Fluid Dynamics for Heavy Ion Collisions

Jean-François Paquet & Chun Shen

JETSCAPE Winter School
Texas A&M
January 9, 2019

Hydrodynamics in heavy-ion collisions

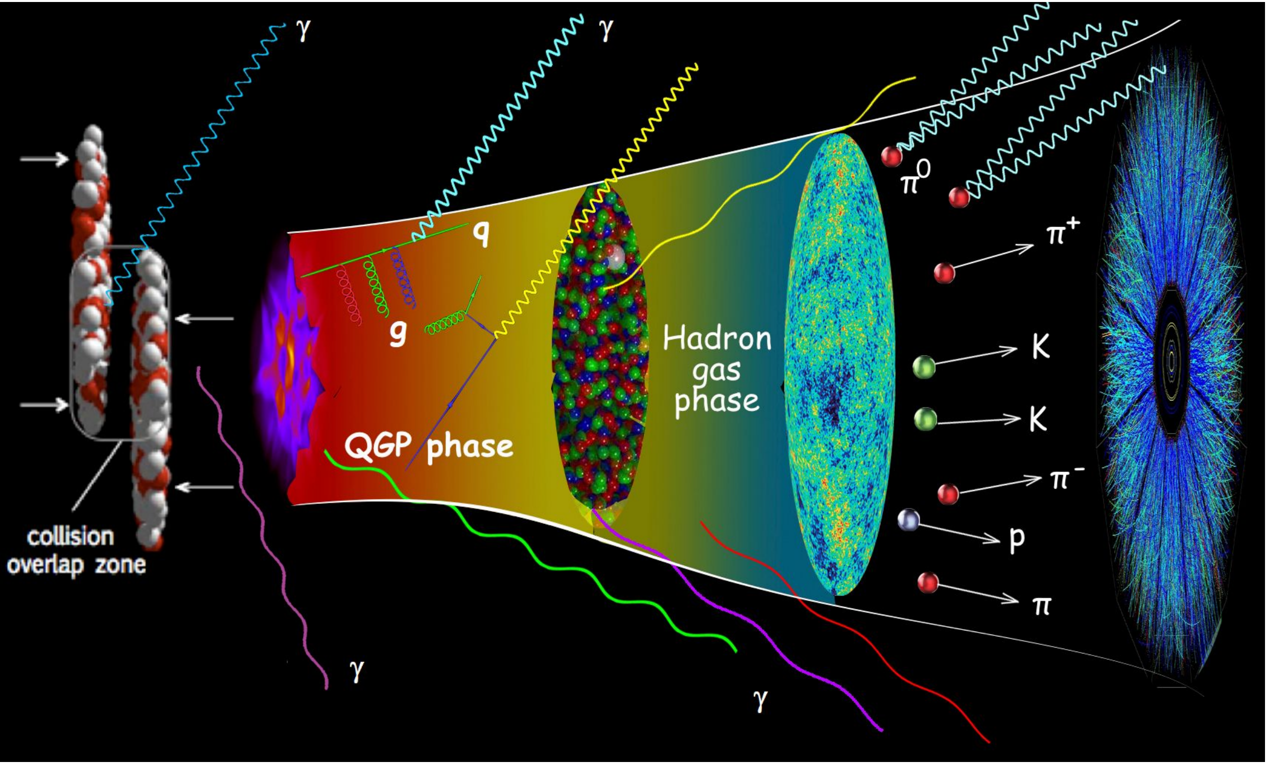
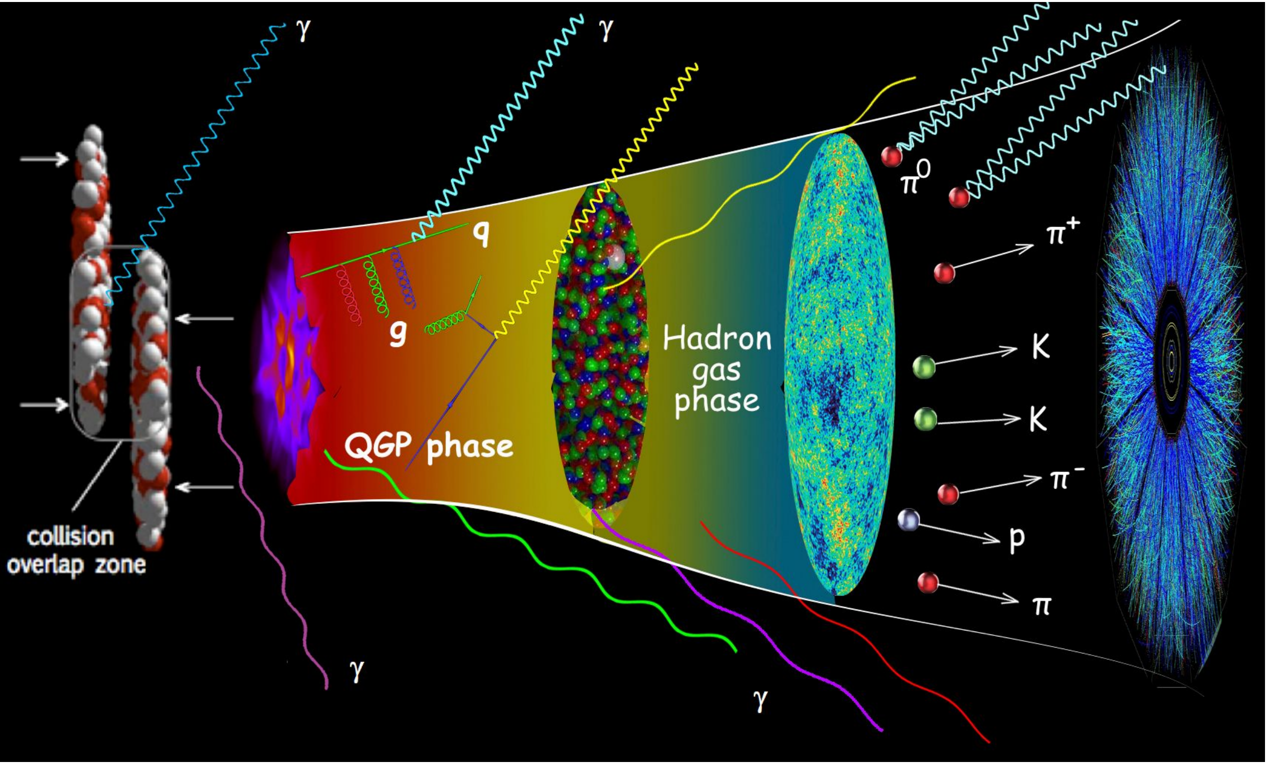


Figure ref.: Chun Shen

Hydrodynamics in heavy-ion collisions



Hydrodynamics used as **effective** description of the **"low energy modes" (long wavelengths)**

<u>Quasi-particles:</u>
(hard) partons, photons, heavy quarks, hadrons, ...

