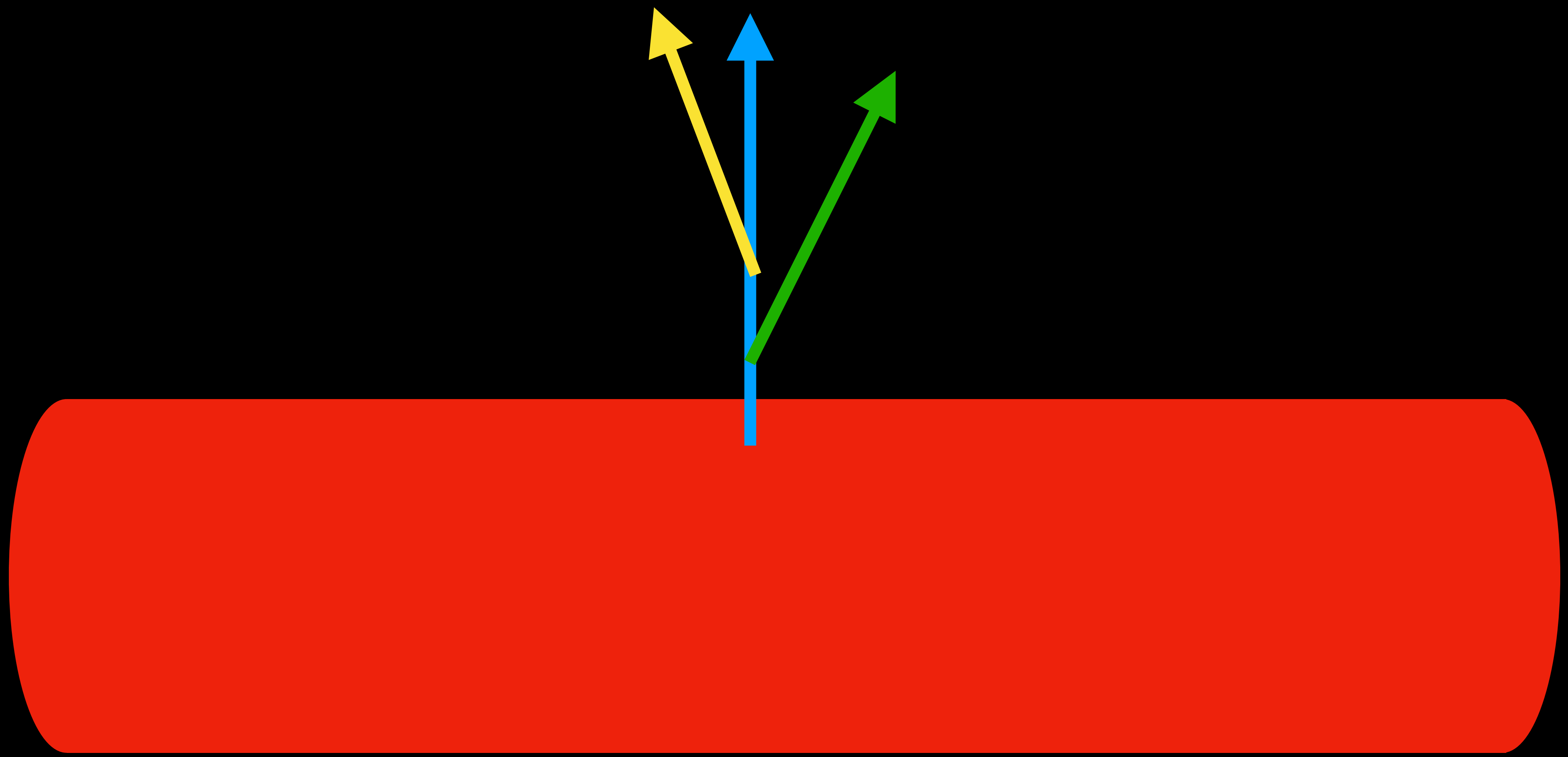


A jet hadronization mechanism that generalizes from p-p to A-A



1) Have separate strings for