Figure ref.: Chun Shen

Example of effective description: the ideal gas

(Non-relativistic) Ideal gas in equilibrium

Microscopic (quasi-particles)

Kinetic theory of gases (Maxwell, Boltzmann, ...)

$$f(\mathbf{p}) = (2\pi m T)^{-3/2} \exp\left[-\frac{\mathbf{p}^2}{2mT}\right]$$

(Distribution of momentum of gas molecules)

Thermodynamics

Pressure “p”, density “n/V”, temperature “T”

Equation of state: $p \propto \frac{n}{V} T$

Quasiparticle interactions are (assumed to be) weak/negligible

Effective description of the quark-gluon plasma

Microscopic (quasi-particles)

↓

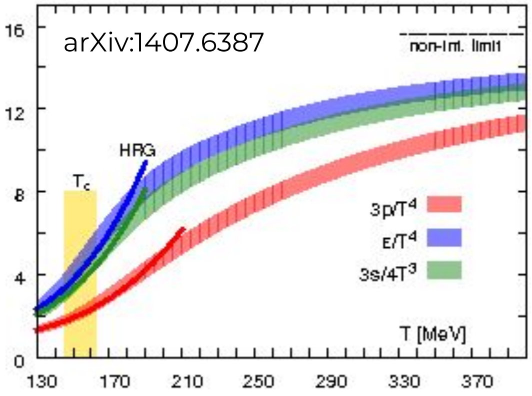
Thermodynamics (equilibrium)

Quark-gluon plasma

- **Low energies (large α_s):** Hadrons as quasi-particles
- **Intermediate energies/α_s:** No general quasi-particle description
- **High energies (small α_s):** Partons (quarks and gluons) as quasi-particles

Pressure "p", temperature "T", entropy density "s", energy density "ε"

Equation of state calculated with lattice QCD

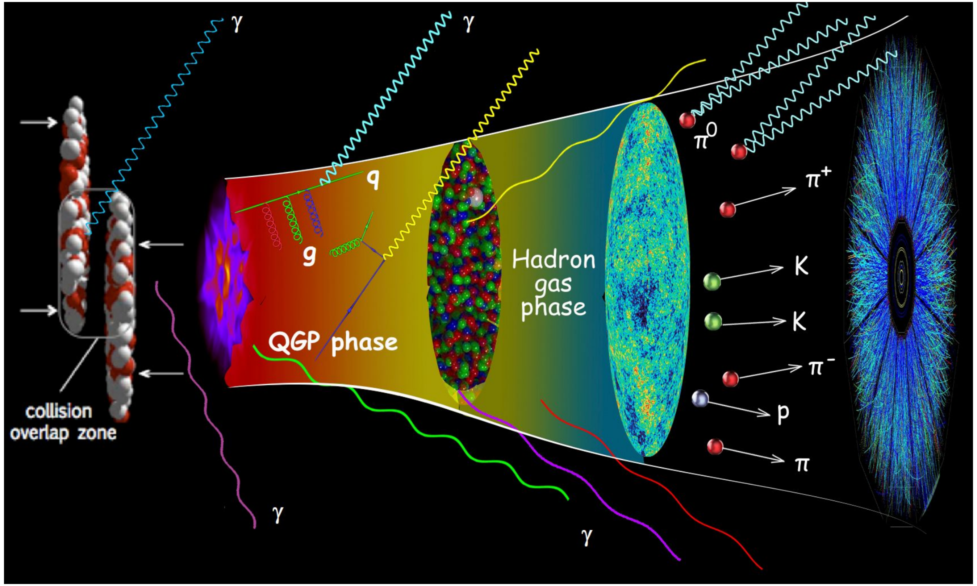


Quasi-particles in heavy ion collisions

Quark-gluon plasma

Microscopic (quasi-particles)

- **Low energies (large α_s):** <u>Hadron</u> as quasi-particles
- **Intermediate energies/α_s:** No general quasi-particle description
- **High energies (small α_s):** <u>Quarks and gluons</u> as quasi-particles



Quasi-particles; thermodynamics; hydrodynamics

Quark-gluon plasma

Microscopic (quasi-particles)

- **Low energies (large α_s):** Hadrons
- **High energies (small α_s):** Partons

↓

Hydrodynamics

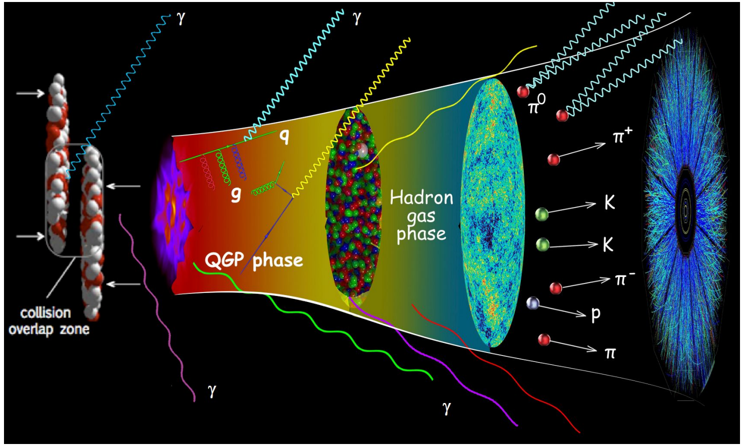
- Describe plasma near **local** equilibrium
- Equations of motion for conserved quantities (energy, momentum, charges)

↓

Thermodynamics (equilibrium)

- **Global equilibrium** properties of plasma
- Equation of state from lattice QCD calculations

Hydrodynamics in heavy ion collisions



Effective description of the plasma?

- ❑ **Global equilibrium/thermodynamics? <u>No</u>**
- ❑ **(Near) local-equilibrium:** Possible from a theory point of view, supported by phenomenological evidence

Hydrodynamics

Coarse-graining

Thermodynamics (equilibrium)

- ❑ Describe plasma near **local** equilibrium
- ❑ Equations of motion for conserved quantities (energy, momentum, charges)

- ❑ **Global equilibrium** properties of plasma
- ❑ Equation of state from lattice QCD calculations

Relativistic ideal hydrodynamics

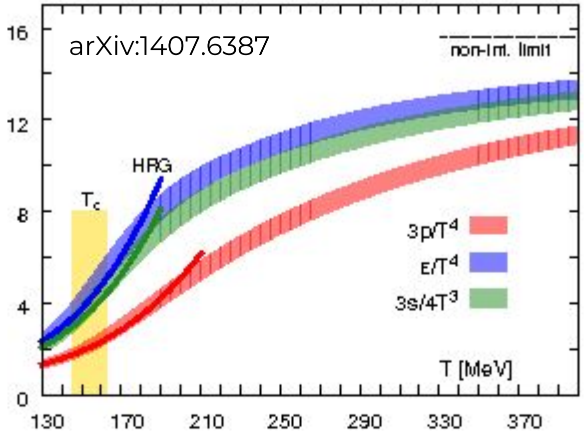
- Assume **local equilibrium**
- Energy-momentum tensor:

$$T^{\mu\nu} = (\epsilon + P(\epsilon))u^\mu u^\nu - P(\epsilon)g^{\mu\nu}$$

with

- "ϵ" the energy density
- "u^μ" the flow velocity ("rest frame")
- "P" the pressure which is related to "ϵ" by the equation of state
- $g^{\mu\nu}$=diag(1,-1,-1,-1)

Equation of state



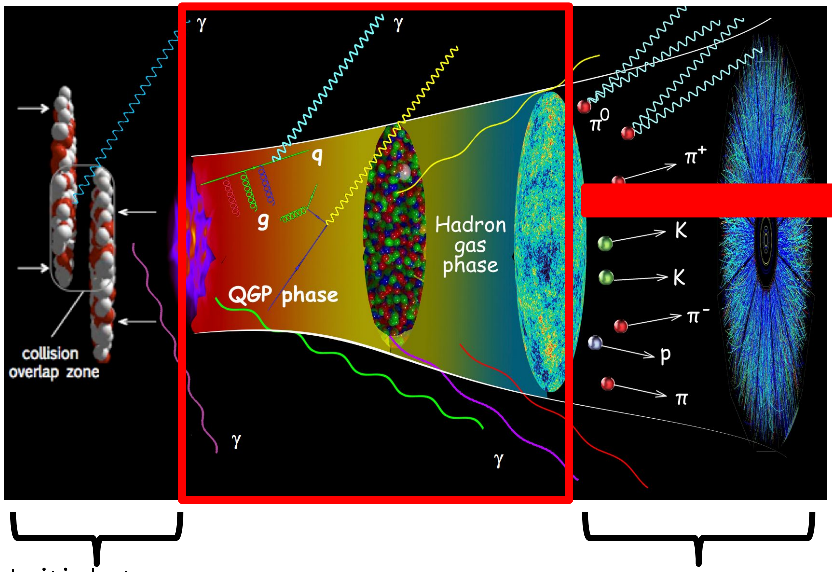
- Local conservation of energy-momentum tensor:

$$\partial_\mu T^{\mu\nu} = \partial_t T^{t\nu} + \partial_i T^{i\nu} = 0$$

Ideal hydrodynamics in heavy ion collisions

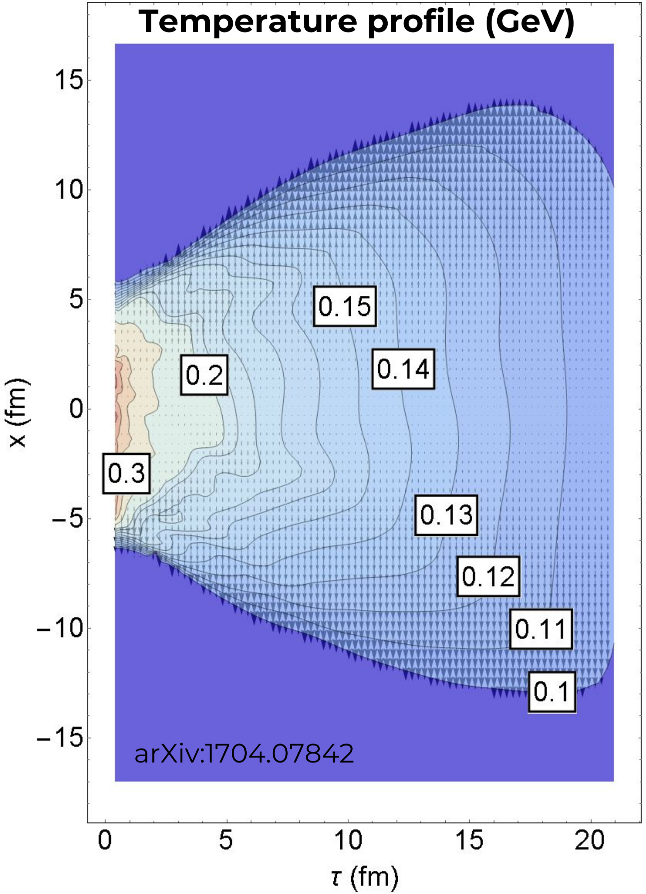
$$\partial_\mu T^{\mu\nu}(X) = 0\,;$$

$$T^{\mu\nu} = (\epsilon + P(\epsilon))u^\mu u^\nu - P(\epsilon)g^{\mu\nu}$$



Initial stage
(c.f. Prithwish's talk this morning)

Late stage hadronic interactions
(c.f. Dima's "SMASH" talk tomorrow)



Relativistic viscous hydrodynamics

- **Near**-local equilibrium
- **Energy-momentum tensor**:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

Bulk pressure (↑ Π) **Shear tensor** (↑ $\pi^{\mu\nu}$)