each shower initiating parton (colored)
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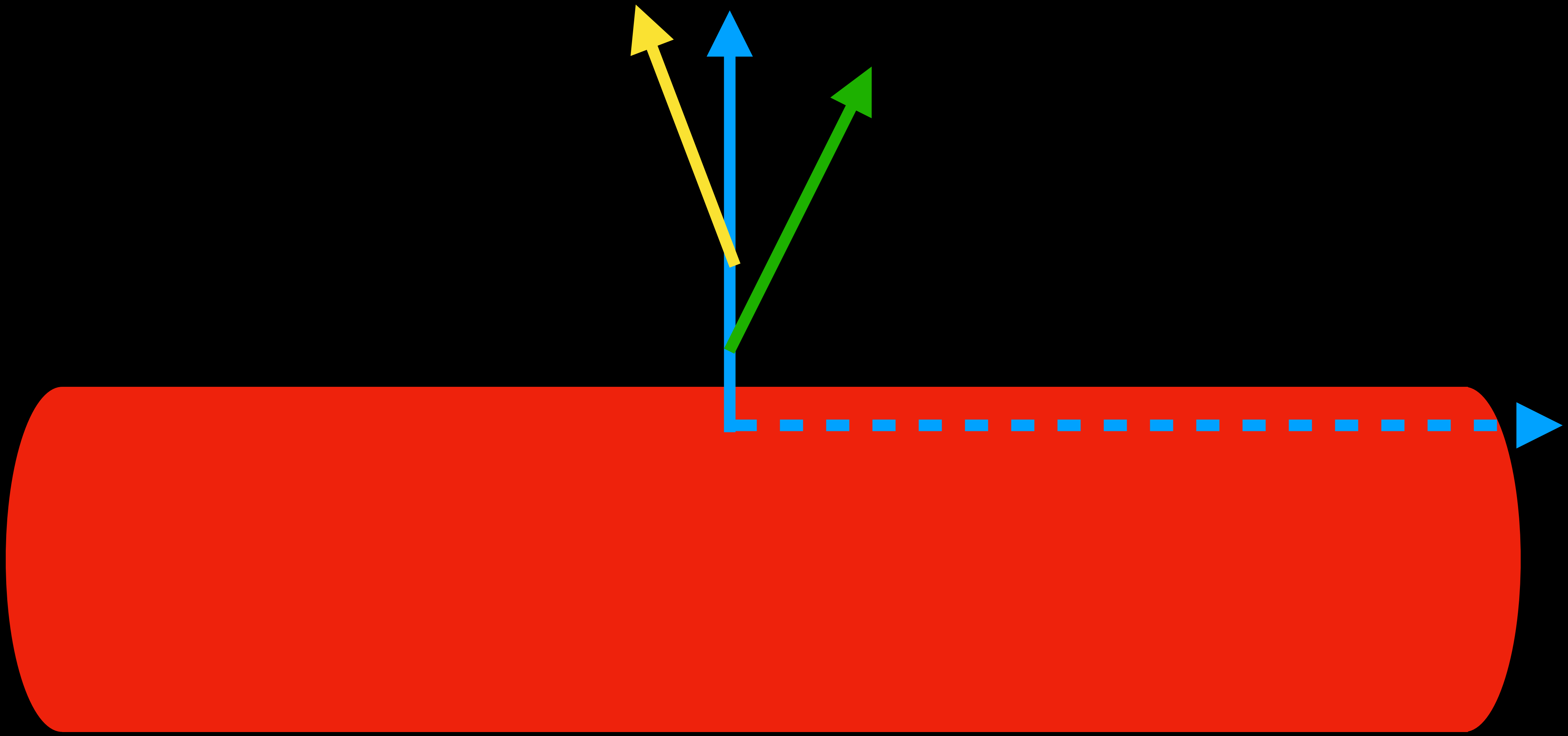

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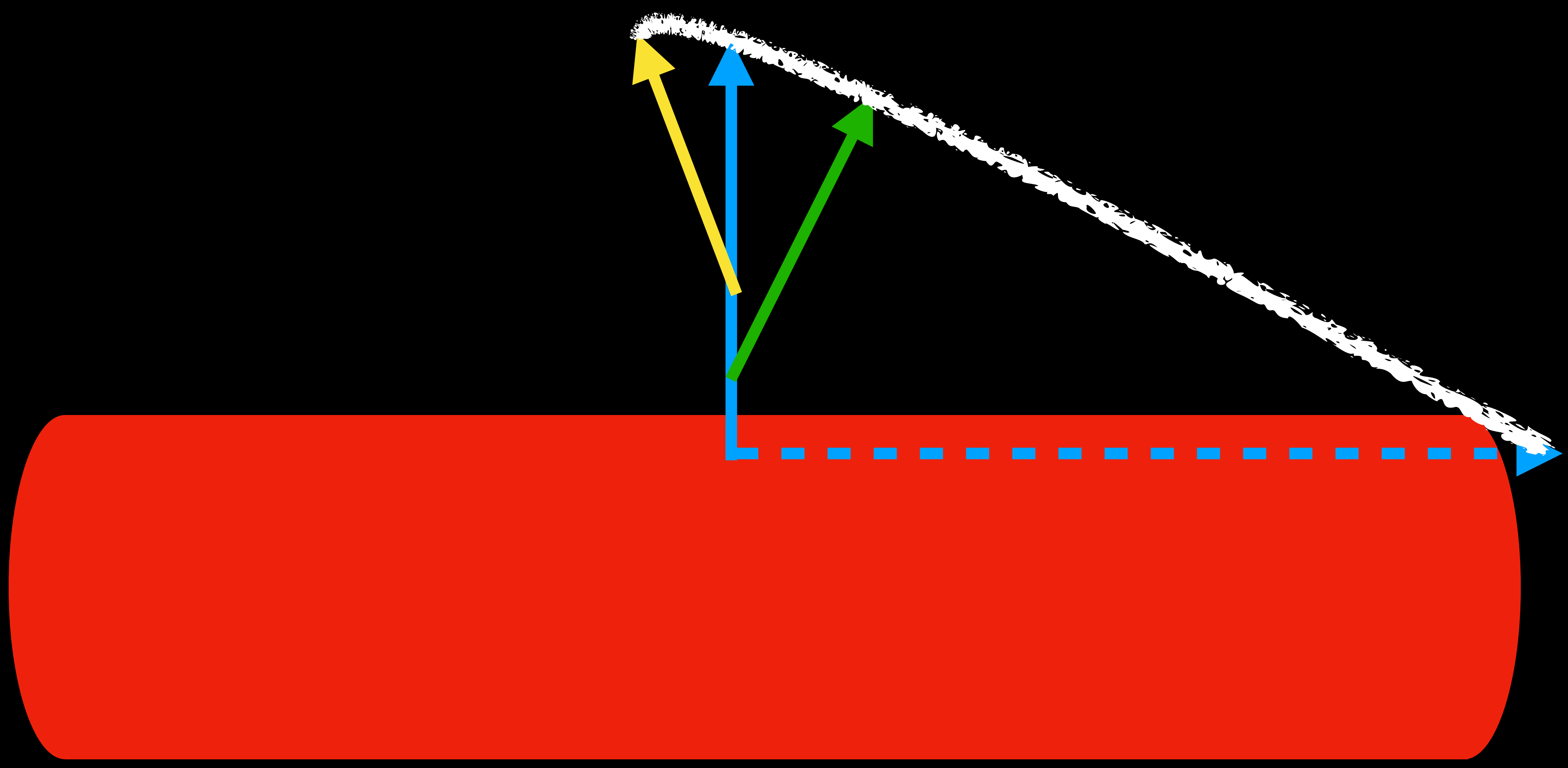

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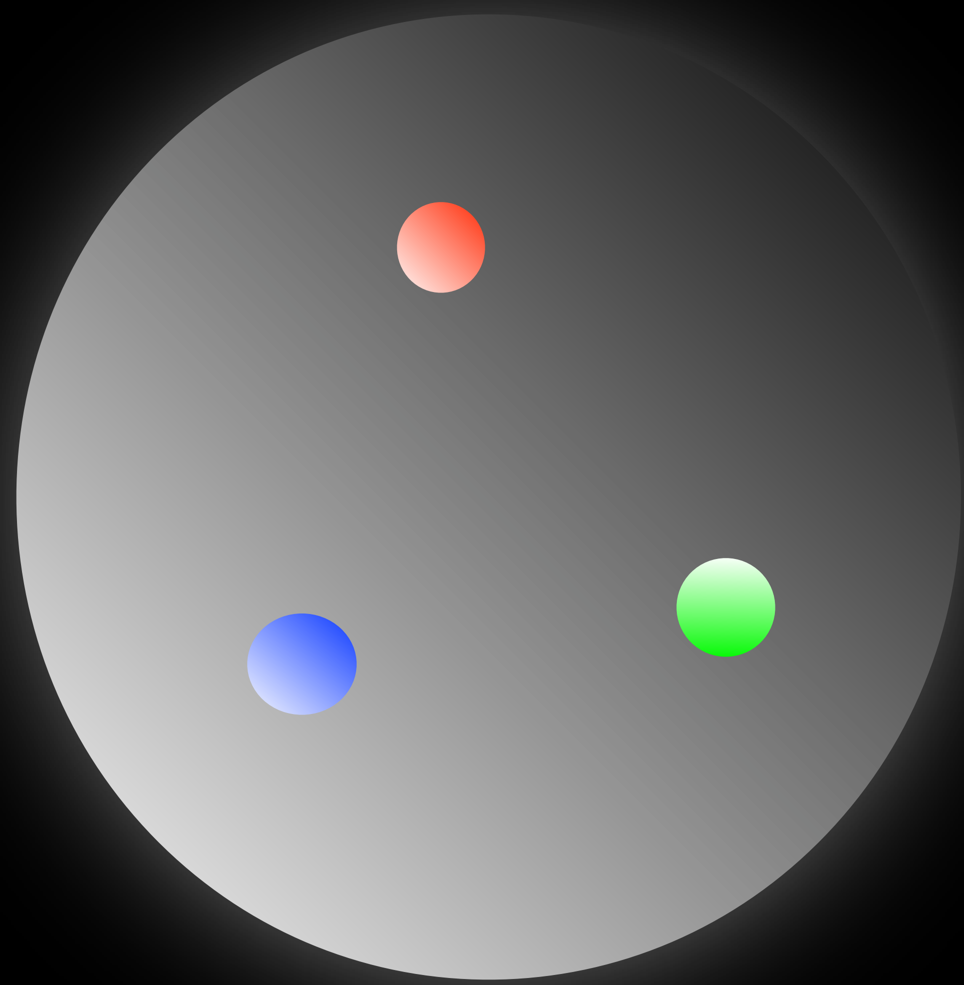
Summary