- **Conservation of energy-momentum** tensor: $\partial_\mu T^{\mu\nu}(X) = 0$

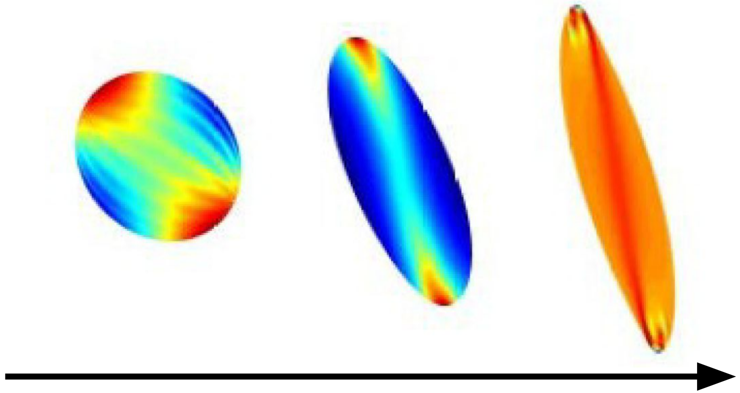
- Equations of motion for **viscous part of energy-momentum tensor**:

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial_\mu u^\mu) + \text{(second order terms)}$$

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} \dot{\pi}^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \text{(second order terms)}$$

Viscosity and transport coefficients

- Viscosities **convert kinetic energy into thermal energy** ("**dissipation**"; generates entropy)
- **Shear and bulk viscosities are characteristic properties of the fluid** that reflect this rate of dissipation



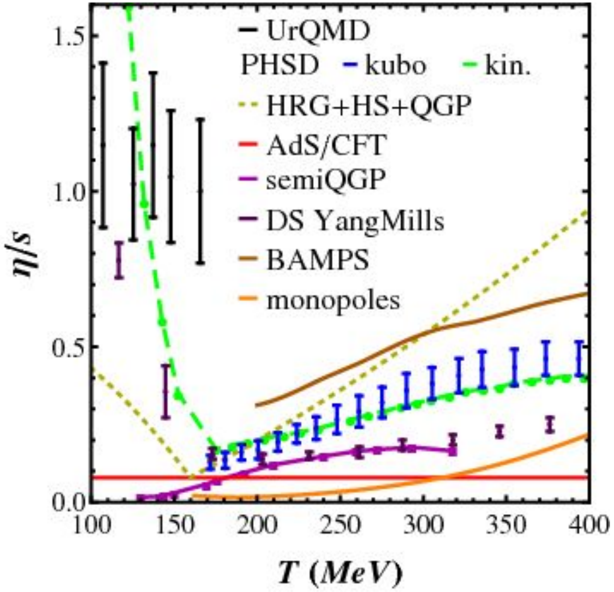
Shear viscosity:
dissipation from **deformations**

Bulk viscosity:
dissipation from **expansion**

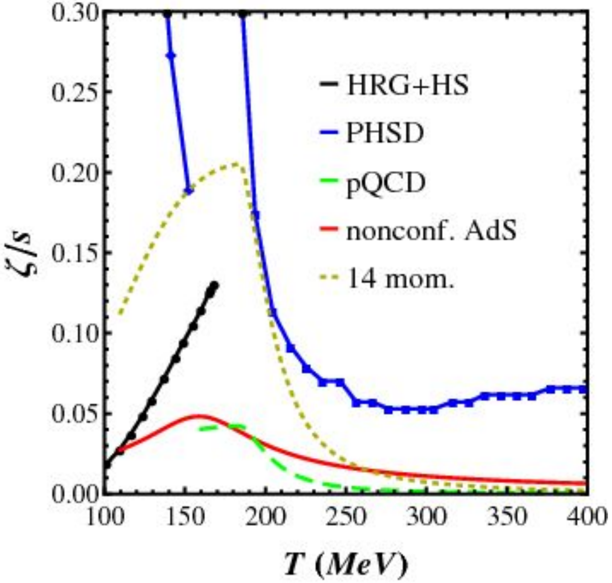
[Figures borrowed from Gabriel Denicol]

Shear and bulk viscosities of QCD

Shear viscosity to entropy density



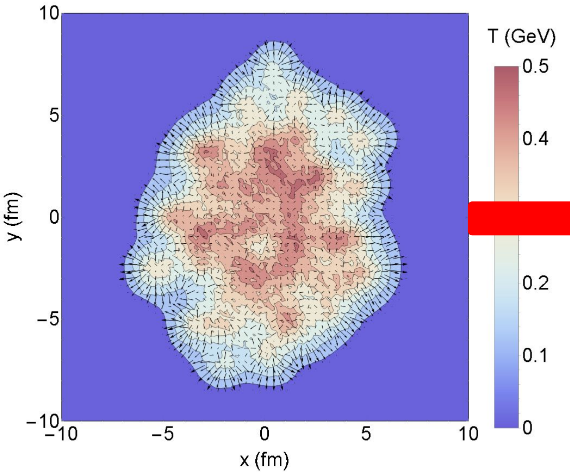
Bulk viscosity to entropy density

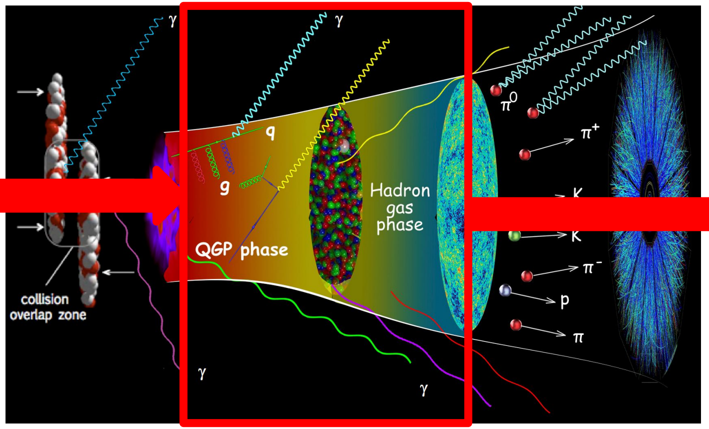


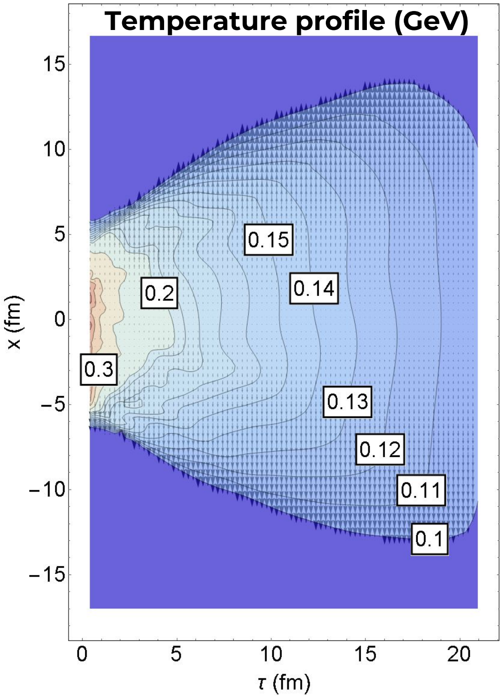
- ❑ Multiple **theoretical** attempt to compute shear and bulk viscosity
- ❑ **Heavy ion collisions** are used to **constrain** them **experimentally**

Figures ref.: Noronha-Hostler, arXiv:1512.06315

Initial conditions





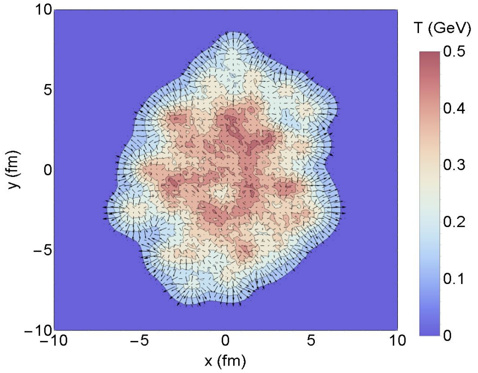


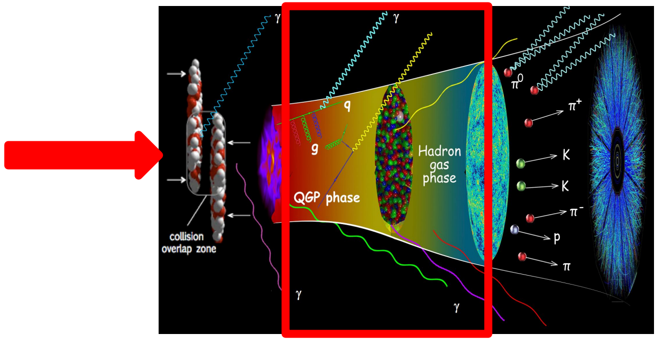
- Hydrodynamics describes the **spacetime evolution** of the plasma **given initial** ("boundary") **conditions**
- The full energy-momentum tensor must be initialized:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

Initial conditions

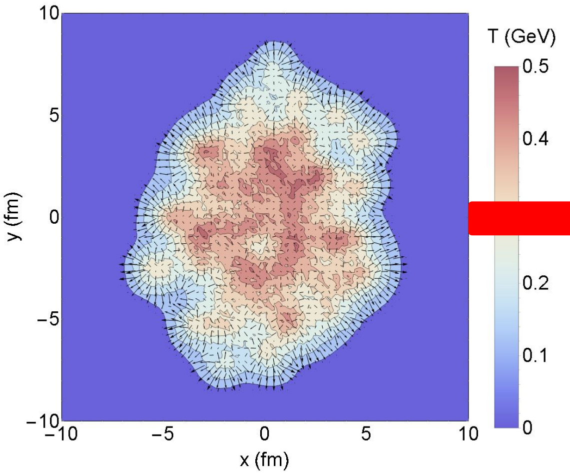
(c.f. Prithwish Tribedy's talk this morning)

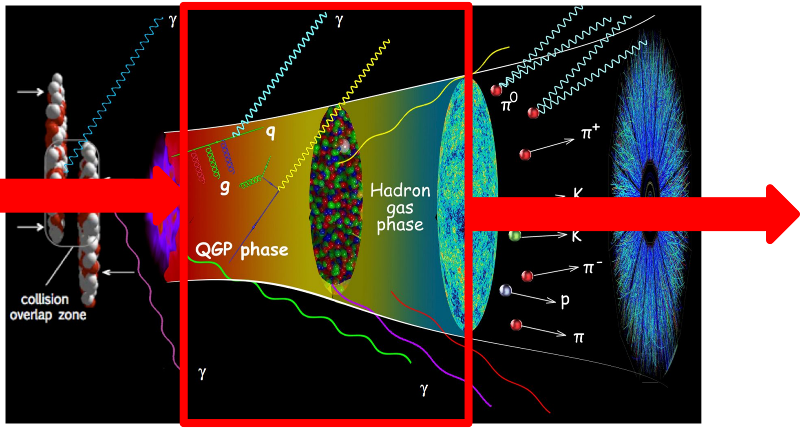


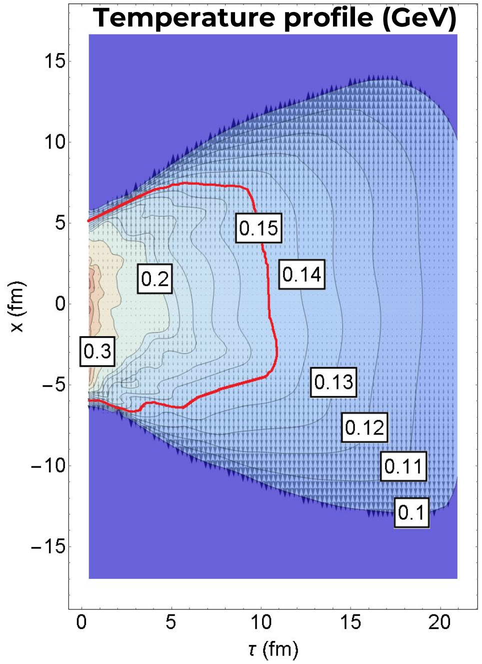


- General approach:
  - Describe **microscopic degrees of freedom** in the early stage of the plasma
  - **Coarse-grain** the degrees of freedom to get initial hydrodynamic $T^{\mu\nu}$
- **"Microscopic" degrees of freedom** can be:
  - Gluons/quarks (e.g. Color Glass Condensate, IP-Glasma)
  - Nucleons (e.g. MC Glauber, "Trento" model)
- Initial conditions can be obtained from **multistage model** themselves e.g. (IP-Glasma or Trento) + (KøMPøST or free-streaming)