- MC methods are key to understanding the fluctuation behavior of quantum systems
- Simulations based on perturbation theory depend on asymptotic freedom and factorization
- Simulations of collisions carried out in a modular - factorized method
- Allows for the set up of elaborate and modular event generators.

What we do in JETSCAPE

- Sample nucleons within the colliding nuclei using the Woods-Saxon distribution
- Generate initial state energy momentum tensor from from deposited energy
- Run a free streaming simulation + hydro + cascade
- Run p-p collision within an A-A collision as a PYTHIA event
- Modify FSR with various methods to understand jet modification

What does this change in G(x) mean?



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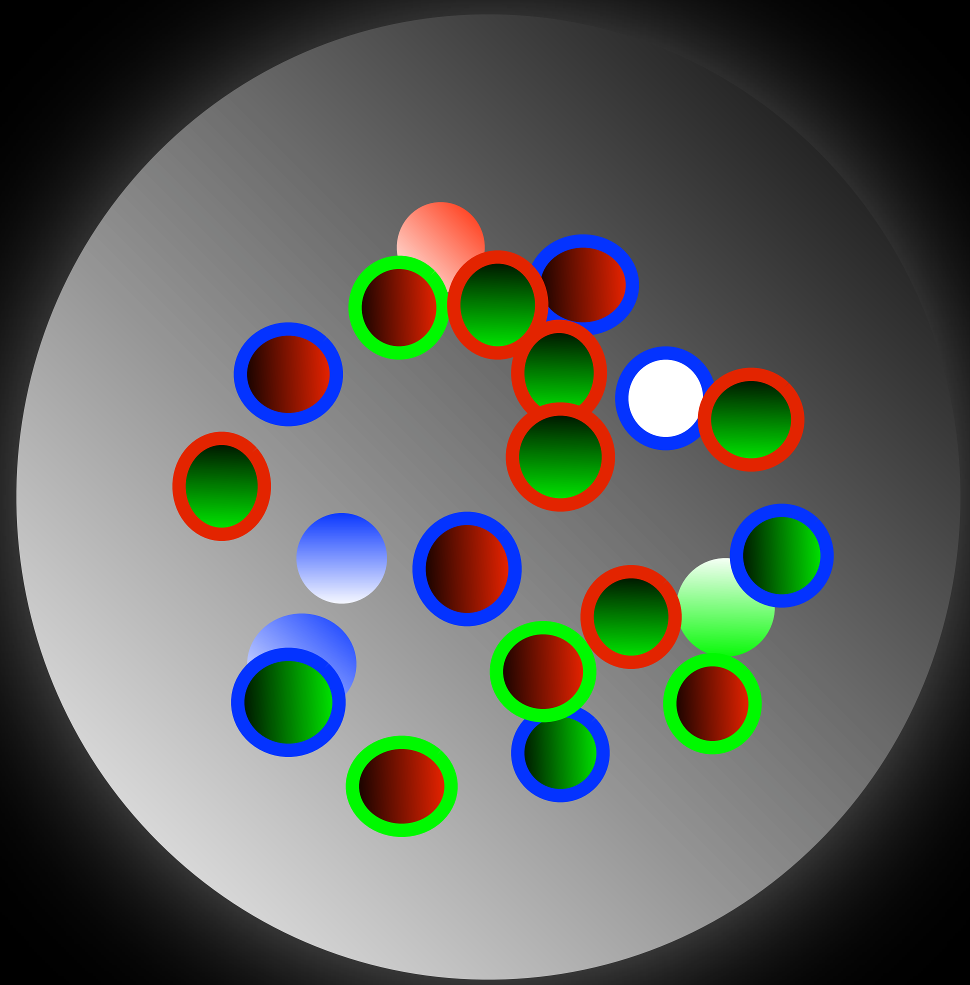