From hydrodynamics to particles





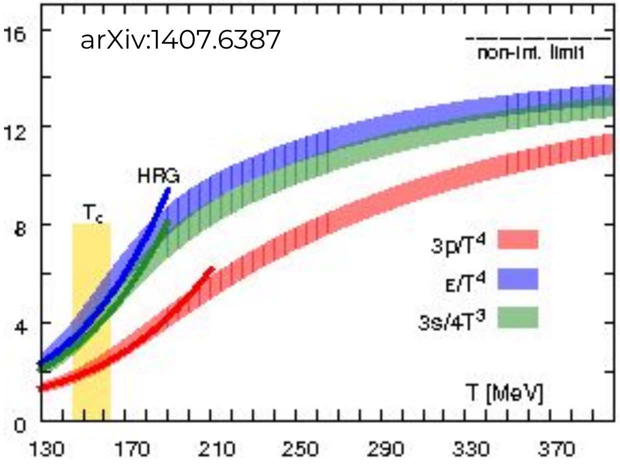


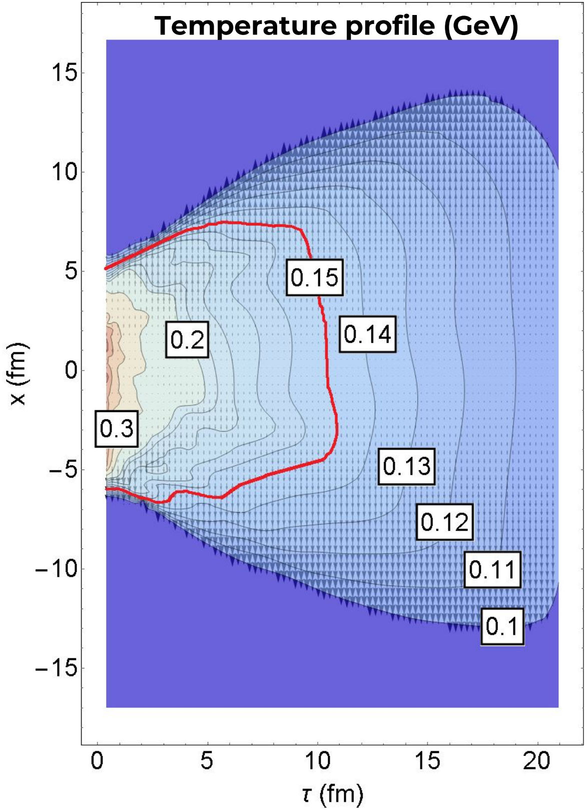
- ❑ The plasma is expanding in vacuum:
  its **energy density/temperature** is **decreasing rapidly**
- ❑ Low enough energy density: can't use a hydrodynamic description

Hydrodynamics **drives the plasma out of equilibrium**

Particlization: from hydrodynamics to particles

- ❑ At temperatures T~160 MeV, the degrees of freedom of the plasma are hadrons





- ❑ At T< ~160 MeV, dual description of the plasma: (i) hydrodynamics and (ii) gas of interacting hadrons [recall the ideal gas example]

From hydrodynamics to hadron resonance gas

- Energy-momentum tensor of the hadron gas:

Sum over all hadron species and resonances

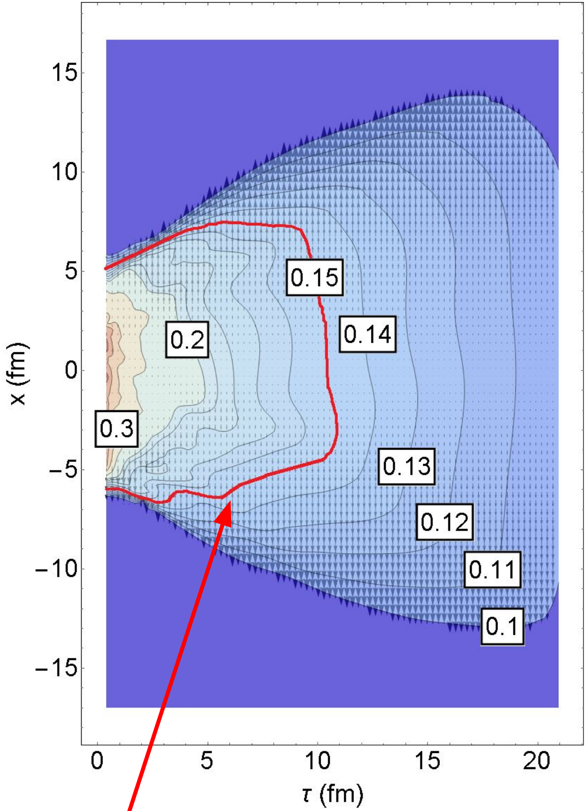
$$T^{\mu\nu}(X) = \sum_h \int \frac{d^3p}{P^0(2\pi)^3} P^\mu P^\nu g_h f_h(P,X)$$

Hadron degeneracy

- In ideal hydrodynamics, $f_h(P,X)$ is the Fermi-Dirac or Bose-Einstein distribution

- Converting the hydrodynamic fluid to particles: **"Cooper-Frye"**

$$E\frac{d^3N}{d\mathbf{p}} = \frac{g_h}{(2\pi)^3}\int_\Sigma d\Sigma_\mu P^\mu f_h(P\cdot u, X)$$

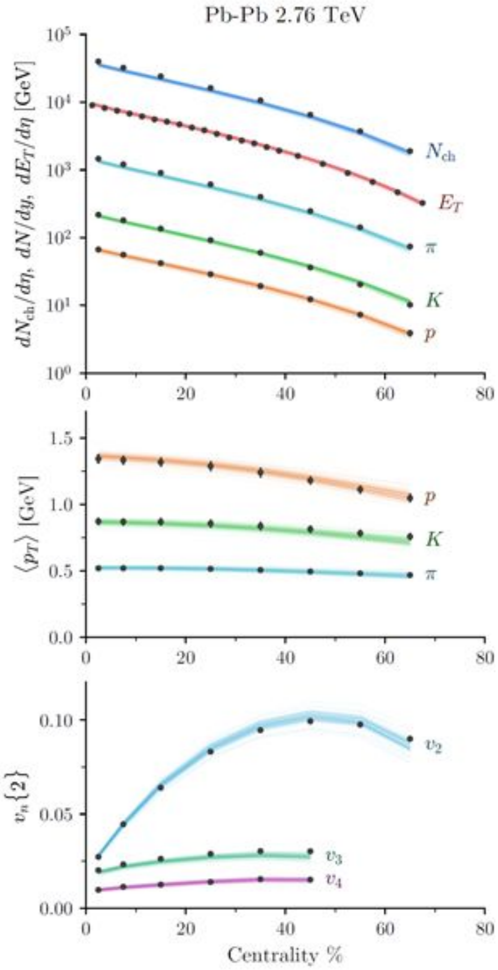


Particle flux flows out a surface Σ

Applications in heavy ion collisions

- By combining relativistic viscous hydrodynamics with
  (i) a model of **initial conditions**
  (ii) a late-stage **hadronic afterburner**

  we can **describe** a wide range of **hadronic measurements** in heavy ion collisions

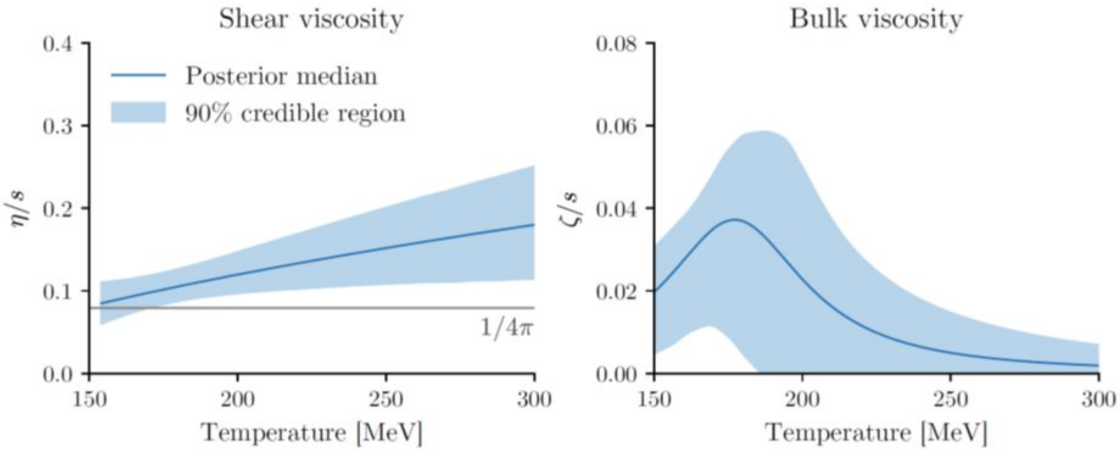


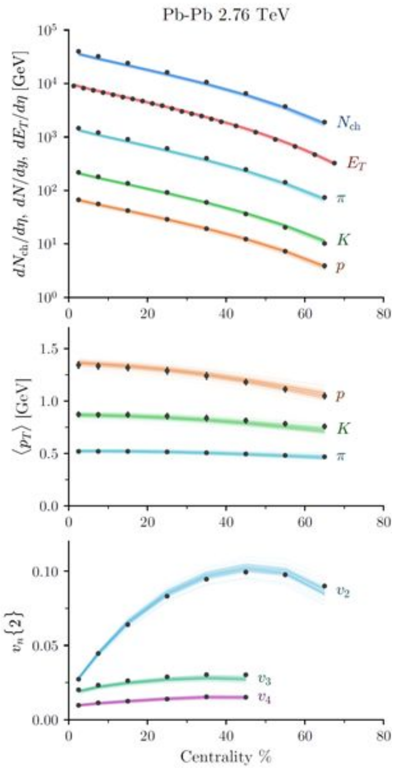
Reference:
Jonah Bernhard,
arXiv:1804.06469

Constraints on shear and bulk viscosities

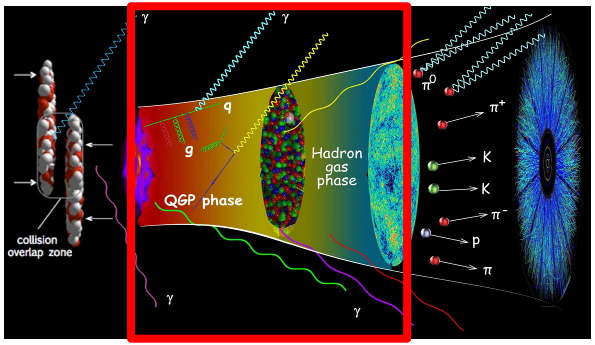
Reference:
Jonah Bernhard,
arXiv:1804.06469

- By combining relativistic viscous hydrodynamics, initial conditions and a late-stage hadronic afterburner, we can
  - **describe hadronic measurements**
  - put **constraints on shear and bulk viscosities**





Summary



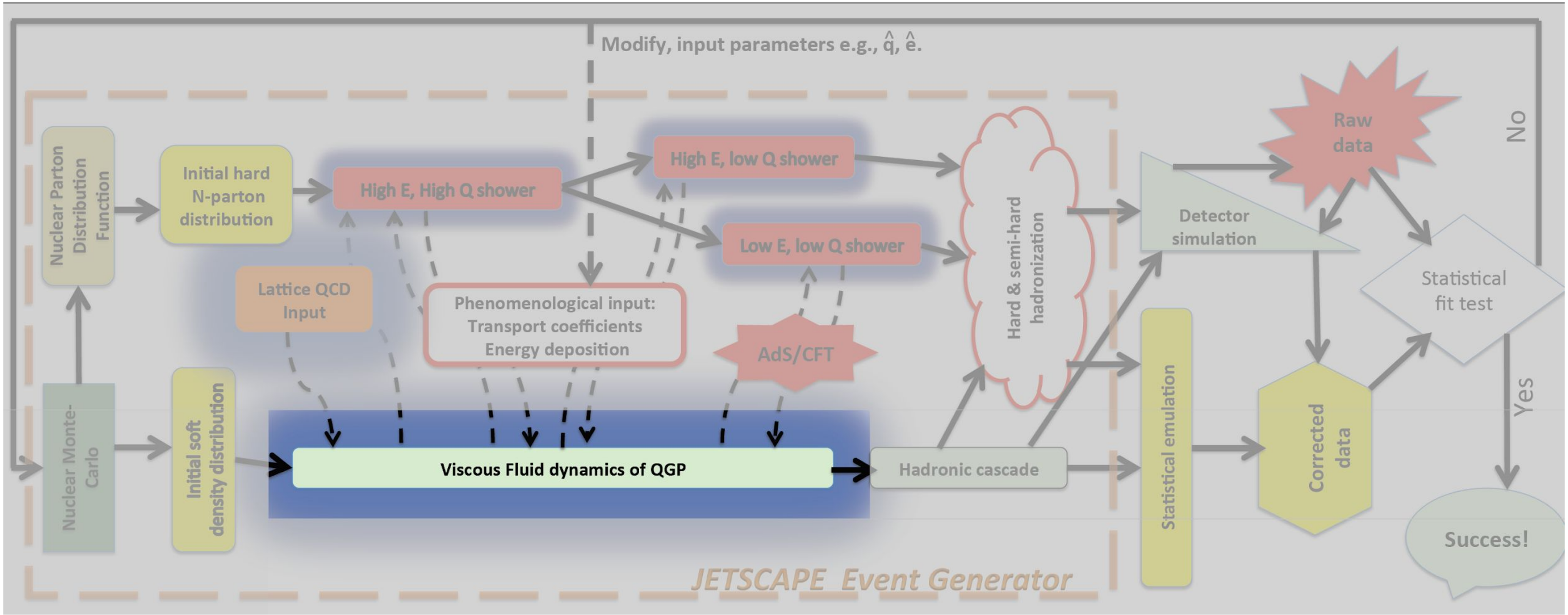
- ❑ Hydrodynamics is an **effective description** of matter created in heavy ion collisions