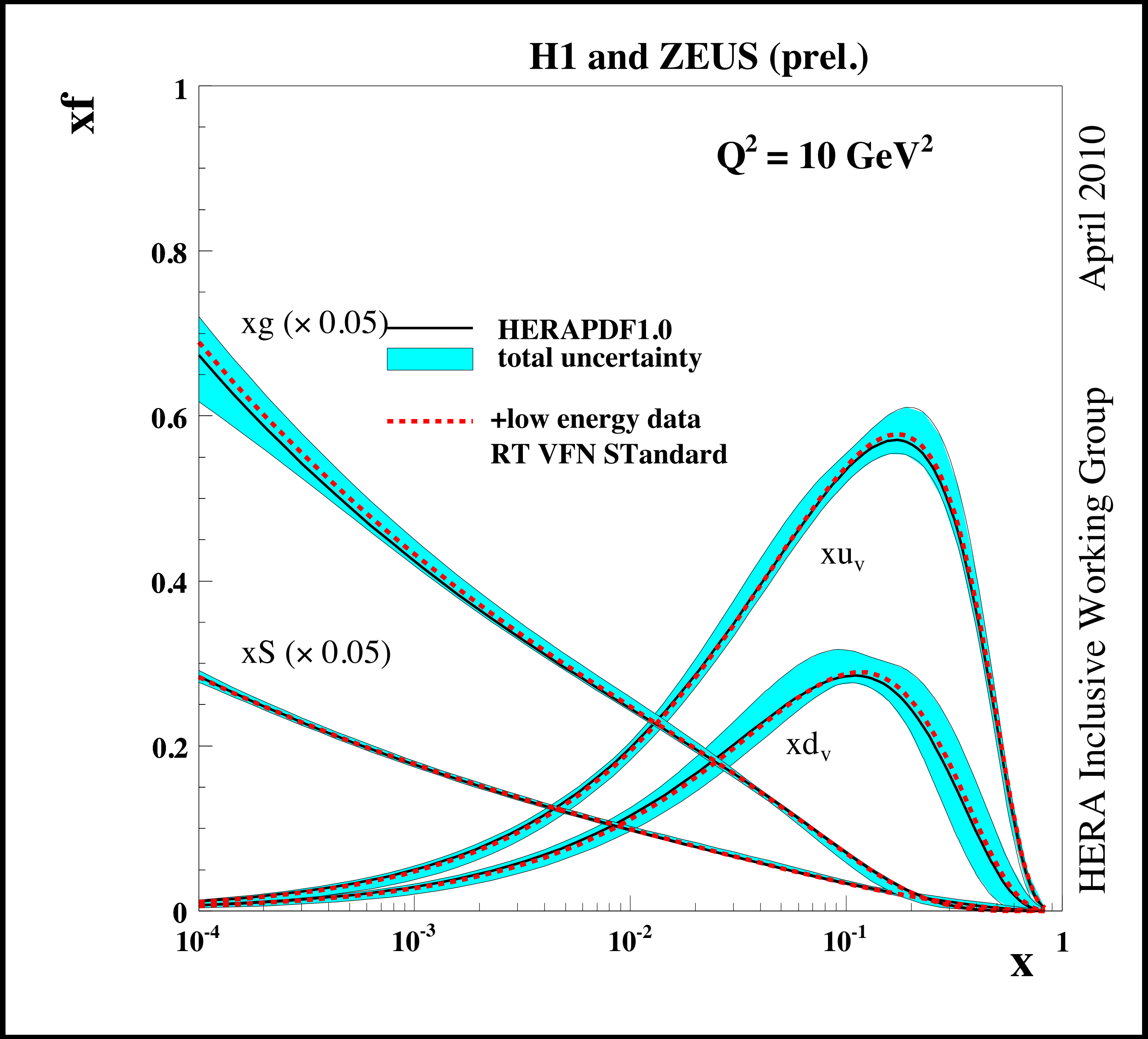

Deep Inelastic Scattering is like
looking inside a strongly coupled state (proton)
at very high resolution, and high boost