- ❑ Hydrodynamics is one key **part of a multistage model** of heavy ion collisions (initial stage and late stage described separately)

- ❑ Hybrid (multistage) model **successful in describing (soft) hadronic observables** --- allowing us to study transport properties of QGP

- ❑ Hydrodynamics provides **realistic medium evolution event-by-event for parton energy loss**

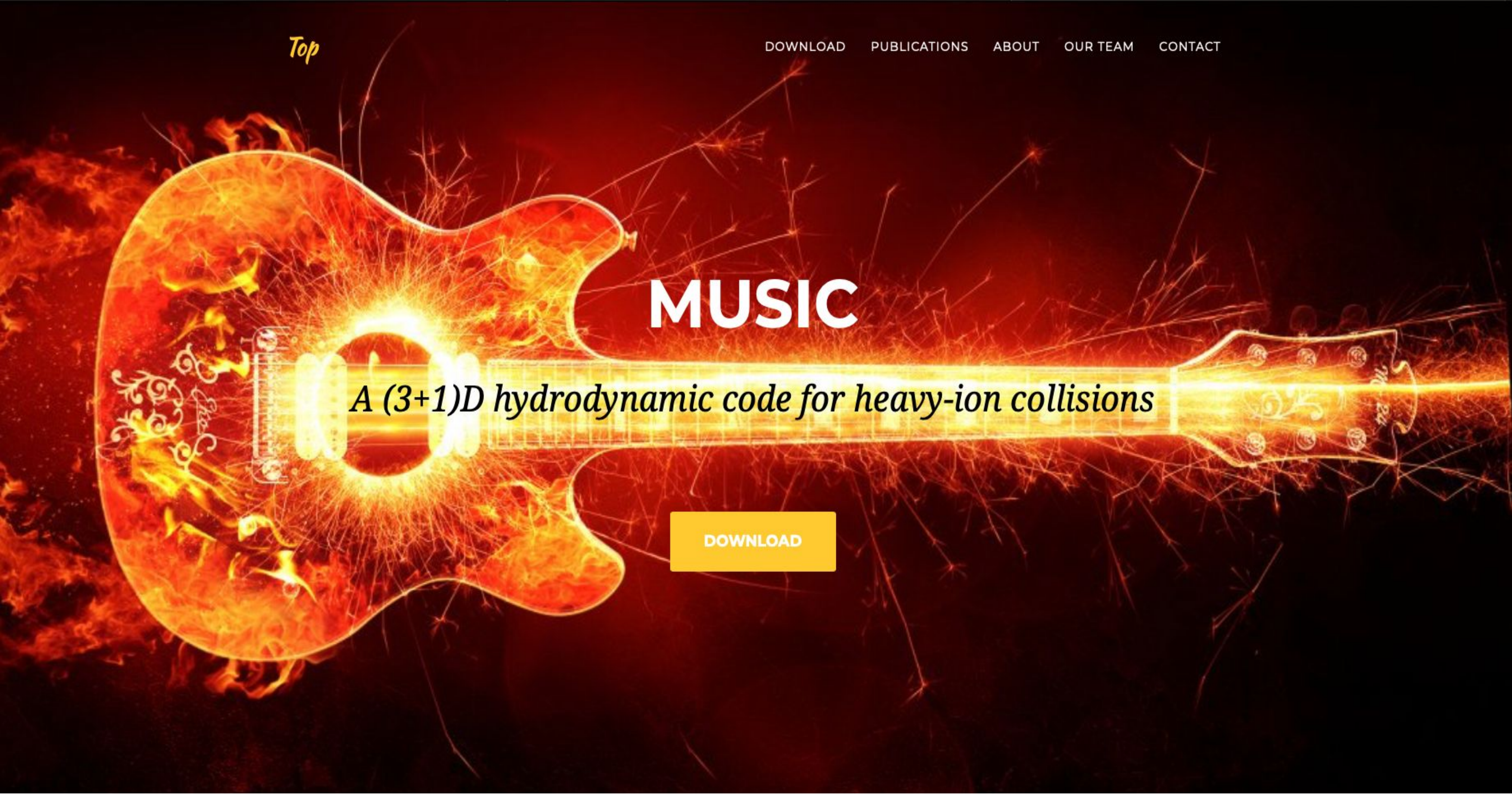
Hydrodynamics with JETSCAPE



Hands on Session

Goals

- Get familiar with running the JETSCAPE framework coupled with a hydrodynamic module (MUSIC)
- Understand the input options and output files from MUSIC_evo
  - Change collision system (type of nucleus, collision centrality, collision energy)
  - Change the transport coefficient in hydrodynamic simulations (shear viscosity)
  - Make plots and animations for a hydrodynamic evolution (temperature, flow velocity)
- (Bonus) Further integrate a particlization module to generate a full event with particles
  - Trento + MUSIC + iSS


Top
DOWNLOAD
PUBLICATIONS
ABOUT
OUR TEAM
CONTACT
MUSIC
A (3+1)D hydrodynamic code for heavy-ion collisions
DOWNLOAD

Instructions

- Download and compile JETSCAPE with MUSIC
  - Go to external_packages/ folder and run ./get_music.sh
  - Go to build/ folder and run cmake .. -Dmusic=ON
  - After the compilation, run MUSIC_evo to generate events
  - Change parameters in the jetscape_init.xml
- Make plots and animations
  - Download the scripts
    - git clone https://github.com/JETSCAPE/WinterSchool2019
  - Open Jupyter notebook
    - Update your running directory
    - Have fun~